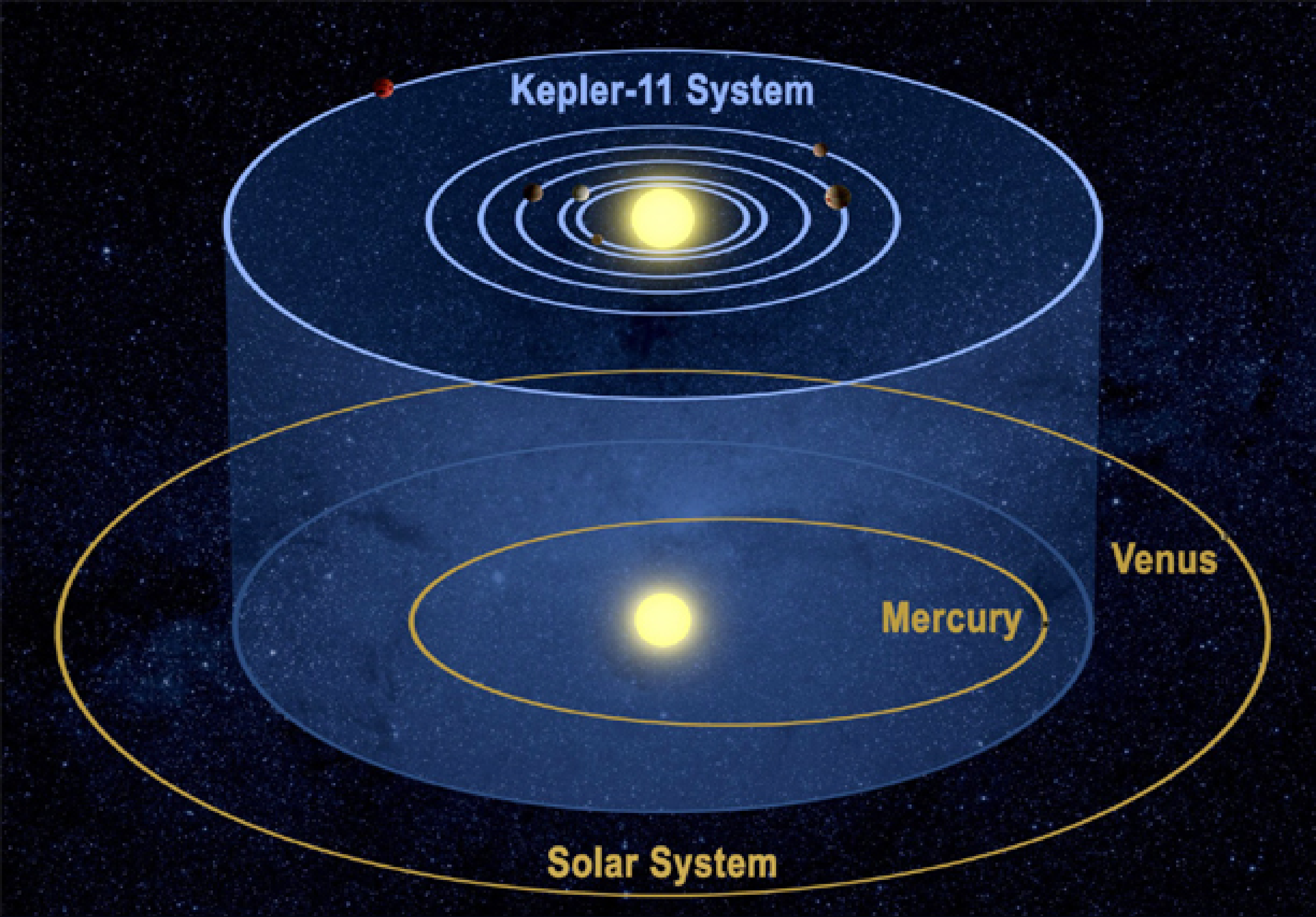

Classification and Types of Geocentric Orbits

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Geocentric Orbit

A **geocentric orbit** involves any object orbiting the Earth, such as the Moon or artificial satellites. Currently there are approximately 2,465 artificial satellites orbiting the Earth and 6,216 pieces of space debris as tracked by the Goddard Space Flight Center. Over 16,291 previously launched objects have decayed into the Earth's atmosphere.

List of terms and concepts

Analemma
: a term in astronomy used to describe the plot of the positions of the Sun on the celestial sphere throughout one year. Closely resembles a figure-eight.

Altitude
: as used here, the height of an object above the average surface of the Earth's oceans.

Apogee
: is the farthest point that a satellite or celestial body can go from Earth, at which the orbital velocity will be at its minimum.

Eccentricity
: a measure of how much an orbit deviates from a perfect circle. Eccentricity is strictly defined for all circular and elliptical orbits, and parabolic and hyperbolic trajectories.

Equatorial plane
: as used here, an imaginary plane extending from the equator on the Earth to the celestial sphere.

Orbital characteristics
: the six parameters of the Keplerian elements needed to specify that orbit uniquely.

Escape velocity
: as used here, the minimum velocity an object without propulsion needs to have to move away indefinitely from the Earth. An object at this velocity will enter a parabolic trajectory; above this velocity it will enter a hyperbolic trajectory.

Impulse
: the integral of a force over the time during which it acts. Measured in (N·sec or lb * sec).

Inclination

the angle between a reference plane and another plane or axis. In the sense discussed here the reference plane is the Earth's equatorial plane.

Orbital period
: as defined here, time it takes a satellite to make one full orbit about the Earth.

Perigee
: is the nearest approach point of a satellite or celestial body from Earth, at which the orbital velocity will be at its maximum.

Sidereal day
: the time it takes for a celestial object to rotate 360°. For the Earth this is: 23 hours, 56 minutes, 4.091 seconds.

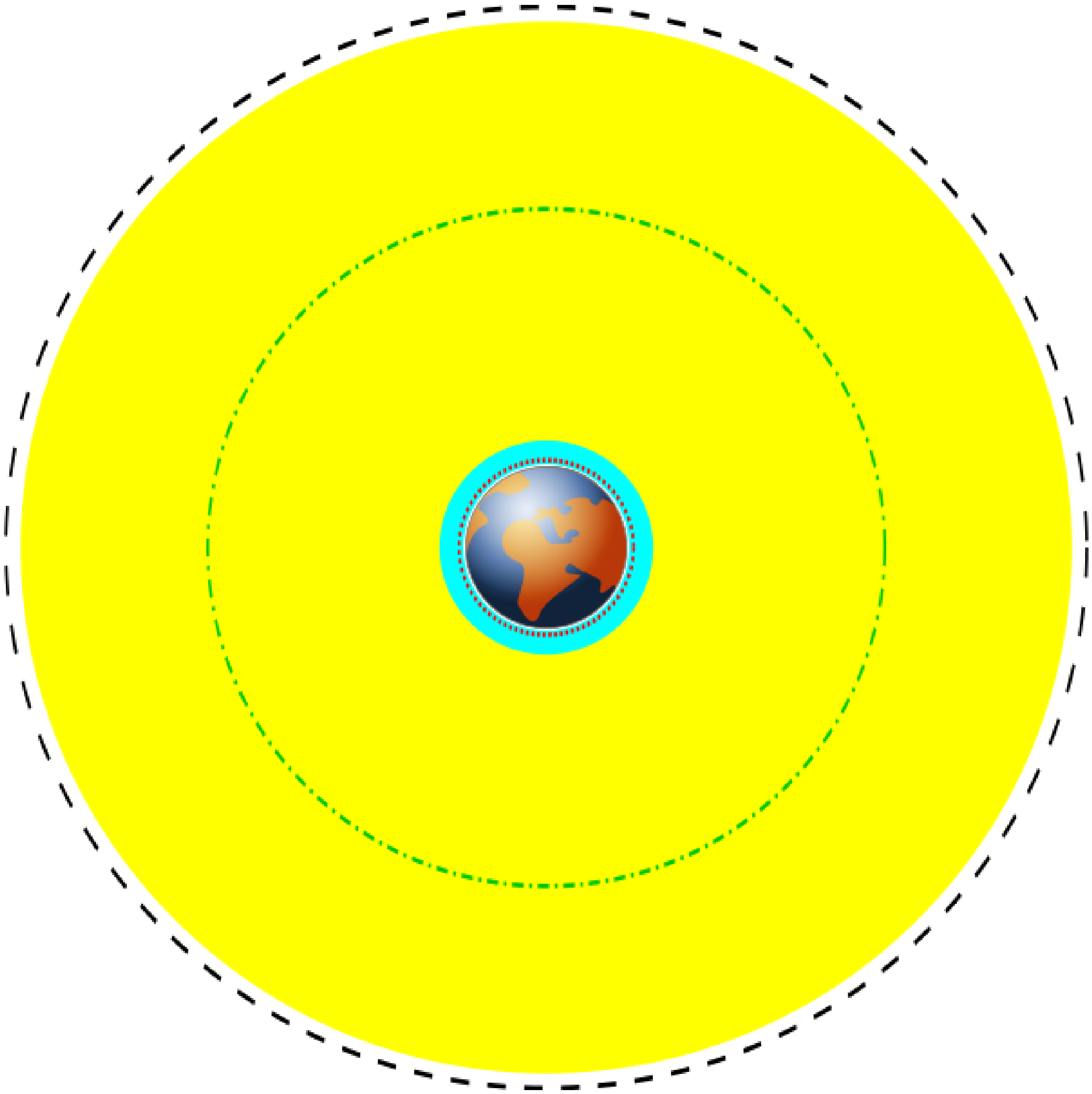
Solar time
: as used here, the local time as measured by a sundial.

Velocity
: an object's speed in a particular direction. Since velocity is defined as a vector, both speed and direction are required to define it.

Geocentric orbit types

The following is a list of different geocentric orbit classifications.

Altitude classifications



Low (cyan) and Medium (yellow) Earth orbit regions to scale. The black dashed line is the geosynchronous orbit. The green dashed line is the 20,230 km orbit used for GPS satellites.

Low Earth Orbit (LEO) - Geocentric orbits ranging in altitude from 160 kilometeres (100 statue miles) to 2,000 kilometres (1,200 mi) above mean sea level. At 160 km, one revolution takes approximately 90 minutes, and the circular orbital speed is 8,000 metres per second (26,000 ft/s).

Medium Earth Orbit (MEO) - Geocentric orbits with altitudes at apogee ranging between 2,000 kilometres (1,200 mi) and that of the geosynchronous orbit at 35,786 kilometres (22,236 mi).

Geosynchronous Orbit (GEO) - Geocentric circular orbit with an altitude of 35,786 kilometres (22,236 mi). The period of the orbit equals one sidereal day, coinciding with the rotation period of the Earth. The speed is approximately 3,000 metres per second (9,800 ft/s).

High Earth Orbit (HEO) - Geocentric orbits with altitudes at apogee higher than that of the geosynchronous orbit. A special case of high Earth orbit is the highly elliptical orbit, where altitude at perigee is less than 2,000 kilometres (1,200 mi).

Inclination classifications

Inclined Orbit - An orbit whose inclination in reference to the equatorial plane is not 0.
Polar Orbit - A satellite that passes above or nearly above both poles of the planet on each revolution. Therefore it has an inclination of (or very close to) 90 degrees.
Polar Sun-synchronous Orbit - A nearly polar orbit that passes the equator at the same local time on every pass. Useful for image taking satellites because shadows will be the same on every pass.

Eccentricity classifications

Circular Orbit - An orbit that has an eccentricity of 0 and whose path traces a circle.
Elliptic Orbit - An orbit with an eccentricity greater than 0 and less than 1 whose orbit traces the path of an ellipse.
Hohmann transfer orbit - An orbital maneuver that moves a spacecraft from one circular orbit to another using two engine impulses. This maneuver was named after Walter Hohmann.
Geosynchronous Transfer Orbit - A geocentric-elliptic orbit where the perigee is at the altitude of a Low Earth Orbit (LEO) and the apogee at the altitude of a geosynchronous orbit.
Geostationary Transfer Orbit - A geocentric-elliptic orbit where the perigee is at the altitude of a Low Earth Orbit (LEO) and the apogee at the altitude of a geostationary orbit.
Highly Elliptical Orbit (HEO) - Geocentric orbit with apogee above 35,786 km and low perigee (about 1,000 km) that result in long dwell times near apogee.
Molniya Orbit - A highly elliptical orbit with inclination of 63.4° and orbital period of ½ of a sidereal day (roughly 12 hours). Such a satellite spends most of its time over a designated area of the Earth.
Tundra Orbit - A highly elliptical orbit with inclination of 63.4° and orbital period of one sidereal day (roughly 24 hours). Such a satellite spends most of its time over a designated area of the Earth.
Hyperbolic trajectory - An "orbit" with eccentricity greater than 1. The object's velocity at perigee reaches some value in excess of the escape velocity, therefore it will escape the gravitational pull of the Earth and continue to travel infinitely with a velocity (relative to Earth) decelerating to some finite value, known as the hyperbolic excess velocity.
Escape Trajectory - This trajectory must be used to launch an interplanetary probe away from Earth, because the excess over escape velocity is what changes its heliocentric orbit from that of Earth.

Capture Trajectory - This is the mirror image of the escape trajectory; an object traveling with sufficient speed, not aimed directly at Earth, will move toward it and accelerate. In the absence of a decelerating engine impulse to put it into orbit, it will follow the escape trajectory after periapsis.

Parabolic trajectory - An "orbit" with eccentricity exactly equal to 1. The object's velocity at perigee equals the escape velocity, therefore it will escape the gravitational pull of the Earth and continue to travel with a velocity (relative to Earth) approaching 0. A spacecraft launched from Earth with this velocity would travel some distance away from it, but follow it around the Sun in the same heliocentric orbit. It is possible, but not likely that an object approaching Earth could follow a parabolic capture trajectory, but speed and direction would have to be precise.

Directional classifications

Prograde orbit - an orbit in which the projection of the object onto the equatorial plane revolves about the Earth in the same direction as the rotation of the Earth.

Retrograde orbit - an orbit in which the projection of the object onto the equatorial plane revolves about the Earth in the direction opposite that of the rotation of the Earth.

Geosynchronous classifications

Semi-Synchronous Orbit (SSO) - An orbit with an altitude of approximately 20,200 km (12544.2 miles) and an orbital period of approximately 12 hours

Geosynchronous Orbit (GEO) - Orbits with an altitude of approximately 35,786 km (22,240 miles). Such a satellite would trace an analemma (figure 8) in the sky.

Geostationary orbit (GSO): A geosynchronous orbit with an inclination of zero. To an observer on the ground this satellite would appear as a fixed point in the sky.

Clarke Orbit - Another name for a geostationary orbit. Named after the writer Arthur C. Clarke.

Earth orbital libration points: The libration points for objects orbiting Earth are at 105 degrees west and 75 degrees east. More than 160 satellites are gathered at these two points.

Supersynchronous orbit - A disposal / storage orbit above GSO/GEO. Satellites will drift west.

Subsynchronous orbit - A drift orbit close to but below GSO/GEO. Satellites will drift east.

Graveyard Orbit - An orbit a few hundred kilometers above geosynchronous that satellites are moved into at the end of their operation.

Disposal Orbit - A synonym for graveyard orbit.

Junk Orbit - A synonym for graveyard orbit.

Special classifications

Sun-synchronous Orbit - An orbit which combines altitude and inclination in such a way that the satellite passes over any given point of the planet's surface at the same local solar time. Such an orbit can place a satellite in constant sunlight and is useful for imaging, spy, and weather satellites.
Moon Orbit - The orbital characteristics of Earth's Moon. Average altitude of 384,403 kilometres (238,857 mi), elliptical-inclined orbit.

Non-geocentric classifications

Horseshoe Orbit - An orbit that appears to a ground observer to be orbiting a planet but is actually in co-orbit with it.
Exo-orbit - A maneuver where a spacecraft approaches the height of orbit but lacks the velocity to sustain it.
Sub-Orbital Spaceflight - A synonym for Exo-orbit.

