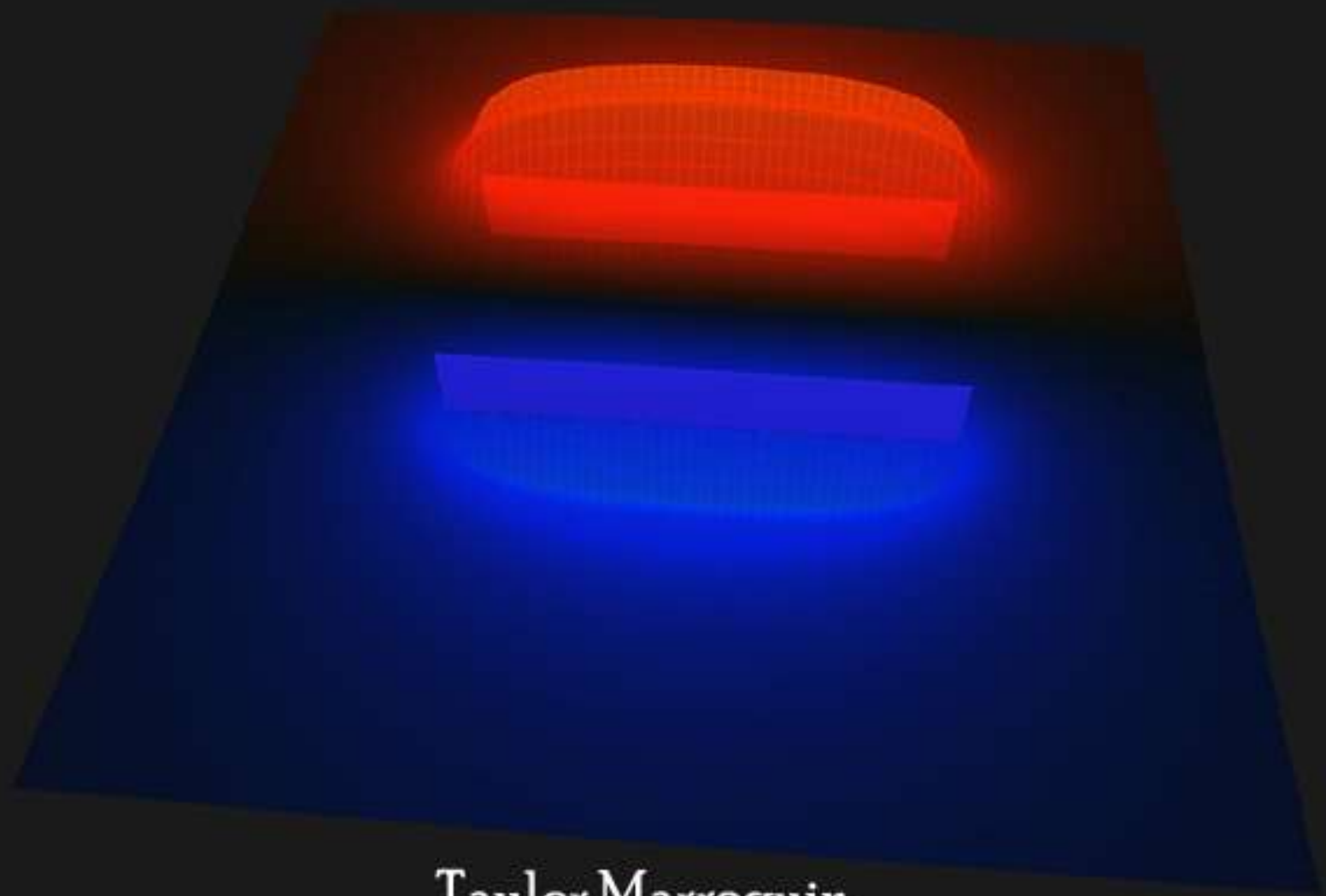
Important Concepts and Elements of

Electromagnetism



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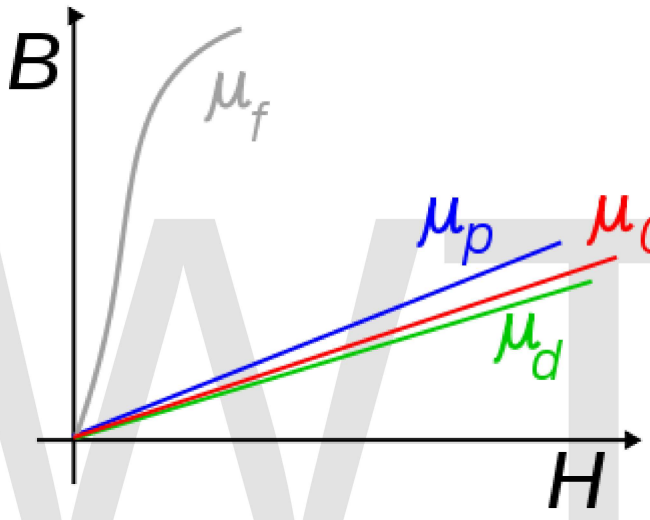
Permeability (electromagnetism)



Simplified comparison of permeabilities for: ferromagnets (μ_f), paramagnets(μ_p), free space(μ_0) and diamagnets (μ_d)

In electromagnetism, **permeability** is the measure of the ability of a material to support the formation of a magnetic field within itself. In other words, it is the degree of magnetization that a material obtains in response to an applied magnetic field. Magnetic permeability is typically represented by the Greek letter μ. The term was coined in September, 1885 by Oliver Heaviside. The reciprocal of magnetic permeability is **magnetic reluctivity**.

In SI units, permeability is measured in the henry per metre (H m^{-1}), or newton per ampere squared (N A^{-2}). The permeability constant (μ_0), also known as the magnetic constant or the permeability of free space, is a measure of the amount of resistance encountered when forming a magnetic field in a classical vacuum. The magnetic constant has the exact (defined) value $\mu_0 = 4\pi\times10^{-7} \approx 1.2566370614...\times10^{-6}$ H·m^{-1} or N·A^{-2}).

Explanation

In electromagnetism, the auxiliary magnetic field **H** represents how a magnetic field **B** influences the organization of magnetic dipoles in a given medium, including dipole migration and magnetic dipole reorientation. Its relation to permeability is

$$\mathbf{B} = \mu \mathbf{H}$$

where the **permeability μ** is a scalar if the medium is isotropic or a second rank tensor for an anisotropic medium.

In general, permeability is not a constant, as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permeability can depend on the strength of the magnetic field. Permeability as a function of frequency can take on real or complex values. In ferromagnetic materials, the relationship between **B** and **H** exhibits both non-linearity and hysteresis: **B** is not a single-valued function of **H**, but depends also on the history of the material. For these materials it is sometimes useful to consider the *incremental permeability* defined as

$$\Delta \mathbf{B} = \mu_{\Delta} \Delta \mathbf{H}.$$

This definition is useful in local linearizations of non-linear material behavior, for example in a Newton-Raphson iterative solution scheme that computes the changing saturation of a magnetic circuit.

Permeability is the inductance per unit length. In SI units, permeability is measured in henries per metre ($H \cdot m^{-1} = J/(A^2 \cdot m) = N\ A^{-2}$). The auxiliary magnetic field **H** has dimensions current per unit length and is measured in units of amperes per metre ($A\ m^{-1}$). The product **μH** thus has dimensions inductance times current per unit area ($H \cdot A/m^2$). But inductance is magnetic flux per unit current, so the product has dimensions magnetic flux per unit area. This is just the magnetic field **B**, which is measured in webers (volt-seconds) per square-metre ($V \cdot s/m^2$), or teslas (T).

B is related to the Lorentz force on a moving charge *q*:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

The charge *q* is given in coulombs (C), the velocity *v* in m/s, so that the force *F* is in newtons (N):

$$q\mathbf{v} \times \mathbf{B} = \mathrm{C} \cdot \frac{\mathrm{m}}{\mathrm{s}} \cdot \frac{\mathrm{V} \cdot \mathrm{s}}{\mathrm{m}^2} = \frac{\mathrm{C} \cdot (\mathrm{J} \,/\, \mathrm{C})}{\mathrm{m}} = \frac{\mathrm{J}}{\mathrm{m}} = \mathrm{N}$$

H is related to the magnetic dipole density. A magnetic dipole is a closed circulation of electric current. The dipole moment has dimensions current times area, units ampere square-metre ($A \cdot m^2$), and magnitude equal to the current around the loop times the area of the loop. The **H** field at a distance from a dipole has magnitude proportional to the dipole moment divided by distance cubed, which has dimensions current per unit length.

Relative permeability

Relative permeability, sometimes denoted by the symbol μ_r, is the ratio of the permeability of a specific medium to the permeability of free space given by the magnetic constant $\mu_0 = 4\pi \times 10^{-7} \frac{N}{A^2}$:

$$\mu_r = \frac{\mu}{\mu_0}.$$

In terms of relative permeability, the magnetic susceptibility is:

$$\chi_m = \mu_r - 1.$$

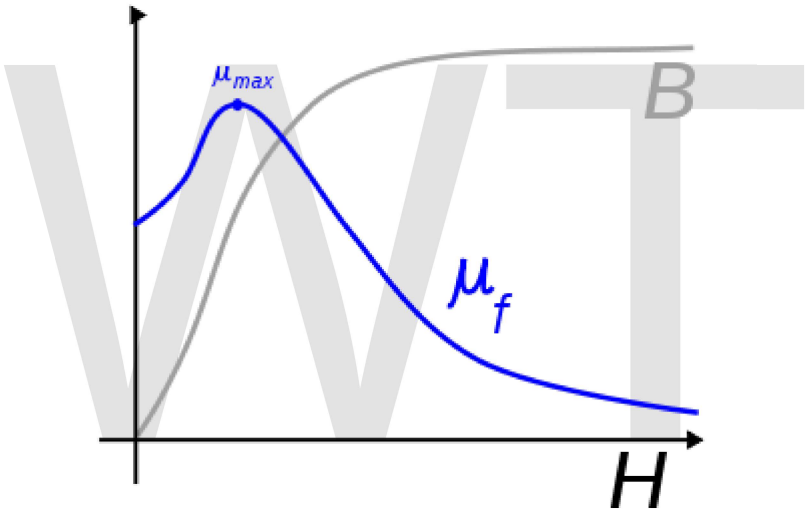
χ_m, a dimensionless quantity, is sometimes called *volumetric* or *bulk* susceptibility, to distinguish it from χ_p (*magnetic mass* or *specific* susceptibility) and χ_M (*molar* or *molar mass* susceptibility).

Values for some common materials

Magnetic susceptibility and permeability data for selected materials

Medium	Susceptibility χ_m (volumetric SI)	Permeability μ [H/m]	Relative Permeability μ/μ_0	Magnetic field	Frequency max.
Mu-metal		2.5×10^{-2}	20,000	at 0.002 T	
Mu-metal			50,000		
Permalloy		1.0×10^{-2}	8,000	at 0.002 T	
Electrical steel		5.0×10^{-3}	4,000	at 0.002 T	
Ferrite (nickel zinc)		2.0×10^{-5} – 8.0×10^{-4}	16–640		100 kHz ~ 1 MHz
Ferrite (manganese zinc)		$>8.0\times10^{-4}$	>640		100 kHz ~ 1 MHz
Steel		8.75×10^{-4}	100	at 0.002 T	
Nickel		1.25×10^{-4}	100 – 600	at 0.002 T	
Concrete			1		

Platinum		1.2569701×10^{-6}	1.000265
Aluminum	2.22×10^{-5}	1.2566650×10^{-6}	1.000022
Air			1.00000037
Vacuum	0	1.2566371×10^{-6} (μ_0)	1
Hydrogen	-2.2×10^{-9}	1.2566371×10^{-6}	1.0000000
Sapphire	-2.1×10^{-7}	1.2566368×10^{-6}	0.99999976
Copper	-6.4×10^{-6} or -9.2×10^{-6}	1.2566290×10^{-6}	0.999994
Water	-8.0×10^{-6}	1.2566270×10^{-6}	0.999992
Bismuth	-1.66×10^{-4}		0.999834
Superconductors	−1	0	0



Magnetisation curve for ferromagnets (and ferrimagnets) and corresponding permeability

A good magnetic core material must have high permeability.

For magnetic levitation a permeability below 1 is needed.

Permeability varies with magnetic field. Values shown above are approximate and valid only at the magnetic fields shown. Moreover, they are given for a zero frequency; in practice, the permeability is generally a function of the frequency. When frequency is considered the permeability can be complex, corresponding to the in phase and out of phase response.

Note that the magnetic constant μ_0 has an exact value in SI units (that is, there is no uncertainty in its value), because the definition of the ampere fixes its value to $4\pi \times 10^{-7}$ H/m exactly.

Complex permeability

A useful tool for dealing with high frequency magnetic effects is the complex permeability. While at low frequencies in a linear material the magnetic field and the auxiliary magnetic field are simply proportional to each other through some scalar permeability, at high frequencies these quantities will react to each other with some lag time. These fields can be written as phasors, such that

$$H = H_0 e^{j\omega t} \qquad B = B_0 e^{j(\omega t - \delta)}$$

where δ is the phase delay of *B* from *H*. Understanding permeability as the ratio of the magnetic field to the auxiliary magnetic field, the ratio of the phasors can be written and simplified as

$$\mu = \frac{B}{H} = \frac{B_0 e^{j(\omega t - \delta)}}{H_0 e^{j\omega t}} = \frac{B_0}{H_0} e^{-j\delta},$$

so that the permeability becomes a complex number. By Euler's formula, the complex permeability can be translated from polar to rectangular form,

$$\mu = \frac{B_0}{H_0}\cos\delta - j\frac{B_0}{H_0}\sin\delta = \mu' - j\mu''.$$

The ratio of the imaginary to the real part of the complex permeability is called the loss tangent,

$$\tan\delta = \frac{\mu''}{\mu'},$$

which provides a measure of how much power is lost in a material versus how much is stored.

Chapter- 2

Electromagnetism

Electromagnetism is one of the four fundamental interactions of nature. The other three are the strong interaction, the weak interaction and gravitation. Electromagnetism is the force that causes the interaction between electrically charged particles; the areas in which this happens are called electromagnetic fields.

Electromagnetism is responsible for practically all the phenomena encountered in daily life, with the exception of gravity. Ordinary matter takes its form as a result of intermolecular forces between individual molecules in matter. Electromagnetism is also the force which holds electrons and protons together inside atoms, which are the building blocks of molecules. This governs the processes involved in chemistry, which arise from interactions between the electrons orbiting atoms.

Electromagnetism manifests as both electric fields and magnetic fields. Both fields are simply different aspects of electromagnetism, and hence are intrinsically related. Thus, a changing electric field generates a magnetic field; conversely a changing magnetic field generates an electric field. This effect is called electromagnetic induction, and is the basis of operation for electrical generators, induction motors, and transformers. Mathematically speaking, magnetic fields and electric fields are convertible with relative motion as a four vector.

Electric fields are the cause of several common phenomena, such as electric potential (such as the voltage of a battery) and electric current (such as the flow of electricity through a flashlight). Magnetic fields are the cause of the force associated with magnets.

In quantum electrodynamics, electromagnetic interactions between charged particles can be calculated using the method of Feynman diagrams, in which we picture messenger particles called virtual photons being exchanged between charged particles. This method can be derived from the field picture through perturbation theory.

The theoretical implications of electromagnetism led to the development of special relativity by Albert Einstein in 1905.

History of electromagnetic theory

Originally electricity and magnetism were thought of as two separate forces. This view changed, however, with the publication of James Clerk Maxwell's 1873 *Treatise on Electricity and Magnetism* in which the interactions of positive and negative charges were shown to be regulated by one force. There are four main effects resulting from these interactions, all of which have been clearly demonstrated by experiments:

1. Electric charges attract or repel one another with a force inversely proportional to the square of the distance between them: unlike charges attract, like ones repel.
2. Magnetic poles (or states of polarization at individual points) attract or repel one another in a similar way and always come in pairs: every north pole is yoked to a south pole.
3. An electric current in a wire creates a circular magnetic field around the wire, its direction depending on that of the current.
4. A current is induced in a loop of wire when it is moved towards or away from a magnetic field, or a magnet is moved towards or away from it, the direction of current depending on that of the movement.

While preparing for an evening lecture on 21 April 1820, Hans Christian Ørsted made a surprising observation. As he was setting up his materials, he noticed a compass needle deflected from magnetic north when the electric current from the battery he was using was switched on and off. This deflection convinced him that magnetic fields radiate from all sides of a wire carrying an electric current, just as light and heat do, and that it confirmed a direct relationship between electricity and magnetism.

At the time of discovery, Ørsted did not suggest any satisfactory explanation of the phenomenon, nor did he try to represent the phenomenon in a mathematical framework. However, three months later he began more intensive investigations. Soon thereafter he published his findings, proving that an electric current produces a magnetic field as it flows through a wire. The CGS unit of magnetic induction (oersted) is named in honor of his contributions to the field of electromagnetism.

His findings resulted in intensive research throughout the scientific community in electrodynamics. They influenced French physicist André-Marie Ampère's developments of a single mathematical form to represent the magnetic forces between current-carrying conductors. Ørsted's discovery also represented a major step toward a unified concept of energy.

This unification, which was observed by Michael Faraday, extended by James Clerk Maxwell, and partially reformulated by Oliver Heaviside and Heinrich Hertz, is one of the key accomplishments of 19th century mathematical physics. It had far-reaching consequences, one of which was the understanding of the nature of light. Light and other electromagnetic waves take the form of quantized, self-propagating oscillatory electromagnetic field disturbances called photons. Different frequencies of oscillation give rise to the different forms of electromagnetic radiation, from radio waves at the

lowest frequencies, to visible light at intermediate frequencies, to gamma rays at the highest frequencies.

Ørsted was not the only person to examine the relation between electricity and magnetism. In 1802 Gian Domenico Romagnosi, an Italian legal scholar, deflected a magnetic needle by electrostatic charges. Actually, no galvanic current existed in the setup and hence no electromagnetism was present. An account of the discovery was published in 1802 in an Italian newspaper, but it was largely overlooked by the contemporary scientific community.

Overview

The electromagnetic force is one of the four fundamental forces. The other fundamental forces are: the strong nuclear force (which holds quarks together, along with its residual strong force effect that holds atomic nuclei together, to form the nucleus), the weak nuclear force (which causes certain forms of radioactive decay), and the gravitational force. All other forces (e.g. friction) are ultimately derived from these fundamental forces.

The electromagnetic force is the one responsible for practically all the phenomena one encounters in daily life, with the exception of gravity. Roughly speaking, all the forces involved in interactions between atoms can be traced to the electromagnetic force acting on the electrically charged protons and electrons inside the atoms. This includes the forces we experience in "pushing" or "pulling" ordinary material objects, which come from the intermolecular forces between the individual molecules in our bodies and those in the objects. It also includes all forms of chemical phenomena, which arise from interactions between electron orbitals.

Classical electrodynamics

Classical electromagnetism (or **classical electrodynamics**) is a branch of theoretical physics that studies consequences of the electromagnetic forces between electric charges and currents. It provides an excellent description of electromagnetic phenomena whenever the relevant length scales and field strengths are large enough that quantum mechanical effects are negligible. Fundamental physical aspects of classical electrodynamics are presented e.g. by Feynman, Leighton and Sands, Panofsky and Phillips, and Jackson.

The theory of electromagnetism was developed over the course of the 19th century, most prominently by James Clerk Maxwell. For a detailed historical account, consult Pauli, Whittaker, and Pais..

