Aircraft Flight Dynamics

Todd Hopkins

First Edition, 2012

ISBN 978-81-323-2826-1

Published by:
Orange Apple
4735/22 Prakashdeep Bldg,
Ansari Road, Darya Ganj,
Delhi - 110002
Email: info@wtbooks.com

Table of Contents

Chapter 1 - Flight Dynamics

Chapter 2 - Axes Conventions

Chapter 3 - Aerodynamics

Chapter 4 - Propeller

Chapter 5 - Phugoid and Dutch Roll

Chapter 6 - Mach Number

Chapter 7 - Reynolds Number

Chapter 8 - Knudsen Number

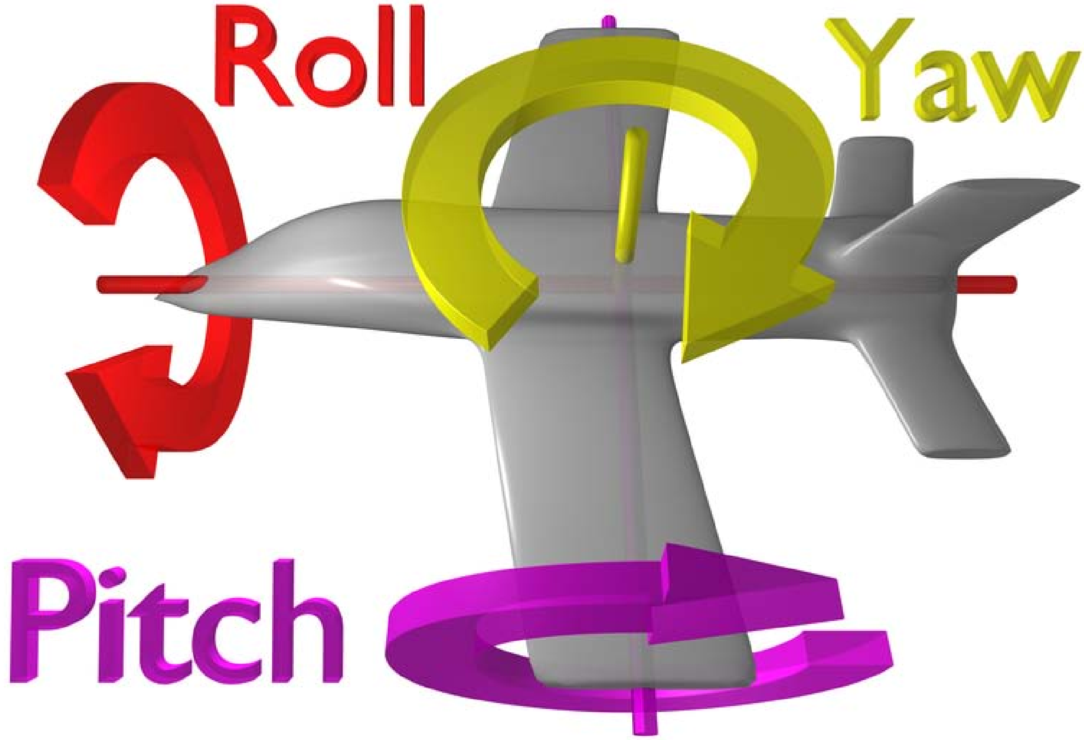
Chapter 9 - Incompressible Flow

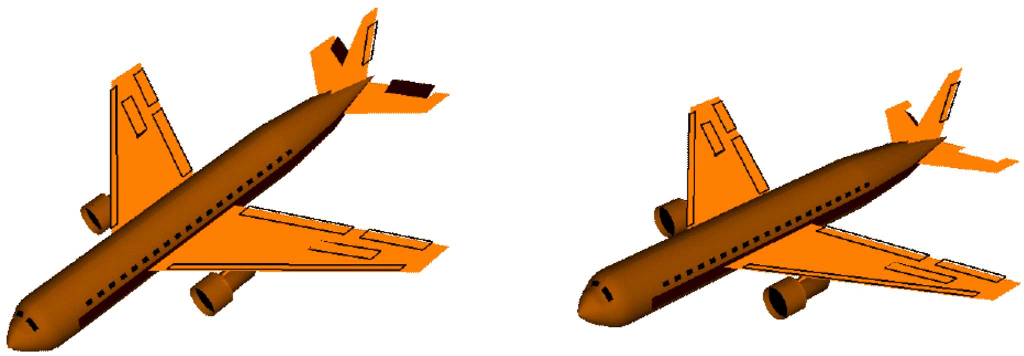
Chapter 10 - Compressible Flow

Chapter 11 - Crosswind Landing

Chapter- 1

Flight Dynamics





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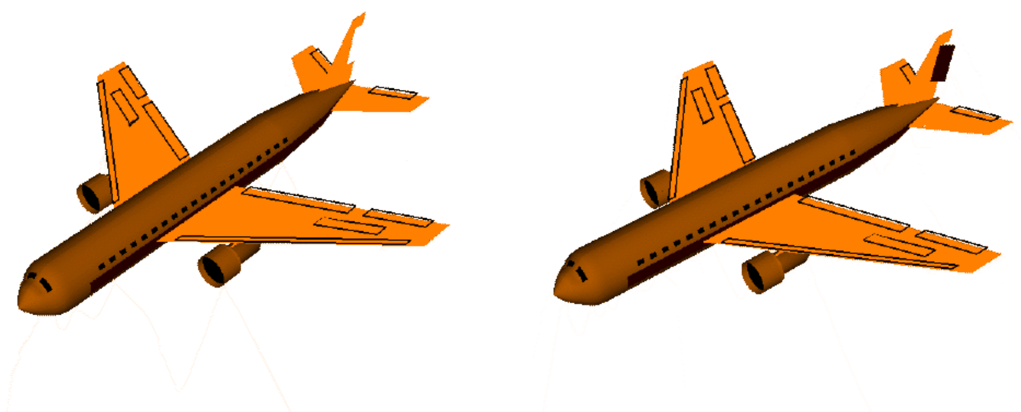

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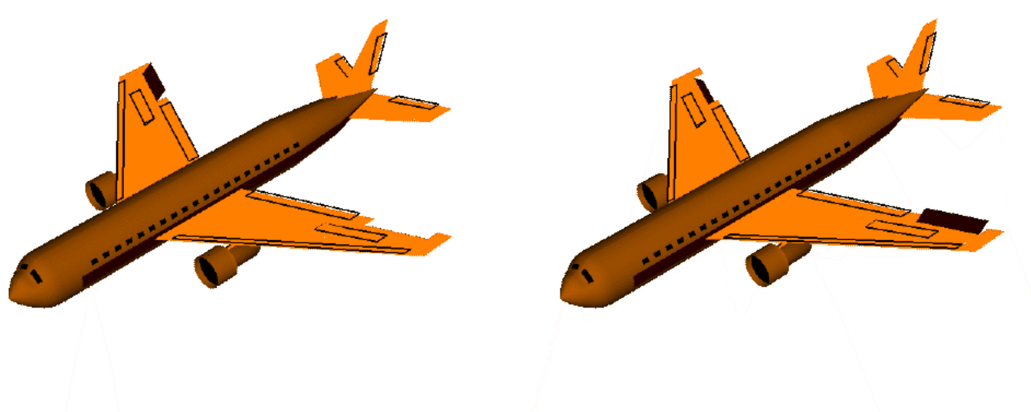


Roll

Flight dynamics is the science of air vehicle orientation and control in three dimensions. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as *pitch*, *roll* and *yaw* (quite different from their use as Tait-Bryan angles).

Aerospace engineers develop control systems for a vehicle's orientation (attitude) about its center of mass. The control systems include actuators, which exert forces in various directions, and generate rotational forces or moments about the aerodynamic center of the aircraft, and thus rotate the aircraft in pitch, roll, or yaw. For example, a pitching moment is a vertical force applied at a distance forward or aft from the aerodynamic center of the aircraft, causing the aircraft to pitch up or down.

Roll, pitch and yaw refer to rotations about the respective axes starting from a defined equilibrium state. The equilibrium roll angle is known as wings level or zero bank angle, equivalent to a level heeling angle on a ship. Yaw is known as "heading". The equilibrium pitch angle in submarine and airship parlance is known as "trim", but in aircraft, this usually refers to angle of attack, rather than orientation. However, common usage ignores this distinction between equilibrium and dynamic cases.

The most common aeronautical convention defines the roll as acting about the longitudinal axis, positive with the starboard (right) wing down. The yaw is about the vertical body axis, positive with the nose to starboard. Pitch is about an axis perpendicular to the longitudinal plane of symmetry, positive nose up.

A fixed-wing aircraft increases or decreases the lift generated by the wings when it pitches nose up or down by increasing or decreasing the angle of attack (AOA). The roll angle is also known as bank angle on a fixed wing aircraft, which usually "banks" to change the horizontal direction of flight. An aircraft is usually streamlined from nose to tail to reduce drag making it typically advantageous to keep the sideslip angle near zero, though there are instances when an aircraft may be deliberately "sideslipped" for example a slip in a fixed wing aircraft.

Introduction

Basic coordinate systems

The position (and hence motion) of an aircraft is generally defined relative to one of 3 sets of co-ordinate systems:

- Wind axes
    - X axis - positive in the direction of the oncoming air (relative wind)
    - Y axis - positive to right of X axis, perpendicular to X axis
    - Z axis - positive downwards, perpendicular to X-Y plane

- Inertial axes (or body axes) - based about aircraft CG
    - X axis - positive forward, through nose of aircraft

  - Y axis - positive to right of X axis, perpendicular to X axis
  - Z axis - positive downwards, perpendicular to X-Y plane

- Earth Axes
  - X axis - positive in the direction of north
  - Y axis - positive in the direction of east (perpendicular to X axis)
  - Z axis - positive towards the center of Earth (perpendicular to X-Y plane)

For flight dynamics applications the earth axes are generally of minimal use, and hence will be ignored. The motions relevant to dynamic stability are usually too short in duration for the motion of the Earth itself to be considered relevant for aircraft.

In flight dynamics, pitch, roll and yaw angles measure both the absolute attitude angles (relative to the horizon/North) and *changes* in attitude angles, relative to the equilibrium orientation of the vehicle. These are defined as:

- Pitch - angle of X body axis (nose) relative to horizon. Also a positive (nose up) rotation about Y body axis
- Roll - angle of Y body axis (wing) relative to horizon. Also a positive (right wing down) rotation about X body axis
- Yaw - angle of X body axis (nose) relative to North. Also a positive (nose right) rotation about Z body axis

In analyzing the dynamics, we are concerned both with rotation and translation of this axis set with respect to a fixed inertial frame. For all practical purposes a local Earth axis set is used, this has X and Y axis in the local horizontal plane, usually with the x-axis coinciding with the projection of the velocity vector at the start of the motion, on to this plane. The z axis is vertical, pointing generally towards the Earth's center, completing an orthogonal set.

In general, the body axes are not aligned with the Earth axes. The body orientation may be defined by three Euler angles, the Tait-Bryan rotations, a quaternion, or a direction cosine matrix (rotation matrix). A rotation matrix is particularly convenient for converting velocity, force, angular velocity, and torque vectors between body and Earth coordinate frames.

Body axes tend to be used with missile and rocket configurations. Aircraft stability uses wind axes in which the x-axis points along the velocity vector. For straight and level flight this is found from body axes by rotating nose down through the angle of attack.

Stability deals with small perturbations in angular displacements about the orientation at the start of the motion. This consists of two components; rotation about each axis, and angular displacements due change in orientation of each axis. The latter term is of second order for the purpose of stability analysis, and is ignored.

Design cases

In analyzing the stability of an aircraft, it is usual to consider perturbations about a nominal equilibrium position. So the analysis would be applied, for example, assuming:

Straight and level flight
Turn at constant speed
Approach and landing
Takeoff

The speed, height and trim angle of attack are different for each flight condition, in addition, the aircraft will be configured differently, e.g. at low speed flaps may be deployed and the undercarriage may be down.

Except for asymmetric designs (or symmetric designs at significant sideslip), the longitudinal equations of motion (involving pitch and lift forces) may be treated independently of the lateral motion (involving roll and yaw).

The following considers perturbations about a nominal straight and level flight path.

To keep the analysis (relatively) simple, the control surfaces are assumed fixed throughout the motion, this is stick-fixed stability. Stick-free analysis requires the further complication of taking the motion of the control surfaces into account.

Furthermore, the flight is assumed to take place in still air, and the aircraft is treated as a rigid body.

Aerodynamic and propulsive forces

Aerodynamic forces

Components of the aerodynamic force

The expression to calculate the aerodynamic force is:

$$\mathbf{F}_A = \int_\Sigma (-\Delta p \mathbf{n} + \mathbf{f})\, d\sigma$$

where:

$\Delta p \equiv$ Difference between static pressure and free current pressure
$\mathbf{n} \equiv$ outer normal vector of the element of area
$\mathbf{f} \equiv$ tangential stress vector practised by the air on the body
$\Sigma \equiv$ adequate reference surface

projected on wind axes we obtain:

$$\mathbf{F}_A = -(\mathbf{i}_w D + \mathbf{j}_w Q + \mathbf{k}_w L)$$

where:

D ≡Drag
Q ≡Lateral force
L ≡Lift

Aerodynamic coefficients

Dynamic pressure of the free current $\equiv q = \frac{1}{2}\,\rho V^2$

Proper reference surface (wing surface, in case of planes) $\equiv S$

Pressure coefficient $\equiv C_p = \dfrac{p - p_\infty}{q}$

Friction coefficient $\equiv C_f = \dfrac{f}{q}$

Drag coefficient $\equiv C_d = \dfrac{D}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{i_w} + C_f \mathbf{t} \bullet \mathbf{i_w}]\, d\sigma$

Lateral force coefficient $\equiv C_Q = \dfrac{Q}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{j_w} + C_f \mathbf{t} \bullet \mathbf{j_w}]\, d\sigma$

Lift coefficient $\equiv C_L = \dfrac{L}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{k_w} + C_f \mathbf{t} \bullet \mathbf{k_w}]\, d\sigma$

It is necessary to know C_p and C_f in every point on the considered surface.

Dimensionless parameters and aerodynamic regimes

In absence of thermal effects, there are three remarkable dimensionless numbers:

- Compressibility of the flow:

  Mach number $\equiv M = \dfrac{V}{a}$

- Viscosity of the flow:

Reynolds number $\equiv Re = \frac{\rho V l}{\mu}$

- Rarefaction of the flow:

Knudsen number $\equiv Kn = \frac{\lambda}{l}$

where:

$a = \sqrt{kR\theta}$ $\equiv$speed of sound
R $\equiv$gas constant by mass unity
θ $\equiv$absolute temperature
$\lambda = \frac{\mu}{\rho}\sqrt{\frac{\pi}{2R\theta}} = \frac{M}{Re}\sqrt{\frac{k\pi}{2}} \equiv$ mean free path

According to λ there are three possible rarefaction grades and their corresponding motions are called:

- Continuum current (negligible rarefaction): $\frac{M}{Re} \ll 1$
- Transition current (moderate rarefaction): $\frac{M}{Re} \approx 1$
- Free molecular current (high rarefaction): $\frac{M}{Re} \gg 1$

The motion of a body through a flow is considered, in flight dynamics, as continuum current. In the outer layer of the space that surrounds the body viscosity will be negligible. However viscosity effects will have to be considered when analysing the flow in the nearness of the boundary layer.

Depending on the compressibility of the flow, different kinds of currents can be considered:

- Incompressible subsonic current: $0 < M < 0.5$
- Compressible subsonic current: $0.5 < M < 0.8$
- Transonic current: $0.8 < M < 1.2$
- Supersonic current: $1.2 < M < 5$
- Hypersonic current: $5 < M$

Drag coefficient equation and aerodynamic efficiency

If the geometry of the body is fixed and in case of symmetric flight (β=0 and Q=0), pressure and friction coefficients are functions depending on:

$C_p = C_p(\alpha, M, Re, P)$
$C_f = C_f(\alpha, M, Re, P)$

where:

$\alpha \equiv$ angle of attack
$P \equiv$ considered point of the surface

Under these conditions, drag and lift coefficient are functions depending exclusively on the angle of attack of the body and Mach and Reynolds numbers. Aerodynamic efficiency, defined as the relation between lift and drag coefficients, will depend on those parameters as well.

$$\begin{cases} C_D = C_D(\alpha, M, Re) \\ C_L = C_L(\alpha, M, Re) \\ E = E(\alpha, M, Re) = \dfrac{C_L}{C_D} \end{cases}$$

It is also possible to get the dependency of the drag coefficient respect to the lift coefficient. This relation is konwn as the drag coefficient equation:

$C_D = C_D(C_L, M, Re) \equiv$ drag coefficient equation

The aerodynamic efficiency has a maximum value, E_{max}, respect to C_L where the tangent line from the coordinate origin touches the drag coefficient equation plot.

The drag coefficient, C_D, can be decomposed in two ways. First typical decomposition separates pressure and friction effects:

$$C_D = C_{Df} + C_{Dp} \begin{cases} C_{Df} = \dfrac{D}{qS} = -\dfrac{1}{S}\int_\Sigma C_f \mathbf{t} \bullet \mathbf{i_w} \, d\sigma \\ C_{Dp} = \dfrac{D}{qS} = -\dfrac{1}{S}\int_\Sigma (-C_p) \mathbf{n} \bullet \mathbf{i_w} \, d\sigma \end{cases}$$

There´s a second typical decomposition taking into account the definition of the drag coefficient ecuation. This decomposition separates the effect of the lift coefficient in the equation, obtaining two terms C_{D0} and C_{Di}. C_{D0} is known as the parasitic drag coefficient and it is the base draft coefficient at zero lift. C_{Di} is known as the induced drag coefficient and it is produced by the body lift.

$$C_D = C_{D0} + C_{Di} \begin{cases} C_{D0} = (C_D)_{C_L=0} \\ C_{Di} \end{cases}$$

Parabolic and generic drag coefficient

A good attempt for the induced drag coefficient is to assume a parabolic dependency of the lift

$$C_{Di} = kC_L^2 \Rightarrow C_D = C_{D0} + kC_L^2$$

Aerodynamic efficiency is now calculated as:

$$E = \frac{C_L}{C_{D0} + kC_L^2} \Rightarrow \begin{cases} E_{max} = \dfrac{1}{2\sqrt{kC_{D0}}} \\ (C_L)_{Emax} = \sqrt{\dfrac{C_{D0}}{k}} \\ (C_{Di})_{Emax} = C_{D0} \end{cases}$$

If the configuration of the pane is symmetrical respect to the XY plane, minimum drag coefficient equals to the parasitic drag of the plane.

$$C_{Dmin} = (C_D)_{CL=0} = C_{D0}$$

In case the configuration is asymmetrical respect to the XY plane, however, minimum flag difers from the parasitic drag. On these cases, a new approximate parabolic drag equation can be traced leaving the minimum drag value at zero lift value.

$$C_{Dmin} = C_{DM} \neq (C_D)_{CL=0}$$
$$C_D = C_{DM} + k(C_L - C_{LM})^2$$

Dynamic stability and control

Longitudinal modes

It is common practice to derive a fourth order characteristic equation to describe the longitudinal motion, and then factorize it approximately into a high frequency mode and a low frequency mode. This requires a level of algebraic manipulation which most readers will doubtless find tedious, and adds little to the understanding of aircraft dynamics. The approach adopted here is to use our qualitative knowledge of aircraft behavior to simplify the equations from the outset, reaching the same result by a more accessible route.

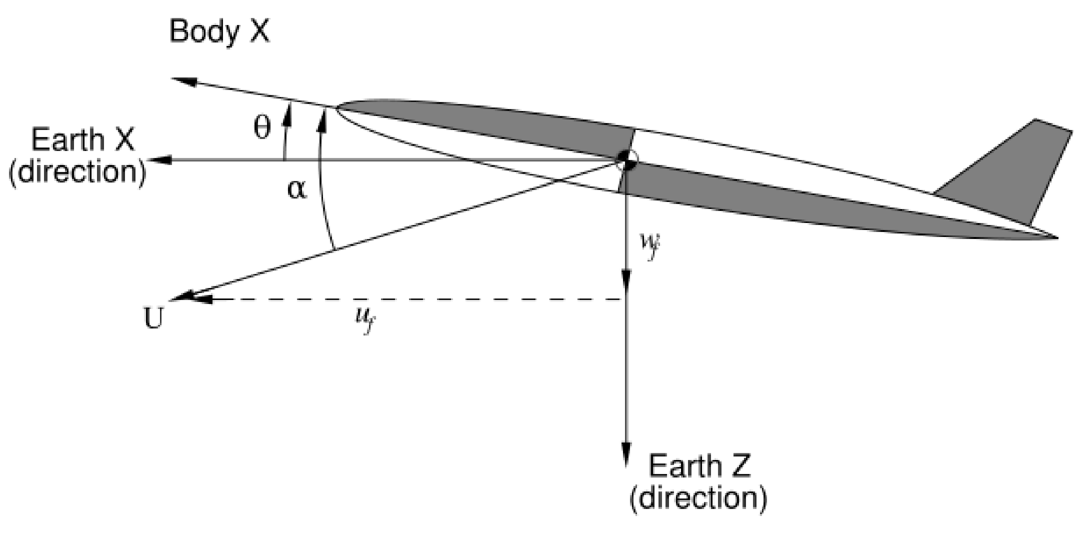
The two longitudinal motions (modes) are called the short period pitch oscillation (SPPO), and the phugoid.

Short-period pitch oscillation

A short input (in control systems terminology an impulse) in pitch (generally via the elevator in a standard configuration fixed wing aircraft) will generally lead to overshoots about the trimmed condition. The transition is characterized by a damped simple harmonic motion about the new trim. There is very little change in the trajectory over the time it takes for the oscillation to damp out.

Generally this oscillation is high frequency (hence short period) and is damped over a period of a few seconds. A real-world example would involve a pilot selecting a new climb attitude, for example 5º nose up from the original attitude. A short, sharp pull back on the control column may be used, and will generally lead to oscillations about the new trim condition. If the oscillations are poorly damped the aircraft will take a long period of time to settle at the new condition, potentially leading to Pilot-induced oscillation. If the short period mode is unstable it will generally be impossible for the pilot to safely control the aircraft for any period of time.