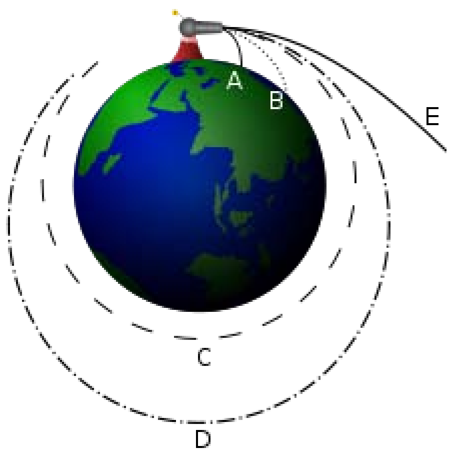
Earth orbits

orbit	center-to-center distance	altitude above the Earth's surface	speed	period/time in space	specific orbital energy
minimum sub-orbital spaceflight (vertical)	6,500 km	100 km	0.0 km/s	just reaching space	1.0 MJ/kg
ICBM	up to 7,600 km	up to 1,200 km	6 to 7 km/s	time in space: 25 min	27 MJ/kg
Low Earth orbit	6,600 to 8,400 km	200 to 2,000 km	circular orbit: 6.9 to 7.8 km/s elliptic orbit: 6.5 to 8.2 km/s	89 to 128 min	32.1 to 38.6 MJ/kg
Molniya orbit	6,900 to 46,300 km	500 to 39,900 km	1.5 to 10.0 km/s	11 h 58 min	54.8 MJ/kg
GEO	42,000 km	35,786 km	3.1 km/s	23 h 56 min	57.5 MJ/kg
Orbit of the Moon	363,000 to 406,000 km	357,000 to 399,000 km	0.97 to 1.08 km/s	27.3 days	61.8 MJ/kg

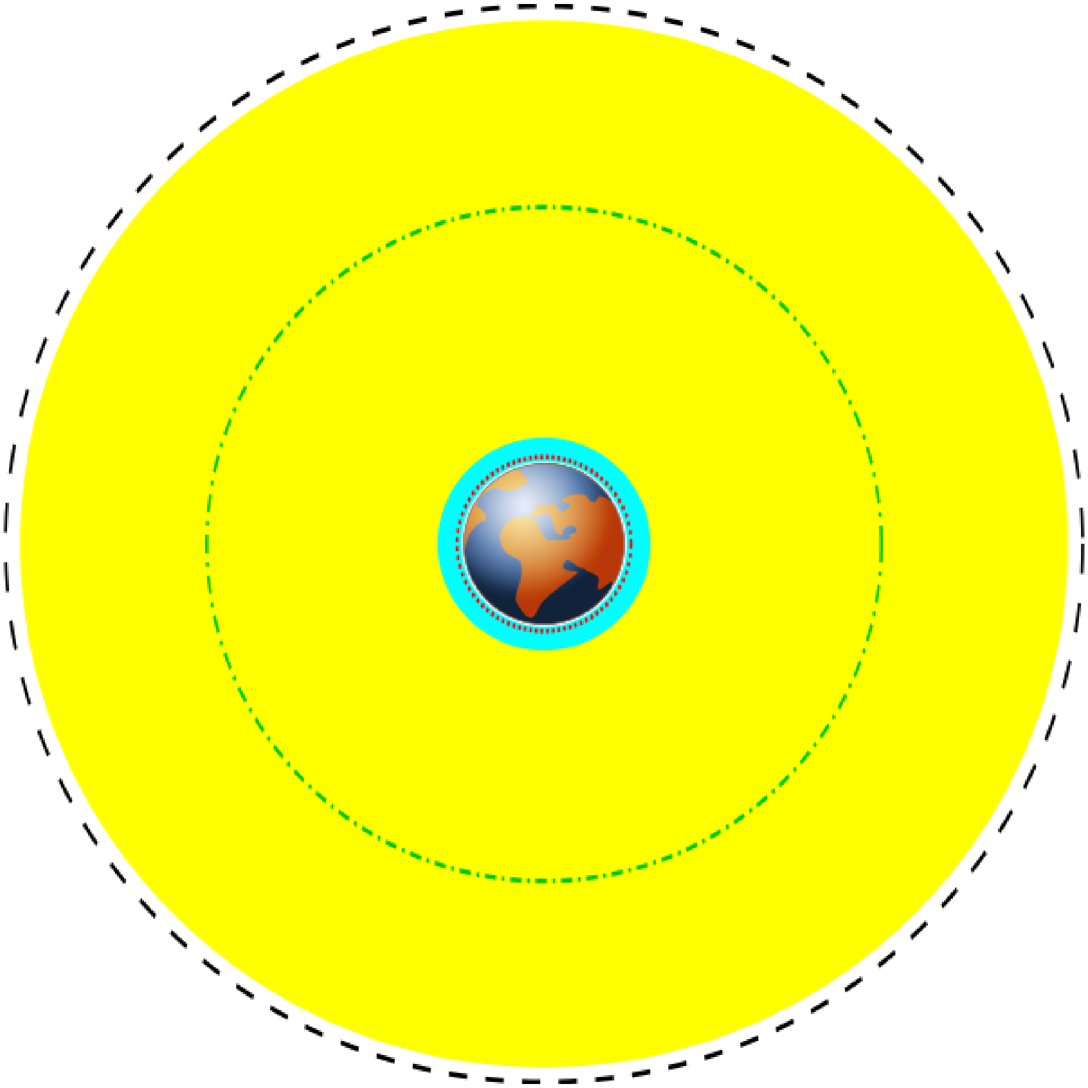
Chapter- 2

Altitude Classification

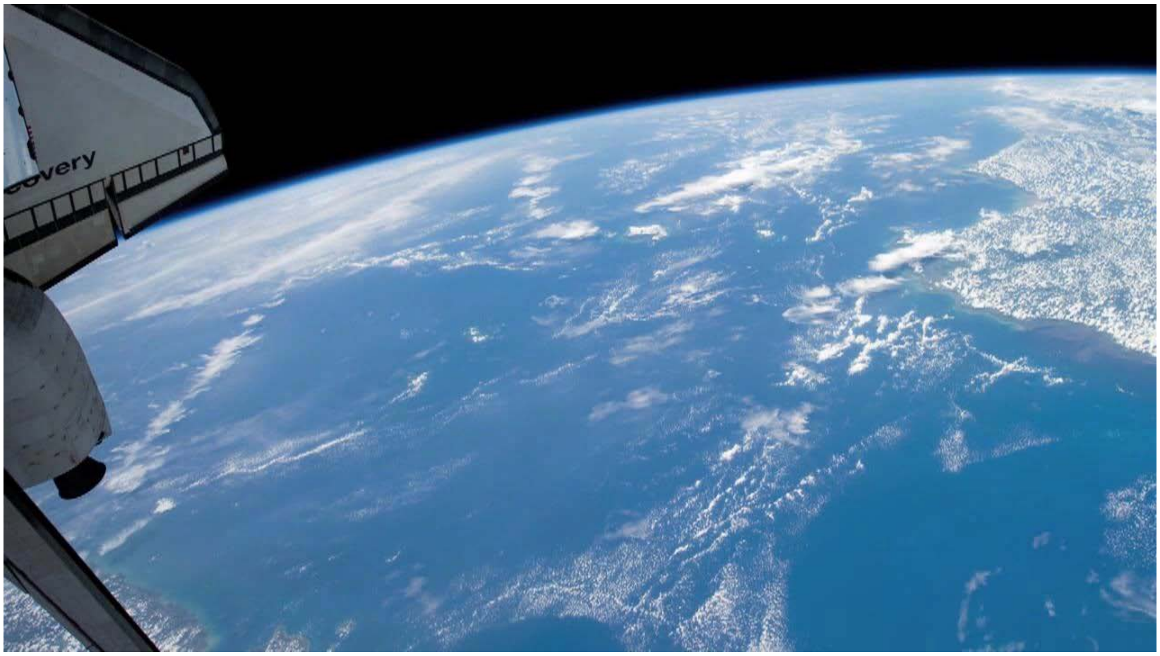
Low Earth orbit



An orbiting cannon ball showing various sub-orbital and orbital possibilities.



Various earth orbits to scale; light blue represents low earth orbit.



Roughly half an orbit of the ISS.

A **low Earth orbit** (**LEO**) is generally defined as an orbit within the locus extending from the Earth’s surface up to an altitude of 2,000 km. Given the rapid orbital decay of objects below approximately 200 km, the commonly accepted definition for LEO is between 160 - 2,000 km (100 - 1,240 miles) above the Earth's surface.

With the exception of the lunar flights of the Apollo program, all human spaceflights have either been orbital in LEO or sub-orbital. The altitude record for a human spaceflight in LEO was Gemini 11 with an apogee of 1,374.1 km.

Orbital characteristics

Objects in LEO encounter atmospheric drag in the form of gases in the thermosphere (approximately 80–500 km up) or exosphere (approximately 500 km and up), depending on orbit height. LEO is an orbit around Earth between the atmosphere and below the inner Van Allen radiation belt. The altitude is usually not less than 300 km because that would be impractical due to the larger atmospheric drag.

Equatorial low Earth orbits (ELEO) are a subset of LEO. These orbits, with low inclination to the Equator, allow rapid revisit times and have the lowest delta-v requirement of any orbit. Orbits with a high inclination angle are usually called polar orbits.

Higher orbits include medium Earth orbit (MEO), sometimes called intermediate circular orbit (ICO), and further above, geostationary orbit (GEO). Orbits higher than low orbit can lead to earlier failure of electronic components due to intense radiation and charge accumulation.

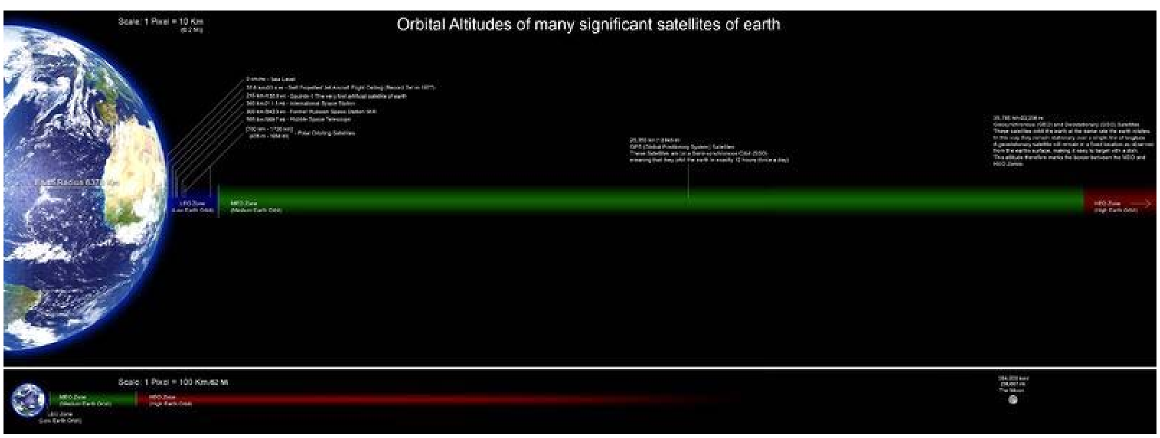
Human use

The International Space Station is in a LEO that varies from 319.6 km (199 mi) to 346.9 km (216 mi) above the Earth's surface.

While a majority of artificial satellites are placed in LEO, where they travel at about 27,400 km/h (8 km/s), making one complete revolution around the Earth in about 90 minutes, many communication satellites require geostationary orbits, and move at the same angular velocity as the Earth. Since it requires less energy to place a satellite into a LEO and the LEO satellite needs less powerful amplifiers for successful transmission, LEO is still used for many communication applications. Because these LEO orbits are not geostationary, a network (or "constellation") of satellites is required to provide continuous coverage. Lower orbits also aid remote sensing satellites because of the added detail that can be gained. Remote sensing satellites can also take advantage of sun-synchronous LEO orbits at an altitude of about 800 km (500 mi) and near polar inclination. ENVISAT is one example of an Earth observation satellite that makes use of this particular type of LEO.

Although the Earth's pull due to gravity in LEO is not much less than on the surface of the Earth, people and objects in orbit experience weightlessness due to the effects of freefall.

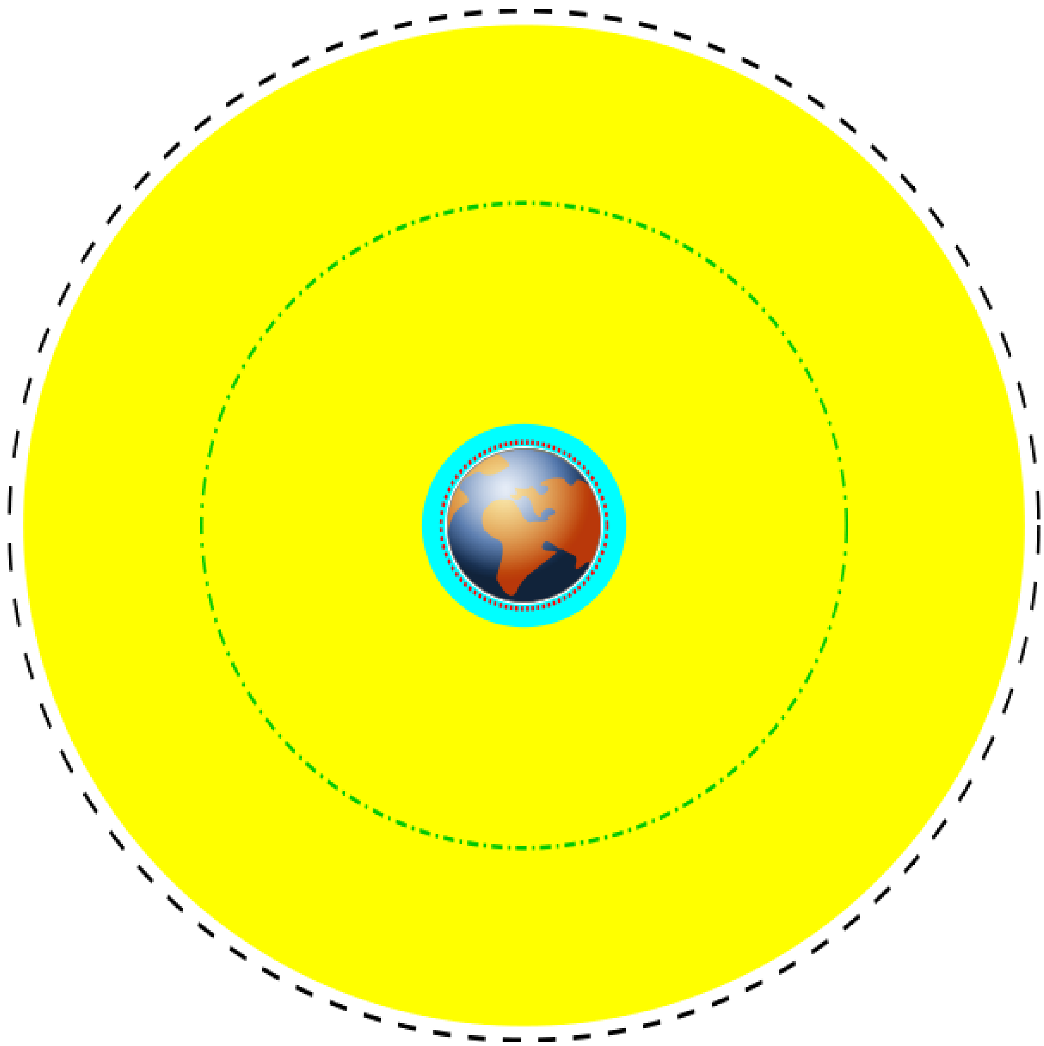
Atmospheric and gravity drag associated with launch typically adds 1,500-2,000 m/s to the delta-v launch vehicle required to reach normal LEO orbital velocity of around 7,800 m/s (17,448 mph).



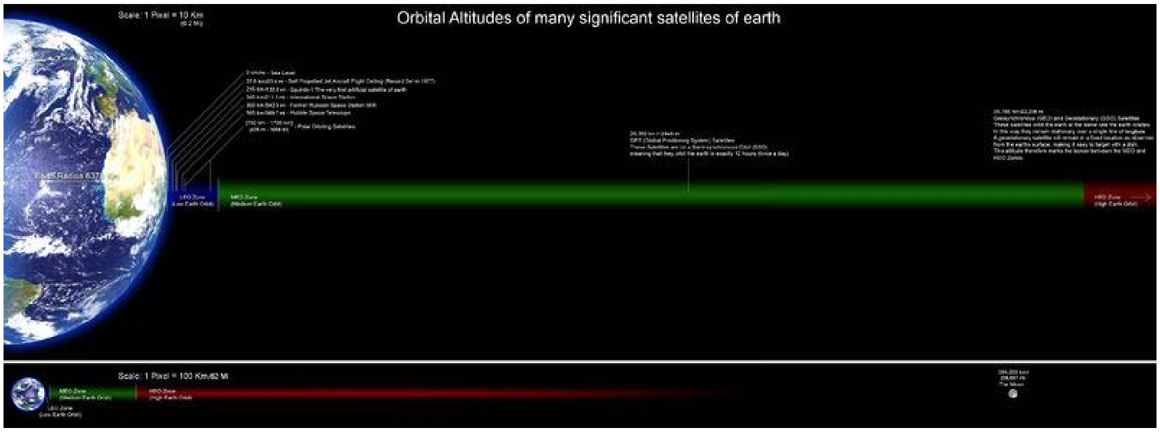
Space debris

The LEO environment is becoming congested with space debris which has caused a growing concern in recent years, since collisions at orbital velocities can be highly damaging or dangerous and can produce even more space debris in the process, called the Kessler Syndrome. The Joint Space Operations Center, part of United States Strategic Command (formerly the United States Space Command), currently tracks more than 8,500 objects larger than 10 cm in LEO, however a limited Arecibo Observatory study suggested there could be about one million objects larger than 2 millimeters, which are too small to be seen from the ground.

Medium Earth orbit



Various earth orbits to scale; Yellow represents medium earth orbit



To-scale diagram of low, medium and high earth orbits

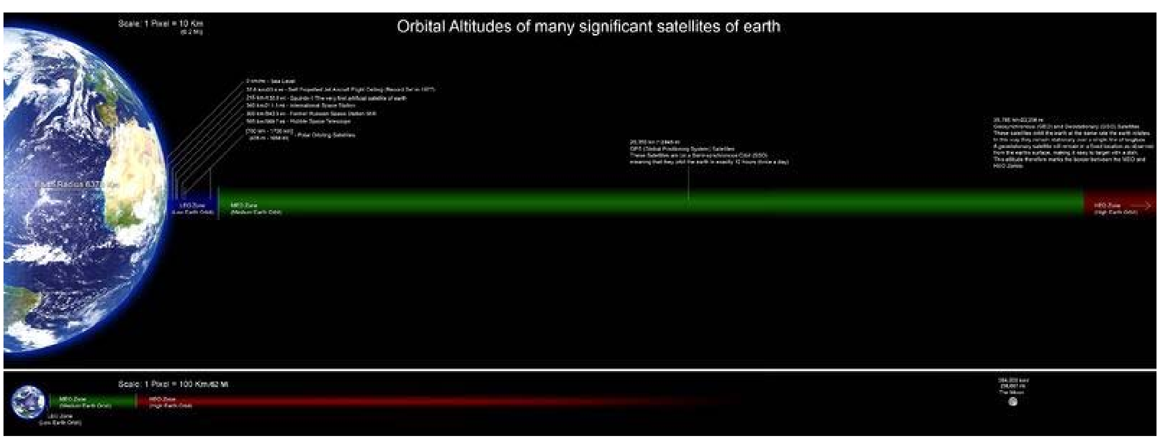
Medium Earth orbit (MEO), sometimes called **intermediate circular orbit (ICO)**, is the region of space around the Earth above low Earth orbit (altitude of 2,000 kilometres (1,243 mi)) and below geostationary orbit (altitude of 35,786 kilometres (22,236 mi)).