Ribarič and Šušteršič considered a dozen open questions in the current understanding of classical electrodynamics; to this end they studied and cited about 240 references from 1903 to 1989. The outstanding problem with classical electrodynamics, as stated by

Jackson, is that we are able to obtain and study relevant solutions of its basic equations only in two limiting cases: »... one in which the sources of charges and currents are specified and the resulting electromagnetic fields are calculated, and the other in which external electromagnetic fields are specified and the motion of charged particles or currents is calculated... Occasionally, ..., the two problems are combined. But the treatment is a stepwise one -- first the motion of the charged particle in the external field is determined, neglecting the emission of radiation; then the radiation is calculated from the trajectory as a given source distribution. It is evident that this manner of handling problems in electrodynamics can be of only approximative validity.« As a consequence, we do not yet have physical understanding of those electromechanical systems where we cannot neglect the mutual interaction between electric charges and currents, and the electromagnetic field emitted by them. In spite of a century long effort, there is as yet no generally accepted classical equation of motion for charged particles, as well as no pertinent experimental data, cf.

Lorentz force

The electromagnetic field exerts the following force (often called the Lorentz force) on charged particles:

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

where all boldfaced quantities are vectors: **F** is the force that a charge *q* experiences, **E** is the electric field at the location of the charge, **v** is the velocity of the charge, **B** is the magnetic field at the location of the charge.

The above equation illustrates that the Lorentz force is the sum of two vectors. One is the cross product of the velocity and magnetic field vectors. Based on the properties of the cross product, this produces a vector that is perpendicular to both the velocity and magnetic field vectors. The other vector is in the same direction as the electric field. The sum of these two vectors is the Lorentz force.

Therefore, in the absence of a magnetic field, the force is in the direction of the electric field, and the magnitude of the force is dependent on the value of the charge and the intensity of the electric field. In the absence of an electric field, the force is perpendicular to the velocity of the particle and the direction of the magnetic field. If both electric and magnetic fields are present, the Lorentz force is thee sum of both of these vectors.

The electric field E

The electric field **E** is defined such that, on a stationary charge:

$$\mathbf{F} = q_0\mathbf{E}$$

where q_0 is what is known as a test charge. The size of the charge doesn't really matter, as long as it is small enough as to not influence the electric field by its mere presence. What is plain from this definition, though, is that the unit of **E** is N/C, or newtons per coulomb. This unit is equal to V/m (volts per meter).

The above definition seems a little bit circular but, in electrostatics, where charges are not moving, Coulomb's law works fine. The result is:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i\left(\mathbf{r}-\mathbf{r}_i\right)}{\left|\mathbf{r}-\mathbf{r}_i\right|^3}$$

where n is the number of charges, q_i is the amount of charge associated with the ith charge, $\mathbf{r}_i$ is the position of the ith charge, **r** is the position where the electric field is being determined, and ε_0 is the electric constant.

Note: the above is just Coulomb's law, divided by q_1, adding up multiple charges.

If the field is instead produced by a continuous distribution of charges, the summation becomes an integral:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(\mathbf{r})\hat{\mathbf{r}}}{r^2}\mathrm{d}V$$

where $\rho(\mathbf{r})$ is the charge density as a function of position, $\hat{\mathbf{r}}$ is the unit vector pointing from dV to the point in space **E** is being calculated at, and r is the distance from the point **E** is being calculated at to the point charge.

Both of the above equations are cumbersome, especially if one wants to calculate **E** as a function of position. There is, however, a scalar function called the electrical potential that can help. Electric potential, also called voltage (the units for which are the volt), which is defined by the line integral

$$\varphi_{\mathbf{E}} = -\int_C \mathbf{E}\cdot\mathrm{d}\mathbf{s}\,,$$

where $\varphi_{\mathbf{E}}$ is the electric potential, and C is the path over which the integral is being taken.

Unfortunately, this definition has a caveat. From Maxwell's equations, it is clear that $\nabla \times \mathbf{E}$ is not always zero, and hence the scalar potential alone is insufficient to define the electric field exactly. As a result, one must resort to adding a correction factor, which is generally done by subtracting the time derivative of the **A** vector potential described below. Whenever the charges are quasistatic, however, this condition will be essentially met, so there will be few problems.

From the definition of charge, one can easily show that the electric potential of a point charge as a function of position is:

$$\varphi = \frac{q}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_q|}$$

where q is the point charge's charge, $\mathbf{r}$ is the position, and $\mathbf{r}_q$ is the position of the point charge. The potential for a general distribution of charge ends up being:

$$\varphi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{r})}{r} \, \mathrm{d}V$$

where $\rho(\mathbf{r})$ is the charge density as a function of position, and r is the distance from the volume element $\mathrm{d}V$.

Note well that φ is a scalar, which means that it will add to other potential fields as a scalar. This makes it relatively easy to break complex problems down in to simple parts and add their potentials. Taking the definition of φ backwards, we see that the electric field is just the negative gradient (the del operator) of the potential. Or:

$$\mathbf{E} = -\nabla\varphi$$

From this formula it is clear that $\mathbf{E}$ can be expressed in V/m (volts per meter).

Electromagnetic waves

A changing electromagnetic field propagates away from its origin in the form of a wave. These waves travel in vacuum at the speed of light and exist in a wide spectrum of wavelengths. Examples of the dynamic fields of electromagnetic radiation (in order of increasing frequency): radio waves, microwaves, light (infrared, visible light and ultraviolet), x-rays and gamma rays. In the field of particle physics this electromagnetic radiation is the manifestation of the electromagnetic interaction between charged particles.

General field equations

As simple and satisfying as Coulomb's equation may be, it is not entirely correct in the context of classical electromagnetism. Problems arise because changes in charge distributions require a non-zero amount of time to be "felt" elsewhere (required by special relativity). Disturbances of the electric field due to a charge propagate at the speed of light.

For the fields of general charge distributions, the retarded potentials can be computed and differentiated accordingly to yield Jefimenko's Equations.

Retarded potentials can also be derived for point charges, and the equations are known as the Liénard-Wiechert potentials. The scalar potential is:

$$\varphi = \frac{1}{4\pi\epsilon_0}\frac{q}{|\mathbf{r} - \mathbf{r}_q(t_{ret})| - \frac{\mathbf{v}_q(t_{ret})}{c} \cdot (\mathbf{r} - \mathbf{r}_q(t_{ret}))}$$

where q is the point charge's charge and $\mathbf{r}$ is the position. $\mathbf{r}_q$ and $\mathbf{v}_q$ are the position and velocity of the charge, respectively, as a function of retarded time. The vector potential is similar:

$$\mathbf{A} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}_q(t_{ret})}{|\mathbf{r} - \mathbf{r}_q(t_{ret})| - \frac{\mathbf{v}_q(t_{ret})}{c} \cdot (\mathbf{r} - \mathbf{r}_q(t_{ret}))}$$

These can then be differentiated accordingly to obtain the complete field equations for a moving point particle.

The photoelectric effect

In the **photoelectric effect**, electrons are emitted from matter (metals and non-metallic solids, liquids or gases) as a consequence of their absorption of energy from electromagnetic radiation of very short wavelength, such as visible or ultraviolet light. Electrons emitted in this manner may be referred to as "photoelectrons". First observed by Heinrich Hertz in 1887, the phenomenon is also known as the "Hertz effect", although the latter term has fallen out of general use. Hertz observed and then showed that electrodes illuminated with ultraviolet light create electric sparks more easily.

The photoelectric effect requires photons with energies from a few electronvolts to, in high atomic number elements, over 1 MeV. At the high photon energies comparable to the electron rest energy of 511 keV, Compton scattering, another process, may take place, and above twice this (1.022 MeV) pair production may take place.

Study of the photoelectric effect led to important steps in understanding the quantum nature of light and electrons and influenced the formation of the concept of wave–particle duality.

The term may also, but incorrectly, refer to related phenomena such as the photoconductive effect (also known as photoconductivity or photoresistivity), the photovoltaic effect, or the photoelectrochemical effect which are, in fact, distinctly different.

Introduction and early historical view

When a surface is exposed to electromagnetic radiation above a certain threshold frequency (typically visible light for alkali metals, near ultraviolet for other metals, and

extreme ultraviolet for non-metals), the radiation is absorbed and electrons are emitted. This phenomenon was first observed by Heinrich Hertz in 1887. Johann Elster (1854-1920) and Hans Geistel (1855-1923), students in Heidelberg, developed the first practical photoelectric cells that could be used to measure the intensity of light. In 1902, Philipp Eduard Anton von Lenard observed that the energy of individual emitted electrons increased with the frequency (which is related to the color) of the light. This appeared to be at odds with James Clerk Maxwell's wave theory of light, which was thought to predict that the electron energy would be proportional to the intensity of the radiation.

In 1905, Albert Einstein solved this apparent paradox by describing light as composed of discrete quanta, now called photons, rather than continuous waves. Based upon Max Planck's theory of black-body radiation, Einstein theorized that the energy in each quantum of light was equal to the frequency multiplied by a constant, later called Planck's constant. A photon above a threshold frequency has the required energy to eject a single electron, creating the observed effect. This discovery led to the quantum revolution in physics and earned Einstein the Nobel Prize in Physics in 1921.

Modern view

It has been shown that it is not necessary for light to be "quantized" to explain the photoelectric effect. The most common method employed by physicists to calculate the probability of an atom ejecting an electron relies on "Fermi's golden rule". Although based upon quantum mechanics, the method treats the incident light as an electromagnetic wave that causes an atom and its constituent electrons to transition from one energy state ("eigenstate") to another.

While one can use the classical electromagnetic theory of light to describe the effect, one may also use the modern quantum theory of light to describe the photoelectric effect. However, the modern quantum theory of light is *not* a "particle model", as it does not always predict results which one would expect from a naïve "particle" interpretation. An example would be in the dependence on polarization with regard to the direction electrons are emitted, a phenomenon that has been considered useful in gathering polarization data from black holes and neutron stars.

Traditional explanation

The photons of a light beam have a characteristic energy determined by the frequency of the light. In the photoemission process, if an electron within some material absorbs the energy of one photon and thus has more energy than the work function (the electron binding energy) of the material, it is ejected. If the photon energy is too low, the electron is unable to escape the material. Increasing the intensity of the light beam increases the number of photons in the light beam, and thus increases the number of electrons emitted, but does not increase the energy that each electron possesses. Thus the energy of the emitted electrons does not depend on the intensity of the incoming light, but only on the energy of the individual photons. (This is true as long as the intensity is low enough for

non-linear effects caused by multiphoton absorption or level shifts such as the AC Stark effect to be insignificant. This was a given in the age of Einstein, well before lasers had been invented.)

Electrons can absorb energy from photons when irradiated, but they usually follow an "all or nothing" principle. All of the energy from one photon must be absorbed and used to liberate one electron from atomic binding, or the energy is re-emitted. If the photon energy is absorbed, some of the energy liberates the electron from the atom, and the rest contributes to the electron's kinetic energy as a free particle.

Experimental results of the photoelectric emission

1. For a given metal and frequency of incident radiation, the rate at which photoelectrons are ejected is directly proportional to the intensity of the incident light.
2. For a given metal, there exists a certain minimum frequency of incident radiation below which no photoelectrons can be emitted. This frequency is called the threshold frequency.
3. For a given metal of particular work function, increase in intensity of incident beam increases the magnitude of the photoelectric current, though stoppage voltage remains the same.
4. For a given metal of particular work function, increase in frequency of incident beam increases the maximum kinetic energy with which the photoelectrons are emitted, but the photoelectric current remains the same, though stoppage voltage increases.
5. Above the threshold frequency, the maximum kinetic energy of the emitted photoelectron depends on the frequency of the incident light, but is independent of the intensity of the incident light so long as the latter is not too high
6. The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than 10^{-9} second.
7. The direction distribution of emitted electrons peaks in the direction of polarization (the direction of the electric field) of the incident light, if it is linearly polarized.

Mathematical description

The maximum kinetic energy K_{max} of an ejected electron is given by

$$K_{\max} = hf - \varphi$$

where h is the Planck constant, f is the frequency of the incident photon, and $\varphi = hf_0$ is the work function (sometimes denoted W), which is the minimum energy required to remove a delocalised electron from the surface of any given metal. The work function, in turn, can be written as

$$\varphi = hf_0,$$

where f_0 is called the threshold frequency for the metal. The maximum kinetic energy of an ejected electron is thus

$$K_{\max} = h(f - f_0)$$

Because the kinetic energy of the electron must be positive, it follows that the frequency f of the incident photon must be greater than f_0 in order for the photoelectric effect to occur.

Three-step model

In the X-ray regime, the photoelectric effect in crystalline material is often decomposed into three steps:

1. Inner photoelectric effect. The hole left behind can give rise to auger effect, which is visible even when the electron does not leave the material. In molecular solids phonons are excited in this step and may be visible as lines in the final electron energy. The inner photoeffect has to be dipole allowed. The transition rules for atoms translate via the tight-binding model onto the crystal. They are similar in geometry to plasma oscillations in that they have to be transversal.
2. Ballistic transport of half of the electrons to the surface. Some electrons are scattered.
3. Electrons escape from the material at the surface.

In the three-step model, an electron can take multiple paths through these three steps. All paths can interfere in the sense of the path integral formulation. For surface states and molecules the three-step model does still make some sense as even most atoms have multiple electrons which can scatter the one electron leaving.

History

Early observations

In 1839, Alexandre Edmond Becquerel discovered the photovoltaic effect while studying the effect of light on electrolytic cells. Though not equivalent to the photoelectric effect, his work on photovoltaics was instrumental in showing a strong relationship between light and electronic properties of materials. In 1873, Willoughby Smith discovered photoconductivity in selenium while testing the metal for its high resistance properties in conjunction with his work involving submarine telegraph cables.

Hertz's spark gaps

In 1887, Heinrich Hertz observed the photoelectric effect and the production and reception of electromagnetic waves. He published these observations in the journal Annalen der Physik. His receiver consisted of a coil with a spark gap, where a spark

would be seen upon detection of electromagnetic waves. He placed the apparatus in a darkened box to see the spark better. However, he noticed that the maximum spark length was reduced when in the box. A glass panel placed between the source of electromagnetic waves and the receiver absorbed ultraviolet radiation that assisted the electrons in jumping across the gap. When removed, the spark length would increase. He observed no decrease in spark length when he substituted quartz for glass, as quartz does not absorb UV radiation. Hertz concluded his months of investigation and reported the results obtained. He did not further pursue investigation of this effect, nor did he make any attempt at explaining how this phenomenon was brought about.

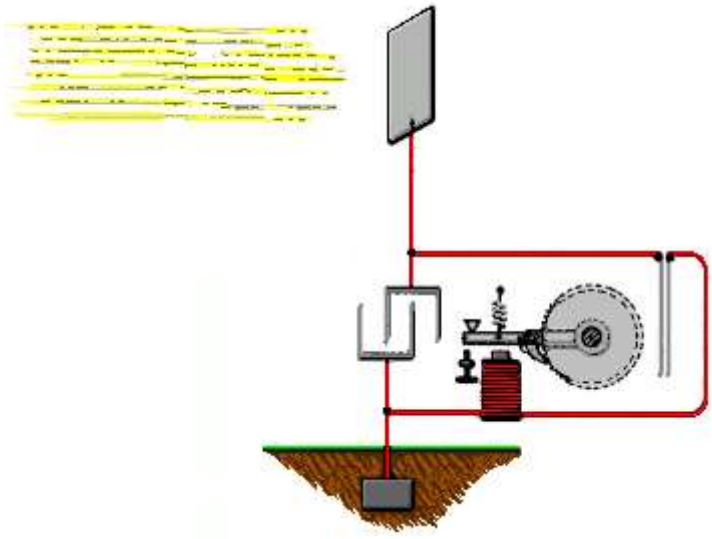
Stoletov: the first law of photoeffect

In the period from February 1888 and until 1891, a detailed analysis of photoeffect was performed by Aleksandr Stoletov with results published in 6 works; four of them in *Comptes Rendus*, one review in *Physikalische Revue* (translated from Russian), and the last work in *Journal de Physique*. First, in these works Stoletov invented a new experimental setup which was more suitable for a quantitative analysis of photoeffect. Using this setup, he discovered the direct proportionality between the intensity of light and the induced photo electric current (the first law of photoeffect or Stoletov's law). One of his other findings resulted from measurements of the dependence of the intensity of the electric photo current on the gas pressure, where he found the existence of an optimal gas pressure P_m corresponding to a maximum photocurrent; this property was used for a creation of solar cells.

JJ Thomson: electrons

In 1899, J. J. Thomson investigated ultraviolet light in Crookes tubes. Influenced by the work of James Clerk Maxwell, Thomson deduced that cathode rays consisted of negatively charged particles, later called electrons, which he called "corpuscles". In the research, Thomson enclosed a metal plate (a cathode) in a vacuum tube, and exposed it to high frequency radiation. It was thought that the oscillating electromagnetic fields caused the atoms' field to resonate and, after reaching a certain amplitude, caused a subatomic "corpuscle" to be emitted, and current to be detected. The amount of this current varied with the intensity and colour of the radiation. Larger radiation intensity or frequency would produce more current.

Radiant energy



Photoelectric motor. Rays falling on insulated conductor connected to a capacitor: the capacitor charges electrically.

Nikola Tesla described the photoelectric effect in 1901. He described such radiation as vibrations of aether of small wavelengths which ionized the atmosphere. On November 5, 1901, he received the patent US685957, *Apparatus for the Utilization of Radiant Energy*, that describes radiation charging and discharging conductors. This was done by using a metal plate or piece of mica exposed to "*radiant energy*". Tesla used this effect to charge a capacitor with energy by means of a conductive plate, making a solar cell precursor. The radiant energy threw off with great velocity minute particles (i.e., electrons) which were strongly electrified. The patent specified that the radiation (or radiant energy) included many different forms. These devices have been referred to as "*Photoelectric alternating current stepping motors*".

In practice, a polished insulated metal plate or other conducting-body in radiant energy (e.g. sunlight) will gain a positive charge as electrons are emitted by the plate. As the plate charges positively, electrons form an electrostatic force on the plate (because of surface emissions of the photoelectrons), and "*drain*" any negatively charged capacitors. In his patent application, Tesla noted that as the rays or radiation fall on the insulated conductor (which is connected to a capacitor), the capacitor will indefinitely charge electrically.

Von Lenard's observations

In 1902, Philipp Lenard observed the variation in electron energy with light frequency. He used a powerful electric arc lamp which enabled him to investigate large changes in intensity, and had sufficient power to enable him to investigate the variation of potential with light frequency. His experiment directly measured potentials, not electron kinetic energy: he found the electron energy by relating it to the maximum stopping potential

(voltage) in a phototube. He found that the calculated maximum electron kinetic energy is determined by the frequency of the light. For example, an increase in frequency results in an increase in the maximum kinetic energy calculated for an electron upon liberation - ultraviolet radiation would require a higher applied stopping potential to stop current in a phototube than blue light. However Lenard's results were qualitative rather than quantitative because of the difficulty in performing the experiments: the experiments needed to be done on freshly cut metal so that the pure metal was observed, but it oxidised in a matter of minutes even in the partial vacuums he used. The current emitted by the surface was determined by the light's intensity, or brightness: doubling the intensity of the light doubled the number of electrons emitted from the surface. Lenard did not know of photons.

Einstein: light quanta

Albert Einstein's mathematical description of how the photoelectric effect was caused by absorption of quanta of light (now called photons), was in one of his 1905 papers, named "*On a Heuristic Viewpoint Concerning the Production and Transformation of Light*". This paper proposed the simple description of "light quanta", or photons, and showed how they explained such phenomena as the photoelectric effect. His simple explanation in terms of absorption of discrete quanta of light explained the features of the phenomenon and the characteristic frequency. Einstein's explanation of the photoelectric effect won him the Nobel Prize in Physics in 1921.

The idea of light quanta began with Max Planck's published law of black-body radiation ("*On the Law of Distribution of Energy in the Normal Spectrum*". Annalen der Physik 4 (1901)) by assuming that Hertzian oscillators could only exist at energies E proportional to the frequency f of the oscillator by $E = hf$, where h is Planck's constant. By assuming that light actually consisted of discrete energy packets, Einstein wrote an equation for the photoelectric effect that fit experiments. It explained why the energy of photoelectrons were dependent only on the *frequency* of the incident light and not on its *intensity*: a low-intensity, high-frequency source could supply a few high energy photons, whereas a high-intensity, low-frequency source would supply no photons of sufficient individual energy to dislodge any electrons. This was an enormous theoretical leap, but the concept was strongly resisted at first because it contradicted the wave theory of light that followed naturally from James Clerk Maxwell's equations for electromagnetic behavior, and more generally, the assumption of infinite divisibility of energy in physical systems. Even after experiments showed that Einstein's equations for the photoelectric effect were accurate, resistance to the idea of photons continued, since it appeared to contradict Maxwell's equations, which were well-understood and verified.