This damped harmonic motion is called the short period pitch oscillation, it arises from the tendency of a stable aircraft to point in the general direction of flight. It is very similar in nature to the weathercock mode of missile or rocket configurations. The motion involves mainly the pitch attitude θ (theta) and incidence α (alpha). The direction of the velocity vector, relative to inertial axes is $\theta - \alpha$. The velocity vector is:



Longitudinal Equations of Motion

$u_f = U\cos(\theta - \alpha)$
$w_f = U\sin(\theta - \alpha)$

where u_f, w_f are the inertial axes components of velocity. According to Newton's Second Law, the accelerations are proportional to the forces, so the forces in inertial axes are:

$$X_f = m\frac{du_f}{dt} = m\frac{dU}{dt}\cos(\theta - \alpha) - mU\frac{d(\theta - \alpha)}{dt}\sin(\theta - \alpha)$$
$$Z_f = m\frac{dw_f}{dt} = m\frac{dU}{dt}\sin(\theta - \alpha) + mU\frac{d(\theta - \alpha)}{dt}\cos(\theta - \alpha)$$

where m is the mass. By the nature of the motion, the speed variation $m\frac{dU}{dt}$ is negligible over the period of the oscillation, so:

$$X_f = -mU\frac{d(\theta - \alpha)}{dt}\sin(\theta - \alpha)$$
$$Z_f = mU\frac{d(\theta - \alpha)}{dt}\cos(\theta - \alpha)$$

But the forces are generated by the pressure distribution on the body, and are referred to the velocity vector. But the velocity (wind) axes set is not an inertial frame so we must resolve the fixed axes forces into wind axes. Also, we are only concerned with the force along the z-axis:

$Z = -Z_f\cos(\theta - \alpha) + X_f\sin(\theta - \alpha)$

Or:

$$Z = -mU\frac{d(\theta - \alpha)}{dt}$$

In words, the wind axes force is equal to the centripetal acceleration.

The moment equation is the time derivative of the angular momentum:

$$M = B\frac{d^2\theta}{dt^2}$$

where M is the pitching moment, and B is the moment of inertia about the pitch axis. Let: $\frac{d\theta}{dt} = q$, the pitch rate. The equations of motion, with all forces and moments referred to wind axes are, therefore:

$$\frac{d\alpha}{dt} = q + \frac{Z}{mU}$$
$$\frac{dq}{dt} = \frac{M}{B}$$

We are only concerned with perturbations in forces and moments, due to perturbations in the states α and q, and their time derivatives. These are characterized by stability derivatives determined from the flight condition. The possible stability derivatives are:

Z_α Lift due to incidence, this is negative because the z-axis is downwards whilst positive incidence causes an upwards force.
Z_q Lift due to pitch rate, arises from the increase in tail incidence, hence is also negative, but small compared with Z_α.
M_α Pitching moment due to incidence - the static stability term. Static stability requires this to be negative.
M_q Pitching moment due to pitch rate - the pitch damping term, this is always negative.

Since the tail is operating in the flowfield of the wing, changes in the wing incidence cause changes in the downwash, but there is a delay for the change in wing flowfield to affect the tail lift, this is represented as a moment proportional to the rate of change of incidence:

$$M_{\dot{\alpha}}$$

Increasing the wing incidence without increasing the tail incidence produces a nose up moment, so $M_{\dot{\alpha}}$ is expected to be positive.

The equations of motion, with small perturbation forces and moments become:

$$\frac{d\alpha}{dt} = \left(1 + \frac{Z_q}{mU}\right) q + \frac{Z_\alpha}{mU}\alpha$$
$$\frac{dq}{dt} = \frac{M_q}{B}q + \frac{M_\alpha}{B}\alpha + \frac{M_{\dot{\alpha}}}{B}\dot{\alpha}$$

These may be manipulated to yield as second order linear differential equation in α:

$$\frac{d^2\alpha}{dt^2} - \left(\frac{Z_\alpha}{mU} + \frac{M_q}{B} + (1 + \frac{Z_q}{mU})\frac{M_{\dot{\alpha}}}{B}\right)\frac{d\alpha}{dt} + \left(\frac{Z_\alpha}{mU}\frac{M_q}{B} - \frac{M_\alpha}{B}(1 + \frac{Z_q}{mU})\right)\alpha = 0$$

This represents a damped simple harmonic motion.

We should expect $\frac{Z_q}{mU}$ to be small compared with unity, so the coefficient of α (the 'stiffness' term) will be positive, provided $M_\alpha < \frac{Z_\alpha}{mU} M_q$. This expression is dominated by M_α, which defines the longitudinal static stability of the aircraft, it must be negative for stability. The damping term is reduced by the downwash effect, and it is difficult to design an aircraft with both rapid natural response and heavy damping. Usually, the response is underdamped but stable.

Phugoid

If the stick is held fixed, the aircraft will not maintain straight and level flight, but will start to dive, level out and climb again. It will repeat this cycle until the pilot intervenes. This long period oscillation in speed and height is called the phugoid mode. This is analyzed by assuming that the SSPO performs its proper function and maintains the angle of attack near its nominal value. The two states which are mainly affected are the climb angle γ (gamma) and speed. The small perturbation equations of motion are:

$$mU\frac{d\gamma}{dt} = -Z$$

which means the centripetal force is equal to the perturbation in lift force.

For the speed, resolving along the trajectory:

$$m\frac{du}{dt} = X - mg\gamma$$

where g is the acceleration due to gravity at the earths surface. The acceleration along the trajectory is equal to the net x-wise force minus the component of weight. We should not expect significant aerodynamic derivatives to depend on the climb angle, so only X_u and Z_u need be considered. X_u is the drag increment with increased speed, it is negative, likewise Z_u is the lift increment due to speed increment, it is also negative because lift acts in the opposite sense to the z-axis.

The equations of motion become:

$$mU\frac{d\gamma}{dt} = -Z_u u$$
$$m\frac{du}{dt} = X_u u - mg\gamma$$

These may be expressed as a second order equation in climb angle or speed perturbation:

$$\frac{d^2u}{dt^2} - \frac{X_u}{m}\frac{du}{dt} - \frac{Z_u g}{mU}u = 0$$

Now lift is very nearly equal to weight:

$$Z = \frac{1}{2}\rho U^2 c_L S_w = W$$

where ρ is the air density, S_w is the wing area, W the weight and c_L is the lift coefficient (assumed constant because the incidence is constant), we have, approximately:

$$Z_u = \frac{2W}{U} = \frac{2mg}{U}$$

The period of the phugoid, T, is obtained from the coefficient of u:

$$\frac{2\pi}{T} = \sqrt{\frac{2g^2}{U^2}}$$

Or:

$$T = \frac{2\pi U}{\sqrt{2}g}$$

Since the lift is very much greater than the drag, the phugoid is at best lightly damped. A propeller with fixed speed would help. Heavy damping of the pitch rotation or a large rotational inertia increase the coupling between short period and phugoid modes, so that these will modify the phugoid.

Lateral modes

With a symmetrical rocket or missile, the directional stability in yaw is the same as the pitch stability; it resembles the short period pitch oscillation, with yaw plane equivalents to the pitch plane stability derivatives. For this reason pitch and yaw directional stability are collectively known as the "weathercock" stability of the missile.

Aircraft lack the symmetry between pitch and yaw, so that directional stability in yaw is derived from a different set of stability derivatives. The yaw plane equivalent to the short period pitch oscillation, which describes yaw plane directional stability is called Dutch roll. Unlike pitch plane motions, the lateral modes involve both roll and yaw motion.

Dutch roll

It is customary to derive the equations of motion by formal manipulation in what, to the engineer, amounts to a piece of mathematical sleight of hand. The current approach follows the pitch plane analysis in formulating the equations in terms of concepts which are reasonably familiar.

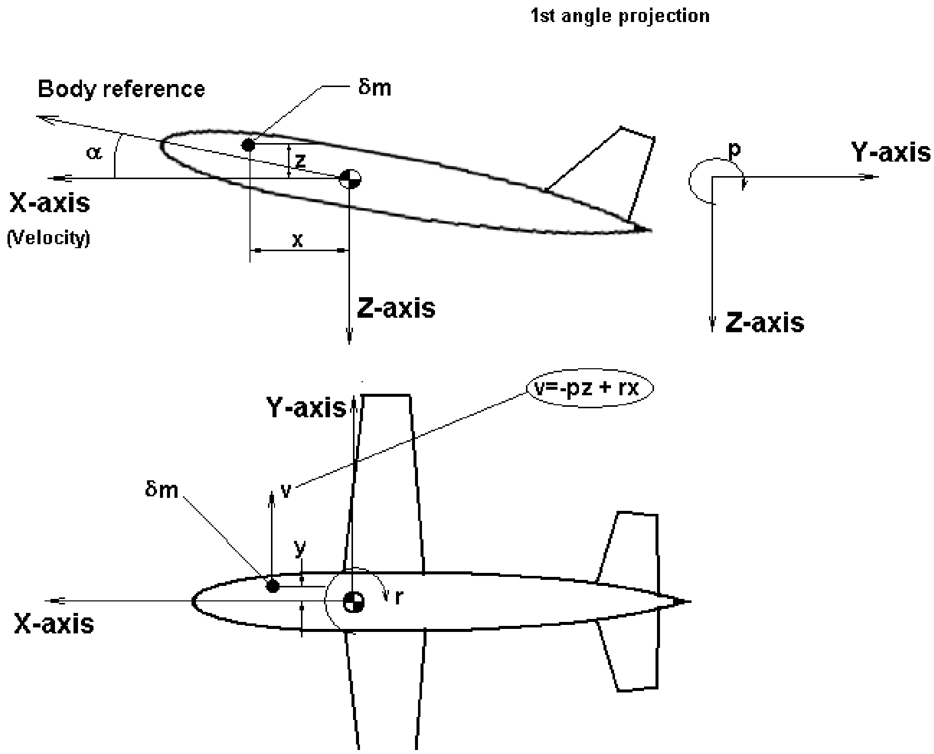
Applying an impulse via the rudder pedals should induce Dutch roll, which is the oscillation in roll and yaw, with the roll motion lagging yaw by a quarter cycle, so that the wing tips follow elliptical paths with respect to the aircraft.

The yaw plane translational equation, as in the pitch plane, equates the centripetal acceleration to the side force.

$$\frac{d\beta}{dt} = \frac{Y}{mU} - r$$

where β (beta) is the sideslip angle, Y the side force and r the yaw rate.

The moment equations are a bit trickier. The trim condition is with the aircraft at an angle of attack with respect to the airflow. The body x-axis does not align with the velocity vector, which is the reference direction for wind axes. In other words, wind axes are not principal axes (the mass is not distributed symmetrically about the yaw and roll axes). Consider the motion of an element of mass in position -z,x in the direction of the y-axis, i.e. into the plane of the paper.



Lateral Equations Product of Inertia

If the roll rate is p, the velocity of the particle is:

$$v = -pz + xr$$

Made up of two terms, the force on this particle is first the proportional to rate of v change, the second is due to the change in direction of this component of velocity as the body moves. The latter terms gives rise to cross products of small quantities (pq,pr,qr), which are later discarded. In this analysis, they are discarded from the outset for the sake of clarity. In effect, we assume that the direction of the velocity of the particle due to the simultaneous roll and yaw rates does not change significantly throughout the motion. With this simplifying assumption, the acceleration of the particle becomes:

$$\frac{dv}{dt} = -\frac{dp}{dt}z + \frac{dr}{dt}x$$

The yawing moment is given by:

$$\delta m x \frac{dv}{dt} = -\frac{dp}{dt}xz\delta m + \frac{dr}{dt}x^2\delta m$$

There is an additional yawing moment due to the offset of the particle in the y direction: $\frac{dr}{dt}y^2\delta m$

The yawing moment is found by summing over all particles of the body:

$$N = -\frac{dp}{dt}\int xzdm + \frac{dr}{dt}\int x^2 + y^2 dm = -E\frac{dp}{dt} + C\frac{dr}{dt}$$

where N is the yawing moment, E is a product of inertia, and C is the moment of inertia about the yaw axis. A similar reasoning yields the roll equation:

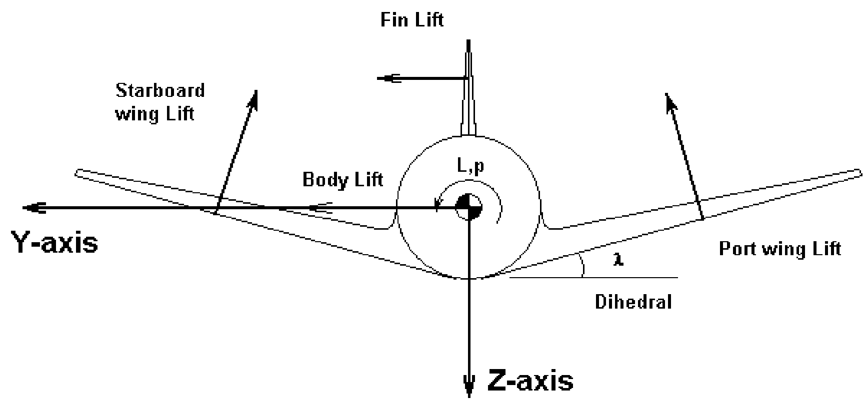
$$L = A\frac{dp}{dt} - E\frac{dr}{dt}$$

where L is the rolling moment and A the roll moment of inertia.

Lateral and longitudinal stability derivatives

The states are β (sideslip), r (yaw rate) and p (roll rate), with moments N (yaw) and L (roll), and force Y (sideways). There are nine stability derivatives relevant to this motion, the following explains how they originate. However a better intuitive understanding is to be gained by simply playing with a model airplane, and considering how the forces on each component are affected by changes in sideslip and angular velocity:

View from nose (negative X direction)



Lateral Stability - Main Sources of Stabilising Forces and Moments

Y_β Side force due to side slip (in absence of yaw).

Sideslip generates a sideforce from the fin and the fuselage. In addition, if the wing has dihedral, side slip at a positive roll angle increases incidence on the starboard wing and reduces it on the port side, resulting in a net force component directly opposite to the sideslip direction. Sweep back of the wings has the same effect on incidence, but since the wings are not inclined in the vertical plane, backsweep alone does not affect Y_β. However, anhedral may be used with high backsweep angles in high performance aircraft to offset the wing incidence effects of sideslip. Oddly enough this does not reverse the sign of the wing configuration's contribution to Y_β (compared to the dihedral case).

Y_p Side force due to roll rate.

Roll rate causes incidence at the fin, which generates a corresponding side force. Also, positive roll (starboard wing down) increases the lift on the starboard wing and reduces it on the port. If the wing has dihedral, this will result in a side force momentarily opposing the resultant sideslip tendency. Anhedral wing and or stabilizer configurations can cause the sign of the side force to invert if the fin effect is swamped.

Y_r Side force due to yaw rate.

Yawing generates side forces due to incidence at the rudder, fin and fuselage.

N_β Yawing moment due to sideslip forces.

Sideslip in the absence of rudder input causes incidence on the fuselage and empennage, thus creating a yawing moment counteracted only by the directional stiffness which would tend to point the aircraft's nose back into the wind in horizontal flight conditions. Under sideslip conditions at a given roll angle N_β will tend to point the nose into the sideslip direction even without rudder input, causing a downward spiraling flight.

N_p Yawing moment due to roll rate.

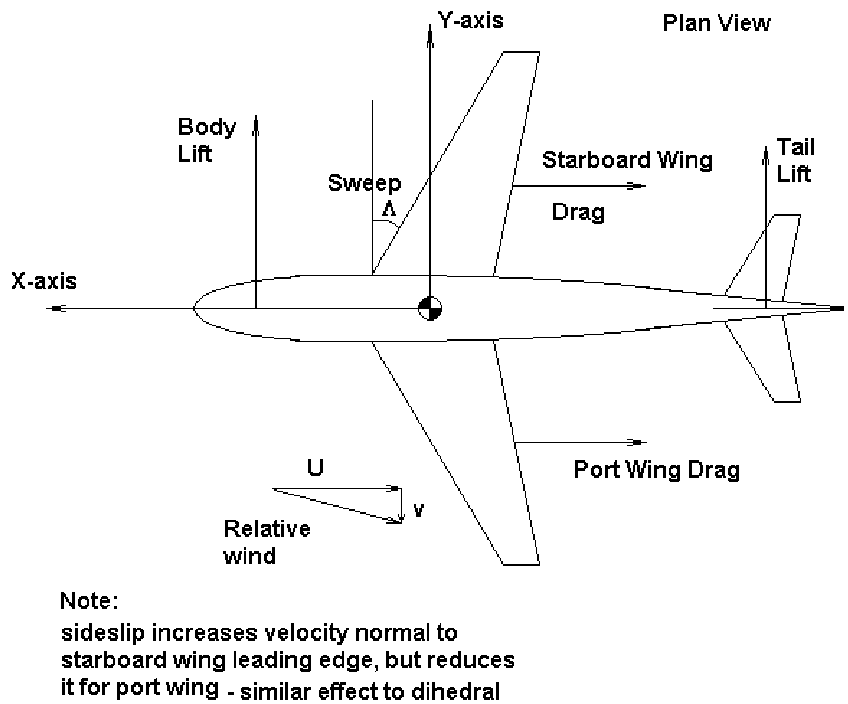
Roll rate generates fin lift causing a yawing moment and also differentially alters the lift on the wings, thus affecting the induced drag contribution of each wing, causing a (small) yawing moment contribution. Positive roll generally causes positive N_p values unless the empennage is anhedral or fin is below the roll axis. Lateral force components resulting from dihedral or anhedral wing lift differences has little effect on N_p because the wing axis is normally closely aligned with the center of gravity.

N_r Yawing moment due to yaw rate.

Yaw rate input at any roll angle generates rudder, fin and fuselage force vectors which dominate the resultant yawing moment. Yawing also increases the speed of the outboard wing whilst slowing down the inboard wing, with corresponding changes in drag causing a (small) opposing yaw moment. N_r opposes the inherent directional stiffness which tends to point the aircraft's nose back into the wind and always matches the sign of the yaw rate input.

L_β Rolling moment due to sideslip.

A positive sideslip angle generates empennage incidence which can cause positive or negative roll moment depending on its configuration. For any non-zero sideslip angle dihedral wings causes a rolling moment which tends to return the aircraft to the horizontal, as does back swept wings. With highly swept wings the resultant rolling moment may be excessive for all stability requirements and anhedral could be used to offset the effect of wing sweep induced rolling moment.



Additional Contributions to Lateral Stability

L_r Rolling moment due to yaw rate.

Yaw increases the speed of the outboard wing whilst reducing speed of the inboard one, causing a rolling moment to the inboard side. The contribution of the fin normally supports this inward rolling effect unless offset by anhedral stabilizer above the roll axis (or dihedral below the roll axis).

L_p Rolling moment due to roll rate.

Roll creates counter rotational forces on both starboard and port wings whilst also generating such forces at the empennage. These opposing rolling moment effects have to be overcome by the aileron input in order to sustain the roll rate. If the roll is stopped at a non-zero roll angle the L_β *upward* rolling moment induced by the ensueing sideslip should return the aircraft to the horizontal unless exceeded in turn by the *downward* L_r rolling moment resulting from sideslip induced yaw rate. Longitudinal stability could be ensured or improved by minimizing the latter effect.

Equations of motion

Since Dutch roll is a handling mode, analogous to the short period pitch oscillation, any effect it might have on the trajectory may be ignored. The body rate r is made up of the rate of change of sideslip angle and the rate of turn. Taking the latter as zero, assuming no effect on the trajectory, for the limited purpose of studying the Dutch roll:

$$\frac{d\beta}{dt} = -r$$

The yaw and roll equations, with the stability derivatives become:

$$C\frac{dr}{dt} - E\frac{dp}{dt} = N_\beta\beta - N_r\frac{d\beta}{dt} + N_p p \text{ (yaw)}$$
$$A\frac{dp}{dt} - E\frac{dr}{dt} = L_\beta\beta - L_r\frac{d\beta}{dt} + L_p p \text{ (roll)}$$

The inertial moment due to the roll acceleration is considered small compared with the aerodynamic terms, so the equations become:

$$-C\frac{d^2\beta}{dt^2} = N_\beta\beta - N_r\frac{d\beta}{dt} + N_p p$$
$$E\frac{d^2\beta}{dt^2} = L_\beta\beta - L_r\frac{d\beta}{dt} + L_p p$$

This becomes a second order equation governing either roll rate or sideslip:

$$\left(\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}\right)\frac{d^2\beta}{dt^2} + \left(\frac{L_p}{A}\frac{N_r}{C} - \frac{N_p}{C}\frac{L_r}{A}\right)\frac{d\beta}{dt} - \left(\frac{L_p}{A}\frac{N_\beta}{C} - \frac{L_\beta}{A}\frac{N_p}{C}\right)\beta = 0$$

The equation for roll rate is identical. But the roll angle, φ (phi) is given by:

$$\frac{d\phi}{dt} = p$$

If p is a damped simple harmonic motion, so is φ, but the roll must be in quadrature with the roll rate, and hence also with the sideslip. The motion consists of oscillations in roll and yaw, with the roll motion lagging 90 degrees behind the yaw. The wing tips trace out elliptical paths.

Stability requires the "stiffness" and "damping" terms to be positive. These are:

$$\frac{\frac{L_p}{A}\frac{N_r}{C} - \frac{N_p}{C}\frac{L_r}{A}}{\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}} \text{ (damping)}$$
$$\frac{\frac{L_\beta}{A}\frac{N_p}{C} - \frac{L_p}{A}\frac{N_\beta}{C}}{\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}} \text{ (stiffness)}$$

The denominator is dominated by L_p, the roll damping derivative, which is always negative, so the denominators of these two expressions will be positive.

Considering the "stiffness" term: $-L_pN_\beta$ will be positive because L_p is always negative and N_β is positive by design. L_β is usually negative, whilst N_p is positive. Excessive dihedral can destabilize the Dutch roll, so configurations with highly swept wings require anhedral to offset the wing sweep contribution to L_β.

The damping term is dominated by the product of the roll damping and the yaw damping derivatives, these are both negative, so their product is positive. The Dutch roll should therefore be damped.

The motion is accompanied by slight lateral motion of the center of gravity and a more "exact" analysis will introduce terms in Y_β etc.

Roll subsidence

Jerking the stick sideways and returning it to center causes a net change in roll orientation.

The roll motion is characterized by an absence of natural stability, there are no stability derivatives which generate moments in response to the inertial roll angle. A roll disturbance induces a roll rate which is only canceled by pilot or autopilot intervention. This takes place with insignificant changes in sideslip or yaw rate, so the equation of motion reduces to:

$$A\frac{dp}{dt} = L_p p.$$

L_p is negative, so the roll rate will decay with time. The roll rate reduces to zero, but there is no direct control over the roll angle.

Spiral mode

Simply holding the stick still, when starting with the wings near level, an aircraft will usually have a tendency to gradually veer off to one side of the straight flightpath. This is the (slightly unstable) **spiral mode**. The opposite holds for a stable spiral mode. The spiral mode is so-named because when it is slightly unstable, and the controls are not moved, the aircraft will tend to increase its bank angle slowly at first, then ever faster. The resulting path through the air is a continuously tightening and ever more rapidly descending *spiral*. An unstable spiral mode is common to most aircraft. It is not dangerous because the times to double the bank angle are large compared to the pilot's ability to respond and correct errors with aileron inputs.

When the spiral mode is stable, it behaves in a way opposite to the exponential divergence of the unstable mode. The stable spiral mode, when starting with the wings at a moderate bank angle, will return to near wings level, first quickly, then more slowly. When the spiral mode is stable and starting at a moderate bank angle, the spiral nature of the flight path is not as obvious. This is because usually only a fraction of a turn is made while the wings are not fully level. The turning starts out (relatively) tight, then becomes less and less so as the wings become more level.