The most common use for satellites in this region is for navigation, such as the GPS (with an altitude of 20,200 kilometres (12,552 mi)), Glonass (with an altitude of 19,100 kilometres (11,868 mi)) and Galileo (with an altitude of 23,222 kilometres (14,429 mi)) constellations. Communications satellites that cover the North and South Pole are also put in MEO.

The orbital periods of MEO satellites range from about 2 to 24 hours. Telstar, one of the first and most famous experimental satellites, orbits in MEO.

The orbit has a moderate number of satellites.

High Earth orbit



To-scale diagram of low, medium and high earth orbits

A **High Earth Orbit** is a geocentric orbit whose apogee lies above that of a geosynchronous orbit (35,786 kilometres (22,236 mi)).

Highly Elliptical Orbits are a subset of High Earth Orbits.

Examples of satellites in High Earth Orbit

Name	NSSDC ID	Launch Date	Perigee	Apogee	Period	Inclination	Eccentricity	Highly Elliptical Orbit
Molniya 1-01	1965-030A	1965-04-23	538 km	39,300 km	708.0 min	65.0°	0.736864	yes
Vela 1A	1963-039A	1963-10-17	101,925 km	116,528 km	6,519.6 min	37.8°	0.063	no

Chapter- 3

Special Classification

Sun-synchronous orbit

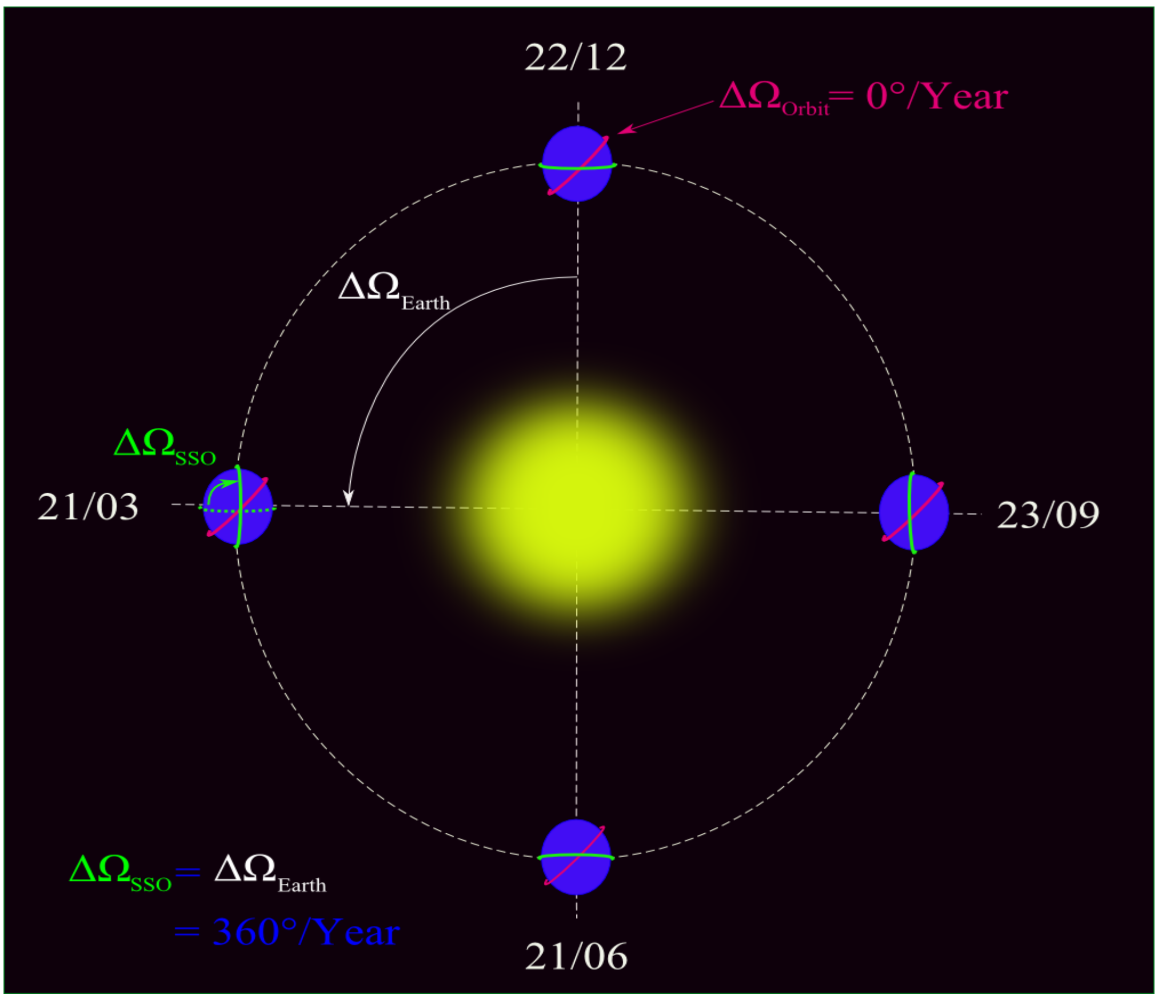


Diagram showing the orientation of a Sun-Synchronous orbit (Green) in four points of the year. A non-sun-synchronous orbit (Magenta) is also shown for reference

A **sun-synchronous orbit** (sometimes incorrectly called a heliosynchronous orbit) is a geocentric orbit which combines altitude and inclination in such a way that an object on that orbit ascends or descends over any given point of the Earth's surface at the same local mean solar time. The surface illumination angle will be nearly the same every time. This consistent lighting is a useful characteristic for satellites that image the Earth's surface in visible or infrared wavelengths (e.g. weather and spy satellites) and for other remote sensing satellites (e.g. those carrying ocean and atmospheric remote sensing instruments that require sunlight). For example, a satellite in sun-synchronous orbit might ascend across the equator twelve times a day each time at approximately 15:00 mean local time. This is achieved by having the osculating orbital plane precess (rotate) approximately one degree each day with respect to the celestial sphere, eastward, to keep pace with the Earth's revolution around the Sun.

The uniformity of sun angle is achieved by tuning the nodal regression–the natural precession of the orbit–to be one full circle per year. Because the Earth rotates, it is slightly oblate (the equator is slightly longer than it would be if Earth were shaped into a perfect sphere), and the extra mass near the equator causes spacecraft that are in inclined orbits to precess: the plane of the orbit is not fixed in space relative to the distant stars, but rotates slowly about the Earth's axis. The speed of the precession depends both on inclination of the orbit and on the altitude of the satellite. By balancing these two effects, it is possible to match a range of precession rates. Typical sun-synchronous orbits are about 600–800 km in altitude, with periods in the 96–100 minute range, and inclinations of around 98° (i.e. slightly retrograde compared to the direction of Earth's rotation: 0° represents an equatorial orbit and 90° represents a polar orbit).

Special cases of the sun-synchronous orbit are the **noon/midnight orbit**, where the local mean solar time of passage for equatorial longitudes is around noon or midnight, and the **dawn/dusk orbit**, where the local mean solar time of passage for equatorial longitudes is around sunrise or sunset, so that the satellite rides the terminator between day and night. Riding the terminator is useful for active radar satellites as the satellites' solar panels can always see the Sun, without being shadowed by the Earth. It is also useful for some satellites with passive instruments which need to limit the Sun's influence on the measurements, as it is possible to always point the instruments towards the night side of the Earth. The dawn/dusk orbit has been used for solar observing scientific satellites such as Yohkoh, TRACE and Hinode, affording them a nearly continuous view of the Sun.

Sun-synchronous orbits are possible around other planets, such as Mars.

Technical details

A sun-synchronous orbit is a retrograde orbit. The node precesses in the same direction as the body's spin (the longitude of the ascending node increases). A good approximation of the precession rate is:

$$\omega_p = \frac{3a^2}{2r^2} J_2 \omega \cos i$$

where

ω_p is the precession rate (rad/s)
a is the Earth's equatorial radius (6 378 137 m)
r is the satellite's orbital radius
ω is its angular frequency (2π radians divided by its period)
i its inclination
J_2 is the Earth's *second dynamic form factor* $(-\sqrt{5}C_{20}=1.08262668\times10^{-3})$.

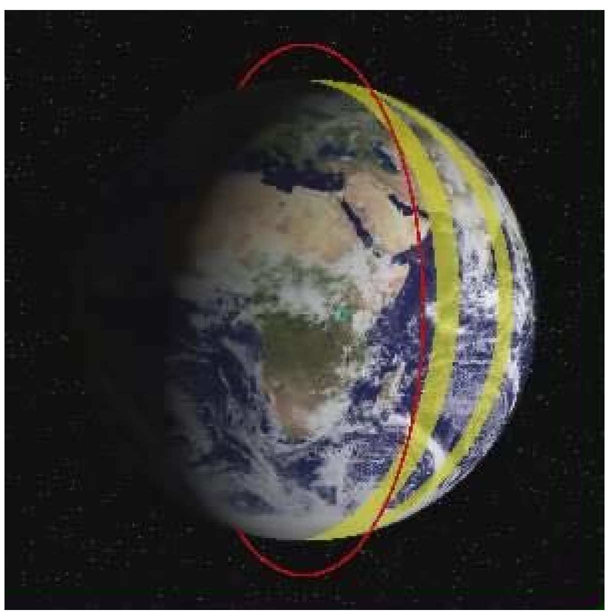
This last quantity is related to the oblateness as follows:

$$J_2 = \frac{2\varepsilon_E}{3} - \frac{a^3\omega_E^2}{3GM_E}$$

where

ε_E is the Earth's oblateness
ω_E is the Earth's rotation rate (7.292115×10^{-5} rad/s)
GM_E is the product of the universal constant of gravitation and the Earth's mass (3.986004418×10^{14} m^3/s^2)

Polar orbit



Polar orbit

A **polar orbit** is an orbit in which a satellite passes above or nearly above both poles of the body (usually a planet such as the Earth, but possibly another body such as the Sun) being orbited on each revolution. It therefore has an inclination of (or very close to) 90 degrees to the equator. Except in the special case of a polar geosynchronous orbit, a satellite in a polar orbit will pass over the equator at a different longitude on each of its orbits.

Polar orbits are often used for earth-mapping, earth observation, and reconnaissance satellites, as well as for some weather satellites. The Iridium satellite constellation also uses a polar orbit to provide telecommunications services. The disadvantage to this orbit is that no one spot on the Earth's surface can be sensed continuously from a satellite in a polar orbit.

It is common for near-polar orbiting satellites to choose a sun-synchronous orbit: meaning that each successive orbital pass occurs at the same local time of day. This can be particularly important for applications such as remote sensing of the atmospheric temperature, where the most important thing to see may well be *changes* over time, which you do not want to see aliased onto changes in local time. To keep the same local time on a given pass, it is desirable for the orbit to be as short as possible, which is to say as low as possible. However, very low orbits of a few hundred kilometers would rapidly decay due to drag from the atmosphere. A commonly used altitude is approximately 1000 km; this produces an orbital period of about 100 minutes. The half-orbit on the sun side then takes only 50 minutes, during which local time of day does not greatly vary.

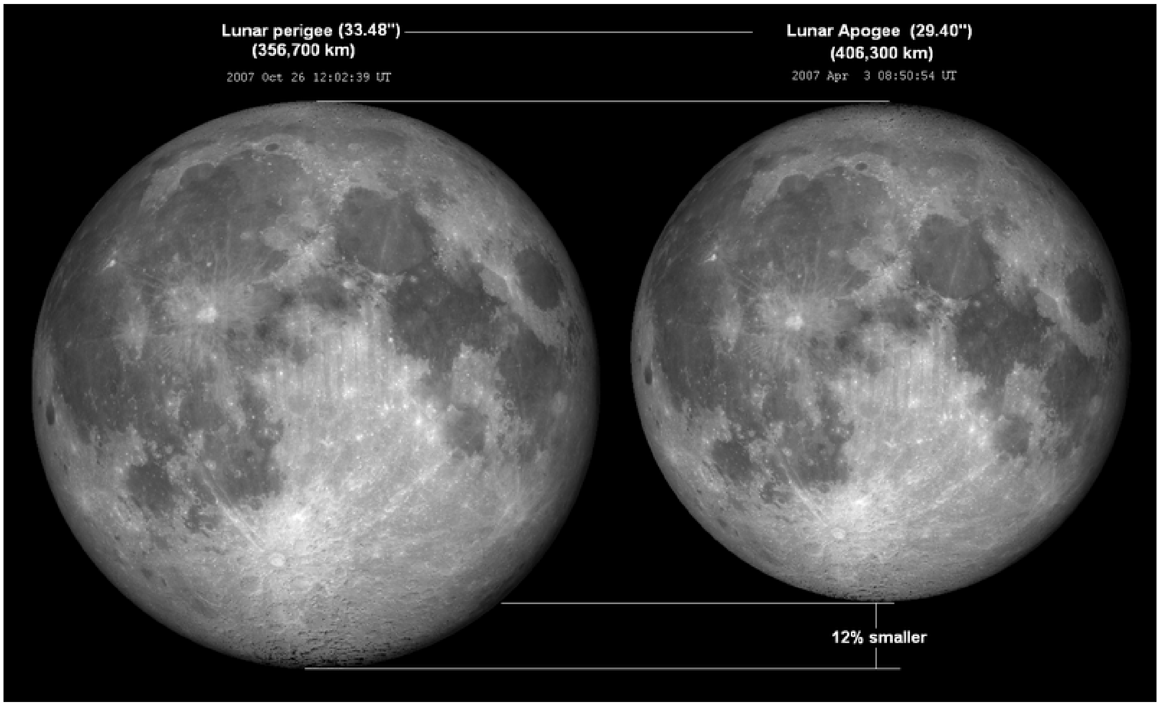
To retain the sun-synchronous orbit as the earth revolves around the sun during the year, the orbit of the satellite must precess at the same rate. Were the satellite to pass exactly over the pole, this would not happen. But because of the earths equatorial bulge, an orbit inclined at a slight angle is subject to a torque which causes precession; it turns out that an angle of about 8 degrees from the pole produces the desired precession in a 100 minute orbit .

A satellite can hover over one polar area a large part of the time, albeit at a large distance, using a polar highly elliptical orbit with its apogee above that area. This is the principle behind a Molniya orbit.

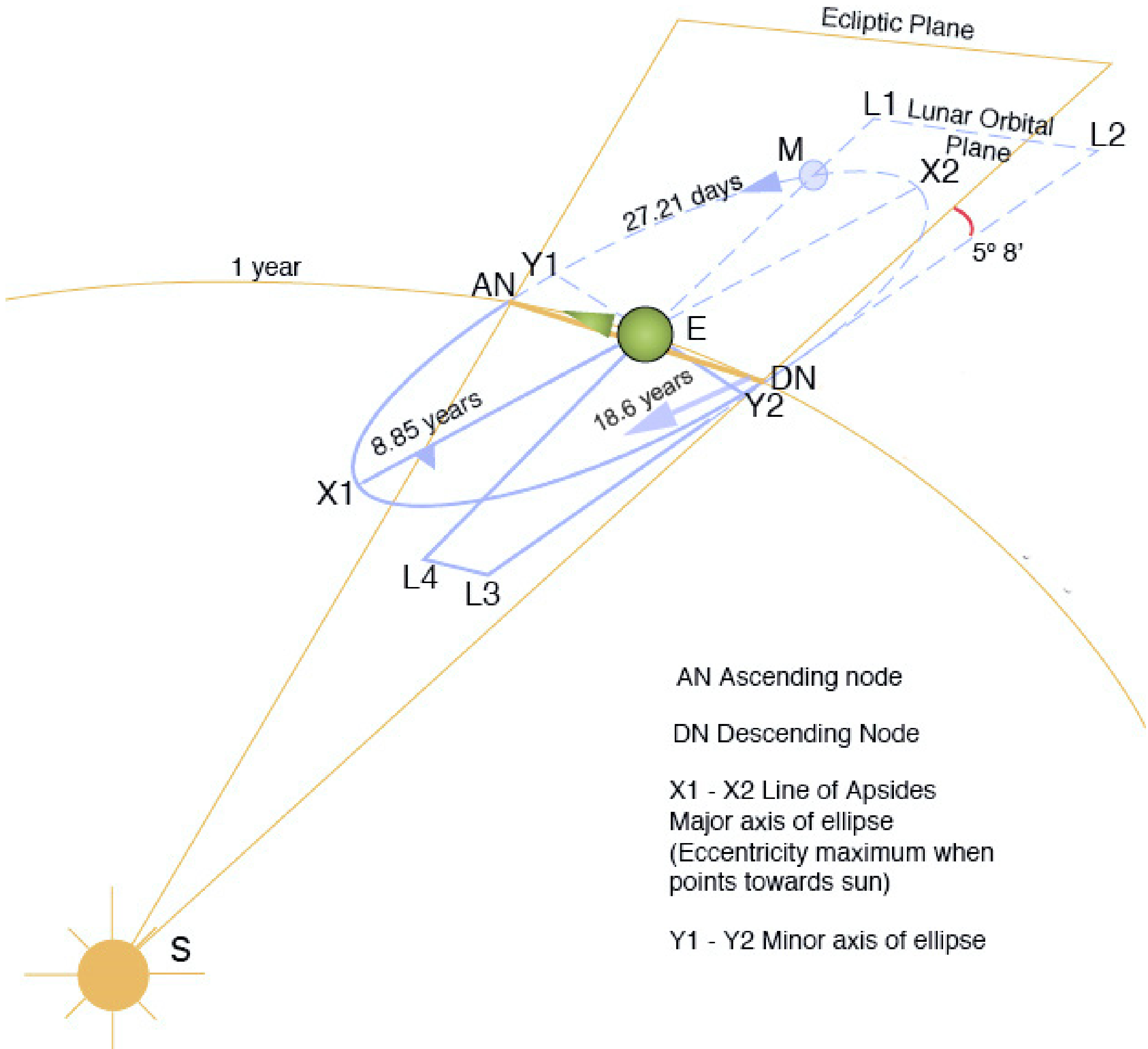
Orbit of the Moon

The Moon completes its orbit around the Earth in approximately 27.3 days (a sidereal month). The Earth and Moon orbit about their barycentre (common centre of mass), which lies about 4700 kilometres from Earth's centre (about three quarters of the Earth's radius). On average, the Moon is at a distance of about 385000 km from the centre of the Earth, which corresponds to about 60 Earth radii. With a mean orbital velocity of 1.023 km/s, the Moon moves relative to the stars each hour by an amount roughly equal to its angular diameter, or by about 0.5°. The Moon differs from most satellites of other planets in that its orbit is close to the plane of the ecliptic, and not to the Earth's equatorial plane. The lunar orbit plane is inclined to the ecliptic by about 5.1°, whereas the Moon's spin axis is inclined by only 1.5°.