Einstein's work predicted that the energy of individual ejected electrons increases linearly with the frequency of the light. Perhaps surprisingly, the precise relationship had not at that time been tested. By 1905 it was known that the energy of photoelectrons increases with increasing *frequency* of incident light and is independent of the *intensity* of the light. However, the manner of the increase was not experimentally determined until 1915 when Robert Andrews Millikan showed that Einstein's prediction was correct.

Effect on wave–particle question

The photoelectric effect helped propel the then-emerging concept of the dualistic nature of light, that light simultaneously possesses the characteristics of both waves and particles, each being manifested according to the circumstances. The effect was impossible to understand in terms of the classical wave description of light, as the energy of the emitted electrons did not depend on the intensity of the incident radiation. Classical theory predicted that the electrons would 'gather up' energy over a period of time, and then be emitted.

Uses and effects

Photodiodes and phototransistors

Solar cells (used in solar power) and light-sensitive diodes use a variant of the photoelectric effect, but not ejecting electrons out of the material. In semiconductors, light of even relatively low energy, such as visible photons, can kick electrons out of the valence band and into the higher-energy conduction band, where they can be harnessed, creating electric current at a voltage related to the bandgap energy.

Photomultipliers

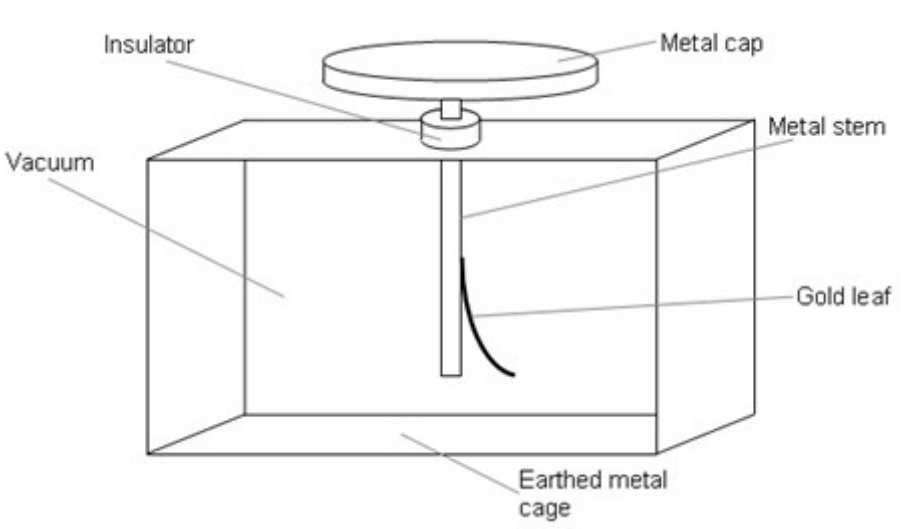
These are extremely light-sensitive vacuum tubes with a photocathode coated onto part (an end or side) of the inside of the envelope. The photocathode contains combinations of materials such as caesium, rubidium and antimony specially selected to provide a low work function, so when illuminated even by very low levels of light, the photocathode readily releases electrons. By means of a series of electrodes (dynodes) at ever-higher potentials, these electrons are accelerated and substantially increased in number through secondary emission to provide a readily detectable output current. Photomultipliers are still commonly used wherever low levels of light must be detected.

Image sensors

Video camera tubes in the early days of television used the photoelectric effect; newer variants used photoconductive rather than photoemissive materials.

Silicon image sensors, such as charge-coupled devices, widely used for photographic imaging, are based on a variant of the photoelectric effect, in which photons knock electrons out of the valence band of energy states in a semiconductor, but not out of the solid itself.

The gold-leaf electroscope



The gold leaf electroscope

Gold-leaf electroscopes are designed to detect static electricity. Charge placed on the metal cap spreads to the stem and the gold leaf of the electroscope. Because they then have the same charge, the stem and leaf repel each other. This will cause the leaf to bend away from the stem. The electroscope is an important tool in illustrating the photoelectric effect. Let us say that the scope is negatively charged throughout. There is an excess of electrons and the leaf is separated from the stem. But if we then shine high-frequency light onto the cap, the scope discharges and the leaf will fall limp. This is because the frequency of the light shining on the cap is above the cap's threshold frequency. The photons in the light have enough energy to liberate electrons from the cap, reducing its negative charge. This will discharge a negatively charged electroscope and further charge a positive electroscope. However, if the electromagnetic radiation hitting the metal cap does not have a high enough frequency (its frequency is below the threshold value for the cap), then the leaf will never discharge, no matter how long one shines the low-frequency light at the cap.

Photoelectron spectroscopy

Since the energy of the photoelectrons emitted is exactly the energy of the incident photon minus the material's work function or binding energy, the work function of a sample can be determined by bombarding it with a monochromatic X-ray source or UV source, and measuring the kinetic energy distribution of the electrons emitted.

Photoelectron spectroscopy is done in a high-vacuum environment, since the electrons would be scattered by gas molecules if they were present. The light source can be a laser, a discharge tube, or a synchrotron radiation source.

The concentric hemispherical analyser (CHA) is a typical electron energy analyzer, and uses an electric field to change the directions of incident electrons, depending on their

kinetic energies. For every element and core (atomic orbital) there will be a different binding energy. The many electrons created from each of these combinations will show up as spikes in the analyzer output, and these can be used to determine the elemental composition of the sample.

Spacecraft

The photoelectric effect will cause spacecraft exposed to sunlight to develop a positive charge. This can get up to the tens of volts. This can be a major problem, as other parts of the spacecraft in shadow develop a negative charge (up to several kilovolts) from nearby plasma, and the imbalance can discharge through delicate electrical components. The static charge created by the photoelectric effect is self-limiting, though, because a more highly charged object gives up its electrons less easily.

Moon dust

Light from the sun hitting lunar dust causes it to become charged through the photoelectric effect. The charged dust then repels itself and lifts off the surface of the Moon by electrostatic levitation. This manifests itself almost like an "atmosphere of dust", visible as a thin haze and blurring of distant features, and visible as a dim glow after the sun has set. This was first photographed by the Surveyor program probes in the 1960s. It is thought that the smallest particles are repelled up to kilometers high, and that the particles move in "fountains" as they charge and discharge.

Night vision devices

Photons hitting a thin film of alkali metal or semiconductor material such as gallium arsenide in an image intensifier tube cause the ejection of photoelectrons due to the photoelectric effect. These are accelerated by an electrostatic field where they strike a phosphor coated screen, converting the electrons back into photons. Intensification of the signal is achieved either through acceleration of the electrons or by increasing the number of electrons through secondary emissions, such as with a Micro-channel plate. Sometimes a combination of both methods are used. It is worth noting that in most cases, the additional kinetic energy imparted to the electron by the frequency of light is required to move it out of the conduction band and to the vacuum level. This is known as the electron affinity of the photocathode and is another barrier to photoemission other than the forbidden band, explained by the band gap model. Some materials such as Gallium Arsenide have an effective electron affinity that is below the level of the conduction band. In these materials, electrons that move to the conduction band are all of sufficient energy to be emitted from the material and as such, the film that absorbs photons can be quite thick. These materials are known as Negative electron affinity materials.

Cross section

The photoelectric effect is simply an interaction mechanism conducted between photons and atoms. However, this mechanism does not have exclusivity in interactions of this

nature and is one of 12 theoretically possible interactions. As noted in the prologue; Compton scattering and pair production are an example of two other competing mechanisms. Indeed, even if the photoelectric effect is the favoured reaction for a particular single-photon bound-electron interaction, the result is also subject to statistical processes and is not guaranteed, albeit the photon has certainly disappeared and a bound electron has been excited (usually K or L shell electrons at nuclear (gamma ray) energies). The probability of the photoelectric effect occurring is measured by the cross section of interaction, σ. This has been found to be a function of the atomic number of the target atom and photon energy. A crude approximation, for photon energies above the highest atomic binding energy, is given by:

$$\sigma = \text{constant} \cdot \frac{Z^n}{E^3}$$

Here Z is atomic number and *n* is a number which varies between 4 and 5. (At lower photon energies a characteristic structure with edges appears, K edge, L edges, M edges, etc.) The obvious interpretation follows that the photoelectric effect rapidly decreases in significance, in the gamma ray region of the spectrum, with increasing photon energy, and that photoelectric effect is directly proportional to atomic number. The corollary is that high-Z materials make good gamma-ray shields, which is the principal reason that lead (Z = 82) is a preferred and ubiquitous gamma radiation shield.

Units

Electromagnetic units are part of a system of electrical units based primarily upon the magnetic properties of electric currents, the fundamental SI unit being the ampere. The units are:

- ampere (current)
- coulomb (charge)
- farad (capacitance)
- henry (inductance)
- ohm (resistance)
- volt (electric potential)
- watt (power)
- tesla (magnetic field)
- weber (flux)

In the electromagnetic cgs system, electric current is a fundamental quantity defined via Ampère's law and takes the permeability as a dimensionless quantity (relative permeability) whose value in a vacuum is unity. As a consequence, the square of the speed of light appears explicitly in some of the equations interrelating quantities in this system.

SI electromagnetism units

Symbol	Name of Quantity	Derived Units	Unit	Base Units
I	Electric current	ampere (SI base unit)	A	A (= W/V = C/s)
Q	Electric charge	coulomb	C	A·s
U, ΔV, $\Delta\varphi$; E	Potential difference; Electromotive force	volt	V	J/C = $kg{\cdot}m^2{\cdot}s^{-3}{\cdot}A^{-1}$
R; Z; X	Electric resistance; Impedance; Reactance	ohm	Ω	V/A = $kg{\cdot}m^2{\cdot}s^{-3}{\cdot}A^{-2}$
ρ	Resistivity	ohm metre	Ω·m	$kg{\cdot}m^3{\cdot}s^{-3}{\cdot}A^{-2}$
P	Electric power	watt	W	V·A = $kg{\cdot}m^2{\cdot}s^{-3}$
C	Capacitance	farad	F	C/V = $kg^{-1}{\cdot}m^{-2}{\cdot}A^2{\cdot}s^4$
$\boldsymbol{E}$	Electric field strength	volt per metre	V/m	N/C = $kg{\cdot}m{\cdot}A^{-1}{\cdot}s^{-3}$
$\boldsymbol{D}$	Electric displacement field	Coulomb per square metre	C/m^2	$A{\cdot}s{\cdot}m^{-2}$
ε	Permittivity	farad per metre	F/m	$kg^{-1}{\cdot}m^{-3}{\cdot}A^2{\cdot}s^4$
χ_e	Electric susceptibility	(dimensionless)	-	-
G; Y; B	Conductance; Admittance; Susceptance	siemens	S	$\Omega^{-1} = kg^{-1}{\cdot}m^{-2}{\cdot}s^3{\cdot}A^2$
κ, γ, σ	Conductivity	siemens per metre	S/m	$kg^{-1}{\cdot}m^{-3}{\cdot}s^3{\cdot}A^2$
$\boldsymbol{B}$	Magnetic flux density, Magnetic induction	tesla	T	$Wb/m^2 = kg{\cdot}s^{-2}{\cdot}A^{-1} = N{\cdot}A^{-1}{\cdot}m^{-1}$
Φ	Magnetic flux	weber	Wb	V·s = $kg{\cdot}m^2{\cdot}s^{-2}{\cdot}A^{-1}$
$\boldsymbol{H}$	Magnetic field strength	ampere per metre	A/m	$A{\cdot}m^{-1}$
L, M	Inductance	henry	H	$Wb/A = V{\cdot}s/A = kg{\cdot}m^2{\cdot}s^{-2}{\cdot}A^{-2}$
μ	Permeability	henry per metre	H/m	$kg{\cdot}m{\cdot}s^{-2}{\cdot}A^{-2}$
χ	Magnetic susceptibility	(dimensionless)	-	-

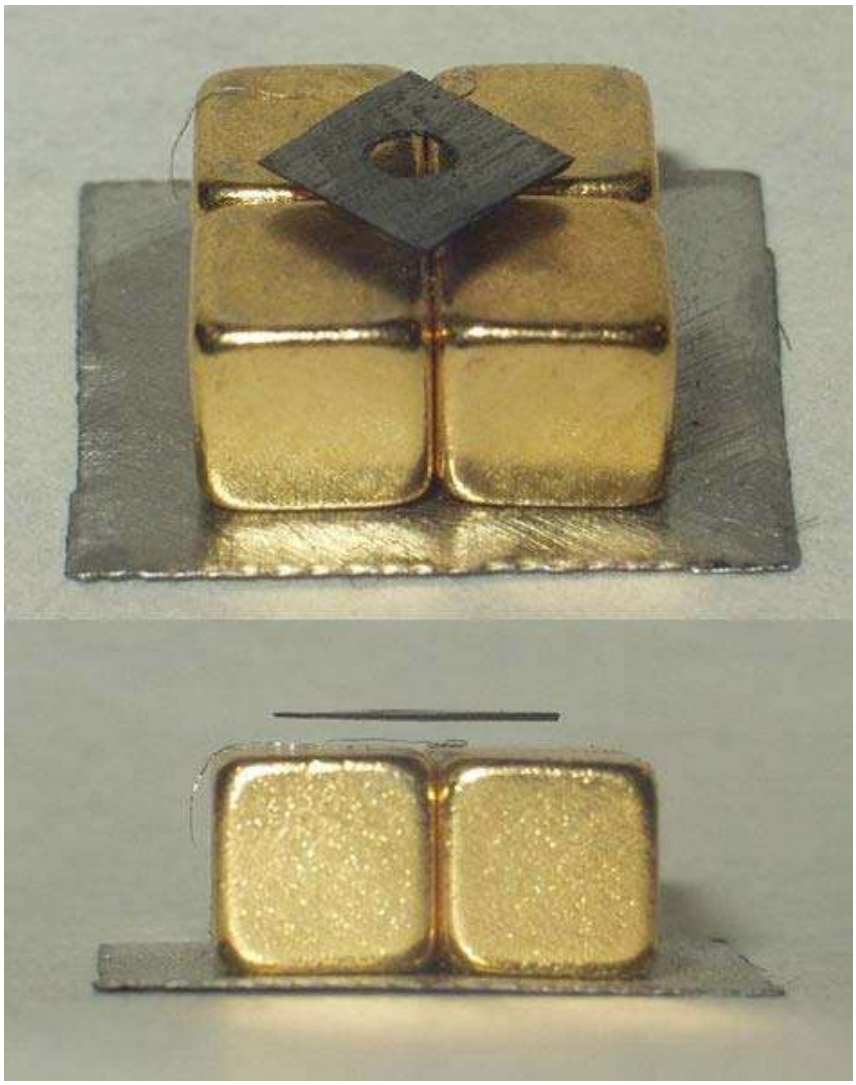
Electromagnetic phenomena

With the exception of gravitation, electromagnetic phenomena as described by quantum electrodynamics (which includes as a limiting case classical electrodynamics) account for almost all physical phenomena observable to the unaided human senses, including light and other electromagnetic radiation, all of chemistry, most of mechanics (excepting gravitation), and of course magnetism and electricity. Magnetic monopoles (and "Gilbert" dipoles) are not strictly electromagnetic phenomena, since in standard electromagnetism, magnetic fields are generated not by true "magnetic charge" but by currents. There are, however, condensed matter analogs of magnetic monopoles in exotic materials (spin ice) created in the laboratory.

Chapter- 3

Diamagnetism and Paramagnetism

Diamagnetism



Levitating pyrolytic carbon

Diamagnetism is the property of an object which causes it to create a magnetic field in opposition to an externally applied magnetic field, thus causing a repulsive effect. Specifically, an external magnetic field alters the orbital velocity of electrons around their nuclei, thus changing the magnetic dipole moment. According to Lenz's law, this opposes the external field. Diamagnets are materials with a magnetic permeability less than μ_0 (a relative permeability less than 1).

Consequently, diamagnetism is a form of magnetism that is only exhibited by a substance in the presence of an externally applied magnetic field. It is generally quite a weak effect in most materials, although superconductors exhibit a strong effect.

Diamagnetic materials cause lines of magnetic flux to curve away from the material, and superconductors can exclude them completely (except for a very thin layer at the surface).

History

In 1778 S. J. Bergman was the first individual to observe that bismuth and antimony were repelled by magnetic fields. However, the term "*diamagnetism*" was coined by Michael Faraday in September 1845, when he realized that all materials in nature possessed some form of diamagnetic response to an applied magnetic field.

Diamagnetic materials

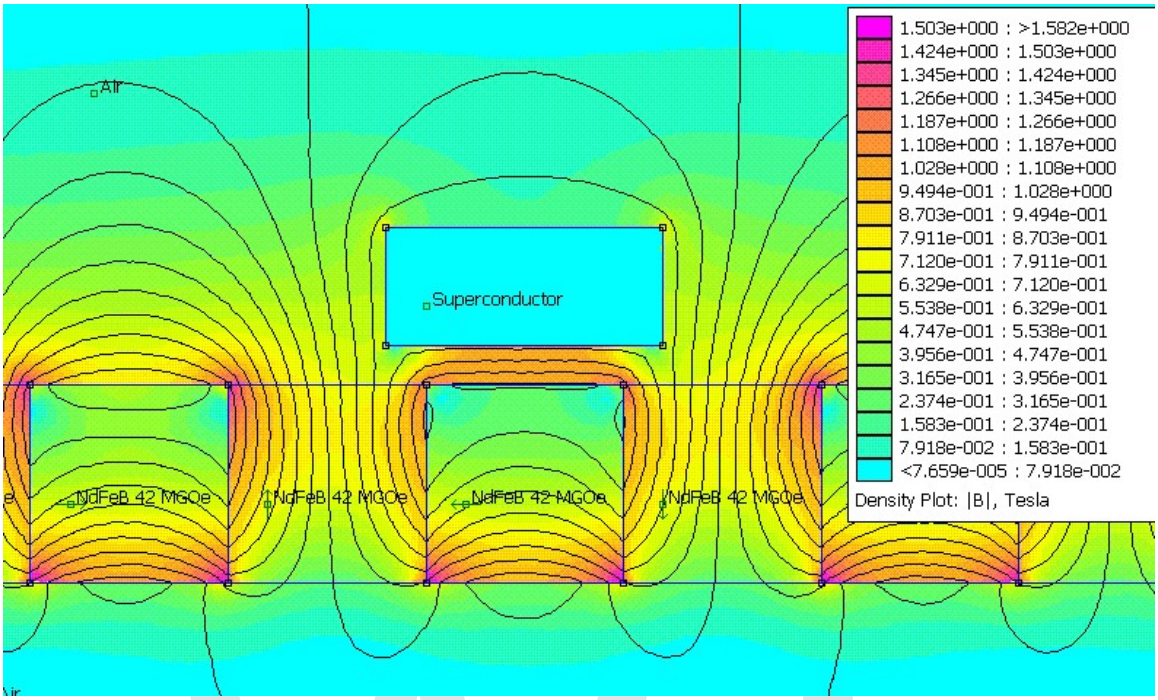
Notable diamagnetic materials

Material	$\chi_v \left(\times 10^{-5}\right)$ **(SI units)**
Bismuth	-16.6
Carbon (diamond)	-2.1
Carbon (graphite)	-1.6

Copper	-1.0
Lead	-1.8
Mercury	-2.9
Silver	-2.6
Water	-0.91
Superconductor	-10^5

Diamagnetism is a very general phenomenon, because all electrons, including the electrons of an atom, will always make a weak contribution to the material's response. However, for materials that show some other form of magnetism (such as ferromagnetism or paramagnetism), the diamagnetism is completely overpowered. Substances that mostly display diamagnetic behaviour are termed diamagnetic materials, or diamagnets. Materials that are said to be diamagnetic are those that are usually considered by non-physicists to be "non-magnetic", and include water, wood, most organic compounds such as petroleum and some plastics, and many metals including copper, particularly the heavy ones with many core electrons, such as mercury, gold and bismuth. The diamagnetism of various molecular fragments are called Pascal's constants.