The divergence rate of the *unstable* spiral mode will be roughly proportional to the roll angle itself (i.e. roughly exponential growth). The *con*vergence rate of the *stable* spiral mode will be roughly proportional to the roll angle itself (i.e. roughly exponential *decay*).

Spiral mode trajectory

In studying the trajectory, it is the direction of the velocity vector, rather than that of the body, which is of interest. The direction of the velocity vector when projected on to the horizontal will be called the track, denoted μ (mu). The body orientation is called the heading, denoted ψ (psi). The force equation of motion includes a component of weight:

$$\frac{d\mu}{dt} = \frac{Y}{mU} + \frac{g}{U}\phi$$

where g is the gravitational acceleration, and U is the speed.

Including the stability derivatives:

$$\frac{d\mu}{dt} = \frac{Y_\beta}{mU}\beta + \frac{Y_r}{mU}r + \frac{Y_p}{mU}p + \frac{g}{U}\phi$$

Roll rates and yaw rates are expected to be small, so the contributions of Y_r and Y_p will be ignored.

The sideslip and roll rate vary gradually, so their time derivatives are ignored. The yaw and roll equations reduce to:

$$N_\beta\beta + N_r\frac{d\mu}{dt} + N_p p = 0 \quad \text{(yaw)}$$

$$L_\beta\beta + L_r\frac{d\mu}{dt} + L_p p = 0 \quad \text{(roll)}$$

Solving for β and p:

$$\beta = \frac{(L_r N_p - L_p N_r)}{(L_p N_\beta - N_p L_\beta)}\frac{d\mu}{dt}$$

$$p = \frac{(L_\beta N_r - L_r N_\beta)}{(L_p N_\beta - N_p L_\beta)}\frac{d\mu}{dt}$$

Substituting for sideslip and roll rate in the force equation results in a first order equation in roll angle:

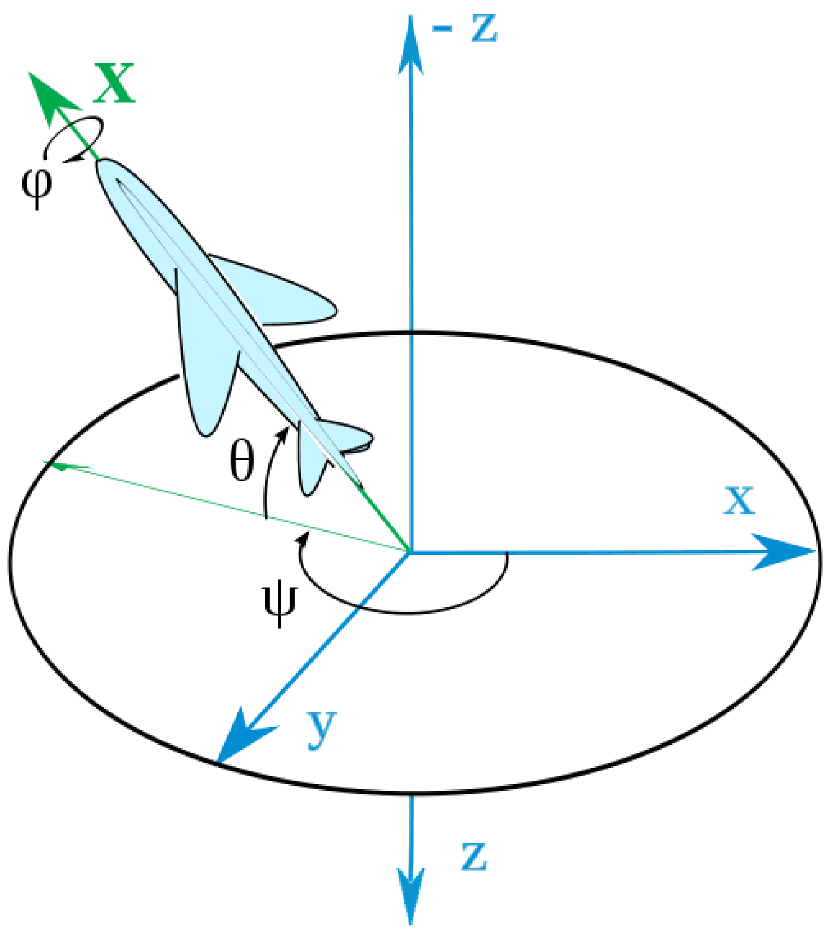
$$\frac{d\phi}{dt} = mg\frac{(L_\beta N_r - N_\beta L_r)}{mU(L_p N_\beta - N_p L_\beta) - Y_\beta(L_r N_p - L_p N_r)}\phi$$

This is an exponential growth or decay, depending on whether the coefficient of φ is positive or negative. The denominator is usually negative, which requires $L_\beta N_r > N_\beta L_r$ (both products are positive). This is in direct conflict with the Dutch roll stability requirement, and it is difficult to design an aircraft for which both the Dutch roll and spiral mode are inherently stable.

Since the spiral mode has a long time constant, the pilot can intervene to effectively stabilize it, but an aircraft with an unstable Dutch roll would be difficult to fly. It is usual to design the aircraft with a stable Dutch roll mode, but slightly unstable spiral mode.

Chapter- 2

Axes Conventions



Heading, elevation and bank angles (Z-Y'-X'') for an aircraft. The aircraft's pitch and yaw axes Y and Z are not shown, and its fixed reference frame xyz has been shifted backwards from its center of gravity (preserving angles) for clarity. Axes named according to the air norm DIN 9300

The orientation of a vehicle is normally referred to as *attitude*. It is described normally by the orientation of a frame fixed in the body relative to a fixed reference frame. The attitude is described by *attitude coordinates*, and consists of at least three coordinates.

While from a geometrical point of view the different methods to describe orientations are defined using only some reference frames, in engineering applications it is important also to describe how these frames are attached to the lab and the body in motion.

Due to the special importance of international conventions in air vehicles, several organizations have published standards to be followed. For example, German DIN has published the DIN 9300 norm for aircrafts (adopted by ISO as ISO 1151–2:1985).

Frames conventions

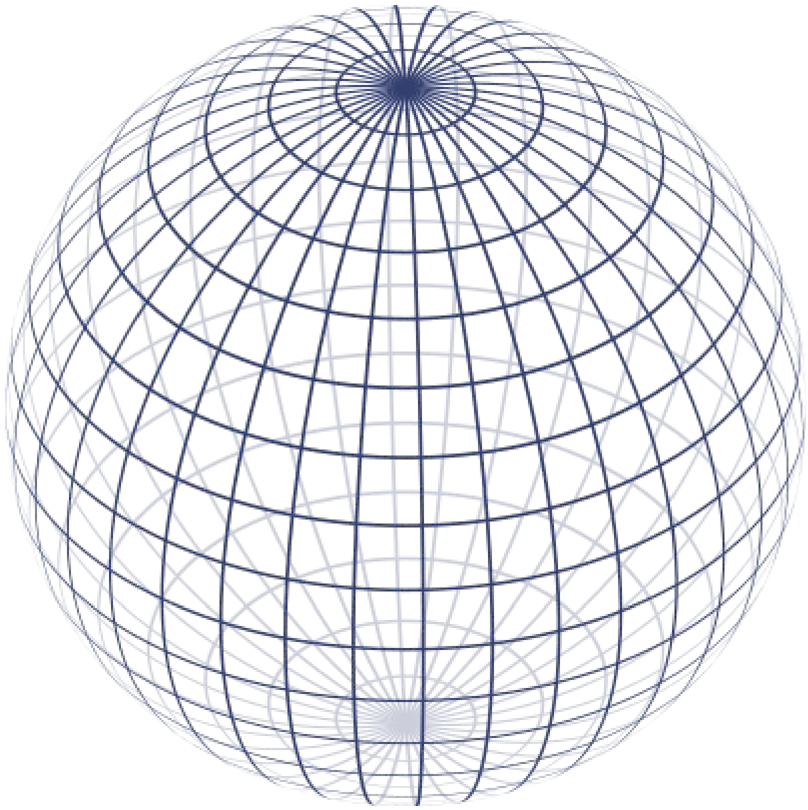
To stablish a standard convention to describe attitudes, it is required to establish at least the axes of the reference system and the axes of the rigid body or vehicle. When an ambiguous notation system is used (such as Euler angles) also the used convention should be stated. Nevertheless most used notations (matrices and quaternions) are unambiguous.

Tait–Bryan angles are often used to describe a vehicle's attitude with respect to a chosen reference frame, though any other notation can be used. The positive *x*-axis in vehicles points always in the direction of movement. For positive *y*- and *z*-axis, we have to face two different conventions:

- In case of land vehicles like cars, tanks etc., which use the ENU-system (East-North-Up) as external reference (*world frame*), the vehicle's positive *y*- or pitch axis always points to its left, and the positive *z*- or yaw axis always points up.

- By contrast, in case of air and sea vehicles like submarines, ships, airplanes etc., which use the NED-system (North-East-Down) as external reference (*world frame*), the vehicle's positive *y*- or pitch axis always points to its right, and its positive *z*- or yaw axis always points down.

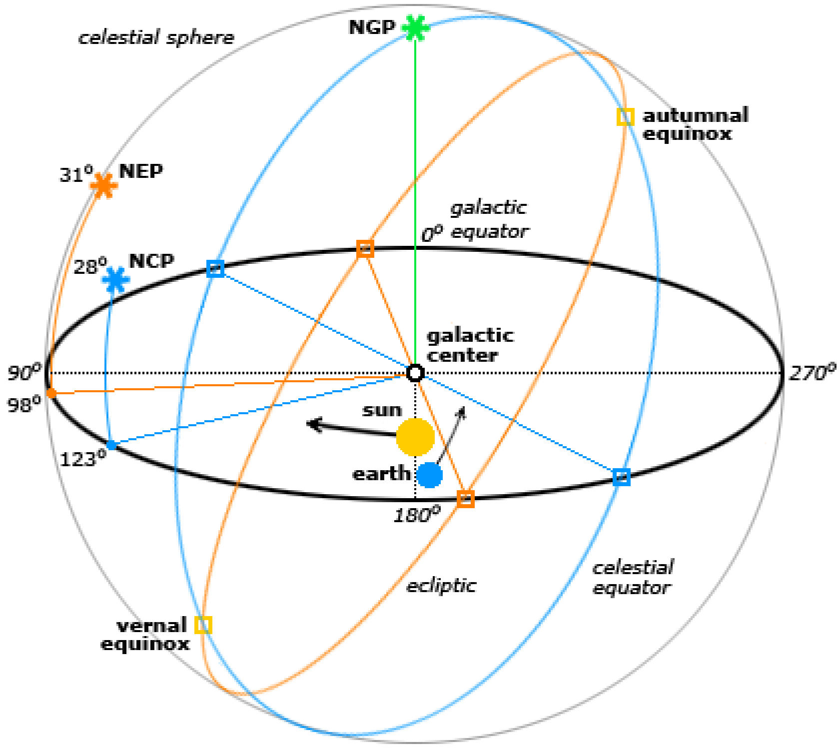
- Finally, in case of space vehicles like space shuttles etc., a modification of the latter convention is used, where the vehicle's positive *y*- or pitch axis again always points to its right, and its positive z- or yaw axis always points down, but “down” now may have two different meanings: If a so-called *local frame* is used as external reference, its positive z-axis points “down” to the center of the earth as it does in case of the earlier mentioned NED-system, but if the *inertial frame* is used as reference, its positive z-axis will point now to the North Celestial Pole, and its positive x-axis to the Vernal Equinox or some other reference meridian.



Representation of the earth with parallels and meridians

World axes conventions

Earth reference frames conventions: ENU and NED



Different reference systems for coordinates in the space

Basically, as lab frame or reference frame, there are two kinds of conventions for the frames:

- East, North, Up, referred as ENU
- North, East, Down, referred as NED, used specially in aerospace

These frames are location dependent. For movements around the globe, like air or see navigation, the frames are defined as tangent to the lines of coordinates.

- East-West tangent to parallels,
- North-South tangent to meridians, and
- Up-Down in the direction to the center of the earth (when using a spherical Earth simplification), or in the direction normal to the local tangent plane (using an oblate spheroidal or geodetic ellipsoidal model of the earth) which does not generally pass through the center of the Earth.

Frames for space study

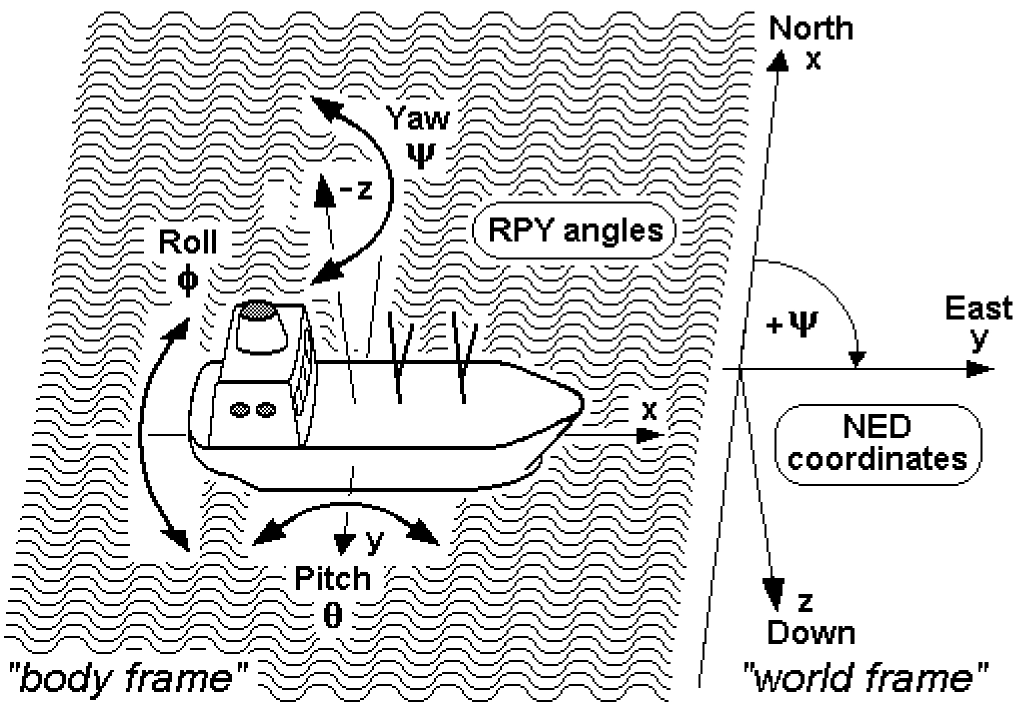
For satellites orbiting the earth it is normal to use the Equatorial coordinate system. The projection of the Earth's equator onto the celestial sphere is called the celestial equator.

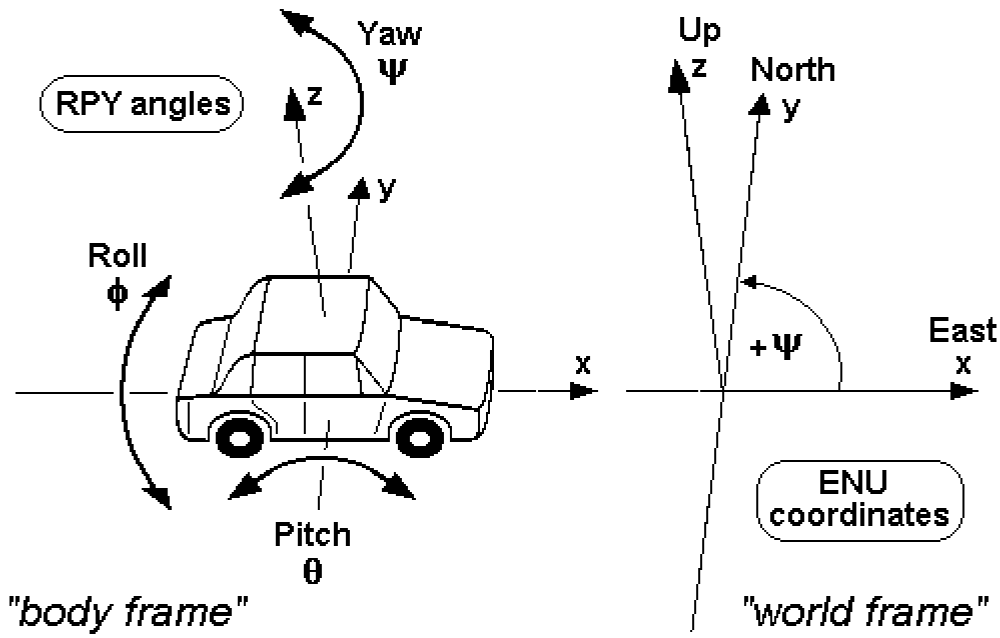
Similarly, the projections of the Earth's north and south geographic poles become the north and south celestial poles, respectively.

Deep space satellites use other Celestial coordinate system, like the Ecliptic coordinate system.

Vehicles conventions

Conventions for ships and land vehicles



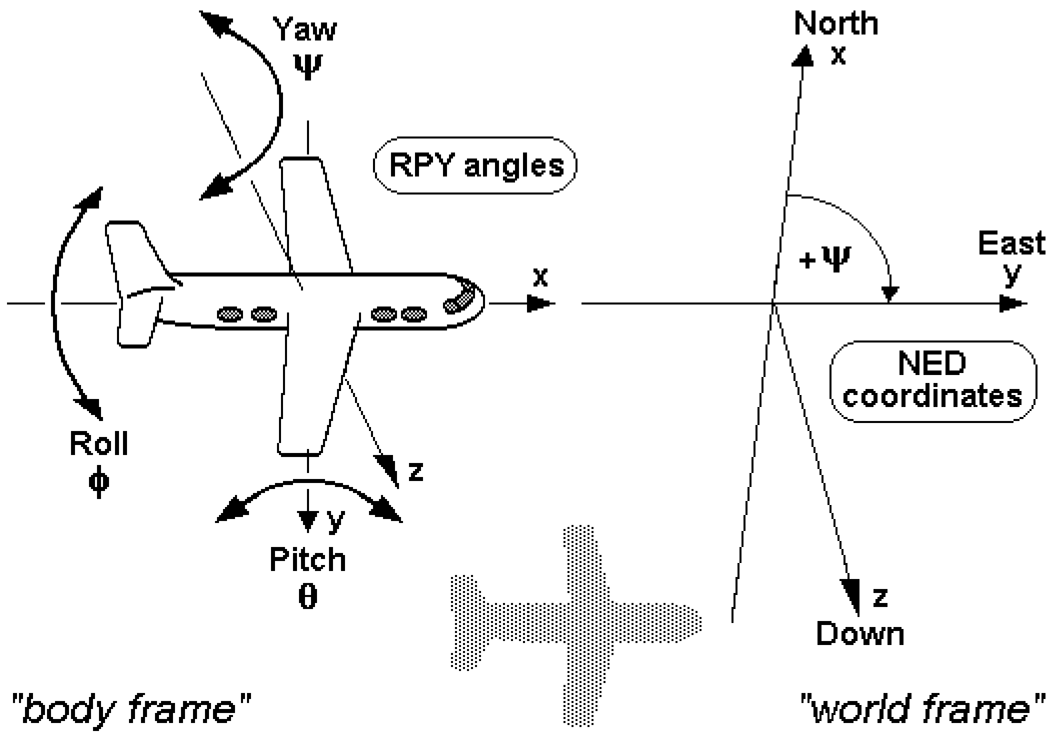


RPY angles of ships, cars and other sea and land vehicles

For land vehicles is rare to describe their complete orientation, except when speaking about electronic stability control. In this case, the convention is normally the one of the adjacent drawing.

As well as aircraft, the same terminology is used for the motion of ships and boats. It is interesting to note that some words commonly used were introduced in maritime navigation. For example, the *yaw* angle or heading, has a nautical origin, with the meaning of "bending out of the course". Etymologically, it is related with the verb 'to go'. It is related to the concept of bearing. It is typically assigned the shorthand notation ψ.

Frames conventions for aircraft attitudes



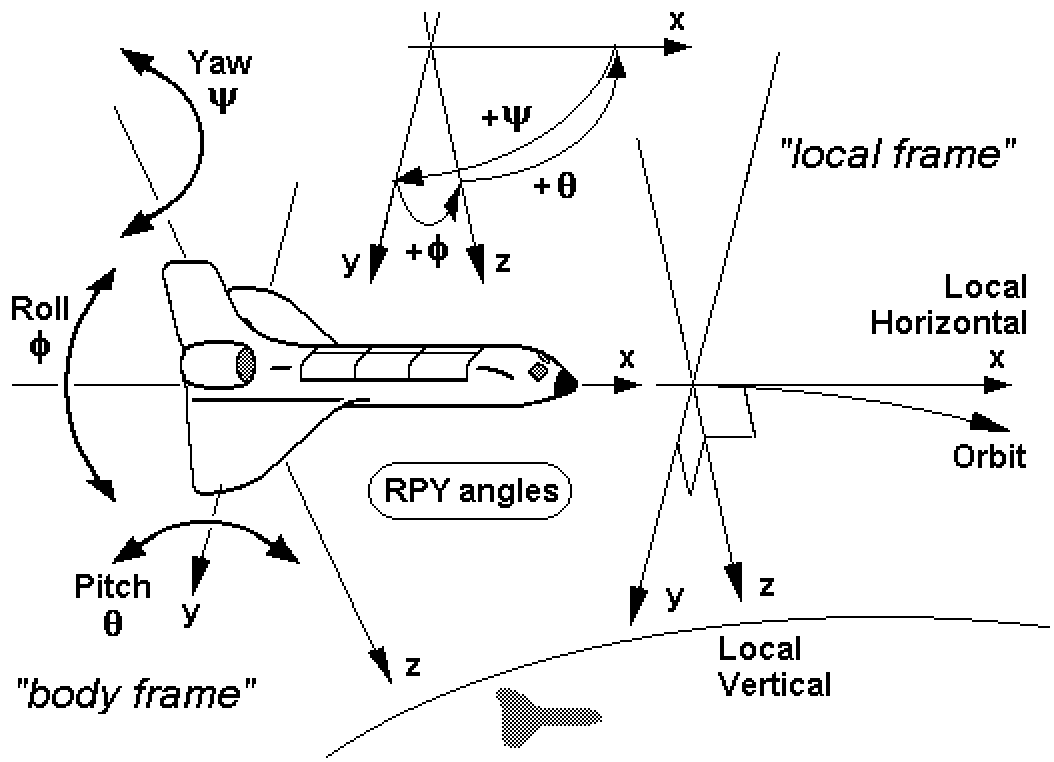
RPY angles of airplanes and other air vehicles

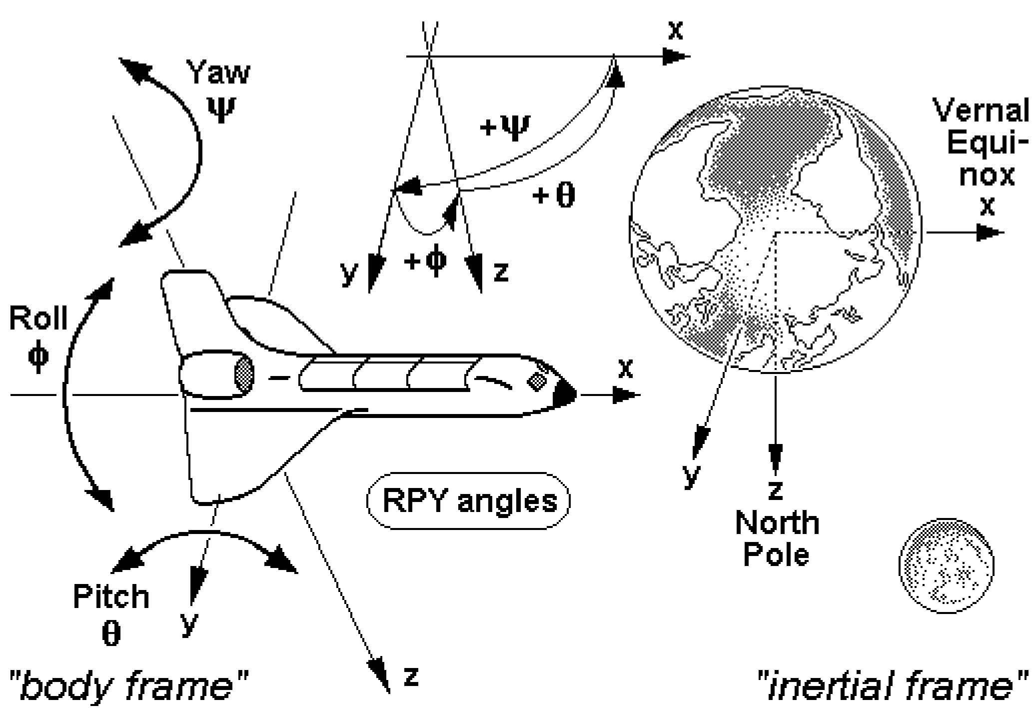
Yaw, pitch and roll are names used in aerospace for a vehicle-fixed axis system (*body frame*), but also as names for the rotations about those axes and also as names for the Tait–Bryan angles of the vehicle respect a *world frame*, which in the context of an aircraft sometimes are called its Heading, Elevation and Bank.