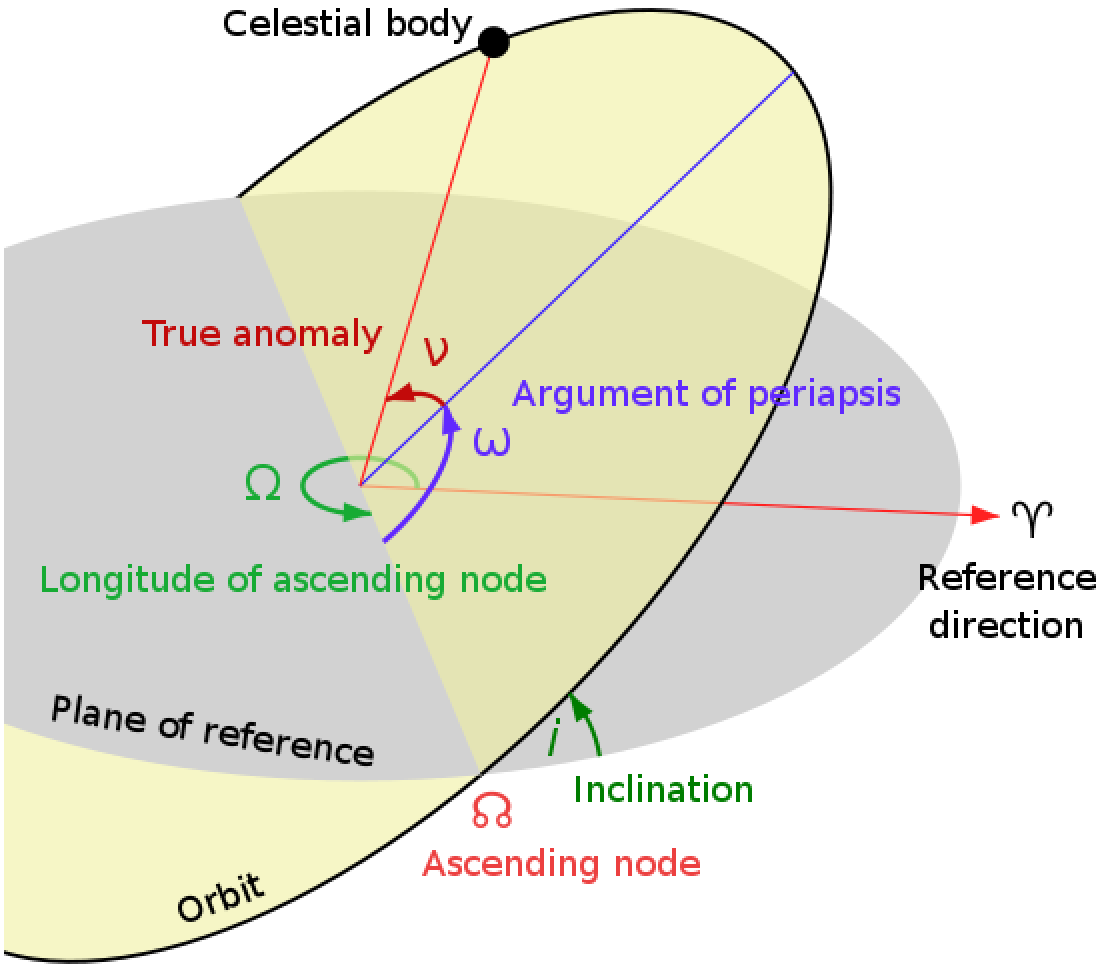
Properties



Lunar perigee–apogee size comparison



Earth's lunar orbit perturbation basics



Definition of orbital parameters.

The properties of the orbit described in this section are approximations. The Moon's orbit around the Earth has many irregularities (perturbations), and their study (lunar theory) has occupied astronomers over a long history.

Elliptic shape

The orbit of the Moon is distinctly elliptical with an average eccentricity of 0.0549. The non-circular form of the lunar orbit causes variations in the Moon's angular speed and apparent size as it moves towards and away from an observer on Earth. The mean angular daily movement relative to an imaginary observer at the barycentre is 13.176358° to the east.

Line of apsides

The orientation of the orbit is not fixed in space, but precesses over time. The nearest and farthest points in the orbit are the perigee and apogee respectively. The line joining these two points (the line of apsides) rotates slowly in the same direction as the Moon itself (direct motion), making one complete revolution in 3232.575 days or about 8.85 years.

Elongation

The Moon's elongation is its angular distance east of the Sun at any time. At new moon it is zero and the Moon is said to be in conjunction. At full moon the elongation is 180° and it is said to be in opposition. In both cases the moon is in syzygy, that is, the Sun, Moon and Earth are nearly aligned. When elongation is either 90° or 270° the Moon is said to be in quadrature.

Nodes

The nodes are points at which the Moon's orbit crosses the ecliptic. The Moon crosses the same node every 27.2122 days, an interval called the *draconic* or *draconitic month*. The line of nodes is the intersection between the two respective planes. It has a retrograde motion: for an observer on Earth it rotates westward along the ecliptic with a period of 18.6 years, or 19°21′ per year. When viewed from celestial north, the nodes move clockwise around the Earth, opposite the Earth's own spin and its rotation around the Sun. Lunar and solar eclipses can only occur when the line of nodes points toward the Sun, roughly every 5.4 months. The type of the eclipse depends on Moon's orbital position in that time window.

Inclination

The mean inclination of the lunar orbit to the ecliptic plane is 5.145°. The rotation axis of the Moon is also not perpendicular to its orbital plane, so the lunar equator is not in the plane of its orbit, but is inclined to it by a constant value of 6.688° (this is the obliquity). One might be tempted to think that as a result of the precession of the Moon's orbital plane, the angle between the lunar equator and the ecliptic would vary between the sum (11.833°) and difference (1.543°) of these two angles. However, as was discovered by Jacques Cassini in 1721, the rotation axis of the Moon precesses with the same rate as its orbital plane, but is 180° out of phase. Thus, although the rotation axis of the Moon is not fixed with respect to the stars, the angle between the ecliptic and the lunar equator is always 1.543°.

Lunar standstill

When the ascending node of the moon's orbit coincides with the vernal equinox in the northern hemisphere, the declination of the moon in the sky reaches a maximum at 23°29′ + 5°9′ or 28°36′. This is called the major standstill. Nine and a half years later, when the descending node has come to the same point, the angle is only 23°28′ − 5°8′ or 18°19′, and the declination of the moon is a minimum. This is the minor standstill.

Double planet?

Some consider the Earth–Moon system to be a double planet because of the relatively large size of the Moon and the small Earth–Moon mass ratio of about 81:1 when compared to other natural satellites in the Solar System. An informal criterion for a double planet system is that its barycentre must be exterior to both bodies. By that criterion, the Earth and Moon are an ordinary planet–moon system, because their mutual distance is about 60 Earth radii but their mass ratio is 81:1 so their centre of mass always lies well within the Earth.

The Moon orbiting Earth with sizes and distances to scale. Each pixel represents 500 km.

History of observations and measurements

About 3000 years ago, the Babylonians were the first human civilization to keep a consistent record of lunar observations. Clay tablets from that time period found over the territory of present-day Iraq are inscribed with cuneiform recording the times and dates of moonrises and moonsets, the stars that the Moon passed close by, and the time differences between rising and setting of both the Sun and the Moon around the time of the full moon. The Babylonians discovered the three main periods of the Moon's motion and used data analysis to build lunar calendars that extended well into the future. This use of detailed, systematic observations to make predictions based on experimental data may be classified as the first scientific study in human history. However, Babylonians seem to have lacked any geometrical or physical interpretation of their data, and they could not predict future lunar eclipses (although "warnings" were issued before likely eclipse times).

Ancient Greek astronomers were the first to introduce and analyze mathematical models of the motion of objects in the sky. Ptolemy described lunar motion by using a well-defined geometric model of epicycles and evection.

Isaac Newton was the first to develop a complete theory of motion, mechanics. The sheer wealth of humanity's observations of the lunar motion was the main testbed of his theory.

Lunar month

The Moon's periods

Name	Value (d)	Definition
sidereal	27.321661	with respect to the distant stars (13.369 passes per year)
synodic	29.530589	with respect to the Sun (phases of the Moon, 12.369 cycles per year)

tropical	27.321582	with respect to the vernal point (precesses in ~26,000 a)
anomalistic	27.554550	with respect to the perigee (recesses in 3232.6 d = 8.8504 a)
draconic (nodical)	27.212221	with respect to the ascending node (precesses in 6793.5 d = 18.5996 a)

There are several ways to measure how much time it takes the Moon to complete one orbit. The sidereal month is the time it takes to make one complete orbit with respect to the fixed stars, which is about 27.3 days. In contrast, the synodic month is the time it takes the Moon to reach the same phase, which takes about 29.5 days. The synodic period is longer than the sidereal period because the Earth–Moon system moves a finite distance in its orbit around the Sun during each sidereal month, and a longer time is required to achieve the same relative geometry. Other definitions for the duration of a lunar month include the time it takes to go from perigee to perigee (the anomalistic month), from ascending node to ascending node (the draconic month), and from two successive passes of the same ecliptic longitude (the tropical month). As a result of the slow precession of the lunar orbit, these latter three periods are only slightly different than the sidereal month. The average length of a calendric month ($^1/_{12}$ of a year) is about 30.4 days.

Tidal evolution

The gravitational attraction that the Moon exerts on Earth is the major cause of tides in the sea; the Sun has a lesser tidal influence. If the Earth possessed a global ocean of uniform depth, the Moon would act to deform both the solid earth (by a small amount) and ocean in the shape of an ellipsoid with high points directly beneath the Moon and on the opposite side of the Earth. However, as a result of the irregular coastline and varying ocean depths, this idealization is only partially realized. While the tidal flow period is generally synchronized to the Moon's orbit around Earth, its phase can vary. In some places on Earth there is only one high tide per day, though this is somewhat rare.

The tidal bulges on Earth are carried ahead of the Earth–Moon axis by a small amount as a result of the Earth's rotation. This is a direct consequence of friction and the dissipation of energy as water moves over the ocean bottom and into or out of bays and estuaries. Each bulge exerts a small amount of gravitational attraction on the Moon, with the bulge closest to the Moon pulling in a direction slightly forward along the Moon's orbit, because the Earth's rotation has carried the bulge forward. The opposing bulge has the opposite effect, but the closer bulge dominates due to its comparative closer distance to the Moon. As a result, some of the Earth's rotational momentum is gradually being transferred to the Moon's orbital momentum, and this causes the Moon to slowly recede from Earth at the rate of approximately 38 millimetres per year. In keeping with the conservation of angular momentum, the Earth's rotation is gradually slowing, and the Earth's day thus lengthens by about 23 microseconds every year (excluding glacial rebound). Both figures are valid only for the current configuration of the continents. Tidal rhythmites from 620 million years ago show that over hundreds of millions of years the Moon receded at an average rate of 22 millimetres per year and the day lengthened at an average rate of 12 microseconds per year, both about half of their current values.

The Moon is gradually receding from the Earth into a higher orbit, and calculations suggest that this would continue for about fifty billion years. By that time, the Earth and Moon would become caught up in what is called a "spin–orbit resonance" in which the Moon will circle the Earth in about 47 days (currently 29 days), and both Moon and Earth would rotate around their axes in the same time, always facing each other with the same side. However, the slowdown of the Earth's rotation is not occurring fast enough for the rotation to lengthen to a month before other effects change the situation: about 2.1 billion years from now, the increase of the Sun's radiation will have caused the Earth's oceans to vaporize, removing the bulk of the tidal friction and acceleration.

Libration

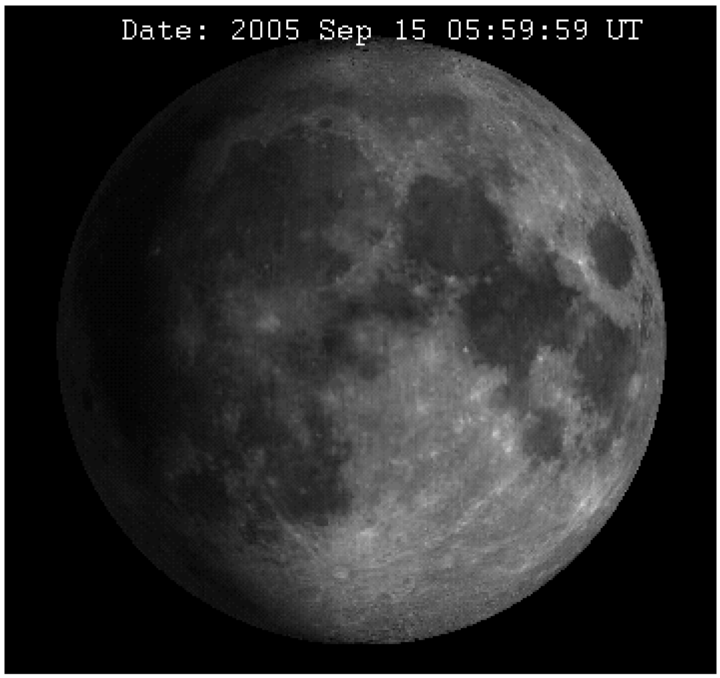


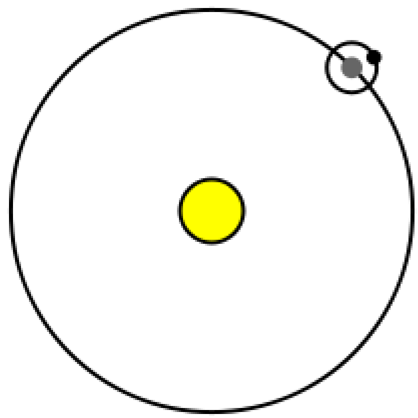
Image of the Moon as it cycles through its phases. The apparent wobbling of the Moon is known as libration.

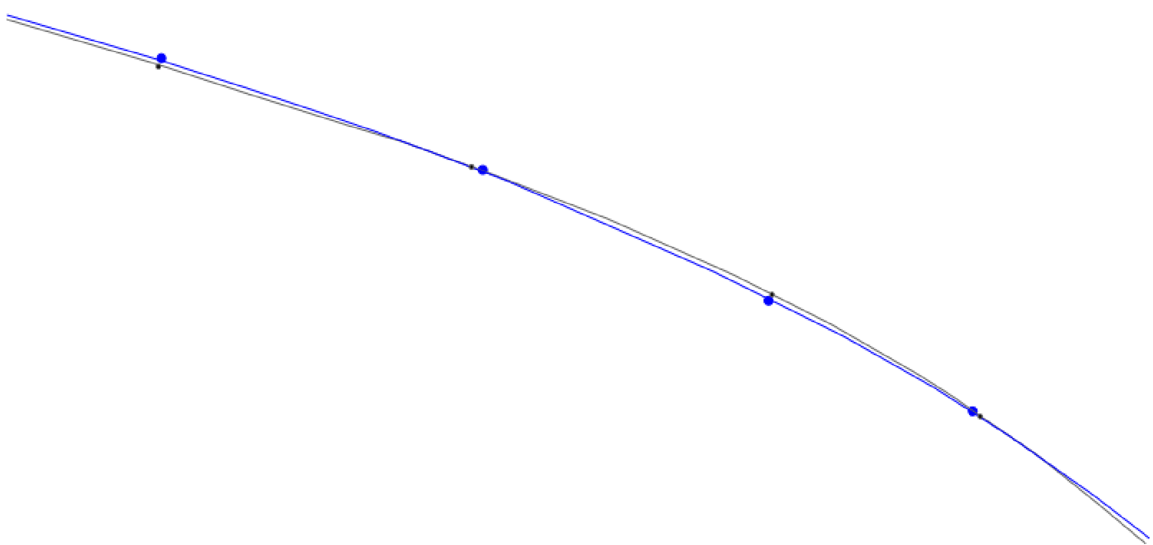
The Moon is in synchronous rotation, meaning that it keeps the same face turned toward the Earth at all times. This synchronous rotation is only true on average, because the Moon's orbit has a definite eccentricity. As a result, the angular velocity of the Moon varies as it moves around the Earth, and is hence not always equal to the Moon's rotational velocity. When the Moon is at its perigee, its rotation is slower than its orbital motion, and this allows us to see up to eight degrees of longitude of its eastern (right) far side. Conversely, when the Moon reaches its apogee, its rotation is faster than its orbital motion and this reveals eight degrees of longitude of its western (left) far side. This is referred to as **longitudinal libration**.

Because the lunar orbit is also inclined to the Earth's ecliptic plane by 5.1°, the rotation axis of the Moon seems to rotate towards and away from us during one complete orbit.