Diamagnetic materials have a relative magnetic permeability that is less than 1, thus a magnetic susceptibility which is less than 0, and are therefore repelled by magnetic fields. However, since diamagnetism is such a weak property its effects are not observable in every-day life. For example, the magnetic susceptibility of diamagnets such as water is $\chi_v = -9.05 \times 10^{-6}$ (SI units). The most strongly diamagnetic material is bismuth, $\chi_v = -1.66 \times 10^{-4}$, although pyrolytic carbon may have a susceptibility of $\chi_v = -4.00 \times 10^{-4}$ in one plane. Nevertheless, these values are orders of magnitudes smaller than the magnetism exhibited by paramagnets and ferromagnets.



A superconductor acts as an essentially perfect diamagnetic material when placed in a magnetic field and it excludes the field, and the flux lines avoid the region

Superconductors may be considered to be perfect diamagnets ($\chi v = -1$), since they expel all fields from their interior due to the Meissner effect. However this effect is not due to eddy currents, as in ordinary diamagnetic materials.

Additionally, all conductors exhibit an effective diamagnetism when they experience a changing magnetic field. The Lorentz force on electrons causes them to circulate around forming eddy currents. The eddy currents then produce an induced magnetic field which opposes the applied field, resisting the conductor's motion.

Demonstrations of diamagnetism

Curving water surfaces

If a powerful magnet (such as a supermagnet) is covered with a layer of water (that is thin compared to the diameter of the magnet) then the field of the magnet significantly repels the water. This causes a slight dimple in the water's surface that may be seen by its reflection.

Diamagnetic levitation

A live frog levitates inside a 32 mm diameter vertical bore of a Bitter solenoid in a magnetic field of about 16 teslas at the Nijmegen High Field Magnet Laboratory.

Diamagnets may be levitated in stable equilibrium in a magnetic field, with no power consumption. Earnshaw's theorem seems to preclude the possibility of static magnetic levitation. However, Earnshaw's theorem only applies to objects with positive moments, such as ferromagnets (which have a permanent positive moment) and paramagnets (which induce a positive moment). These are attracted to field maxima, which do not exist in free space. Diamagnets (which induce a negative moment) are attracted to field minima, and there can be a field minimum in free space.

A thin slice of pyrolytic graphite, which is an unusually strong diamagnetic material, can be stably floated in a magnetic field, such as that from rare earth permanent magnets. This can be done with all components at room temperature, making a visually effective demonstration of diamagnetism.

The Radboud University Nijmegen, the Netherlands, has conducted experiments where water and other substances were successfully levitated. Most spectacularly, a live frog (see figure) was levitated.

In September 2009, NASA's Jet Propulsion Laboratory in Pasadena, California announced they had successfully levitated mice using a superconducting magnet, an important step forward since mice are closer biologically to humans than frogs. They hope to perform experiments regarding the effects of microgravity on bone and muscle mass.

Recent experiments studying the growth of protein crystals has led to a technique using powerful magnets to allow growth in ways that counteract Earth's gravity.

A simple homemade device for demonstration can be constructed out of bismuth plates and a few permanent magnets that will levitate a permanent magnet.

Theory of Diamagnetism

The Bohr–van Leeuwen theorem proves that there cannot be any diamagnetism or paramagnetism in a purely classical system. Yet the classical theory for Langevin diamagnetism gives the same prediction as the quantum theory. The classical theory is given below.

Langevin diamagnetism

The Langevin theory of diamagnetism applies to materials containing atoms with closed shells. A field with intensity B, applied to an electron with charge e and mass m, gives rise to Larmor precession with frequency $\omega = eB / 2m$. The number of revolutions per unit time is $\omega / 2\pi$, so the current for an atom with Z electrons is (in SI units)

$$I = -\frac{Ze^2B}{4\pi m}.$$

The magnetic moment of a current loop is equal to the current times the area of the loop. Suppose the field is aligned with the z axis. The average loop area can be given as $\pi\langle\rho^2\rangle$, where $\langle\rho^2\rangle$ is the mean square distance of the electrons perpendicular to the z axis. The magnetic moment is therefore

$$\mu = -\frac{Ze^2B}{4m}\langle\rho^2\rangle.$$

If the distribution of charge is spherically symmetric, we can suppose that the distribution of x,y,z coordinates are independent and identically distributed. Then $\langle x^2\rangle = \langle y^2\rangle = \langle z^2\rangle = \langle r^2\rangle/3$, where $\langle r^2\rangle$ is the mean square distance of the

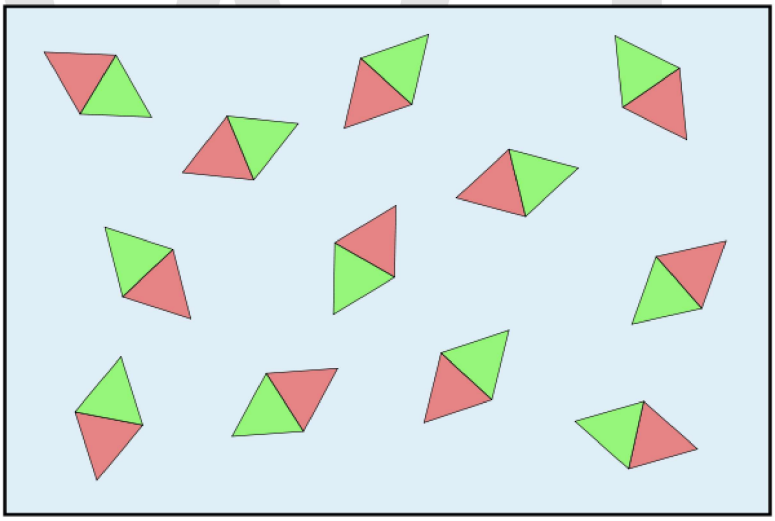
electrons from the nucleus. Therefore $\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = (2/3)\langle r^2 \rangle$. If N is the number of atoms per unit volume, the diamagnetic susceptibility is

$$\chi = \frac{\mu_0 N \mu}{B} = -\frac{\mu_0 N Z e^2}{6m} \langle r^2 \rangle.$$

Diamagnetism in metals

The Langevin theory does not apply to metals because they have non-localized electrons. The theory for the diamagnetism of a free electron gas is called Landau diamagnetism.

Paramagnetism



Simple Illustration of a paramagnetic probe made up from miniature magnets

Paramagnetism is a form of magnetism that occurs only in the presence of an externally applied magnetic field. Paramagnetic materials are attracted to magnetic fields and hence have a relative magnetic permeability of ≥1 (a positive magnetic susceptibility). The magnetic moment induced by the applied field is *linear* in the field strength and rather *weak*. It typically requires a sensitive analytical balance to detect the effect and modern measurements on paramagnetic materials are often conducted with a SQUID magnetometer.

Unlike ferromagnets, paramagnets do not retain any magnetization in the absence of an externally applied magnetic field, because thermal motion causes the spins to become *randomly oriented* without it. Thus the total magnetization will drop to zero when the applied field is removed. Even in the presence of the field there is only a small *induced* magnetization because only a small fraction of the spins will be oriented by the field. This fraction is proportional to the field strength and this explains the linear dependency. The attraction experienced by ferromagnets is non-linear and much stronger, so that it is easily observed, for instance, in magnets on one's refrigerator.

Relation to electron spins

Constituent atoms or molecules of paramagnetic materials have permanent magnetic moments (dipoles), even in the absence of an applied field. This generally occurs due to the spin of unpaired electrons in the atomic/molecular electron orbitals. In pure paramagnetism, the dipoles do not interact with one another and are randomly oriented in the absence of an external field due to thermal agitation, resulting in zero net magnetic moment. When a magnetic field is applied, the dipoles will tend to align with the applied field, resulting in a net magnetic moment in the direction of the applied field. In the classical description, this alignment can be understood to occur due to a torque being provided on the magnetic moments by an applied field, which tries to align the dipoles parallel to the applied field. However, the true origins of the alignment can only be understood via the quantum-mechanical properties of spin and angular momentum.

If there is sufficient energy exchange between neighbouring dipoles they will interact, and may spontaneously align or anti-align and form magnetic domains, resulting in ferromagnetism (permanent magnets) or antiferromagnetism, respectively. Paramagnetic behavior can also be observed in ferromagnetic materials that are above their Curie temperature, and in antiferromagnets above their Néel temperature. At these temperatures the available thermal energy simply overcomes the interaction energy between the spins.

In general paramagnetic effects are quite small: the magnetic susceptibility is of the order of 10^{-3} to 10^{-5} for most paramagnets, but may be as high as 10^{-1} for synthetic paramagnets such as ferrofluids.

Delocalization

Selected Pauli-paramagnetic metals

Material	Magnetic susceptibility ($\times 10^{-5}$)
Tungsten	6.8

Cesium	5.1
Aluminium	2.2
Lithium	1.4
Magnesium	1.2
Sodium	0.72

In many metallic materials the electrons are itinerant, i.e. they travel through the solid more or less as an electron gas. This is the result of very strong interactions (overlap) between the wave functions of neighboring atoms in the extended lattice structure. The wave functions of the valence electrons thus form a band with equal numbers of spins up and down. When exposed to an external field only those electrons close to the Fermi level will respond and a small surplus of one type of spins will result. This effect is a weak form of paramagnetism known as Pauli-paramagnetism. The effect always competes with a diamagnetic response of opposite sign due to all the core electrons of the atoms. Stronger forms of magnetism usually require localized rather than itinerant electrons. However in some cases a bandstructure can result in which there are two delocalized subbands with states of opposite spins that have different energies. If one subband is preferentially filled over the other, one can have itinerant ferromagnetic order. This usually only happens in relatively narrow (d-)bands, which are poorly delocalized.

s and p electrons

In general one can say that strong delocalization in a solid due to large overlap with neighboring wave functions tends to lead to pairing of spins (*quenching*) and thus weak magnetism. This is why s- and p-type metals are typically either Pauli-paramagnetic or as in the case of gold even diamagnetic. In the latter case the diamagnetic contribution from the closed shell inner electrons simply wins from the weak paramagnetic term of the almost free electrons.

d and f electrons

Stronger magnetic effects are typically only observed when d- or f-electrons are involved. Particularly the latter are usually strongly localized. Moreover the size of the magnetic moment on a lanthanide atom can be quite large as it can carry up to 7 unpaired electrons.

This is one reason why superstrong magnets are typically based on lanthanide elements like neodymium or samarium.

Molecular localization

Of course the above picture is a *generalization* as it pertains to materials with an extended lattice rather than a molecular structure. Molecular structure can also lead to localization of electrons. Although there are usually energetic reasons why a molecular structure results such that it does not exhibit partly filled orbitals (i.e. unpaired spins), some non-closed shell moieties do occur in nature. Molecular oxygen is a good example. Even in the frozen solid it contains di-radical molecules resulting in paramagnetic behavior. The unpaired spins reside in orbitals derived from oxygen p wave functions, but the overlap is limited to the one neighbor in the O_2 molecules. The distances to other oxygen atoms in the lattice remain too large to lead to delocalization and the magnetic moments remain unpaired.

Curie's law

For low levels of magnetization, the magnetization of paramagnets follows Curie's law to good approximation:

$$\boldsymbol{M} = \chi \cdot \boldsymbol{H} = C \cdot \frac{\boldsymbol{H}}{T}$$

where

M is the resulting magnetization
χ is the magnetic susceptibility
H is the auxiliary magnetic field, measured in amperes/meter
T is absolute temperature, measured in kelvins
C is a material-specific Curie constant

This law indicates that the susceptibility χ of paramagnetic materials is inversely proportional to their temperature. Curie's law is valid under the commonly encountered conditions of low magnetization ($\mu_B H \lesssim k_B T$), but does not apply in the high-field/low-temperature regime where saturation of magnetization occurs ($\mu_B H \gtrsim k_B T$) and magnetic dipoles are all aligned with the applied field. When the dipoles are aligned, increasing the external field will not increase the total magnetization since there can be no further alignment.

For a paramagnetic ion with noninteracting magnetic moments with angular momentum J, the Curie constant is related the individual ions' magnetic moments,

$$C = \frac{N_A}{3k_B}\mu_{\text{eff}}^2 \text{ where } \mu_{\text{eff}} = g_J\sqrt{J(J+1)}\mu_B$$

The parameter μ_{eff} is interpreted as the effective magnetic moment per paramagnetic ion. If one uses a classical treatment with molecular magnetic moments represented as discrete magnetic dipoles, μ, a Curie Law expression of the same form will emerge with μ appearing in place of μ_{eff}.

When orbital angular momentum contributions to the magnetic moment are small, as occurs for most organic radicals or for octahedral transition metal complexes with d^3 or high-spin d^5 configurations, the effective magnetic moment takes the form ($g_e = 2.0023... \approx 2$),

$\mu_{\text{eff}} \simeq 2\sqrt{S(S+1)}\mu_B = \sqrt{n(n+2)}\mu_B$, where *n* is the number of unpaired electrons. In other transition metal complexes this yields a useful, if somewhat cruder, estimate.

Examples of paramagnets

Materials that are called 'paramagnets', are most often those that exhibit, at least over an appreciable temperature range, magnetic susceptibilities that adhere to the Curie or Curie–Weiss laws. In principle any system that contains atoms, ions or molecules with unpaired spins can be called a paramagnet, but the interactions between them need to be carefully considered.

Systems with minimal interactions

The narrowest definition would be: a system with unpaired spins that *do not interact* with each other. In this narrowest sense, the only pure paramagnet is a dilute gas of monatomic hydrogen atoms. Each atom has one non-interacting unpaired electron. Of course, the latter could be said about a gas of lithium atoms but these already possess two paired core electrons that produce a diamagnetic response of opposite sign. Strictly speaking Li is a mixed system therefore, although admittedly the diamagnetic component is weak and often neglected. In the case of heavier elements the diamagnetic contribution becomes more important and in the case of metallic gold it dominates the properties. Of course, the element hydrogen is virtually never called 'paramagnetic' because the monatomic gas is stable only at extremely high temperature; H atoms combine to form molecular H_2 and in so doing, the magnetic moments are lost (*quenched*), because the spins pair. Hydrogen is therefore *diamagnetic* and the same holds true for most elements. Although the electronic configuration of the individual atoms (and ions) of most elements contain unpaired spins, it is not correct to call these elements 'paramagnets' because at ambient temperature quenching is very much the rule rather than the exception. However, the quenching tendency is weakest for f-electrons because *f* (especially 4*f*) orbitals are radially contracted and they overlap only weakly with orbitals on adjacent atoms.

Consequently, the lanthanide elements with incompletely filled 4f-orbitals are paramagnetic or magnetically ordered.

μ_{eff} values for typical d^3 and d^5 transition metal complexes.

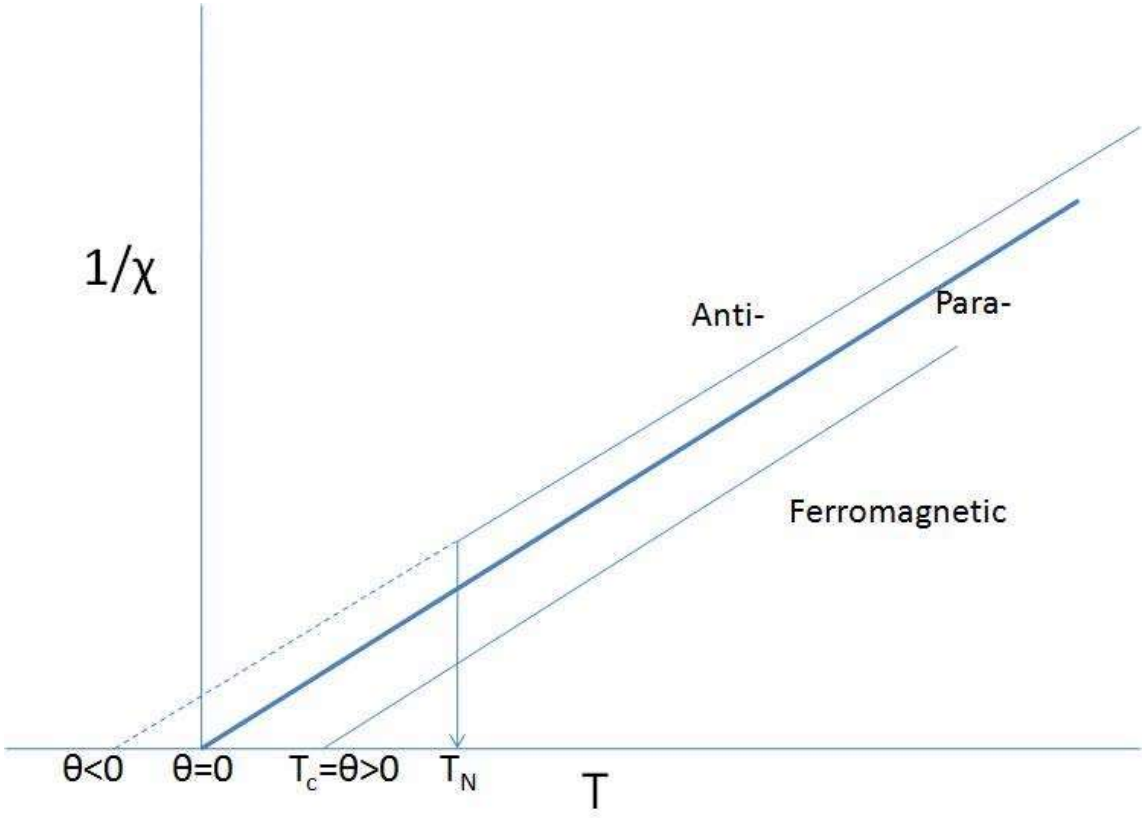
Material	μ_{eff}/μ_B
$[Cr(NH_3)_6]Br_3$	3.77
$K_3[Cr(CN)_6]$	3.87
$K_3[MoCl_6]$	3.79
$K_4[V(CN)_6]$	3.78
$[Mn(NH_3)_6]Cl_2$	5.92
$(NH_4)_2[Mn(SO_4)_2]\cdot 6H_2O$	5.92
$NH_4[Fe(SO_4)_2]\cdot 12H_2O$	5.89

Thus, condensed phase paramagnets are only possible if the interactions of the spins that lead either to quenching or to ordering are kept at bay by structural isolation of the magnetic centers. There are two classes of materials for which this holds:

1. Molecular materials with a (isolated) paramagnetic center.
    1. Good examples are coordination complexes of d- or f-metals or proteins with such centers, e.g. myoglobin. In such materials the organic part of the molecule acts as an envelope shielding the spins from their neighbors.
    2. Small molecules can be stable in radical form, oxygen O_2 is a good example. Such systems are quite rare because they tend to be rather reactive.
2. Dilute systems.
    1. Dissolving a paramagnetic species in a diamagnetic lattice at small concentrations, e.g. Nd^{3+} in $CaCl_2$ will separate the neodymium ions at

large enough distances that they do not interact. Such systems are of prime importance for what can be considered the most sensitive method to study paramagnetic systems: EPR.

Systems with interactions



Idealized Curie–Weiss behavior; N.B. $T_C=\theta$, but T_N is not θ. Paramagnetic regimes are denoted by solid lines. Close to T_N or T_C the behavior usually deviates from ideal.