These axes are normally taken so that X axis is the longitudinal axis pointing ahead, Z axis is the vertical axis pointing downwards, and the Y axis is the lateral one, pointing in such a way that the frame is right handed.

The *motion* of an aircraft is often described in terms of rotation about these axes, so rotation about the *X*-axis is called rolling, rotation about the *Y*-axis is called pitching, and rotation about the *Z*-axis is called yawing.

Frames conventions for space ships





RPY angles of space shuttles and other space vehicles, first using a local frame as reference and second using an inertial frame as reference.

If the goal is to keep the shuttle during its orbits in a constant attitude with respect to the sky, e.g. in order to perform certain astronomical observations, the preferred reference is the *inertial frame*, and the RPY angle vector (0|0|0) describes an attitude then, where the shuttle's wings are kept permanently parallel to the earth's equator, its nose points permanently to the vernal equinox, and its belly towards the Northern polar star. (Note that rockets and missiles more commonly follow the conventions for aircraft where the RPY angle vector (0|0|0) points north, rather than toward the vernal equinox).

On the other hand, if it's the goal to keep the shuttle during its orbits in an constant attitude with respect to the surface of the earth, the preferred reference will be the *local frame*, with the RPY angle vector (0|0|0) describing an attitude, where the shuttle's wings are parallel to the earth's surface, its nose points to its heading, and its belly down towards the centre of the earth.

Chapter- 3

Aerodynamics

A vortex is created by the passage of an aircraft wing, revealed by smoke. Vortices are one of the many phenomena associated to the study of aerodynamics. The equations of aerodynamics show that the vortex is created by the difference in pressure between the upper and lower surface of the wing. At the end of the wing, the lower surface effectively tries to 'reach over' to the low pressure side, creating rotation and the vortex.

Aerodynamics is a branch of dynamics concerned with studying the motion of air, particularly when it interacts with a moving object. Aerodynamics is a subfield of fluid dynamics and gas dynamics, with much theory shared between them. Aerodynamics is often used synonymously with gas dynamics, with the difference being that gas dynamics applies to all gases. Understanding the motion of air (often called a flow field) around an object enables the calculation of forces and moments acting on the object. Typical properties calculated for a flow field include velocity, pressure, density and temperature as a function of position and time. By defining a control volume around the flow field, equations for the conservation of mass, momentum, and energy can be defined and used to solve for the properties. The use of aerodynamics through mathematical analysis, empirical approximations, wind tunnel experimentation, and computer simulations form the scientific basis for heavier-than-air flight.

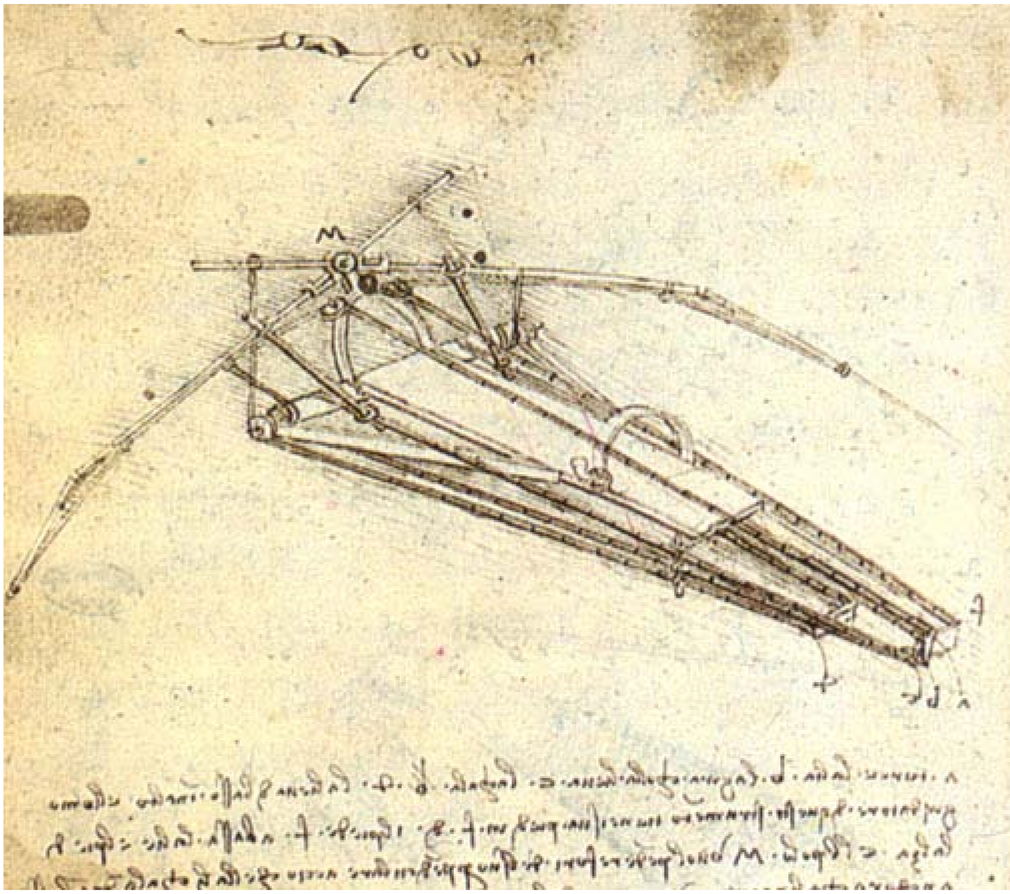
Aerodynamic problems can be identified in a number of ways. The flow environment defines the first classification criterion. *External* aerodynamics is the study of flow around solid objects of various shapes. Evaluating the lift and drag on an airplane or the shock waves that form in front of the nose of a rocket are examples of external aerodynamics. *Internal* aerodynamics is the study of flow through passages in solid objects. For instance, internal aerodynamics encompasses the study of the airflow through a jet engine or through an air conditioning pipe.

The ratio of the problem's characteristic flow speed to the speed of sound comprises a second classification of aerodynamic problems. A problem is called subsonic if all the speeds in the problem are less than the speed of sound, transonic if speeds both below and above the speed of sound are present (normally when the characteristic speed is approximately the speed of sound), supersonic when the characteristic flow speed is greater than the speed of sound, and hypersonic when the flow speed is much greater than the speed of sound. Aerodynamicists disagree over the precise definition of hypersonic flow; minimum Mach numbers for hypersonic flow range from 3 to 12.

The influence of viscosity in the flow dictates a third classification. Some problems may encounter only very small viscous effects on the solution, in which case viscosity can be considered to be negligible. The approximations to these problems are called inviscid flows. Flows for which viscosity cannot be neglected are called viscous flows.

History

Early ideas - ancient times to the 17th century



A drawing of a design for a flying machine by Leonardo da Vinci (c. 1488). This machine was an ornithopter, with flapping wings similar to a bird, first presented in his Codex on the Flight of Birds in 1505.

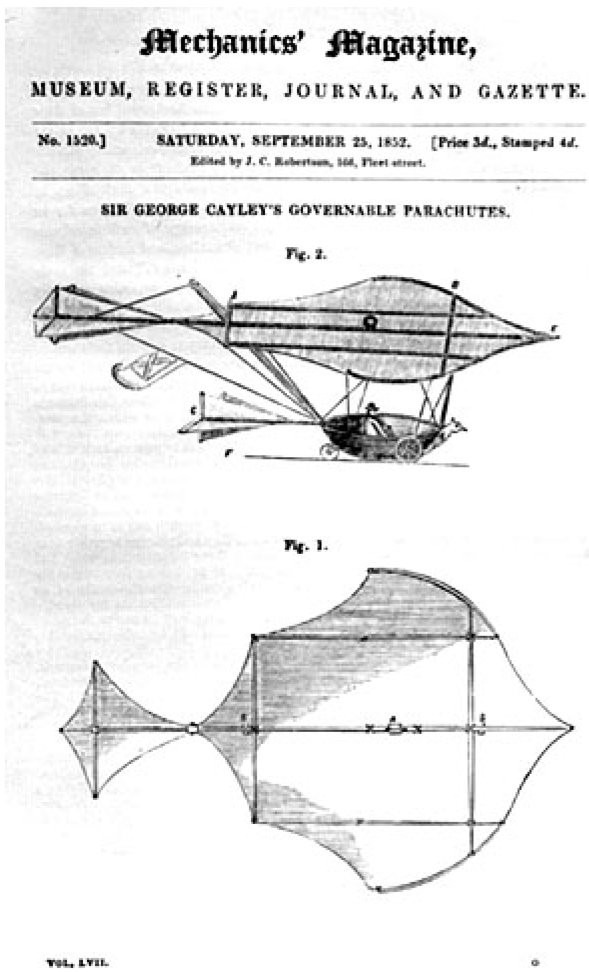
Images and stories of flight have appeared throughout recorded history, such as the legendary story of Icarus and Daedalus. Although observations of some aerodynamic effects like wind resistance (a.k.a. drag) were recorded by the likes of Aristotle, Leonardo da Vinci and Galileo Galilei, very little effort was made to develop governing laws for understanding the nature of flight prior to the 17th century.

In 1505, Leonardo da Vinci wrote the *Codex on the Flight of Birds*, one of the earliest treatises on aerodynamics. He notes for the first time that the center of gravity of a flying bird does not coincide with its center of pressure, and he describes the construction of an ornithopter, with flapping wings similar to a bird's.

Sir Isaac Newton was the first person to develop a theory of air resistance, making him one of the first aerodynamicists. As part of that theory, Newton believed that drag was due to the dimensions of a body, the density of the fluid, and the velocity raised to the second power. These beliefs all turned out to be correct for low flow speeds. Newton also developed a law for the drag force on a flat plate inclined towards the direction of the fluid flow. Using F for the drag force, ρ for the density, S for the area of the flat plate, V for the flow velocity, and θ for the inclination angle, his law was expressed as $F = \rho S V^2 \sin^2(\theta)$

Unfortunately, this equation is incorrect for the calculation of drag in most cases. Drag on a flat plate is closer to being linear with the angle of inclination as opposed to acting quadratically at low angles. The Newton formula can lead one to believe that flight is more difficult than it actually is, and it may have contributed to a delay in human flight. However, it is correct for a very slender plate when the angle becomes large and flow separation occurs, or if the flow speed is supersonic.

Modern beginnings - 18th to 19th century



A drawing of a glider by Sir George Cayley, one of the early attempts at creating an aerodynamic shape.

In 1738 The Dutch-Swiss mathematician Daniel Bernoulli published his book *Hydrodynamica*, in which he first set out the principle, named after him, by which aerodynamic lift may be derived.

Sir George Cayley is credited as the first person to identify the four aerodynamic forces of flight—weight, lift, drag, and thrust—and the relationship between them. Cayley believed that the drag on a flying machine must be counteracted by a means of propulsion in order for level flight to occur. Cayley also looked to nature for aerodynamic shapes with low drag. Among the shapes he investigated were the cross-sections of trout. This may appear counterintuitive, however, the bodies of fish are shaped to produce very low resistance as they travel through water. Their cross-sections are sometimes very close to that of modern low drag airfoils.

Air resistance experiments were carried out by investigators throughout the 18th and 19th centuries. Drag theories were developed by Jean le Rond d'Alembert, Gustav Kirchhoff, and Lord Rayleigh. Equations for fluid flow with friction were developed by Claude-Louis Navier and George Gabriel Stokes. To simulate fluid flow, many experiments involved immersing objects in streams of water or simply dropping them off the top of a tall building. Towards the end of this time period Gustave Eiffel used his Eiffel Tower to assist in the drop testing of flat plates.

Of course, a more precise way to measure resistance is to place an object within an artificial, uniform stream of air where the velocity is known. The first person to experiment in this fashion was Francis Herbert Wenham, who in doing so constructed the first wind tunnel in 1871. Wenham was also a member of the first professional organization dedicated to aeronautics, the Royal Aeronautical Society of the United Kingdom. Objects placed in wind tunnel models are almost always smaller than in practice, so a method was needed to relate small scale models to their real-life counterparts. This was achieved with the invention of the dimensionless Reynolds number by Osborne Reynolds. Reynolds also experimented with laminar to turbulent flow transition in 1883.

By the late 19th century, two problems were identified before heavier-than-air flight could be realized. The first was the creation of low-drag, high-lift aerodynamic wings. The second problem was how to determine the power needed for sustained flight. During this time, the groundwork was laid down for modern day fluid dynamics and aerodynamics, with other less scientifically inclined enthusiasts testing various flying machines with little success.

A replica of the Wright Brothers' wind tunnel is on display at the Virginia Air and Space Center. Wind tunnels were key in the development and validation of the laws of aerodynamics.

In 1889, Charles Renard, a French aeronautical engineer, became the first person to reasonably predict the power needed for sustained flight. Renard and German physicist Hermann von Helmholtz explored the wing loading of birds, eventually concluding that humans could not fly under their own power by attaching wings onto their arms. Otto Lilienthal, following the work of Sir George Cayley, was the first person to become highly successful with glider flights. Lilienthal believed that thin, curved airfoils would produce high lift and low drag.

Octave Chanute provided a great service to those interested in aerodynamics and flying machines by publishing a book outlining all of the research conducted around the world up to 1893.

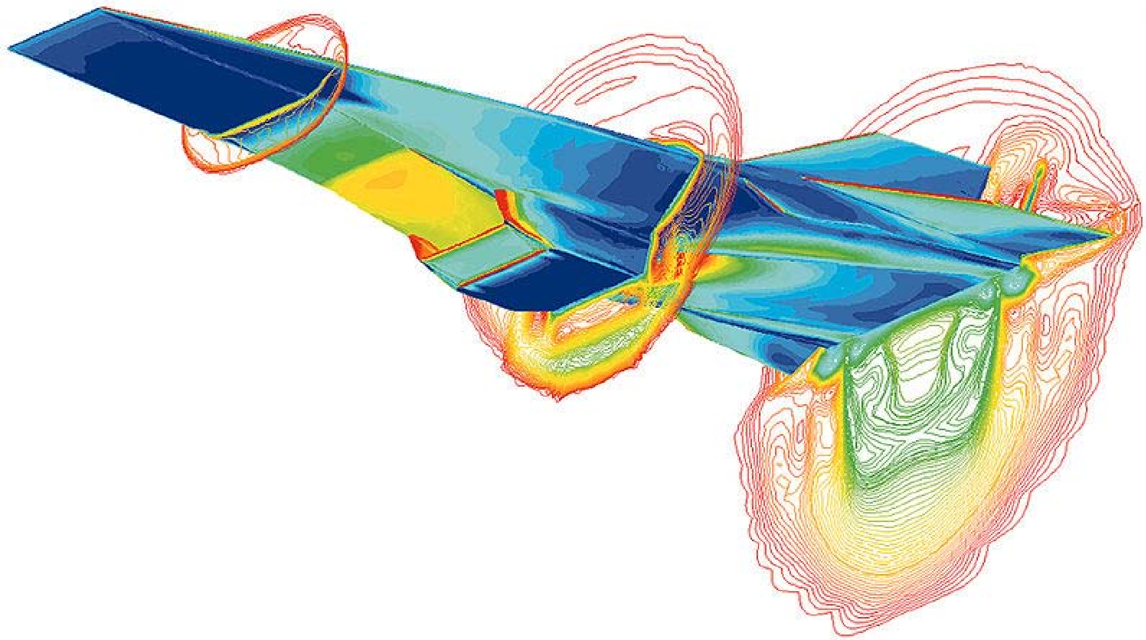
Practical flight - early 20th century

With the information contained in Chanute's book, the personal assistance of Chanute himself, and research carried out in their own wind tunnel, the Wright brothers gained just enough knowledge of aerodynamics to fly the first powered aircraft on December 17, 1903, just in time to beat the efforts of Samuel Pierpont Langley. The Wright brothers' flight confirmed or disproved a number of aerodynamics theories. Newton's drag force theory was finally proved incorrect. This first widely-publicised flight led to a more organized effort between aviators and scientists, leading the way to modern aerodynamics.

During the time of the first flights, Frederick W. Lanchester, Martin Wilhelm Kutta, and Nikolai Zhukovsky independently created theories that connected circulation of a fluid flow to lift. Kutta and Zhukovsky went on to develop a two-dimensional wing theory. Expanding upon the work of Lanchester, Ludwig Prandtl is credited with developing the mathematics behind thin-airfoil and lifting-line theories as well as work with boundary layers. Prandtl, a professor at the University of Göttingen, instructed many students who would play important roles in the development of aerodynamics like Theodore von Kármán and Max Munk.

Faster than sound - latter 20th century

As aircraft began to travel faster, aerodynamicists realized that the density of air began to change as it came into contact with an object, leading to a division of fluid flow into the incompressible and compressible regimes. In compressible aerodynamics, density and pressure both change, which is the basis for calculating the speed of sound. Newton was the first to develop a mathematical model for calculating the speed of sound, but it was not correct until Pierre-Simon Laplace accounted for the molecular behavior of gases and introduced the heat capacity ratio. The ratio of the flow speed to the speed of sound was named the Mach number after Ernst Mach, who was one of the first to investigate the properties of supersonic flow which included Schlieren photography techniques to visualize the changes in density. William John Macquorn Rankine and Pierre Henri Hugoniot independently developed the theory for flow properties before and after a shock wave. Jakob Ackeret led the initial work on calculating the lift and drag on a supersonic airfoil. Theodore von Kármán and Hugh Latimer Dryden introduced the term transonic to describe flow speeds around Mach 1 where drag increases rapidly. Because of the increase in drag approaching Mach 1, aerodynamicists and aviators disagreed on whether supersonic flight was achievable.



A computer generated model of NASA's X-43A hypersonic research vehicle flying at Mach 7 using a computational fluid dynamics code.

On September 30, 1935 an exclusive conference was held in Rome with the topic of high velocity flight and the possibility of breaking the sound barrier. Participants included Theodore von Kármán, Ludwig Prandtl, Jakob Ackeret, Eastman Jacobs, Adolf Busemann, Geoffrey Ingram Taylor, Gaetano Arturo Crocco, and Enrico Pistolesi. The new research presented was impressive. Ackeret presented a design for a supersonic wind tunnel. Busemann gave perhaps the best presentation on the need for aircraft with swept wings for high speed flight. Eastman Jacobs, working for NACA, presented his optimized airfoils for high subsonic speeds which led to some of the high performance American aircraft during World War II. Supersonic propulsion was also discussed. The sound barrier was broken using the Bell X-1 aircraft twelve years later, thanks in part to those individuals.

By the time the sound barrier was broken, much of the subsonic and low supersonic aerodynamics knowledge had matured. The Cold War fueled an ever evolving line of high performance aircraft. Computational fluid dynamics was started as an effort to solve for flow properties around complex objects and has rapidly grown to the point where entire aircraft can be designed using a computer.

With some exceptions, the knowledge of hypersonic aerodynamics has matured between the 1960s and the present decade. Therefore, the goals of an aerodynamicist have shifted from understanding the behavior of fluid flow to understanding how to engineer a vehicle to interact appropriately with the fluid flow. For example, while the behavior of hypersonic flow is understood, building a scramjet aircraft to fly at hypersonic speeds has seen very limited success. Along with building a successful scramjet aircraft, the desire to improve the aerodynamic efficiency of current aircraft and propulsion systems will continue to fuel new research in aerodynamics.

Introductory terminology

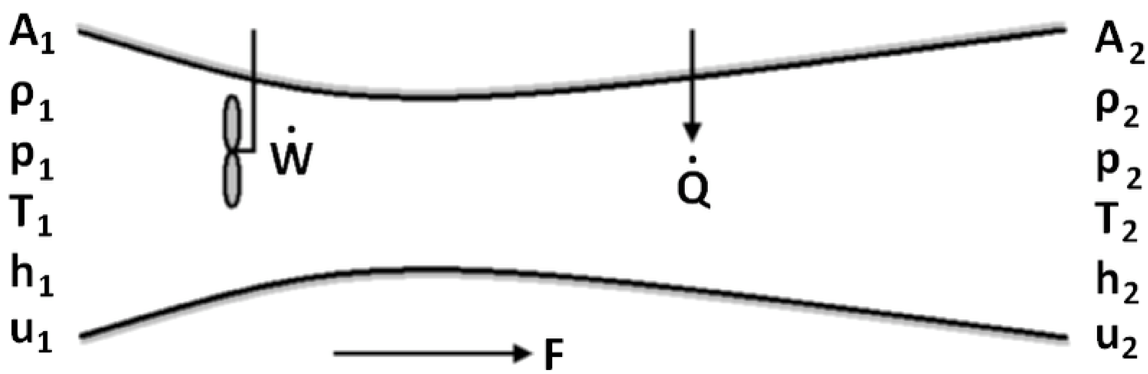
- Lift
- Drag
- Reynolds number
- Mach number

Continuity assumption

Gases are composed of molecules which collide with one another and solid objects. If density and velocity are taken to be well-defined at infinitely small points, and are assumed to vary continuously from one point to another, the discrete molecular nature of a gas is ignored.