This is referred to as **latitudinal libration**, which allows one to see almost 7° of latitude beyond the pole on the far side. Finally, because the Moon is only about 60 Earth radii away from the Earth's centre of mass, an observer at the equator who observes the Moon throughout the night moves laterally by one Earth diameter. This gives rise to a **diurnal libration**, which allows one to view an additional one degree's worth of lunar longitude. For the same reason, observers at both geographical poles of the Earth would be able to see one additional degree's worth of libration in latitude.

Path of Earth and Moon around Sun





The Moon's orbital path around the Sun (accompanying the Earth in its own path around the Sun) is always convex outwards. (In this diagram, the Sun is below and to the left of the frame.)

In representations of the solar system, it is common to draw the trajectory of the Earth from the point of view of the Sun, and the trajectory of the Moon from the point of view of the Earth. This could give the impression that the Moon circles around the Earth in such a way that sometimes it goes backwards when viewed from the Sun's perspective.

Since the orbital velocity of the Moon about the Earth (1 km/s) is small compared to the orbital velocity of the Earth about the Sun (30 km/s), this never occurs.

Considering the Earth-Moon system as a binary planet, their mutual centre of gravity is within the Earth, about 4624 km from its centre or 72.6% of its radius. This centre of gravity remains in line towards the Moon as the Earth completes its diurnal rotation. It is this mutual centre of gravity which defines the path of the Earth-Moon system in solar orbit. Consequently the Earth's centre veers inside and outside the orbital path during each synodic month as the Moon moves in the opposite direction.

Unlike most other moons in the solar system, the trajectory of the Moon is very similar to that of the Earth. The Sun's gravitational pull on the Moon is over twice as great as the Earth's pull on the Moon; consequently, the Moon's trajectory is always convex (as seen when looking inward at the entire Moon/Earth/Sun system from a great distance off), and is nowhere concave (from the perspective just mentioned) or looped. If the gravitational attraction of the Sun could be "turned off" while maintaining the Earth-Moon gravitational attraction, the Moon would continue to orbit the Earth once every sidereal month.

Chapter- 4

Non- Geocentric Classification

Horseshoe orbit

A **horseshoe orbit** appears when a viewer on an orbiting body (like Earth) watches the movement of another orbiting body, whose orbit is skinnier (more eccentric), but has about the same period. As a result, the path appears to have the shape of a kidney bean.

The loop is not closed but will drift forward or backward slightly each time, so that the point it circles will appear to move smoothly along Earth's orbit over a long period of time. When the object approaches Earth closely at either end of its trajectory, its apparent direction changes. Over an entire cycle the center traces the outline of a horseshoe, with the Earth between the 'horns'.

Several asteroids (such as 3753 Cruithne, 54509 YORP, (85770) 1998 UP_1, 2002 AA_{29}, and 2003 YN_{107}) are known to occupy horseshoe orbits with respect to Earth. Saturn's moons Epimetheus and Janus occupy horseshoe orbits with respect to each other (in their case, there is no repeated looping: each one traces a full horseshoe with respect to the other).

Explanation of horseshoe orbital cycle

Background

The following explanation relates to an asteroid which is in such an orbit around the Sun, and is also affected by the Earth.

The asteroid is in almost the same solar orbit as Earth. Both take approximately one year to orbit the Sun.

It is also necessary to grasp two rules of orbit dynamics:

1. A body closer to the Sun completes an orbit more quickly than a body further away.

2. If a body accelerates along its orbit, its orbit moves outwards from the Sun. If it decelerates, the orbital radius decreases.

The horseshoe orbit arises because the gravitational attraction of the Earth changes the shape of the elliptical orbit of the asteroid. The shape changes are very small but result in significant changes relative to the Earth.

The horseshoe only becomes apparent when mapping the movement of the asteroid relative to both the Sun and the Earth. The asteroid always orbits the sun in the same direction. However, it goes through a cycle of catching up with the Earth and falling behind, so that its movement relative to both the Sun and the Earth traces a shape like the outline of a horseshoe.

Stages of the orbit

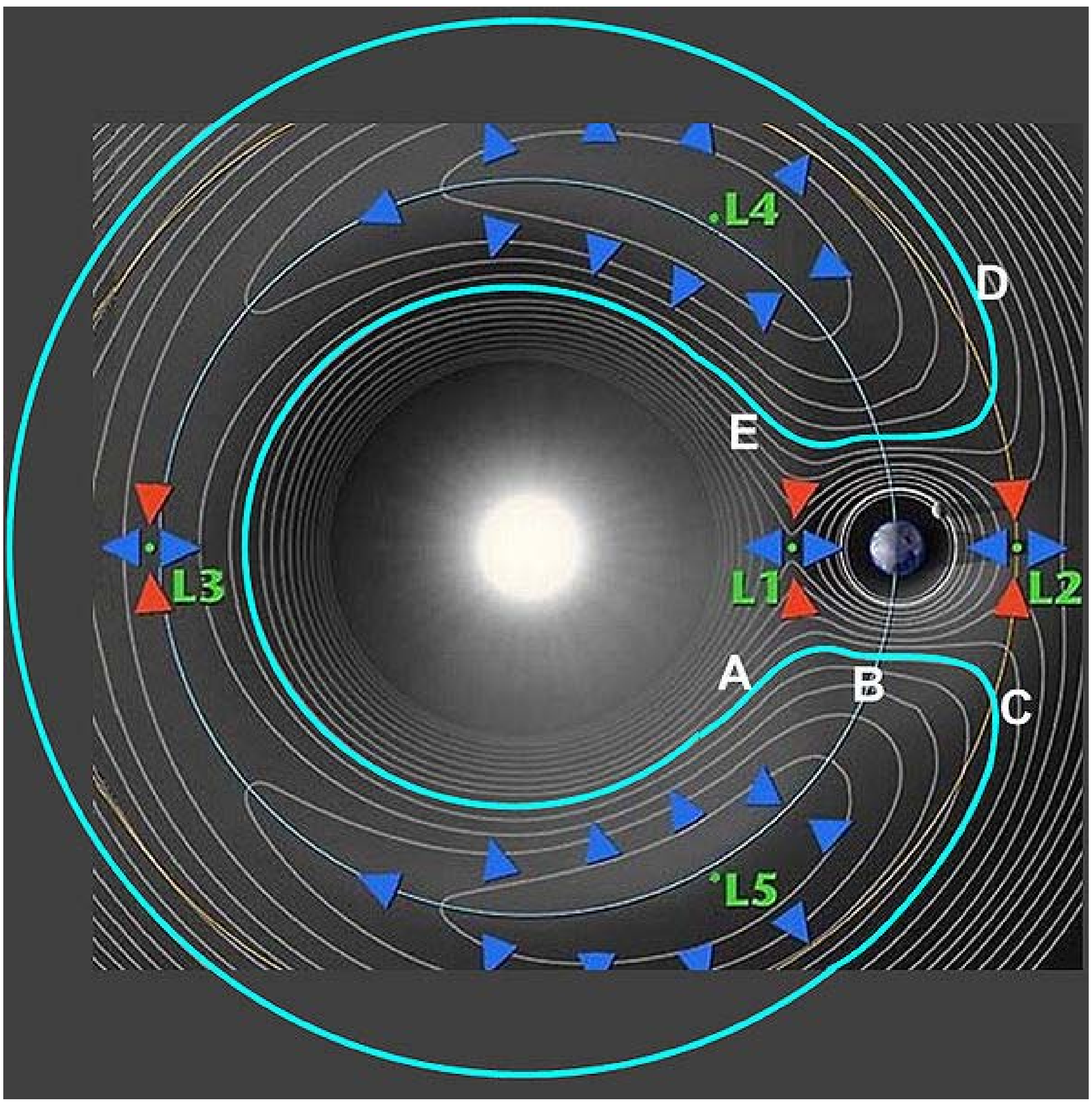


Figure 1. Plan showing possible orbits along gravitational contours

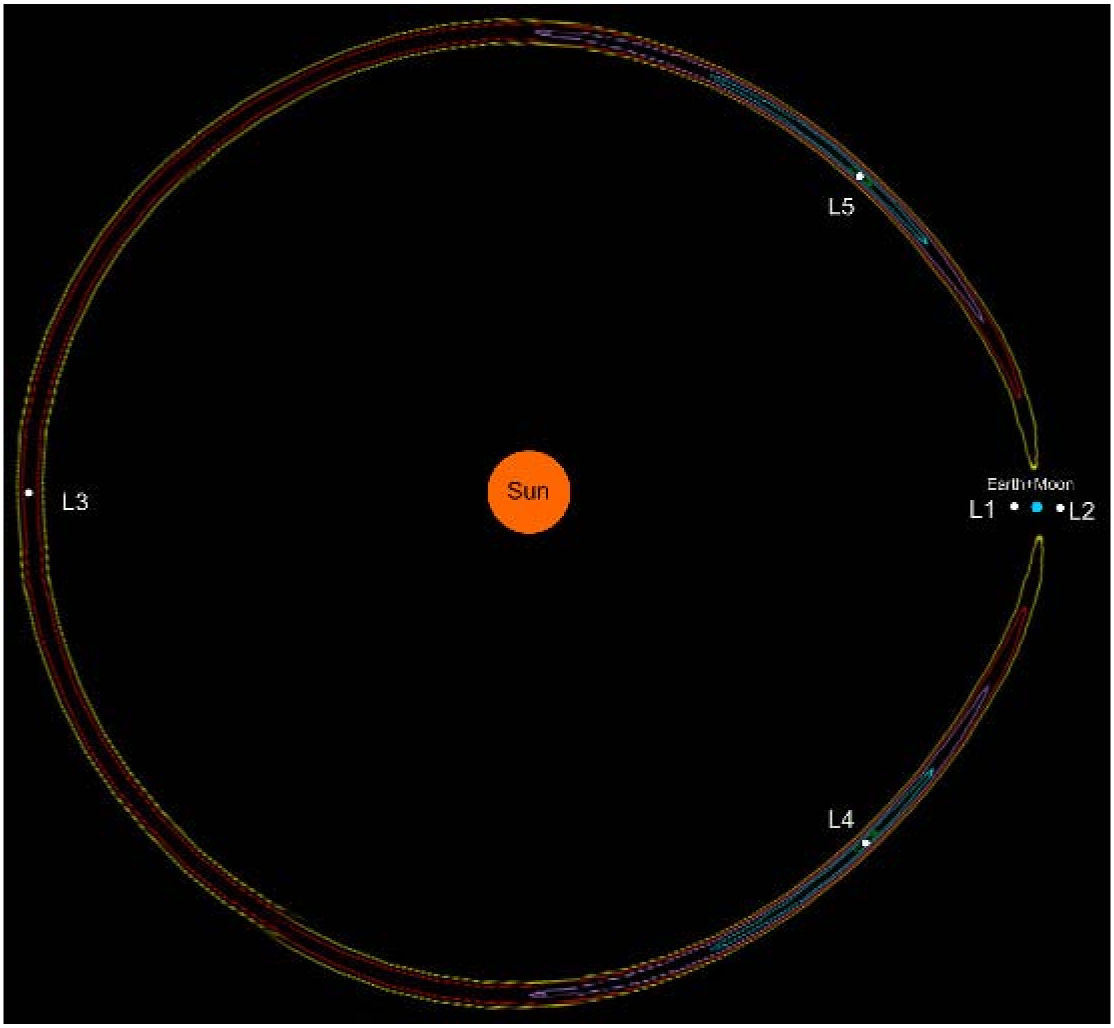


Figure 2. Thin horseshoe orbit

Starting out at point A on the inner ring between L_5 and Earth, the satellite is orbiting faster than the Earth. It's on its way toward passing between the Earth and the Sun. But Earth's gravity exerts an outward accelerating force, pulling the satellite into a higher orbit which – counter-intuitively – decreases its speed.

When the satellite gets to point B, it is traveling at the same speed as Earth. Earth's gravity is still accelerating the satellite along the orbital path. The twisted rule above continues to pull the satellite into a higher orbit. Eventually, at C, the satellite reaches a high enough, slow enough orbit and starts to fall behind Earth. It then spends the next century or more appearing to drift 'backwards' around the orbit. Remember that its orbit around the Sun still takes only slightly more than one Earth year and the apparent backwards motion is only relative to the Earth.

Eventually the satellite comes around to point D. Earth's gravity is now reducing the satellite's orbital velocity, causing it to fall into a lower orbit, which actually speeds up the satellite. This continues until the satellite's orbit is lower and faster than Earth's orbit. It begins moving out ahead of the earth. Over the next few centuries it completes its journey back to point A.

Energy viewpoint

A somewhat different, but equivalent, view of the situation may be noted by considering conservation of energy. It is a theorem of classical mechanics that a body moving in a time-independent potential field will have its total energy, $E = T + V$, conserved, where E is total energy, T is kinetic energy (always non-negative) and V is potential energy, which is negative. It is apparent then, since $V = -GM/R$ near a gravitating body of mass M, that seen from a *stationary* frame, V will be increasing for the region behind M, and decreasing for the region in front of it. However, orbits with lower total energy have shorter periods, and so a body moving slowly on the forward side of a planet will lose energy, fall into a shorter-period orbit, and thus slowly move away, or be "repelled" from it. Bodies moving slowly on the trailing side of the planet will gain energy, rise to a higher, slower, orbit, and thereby fall behind, similarly repelled. Thus a small body can move back and forth between a leading and a trailing position, never approaching too close to the planet that dominates the region.

Tadpole orbit

Figure 1 above shows shorter orbits around the Lagrangian points L_4 and L_5 (e.g. the lines close to the blue triangles). These are called **tadpole orbits** and can be explained in a similar way, except that the asteroid's distance from the Earth does not oscillate as far as the L_3 point on the other side of the Sun. As it moves closer to or farther from the Earth, the changing pull of Earth's gravitational field causes it to accelerate or decelerate, causing a change in its orbit known as libration.

An example of a body in a tadpole orbit is Polydeuces, a small moon of Saturn which librates around the trailing L_5 point relative to a larger moon, Dione.

Orbital spaceflight

The International Space Station as it appeared during its construction in orbit around the Earth on 2001-08-20

An **orbital spaceflight** (or orbital flight) is a spaceflight in which a spacecraft is placed on a trajectory where it could remain in space for at least one orbit. To do this around the Earth, it must be on a free trajectory which has an altitude at perigee (altitude at closest approach) above 100 kilometers (62 mi) (this is, by at least one convention, the boundary of space). To remain in orbit at this altitude requires an orbital speed of ~7.8 km/s. Orbital speed is slower for higher orbits, but attaining them requires higher delta-v.

The expression "orbital spaceflight" is mostly used to distinguish from sub-orbital spaceflights, which are flights where apogee of a spacecraft reaches space but perigee is too low.

Orbital launch

Orbital spaceflight from Earth has only been achieved by launch vehicles that use rocket engines for propulsion. To reach orbit, the rocket must impart to the payload a delta-v of about 9.3-10 km/s. This figure is mainly (~7.8 km/s) for horizontal acceleration needed to reach orbital speed, but allows for atmospheric drag (approximately 300 m/s with the ballistic coefficient of a 20 m long dense fuelled vehicle), gravity losses (depending on burn time and details of the trajectory and launch vehicle), and gaining altitude.

The main proven technique involves launching nearly vertically for a few kilometers while performing a gravity turn, and then progressively flattening the trajectory out at an altitude of 170+ km and accelerating on a horizontal trajectory (with the rocket angled upwards to fight gravity and maintain altitude) for a 5-8 minute burn until orbital velocity is achieved. Currently, 2-4 stages are needed to achieve the required delta-v. Most launches are by expendable launch systems.

The Pegasus rocket for small satellites instead launches from an aircraft at an altitude of 12 km.

Other techniques, such as use of a launch loop, have been proposed for non-rocket spacelaunch. These techniques are theoretical: no attempts have been made to orbit a vehicle using any of them.

Stability

An object in orbit at an altitude of less than roughly 200 km is considered unstable due to atmospheric drag. For a satellite to be in a stable orbit (i.e. sustainable for more than a few months), 350 km is a more standard altitude for low Earth orbit. For example, on 1958-02-01 the Explorer 1 satellite was launched into an orbit with a perigee of 358 kilometers (222 mi). It remained in orbit for more than 12 years before its atmospheric reentry over the Pacific Ocean on 1970-03-31.

However, the exact behaviour of objects in orbit depends on altitude, their ballistic coefficient, and details of space weather which can affect the height of the upper atmosphere.

Orbits

There are three main 'bands' of orbit around the Earth: low Earth orbit (LEO), medium Earth orbit (MEO) and geostationary orbit (GEO).

Due to Orbital mechanics orbits are in a particular, largely fixed plane around the Earth, which coincides with the center of the Earth, and may be tilted with respect to the equator. The Earth rotates about its axis within this orbit, and the relative motion of the spacecraft and the movement of the Earths surface determines the position that the spacecraft appears in the sky from the ground, and which parts of the Earth are visible from the spacecraft.

By dropping a vertical down to the Earth's surface it is possible to calculate a ground track which shows which part of the Earth a spacecraft is immediately above, and this is useful for helping to visualise the orbit.

NASA provides real-time tracking of the over 500 artificial satellites maintained in orbit around Earth.

Re-entry

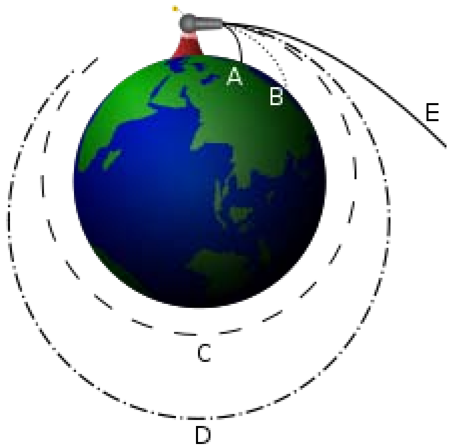
Due to the high speeds of orbital spaceflight, atmospheric reentry is much more difficult compared to sub-orbital flights.