As stated above many materials that contain d- or f-elements do retain unquenched spins. Salts of such elements often show paramagnetic behavior but at low enough temperatures the magnetic moments may order. It is not uncommon to call such materials 'paramagnets', when referring to their paramagnetic behavior above their Curie or Néel-points, particularly if such temperatures are very low or have never been properly measured. Even for iron it is not uncommon to say that *iron becomes a paramagnet* above its relatively high Curie-point. In that case the Curie-point is seen as a phase transition between a ferromagnet and a 'paramagnet'. The word paramagnet now merely refers to the linear response of the system to an applied field, the temperature dependence of which requires an amended version of Curie's law, known as the Curie–Weiss law:

$$\boldsymbol{M} = C\frac{\boldsymbol{H}}{T-\theta}$$

This amended law includes a term θ that describes the exchange interaction that is present albeit overcome by thermal motion. The sign of θ depends on whether ferro- or antiferromagnetic interactions dominate and it is seldom exactly zero, except in the dilute, isolated cases mentioned above.

Obviously, the paramagnetic Curie–Weiss description above T_N or T_C is a rather different interpretation of the word 'paramagnet' as it does *not* imply the *absence* of interactions, but rather that the magnetic structure is random in the absence of an external field at these sufficiently high temperatures. Even if θ is close to zero this does not mean that there are no interactions, just that the aligning ferro- and the anti-aligning antiferromagnetic ones cancel. An additional complication is that the interactions are often different in different directions of the crystalline lattice (anisotropy), leading to complicated magnetic structures once ordered.

Randomness of the structure also applies to the many metals that show a net paramagnetic response over a broad temperature range. They do not follow a Curie type law as function of temperature however, often they are more or less temperature independent. This type of behavior is of an itinerant nature and better called Pauli-paramagnetism, but it is not unusual to see e.g. the metal aluminium called a 'paramagnet', even though interactions are strong enough to give this element very good electrical conductivity.

Superparamagnets

There are materials that show induced magnetic behavior that follows a Curie type law but with exceptionally large values for the Curie constants. These materials are known as superparamagnets. They are characterized by a strong ferro- or ferrimagnetic type of coupling into domains of a limited size that behave independently from one another. The bulk properties of such a system resembles that of a paramagnet, but on a microscopic level they are ordered. The materials do show an ordering temperature above which the behavior reverts to ordinary paramagnetism (with interaction). Ferrofluids are a good example, but the phenomenon can also occur inside solids, e.g. when dilute paramagnetic centers are introduced in a strong itinerant medium of ferromagnetic coupling such as when Fe is substituted in $TlCu_2Se_2$ or the alloy AuFe. Such systems contain ferromagnetically coupled clusters that freeze out at lower temperatures. They are also called mictomagnets.

Superparamagnetism

Superparamagnetism is a form of magnetism, which appears in small ferromagnetic or ferrimagnetic nanoparticles. In small enough nanoparticles, magnetization can randomly flip direction under the influence of temperature. The typical time between two flips is called the Néel relaxation time. In the absence of external magnetic field, when the time used to measure the magnetization of the nanoparticles is much longer than the *Néel*

relaxation time, their magnetization appears to be in average zero: they are said to be in the superparamagnetic state. In this state, an external magnetic field is able to magnetize the nanoparticles, similarly to a paramagnet. However, their magnetic susceptibility is much larger than the one of paramagnets.

The Néel relaxation in the absence of magnetic field

Normally, any ferromagnetic or ferrimagnetic material undergoes a transition to a paramagnetic state above its Curie temperature. Superparamagnetism is different from this standard transition since it occurs below the Curie temperature of the material.

Superparamagnetism occurs in nanoparticles which are single-domain, i.e. composed of a single magnetic domain. This is possible when their diameter is below 3–50 nm, depending on the materials. In this condition, it is considered that the magnetization of the nanoparticles is a single giant magnetic moment, sum of all the individual magnetic moments carried by the atoms of the nanoparticle. This is what people working in the field of superparamagnetism call the "macro-spin approximation".

Because of the nanoparticle's magnetic anisotropy, the magnetic moment has usually only two stable orientations antiparallel to each other, separated by an energy barrier. The stable orientations define the nanoparticle's so called "easy axis". At finite temperature, there is a finite probability for the magnetization to flip and reverse its direction. The mean time between two flips is called the Néel relaxation time τ_N and is given by the following Néel-Arrhenius equation:

$$\tau_N = \tau_0 \exp\left(\frac{KV}{k_B T}\right),$$

where:

- τ_N is thus the average length of time that it takes for the nanoparticle's magnetization to randomly flip as a result of thermal fluctuations.
- τ_0 is a length of time, characteristic of the material, called the *attempt time* or *attempt period* (its reciprocal is called the *attempt frequency*); its typical value is 10^{-9}–10^{-10} second.
- K is the nanoparticle's magnetic anisotropy energy density and V its volume. KV is therefore the energy barrier associated with the magnetization moving from its initial easy axis direction, through a "hard plane", to the other easy axis direction.
- k_B is the Boltzmann constant.
- T is the temperature.

This length of time can be anywhere from a few nanoseconds to years or much longer. In particular, it can be seen that the Néel relaxation time is an exponential function of the grain volume, which explains why the flipping probability becomes rapidly negligible for bulk materials or large nanoparticles.

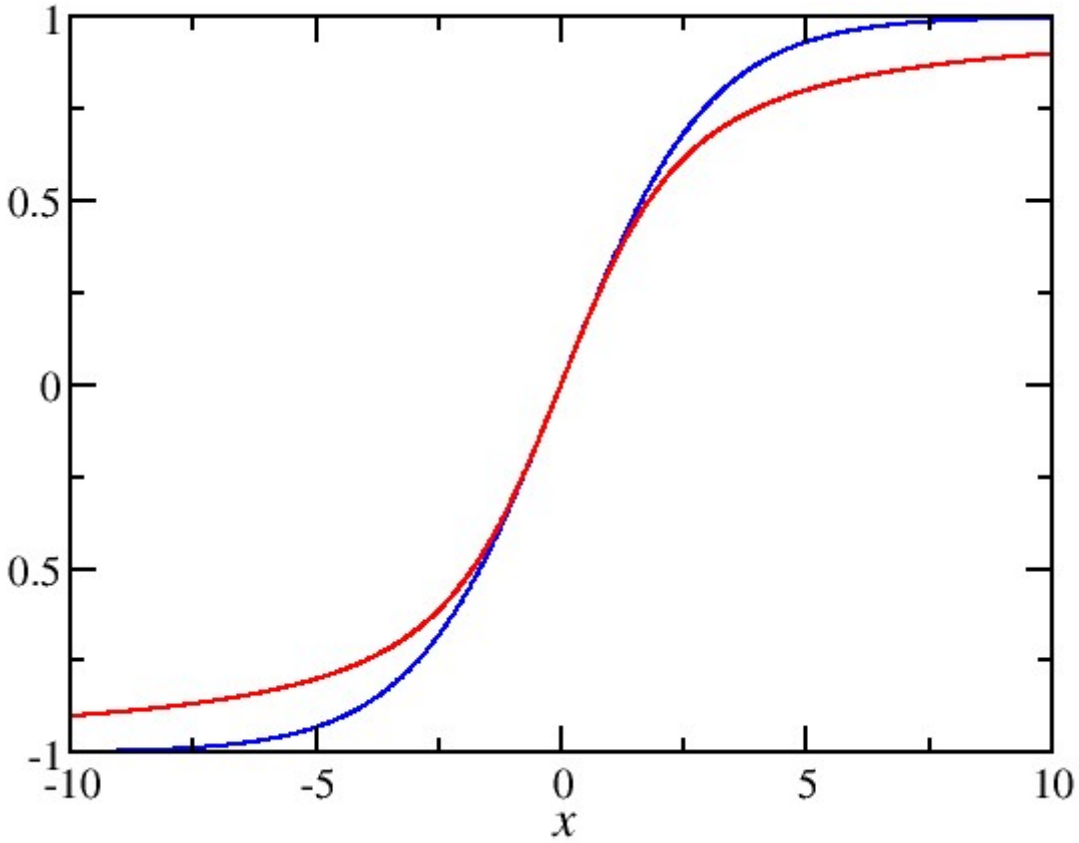
Blocking temperature

Let us imagine that the magnetization of a single superparamagnetic nanoparticle is measured and let us call τ_m the measurement time. If $\tau_m \gg \tau_N$, the nanoparticle magnetization will flip several times during the measurement, then the measured magnetization will average to zero. If $\tau_m \ll \tau_N$, the magnetization will not flip during the measurement, so the measured magnetization will be what the instantaneous magnetization was at the beginning of the measurement. In the former case, the nanoparticle will appear to be in the superparamagnetic state whereas in the latter case it will appear to be "blocked" in its initial state. **The state of the nanoparticle (superparamagnetic or blocked) depends on the measurement time.** A transition between superparamagnetism and blocked state occurs when $\tau_m = \tau_N$. In several experiments, the measurement time is kept constant but the temperature is varied, so the transition between superparamagnetism and blocked state is seen as a function of the temperature. The temperature for which $\tau_m = \tau_N$ is called the **blocking temperature**:

$$T_B = \frac{KV}{k_B \ln\left(\frac{\tau_m}{\tau_0}\right)}$$

For typical laboratory measurements, the value of the logarithm in the previous equation is in the order of 20–25.

Effect of a magnetic field



Langevin function (red line), compared with *tanh*(*x* / 3) (blue line)

When an external magnetic field is applied to an assembly of superparamagnetic nanoparticles, their magnetic moments tend to align along the applied field, leading to a net magnetization. The magnetization curve of the assembly, i.e. the magnetization as a function of the applied field, is a reversible S-shaped increasing function. This function is quite complicated but for some simple cases:

1. If all the particles are identical (same energy barrier and same magnetic moment), their easy axes are all oriented parallel to the applied field and the temperature is low enough ($T_B < T \lesssim KV/(10\ k_B)$), then the magnetization of the assembly is
$$M(H) \approx n\mu \tanh\left(\frac{\mu_0 H \mu}{k_B T}\right).$$
2. If all the particles are identical and the temperature is high enough ($T \gtrsim KV/k_B$), then, irrespective of the orientations of the easy axes:
$$M(H) \approx n\mu L\left(\frac{\mu_0 H \mu}{k_B T}\right)$$

In the above equations:

- n in the density of nanoparticles in the sample
- μ_0 is the magnetic permeability of vacuum
- μ is the magnetic moment of a nanoparticle
- $L(x) = 1 / \tanh(x) - 1 / x$ is the Langevin function

The initial slope of the $M(H)$ function is the magnetic susceptibility of the sample χ:

$$\chi = \frac{n\mu_0\mu^2}{k_B T}$$ in the first case

$$\chi = \frac{n\mu_0\mu^2}{3k_B T}$$ in the second case.

The later susceptibility is also valid for all temperatures $T > T_B$ if the the easy axes of the nanoparticles are randomly oriented.

It can be seen from these equations that large nanoparticles have a larger μ and so a larger susceptibility. This explains why superparamagnetic nanoparticles have a much larger susceptibility than standard paramagnets: they behave exactly as a paramagnet with a huge magnetic moment.

Time dependence of the magnetization

There is no time-dependence of the magnetization when the nanoparticles are either completely blocked ($T \ll T_B$) or completely superparamagnetic ($T \gg T_B$). There is, however, a narrow window around T_B where the measurement time and the relaxation time have comparable magnitude. In this case, a frequency-dependence of the susceptibility can be observed. For a randomly-oriented sample, the complex susceptibility is:

$$\chi(\omega) = \frac{\chi_{sp} + i\omega\tau\chi_b}{1 + i\omega\tau}$$

where

- $\frac{\omega}{2\pi}$ is the frequency of the applied field
- $\chi_{sp} = \frac{n\mu_0\mu^2}{3k_B T}$ is the susceptibility in the superparamagnetic state
- $\chi_b = \frac{n\mu_0\mu^2}{3KV}$ is the susceptibility in the blocked state
- $\tau = \frac{\tau_N}{2}$ is the relaxation time of the assembly

From this frequency-dependent susceptibility, the time-dependence of the magnetization for low-fields can be derived:

$$\tau \frac{\mathrm{d}M}{\mathrm{d}t} + M = \tau \chi_b \frac{\mathrm{d}H}{\mathrm{d}t} + \chi_{sp} H$$

Measurements

A superparamagnetic system can be measured with AC susceptibility measurements, where an applied magnetic field varies in time, and the magnetic response of the system is measured. A superparamagnetic system will show a characteristic frequency dependence: When the frequency is much higher than $1/\tau_N$, there will be a different magnetic response than when the frequency is much lower than $1/\tau_N$, since in the latter case, but not the former, the ferromagnetic clusters will have time to respond to the field by flipping their magnetization. The precise dependence can be calculated from the Néel-Arrhenius equation, assuming that the neighboring clusters behave independently of one another (if clusters interact, their behavior becomes more complicated).

Effect on hard drives

Superparamagnetism sets a limit on the storage density of hard disk drives due to the minimum size of particles that can be used. This limit is known as the **superparamagnetic limit**. Current hard disk technology with longitudinal recording has an estimated limit of 100 to 200 Gbit/in², though this estimate is constantly changing.

One suggested technique to further extend recording densities on hard disks is to use perpendicular recording rather than the conventional longitudinal recording. This changes the geometry of the disk and alters the strength of the superparamagnetic effect. Perpendicular recording is predicted to allow information densities of up to around 1 Tbit/in² (1024 Gbit/in²).

Another technique in development is the use of HAMR drives, which use materials that are stable at much smaller sizes. But, they require heating before the magnetic orientation of a bit can be changed.

Applications of superparamagnetism

General Applications

- Ferrofluid: tunable viscosity
- Data analysis: superparamagnetic clustering (SPC) and its extension global SPC (gSPC)

Biomedical applications

- Imaging: Contrast agents in Magnetic Resonance Imaging (MRI)
- Magnetic separation: cell-, DNA-, protein- separation, RNA fishing
- Treatments: targeted drug delivery, magnetic hyperthermia, magnetofection

Chapter- 4

Electric Field

In physics, an **electric field** surrounds electrically charged particles and time-varying magnetic fields. This electric field exerts a force on other electrically charged objects. Michael Faraday introduced the concept of an electric field.

The electric field is a vector field with SI units of newtons per coulomb (N C^{-1}) or, equivalently, volts per metre (V m^{-1}). The SI base units of the electric field are $kg \cdot m \cdot s^{-3} \cdot A^{-1}$. The strength or magnitude of the field at a given point is defined as the force that would be exerted on a positive test charge of 1 coulomb placed at that point; the direction of the field is given by the direction of that force. Electric fields contain electrical energy with energy density proportional to the square of the field amplitude. The electric field is to charge as gravitational acceleration is to mass and force density is to volume.

An electric field that changes with time (such as due to the motion of charged particles in the field) influences the local magnetic field. That is, the electric and magnetic fields are not completely separate phenomena; what one observer perceives as an electric field, another observer in a different frame of reference perceives as a mixture of electric and magnetic fields. For this reason, one speaks of "electromagnetism" or "electromagnetic fields". In quantum mechanics, disturbances in the electromagnetic fields are called photons, and the energy of photons is quantized.

Definition

The electric field is defined as the force per unit charge that would be experienced by a stationary point charge at a given location in the field:

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

where

F is the **electric force** experienced by the particle
q is its charge
E is the electric field wherein the particle is located

Taken literally, this equation only defines the electric field at the places where there are stationary charges present to experience it. Furthermore, the force exerted by another charge q will alter the source distribution, which means the electric field in the presence of q differs from itself in the absence of q. However, the electric field of a given source distribution remains defined in the absence of any charges with which to interact. This is achieved by measuring the force exerted on successively smaller *test charges* placed in the vicinity of the source distribution. By this process, the electric field created by a given source distribution is defined as the limit as the test charge approaches zero of the force per unit charge exerted thereupon.

$$\mathbf{E} = \lim_{q \to 0} \frac{\mathbf{F}}{q}$$

This allows the electric field to be dependent on the source distribution alone.

As is clear from the definition, the direction of the electric field is the same as the direction of the force it would exert on a positively-charged particle, and opposite the direction of the force on a negatively-charged particle. Since like charges repel and opposites attract (as quantified below), the electric field tends to point away from positive charges and towards negative charges.

Based on Coulomb's Law for interacting point charges, the contribution to the E-field at a point in space due to a single, discrete charge located at another point in space is given by the following:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{\mathbf{r}}$$

where

q is the charge of the particle creating the electric force,
r is the distance from the particle with charge q to the E-field evaluation point,
$\hat{\mathbf{r}}$ is the unit vector pointing from the particle with charge q to the E-field evaluation point,
ε_0 is the electric constant.

The total E-field due to a quantity of point charges, n_q, is simply the superposition of the contribution of each individual point charge:

$$\mathbf{E} = \sum_{i=1}^{n_q} \mathbf{E}_i = \sum_{i=1}^{n_q} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i .$$

Alternately, Gauss's Law allows the E-field to be calculated in terms of a continuous distribution of charge density in space, ρ:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}.$$

Coulomb's law is actually a special case of Gauss's Law, a more fundamental description of the relationship between the distribution of electric charge in space and the resulting electric field. Gauss's law is one of Maxwell's equations, a set of four laws governing electromagnetics.

Time-varying fields

Charges do not only produce electric fields. As they move, they generate magnetic fields, and if the magnetic field changes, it generates electric fields. A changing magnetic field gives rise to an electric field,

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$$

which yields Faraday's law of induction,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

where

$\nabla \times \mathbf{E}$ indicates the curl of the electric field,

$-\frac{\partial \mathbf{B}}{\partial t}$ represents the vector rate of decrease of magnetic field with time.

This means that a magnetic field changing in time produces a curled electric field, possibly also changing in time. The situation in which electric or magnetic fields change in time is no longer electrostatics, but rather electrodynamics or electromagnetics.

Properties (in electrostatics)

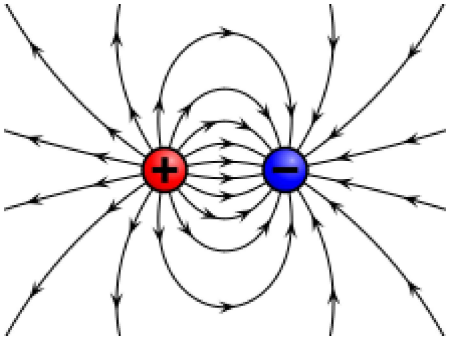


Illustration of the electric field surrounding a positive (red) and a negative (blue) charge

According to equation (1) above, electric field is dependent on position. The electric field due to any single charge falls off as the square of the distance from that charge.

Electric fields follow the superposition principle. If more than one charge is present, the total electric field at any point is equal to the vector sum of the respective electric fields that each object would create in the absence of the others.

$$\mathbf{E}_{\text{total}} = \sum_i \mathbf{E}_i = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \ldots$$

If this principle is extended to an infinite number of infinitesimally small elements of charge, the following formula results:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho}{r^2} \hat{\mathbf{r}} \, \mathrm{d}V$$

where

ρ is the charge density, or the amount of charge per unit volume.

The electric field at a point is equal to the negative gradient of the electric potential there. In symbols,

$$\mathbf{E} = -\nabla \Phi$$

where

$\Phi(x,y,z)$ is the scalar field representing the electric potential at a given point.

If several spatially distributed charges generate such an electric potential, e.g. in a solid, an electric field gradient may also be defined.

Considering the permittivity ε of a linear material, which may differ from the permittivity of free space ε_0, the electric displacement field is:

$$\mathbf{D} = \varepsilon\mathbf{E}.$$

Energy in the electric field

The electric field stores energy. The energy density of the electric field is given by

$$u = \frac{1}{2}\varepsilon|\mathbf{E}|^2,$$

where ε is the permittivity of the medium in which the field exists, and **E** is the electric field vector.

The total energy stored in the electric field in a given volume V is therefore

$$\frac{1}{2}\varepsilon\int_V |\mathbf{E}|^2\,\mathrm{d}V\,,$$

where dV is the differential volume element.

Parallels between electrostatics and gravity

Coulomb's law, which describes the interaction of electric charges:

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}} = q\mathbf{E}$$

is similar to Newton's law of universal gravitation:

$$\mathbf{F} = G\frac{Mm}{r^2}\hat{\mathbf{r}} = m\mathbf{g}.$$

This suggests similarities between the electric field **E** and the gravitational field **g**, so sometimes mass is called "gravitational charge".