The continuity assumption becomes less valid as a gas becomes more rarefied. In these cases, statistical mechanics is a more valid method of solving the problem than continuous aerodynamics. The Knudsen number can be used to guide the choice between statistical mechanics and the continuous formulation of aerodynamics.

Laws of conservation



Control volume schematic of internal flow with one inlet and exit including an axial force, work, and heat transfer. State 1 is the inlet and state 2 is the exit.

Aerodynamics problems are often solved using conservation laws as applied to a fluid continuum. The conservation laws can be written in integral or differential form. In many basic problems, three conservation principles are used:

- Continuity: If a certain mass of fluid enters a volume, it must either exit the volume or change the mass inside the volume. In fluid dynamics, the continuity equation is analogous to Kirchhoff's Current Law in electric circuits. The differential form of the continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Above, ρ is the fluid density, **u** is a velocity vector, and *t* is time. Physically, the equation also shows that mass is neither created nor destroyed in the control volume. For a steady state process, the rate at which mass enters the volume is equal to the rate at which it leaves the volume. Consequently, the first term on the left is then equal to zero. For flow through a tube with one inlet (state 1) and exit (state 2) as shown in the figure in this section, the continuity equation may be written and solved as:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

Above, *A* is the variable cross-section area of the tube at the inlet and exit. For incompressible flows, density remains constant.

- Conservation of Momentum: This equation applies Newton's second law of motion to a continuum, whereby force is equal to the time derivative of momentum. Both surface and body forces are accounted for in this equation. For instance, *F* could be expanded into an expression for the frictional force acting on an internal flow.

$$\frac{D\mathbf{u}}{Dt} = \mathbf{F} - \frac{\nabla p}{\rho}$$

For the same figure, a control volume analysis yields:

$$p_1 A_1 + \rho_1 A_1 u_1^2 + F = p_2 A_2 + \rho_2 A_2 u_2^2$$

Above, the force *F* is placed on the left side of the equation, assuming it acts with the flow moving in a left-to-right direction. Depending on the other properties of the flow, the resulting force could be negative which means it acts in the opposite direction as depicted in the figure.

- Conservation of Energy: Although energy can be converted from one form to another, the total energy in a given system remains constant.

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \nabla \cdot (k \nabla T) + \Phi$$

Above, *h* is enthalpy, *k* is the thermal conductivity of the fluid, *T* is temperature, and Φ is the viscous dissipation function. The viscous dissipation function governs the rate at which mechanical energy of the flow is converted to heat. The expression on the left side is a material derivative. The term is always positive since, according to the second law of thermodynamics, viscosity cannot add energy to the control volume. Again using the figure, the energy equation in terms of the control volume may be written as:

$$\rho_1 u_1 A_1 \left(h_1 + \frac{u_1^2}{2} \right) + \dot{W} + \dot{Q} = \rho_2 u_2 A_2 \left(h_2 + \frac{u_2^2}{2} \right)$$

Above, the shaft work and heat transfer are assumed to be acting on the flow. They may be positive (to the flow from the surroundings) or negative (to the surroundings from the flow) depending on the problem. The ideal gas law or another equation of state is often used in conjunction with these equations to form a system to solve for the unknown variables.

Incompressible aerodynamics

An incompressible flow is characterized by a constant density despite flowing over surfaces or inside ducts. A flow can be considered incompressible as long as its speed is low. For higher speeds, the flow will begin to compress as it comes into contact with surfaces. The Mach number is used to distinguish between incompressible and compressible flows.

Subsonic flow

Subsonic (or low-speed) aerodynamics is the study of fluid motion which is everywhere much slower than the speed of sound through the fluid or gas. There are several branches of subsonic flow but one special case arises when the flow is inviscid, incompressible and irrotational. This case is called Potential flow and allows the differential equations used to be a simplified version of the governing equations of fluid dynamics, thus making available to the aerodynamicist a range of quick and easy solutions. It is a special case of Subsonic aerodynamics.

In solving a subsonic problem, one decision to be made by the aerodynamicist is whether to incorporate the effects of compressibility. Compressibility is a description of the amount of change of density in the problem. When the effects of compressibility on the solution are small, the aerodynamicist may choose to assume that density is constant. The problem is then an incompressible low-speed aerodynamics problem. When the density is allowed to vary, the problem is called a compressible problem. In air, compressibility effects are usually ignored when the Mach number in the flow does not exceed 0.3 (about 335 feet (102m) per second or 228 miles (366 km) per hour at 60^{o}F). Above 0.3, the problem should be solved by using compressible aerodynamics.

Compressible aerodynamics

According to the theory of aerodynamics, a flow is considered to be compressible if its change in density with respect to pressure is non-zero along a streamline. This means that

- unlike incompressible flow - changes in density must be considered. In general, this is the case where the Mach number in part or all of the flow exceeds 0.3. The Mach .3 value is rather arbitrary, but it is used because gas flows with a Mach number below that value demonstrate changes in density with respect to the change in pressure of less than 5%. Furthermore, that maximum 5% density change occurs at the stagnation point of an object immersed in the gas flow and the density changes around the rest of the object will be significantly lower. Transonic, supersonic, and hypersonic flows are all compressible.

Transonic flow

The term Transonic refers to a range of velocities just below and above the local speed of sound (generally taken as Mach 0.8–1.2). It is defined as the range of speeds between the critical Mach number, when some parts of the airflow over an aircraft become supersonic, and a higher speed, typically near Mach 1.2, when all of the airflow is supersonic. Between these speeds some of the airflow is supersonic, and some is not.

Supersonic flow

Supersonic aerodynamic problems are those involving flow speeds greater than the speed of sound. Calculating the lift on the Concorde during cruise can be an example of a supersonic aerodynamic problem.

Supersonic flow behaves very differently from subsonic flow. Fluids react to differences in pressure; pressure changes are how a fluid is "told" to respond to its environment. Therefore, since sound is in fact an infinitesimal pressure difference propagating through a fluid, the speed of sound in that fluid can be considered the fastest speed that "information" can travel in the flow. This difference most obviously manifests itself in the case of a fluid striking an object. In front of that object, the fluid builds up a stagnation pressure as impact with the object brings the moving fluid to rest. In fluid traveling at subsonic speed, this pressure disturbance can propagate upstream, changing the flow pattern ahead of the object and giving the impression that the fluid "knows" the object is there and is avoiding it. However, in a supersonic flow, the pressure disturbance cannot propagate upstream. Thus, when the fluid finally does strike the object, it is forced to change its properties -- temperature, density, pressure, and Mach number -- in an extremely violent and irreversible fashion called a shock wave. The presence of shock waves, along with the compressibility effects of high-velocity fluids, is the central difference between supersonic and subsonic aerodynamics problems.

Hypersonic flow

In aerodynamics, hypersonic speeds are speeds that are highly supersonic. In the 1970s, the term generally came to refer to speeds of Mach 5 (5 times the speed of sound) and above. The hypersonic regime is a subset of the supersonic regime. Hypersonic flow is characterized by high temperature flow behind a shock wave, viscous interaction, and chemical dissociation of gas.

Associated terminology

The incompressible and compressible flow regimes produce many associated phenomena, such as boundary layers and turbulence.

Boundary layers

The concept of a boundary layer is important in many aerodynamic problems. The viscosity and fluid friction in the air is approximated as being significant only in this thin layer. This principle makes aerodynamics much more tractable mathematically.

Turbulence

In aerodynamics, turbulence is characterized by chaotic, stochastic property changes in the flow. This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time. Flow that is not turbulent is called laminar flow.

Aerodynamics in other fields

Aerodynamics is important in a number of applications other than aerospace engineering. It is a significant factor in any type of vehicle design, including automobiles. It is important in the prediction of forces and moments in sailing. It is used in the design of large components such as hard drive heads. Structural engineers also use aerodynamics, and particularly aeroelasticity, to calculate wind loads in the design of large buildings and bridges. Urban aerodynamics seeks to help town planners and designers improve comfort in outdoor spaces, create urban microclimates and reduce the effects of urban pollution. The field of environmental aerodynamics studies the ways atmospheric circulation and flight mechanics affect ecosystems. The aerodynamics of internal passages is important in heating/ventilation, gas piping, and in automotive engines where detailed flow patterns strongly affect the performance of the engine.

Chapter- 4

Propeller

The feathered propellers of an RAF Hercules C.4

Aircraft propellers convert rotary motion from piston engines or turboprops to provide propulsive force. They may be fixed or variable pitch. Early aircraft propellers were carved by hand from solid or laminated wood with later propellers being constructed from metal. The most modern propeller designs use high-technology composite materials.

The propeller is usually attached to the crankshaft of a piston engine, either directly or through a reduction unit. Light aircraft engines often do not require the complexity of gearing but on larger engines and turboprop aircraft it is essential.

History

The twisted airfoil (aerofoil) shape of modern aircraft propellers was pioneered by the Wright brothers. They found that a propeller is essentially the same as a wing and so were able to use data collated from their earlier wind tunnel experiments on wings. They also found that the relative angle of attack from the forward movement of the aircraft was different for all points along the length of the blade, thus it was necessary to introduce a twist along its length. Their original propeller blades were only about 5% less efficient than the modern equivalent, some 100 years later.

Alberto Santos Dumont was another early pioneer, having designed propellers before the Wright Brothers (albeit not as efficient) for his airships. He applied the knowledge he gained from experiences with airships to make a propeller with a steel shaft and aluminium blades for his 14 bis biplane. Some of his designs used a bent aluminium sheet for blades, thus creating an airfoil shape. These are heavily undercambered because of this and combined with the lack of a lengthwise twist made them less efficient than the Wright propellers. Even so, this was perhaps the first use of aluminium in the construction of an airscrew.

Theory and design of aircraft propellers

A well-designed propeller typically has an efficiency of around 80% when operating in the best regime.Changes to a propeller's efficiency are produced by a number of factors, notably adjustments to the helix angle(θ), the angle between the resultant relative velocity and the blade rotation direction, and to blade pitch (where $\theta = \Phi + \alpha$) . Very small pitch and helix angles give a good performance against resistance but provide little thrust, while larger angles have the opposite effect. The best helix angle is when the blade is acting as a wing producing much more lift than drag.

A propeller's efficiency is determined by

$$\eta = \frac{\text{propulsive power out}}{\text{shaft power in}} = \frac{\text{thrust} \cdot \text{axial speed}}{\text{resistance torque} \cdot \text{rotational speed}}.$$

Propellers are similar in aerofoil section to a low drag wing and as such are poor in operation when at other than their optimum angle of attack. Control systems are required to counter the need for accurate matching of pitch to flight speed and engine speed.

The three-bladed propeller of a light aircraft: the Vans RV-7A

A further consideration is the number and the shape of the blades used. Increasing the aspect ratio of the blades reduces drag but the amount of thrust produced depends on blade area, so using high aspect blades can lead to the need for a propeller diameter which is unusable. A further balance is that using a smaller number of blades reduces interference effects between the blades, but to have sufficient blade area to transmit the available power within a set diameter means a compromise is needed. Increasing the number of blades also decreases the amount of work each blade is required to perform, limiting the local Mach number - a significant performance limit on propellers.

A propeller's performance suffers as the blade speed exceeds the speed of sound. As the relative air speed at the blade is rotation speed plus axial speed, a propeller blade tip will reach sonic speed sometime before the rest of the aircraft (with a theoretical blade the maximum aircraft speed is about 845 km/h (Mach 0.7) at sea-level, in reality it is rather lower). When a blade tip becomes supersonic, drag and torque resistance increase suddenly and shock waves form creating a sharp increase in noise. Aircraft with conventional propellers, therefore, do not usually fly faster than Mach 0.6. There are certain propeller-driven aircraft, usually military, which do operate at Mach 0.8 or higher, although there is considerable fall off in efficiency.

There have been efforts to develop propellers for aircraft at high subsonic speeds. The 'fix' is similar to that of transonic wing design. The maximum relative velocity is kept as low as possible by careful control of pitch to allow the blades to have large helix angles;

thin blade sections are used and the blades are swept back in a scimitar shape (Scimitar propeller); a large number of blades are used to reduce work per blade and so circulation strength; contra-rotation is used. The propellers designed are more efficient than turbo-fans and their cruising speed (Mach 0.7–0.85) is suitable for airliners, but the noise generated is tremendous.

Forces acting on a propeller

Five forces act on the blades of an aircraft propeller in motion, they are:

Thrust bending force
: Thrust loads on the blades act to bend them forwards, opposite to the direction of flight.

Centrifugal twisting force
: Acts to twist the blades to a low or fine pitch angle.

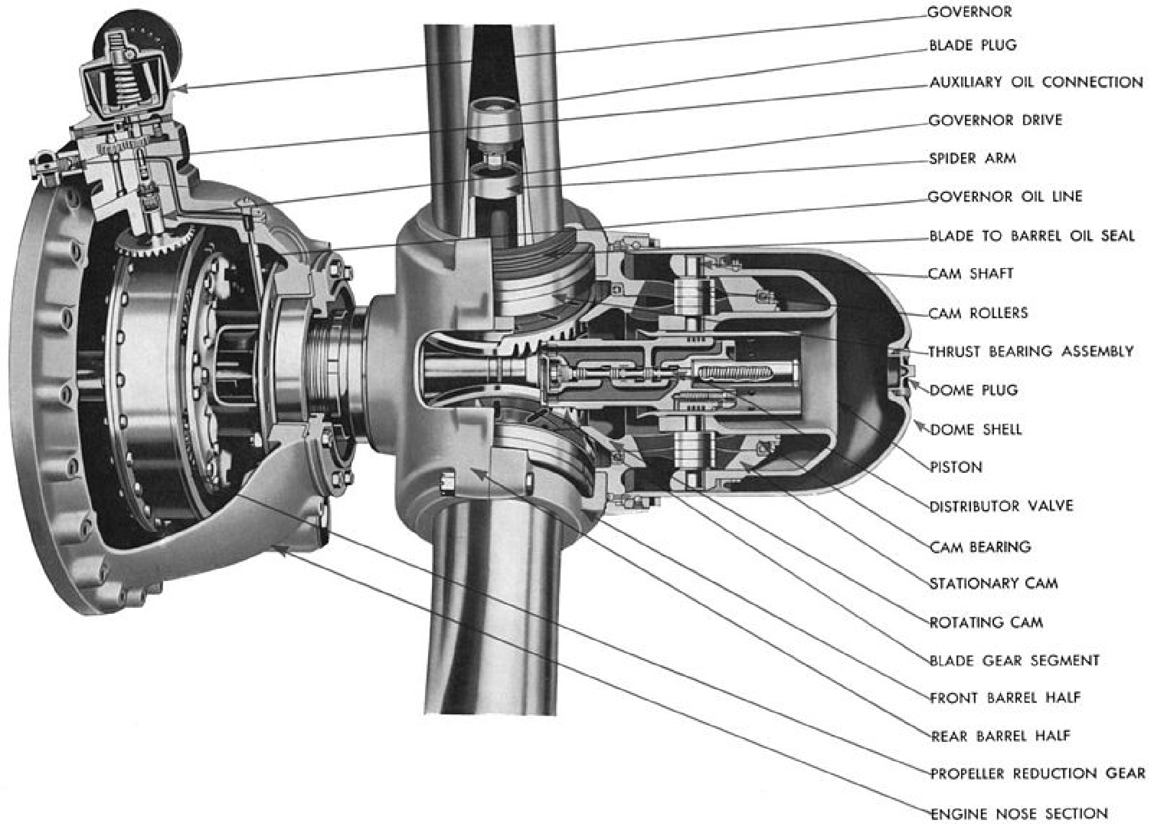
Aerodynamic twisting force
: As the centre of pressure of a propeller blade is forward of its centreline the blade is twisted towards a coarse pitch position.

Centrifugal force
: The force felt by the blades acting to pull them away from the hub when turning.

Torque bending force
: Air resistance acting against the blades, combined with inertial effects causes propeller blades to bend away from the direction of rotation.

Propeller control

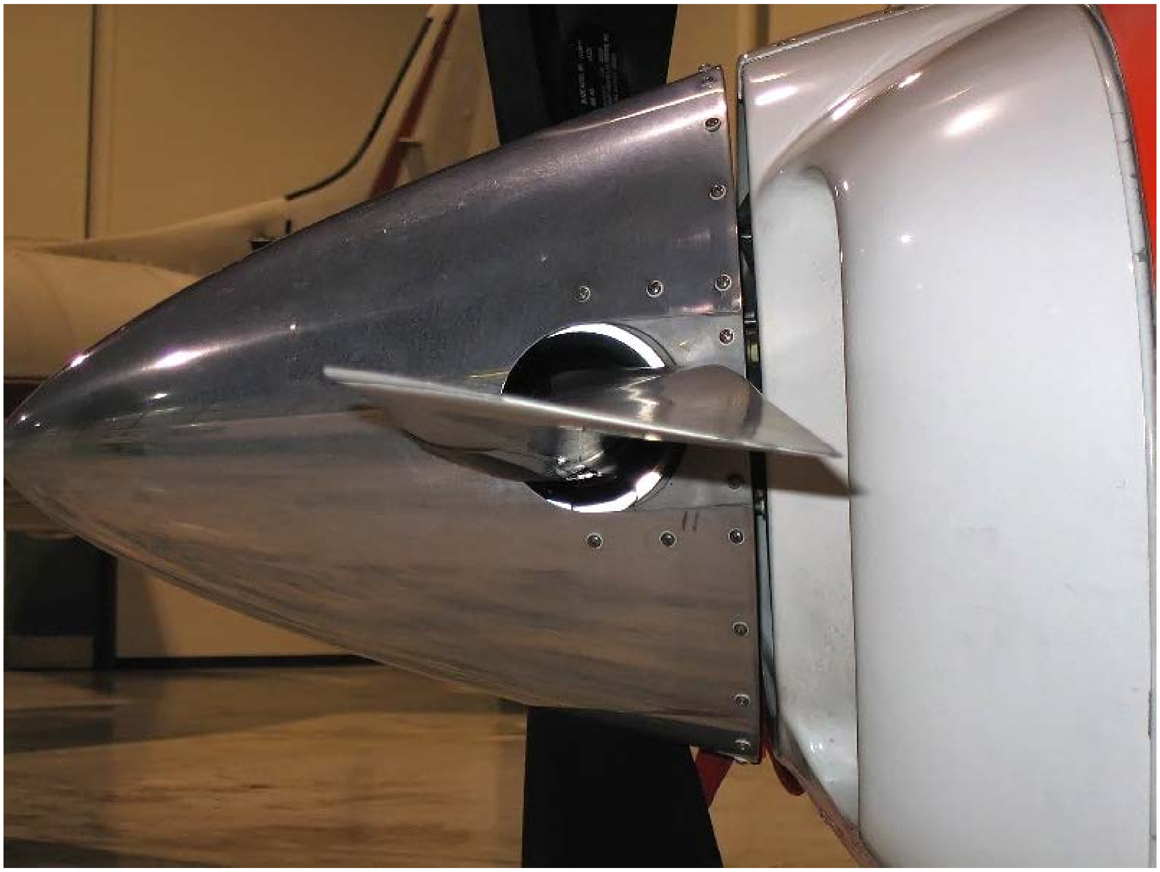
Variable pitch



Cut away view of a Hamilton Standard propeller. The engineers at Hamilton came up with a propeller design that was very reliable and maintenance friendly. With an auxiliary oil connection to feather the propeller in flight, gave multi-engined aircraft a huge safety factor. This type of propeller was used on most American fighters, bombers and transport aircraft of WWII.

The purpose of varying pitch angle with a variable pitch propeller is to maintain an optimal angle of attack (maximum lift to drag ratio) on the propeller blades as aircraft speed varies. Early pitch control settings were pilot operated, either two-position or manually variable. Following World War II, automatic propellers were developed to maintain an optimum angle of attack. This was done by balancing the centripetal twisting moment on the blades and a set of counterweights against a spring and the aerodynamic forces on the blade. Automatic props had the advantage of being simple, lightweight, and requiring no external control, but a particular propeller's performance was difficult to match with that of the aircraft's powerplant. An improvement on the automatic type was the constant-speed propeller. Constant speed propellers allow the pilot to select a rotational speed for maximum engine power or maximum efficiency, and a propeller governor acts as a closed-loop controller to vary propeller pitch angle as required to maintain the RPM commanded by the pilot. In most aircraft this system is hydraulic, with engine oil serving as the hydraulic fluid. However, electrically controlled propellers were developed during World War II and saw extensive use on military aircraft, and have recently seen a revival in use on homebuilt aircraft.

Feathering



A propeller blade in feathered position

On some variable-pitch propellers, the blades can be rotated parallel to the airflow to reduce drag in case of an engine failure. This is called *feathering*. Feathering propellers were developed for military fighter aircraft prior to World War II, as a fighter is more likely to experience an engine failure due to the inherent danger of combat. On single-engined aircraft, whether a powered glider or turbine powered aircraft, the effect is to increase the gliding distance. On a multi-engine aircraft, feathering the propeller on a failed engine reduces drag.

Most feathering systems for reciprocating engines sense a drop in oil pressure and move the blades toward the feather position, and require the pilot to pull the propeller control back to disengage the high-pitch stop pins before the engine reaches idle RPM. Turboprop control systems usually utilize a *negative torque sensor* in the reduction gearbox which moves the blades toward feather when the engine is no longer providing power to the propeller. Depending on design, the pilot may have to push a button to override the high-pitch stops and complete the feathering process, or the feathering process may be totally automatic.

Reverse pitch

Contra-rotating propellers of a modified
P-51 Mustang fitted with a Rolls-Royce Griffon.

In some aircraft (e.g., the C-130 Hercules), the pilot can manually override the constant speed mechanism to reverse the blade pitch angle, and thus the thrust of the engine (although the rotation of the engine itself does not reverse). This is used to help slow the plane down after landing in order to save wear on the brakes and tires, but in some cases also allows the aircraft to back up on its own. This is known as "Beta Range" operation.

Contra-rotating propellers

Contra-rotating propellers use a second propeller rotating in the opposite direction immediately 'downstream' of the main propeller so as to recover energy lost in the swirling motion of the air in the propeller slipstream. Contra-rotation also increases power without increasing propeller diameter and provides a counter to the torque effect of high-power piston engine as well as the gyroscopic precession effects, and of the slipstream swirl. However on small aircraft the added cost, complexity, weight and noise of the system rarely make it worthwhile.

Counter-rotating propellers

Counter-rotating propellers, are found on twin-, and multi-engine, propeller-driven aircraft and have propellers that spin in opposite directions. Generally, the propellers on both engines of most conventional twin-engined aircraft spin clockwise (as viewed from the rear of the aircraft). Counter-rotating propellers generally spin clockwise on the left engine, and counter-clockwise on the right. The advantage of counter-rotating propellers is to balance out the effects of torque and p-factor, eliminating the problem of the critical engine.