Even if the vehicle is a satellite that is ultimately expendable, most space authorities are pushing towards controlled re-entry techniques to avoid issues of space debris reaching the ground and causing a hazard to lives and property. In addition, this minimises the creation of orbital space junk.

Returning craft though (including all potentially manned craft), have to find a way of slowing down as much as possible while still in higher atmospheric layers and avoid hitting the ground (lithobraking) or burning up. The problem of deceleration from orbital speeds is solved through using atmospheric drag (aerobraking) to lose nearly all of the speed. On an orbital space flight initial deceleration is provided by the retrofiring of the craft's rocket engines, perturbing the orbit (by lowering perigee down into the atmosphere) onto a suborbital trajectory.

Aerobraking is achieved by orienting the returning space craft to fly so as to present the heat shields forwards towards the atmosphere so as to protect against the high temperatures generated by atmospheric compression and friction caused by passing through the atmosphere at hypersonic speeds. The thermal energy is dissipated mainly by compression heating the air in a shockwave ahead of the vehicle using a blunt heat shield shape, with the aim of minimising the heat entering the vehicle. Sub-orbital space flights, being at a much lower speed, do not generate anywhere near as much heat upon re-entry.

Sub-orbital spaceflight



Isaac Newton's Cannonball

A **sub-orbital space flight** is a spaceflight in which the spacecraft reaches space, but its trajectory intersects the atmosphere or surface of the gravitating body from which it was launched, so that it does not complete one orbital revolution.

For example, the path of an object launched from Earth that reaches 100 km (62 mi) above sea level, and then falls back to Earth, is considered a sub-orbital spaceflight. Some sub-orbital flights have been undertaken to test spacecraft and launch vehicles later intended for orbital spaceflight. Other vehicles are specifically designed only for sub-orbital flight; examples include manned vehicles such as the X-15 and SpaceShipOne, and unmanned ones such as ICBMs and sounding rockets.

Sub-orbital spaceflights are distinct from flights that attain orbit but use retro-rockets to deorbit after less than one full orbital period. Thus the flights of the Fractional Orbital Bombardment System would not be considered sub-orbital; instead these are simply considered flights to low Earth orbit.

Usually a rocket is used, but experimentally a sub-orbital spaceflight has also been achieved with a space gun.

Altitude requirement

By one definition a sub-orbital spaceflight reaches an altitude higher than 100 km above sea level. This altitude, known as the Kármán line, was chosen by the Fédération Aéronautique Internationale because it is roughly the point where a vehicle flying fast enough to support itself with aerodynamic lift from the Earth's atmosphere would be flying faster than orbital speed. The US military and NASA award astronaut wings to those flying above 50 miles (80.47 km), although the US State Department appears to not support a distinct boundary between atmospheric flight and space flight.

Orbit

During freefall the trajectory is part of an elliptic orbit as given by the orbit equation. The perigee distance is less than the radius of the Earth *R* including atmosphere, hence the ellipse intersects the Earth, and hence the spacecraft will fail to complete an orbit. The major axis is vertical, the semi-major axis *a* is more than *R*/2. The specific orbital energy ε is given by:

$$\epsilon = -\frac{\mu}{2a} > -\frac{\mu}{R}$$

where μ is the standard gravitational parameter.

Almost always $a < R$, corresponding to a lower ε than the minimum for a full orbit, which is $-\frac{\mu}{2R}$

Thus the net extra specific energy needed compared to just raising the spacecraft into space is between 0 and $\frac{\mu}{2R}$.

Speed, range, altitude

To minimize the required delta-v (an astrodynamical measure which strongly determines the required fuel), the high-altitude part of the flight is made with the rockets off (this is technically called free-fall even for the upward part of the trajectory). The maximum speed in a flight is attained at the lowest altitude of this free-fall trajectory, both at the start and at the end of it.

If one's goal is simply to "reach space", for example in competing for the Ansari X Prize, horizontal motion is not needed. In this case the lowest required delta-v is about 1.4 km/s, for a sub-orbital flight with a maximum speed of about 1 km/s. Moving slower, with less free-fall, would require more delta-v.

Compare this with orbital spaceflights: a low Earth orbit (LEO), with an altitude of about 300 km), needs a speed around 7.7 km/s, requiring a delta-v of about 9.2 km/s.

For sub-orbital spaceflights covering a horizontal distance the maximum speed and required delta-v are in between those of a vertical flight and a LEO. The maximum speed at the lower ends of the trajectory are now composed of a horizontal and a vertical component. The higher the horizontal distance covered, the more are both speeds, and the more is the maximum altitude. For the V-2 rocket, just reaching space but with a range of about 330 km, the maximum speed was 1.6 km/s. Scaled Composites SpaceShipTwo which is under development will have a similar free-fall orbit but the announced maximum speed is 1.1 km/s (perhaps because of engine shut-off at a higher altitude).

For larger ranges, due to the elliptic orbit the maximum altitude can even be considerably more than for a LEO. On an intercontinental flight, such as that of an intercontinental ballistic missile or possible future commercial spaceflight, the maximum speed is about 7 km/s, and the maximum altitude about 1200 km. Note that an intercontinental flight at an altitude of 300 km would require a larger delta-v, that of a LEO. It should be noted that any spaceflight that returns to the surface, including sub-orbital ones, will undergo atmospheric reentry. The speed at the start of that is basically the maximum speed of the flight. The aerodynamic heating caused will vary accordingly: it is much less for a flight with a maximum speed of only 1 km/s than for one with a maximum speed of 7 or 8 km/s.

Flight duration

In a vertical flight of not too high altitudes, the time of the free-fall is both for the upward and for the downward part the maximum speed divided by the acceleration of gravity, so with a maximum speed of 1 km/s together 3 minutes and 20 seconds. The duration of the flight phases before and after the free-fall can vary.

For an intercontinental flight the boost phase takes 3 to 5 minutes, the free-fall (midcourse phase) about 25 minutes. For an ICBM the atmospheric reentry phase takes about 2 minutes; this will be longer for any soft landing, such as for a possible future commercial flight.

Suborbital flights can last many hours. Pioneer 1 was NASA's first space probe, intended to reach the Moon. A partial failure caused it to instead follow a suborbital trajectory, reentering the Earth's atmosphere 43 hours after launch.

Flight profiles

While there are a great many possible sub-orbital flight profiles, it is expected that some will be more common than others.

Ballistic missiles

The first suborbital vehicles which reached space were ballistic missiles. The very first ballistic missile to reach space was the German V-2 on October 3, 1942 which reached an altitude of 60 miles (97 km). That in fact was the first man-made object of any kind to reach space. Then in the 1950s the USA and USSR concurrently developed much longer range Intercontinental Ballistic Missiles (ICBM)s all of which were based on the V-2 Rocket and the work of the scientists at Peenemunde. There are now many countries who possess ICBMs and even more with shorter range IRBMs (Intermediate Range Ballistic Missiles).

Tourist flights

Sub-orbital tourist flights will initially focus on attaining the altitude required to qualify as reaching space. The flight path will probably be either vertical or very steep, with the spacecraft landing back at its take-off site.

The spacecraft will probably shut off its engines well before reaching maximum altitude, and then coast up to its highest point. During a few minutes, from the point when the engines are shut off to the point where the atmosphere begins to slow down the downward acceleration, the passengers will experience weightlessness.

In 2004, a number of companies worked on vehicles in this class as entrants to the Ansari X Prize competition. The Scaled Composites SpaceShipOne was officially declared by Rick Searfoss to have won the competition on October 4, 2004 after completing two flights within a two week period.

In 2005, Sir Richard Branson of the Virgin Group announced the creation of Virgin Galactic and his plans for a 9 seat capacity SpaceShipTwo named VSS *Enterprise*. It has since been completed with eight seats (one pilot, one co-pilot and six passengers) and has taken part in captive-carry tests and with the first mother-ship WhiteKnightTwo, or VMS Eve. It has also completed solitary glides, although the hybrid rocket motor has not yet been fired. Four more of each have been ordered and will operate from the new Spaceport America. Flights have been scheduled for 2011 driven by a "safety-driven schedule".

Scientific experiments

A major use of suborbital vehicles today are as scientific sounding rockets. Scientific suborbital flights began in the 1920s when Robert H. Goddard launched the first liquid fueled rockets, however they did not reach space altitude. Modern sounding rocket flights began in the late 1940s using vehicles derived from German V-2 ballistic missiles. Today there are dozens of different sounding rockets on the market, from a variety of suppliers in various countries. Typically, researchers wish to conduct experiments in microgravity or above the atmosphere. There have reportedly been several offers from researchers to launch experiments on SpaceShipOne, which have been turned down until the next version of the vehicle .

Suborbital transportation

Research, such as that done for the X-20 Dyna-Soar project suggests that a semi-ballistic sub-orbital flight could travel from Europe to North America in less than an hour.

However, the size of rocket, relative to the payload, necessary to achieve this, is similar to an ICBM. ICBMs have delta-v's somewhat less than orbital; and therefore would be somewhat cheaper than the costs for reaching orbit, but the difference is not large.

Thus due to the high cost, this is likely to be initially limited to high value, very high urgency cargo such as courier flights, or as the ultimate business jet; or possibly as an extreme sport, or for military fast-response.

Tether launch assist

There have been proposals to use tethers (commonly referred to as skyhooks) to put suborbital payloads into orbit. For example, an orbiting space station could extend a tether, and a suborbital vehicle rendezvous with the end of the tether and dock to it. If practical, this would be considerably less expensive than launching payloads directly into orbit on rockets on a per flight basis..

Notable unmanned sub-orbital spaceflights

- The first sub-orbital space flight was in early 1944, when a V-2 test rocket launched from Peenemünde in Germany reached 189 kilometres altitude.
- 8 September 1944, the world's first successful ballistic missile (V-2, launched by Germany) hits its target for the first time, Chiswick in London, England. Three civilians were killed and seventeen injured, a massive crater was left. By September 1944, the V-2s routinely achieved Mach-4 during terminal descent.
- Bumper 5, a two stage rocket launched from the White Sands Proving Grounds. On 24 February 1949 the upper stage reached an altitude of 248 miles (399 km) and a speed of 7,553 feet per second (2300 meters per second approx.) which is nearly Mach-7.
- USSR — Energia, 1986, Polyus payload failed to reach orbit; this was the most massive object launched into suborbital spaceflight to date
- USA USAF/NASA/DARPA´s X-37 B, intended to demonstrate reusable space technologies, The X-37 began as a NASA project in 1999, then was transferred to the US Department of Defense in 2004. It had its first flight as a drop test on April 7, 2006, at Edwards AFB. The spaceplane's first orbital mission, USA-212 was launched on April 22, 2010 using an Atlas V rocket and landed on Dec. 3rd 2010.

Manned sub-orbital spaceflights

	Date (GMT)	Mission	Crew	Country	Remarks
1	1961-05-05	Mercury-Redstone 3	Alan Shepard	United States	First manned sub-orbital spaceflight, first American in space
2	1961-07-21	Mercury-Redstone 4	Virgil Grissom	United States	
3	1963-07-19	X-15 Flight 90	Joseph A. Walker	United States	First winged craft in space
4	1963-08-22	X-15 Flight 91	Joseph A. Walker	United States	First person and spacecraft to make two flights into space

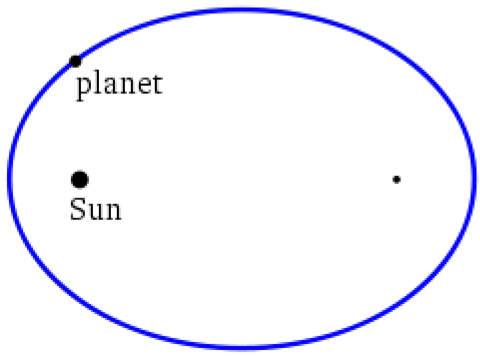
5	1975-04-05	Soyuz 18a	Vasili Lazarev Oleg Makarov	Soviet Union	Failed orbital launch. Aborted after malfunction during stage separation
6	2004-06-21	SpaceShipOne flight 15P	Mike Melvill	United States	First commercial spaceflight
7	2004-09-29	SpaceShipOne flight 16P	Mike Melvill	United States	First of two flights to win Ansari X-Prize
8	2004-10-04	SpaceShipOne flight 17P	Brian Binnie	United States	

Future of manned sub-orbital spaceflight

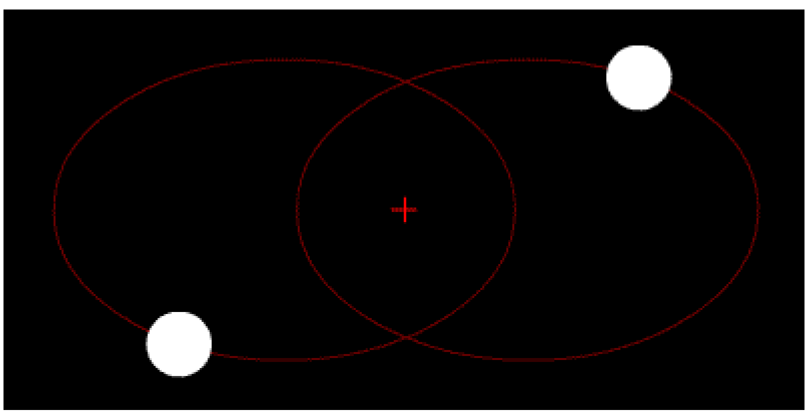
Private companies such as Rocketplane Limited and Blue Origin are taking an interest in sub-orbital spaceflight, due in part to ventures like the Ansari X Prize. NASA and others are experimenting with scramjet based hypersonic aircraft which may well be used with flight profiles that qualify as sub-orbital spaceflight. In addition, the VSS Enterprise is expected to begin manned sub-orbital flights starting in 2011. Non-profit entities like Copenhagen Suborbitals also attempt launches.

Chapter- 5

Elliptic Orbit



A small body in space orbits a large one (like a planet around the sun) along an elliptical path, with the large body being located at one of the ellipse foci.



Two bodies with similar mass orbiting around a common barycenter with elliptic orbits.

In astrodynamics or celestial mechanics an **elliptic orbit** is a Kepler orbit with the eccentricity less than 1; this includes the special case of a circular orbit, with eccentricity equal to zero. In a stricter sense, it is a Kepler orbit with the eccentricity greater than 0 and less than 1 (thus excluding the circular orbit). In a wider sense it is a Kepler orbit with negative energy. This includes the radial elliptic orbit, with eccentricity equal to 1.

In a gravitational two-body problem with negative energy both bodies follow similar elliptic orbits with the same orbital period around their common barycenter. Also the relative position of one body with respect to the other follows an elliptic orbit.

Examples of elliptic orbits include: Hohmann transfer orbit, Molniya orbit and tundra orbit.

Velocity

Under standard assumptions the orbital speed (v) of a body traveling along **elliptic orbit** can be computed from the Vis-viva equation as:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

where:

- μ is the standard gravitational parameter,
- r is the distance between the orbiting bodies.
- a is the length of the semi-major axis.

The velocity equation for a hyperbolic trajectory has either + $\frac{1}{a}$, or it is the same with the convention that in that case *a* is negative.

Orbital period

Under standard assumptions the orbital period (P) of a body traveling along an elliptic orbit can be computed as:

$$P = 2\pi \sqrt{\frac{a^3}{\mu}}$$

where:

- μ is standard gravitational parameter,
- a is length of semi-major axis.

Conclusions:

- The orbital period is equal to that for a circular orbit with the orbit radius equal to the semi-major axis (a),
- For a given semi-major axis the orbital period does not depend on the eccentricity.

Energy

Under standard assumptions, specific orbital energy (ϵ) of elliptic orbit is negative and the orbital energy conservation equation (the Vis-viva equation) for this orbit can take the form:

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} = \epsilon < 0$$

where:

- v is orbital speed of orbiting body,
- r is distance of orbiting body from central body,
- a is length of semi-major axis,
- μ is standard gravitational parameter.

Conclusions:

- For a given semi-major axis the specific orbital energy is independent of the eccentricity.

Using the virial theorem we find:

- the time-average of the specific potential energy is equal to 2ε
  - the time-average of r^{-1} is a^{-1}
- the time-average of the specific kinetic energy is equal to -ε

Flight path angle

The flight path angle is the angle between the orbiting body's velocity vector (= the vector tangent to the instantaneous orbit) and the local horizontal. Under standard assumptions the flight path angle φ satisfies the equation:

$$h = r\,v\,\cos\phi$$

where:

- h is the specific relative angular momentum of the orbit,
- v is orbital speed of orbiting body,
- r is radial distance of orbiting body from central body,
- ϕ is the flight path angle

Orbital parameters

The state of an orbiting body at any given time is defined by the orbiting body's position and velocity with respect to the central body, which can be represented by the three-dimensional Cartesian coordinates (position of the orbiting body represented by x, y, and z) and the similar Cartesian components of the orbiting body's velocity. This set of six variables, together with time, are called the orbital state vectors. Given the masses of the two bodies they determine the full orbit. The two most general cases with these 6 degrees of freedom are the elliptic and the hyperbolic orbit. Special cases with less degrees of freedom are the circular and parabolic orbit.

Because at least six variables are absolutely required to completely represent an elliptic orbit with this set of parameters, then six variables are required to represent an orbit with any set of parameters. Another set of six parameters that are commonly used are the orbital elements.

Solar system

In the Solar System, planets, asteroids, most comets and some pieces of space debris have approximately elliptical orbits around the Sun.

Radial elliptic trajectory

A radial trajectory can be a double line segment, which is a degenerate ellipse with semi-minor axis = 0 and eccentricity = 1. Although the eccentricity is 1 this is not a parabolic orbit. Most properties and formulas of elliptic orbits apply. However, the orbit cannot be closed. It is an open orbit corresponding to the part of the degenerate ellipse from the moment the bodies touch each other and move away from each other until they touch each other again. In the case of point masses one full orbit is possible, starting and ending with a singularity. The velocities at the start and end are infinite in opposite directions, and the potential energy is equal to minus infinity.