Similarities between electrostatic and gravitational forces:

1. Both act in a vacuum.
2. Both are central and conservative.
3. Both obey an inverse-square law (both are inversely proportional to square of r).
4. Both propagate with finite speed c, the speed of light.

5. Electric charge and relativistic mass are conserved; note, though, that rest mass is not conserved.

Differences between electrostatic and gravitational forces:

1. Electrostatic forces are much greater than gravitational forces (by about 10^{36} times).
2. Gravitational forces are attractive for like charges, whereas electrostatic forces are repulsive for like charges.
3. There are no negative gravitational charges (no negative mass) while there are both positive and negative electric charges. This difference combined with previous implies that gravitational forces are always attractive, while electrostatic forces may be either attractive or repulsive.

Electric potential energy

Electric potential energy, or **electrostatic potential energy**, is a potential energy associated with the conservative Coulomb forces within a defined system of point charges. The term "electrostatic potential energy" is preferred here because it seems less likely to be misunderstood. The reference zero is usually taken to be a state in which the individual point charges are very well separated ("are at infinite separation") and are at rest.[:§25-1] The electrostatic potential energy of the system (U_{E}), relative to this zero, is equal to the total work W that must be done by a hypothetical external agent in order to bring the charges slowly, one by one, from infinite separation to the desired system configuration:

$$U_{\mathrm{E}} = W \ .$$

In this process the external agent is deemed to provide or absorb any relevant work, and the point charge being slowly moved gains no kinetic energy.

Sometimes reference is made to the potential energy of a charge in an electrostatic field. This actually refers to the potential energy of the system containing the charge and the *other* charges that created the electrostatic field.[:§25-1]

To calculate the work required to bring a point charge into the vicinity of other (stationary) point charges, it is sufficient to know only (a) the total field generated by the other charges and (b) the charge of the point charge being moved. The field due to the charge being moved and the values of the other charges are not required. Nonetheless, in many circumstances it is mathematically easier to add up all the pairwise potential energies (as below).

It is important to understand that electrostatics is a 18th-19th-century theory of hypothetical entities called "point charges". Electrostatics is categorically *not* a complete

theory of the charged physical particles that make up the physical world, and are subject to the Heisenberg uncertainty principle and other laws of quantum mechanics.

Electrostatic potential energy stored in a configuration of discrete point charges

The mutual electrostatic potential energy of two charges is equal to the potential energy of a charge in the electrostatic potential generated by the other. That is to say, if charge q_1 generates an electrostatic potential $\Phi_1(\mathbf{r})$, which is a function of position $\mathbf{r}$, then $U_E = q_2\Phi_1(\mathbf{r}_2)$. Also, a similar development gives $U_E = q_1\Phi_2(\mathbf{r}_1)$.

This can be generalized to give an expression for a group of N charges, q_i at positions $\mathbf{r}_i$:

$$U_E = \frac{1}{2}\sum_{i}^{N} q_i\Phi(\mathbf{r}_i)$$

where, for each i value, $\Phi(\mathbf{r}_i)$ is the electrostatic potential due to all point charges except the one at $\mathbf{r}_i$

Note: The factor of one half accounts for the 'double counting' of charge pairs. For example, consider the case of just two charges.

Alternatively, the factor of one half may be dropped if the sum is only performed once per charge pair. This is done in the examples below to cut down on the math.

One point charge

The electrostatic potential energy of a system containing only one point charge is zero, as there are no other sources of electrostatic potential against which an external agent must do work in moving the point charge from infinity to its final location. One should carefully consider the possibility of the point charge interacting with its own electrostatic potential. However, since such a potential at the location of the point charge itself is infinite, this "self-energy" is intentionally excluded from an evaluation of the total (finite) electrostatic potential energy of the system. Moreover, one may argue that since the electrostatic potential due to the point charge itself provides no work in moving the point charge around this interaction is unimportant for most purposes.

Two point charges

Consider bringing a second point charge, q_2, into its final position in the vicinity of the first point charge, q_1. The electrostatic potential $\Phi(r)$ due to q_1 is

$$\Phi(r_1) = k_e \frac{q_1}{r}$$

where k_e is Coulomb's constant. In the International System of Quantities, which has been the preferred international system since the 1970s and forms the basis for the definition of SI units, the Coulomb constant is given by

$$k_e = \frac{1}{4\pi\varepsilon_0},$$

where $\varepsilon_0$is the electric constant. Hence we obtain,

$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where r_{12} is the distance between the two point charges.

The electrostatic potential energy is negative if the charges have opposite sign and positive if the charges have the same sign. Negative mutual potential energy corresponds to attraction between two point charges; positive mutual potential energy to repulsion between two point charges.

Three or more point charges

For three or more point charges, the electrostatic potential energy of the system may be calculated by the total amount of work done by an external agent in bringing individual point charges into their final positions one after another. Thus,

$$U_E = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \cdots + \frac{q_i q_j}{r_{ij}}\right)$$

where

q_1, q_2, ..., are the point charges
r_{ij} is the distance between the ith and jth point charges.

NOTE Here, $\varepsilon_0$is the relative permittivity of free space. When the charge is in a medium other than free space / air,the relative permittivity, $\varepsilon = k\varepsilon_0$, has to be taken into account where **k** is the dielectric constant of the medium. K is the ratio of the electrostatic force on the charges in free space to the electrostatic force on the charges in the respective medium

Energy stored in an electrostatic field distribution

One may take the equation for the electrostatic potential energy of a continuous charge distribution and put it in terms of the electrostatic field.

Since Gauss' law for electrostatic field in differential form states

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

where

- $\mathbf{E}$ is the electric field vector
- ρ is the total charge density including dipole charges bound in a material.

then,

$$U = \frac{1}{2} \int\limits_{\text{all space}} \rho(r)\Phi(r)d^3r$$
$$= \frac{1}{2} \int\limits_{\text{all space}} \epsilon_0(\nabla \cdot \mathbf{E})\Phi(r)d^3r$$

so, now using the following divergence vector identity

$$\nabla \cdot (\vec{A}B) = (\nabla \cdot \vec{A})B + \vec{A} \cdot (\nabla B) \Rightarrow (\nabla \cdot \vec{A})B = \nabla \cdot (\vec{A}B) - \vec{A} \cdot (\nabla B)$$

we have

$$U = \frac{\epsilon_0}{2} \int\limits_{\text{all space}} \nabla \cdot (\mathbf{E}\Phi)d^3r - \frac{\epsilon_0}{2} \int\limits_{\text{all space}} (\nabla\Phi) \cdot \mathbf{E}d^3r$$

using the divergence theorem and taking the area to be at infinity where $\Phi(\infty) = 0$

$$U = \frac{\epsilon_0}{2} \int \Phi\mathbf{E} \cdot dA - \frac{\epsilon_0}{2} \int\limits_{\text{all space}} (-\mathbf{E}) \cdot \mathbf{E}d^3r$$
$$= \int\limits_{\text{all space}} \frac{1}{2}\epsilon_0 |\mathbf{E}|^2 d^3r.$$

So, the energy density, or energy per unit volume of the electrostatic field is:

$$u_e = \frac{1}{2}\epsilon_0 |\mathbf{E}|^2 .$$

Energy in electronic elements

Some elements in a circuit can convert energy from one form to another. For example, a resistor converts electrical energy to heat, and a capacitor stores it in its electric field.

The total electrostatic potential energy stored in a capacitor is given by

$$U_{\mathrm{E}} = \frac{1}{2}CV^2$$

where C is the capacitance and V is the total electrostatic potential.

Chapter- 5

Permittivity

In electromagnetism, **permittivity** is the measure of how much resistance is encountered when forming an electric field in a medium. In other words, permittivity is a measure of how an electric field affects, and is affected by, a dielectric medium. Permittivity is determined by the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material. Thus, permittivity relates to a material's ability to transmit (or "permit") an electric field.

It is directly related to **electric susceptibility**, which is a measure of how easily a dielectric polarizes in response to an electric field.

In SI units, permittivity ε is measured in farads per meter (F/m); electric susceptibility χ is dimensionless. They are related to each other through

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi)\varepsilon_0$$

where ε_r is the relative permittivity of the material, and $\varepsilon_0 = 8.85\ldots \times 10^{-12}$ F/m is the vacuum permittivity.

Explanation

In electromagnetism, the electric displacement field **D** represents how an electric field **E** influences the organization of electrical charges in a given medium, including charge migration and electric dipole reorientation. Its relation to permittivity in the very simple case of *linear, homogeneous, isotropic* materials with *"instantaneous" response* to changes in electric field is

$$\mathbf{D} = \varepsilon \mathbf{E}$$

where the permittivity ε is a scalar. If the medium is anisotropic, the permittivity is a second rank tensor.

In general, permittivity is not a constant, as it can vary with the position in the medium, the frequency of the field applied, humidity, temperature, and other parameters. In a nonlinear medium, the permittivity can depend on the strength of the electric field. Permittivity as a function of frequency can take on real or complex values.

In SI units, permittivity is measured in farads per meter (F/m or $A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}$). The displacement field **D** is measured in units of coulombs per square meter (C/m^2), while the electric field **E** is measured in volts per meter (V/m). **D** and **E** describe the interaction between charged objects. **D** is related to the *charge densities* associated with this interaction, while **E** is related to the *forces* and *potential differences*.

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field, and so the more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi(t - t')\mathbf{E}(t')\, dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi(\Delta t) = \chi\delta(\Delta t)$.

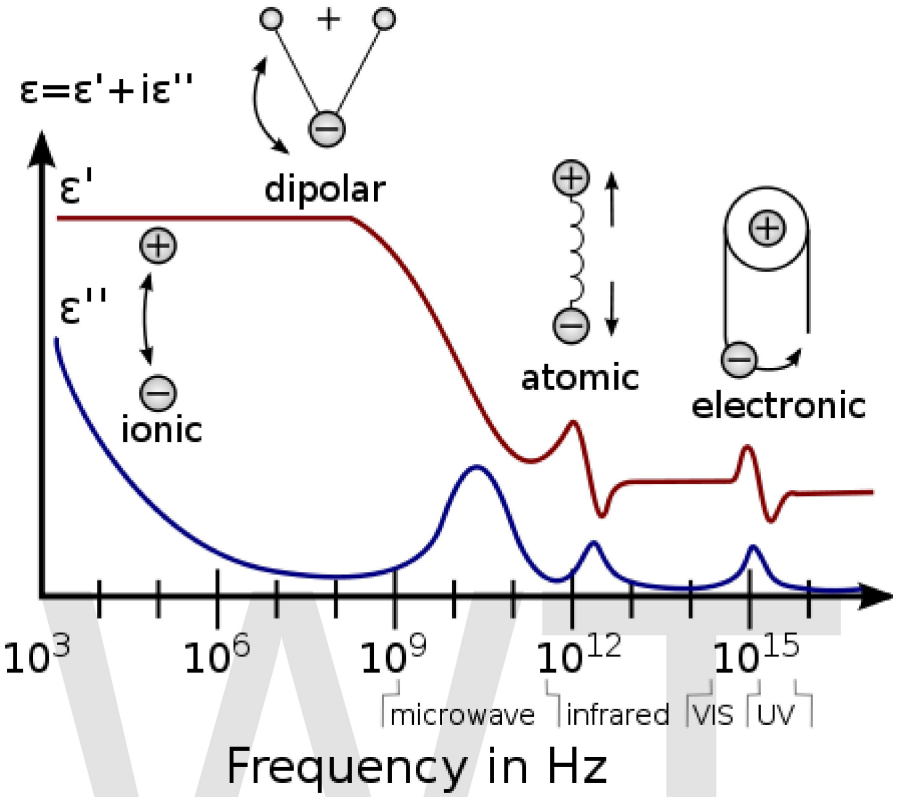
It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

$$\mathbf{P}(\omega) = \varepsilon_0 \chi(\omega)\mathbf{E}(\omega).$$

This frequency dependence of the susceptibility leads to frequency dependence of the permittivity. The shape of the susceptibility with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers–Kronig constraints on the susceptibility $\chi(0)$.

Complex permittivity



A dielectric permittivity spectrum over a wide range of frequencies. ε′ and ε″ denote the real and the imaginary part of the permittivity, respectively. Various processes are labeled on the image: ionic and dipolar relaxation, and atomic and electronic resonances at higher energies.

As opposed to the response of a vacuum, the response of normal materials to external fields generally depends on the frequency of the field. This frequency dependence reflects the fact that a material's polarization does not respond instantaneously to an applied field. The response must always be *causal* (arising after the applied field) which can be represented by a phase difference. For this reason permittivity is often treated as a complex function (since complex numbers allow specification of magnitude and phase) of the (angular) frequency of the applied field ω, $\varepsilon \rightarrow \widehat{\varepsilon}(\omega)$. The definition of permittivity therefore becomes

$$D_0 e^{-i\omega t} = \widehat{\varepsilon}(\omega) E_0 e^{-i\omega t},$$

where

D_0 and E_0 are the amplitudes of the displacement and electrical fields, respectively,
i is the imaginary unit, $i^2 = -1$.

It is important to realize that the choice of sign for time-dependence dictates the sign convention for the imaginary part of permittivity. The signs used here correspond to those commonly used in physics, whereas for the engineering convention one should reverse all imaginary quantities.

The response of a medium to static electric fields is described by the low-frequency limit of permittivity, also called the static permittivity ε_s (also ε_{DC}):

$$\varepsilon_s = \lim_{\omega \to 0} \widehat{\varepsilon}(\omega).$$

At the high-frequency limit, the complex permittivity is commonly referred to as ε_∞. At the plasma frequency and above, dielectrics behave as ideal metals, with electron gas behavior. The static permittivity is a good approximation for altering fields of low frequencies, and as the frequency increases a measurable phase difference δ emerges between **D** and **E**. The frequency at which the phase shift becomes noticeable depends on temperature and the details of the medium. For moderate fields strength (E_0), **D** and **E** remain proportional, and

$$\widehat{\varepsilon} = \frac{D_0}{E_0} = |\varepsilon| e^{i\delta}.$$

Since the response of materials to alternating fields is characterized by a complex permittivity, it is natural to separate its real and imaginary parts, which is done by convention in the following way:

$$\widehat{\varepsilon}(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) = \frac{D_0}{E_0} (\cos\delta + i \sin\delta).$$

where

ε" is the imaginary part of the permittivity, which is related to the dissipation (or loss) of energy within the medium.
ε' is the real part of the permittivity, which is related to the stored energy within the medium.

The complex permittivity is usually a complicated function of frequency ω, since it is a superimposed description of dispersion phenomena occurring at multiple frequencies. The dielectric function ε(ω) must have poles only for frequencies with positive imaginary parts, and therefore satisfies the Kramers–Kronig relations. However, in the narrow frequency ranges that are often studied in practice, the permittivity can be approximated as frequency-independent or by model functions.

At a given frequency, the imaginary part of $\widehat{\varepsilon}$ leads to absorption loss if it is positive (in the above sign convention) and gain if it is negative. More generally, the imaginary parts of the eigenvalues of the anisotropic dielectric tensor should be considered.

In the case of solids, the complex dielectric function is intimately connected to band structure. The primary quantity that characterizes the electronic structure of any crystalline material is the probability of photon absorption, which is directly related to the imaginary part of the optical dielectric function ε(ω). The optical dielectric function is given by the fundamental expression:

$$\varepsilon(\omega) = 1 + \frac{8\pi^2 e^2}{m^2} \sum_{c,v} \int W_{cv}(E) \left[\varphi(\hbar\omega - E) - \varphi(\hbar\omega + E)\right] dx.$$

In this expression, $W_{cv}(E)$ represents the product of the Brillouin zone-averaged transition probability at the energy E with the joint density of states, $J_{cv}(E)$; φ is a broadening function, representing the role of scattering in smearing out the energy levels. In general, the broadening is intermediate between Lorentzian and Gaussian; for an alloy it is somewhat closer to Gaussian because of strong scattering from statistical fluctuations in the local composition on a nanometer scale.

Classification of materials

Materials can be classified according to their permittivity and conductivity, σ. Materials with a large amount of loss inhibit the propagation of electromagnetic waves. In this case, generally when $\sigma/(\omega\varepsilon')$ >> 1, we consider the material to be a good conductor. Dielectrics are associated with lossless or low-loss materials, where $\sigma/(\omega\varepsilon')$ << 1. Those that do not fall under either limit are considered to be general media. A *perfect dielectric* is a material that has no conductivity, thus exhibiting only a displacement current. Therefore it stores and returns electrical energy as if it were an ideal capacitor.

Lossy medium

In the case of lossy medium, i.e. when the conduction current is not negligible, the total current density flowing is:

$$J_{\text{tot}} = J_c + J_d = \sigma E - i\omega\varepsilon' E = -i\omega\widehat{\varepsilon} E$$

where

σ is the conductivity of the medium;
ε' is the real part of the permittivity.
$\widehat{\varepsilon}$ is the complex permittivity

The size of the displacement current is dependent on the frequency ω of the applied field E; there is no displacement current in a constant field.

In this formalism, the complex permittivity is defined as:

$$\hat{\varepsilon} = \varepsilon' + i\frac{\sigma}{\omega}$$

In general, the absorption of electromagnetic energy by dielectrics is covered by a few different mechanisms that influence the shape of the permittivity as a function of frequency:

- First, are the relaxation effects associated with permanent and induced molecular dipoles. At low frequencies the field changes slowly enough to allow dipoles to reach equilibrium before the field has measurably changed. For frequencies at which dipole orientations cannot follow the applied field due to the viscosity of the medium, absorption of the field's energy leads to energy dissipation. The mechanism of dipoles relaxing is called dielectric relaxation and for ideal dipoles is described by classic Debye relaxation.

- Second are the resonance effects, which arise from the rotations or vibrations of atoms, ions, or electrons. These processes are observed in the neighborhood of their characteristic absorption frequencies.

The above effects often combine to cause non-linear effects within capacitors. For example, dielectric absorption refers to the inability of a capacitor that has been charged for a long time to completely discharge when briefly discharged. Although an ideal capacitor would remain at zero volts after being discharged, real capacitors will develop a small voltage, a phenomenon that is also called *soakage* or *battery action*. For some dielectrics, such as many polymer films, the resulting voltage may be less than 1-2% of the original voltage. However, it can be as much as 15 - 25% in the case of electrolytic capacitors or supercapacitors.

Quantum-mechanical interpretation

In terms of quantum mechanics, permittivity is explained by atomic and molecular interactions.

At low frequencies, molecules in polar dielectrics are polarized by an applied electric field, which induces periodic rotations. For example, at the microwave frequency, the microwave field causes the periodic rotation of water molecules, sufficient to break hydrogen bonds. The field does work against the bonds and the energy is absorbed by the material as heat. This is why microwave ovens work very well for materials containing water. There are two maxima of the imaginary component (the absorptive index) of water, one at the microwave frequency, and the other at far ultraviolet (UV) frequency. Both of these resonances are at higher frequencies than the operating frequency of microwave ovens.

At moderate frequencies, the energy is too high to cause rotation, yet too low to affect electrons directly, and is absorbed in the form of resonant molecular vibrations. In water, this is where the absorptive index starts to drop sharply, and the minimum of the

imaginary permittivity is at the frequency of blue light (optical regime). This is why sunlight does not damage water-containing organs such as the eye.

At high frequencies (such as UV and above), molecules cannot relax, and the energy is purely absorbed by atoms, exciting electron energy levels. Thus, these frequencies are classified as ionizing radiation.

While carrying out a complete *ab initio* (that is, first-principles) modelling is now computationally possible, it has not been widely applied yet. Thus, a phenomenological model is accepted as being an adequate method of capturing experimental behaviors. The Debye model and the Lorentz model use a 1st-order and 2nd-order (respectively) lumped system parameter linear representation (such as an RC and an LRC resonant circuit).