Aircraft fans

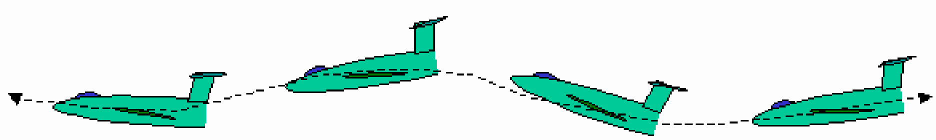
A fan is a propeller with a large number of blades. A fan therefore produces a lot of thrust for a given diameter but the closeness of the blades means that each strongly affects the flow around the others. If the flow is supersonic, this interference can be beneficial if the flow can be compressed through a series of shock waves rather than one. By placing the fan within a shaped duct, specific flow patterns can be created depending on flight speed and engine performance. As air enters the duct, its speed is reduced while its pressure and temperature increases. If the aircraft is at a high subsonic speed this creates two advantages: the air enters the fan at a lower Mach speed; and the higher temperature increases the local speed of sound. While there is a loss in efficiency as the fan is drawing on a smaller area of the free stream and so using less air, this is balanced by the ducted fan retaining efficiency at higher speeds where conventional propeller efficiency would be poor. A ducted fan or propeller also has certain benefits at lower speeds but the duct needs to be shaped in a different manner than one for higher speed flight. More air is taken in and the fan therefore operates at an efficiency equivalent to a larger un-ducted propeller. Noise is also reduced by the ducting and should a blade become detached the duct would contain the damage. However the duct adds weight, cost, complexity and (to a certain degree) drag.

Chapter- 5

Phugoid and Dutch Roll

Phugoid

A **phugoid** is an aircraft motion where the vehicle pitches up and climbs, and then pitches down and descends, accompanied by speeding up and slowing down as it goes "uphill" and "downhill." This is one of the basic flight dynamics modes of an aircraft (others include short period, dutch roll, and spiral divergence), and a classic example of a positive feedback system.



A diagrammatic representation of a fixed-wing airplane in phugoid

Detailed description

The phugoid has a nearly constant angle of attack but varying pitch, caused by a repeated exchange of airspeed and altitude. It can be excited by an elevator singlet (a short, sharp deflection followed by a return to the centered position) resulting in a pitch increase with no change in trim from the cruise condition. As speed decays, the nose will drop below the horizon. Speed will increase, and the nose will climb above the horizon. Periods can vary from under 30 seconds for light aircraft to minutes for larger aircraft. Microlight aircraft typically show a phugoid period of 15–25 seconds, and it has been suggested that birds and model airplanes show convergence between the phugoid and short period modes. A classical model for the phugoid period can be simplified to about (0.85 × speed in knots) seconds, but this only really works for larger aircraft.

Phugoids are often demonstrated to student pilots as an example of the speed stability of the aircraft and the importance of proper trimming. When it occurs, it is considered a nuisance, and in lighter airplanes (typically showing a shorter period) it can be a cause of pilot-induced oscillation.

The phugoid, for moderate amplitude, occurs at an effectively constant angle of attack, although in practice the angle of attack actually varies by a few tenths of a degree. This means that the stalling angle of attack is never exceeded, and it is possible (in the $<1g$ section of the cycle) to fly at speeds below the known stalling speed. Free flight models with badly unstable phugoid typically stall or loop, depending on thrust.

An unstable or divergent phugoid is caused, mainly, by a large difference between the incidence angles of the wing and tail. A stable, decreasing phugoid can be attained by building a smaller stabilizer on a longer tail, or, at the expense of pitch and yaw "static" stability, by shifting the center of gravity to the rear.

The term "phugoid" was coined by Frederick W. Lanchester, the British aerodynamicist who first characterized the phenomenon. He derived the word from the Greek words φυγή and εἶδος to mean "flight-like" but recognized the diminished appropriateness of the derivation given that φυγή meant flight in the sense of "escape" rather than vehicle flight.

Aviation incidents

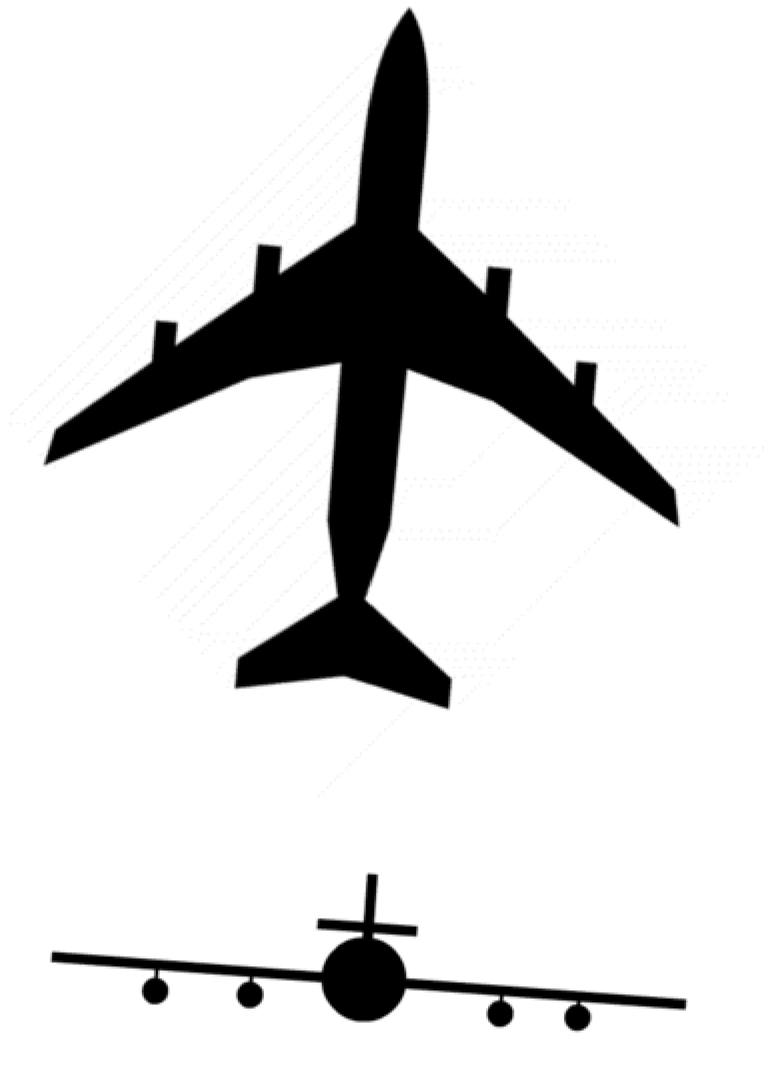
Japan Airlines Flight 123 lost all hydraulic controls and its vertical stabiliser, and went into a phugoid before crashing into a mountain. With 520 deaths it remains the deadliest single-aircraft disaster in history.

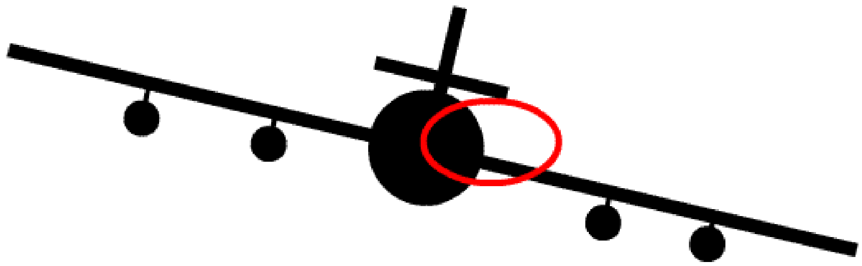
United Airlines Flight 232 suffered an engine failure which caused total hydraulic system failure. The crew flew the aircraft with throttle only. Suppressing the phugoid tendency was particularly difficult. The pilots were able to reach Sioux Gateway Airport but crashed during the landing attempt. The pilots and a majority of the passengers survived.

Another aircraft that lost all hydraulics was a DHL operated Airbus A300B4 that was hit by a surface-to-air missile fired by the Iraqi resistance in the 2003 Baghdad DHL attempted shootdown incident. This was the first time that a crew was able to land an air transport aircraft safely only adjusting engine thrust.

The crash of the Helios solar powered aircraft was precipitated by reacting to an inappropriately diagnosed phugoid oscillation the results of which ultimately resulted in the aircraft structure exceeding design loads.

Dutch roll





Dutch roll is a type of aircraft motion, consisting of an out-of-phase combination of "tail-wagging" and rocking from side to side. This yaw-roll coupling is one of the basic flight dynamic modes (others include phugoid, short period, and spiral divergence). This motion is normally well damped in most light aircraft, though some aircraft with well-damped Dutch roll modes can experience a degradation in damping as airspeed decreases and altitude increases. Dutch roll stability can be artificially increased by the installation of a yaw damper. Wings placed well above the center of mass, sweepback (swept wings) and dihedral wings tend to increase the roll restoring force, and therefore increase the Dutch roll tendencies; this is why high-winged aircraft often are slightly anhedral, and transport category swept wing aircraft are equipped with yaw dampers.

In aircraft design, Dutch roll results from relatively weaker positive directional stability as opposed to positive lateral stability. When an aircraft rolls around the longitudinal axis, a sideslip is introduced into the relative wind in the direction of the rolling motion. Strong lateral stability begins to restore the aircraft to level flight. At the same time, somewhat weaker directional stability attempts to correct the sideslip by aligning the aircraft with the perceived relative wind. Since directional stability is weaker than lateral stability for the particular aircraft, the restoring yaw motion lags significantly behind the restoring roll motion. As such, the aircraft passes through level flight as the yawing motion is continuing in the direction of the original roll. At that point, the sideslip is introduced in the opposite direction and the process is reversed.

The Dutch roll mode can be excited by any use of aileron or rudder, but for flight test purposes it is usually excited with a rudder singlet (short, sharp motions of the rudder to a specified angle, and then back to the centered position) or doublet (a pair of such motions in opposite directions). Some larger aircraft are better excited with aileron inputs. Periods can range from a few seconds for light aircraft to a minute or more for airliners.

Dutch roll is also the name (considered by professionals to be a misnomer) given to a coordination maneuver generally taught to student pilots to help them improve their "stick-and-rudder" technique. The aircraft is alternately rolled as much as 60 degrees left and right while rudder is applied to keep the nose of the aircraft pointed at a fixed point.

This coordination technique is better referred to as "rolling on a heading", where the aircraft is rolled in such a way as to maintain an accurate heading without the nose moving from side-to-side (or yawing). The yaw motion is induced through the use of ailerons alone due to aileron drag where the lifting wing (aileron down) is doing more work than the descending wing (aileron up) and therefore creates more drag, forcing the lifting wing back, yawing the aircraft toward it. This yawing effect produced by rolling motion is known as adverse yaw. This has to be countered precisely by application of rudder *in the same direction* as the aileron control (left stick, left rudder - right stick, right rudder). This is known as synchronised controls when done properly, and is difficult to learn and apply well. As each aircraft is different, learning the correct amount of rudder to apply with aileron is different for each aircraft.

The origin of the name Dutch roll is uncertain. However, it is likely that this term, describing a lateral asymmetric motion of an airplane, was borrowed from a reference to similar appearing motion in ice skating. In 1916, aeronautical engineer Jerome C. Hunsaker published the following quote: “Dutch roll – the third element in the [lateral] motion [of an airplane] is a yawing to the right and left, combined with rolling. The motion is oscillatory of period for 5 to 12 seconds which may or may not be damped. The analogy to ‘Dutch Roll’ or ‘Outer Edge’ in ice skating is obvious.” In 1916, Dutch Roll was the term used for skating repetitively to right and left (by analogy to the motion described for the aircraft) on the outer edge of one’s skates. By 1916, the term had been imported from skating to aeronautical engineering, perhaps by Hunsaker himself. 1916 was only five years after G.H. Bryan did the first mathematics of lateral motion of aircraft in 1911.

Chapter- 6

Mach Number

An F/A-18 Hornet at transonic speed and displaying the Prandtl–Glauert singularity just before reaching the speed of sound

Mach number (Ma or *M*) is the speed of an object moving through air, or any other fluid substance, divided by the speed of sound as it is in that substance for its particular physical conditions, including those of temperature and pressure. It is commonly used to represent the speed of an object when it is traveling close to or above the speed of sound.

$$M = \frac{V}{a}$$

where

M is the Mach number
V is the relative velocity of the source to the medium and
a is the speed of sound in the medium

The Mach number is named after Austrian physicist and philosopher Ernst Mach, a designation proposed by aeronautical engineer Jakob Ackeret. Because the Mach number is often viewed as a dimensionless quantity rather than a unit of measure, with Mach, the number comes *after* the unit; the second Mach number is "Mach 2" instead of "2 Mach" (or Machs). This is somewhat reminiscent of the early modern ocean sounding unit "mark" (a synonym for fathom), which was also unit-first, and may have influenced the use of the term Mach. In the decade preceding faster-than-sound human flight, aeronautical engineers referred to the speed of sound as *Mach's number*, never "Mach 1."

In French, the Mach number is sometimes called the "nombre de Sarrau" ("Sarrau number") after Émile Sarrau, researching on explosions in the 1870s and 1880s.

Overview

The Mach number is commonly used both with objects traveling at high speed in a fluid, and with high-speed fluid flows inside channels such as nozzles, diffusers or wind tunnels. As it is defined as a ratio of two speeds, it is a dimensionless number. At Standard Sea Level conditions (corresponding to a temperature of 15 degrees Celsius), the speed of sound is 340.3 m/s (1225 km/h, or 761.2 mph, or 661.5 knots, or 1116 ft/s) in the Earth's atmosphere. The speed represented by Mach 1 is not a constant; for example, it is mostly dependent on temperature and atmospheric composition and largely independent of pressure. In the stratosphere, where the temperatures are constant, it does not vary with altitude even though the air pressure changes significantly with altitude.

Since the speed of sound increases as the temperature increases, the actual speed of an object traveling at Mach 1 will depend on the fluid temperature around it. Mach number is useful because the fluid behaves in a similar way at the same Mach number. So, an aircraft traveling at Mach 1 at 20°C or 68°F will experience shock waves in much the same manner as when it is traveling at Mach 1 at 11,000 m (36,000 ft) at -50°C or -58F, even though it is traveling at only 86% of its speed at higher temperature like 20°C or 68°F.

High-speed flow around objects

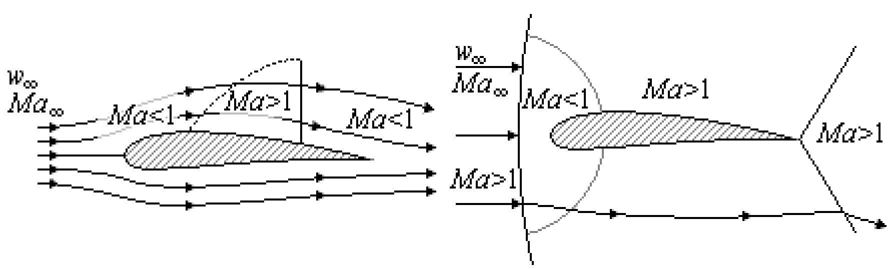
Flight can be roughly classified in six categories:

Regime	Subsonic	Transonic	Sonic	Supersonic	Hypersonic	High-hypersonic
Mach	<0.75	0.75–1.2	1.0	1.2–5.0	5.0–10.0	>10.0

For comparison: the required speed for low Earth orbit is approximately 7.5 km/s = Mach 25.4 in air at high altitudes. The speed of light in a vacuum corresponds to a Mach number of approximately 881,000 (relative to air at sea level).

At transonic speeds, the flow field around the object includes both sub- and supersonic parts. The transonic period begins when first zones of M>1 flow appear around the object. In case of an airfoil (such as an aircraft's wing), this typically happens above the wing. Supersonic flow can decelerate back to subsonic only in a normal shock; this typically happens before the trailing edge. (Fig.1a)

As the speed increases, the zone of *M*>1 flow increases towards both leading and trailing edges. As *M*=1 is reached and passed, the normal shock reaches the trailing edge and becomes a weak oblique shock: the flow decelerates over the shock, but remains supersonic. A normal shock is created ahead of the object, and the only subsonic zone in the flow field is a small area around the object's leading edge. (Fig.1b)



(a) (b)

Fig. 1. Mach number in transonic airflow around an airfoil; M<1 (a) and M>1 (b).

When an aircraft exceeds Mach 1 (i.e. the sound barrier) a large pressure difference is created just in front of the aircraft. This abrupt pressure difference, called a shock wave, spreads backward and outward from the aircraft in a cone shape (a so-called Mach cone). It is this shock wave that causes the sonic boom heard as a fast moving aircraft travels overhead. A person inside the aircraft will not hear this. The higher the speed, the more narrow the cone; at just over *M*=1 it is hardly a cone at all, but closer to a slightly concave plane.

At fully supersonic speed, the shock wave starts to take its cone shape and flow is either completely supersonic, or (in case of a blunt object), only a very small subsonic flow area remains between the object's nose and the shock wave it creates ahead of itself. (In the case of a sharp object, there is no air between the nose and the shock wave: the shock wave starts from the nose.)

As the Mach number increases, so does the strength of the shock wave and the Mach cone becomes increasingly narrow. As the fluid flow crosses the shock wave, its speed is reduced and temperature, pressure, and density increase. The stronger the shock, the greater the changes. At high enough Mach numbers the temperature increases so much over the shock that ionization and dissociation of gas molecules behind the shock wave begin. Such flows are called hypersonic.

It is clear that any object traveling at hypersonic speeds will likewise be exposed to the same extreme temperatures as the gas behind the nose shock wave, and hence choice of heat-resistant materials becomes important.

High-speed flow in a channel

As a flow in a channel crosses M=1 becomes supersonic, one significant change takes place. The conservation of mass flow rate leads one to expect that contracting the flow channel would increase the flow speed (i.e. making the channel narrower results in faster air flow) and at subsonic speeds this holds true. However, once the flow becomes supersonic, the relationship of flow area and speed is reversed: expanding the channel actually increases the speed.

The obvious result is that in order to accelerate a flow to supersonic, one needs a convergent-divergent nozzle, where the converging section accelerates the flow to M=1, sonic speeds, and the diverging section continues the acceleration. Such nozzles are called de Laval nozzles and in extreme cases they are able to reach incredible, hypersonic speeds (Mach 13 at 20°C).

An aircraft Machmeter or electronic flight information system (EFIS) can display Mach number derived from stagnation pressure (pitot tube) and static pressure.

Calculating Mach Number

Assuming air to be an ideal gas, the formula to compute Mach number in a subsonic compressible flow is derived from Bernoulli's equation for M<1:

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{q_c}{p} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

where:

M is Mach number
q_c is impact pressure and
p is static pressure
γ is the ratio of specific heat of a gas at a constant pressure to heat at a constant volume (1.4 for air).

The formula to compute Mach number in a supersonic compressible flow is derived from the Rayleigh Supersonic Pitot equation:

$$\frac{q_c}{p} = \left[\left(\frac{\gamma+1}{2}\right)M^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)} \cdot \left[\frac{\gamma+1}{(1-\gamma+2\gamma\cdot M^2)^{\left(\frac{1}{\gamma-1}\right)}}\right]^{\left(\frac{1}{\gamma-1}\right)} - 1$$

or for air, a simplfied formula:

$$M = 0.88128485\sqrt{\left[\left(\frac{q_c}{p}+1\right)\left(1-\frac{1}{[7M^2]}\right)^{2.5}\right]}$$

where:

q_c is now impact pressure measured behind a normal shock.

The Mach number at which an aircraft is flying at can be calculated by

$$M = \frac{V}{a}$$

where:

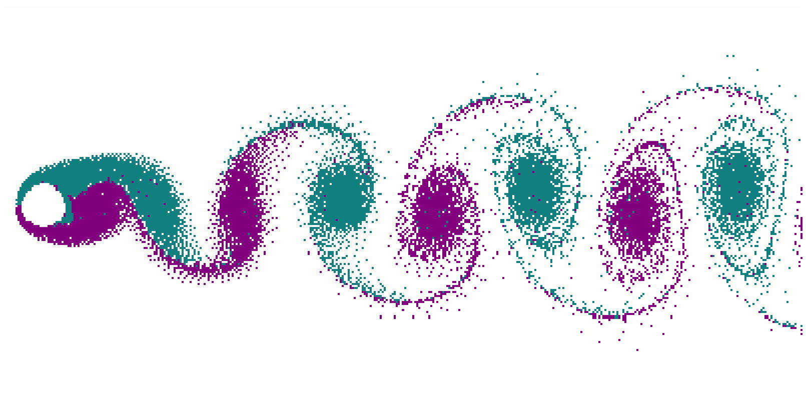
M is Mach number
V is velocity of the moving aircraft and
a is the speed of sound at the given altitude

Note that the dynamic pressure can be found as:

$$q = \frac{\gamma}{2}pM^2$$

Chapter- 7

Reynolds Number



A vortex street around a cylinder. This occurs around cylinders, independently of the fluid, the cylinder size and the fluid speed, provided that there is a Reynolds number of between 250 and 200,000.

In fluid mechanics, the **Reynolds number** Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842–1912), who popularized its use in 1883.

Reynolds numbers frequently arise when performing dimensional analysis of fluid dynamics problems, and as such can be used to determine dynamic similitude between different experimental cases. They are also used to characterize different flow regimes, such as laminar or turbulent flow: laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

Definition

Reynolds number can be defined for a number of different situations where a fluid is in relative motion to a surface (the definition of the Reynolds number is not to be confused with the Reynolds Equation or lubrication equation). These definitions generally include the fluid properties of density and viscosity, plus a velocity and a *characteristic length* or *characteristic dimension*. This dimension is a matter of convention - for example a radius or diameter are equally valid for spheres or circles, but one is chosen by convention. For aircraft or ships, the length or width can be used. For flow in a pipe or a sphere moving in a fluid the internal diameter is generally used today. Other shapes (such as rectangular pipes or non-spherical objects) have an *equivalent diameter* defined. For fluids of variable density (e.g. compressible gases) or variable viscosity (non-Newtonian fluids) special rules apply. The velocity may also be a matter of convention in some circumstances, notably stirred vessels.

$$\mathrm{Re} = \frac{\rho \mathrm{V} L}{\mu} = \frac{\mathrm{V} L}{\nu}$$

where:

- V is the mean velocity of the object relative to the fluid (SI units: m/s)
- L is a characteristic linear dimension, (travelled length of the fluid; hydraulic diameter when dealing with river systems) (m)
- μ is the dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/(m·s))
- ν is the kinematic viscosity (ν = μ / ρ) (m²/s)
- ρ is the density of the fluid (kg/m³)

Note that multiplying the Reynolds number, $\frac{\rho V L}{\mu}$ by $\frac{VL}{VL}$ yields $\frac{\rho V^2 L^2}{\mu V L}$ which is the ratio, $\frac{inertia force (drag)}{viscous force}$.