The radial elliptic trajectory is the solution of a two-body problem with at some instant zero speed, as in the case of dropping an object (neglecting air resistance).

History

Babylonian astronomers in the first millennium BC observed that the Sun's motion along the ecliptic was not uniform, though they were unaware of why this was; it is today known that this is due to the Earth moving in an elliptic orbit around the Sun, with the Earth moving faster when it is nearer to the Sun at perihelion and moving slower when it is farther away at aphelion.

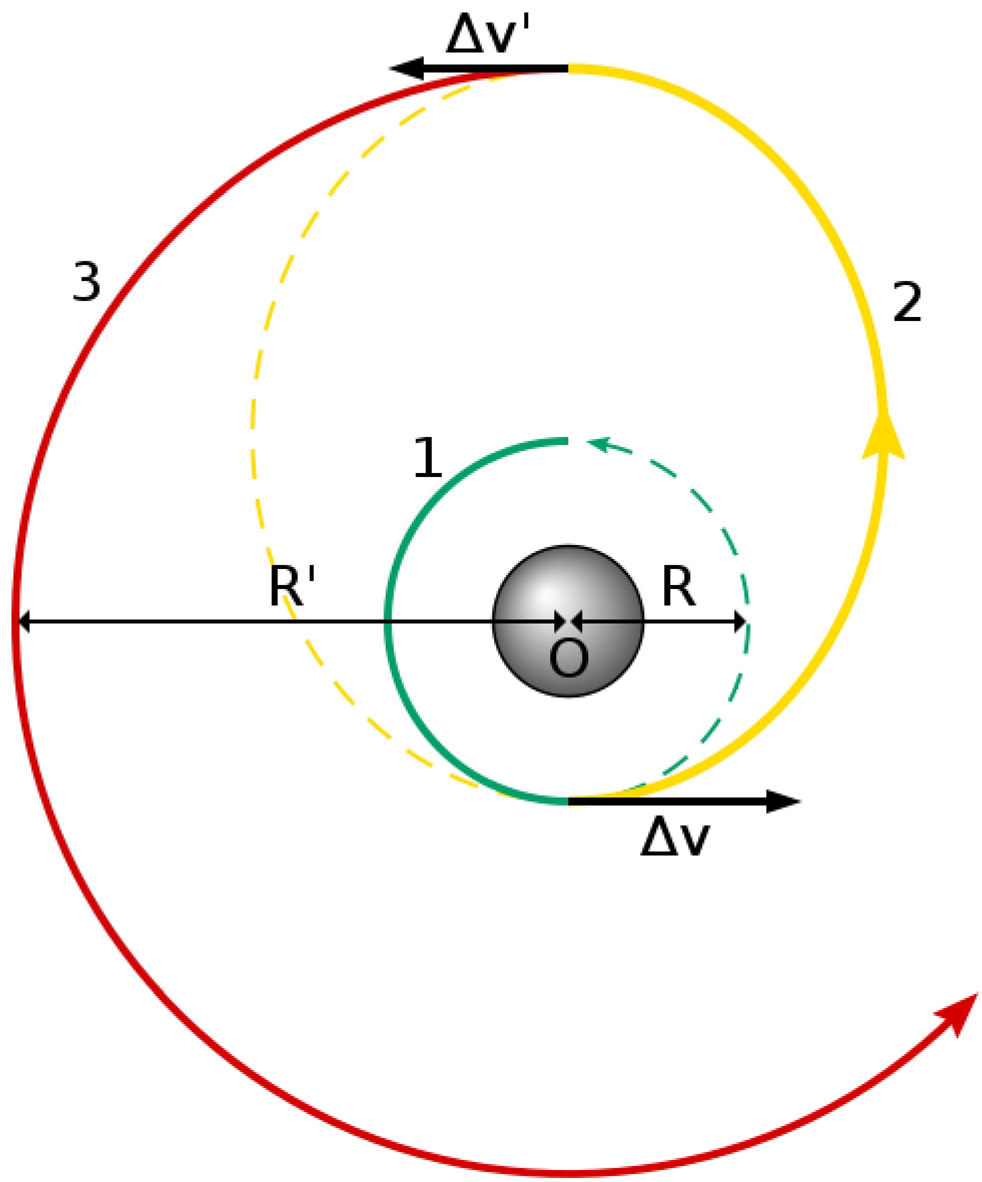
In the 17th century, Johannes Kepler discovered that the orbits along which the planets travel around the Sun are ellipses with the Sun at one focus, in his first law of planetary motion. Later, Isaac Newton explained this as a corollary of his law of universal gravitation.

Chapter- 6

Hohmann Transfer Orbit

In orbital mechanics, the **Hohmann transfer orbit** is an orbital maneuver using two engine impulses which, under standard assumptions, move a spacecraft between two coplanar circular orbits. This maneuver was named after Walter Hohmann, the German scientist who published a description of it in 1925. Hohmann was influenced in part by the German science fiction author Kurd Laßwitz and his 1897 book *Two Planets*.

Soviet literature also refer to this aspect of celestial mechanics; for example, *Pionery Raketnoi Tekhniki*.



Hohmann Transfer Orbit

Explanation

The diagram shows a Hohmann transfer orbit to bring a spacecraft from a lower circular orbit into a higher one. It is one half of an elliptic orbit that touches both the lower circular orbit that one wishes to leave (labeled *1* on diagram) and the higher circular orbit that one wishes to reach (*3* on diagram). The transfer (*2* on diagram) is initiated by firing the spacecraft's engine in order to accelerate it so that it will follow the elliptical orbit; this adds energy to the spacecraft's orbit. When the spacecraft has reached its destination orbit, its orbital speed (and hence its orbital energy) must be increased again in order to change the elliptic orbit to the larger circular one.

Due to the reversibility of orbits, Hohmann transfer orbits also work to bring a spacecraft from a higher orbit into a lower one; in this case, the spacecraft's engine is fired in the opposite direction to its current path, decelerating the spacecraft and causing it to drop into the lower-energy elliptical transfer orbit. The engine is then fired again at the lower distance to decelerate the spacecraft into the lower circular orbit.

The Hohmann transfer orbit is theoretically based on two impulsive (i.e. instantaneous) velocity changes. Extra fuel is required to compensate for the fact that in reality the bursts take time; this is minimized by minimizing the duration of the bursts, i.e., by using high thrust engines. Low thrust engines can perform an approximation of a Hohmann transfer orbit, by creating a gradual enlargement of the initial circular orbit through carefully timed engine firings. This requires a delta-v that is up to 141% greater than the 2 impulse transfer orbit, and takes longer to complete.

Calculation

For a small body orbiting another (such as a satellite orbiting the earth), the total energy of the body is just the sum of its kinetic energy and potential energy, and this total energy also equals half the potential at the average distance *a*, (the semi-major axis):

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} = \frac{-GMm}{2a}$$

Solving this equation for velocity results in the Vis-viva equation,

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a}\right)$$

where:

- v is the speed of an orbiting body
- $\mu = GM$ is the standard gravitational parameter of the primary body
- r is the distance of the orbiting body from the primary focus
- a is the semi-major axis of the body's orbit

Therefore the delta-v required for the Hohmann transfer can be computed as follows (this is only valid for instantaneous burns):

$$\Delta v = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right),$$

$$\Delta v' = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right),$$

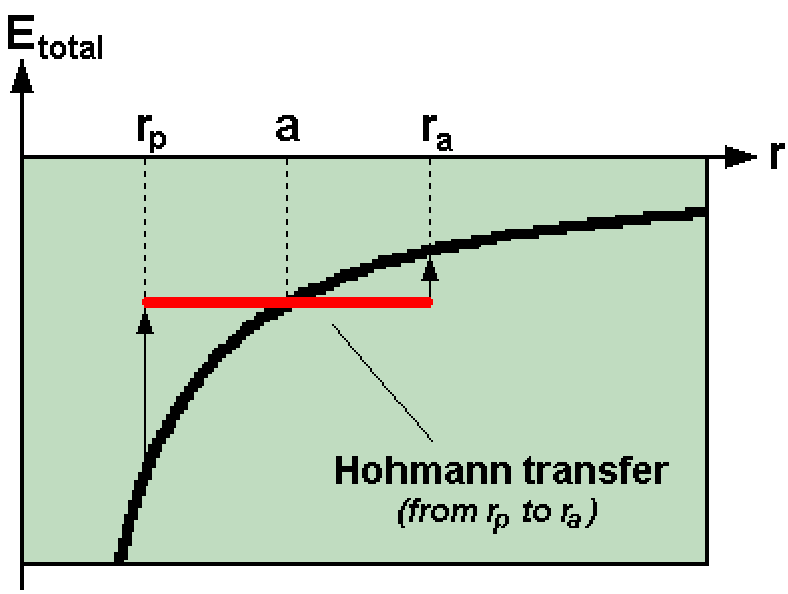
where r_1 and r_2 are, respectively, the radii of the departure and arrival circular orbits; the smaller (greater) of r_1 and r_2 corresponds to the periapsis distance (apoapsis distance) of the Hohmann elliptical transfer orbit.

Whether moving into a higher or lower orbit, by Kepler's third law, the time taken to transfer between the orbits is:

$$t_H = \frac{1}{2}\sqrt{\frac{4\pi^2 a_H^3}{\mu}} = \pi\sqrt{\frac{(r_1+r_2)^3}{8\mu}}$$

(one half of the orbital period for the whole ellipse), where a_H is length of semi-major axis of the Hohmann transfer orbit.

Example



Total energy balance during a Hohmann transfer between two circular orbits with first radius r_p and second radius r_a

For the geostationary transfer orbit we have r_2 = 42,164 km and e.g. r_1 = 6,678 km (altitude 300 km).

In the smaller circular orbit the speed is 7.73 km/s, in the larger one 3.07 km/s. In the elliptical orbit in between the speed varies from 10.15 km/s at the perigee to 1.61 km/s at the apogee.

The delta-v's are 10.15 − 7.73 = 2.42 and 3.07 − 1.61 = 1.46 km/s, together 3.88 km/s.

Compare with the delta-v for an escape orbit: 10.93 − 7.73 = 3.20 km/s. Applying a delta-v at the LEO of only 0.68 km/s more would give the rocket the escape speed, while at the geostationary orbit a delta-v of 1.46 km/s is needed for reaching the sub-escape speed of this circular orbit. This illustrates that at large speeds the same delta-v provides more specific orbital energy, and, as explained in gravity drag, energy increase is maximized if one spends the delta-v as soon as possible, rather than spending some, being decelerated by gravity, and then spending some more (of course, the objective of a Hohmann transfer orbit is different).

Worst case, maximum delta-v

A Hohmann transfer orbit from a given circular orbit to a larger circular orbit, in the case of a single central body, costs the largest delta-v (53.6% of the original orbital speed) if the radius of the target orbit is 15.6 (positive root of $x^3 - 15x^2 - 9x - 1 = 0$) times as large as that of the original orbit. For higher target orbits the delta-v decreases again, and tends to $\sqrt{2} - 1$ times the original orbital speed (41.4%). (The first burst tends to accelerate to the escape speed, which is $\sqrt{2}$ times the circular orbit speed, and the second tends to zero.)

Low-thrust transfer

It can be shown that going from one circular orbit to another by gradually changing the radius costs a delta-v of simply the absolute value of the difference between the two speeds. Thus for the geostationary transfer orbit 7.73 - 3.07 = 4.66 km/s, the same as, in the absence of gravity, the *deceleration* would cost. In fact, *acceleration* is applied to compensate half of the deceleration due to moving outward. Therefore the acceleration due to thrust is equal to the deceleration due to the combined effect of thrust and gravity.

Such a low-thrust maneuver requires *more* delta-v than a 2-burn Hohmann transfer maneuver, requiring more fuel for a given engine design. However, if only low-thrust maneuvers are required on a mission, then continuously firing a very high-efficiency, low-thrust engine with a high effective exhaust velocity might generate this higher delta-v using less total mass than a high-thrust engine using a "more efficient" Hohmann transfer maneuver. This is more efficient for a small satellite because the additional mass of the propellant, especially for electric propulsion systems, is lower than the added mass would be for a separate high-thrust system.

Application to interplanetary travel

When used to move a spacecraft from orbiting one planet to orbiting another, the situation becomes somewhat more complex. For example, consider a spacecraft travelling from the Earth to Mars. At the beginning of its journey, the spacecraft will already have a certain velocity associated with its orbit around Earth – this is velocity that will not need to be found when the spacecraft enters the transfer orbit (around the Sun). At the other end, the spacecraft will need a certain velocity to orbit Mars, which will actually be less than the velocity needed to continue orbiting the Sun in the transfer orbit, let alone attempting to orbit the Sun in a Mars-like orbit. Therefore, the spacecraft will have to *decelerate* and allow Mars' gravity to capture it. Therefore, relatively small amounts of thrust at either end of the trip are all that are needed to arrange the transfer. Note, however, that the alignment of the two planets in their orbits is crucial – the destination planet and the spacecraft must arrive at the same point in their respective orbits around the Sun at the same time.

Interplanetary Transport Network

In 1997, a set of orbits known as the Interplanetary Transport Network was published, providing even lower-energy (though much slower) paths between different orbits than Hohmann transfer orbit.

Chapter- 7

Geosynchronous Orbit

A **geosynchronous orbit** (short **GSO**) is an orbit around the Earth with an orbital period that matches the Earth's sidereal rotation period. The synchronization of rotation and orbital period means that for an observer on the surface of the Earth, the satellite appears to constantly hover over the same meridian (north-south line) on the surface, moving in a slow oscillation alternately north and south with a period of one day, so it returns to exactly the same place in the sky at exactly the same time each day

However, the term is often used to refer to the special case of a geosynchronous orbit that is circular (or nearly circular) and at zero (or nearly zero) inclination, that is, directly above the equator. This is technically called a geostationary orbit. A satellite in a geostationary orbit appears stationary, always at the same point in the sky, to ground observers. Communications satellites are often given geostationary orbits, so that the satellite antennas that communicate with them don't have to move, but can be pointed permanently at the fixed location in the sky where the satellite appears.

A **semisynchronous orbit** has an orbital period of 0.5 sidereal days, i.e., 11 h 58 min. Relative to the Earth's surface it has twice this period, and hence appears to go around the Earth once every day. Examples include the Molniya orbit and the orbits of the satellites in the Global Positioning System.

Orbital characteristics

All Earth geosynchronous orbits have a semi-major axis of 42,164 km (26,199 mi). In fact, orbits with the same period share the same semi-major axis:

$$a = \sqrt[3]{\mu \left(\frac{P}{2\pi}\right)^2}$$

where a=semi-major axis, P = orbital period, μ = geocentric gravitational constant ≈ $5.165861726 \times 10^{12}$ km^3/hr^2 .

In the special case of a geostationary orbit, the ground track of a satellite is a single point on the equator. In the general case of a geosynchronous orbit with a non-zero inclination or eccentricity, the ground track is a more or less distorted figure-eight, returning to the same places once per solar day.

Geostationary orbit

A circular geosynchronous orbit in the plane of the Earth's equator has a radius of approximately 42,164 km (26,199 mi) from the center of the Earth. A satellite in such an orbit is at an altitude of approximately 35,786 km (22,236 mi) above mean sea level. It maintains the same position relative to the Earth's surface. If one could see a satellite in geostationary orbit, it would appear to hover at the same point in the sky, i.e., not exhibit diurnal motion, while the Sun, Moon, and stars would traverse the heavens behind it. This is sometimes called a Clarke orbit. Such orbits are useful for telecommunications satellites.

A perfect stable geostationary orbit is an ideal that can only be approximated. In practice the satellite drifts out of this orbit (because of perturbations such as the solar wind, radiation pressure, variations in the Earth's gravitational field, and the gravitational effect of the Moon and Sun), and thrusters are used to maintain the orbit in a process known as station-keeping.

Other synchronous orbits

Synchronous orbits exist around all moons, planets, stars and black holes — unless they rotate so slowly that the orbit would be outside their Hill sphere or so fast that such an orbit would be inside the body. Most inner moons of planets have synchronous rotation, so their synchronous orbits are, in practice, limited to their leading and trailing (L_4 and L_5) Lagrange points, as well as the L_1 and L_2 Lagrange points, assuming they don't fall within the body of the moon. Objects with chaotic rotations (such as Hyperion) are also problematic, as their synchronous orbits keep changing unpredictably.

Other geosynchronous orbits

Elliptical orbits can be and are designed for communications satellites that keep the satellite within view of its assigned ground stations or receivers. A satellite in an elliptical geosynchronous orbit appears to oscillate in the sky from the viewpoint of a ground station, tracing an analemma in the sky. Satellites in highly elliptical orbits must be tracked by steerable ground stations.

The Infrared Space Observatory was in a highly-elliptical geosynchronous orbit with apogee 70,600 km and perigee 1,000 km. It was controlled by two ground stations.

Theoretically an *active geosynchronous* orbit can be maintained if forces other than gravity are also used to maintain the orbit, such as a solar sail. Such a statite can be geosynchronous in an orbit different (higher, lower, more or less elliptical, or some other path) from the conic section orbit formed by a gravitational body.

Surveillance satellites use active geosynchronous orbits to maintain position and track above a fixed point on the Earth's surface. They are directed by controllers on the ground.

A further form of geosynchronous orbit is proposed for the theoretical space elevator, in which one end of the structure is tethered to the ground, maintaining a longer orbital period than by gravity alone if under tension.

The acronym GEO can mean geosynchronous Earth orbit, geostationary Earth orbit or geostationary orbit.

The following orbits are special orbits that are also used to categorize orbits:

- Geostationary orbit (GEO): zero inclination geosynchronous orbit
- Supersynchronous orbit - a disposal / storage orbit above GSO/GEO. Satellites drift in a westerly direction.
- Subsynchronous orbit - a drift orbit close to but below GSO/GEO. Used for satellites undergoing station changes in an eastern direction.
- Graveyard orbit - a supersynchronous orbit where spacecraft are intentionally placed at the end of their operational life.