Measurement

The dielectric constant of a material can be found by a variety of static electrical measurements. The complex permittivity is evaluated over a wide range of frequencies by using different variants of dielectric spectroscopy, covering nearly 21 orders of magnitude from 10^{-6} to 10^{15} Hz. Also, by using cryostats and ovens, the dielectric properties of a medium can be characterized over an array of temperatures. In order to study systems for such diverse exciting fields, a number of measurement setups are used, each adequate for a special frequency range.

Various microwave measurement techniques are outlined in Chen *et al.*. Typical errors for the Hakki-Coleman method employing a puck of material between conducting planes are about 0.3%.

- Low-frequency time domain measurements (10^{-6}-10^{3} Hz)
- Low-frequency frequency domain measurements (10^{-5}-10^{6} Hz)
- Reflective coaxial methods (10^{6}-10^{10} Hz)
- Transmission coaxial method (10^{8}-10^{11} Hz)
- Quasi-optical methods (10^{9}-10^{10} Hz)
- Fourier-transform methods (10^{11}-10^{15} Hz)

At infrared and optical frequencies, a common technique is ellipsometry. Dual polarisation interferometry is also used to measure the complex refractive index for very thin films at optical frequencies.

Chapter- 6

Vacuum and Relative Permittivity

Vacuum Permittivity

The constant ε_0, commonly called the **vacuum permittivity**, **permittivity of free space** or **electric constant**, relates the units for electric charge to mechanical quantities such as length and force. For example, the force between two separated electric charges (in vacuum) is given by Coulomb's law:

$$F_C = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

where q_1 and q_2 are the charges, and r is the distance between them. Likewise, ε_0 appears in Maxwell's equations, which describe the properties of electric and magnetic fields and electromagnetic radiation, and relate them to their sources.

The value of ε_0 is *defined* in the International System of Units by the formula

$$\varepsilon_0 = \frac{1}{\mu_0 c_0^2} = 8.854187817...\times10^{-12}\ \text{A·s/(V·m)} = 8.854187817...\times10^{-12}\ \text{F/m}$$

where c_0 is the speed of light in vacuum and μ_0 is the magnetic constant or vacuum permeability.

Value

The value of ε_0 is *defined* by the formula

$$\varepsilon_0 = \frac{1}{\mu_0 c_0^2}$$

where c_0 is the speed of light in vacuum, and μ_0 is the parameter that international Standards Organizations call the "magnetic constant" (commonly called vacuum

permeability). Since μ_0 has the *defined* value $4\pi \times 10^{-7}$ H m^{-1}, and c_0 has the *defined* value 299792458 $m \cdot s^{-1}$, it follows that ε_0 has a *defined* value given approximately by

$\varepsilon_0 \approx 8.854187817... \times 10^{-12}$ $F \cdot m^{-1}$ (or $A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}$ in SI base units, or $C^2 \cdot N^{-1} \cdot m^{-2}$ or $C \cdot V^{-1} \cdot m^{-1}$ using other SI coherent units).

The ellipsis (...) does not indicate experimental uncertainty, but the arbitrary termination of a nonrecurring decimal. The historical origins of the electric constant ε_0, and its value, are explained in more detail below.

Under the proposals to redefine the ampere as a fixed number of elementary charges per second, the electric constant would no longer have an exact fixed value. Instead, it would be defined by the equation

$$\varepsilon_0 = \frac{e^2}{2\alpha h c_0}$$

where e is the elementary charge, α is the fine structure constant and h is the Planck constant. The relative uncertainty in the value would be the same as that of the fine structure constant, currently 6.8×10^{-10}.

Terminology

Historically, the parameter ε_0 has been known by many different names. The terms "vacuum permittivity" or its variants, such as "permittivity in/of vacuum", "permittivity of empty space", or "permittivity of free space" are widespread. Standards Organizations worldwide now use "electric constant" as a uniform term for this quantity, and official standards documents have adopted the term (although they continue to list the older terms as synonyms).

Another historical synonym was "dielectric constant of vacuum", as "dielectric constant" was sometimes used in the past for the absolute permittivity. However, in modern usage "dielectric constant" typically refers exclusively to a relative permittivity $\varepsilon/\varepsilon_0$ and even this usage is considered "obsolete" by some standards bodies in favor of relative static permittivity. Hence, the term "dielectric constant of vacuum" for the electric constant ε_0 is considered obsolete by most modern authors, although occasional examples of continuing usage can be found.

As for notation, the constant can be denoted by either ε_0 or ϵ_0, using either of the common glyphs for the letter epsilon.

Historical origin of the parameter ε_0

As indicated above, the parameter ε_0 is a measurement-system constant. Its presence in the equations now used to define electromagnetic quantities is the result of the so-called

"rationalization" process described below. But the method of allocating a value to it is a consequence of the result that Maxwell's equations predict that, in free space, electromagnetic waves move with the speed of light. Understanding why ε_0 has the value it does requires a brief understanding of the history of how electromagnetic measurement systems developed.

Rationalization of units

The experiments of Coulomb and others showed that the force F between two equal point-like "amounts" of electricity, situated a distance r apart in free space, should be given by a formula that has the form

$$F = k_e Q^2 / r^2,$$

where Q is a quantity that represents the amount of electricity present at each of the two points, and k_e is Coulomb's constant. If one is starting with no constraints, then the value of k_e may be chosen arbitrarily. For each different choice of k_e there is a different "interpretation" of Q: to avoid confusion, each different "interpretation" has to be allocated a distinctive name and symbol.

In one of the systems of equations and units agreed in the late 19th century, called the "centimetre-gram-second electrostatic system of units" (the cgs esu system), the constant k_e was taken equal to 1, and a quantity now called "gaussian electric charge" q_s was defined by the resulting equation

$$F = {q_s}^2 / r^2.$$

The unit of gaussian charge, the statcoulomb, is such that two units, a distance of 1 centimetre apart, repel each other with a force equal to the cgs unit of force, the dyne. Thus the unit of gaussian charge can also be written 1 $\text{dyne}^{1/2}$ cm. "Gaussian electric charge" is not the same mathematical quantity as modern (rmks) electric charge and is not measured in coulombs.

The idea subsequently developed that it would be better, in situations of spherical geometry, to include a factor 4π in equations like Coulomb's law, and write it in the form:

$$F = k_e' {q_s'}^2 / 4\pi r^2.$$

This idea is called "rationalization". The quantities q'_s and k_e' are not the same as those in the older convention. Putting $k_e'=1$ generates a unit of electricity of different size, but it still has the same dimensions as the cgs esu system.

The next step was to treat the quantity representing "amount of electricity" as a fundamental quantity in its own right, denoted by the symbol q, and to write Coulomb's Law in its modern form:

$$F = q^2/4\pi\epsilon_0 r^2.$$

The system of equations thus generated is known as the rationalized metre-kilogram-second (rmks) equation system, or "metre-kilogram-second-ampere (mksa)" equation system. This is system used to define the SI units. The new quantity q is given the name "rmks electric charge", or (nowadays) just "electric charge". Clearly, the quantity q_s used in the old cgs esu system is related to the new quantity q by

$$q_s = q/(k'_e\epsilon_0)^{1/2}.$$

Determination of a value for ε_0

One now adds the requirement that one wants force to be measured in newtons, distance in metres, and charge to be measured in the engineers' practical unit, the coulomb, which is defined as the charge accumulated when a current of 1 ampere flows for one second. This shows that the parameter ε_0 should be allocated the unit $C^2{\cdot}N^{-1}{\cdot}m^{-2}$ (or equivalent units - in practice "farads per metre").

In order to establish the numerical value of ε_0, one makes use of the fact that if one uses the rationalized forms of Coulomb's law and Ampère's force law (and other ideas) to develop Maxwell's equations, then the relationship stated above is found to exist between ε_0, μ_0 and c_0. In principle, one has a choice of deciding whether to make the coulomb or the ampere the fundamental unit of electricity and magnetism. The decision was taken internationally to use the ampere.

Permittivity of real media

By convention, the electric constant ε_0 appears in the relationship that defines the electric displacement field **D** in terms of the electric field **E**. In real media this relationship has the form

$$\mathbf{D} = \varepsilon\mathbf{E} = \varepsilon_r\varepsilon_0\mathbf{E} = \varepsilon_0\mathbf{E} + \mathbf{P},$$

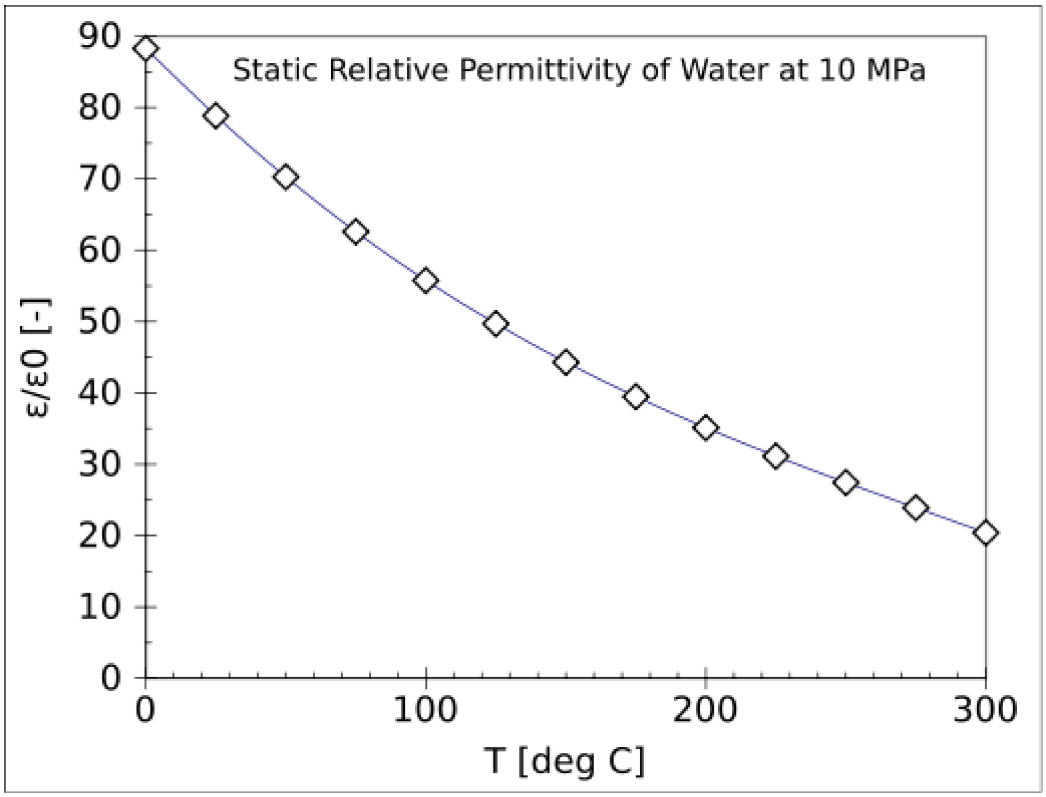
where ε is the permittivity, ε_r the relative static permittivity, and **P** is the classical electrical polarization density of the medium. In vacuum, the polarization **P** = **0**.

Relative permittivity

Relative static permittivities of some materials at room temperature under 1 kHz

Material	ε_r
Vacuum	1 (by definition)
Air	1.00058986 ± 0.00000050 (at STP, for 0.9 MHz),
PTFE/Teflon	2.1
Polyethylene	2.25
Polyimide	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Carbon disulfide	2.6
Paper	3.5
Electroactive polymers	2–12
Silicon dioxide	3.9
Concrete	4.5
Pyrex (Glass)	4.7 (3.7–10)
Rubber	7
Diamond	5.5–10
Salt	3–15
Graphite	10–15
Silicon	11.68
Ammonia	26, 22, 20, 17 (−80, −40, 0, 20 °C)
Methanol	30
Ethylene Glycol	37
Furfural	42.0
Glycerol	41.2, 47, 42.5 (0, 20, 25 °C)
Water	88, 80.1, 55.3, 34.5 (0, 20, 100, 200 °C)
Hydrofluoric acid	83.6 (0 °C)
Formamide	84.0 (20 °C)
Sulfuric acid	84–100 (20–25 °C)
Hydrogen peroxide	128 aq–60 (−30–25 °C)
Hydrocyanic acid	158.0–2.3

	(0–21 °C)
Titanium dioxide	86–173
Strontium titanate	310
Barium strontium titanate	500
Barium titanate	1250–10,000 (20–120 °C)
Lead zirconate titanate	500–6000
Conjugated polymers	1.8-6 up to 100000



Temperature dependence of the relative static permittivity of water

The **relative static permittivity** (or **static relative permittivity**) of a material under given conditions reflects the extent to which it concentrates electrostatic lines of flux. It is the ratio of the amount of electrical energy stored in a material by an applied voltage, relative to that stored in a vacuum; similarly, it is the ratio of the capacitance of a capacitor using that material as a dielectric, compared to a similar capacitor which has a vacuum as its dielectric. The relative static permittivity is the same as the relative permittivity evaluated for a frequency of zero.

The static relative permittivity is a special case of the more general relative permittivity. The latter is denoted $\varepsilon_r(\omega)$ (sometimes κ or K) and is defined as

$$\varepsilon_r(\omega) = \frac{\varepsilon(\omega)}{\varepsilon_0},$$

where $\varepsilon(\omega)$ is the complex frequency-dependent absolute permittivity of the material, and ε_0 is the electric constant. The former is simply the latter evaluated at the limit $\omega \to 0$:

$$\varepsilon_r = \lim_{\omega \to 0} \varepsilon_r(\omega) = \frac{\varepsilon_s}{\varepsilon_0},$$

where ε_s is the static absolute permittivity.

Other terms for the relative static permittivity are the **dielectric constant**, or **relative dielectric constant**, or **static dielectric constant**. These terms, while they remain very common, are ambiguous and have been deprecated by some standards organizations. The reason for the potential ambiguity is twofold. First, some older authors used "dielectric constant" or "absolute dielectric constant" for the absolute permittivity ε rather than the relative permittivity. Second, while in most modern usage "dielectric constant" refers to a relative permittivity, it may be either the static or the frequency-dependent relative permittivity depending on context.

Relative permittivity is a dimensionless complex number. By definition, the linear relative permittivity of vacuum is equal to 1, that is $\varepsilon = \varepsilon_0$, although there are theoretical nonlinear quantum effects in vacuum that have been predicted at high field strengths (but not yet observed).

The static relative permittivity of a medium is related to its static electric susceptibility, χ_e, as $\varepsilon_r(\omega) = 1 + \chi_e$

Measurement

The relative static permittivity, ε_r, can be measured for static electric fields as follows: first the capacitance of a test capacitor, C_0, is measured with vacuum between its plates. Then, using the same capacitor and distance between its plates the capacitance C_x with a dielectric between the plates is measured. The relative dielectric constant can be then calculated as

$$\varepsilon_r = \frac{C_x}{C_0}.$$

For time-variant electromagnetic fields, this quantity becomes frequency dependent and in general is called *relative permittivity*.

Practical relevance

The dielectric constant is an essential piece of information when designing capacitors, and in other circumstances where a material might be expected to introduce capacitance into a circuit. If a material with a high dielectric constant is placed in an electric field, the magnitude of that field will be measurably reduced within the volume of the dielectric. This fact is commonly used to increase the capacitance of a particular capacitor design. The layers beneath etched conductors in printed circuit boards (PCBs) also act as dielectrics.

Dielectrics are used in RF transmission lines. In a coaxial cable, polyethylene can be used between the center conductor and outside shield. It can also be placed inside waveguides to form filters. Optical fibers are examples of *dielectric waveguides*. They consist of dielectric materials that are purposely doped with impurities so as to control the precise value of ε_r within the cross-section. This controls the refractive index of the material and therefore also the optical modes of transmission. However, in these cases it is technically the relative permittivity that matters, as they are not operated in the electrostatic limit.

Chemical applications

The relative static permittivity of a solvent is a relative measure of its polarity. For example, water (very polar) has a dielectric constant of 80.10 at 20 °C while n-hexane (very non-polar) has a dielectric constant of 1.89 at 20 °C. This information is of great value when designing separation, sample preparation and chromatography techniques in analytical chemistry.

Complex permittivity

Similar as for absolute permittivity, relative permittivity can be decomposed into real and imaginary parts:

$$\varepsilon_r(\omega) = \varepsilon'_r(\omega) + i\varepsilon''_r(\omega)$$

Lossy medium

Again, similar as for absolute permittivity, relative permittivity for lossy materials can be formulated as:

$$\varepsilon_r = \varepsilon'_r + i\frac{\sigma}{\omega\varepsilon_0},$$

in terms of a "dielectric conductivity" σ (units S/m, siemens per meter) which "sums over all the dissipative effects of the material; it may represent an actual [electrical] conductivity caused by migrating charge carriers and it may also refer to an energy loss

associated with the dispersion of ε' [the real-valued permittivity]" (, p. 8). Expanding the angular frequency $\omega = 2\pi c/\lambda$ and the electric constant $\varepsilon_0 = 1/(\mu_0 c^2)$, it reduces to:

$$\varepsilon_r = \varepsilon'_r + i\sigma\lambda\kappa,$$

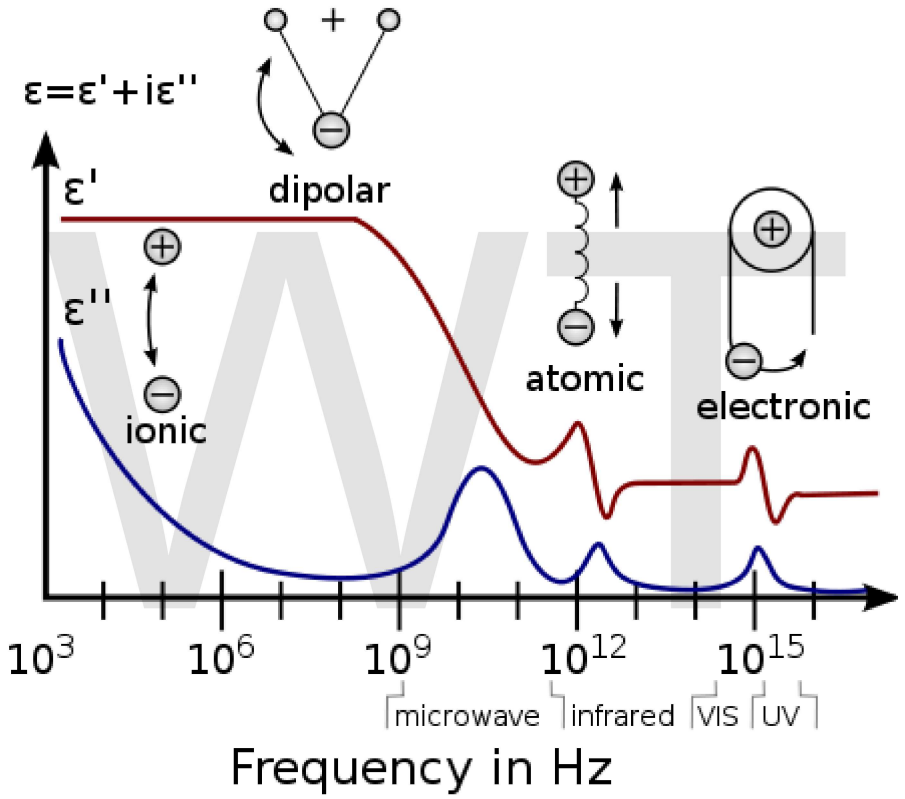
where λ is the wavelength, c is the speed of light in vacuum and $\kappa = \mu_0 c/2\pi \approx 60.0\ \mathrm{S}^{-1}$ is a newly-introduced constant (units reciprocal of siemens, such that $\sigma\lambda\kappa = \varepsilon_r$" remains unitless).

Metals

Although permittivity is typically associated with dielectric materials, we may still speak of an effective permittivity of a metal, with real relative permittivity equal to one (, eq.(4.6), p.121). In the low-frequency region (which extends from radiofrequencies to the far infrared region), the plasma frequency of the electron gas is much greater than the electromagnetic propagation frequency, so the complex permittivity ε of a metal is practically a purely imaginary number, expressed in terms of the imaginary unit and a real-valued electrical conductivity.