Significance

$$\mathrm{Re} = \frac{\text{total momentum transfer}}{\text{molecular momentum transfer}}$$

Flow in Pipe

For flow in a pipe or tube, the Reynolds number is generally defined as:

$$\mathrm{Re} = \frac{\rho \mathrm{V} D_H}{\mu} = \frac{\mathrm{V} D_H}{\nu} = \frac{\mathrm{Q} D_H}{\nu A}$$

where:

- D_H is the hydraulic diameter of the pipe (m).
- Q is the volumetric flow rate (m^3/s)
- A is the pipe *cross-sectional* area (m^2).

Flow in a non-circular duct (annulus)

For shapes such as squares, rectangular or annular ducts (where the height and width are comparable) the characteristic dimension for internal flow situations is taken to be the *hydraulic diameter*, D_H, defined as 4 times the cross-sectional area (of the fluid), divided by the **wetted perimeter**. The wetted perimeter for a channel is the total perimeter of all channel walls that are in contact with the flow. This means the length of the water exposed to air is NOT included in the wetted perimeter

$$D_H = \frac{4A}{P}.$$

For a circular pipe, the hydraulic diameter is exactly equal to the inside pipe diameter, as can be shown mathematically.

For an annular duct, such as the outer channel in a tube-in-tube heat exchanger, the hydraulic diameter can be shown algebraically to reduce to

$$D_{H,\text{annulus}} = D_o - D_i$$

where

D_o is the inside diameter of the outside pipe, and
D_i is the outside diameter of the inside pipe.

For calculations involving flow in non-circular ducts, the hydraulic diameter can be substituted for the diameter of a circular duct, with reasonable accuracy.

Flow in a Wide Duct

For a fluid moving between two plane parallel surfaces (where the width is much greater than the space between the plates) then the characteristic dimension is twice the distance between the plates.

Flow in an Open Channel

For flow of liquid with a free surface, the *hydraulic radius* must be determined. This is the cross-sectional area of the channel divided by the wetted perimeter. For a semi-circular channel, it is half the radius. For a rectangular channel, the hydraulic radius is the cross-sectional area divided by the wetted perimeter. Some texts then use a characteristic dimension that is 4 times the hydraulic radius (chosen because it gives the same value of Re for the onset of turbulence as in pipe flow), while others the hydraulic radius as the characteristic length-scale with consequently different values of Re for transition and turbulent flow.

Object in a fluid

The Reynolds number for an object in a fluid, called the particle Reynolds number and often denoted $\mathbf{Re_p}$, is important when considering the nature of flow around that grain, whether or not vortex shedding will occur, and its fall velocity.

Sphere in a fluid

For a sphere in a fluid, the characteristic length-scale is the diameter of the sphere and the characteristic velocity is that of the sphere relative to the fluid some distance away from the sphere (such that the motion of the sphere does not disturb that reference parcel of fluid). The density and viscosity are those belonging to the fluid. Note that purely laminar flow only exists up to Re = 0.1 under this definition.

Under the condition of low Re, the relationship between force and speed of motion is given by Stokes' law.

Oblong object in a fluid

The equation for an oblong object is identical to that of a sphere, with the object being approximated as an ellipsoid and the axis of length being chosen as the characteristic length scale. Such considerations are important in natural streams, for example, where there are few perfectly spherical grains. For grains in which measurement of each axis is impractical (e.g., because they are too small), sieve diameters are used instead as the characteristic particle length-scale. Both approximations alter the values of the critical Reynolds number.

Fall velocity

The particle Reynolds number is important in determining the fall velocity of a particle. When the particle Reynolds number indicates laminar flow, Stokes' law can be used to calculate its fall velocity. When the particle Reynolds number indicates turbulent flow, a turbulent drag law must be constructed to model the appropriate settling velocity.

Packed Bed

For flow of fluid through a bed of approximately spherical particles of diameter D in contact, if the voidage (fraction of the bed not filled with particles) is $\boldsymbol{\varepsilon}$ and the superficial velocity $\mathbf{V}$ (i.e. the velocity through the bed as if the particles were not there - the actual velocity will be higher) then a Reynolds number can be defined as:

$$\mathrm{Re} = \frac{\rho V D}{\mu(1-\epsilon)}.$$

Laminar conditions apply up to Re = 10, fully turbulent from 2000.

Stirred Vessel

In a cylindrical vessel stirred by a central rotating paddle, turbine or propellor, the characteristic dimension is the diameter of the agitator *D*. The velocity is *ND* where *N* is the rotational speed (revolutions per second). Then the Reynolds number is:

$$\mathrm{Re} = \frac{\rho N D^2}{\mu}.$$

The system is fully turbulent for values of Re above 10 000.

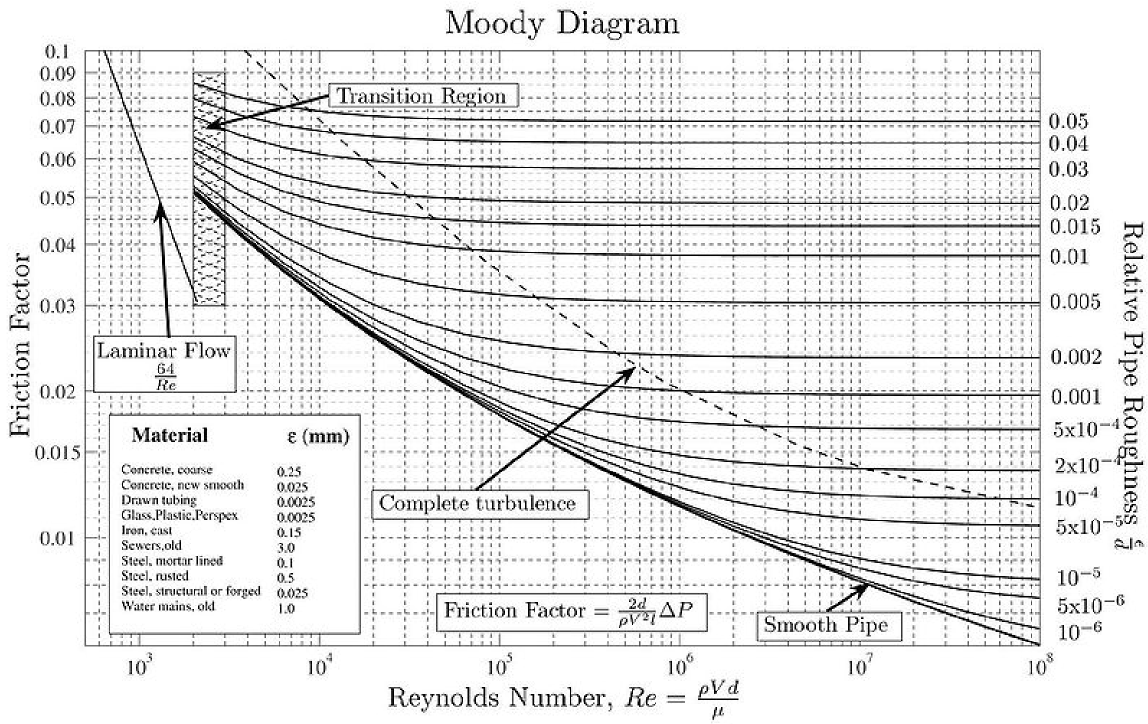
Transition Reynolds number

In boundary layer flow over a flat plate, experiments can confirm that, after a certain length of flow, a laminar boundary layer will become unstable and become turbulent. This instability occurs across different scales and with different fluids, usually when $\mathrm{Re}_x \approx 5 \times 10^5$, where *x* is the distance from the leading edge of the flat plate, and the flow velocity is the freestream velocity of the fluid outside the boundary layer.

For flow in a pipe of diameter *D*, experimental observations show that for 'fully developed' flow (Note:), laminar flow occurs when $\mathrm{Re}_D < 2300$ and turbulent flow occurs when $\mathrm{Re}_D > 4000$. In the interval between 2300 and 4000, laminar and turbulent flows are possible ('transition' flows), depending on other factors, such as pipe roughness and flow uniformity). This result is generalised to non-circular channels using the hydraulic diameter, allowing a transition Reynolds number to be calculated for other shapes of channel.

These transition Reynolds numbers are also called *critical Reynolds numbers*, and were studied by Osborne Reynolds around 1895.

Reynolds number in pipe friction



Pressure drops seen for fully-developed flow of fluids through pipes can be predicted using the Moody diagram which plots the Darcy–Weisbach friction factor *f* against Reynolds number Re and relative roughness ε / *D*. The diagram clearly shows the laminar, transition, and turbulent flow regimes as Reynolds number increases. The nature of pipe flow is strongly dependent on whether the flow is laminar or turbulent

The similarity of flows

In order for two flows to be similar they must have the same geometry, and have equal Reynolds numbers and Euler numbers. When comparing fluid behaviour at corresponding points in a model and a full-scale flow, the following holds:

$$\mathrm{Re}_m = \mathrm{Re}$$
$$\mathrm{Eu}_m = \mathrm{Eu} \qquad \text{i.e.} \quad \frac{p_m}{\varrho_m {v_m}^2} = \frac{p}{\varrho v^2}\ ,$$

quantities marked with 'm' concern the flow around the model and the others the actual flow. This allows engineers to perform experiments with reduced models in water channels or wind tunnels, and correlate the data to the actual flows, saving on costs during experimentation and on lab time. Note that true dynamic similitude may require matching other dimensionless numbers as well, such as the Mach number used in compressible flows, or the Froude number that governs open-channel flows. Some flows involve more dimensionless parameters than can be practically satisfied with the available apparatus and fluids (for example air or water), so one is forced to decide which parameters are most important. For experimental flow modeling to be useful, it requires a fair amount of experience and judgement of the engineer.

Typical values of Reynolds number

- Ciliate ~ 1×10^{-1}
- Smallest Fish ~ 1

- Blood flow in brain ~ 1×10^2
- Blood flow in aorta ~ 1×10^3

Onset of turbulent flow ~ 2.3×10^3 to 5.0×10^4 for pipe flow to 10^6 for boundary layers

- Typical pitch in Major League Baseball ~ 2×10^5
- Person swimming ~ 4×10^6
- Fastest Fish ~ 1×10^6
- Blue Whale ~ 3×10^8
- A large ship (RMS Queen Elizabeth 2) ~ 5×10^9

Reynolds number sets the smallest scales of turbulent motion

In a turbulent flow, there is a range of scales of the time-varying fluid motion. The size of the largest scales of fluid motion (sometimes called eddies) are set by the overall geometry of the flow. For instance, in an industrial smoke stack, the largest scales of fluid motion are as big as the diameter of the stack itself. The size of the smallest scales is set by the Reynolds number. As the Reynolds number increases, smaller and smaller scales of the flow are visible. In a smoke stack, the smoke may appear to have many very small velocity perturbations or eddies, in addition to large bulky eddies. In this sense, the Reynolds number is an indicator of the range of scales in the flow. The higher the Reynolds number, the greater the range of scales. The largest eddies will always be the same size; the smallest eddies are determined by the Reynolds number.

What is the explanation for this phenomenon? A large Reynolds number indicates that viscous forces are not important at large scales of the flow. With a strong predominance of inertial forces over viscous forces, the largest scales of fluid motion are undamped—there is not enough viscosity to dissipate their motions. The kinetic energy must "cascade" from these large scales to progressively smaller scales until a level is reached for which the scale is small enough for viscosity to become important (that is, viscous forces become of the order of inertial ones). It is at these small scales where the dissipation of energy by viscous action finally takes place. The Reynolds number indicates at what scale this viscous dissipation occurs. Therefore, since the largest eddies are dictated by the flow geometry and the smallest scales are dictated by the viscosity, the Reynolds number can be understood as the ratio of the largest scales of the turbulent motion to the smallest scales.

Example of the importance of the Reynolds number

If an airplane wing needs testing, one can make a scaled down model of the wing and test it in a wind tunnel using the same Reynolds number that the actual airplane is subjected to. If for example the scale model has linear dimensions one quarter of full size, the flow velocity of the model would have to be multiplied by a factor of 4 to obtain similar flow behavior.

Alternatively, tests could be conducted in a water tank instead of in air (provided the compressibility effects of air are not significant). As the kinematic viscosity of water is around 13 times less than that of air at 15 °C, in this case the scale model would need to be about one thirteenth the size in all dimensions to maintain the same Reynolds number, assuming the full-scale flow velocity was used.

The results of the laboratory model will be similar to those of the actual plane wing results. Thus there is no need to bring a full scale plane into the lab and actually test it. This is an example of "dynamic similarity".

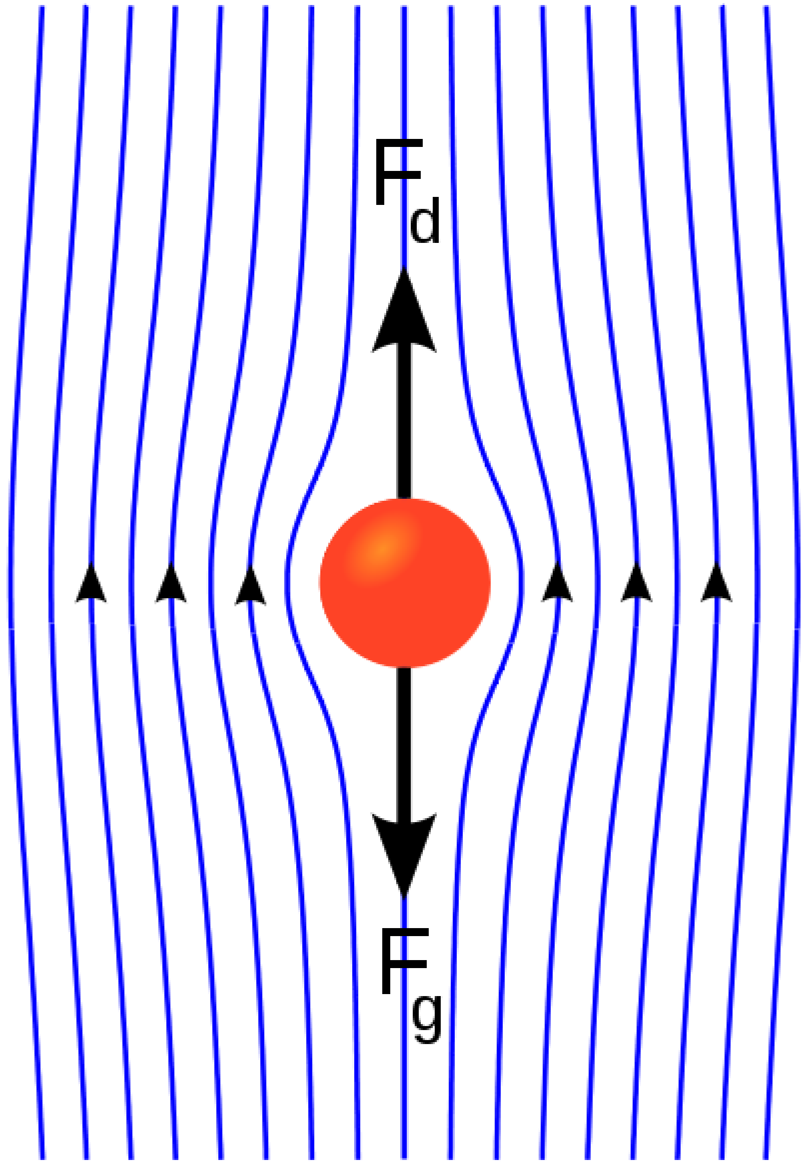
Reynolds number is important in the calculation of a body's drag characteristics. A notable example is that of the flow around a cylinder. Above roughly 3×10^6 Re the drag coefficient drops considerably. This is important when calculating the optimal cruise speeds for low drag (and therefore long range) profiles for airplanes.

Reynolds number in physiology

Poiseuille's law on blood circulation in the body is dependent on laminar flow. In turbulent flow the flow rate is proportional to the square root of the pressure gradient, as opposed to its direct proportionality to pressure gradient in laminar flow.

Using the definition of the Reynolds number we can see that a large diameter with rapid flow, where the density of the blood is high, tends towards turbulence. Rapid changes in vessel diameter may lead to turbulent flow, for instance when a narrower vessel widens to a larger one. Furthermore, an atheroma may be the cause of turbulent flow, and as such detecting turbulence with a stethoscope may be a sign of such a condition.

Reynolds number in viscous fluids



Creeping flow past a sphere: streamlines, drag force F_d and force by gravity F_g.

Where the viscosity is naturally high, such as polymer solutions and polymer melts, flow is normally laminar. The Reynolds number is very small and Stokes' Law can be used to measure the viscosity of the fluid. Spheres are allowed to fall through the fluid and they reach the terminal velocity quickly, from which the viscosity can be determined.

The laminar flow of polymer solutions is exploited by animals such as fish and dolphins, who exude viscous solutions from their skin to aid flow over their bodies while swimming. It has been used in yacht racing by owners who want to gain a speed advantage by pumping a polymer solution such as low molecular weight polyoxyethylene in water, over the wetted surface of the hull. It is however, a problem for mixing of polymers, because turbulence is needed to distribute fine filler (for example) through the material. Inventions such as the "cavity transfer mixer" have been developed to produce multiple folds into a moving melt so as to improve mixing efficiency. The device can be fitted onto extruders to aid mixing.

Where does it come from?

The Reynolds number can be obtained when one uses the nondimensional form of the incompressible Navier-Stokes equations:

$$\varrho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\mathbf{f}.$$

Each term in the above equation has the units of a volume force or, equivalently, an acceleration times a density. Each term is thus dependent on the exact measurements of a flow. When one renders the equation nondimensional, that is that we multiply it by a factor with inverse units of the base equation, we obtain a form which does not depend directly on the physical sizes. One possible way to obtain a nondimensional equation is to multiply the whole equation by the following factor:

$$\frac{D}{\varrho V^2}$$

where the symbols are the same as those used in the definition of the Reynolds number. If we now set:

$$\mathbf{v}'=\frac{\mathbf{v}}{V},\ p'=p\frac{1}{\varrho V^2},\ \mathbf{f}'=\mathbf{f}\frac{D}{\varrho V^2},\ \frac{\partial}{\partial t'}=\frac{D}{V}\frac{\partial}{\partial t},\ \nabla'=D\nabla$$

we can rewrite the Navier-Stokes equation without dimensions:

$$\frac{\partial \mathbf{v}'}{\partial t'}+\mathbf{v}'\cdot\nabla'\mathbf{v}'=-\nabla'p'+\frac{\mu}{\varrho DV}\nabla'^2\mathbf{v}'+\mathbf{f}'$$

where the term : $\frac{\mu}{\varrho DV}=\frac{1}{\mathrm{Re}}.$

Finally, dropping the primes for ease of reading:

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}=-\nabla p+\frac{1}{\mathrm{Re}}\nabla^2\mathbf{v}+\mathbf{f}.$$

This is why mathematically all flows with the same Reynolds number are comparable. Notice also, in the above equation, as :$\mathrm{Re} \to \infty$ the viscous terms vanish. Thus, high Reynolds number flows are approximately inviscid in the free-stream.

Chapter- 8

Knudsen Number

The **Knudsen number** (***Kn***) is a dimensionless number defined as the ratio of the molecular mean free path length to a representative physical length scale. This length scale could be, for example, the radius of a body in a fluid. The number is named after Danish physicist Martin Knudsen (1871–1949).

Definition

The Knudsen number is a dimensionless number defined as:

$$Kn = \frac{\lambda}{L}$$

where

- λ = mean free path $[L^1]$
- L = representative physical length scale $[L^1]$.

For an ideal gas, the mean free path may be readily calculated so that:

$$Kn = \frac{k_B T}{\sqrt{2}\pi\sigma^2 p L}$$

where

- k_B is the Boltzmann constant (1.3806504(24) × 10^{-23} J/K in SI units), $[M^1\ L^2\ T^{-2}\ \theta^{-1}]$
- T is the thermodynamic temperature, $[\theta^1]$
- σ is the particle hard shell diameter, $[L^1]$
- p is the total pressure, $[M^1\ L^{-1}\ T^{-2}]$.

For particle dynamics in the atmosphere, and assuming standard temperature and pressure, i.e. 25 °C and 1 atm, we have $\lambda \approx 8 \times 10^{-8}$ m, or approximately 2.6×10^{-9} ft.

Relationship to Mach and Reynolds numbers

The Knudsen number can be related to the Mach number and the Reynolds number:

Noting the following:

Dynamic viscosity,

$$\mu = \frac{1}{2}\rho\bar{c}\lambda.$$

Average molecule speed (from Maxwell-Boltzmann distribution),

$$\bar{c} = \sqrt{\frac{8k_BT}{\pi m}}$$

thus the mean free path,

$$\lambda = \frac{\mu}{\rho}\sqrt{\frac{\pi m}{2k_BT}}$$

dividing through by *L* (some characteristic length) the Knudsen number is obtained:

$$\frac{\lambda}{L} = \frac{\mu}{\rho L}\sqrt{\frac{\pi m}{2k_BT}}$$

where

- $\bar{c}$ is the average molecular speed from the Maxwell–Boltzmann distribution, $[L^1\ T^{-1}]$
- T is the thermodynamic temperature, $[\theta^1]$
- μ is the dynamic viscosity, $[M^1\ L^{-1}\ T^{-1}]$
- m is the molecular mass, $[M^1]$
- k_B is the Boltzmann constant, $[M^1\ L^2\ T^{-2}\ \theta^{-1}]$
- ρ is the density, $[M^1\ L^{-3}]$.

The dimensionless Mach number can be written:

$$Ma = \frac{U_\infty}{c_s}$$

where the speed of sound is given by

$$c_s = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma k_B T}{m}}$$

where

- U_∞ is the freestream speed, [L^1 T^{-1}]
- R is the Universal gas constant, (in SI, 8.314 47215 J K^{-1} mol^{-1}), [M^1 L^2 T^{-2} θ^{-1} 'mol'$^{-1}$]
- M is the molar mass, [M^1 'mol'$^{-1}$]
- γ is the ratio of specific heats, and is dimensionless.

The dimensionless Reynolds number can be written:

$$Re = \frac{\rho U_\infty L}{\mu}.$$

Dividing the Mach number by the Reynolds number,

$$\frac{Ma}{Re} = \frac{U_\infty \div c_s}{\rho U_\infty L \div \mu} = \frac{\mu}{\rho L c_s} = \frac{\mu}{\rho L \sqrt{\frac{\gamma k_B T}{m}}} = \frac{\mu}{\rho L}\sqrt{\frac{m}{\gamma k_B T}}$$

and by multiplying by $\sqrt{\frac{\gamma\pi}{2}}$,

$$\frac{\mu}{\rho L}\sqrt{\frac{m}{\gamma k_B T}}\sqrt{\frac{\gamma\pi}{2}} = \frac{\mu}{\rho L}\sqrt{\frac{\pi m}{2k_B T}}$$

the Knudsen number is obtained.