History

Author Arthur C. Clarke is credited with proposing the notion of using a geostationary orbit for communications satellites. The orbit is also known as the **Clarke Orbit**. Together, the collection of artificial satellites in these orbits is known as the **Clarke Belt**.

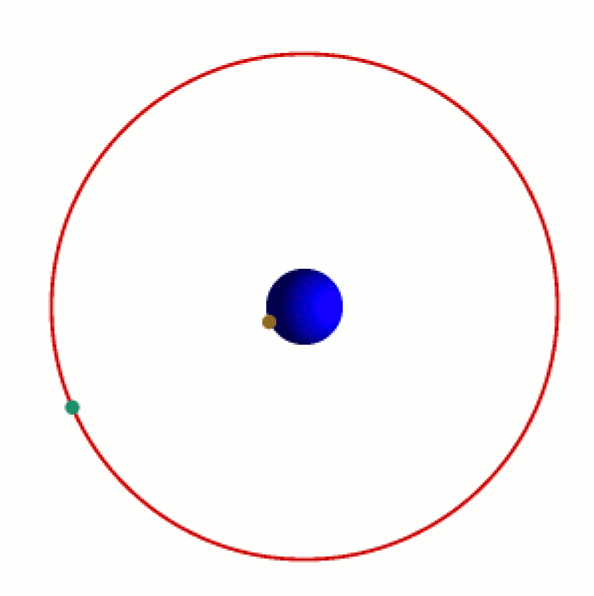
The first communications satellite placed in a geosynchronous orbit was Syncom 2, launched in 1963. However, it was in an inclined orbit, still requiring the use of moving antennas. The first communications satellite placed in a geostationary orbit was Syncom 3. Geostationary orbits have been in common use ever since, in particular for satellite television.

Geostationary satellites also carry international telephone traffic but they are being replaced by fiber optic cables in heavily populated areas and along the coasts of less developed regions, because of the greater bandwidth available and lower latency, due to the inherent disconcerting delay in communicating via a satellite in such a high orbit. It takes electromagnetic waves about a quarter of a second to travel from one end to the other end of the link. Thus, two parties talking via satellite are subject to about a half second delay in a round-trip message/response sequence.

Although many populated land locations on the planet now have terrestrial communications facilities (microwave, fiber-optic), even undersea, with more than sufficient capacity, satellite telephony and Internet access is still the only service available for many places in Africa, Latin America, and Asia, as well as isolated locations that have no terrestrial facilities, such as Canada's Arctic islands, Antarctica, the far reaches of Alaska and Greenland, and ships at sea.

Chapter- 8

Geostationary Orbit



Geostationary orbit

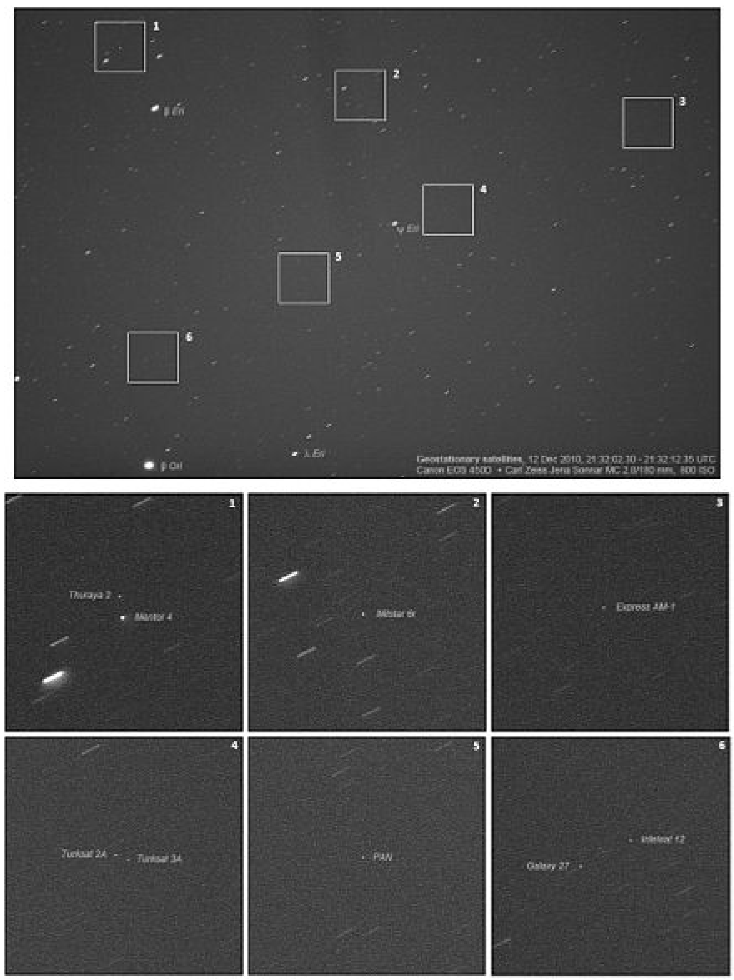
A **geostationary orbit** (or **Geostationary Earth Orbit - GEO**) is a geosynchronous orbit directly above the Earth's equator (0° latitude), with a period equal to the Earth's rotational period and an orbital eccentricity of approximately zero. An object in a geostationary orbit appears motionless, at a fixed position in the sky, to ground observers. Communications satellites and weather satellites are often given geostationary orbits, so that the satellite antennas that communicate with them do not have to move to track them, but can be pointed permanently at the position in the sky where they stay. Due to the constant 0° latitude and circularity of geostationary orbits, satellites in GEO differ in location by longitude only.

The notion of a geosynchronous satellite for communication purposes was first published in 1928 (but not widely so) by Herman Potočnik. The idea of a geostationary orbit was first disseminated on a wide scale in a 1945 paper entitled "Extra-Terrestrial Relays — Can Rocket Stations Give Worldwide Radio Coverage?" by British science fiction writer

Arthur C. Clarke, published in *Wireless World* magazine. The orbit, which Clarke first described as useful for broadcast and relay communications satellites, is sometimes called the **Clarke Orbit**. Similarly, the **Clarke Belt** is the part of space about 36,000 km (22,000 mi) above sea level, in the plane of the equator, where near-geostationary orbits may be implemented. The Clarke Orbit is about 265,000 km (165,000 mi) long.

Geostationary orbits are useful because they cause a satellite to appear stationary with respect to a fixed point on the rotating Earth, allowing a fixed antenna to maintain a link with the satellite. The satellite orbits in the direction of the Earth's rotation, at an altitude of 35,786 km (22,236 mi) above ground, producing an orbital period equal to the Earth's period of rotation, known as the sidereal day.

Introduction



A 5 x 6 degrees view of a part of the geostationary belt, showing several geostationary satellites. Those with inclination 0 degrees form a diagonal belt across the image: a few objects with small inclinations to the equator are visible above this line. Note how the satellites are pinpoint, while stars have created small trails due to the Earth's rotation.

A geostationary orbit can only be achieved at an altitude very close to 35,786 km (22,236 mi), and directly above the equator. This equates to an orbital velocity of 3.07 km/s (1.91 mi/s) or a period of 1,436 minutes, which equates to almost exactly one sidereal day or 23.934461223 hours. This makes sense considering that the satellite must be locked to the Earth's rotational period in order to have a stationary footprint on the ground. In practice, this means that all geostationary satellites have to exist on this ring, which poses problems for satellites that will be decommissioned at the end of their service lives (e.g., when they run out of thruster fuel). Such satellites will either continue to be used in inclined orbits (where the orbital track appears to follow a figure-eight loop centered on the equator), or else be elevated to a "graveyard" disposal orbit.

A geostationary transfer orbit is used to move a satellite from low Earth orbit (LEO) into a geostationary orbit.

A worldwide network of operational geostationary meteorological satellites is used to provide visible and infrared images of Earth's surface and atmosphere. These satellite systems include:

- the United States GOES
- Meteosat, launched by the European Space Agency and operated by the European Weather Satellite Organization, EUMETSAT
- the Japanese MTSAT
- India's INSAT series

Most commercial communications satellites, broadcast satellites and SBAS satellites operate in geostationary orbits. (Russian television satellites have used elliptical Molniya and Tundra orbits due to the high latitudes of the receiving audience.) The first satellite placed into a geostationary orbit was the Syncom-3, launched by a Delta-D rocket in 1964.

A statite, a hypothetical satellite that uses a solar sail to modify its orbit, could theoretically hold itself in a "geostationary" orbit with different altitude and/or inclination from the "traditional" equatorial geostationary orbit.

Derivation of geostationary altitude

In any circular orbit, the centripetal force required to maintain the orbit is provided by the gravitational force on the satellite. To calculate the geostationary orbit altitude, one begins with this equivalence, and uses the fact that the orbital period is one sidereal day.

$$\mathbf{F}_\mathrm{c} = \mathbf{F}_\mathrm{g}$$

By Newton's second law of motion, we can replace the forces **F** with the mass *m* of the object multiplied by the acceleration felt by the object due to that force:

$$m\mathbf{a}_\mathrm{c} = m\mathbf{g}$$

We note that the mass of the satellite *m* appears on both sides — geostationary orbit is independent of the mass of the satellite. So calculating the altitude simplifies into calculating the point where the magnitudes of the centripetal acceleration required for orbital motion and the gravitational acceleration provided by Earth's gravity are equal.

The centripetal acceleration's magnitude is:

$$|\mathbf{a}_c| = \omega^2 r$$

where ω is the angular speed, and r is the orbital radius as measured from the Earth's center of mass.

The magnitude of the gravitational acceleration is:

$$|\mathbf{g}| = \frac{GM}{r^2}$$

where M is the mass of Earth, 5.9736×10^{24} kg, and G is the gravitational constant, $6.67428 \pm 0.00067 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.

Equating the two accelerations gives:

$$r^3 = \frac{GM}{\omega^2} \to r = \sqrt[3]{\frac{GM}{\omega^2}}$$

The product GM is known with much greater accuracy than either factor; it is known as the geocentric gravitational constant $\mu = 398{,}600.4418 \pm 0.0008$ km^3 s^{-2}:

$$r = \sqrt[3]{\frac{\mu}{\omega^2}}$$

The angular speed ω is found by dividing the angle travelled in one revolution (360° = 2π rad) by the orbital period (the time it takes to make one full revolution: one sidereal day, or 86,164.09054 seconds). This gives:

$$\omega \approx \frac{2\pi \text{ rad}}{86\,164 \text{ s}} \approx 7.2921 \times 10^{-5} \text{ rad/s}$$

The resulting orbital radius is 42,164 kilometres (26,199 mi). Subtracting the Earth's equatorial radius, 6,378 kilometres (3,963 mi), gives the altitude of **35,786 kilometres (22,236 mi)**.

Orbital speed (how fast the satellite is moving through space) is calculated by multiplying the angular speed by the orbital radius:

$$v = \omega r \approx 3.0746 \text{ km/s} \approx 11068 \text{ km/h} \approx 6877.8 \text{ mph}.$$

Now, by the same formula, let us find the geostationary orbit of an object in relation to Mars. The geocentric gravitational constant GM (which is μ) for Mars has the value of 42,828 $km^3 s^{-2}$, and the known rotational period (T) of Mars is 88,642.66 seconds. Since $\omega = 2\pi/T$, using the formula above, the value of ω is found to be approx 7.088218×10^{-5} s^{-1}. Thus, $r^3 = 8.5243 \times 10^{12}$ km^3, whose cube root of is 20,427 km; subtracting the equatorial radius of Mars (3396.2 km) we have 17,031 km.

Practical limitations

A combination of lunar gravity, solar gravity, and the flattening of the Earth at its poles is causing a precession motion of the orbit plane of any geostationary object with a period of about 53 years and an initial inclination gradient of about 0.85 degrees per year, achieving a maximum inclination of 15 degrees after 26.5 years. To correct for this orbital perturbation, regular orbital stationkeeping maneuvers are necessary, amounting to a delta-v of approximately 50 m/s per year.

The second effect to be taken into account is the longitude drift, caused by the asymmetry of the earth - the equator is slightly elliptical. There are two stable (at 75.3°E, and at 104.7°W) and two unstable (at 165.3°E, and at 14.7°W) equilibrium points. Any geostationary object placed between the equilibrium points would (without any action) be slowly accelerated towards the stable equilibrium position, causing a periodic longitude variation. The correction of this effect requires orbit control maneuvers with a maximum delta-v of about 2 m/s per year, depending on the desired longitude.

In the absence of servicing missions from the Earth, the consumption of thruster propellant for station-keeping places a limitation on the lifetime of the satellite.

Communications

Satellites in geostationary orbits are far enough away from Earth that communication latency becomes very high — about a quarter of a second for a one-way trip from a ground based transmitter to a geostationary satellite and then on to a ground based receiver, and close to half a second for round-trip communication between two earth stations.

For example, for ground stations at latitudes of $\varphi = \pm 45°$ on the same meridian as the satellite, the one-way delay can be computed by using the cosine rule, given the above derived geostationary orbital radius r, the Earth's radius R and the speed of light c, as

$$2\frac{\sqrt{R^2 + r^2 - 2Rr\cos\varphi}}{c} \approx 253\,\text{ms}$$

This presents problems for latency-sensitive applications such as voice communication or online gaming.

Orbit allocation

Satellites in geostationary orbit must all occupy a single ring above the equator. The requirement to space these satellites apart to avoid harmful radio-frequency interference during operations means that there are a limited number of orbital "slots" available, thus only a limited number of satellites can be operated in geostationary orbit. This has led to conflict between different countries wishing access to the same orbital slots (countries at the same longitude but differing latitudes) and radio frequencies. These disputes are addressed through the International Telecommunication Union's allocation mechanism. Countries located at the Earth's equator have also asserted their legal claim to control the use of space above their territory.

Geostationary transfer orbit

A **geosynchronous transfer orbit** or **geostationary transfer orbit** (**GTO**) is an intermediate orbit used to reach geosynchronous or geostationary orbit. It is a highly elliptical Earth orbit with apogee at about 35,700 km, geostationary (GEO) altitude, and an argument of perigee such that apogee occurs on or near the equator. Perigee can be anywhere above the atmosphere, but is usually limited to only a few hundred km to reduce launcher delta-v (ΔV) requirements and to limit the orbital lifetime of the spent booster.

The inclination of a GTO is the angle between the orbit plane and the Earth's equatorial plane. It is determined by the latitude of the launch site and the launch azimuth (direction). The inclination and eccentricity must both be reduced to zero to obtain a geostationary orbit. If only the eccentricity of the orbit is reduced to zero, the result is a geosynchronous orbit. Because the ΔV required for a plane change is proportional to the instantaneous velocity, the inclination and eccentricity are usually changed together in a single maneuver at apogee where velocity is lowest. The required ΔV for an inclination change at either the ascending or descending node of the orbit is calculated as follows:

$$\Delta V = 2V \sin \frac{\Delta i}{2}$$

For a typical GTO with a semimajor axis of 24,582 km, perigee velocity is 9.88 km/s and apogee velocity is 1.64 km/s, clearly making the inclination change far less costly at apogee. In practice, the inclination change is combined with the orbital circularization (or "apogee kick") burn, so additional ΔV is required.

Even at apogee, the fuel needed to reduce inclination to zero can be significant, giving equatorial launch sites a substantial advantage over those at higher latitudes. Kennedy Space Center is at 28.5 degrees north, the Guiana Space Centre, the Ariane launch facility, is at 5 degrees north latitude and Sea Launch launches from a floating platform directly on the equator in the Pacific Ocean. All have a significant advantage over Russia's high latitude launch sites.

Expendable launchers generally reach GTO directly, but a spacecraft already in a low Earth orbit (LEO) can enter GTO by firing a rocket along its orbital direction to increase its velocity. This is done when a geostationary spacecraft is launched from the space shuttle; a "perigee kick motor" attached to the spacecraft ignites after the shuttle has released it and withdrawn to a safe distance.

Although some launchers can take their payloads all the way to geostationary orbit, most end their missions by releasing their payloads into GTO. The spacecraft and its operator are then responsible for the maneuver into the final geostationary orbit. The five hour coast to first apogee can be longer than the launcher's battery lifetime, and the maneuver is sometimes performed at a later apogee. The solar power available on the spacecraft supports the mission after launcher separation. Also, many launchers now carry several satellites in each launch to reduce overall costs, and this practice simplifies the mission when the payloads may be destined for different orbital positions.

Because of this practice launcher capacity is usually quoted as separated spacecraft mass to GTO, and this number will be higher than the payload that could be delivered directly into GEO.

For example, the capacity (separated spacecraft mass) of the Delta IV Heavy:

- GTO 12,757 kg (185 km x 35,786 km at 27.0 deg inclination), theoretically more than any other currently available launch vehicle (has not flown with such a payload yet)
- GEO 6,276 kg

If the manoeuver from GTO to GEO is to be performed with a single impulse, as with a single solid rocket motor, apogee must occur at an equatorial crossing. This implies an argument of perigee of either 0 or 180 degrees. Because the argument of perigee is slowly perturbed by the oblateness of the Earth, it is usually biased at launch so that it reaches the desired value at the appropriate time. (If the GTO inclination is zero, as with Sea Launch, then this does not apply.)

The preceding discussion has primarily focused on the case where the transfer between LEO and GEO is done with a single intermediate transfer orbit. More complicated trajectories are sometimes used. For example, the Proton M uses a set of four intermediate orbits, requiring five rocket firings, to place a satellite into GEO from the high-inclination site of Baikonur Cosmodrome, in Kazakhstan.

Transfer stage

The spent upper stage is usually left in GTO (some are occasionally left in GEO, like the Proton Block DM). An empty rocket stage generally has a low ballistic coefficient, so atmospheric drag at the low GTO perigee will quickly lower its apogee, removing it as a collision threat to satellites at GEO. The orbit will continue to decay and eventually the stage will re-enter the atmosphere. Most such stages decay within a few years.