The Mach, Reynolds and Knudsen numbers are therefore related by:

$$Kn = \frac{Ma}{Re}\sqrt{\frac{\gamma\pi}{2}}.$$

Application

The Knudsen number is useful for determining whether statistical mechanics or the continuum mechanics formulation of fluid dynamics should be used: If the Knudsen number is near or greater than one, the mean free path of a molecule is comparable to a length scale of the problem, and the continuum assumption of fluid mechanics is no longer a good approximation. In this case statistical methods must be used.

Problems with high Knudsen numbers include the calculation of the motion of a dust particle through the lower atmosphere, or the motion of a satellite through the exosphere. One of the most widely used applications for the Knudsen number is in microfluidics and MEMS device design. The solution of the flow around an aircraft has a low Knudsen number, making it firmly in the realm of continuum mechanics. Using the Knudsen number an adjustment for Stokes' Law can be used in the Cunningham correction factor, this is a drag force correction due to slip in small particles (i.e. $d_p < 5$ µm).

Chapter- 9

Incompressible Flow

In fluid mechanics or more generally continuum mechanics, incompressible (isochoric) flow refers to flow in which the material density is constant within an infinitesimal volume that moves with the velocity of the fluid. An equivalent statement that implies incompressible flow is that the divergence of the fluid velocity is zero.

Incompressible flow does not imply that the fluid itself is compressible. It is shown in the derivation below that even compressible fluids can undergo incompressible flow. Incompressible fluids must have a constant density everywhere, while incompressible flow only requires that the density remain constant within a parcel of fluid which moves with the fluid velocity.

Derivation

The fundamental requirement for incompressible flow is that the density, ρ, is constant within an infinitesimal volume, dV, which moves at the velocity of the fluid, **v**. Mathematically, this constraint implies that the material derivative (discussed below) of the density must vanish to ensure incompressible flow. Before introducing this constraint, we must apply the conservation of mass to generate the necessary relations. The mass is calculated by a volume integral of the density, ρ:

$$m = \iiint\limits_V \rho dV .$$

The conservation of mass requires that the time derivative of the mass inside a control volume be equal to the mass flux, **J**, across its boundaries. Mathematically, we can represent this constraint in terms of a surface integral:

$$\frac{\partial m}{\partial t} = - \oiint\limits_S (\mathbf{J} \cdot d\mathbf{S}) .$$

The negative sign in the above expression ensures that outward flow results in a decrease in the mass with respect to time, using the convention that the surface area vector points outward. Now, using the divergence theorem we can derive the relationship between the flux and the partial time derivative of the density:

$$\iiint_V \frac{\partial \rho}{\partial t} dV = -\iiint_V (\nabla \cdot \mathbf{J})\, dV ,$$

therefore:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \mathbf{J} .$$

The partial derivative of the density with respect to time need not vanish to ensure incompressible *flow*. When we speak of the partial derivative of the density with respect to time, we are referring to this rate of change within a control volume of *fixed position*. By allowing the partial time derivative of the density to be non-zero, we are not restricting ourselves to incompressible fluids because the density is allowed to change as observed from a fixed position as fluid flows through the control volume. This approach maintains generality, and not requiring that the partial time derivative of the density vanishes illustrates that compressible fluids can still undergo incompressible flow. What we are interested in now is the change in density of a control volume which moves along with the fluid velocity, **v**. The flux is related to the fluid velocity through the following function:

$$\mathbf{J} = \rho \mathbf{v} .$$

So that the conservation of mass implies that:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \frac{\partial \rho}{\partial t} + \nabla \rho \cdot \mathbf{v} + \rho (\nabla \cdot \mathbf{v}) = 0 .$$

The previous relation (where we have used the appropriate product rule) is known as the continuity equation. Now, we need the following relation about the total derivative of the density (where we apply the chain rule):

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{dx}{dt} + \frac{\partial \rho}{\partial y}\frac{dy}{dt} + \frac{\partial \rho}{\partial z}\frac{dz}{dt} .$$

So if we choose a control volume that is moving at the same rate as the fluid (i.e. (dx/dt,dy/dt,dz/dt)=**v**), then this expression simplifies to the material derivative:

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \nabla\rho\cdot\mathbf{v}.$$

And so using the continuity equation derived above, we see that:

$$\frac{D\rho}{Dt} = -\rho\left(\nabla\cdot\mathbf{v}\right).$$

A change in the density over time would imply that the fluid had either compressed or expanded (or that the mass contained in our constant volume, *dV*, had changed), which we have prohibited. We must then require that the material derivative of the density vanishes, and equivalently (for non-zero density) so must the divergence of the fluid velocity:

$$\nabla\cdot\mathbf{v} = 0.$$

And so beginning with the conservation of mass and the constraint that the density within a moving volume of fluid remains constant, it has been shown that an equivalent condition required for incompressible flow is that the divergence of the fluid velocity vanishes.

Relation to compressibility factor

In some fields, a measure of the incompressibility of a flow is the change in density as a result of the pressure variations. This is best expressed in terms of the compressibility factor

$$Z = \frac{1}{\rho}\frac{d\rho}{dp}.$$

If the compressibility factor is acceptably small, the flow is considered to be incompressible.

Relation to solenoidal field

An incompressible flow is described by a velocity field which is solenoidal. But a solenoidal field, besides having a zero divergence, also has the additional connotation of having non-zero curl (i.e., rotational component).

Otherwise, if an incompressible flow also has a curl of zero, so that it is also irrotational, then the velocity field is actually Laplacian.

Difference between incompressible flow and material

As defined earlier, an incompressible (isochoric) flow is the one in which

$$\nabla \cdot \mathbf{u} = 0.$$

This is equivalent to saying that

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0$$

i.e. the material derivative of the density is zero. Thus if we follow a material element, its mass density will remain constant. Note that the material derivative consists of two terms. The first term $\frac{\partial \rho}{\partial t}$ describes how the density of the material element changes with time. This term is also known as the *unsteady term*. The second term, $\mathbf{u} \cdot \nabla \rho$ describes the changes in the density as the material element moves from one point to another. This is the *convection* or the *advection term*. For a flow to be incompressible the sum of these terms should be zero.

On the other hand, a **homogeneous, incompressible material** is defined as one which has constant density throughout. For such a material, ρ = *constant*. This implies that,

$\frac{\partial \rho}{\partial t} = 0$ and
$\nabla \rho = 0$ *independently*.

From the continuity equation it follows that

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = 0 \implies \nabla \cdot \mathbf{u} = 0$$

Thus homogeneous materials always undergo flow that is incompressible, but the converse is not true.

It is common to find references where the author mentions incompressible flow and assumes that density is constant. Even though this is technically incorrect, it is an accepted practice. One of the advantages of using the incompressible material assumption over the incompressible flow assumption is in the momentum equation where the kinematic viscosity ($\nu = \frac{\mu}{\rho}$) can be assumed to be constant. The subtlety above is frequently a source of confusion. Therefore many people prefer to refer explicitly to *incompressible materials* or *isochoric flow* when being descriptive about the mechanics.

Related flow constraints

In fluid dynamics, a flow is considered to be incompressible if the divergence of the velocity is zero. However, related formulations can sometimes be used, depending on the flow system to be modelled. Some versions are described below:

1. *Incompressible flow*: $\nabla \cdot \mathbf{u} = 0$. This can assume either constant density (strict incompressible) or varying density flow. The varying density set accepts solutions involving small perturbations in density, pressure and/or temperature fields, and can allow for pressure stratification in the domain.
2. *Anelastic flow*: $\nabla \cdot (\rho_o \mathbf{u}) = 0$. Principally used in the field of atmospheric sciences, the anelastic constraint extend incompressible flow validity to stratified density and/or temperature as well as pressure. This allow the thermodynamic variables to relax to an 'atmospheric' base state seen in the lower atmosphere when used in the field of meteorology, for example. This condition can also be used for various astrophysical systems.
3. *Low Mach-number flow / Pseudo-incompressibility*: $\nabla \cdot (\alpha \mathbf{u}) = \beta$. The low Mach-number constraint can be derived from the compressible Euler equations using scale analysis of non-dimensional quantities. The restraint, like the previous in this section, allows for the removal of acoustic waves, but also allows for *large* perturbations in density and/or temperature. The assumption is that the flow remains within a Mach number limit (normally less than 0.3) for any solution using such a constraint to be valid. Again, in accordance with all incompressible flows the pressure deviation must be small in comparison to the pressure base state.

These methods make differing assumptions about the flow, but all take into account the general form of the constraint $\nabla \cdot (\alpha \mathbf{u}) = \beta$ for general flow dependent functions α and β.

Numerical approximations of incompressible flow

The stringent nature of the incompressible flow equations means that specific mathematical techniques have been devised to solve them. Some of these methods include:

1. The projection method (both approximate and exact)
2. Artificial compressibility technique (approximate)
3. Compressibility pre-conditioning

Chapter- 10

Compressible Flow

Compressible flow is the area of fluid mechanics that deals with fluids in which the fluid density varies significantly in response to a change in pressure. Compressibility effects are typically considered significant if the Mach number (the ratio of the flow velocity to the local speed of sound) of the flow exceeds 0.3, or if the fluid undergoes very large pressure changes. The most distinct differences between the compressible and incompressible flow models are that the compressible flow model allows for the existence of shock waves and choked flow.

Definition

Compressible flow describes the behaviour of fluids that experience significant variations in density. For flows in which the density does not vary significantly, the analysis of the behaviour of such flows may be simplified greatly by assuming a constant density. This is an idealization, which leads to the theory of incompressible flow. However, in the many cases dealing with gases (especially at higher velocities) and those cases dealing with liquids with large pressure changes, the significant variations in density can occur, and the flow should be analysed as a compressible flow if accurate results are to be obtained.

Allowing for a change in density brings an additional variable into the analysis. In contrast to incompressible flows, which can usually be solved by considering only the equations from conservation of mass and conservation of momentum. Usually, the principle of conservation of energy is included. However, this introduces another variable (temperature), and so a fourth equation (such as the ideal gas equation) is required to relate the temperature to the other thermodynamic properties in order to fully describe the flow.

When defining what is meant by a compressible flow, it is useful to compare the density to a reference value, such as the stagnation density, ρ_0, which is the density of the fluid if it were to be slowed down isentropically to stationary. As a general rule of thumb, if the change in density relative to the stagnation density is greater than 5%, then the fluid should be analysed as a compressible flow. For an ideal gas with a ratio of specific heats of 1.4, this occurs at a Mach number greater than approximately 0.3. Below this value, however, whether or not a specific case should be treated as compressible or incompressible depends largely on the level of accuracy that is required .

Compressible Flow Phenomena

Two of the most distinctive phenomena which occur in compressible are the possibility of choked flow and the presence of acoustic waves, which may also be referred to as either compression or expansion waves, depending on whether the lead to an increase or decrease in pressure.

Shock Waves

Shock waves are one of the most common examples of compressible flow phenomena. A shock is characterised by a discontinuous change in the thermodynamic properties. In one dimensional flows, shock waves can form when a series of compression waves coalesce, or when a membrane separating two regions of differing pressure is suddenly removed. This is the technique often used to produce shock waves in shock tubes.

In two and three dimensional supersonic flows, oblique shock waves occur as a result of a change in direction of the flow. A classic example of these shock waves are those shock waves that form off the nose of a supersonic aircraft.

Aerodynamics

Aerodynamics is a subfield of fluid dynamics and gas dynamics, and is primarily concerned with obtaining the forces that air exerts on an object. For Mach numbers greater than about 0.3, density changes are significant, and the flow should be considered compressible for an accurate representation of reality.

Subsonic Aerodynamics

Due to the complexities of compressible flow theory, it is often easier to calculate the incompressible flow characteristics first, and then employ a correction factor to obtain the actual flow properties. Several correction factors exist with varying degrees of complexity and accuracy.

Prandtl–Glauert transformation

The Prandtl-Glauert transformation is found by linearizing the potential equations associated with compressible, inviscid flow. The Prandtl–Glauert transformation or Prandtl–Glauert rule (also Prandtl–Glauert–Ackeret rule) is an approximation function which allows comparison of aerodynamical processes occurring at different Mach numbers. It was discovered that the linearized pressures in such a flow were equal to those found from incompressible flow theory multiplied by a correction factor. This correction factor is given below. :

$$c_p = \frac{c_{p0}}{\sqrt{1 - M^2}}.$$

where

- c_p is the compressible pressure coefficient
- c_{p0} is the incompressible pressure coefficient
- M is the Mach number.

This correction factor works well for all Mach numbers between 0.3 and 0.7. It should be noted that since this correction factor is derived from linearized equations, the pressures calculated is always less in magnitude than the actual pressures within the fluid.

Karman-Tsien correction factor

The Karman-Tsien transformation is a nonlinear correction factor to find the pressure coefficient of a compressible, inviscid flow. It is an empirically derived correction factor that tends to slightly overestimate the magnitude of the fluid's pressure. In order to employ this correction factor, the incompressible, inviscid fluid pressure must be known from previous investigation.

$$C_P = \frac{C_{P0}}{\sqrt{1 - M^2} + \frac{C_{P0}}{2}\left(1 - \sqrt{1 - M^2}\right)}$$

where

- c_p is the compressible pressure coefficient
- c_{p0} is the incompressible pressure coefficient
- M is the Mach number.

This correction factor is valid for $M<0.8$.

Supersonic Aerodynamics

As with subsonic aerodynamics, a compressibility correction factor can be derived by linearising the governing equations. The supersonic correction factor is similar to the Prandtl-Glauert transformation, but the terms under the square root sign are reversed.

$$c_p = \frac{c_{p0}}{\sqrt{M^2 - 1}}.$$

where

- c_p is the compressible pressure coefficient
- c_{p0} is the incompressible pressure coefficient
- M is the Mach number.

This is considered a valid approximation for Mach numbers greater than 1.3.

Transonic Aerodynamics

Transonic flow typically occurs in flows with Mach numbers between 0.8 and 1.2. Under these conditions, some of the flow is supersonic and some is subsonic. At these velocities, the correction factors derived using linearized theory breaks down due to a singularity that occurs at a Mach number of 1. In addition, severe instabilities caused by the formation of local shock waves and the existence of of both subsonic and supersonic flow (which behave completely differently) makes the solution of the governing equations rather difficult. However, the analysis of compressible flows in the transonic regime has led to some developments which help reduce the increases in drag caused by compressibility effects, including the use of swept wings and the Whitcomb area rule.

Internal Flows

If the flow of a fluid the fluid is confined by a surface, it is referred to as an internal flow. This includes the flow of fluids through pipes and ducts, and often arise in industrial and manufacturing processes, and is vital in the analysis of propulsion systems.

One example is in die casting or injection molding processes. This involves injecting a liquid material (such as a thermosetting plastic for injection molding or molten metal for die casting) at very high pressures into a cavity. The air that is already in the cavity is displaced very rapidly, and compressibility needs to be considered in the design of the die if problems with air entrapment are to be avoided.

Effect of area changes

Compressible flows play a big role in determining the behaviour of nozzles. Subsonic and supersonic flow react differently to changes in cross sectional area. While subsonic flow flowing through a converging duct (narrowing down from a wide diameter to a smaller diameter in the direction of the flow) will experience an increase in velocity, a supersonic flow through an identical duct will experience a decrease in velocity. In general, flow through a converging nozzle will always tend towards Mach 1. If the area convergence is great enough that the sound speed is reached, a phenomenon known as "choking" occurs. In this case, the flow is choked, and either the flow rate of the fluid entering the pipe is limited, or shock waves form in the nozzle such that the Mach number at the point of minimum area (called the throat) remains unity. Similarly, subsonic flow through a diverging nozzle will always be slowed, and supersonic flow will accelerate. The Mach number of the flow can be directly related to the area by the relation

$$\frac{A}{A^*} = \frac{1}{M}\left[\frac{\left(\frac{\gamma+1}{2}\right)}{1+\left(\frac{\gamma-1}{2}\right)M^2}\right]^{\frac{\gamma+1}{2-2\gamma}}$$

where A and M are the area and Mach number at a point in the nozzle, γ is the ratio of specific heats, and A^* is the area that would cause the flow velocity to reach a Mach number of 1 (i.e. the area at the throat, provided that the nozzle is choked).

Thus, for a subsonic flow to be accelerated to supersonic velocities, the nozzle needs to have a converging section in which the flow is subsonic, a throat, at which the flow velocity is the local speed of sound, and a diverging section with supersonic flow. Such an arrangement is called a de Laval nozzle, and is commonly used in propulsion systems such as rocket and supersonic jet engines.

Note that Mach 1 can be a very high speed for a hot gas, since the speed of sound varies as the square root of absolute temperature. Thus the speed reached at a nozzle throat can be far higher than the speed of sound under standard atmospheric conditions. This fact is used extensively in rocketry where hypersonic flows are required, and where propellant mixtures are deliberately chosen to further increase the sonic speed.

Effect of friction

Friction has a similar effect as an area change on compressible flow. In a pipe of constant cross sectional area in which the walls exert a frictional force on the flow, the flow velocity will tend toward the speed of sound. In other words, subsonic flow through a pipe with friction will accelerate, and supersonic flow will decelerate. If the pipe length is long enough that the flow velocity would pass through unity, then the flow chokes such that the flow exiting the pipe is at Mach 1. As with the nozzle, this is achieved either through the flow rate at the inlet being limited, or the formation of shock waves in the pipe (for supersonic flows). For the adiabatic flow of an ideal gas model, the effects of friction may be calculated using the Fanno flow model. For a constant friction factor, the model is given by

$$4\frac{fL^*}{D_h} = \left(\frac{1-M^2}{\gamma M^2}\right) + \left(\frac{\gamma+1}{2\gamma}\right)\ln\left[\frac{M^2}{\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)}\right]$$

where f is the Fanning friction factor, L^* is the required pipe length passed the point being considered that would result in the flow choking, and D_h is the hydraulic diameter of the pipe.

Effect of heat transfer

Adding heat to a fluid flowing at subsonic velocities in a pipe will cause the flow to accelerate, and adding heat to supersonic flow in a pipe will cause the flow to decelerate.

As with the cases of friction and area change discussed above, adding more heat than that required to reach a Mach number of 1 will result in the flow choking.

For an ideal gas in a constant area pipe, the effect of heat addition to the pipe may be calculated using the Rayleigh flow model, which describes how the Mach number varies with changes in the stagnation temperature. The stagnation temperature at a point is the temperature that the fluid would reach if it were to be slowed isentropically to stationary. As heat is added to the system, the stagnation temperature increases. The Rayleigh flow model is given by

$$\frac{T_0}{T_0^*} = \frac{2(\gamma+1)M^2}{(1+\gamma M^2)^2}\left(1+\frac{\gamma-1}{2}M^2\right)$$

where T_0 and T_0^* represent the stagnation temperatures at the point under consideration, and at the point at which the Mach number is 1 respectively.

Shock Tubes

In addition to measurements of rates of chemical kinetics, shock tubes have been used to measure dissociation energies and molecular relaxation rates, investigate shock wave behaviour, and they have been used in aerodynamic tests. The fluid flow in the driven gas (the gas behind the shock wave) can be used much as a wind tunnel, allowing higher temperatures and pressures replicating the conditions in the turbine sections of jet engines. However, test times are limited to a few milliseconds, either by the arrival of the contact surface or the reflected shock wave.

They have been further developed into shock tunnels, with an added nozzle and dump tank. The resultant high temperature hypersonic flow can be used to simulate atmospheric re-entry of spacecraft or hypersonic craft, again with limited testing times.

Chapter- 11

Crosswind Landing

A **crosswind landing** is a landing maneuver in which a significant component of the prevailing wind is perpendicular to the runway center line.

Significance

Aircraft in flight are subject to the direction of the winds in which the aircraft is operating. For example, an aircraft in flight that is pointed directly north along its longitudinal axis will, generally, fly in that northerly direction. However, if there is a west wind in the air in which the aircraft is flying, then the actual trajectory of the aircraft will be slightly to the east of north. If the aircraft was landing on a northbound runway, it would need to compensate for this easterly component of velocity caused by the west crosswind.

XB-52 performing a crab landing having nose pointed toward incoming wind, but undercarriage aligned along the runway

In situations where a crosswind is present, the aircraft will adopt a yaw orientation with respect to the runway and will drift laterally as it approaches the runway. These pose significant safety issues when safe operation of the undercarriage requires the body and the velocity of the aircraft to be aligned with the runway at touch down. The landing gear on the B-52 included an unusual feature to counteract the problem: all landing gear bogies could be steered, allowing the aircraft to land with the wheels facing the direction of travel even if the nose was not pointed in the same direction.

To meet these conflicting requirements, three standard procedures are used for executing a safe landing in a cross wind situation. These landings are called the Crab, De-Crab and Sideslip techniques.

If the crosswind landing is not executed safely, the aircraft may experience wingstrike, where a wing hits the runway.

Techniques

The following guidelines are advised by Boeing for a crosswind landing. These guidelines assume steady wind (no gusting). These winds are measured at 10 m (33 feet) tower height for a runway 45 m (148 feet) in width. Basically, there are 3 landing techniques which may be used to correct for cross winds: De-Crab, Crab, and Sideslip.

De-Crab

The objective of this technique is to maintain wings level and the aircraft position near the runway centerline during approach. The nose points into the wind so that the aircraft approaches the runway slightly skewed with respect to the runway centerline (crabbing). This gives the impression of approaching the runway flying sideways, which can be disorienting for the pilot. Position is maintained by balancing the crosswind component, or more accurately the drag force arising from it, with engine thrust. Wings are maintained level throughout the approach. Just before the flare, opposite rudder (downwind rudder) is applied to eliminate the crab, with a simultaneous application of opposite aileron to maintain a wings-level attitude, so that at touch down, the body, velocity vector, and bank angle are all aligned with the runway, and the aircraft is positioned near the center.

Crab

On dry runways, upon touchdown the airplane tracks towards the upwind edge of the runway while de-crabbing to align with the runway. Immediate upwind aileron is needed to ensure the wings remain level while rudder is needed to track center line. The greater the amount of crab at touchdown, the larger the lateral deviation from the point of touchdown. For this reason, touchdown in a crab only condition is not recommended when landing on a dry runway.

On very slippery runways, landing the airplane using crab only reduces drift towards the downwind side of a touchdown, and may reduce pilot workload since the airplane does not have to be de-crabbed before touchdown. However, proper rudder and upwind aileron must be applied after touchdown to ensure directional control is maintained.

Sideslip

This sideslip crosswind technique is to maintain the aircraft's heading aligned with the runway centerline. The initial phase of the approach is flown using the Crab technique to correct for drift. The aircraft heading is adjusted using rudder and ailerons to align with the runway. This places the aircraft at a constant sideslip angle, which its natural stability will tend to correct. Sufficient rudder and aileron must be applied continuously to maintain the sideslip at this value. The dihedral action of the wings has a tendency to cause the aircraft to roll, so aileron must be applied to check the bank angle.

With a slight residual bank angle, a touchdown is typically accomplished with the upwind main wheels touching down just before the downwind wheels. Excessive control must be avoided because over-banking could cause the engine nacelle or outboard wing flap to contact the runway/ground.