Chapter- 7

Dielectric Spectroscopy



A dielectric permittivity spectrum over a wide range of frequencies. The real and imaginary parts of permittivity are shown, and various processes are depicted: ionic and dipolar relaxation, and atomic and electronic resonances at higher energies. From the Dielectric spectroscopy page of the research group of Dr. Kenneth A. Mauritz.

Dielectric spectroscopy (sometimes called **impedance spectroscopy**), and also known as **Electrochemical Impedance Spectroscopy**, measures the dielectric properties of a medium as a function of frequency. It is based on the interaction of an external field with the electric dipole moment of the sample, often expressed by permittivity.

It is also an experimental method of characterizing electrochemical systems. This technique measures the impedance of a system over a range of frequencies, and therefore the frequency response of the system, including the energy storage and dissipation

properties, is revealed. Often, data obtained by EIS is expressed graphically in a Bode plot or a Nyquist plot.

Impedance is the opposition to the flow of alternating current (AC) in a complex system. A passive complex electrical system comprises both energy dissipater (resistor) and energy storage (capacitor) elements. If the system is purely resistive, then the opposition to AC or direct current (DC) is simply resistance.

Almost any physico-chemical system, such as electrochemical cells, mass-beam oscillators, and even biological tissue possesses energy storage and dissipation properties. EIS examines them.

This technique has grown tremendously in stature over the past few years and is now being widely employed in a wide variety of scientific fields such as fuel cell testing, biomolecular interaction, and microstructural characterization. Often, EIS reveals information about the reaction mechanism of an electrochemical process: different reaction steps will dominate at certain frequencies, and the frequency response shown by EIS can help identify the rate limiting step.

Dielectric mechanisms

There are a number of different dielectric mechanisms, connected to the way a studied medium reacts to the applied field. Each dielectric mechanism is centered around its characteristic frequency, which is the reciprocal of the characteristic time of the process. In general, dielectric mechanisms can be divided into relaxation and resonance processes. The most common, starting from high frequencies, are:

Electronic polarization

This resonant process occurs in a neutral atom when the electric field displaces the electron density relative to the nucleus it surrounds.

This displacement occurs due to the equilibrium between restoration and electric forces. Electronic polarization may be understood by assuming an atom as a point nucleus surrounded by spherical electron cloud of uniform charge density.

Atomic polarization

Atomic polarization is observed when the electronic cloud is deformed under the force of the applied field, so that the negative and positive charge are formed. This is a resonant process.

Dipole relaxation

This originates from permanent and induced dipoles aligning to an electric field. Their orientation polarisation is disturbed by thermal noise (which mis-aligns the dipole vectors

from the direction of the field), and the time needed for dipoles to relax is determined by the local viscosity. These two facts make dipole relaxation heavily dependent on temperature and chemical surrounding.

Ionic relaxation

Ionic relaxation comprises ionic conductivity and interfacial and space charge relaxation. Ionic conductivity predominates at low frequencies and introduces only losses to the system. Interfacial relaxation occurs when charge carriers are trapped at interfaces of heterogeneous systems. A related effect is Maxwell-Wagner-Sillars polarization, where charge carriers blocked at inner dielectric boundary layers (on the mesoscopic scale) or external electrodes (on a macroscopic scale) lead to a separation of charges. The charges may be separated by a considerable distance and therefore make contributions to the dielectric loss that are orders of magnitude larger than the response due to molecular fluctuations.

Dielectric relaxation

Dielectric relaxation as a whole is the result of the movement of dipoles (dipole relaxation) and electric charges (ionic relaxation) due to an applied alternating field, and is usually observed in the frequency range 10^2-10^{10} Hz. Relaxation mechanisms are relatively slow compared to resonant electronic transitions or molecular vibrations, which usually have frequencies above 10^{12} Hz.

Dielectric

A **dielectric** is an electrical insulator that may be polarized by an applied electric field. When a dielectric is placed in an electric field, electric charges do not flow through the material, as in a conductor, but only slightly shift from their average equilibrium positions causing **dielectric polarization**. Because of dielectric polarization, positive charges are displaced toward the field and negative charges shift in the opposite direction. This creates an internal electric field that partly compensates the external field inside the dielectric. If a dielectric is composed of weakly bonded molecules, those molecules not only become polarized, but also reorient so that their symmetry axis aligns to the field.

Although the term "insulator" refers to a low degree of electrical conduction, the term "dielectric" is typically used to describe materials with a high polarizability. The latter is expressed by a number called the dielectric constant. A common, yet notable example of a dielectric is the electrically insulating material between the metallic plates of a capacitor. The polarization of the dielectric by the applied electric field increases the capacitor's surface charge.

The study of dielectric properties is concerned with the storage and dissipation of electric and magnetic energy in materials. It is important to explain various phenomena in electronics, optics, and solid-state physics.

The term "dielectric" was coined by William Whewell (from "dia-electric") in response to a request from Michael Faraday.

Electric susceptibility

The **electric susceptibility** χ_e of a dielectric material is a measure of how easily it polarizes in response to an electric field. This, in turn, determines the electric permittivity of the material and thus influences many other phenomena in that medium, from the capacitance of capacitors to the speed of light.

It is defined as the constant of proportionality (which may be a tensor) relating an electric field **E** to the induced dielectric polarization density **P** such that

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E},$$

where ε_0 is the electric permittivity of free space.

The susceptibility of a medium is related to its relative permittivity ε_r by

$$\chi_e = \varepsilon_r - 1.$$

So in the case of a vacuum,

$$\chi_e = 0.$$

The electric displacement **D** is related to the polarization density **P** by

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E}.$$

Dispersion and causality

In general, a material cannot polarize instantaneously in response to an applied field. The more general formulation as a function of time is

$$\mathbf{P}(t) = \varepsilon_0 \int_{-\infty}^{t} \chi_e(t - t') \mathbf{E}(t') \, dt'.$$

That is, the polarization is a convolution of the electric field at previous times with time-dependent susceptibility given by $\chi_e(\Delta t)$. The upper limit of this integral can be extended to infinity as well if one defines $\chi_e(\Delta t) = 0$ for $\Delta t < 0$. An instantaneous response corresponds to Dirac delta function susceptibility $\chi_e(\Delta t) = \chi_e \delta(\Delta t)$.

It is more convenient in a linear system to take the Fourier transform and write this relationship as a function of frequency. Due to the convolution theorem, the integral becomes a simple product,

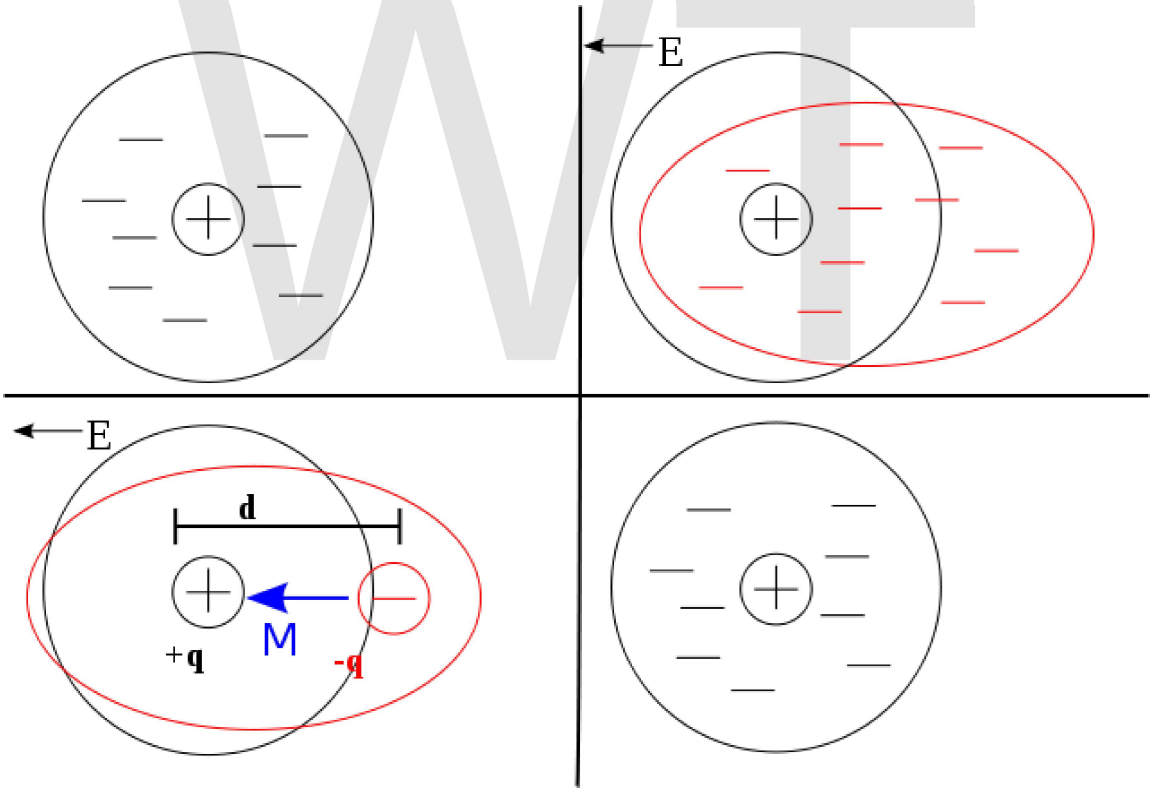
$$\mathbf{P}(\omega) = \varepsilon_0 \chi_e(\omega) \mathbf{E}(\omega).$$

Note the simple frequency dependence of the susceptibility, or equivalently the permittivity. The shape of the susceptibility with respect to frequency characterizes the dispersion properties of the material.

Moreover, the fact that the polarization can only depend on the electric field at previous times (i.e. $\chi_e(\Delta t) = 0$ for $\Delta t < 0$), a consequence of causality, imposes Kramers–Kronig constraints on the susceptibility $\chi_e(0)$.

Dielectric polarization

Basic atomic model



Electric field interaction with an atom under the classical dielectric model

In the classical approach to the dielectric model, a material is made up of atoms. Each atom consists of a cloud of negative charge (Electrons) bound to and surrounding a positive point charge at its center. Because of the comparatively huge distance between them, none of the atoms in the dielectric material interact with one another. (Note that

this model is attempting to describe not the structure of matter, but the interaction between an electric field and matter.)

In the presence of an electric field the charge cloud is distorted, as shown in the top right of the figure.

This can be reduced to a simple dipole using the superposition principle. A dipole is characterized by its dipole moment, a vector quantity shown in the figure as the blue arrow labeled *M*. It is the relationship between the electric field and the dipole moment that gives rise to the behavior of the dielectric. (Note that the dipole moment is shown to be pointing in the same direction as the electric field. This isn't always correct, and it is a major simplification, but it is suitable for many materials.)

When the electric field is removed the atom returns to its original state. The time required to do so is the so-called relaxation time; an exponential decay.

This is the essence of the model in physics. The behavior of the dielectric now depends on the situation. The more complicated the situation the richer the model has to be in order to accurately describe the behavior. Important questions are:

- Is the electric field constant or does it vary with time?
  - If the electric field does vary, at what rate?
- What are the characteristics of the material?
  - Is the direction of the field important (isotropy)?
  - Is the material the same all the way through (homogeneous)?
  - Are there any boundaries/interfaces that have to be taken into account?
- Is the system linear or do nonlinearities have to be taken into account?

The relationship between the electric field **E** and the dipole moment **M** gives rise to the behavior of the dielectric, which, for a given material, can be characterized by the function **F** defined by the equation:

$$\mathbf{M} = \mathbf{F}(\mathbf{E}),$$

When both the type of electric field and the type of material have been defined, one then chooses the simplest function *F* that correctly predicts the phenomena of interest. Examples of phenomena that can be so modeled include:

- Refractive index
- Group velocity dispersion
- Birefringence
- Self-focusing
- Harmonic generation

Dipolar polarization

Dipolar polarization is a polarization that is particular to polar molecules. This polarization results from permanent dipoles, e.g. asymmetric bonds between oxygen and hydrogen atoms, which retain polarization in the absence of an external electric field. The assembly of these dipoles forms a macroscopic polarization.

When an external electric field is applied, the distance between charges, which is related to chemical bonding, remains constant in the polarization; however, the polarization itself rotates. This rotation occurs on a timescale which depends on the torque and the surrounding local viscosity of the molecules. Because the rotation is not instantaneous, dipolar polarizations lose the response to electric fields at the lowest frequency in polarizations. The delay of the response to the change of the electric field causes friction and heat.

Ionic polarization

Ionic polarization is polarization which is caused by relative displacements between positive and negative ions in ionic crystals (for example, NaCl).

If crystals or molecules do not consist of only atoms of the same kind, the distribution of charges around an atom in the crystals or molecules leans to positive or negative. As a result, when lattice vibrations or molecular vibrations induce relative displacements of the atoms, the centers of positive and negative charges might be in different locations. These center positions are affected by the symmetry of the displacements. When the centers don't correspond, polarizations arise in molecules or crystals. This polarization is called **ionic polarization**.

Ionic polarization causes ferroelectric transition as well as dipolar polarization. The transition, which is caused by the order of the directional orientations of permanent dipoles along a particular direction, is called **order-disorder phase transition**. The transition which is caused by ionic polarizations in crystals is called **displacive phase transition**.

Dielectric dispersion

In physics, **dielectric dispersion** is the dependence of the permittivity of a dielectric material on the frequency of an applied electric field. Because there is always a lag between changes in polarization and changes in an electric field, the permittivity of the dielectric is a complicated, complex-valued function of frequency of the electric field. It is very important for the application of dielectric materials and the analysis of polarization systems.

This is one instance of a general phenomenon known as material dispersion: a frequency-dependent response of a medium for wave propagation.

When the frequency becomes higher:

1. it becomes impossible for dipolar polarization to follow the electric field in the microwave region around 10^{10} Hz;
2. in the infrared or far-infrared region around 10^{13} Hz, ionic polarization loses the response to the electric field;
3. electronic polarization loses its response in the ultraviolet region around 10^{15} Hz.

In the wavelength region below ultraviolet, permittivity approaches the constant ε_0 in every substance, where ε_0 is the permittivity of the free space. Because permittivity indicates the strength of the relation between an electric field and polarization, if a polarization process loses its response, permittivity decreases.

Dielectric relaxation

Dielectric relaxation is the momentary delay (or lag) in the dielectric constant of a material. This is usually caused by the delay in molecular polarization with respect to a changing electric field in a dielectric medium (e.g. inside capacitors or between two large conducting surfaces). Dielectric relaxation in changing electric fields could be considered analogous to hysteresis in changing magnetic fields (for inductors or transformers). Relaxation in general is a delay or lag in the response of a linear system, and therefore dielectric relaxation is measured relative to the expected linear steady state (equilibrium) dielectric values. The time lag between electrical field and polarization implies an irreversible degradation of free energy(G).

In physics, **dielectric relaxation** refers to the relaxation response of a dielectric medium to an external electric field of microwave frequencies. This relaxation is often described in terms of permittivity as a function of frequency, which can, for ideal systems, be described by the Debye equation. On the other hand, the distortion related to ionic and electronic polarization shows behavior of the resonance or oscillator type. The character of the distortion process depends on the structure, composition, and surroundings of the sample.

The number of possible wavelengths of emitted radiation due to dielectric relaxation can be equated using Hemmings 1st Law

$$n = \frac{l^2 - l}{2}$$

where

n is the number of different possible wavelengths of emitted radiation
l is the number of energy levels (including ground level).

Debye relaxation

Debye relaxation is the dielectric relaxation response of an ideal, noninteracting population of dipoles to an alternating external electric field. It is usually expressed in the complex permittivity $\varepsilon$of a medium as a function of the field's frequency ω:

$$\hat{\varepsilon}(\omega) = \varepsilon_\infty + \frac{\Delta\varepsilon}{1 + i\omega\tau},$$

where $\varepsilon_\infty$is the permittivity at the high frequency limit, $\Delta\varepsilon = \varepsilon_s - \varepsilon_\infty$where $\varepsilon_s$is the static, low frequency permittivity, and τ is the characteristic relaxation time of the medium.

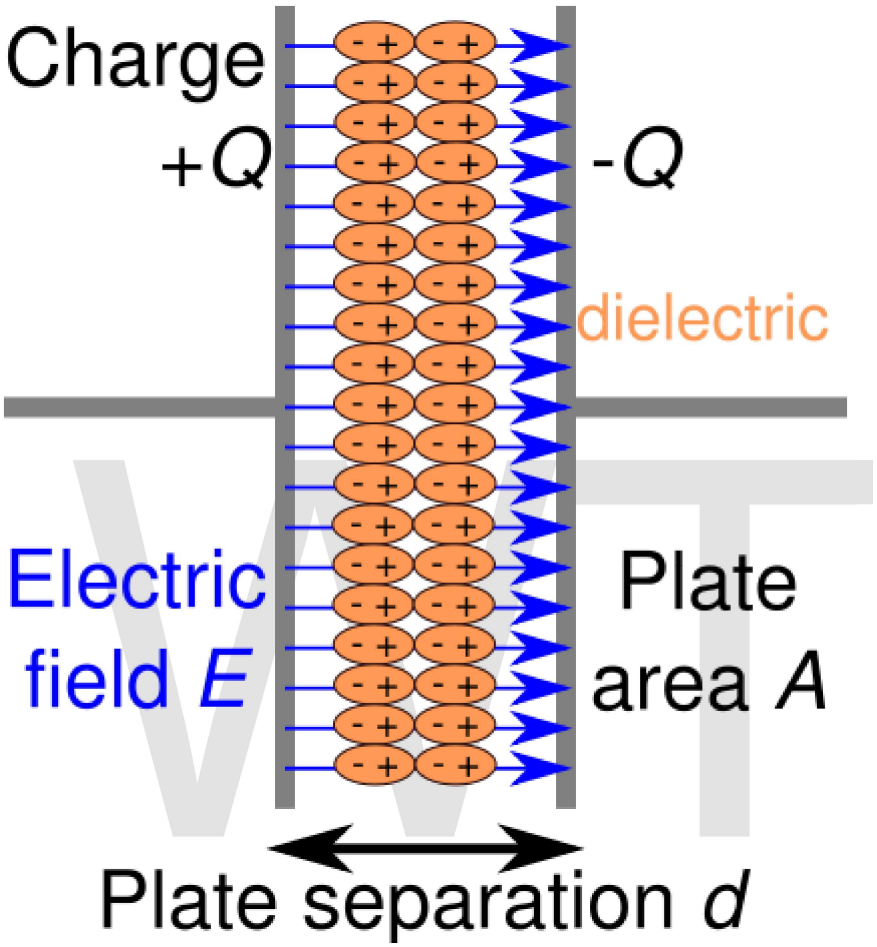
This relaxation model was named after the chemist Peter Debye.

Variants of the Debye equation

- Cole-Cole equation
- Cole-Davidson equation
- Havriliak-Negami relaxation
- Kohlrausch-Williams-Watts function (Fourier transform of stretched exponential function)

Applications

Capacitors



Charge separation in a parallel-plate capacitor causes an internal electric field. A dielectric (orange) reduces the field and increases the capacitance.

Commercially manufactured capacitors typically use a solid dielectric material with high permittivity as the intervening medium between the stored positive and negative charges. This material is often referred to in technical contexts as the "capacitor dielectric". The most obvious advantage to using such a dielectric material is that it prevents the conducting plates on which the charges are stored from coming into direct electrical contact. More significant, however, a high permittivity allows a greater charge to be stored at a given voltage. This can be seen by treating the case of a linear dielectric with permittivity ε and thickness d between two conducting plates with uniform charge density σ_ε. In this case the charge density is given by

$$\sigma_\epsilon = \epsilon \frac{V}{d}$$

and the capacitance per unit area by

$$c = \frac{\sigma_\epsilon}{V} = \frac{\epsilon}{d}$$

From this, it can easily be seen that a larger ε leads to greater charge stored and thus greater capacitance.

Dielectric materials used for capacitors are also chosen such that they are resistant to ionization. This allows the capacitor to operate at higher voltages before the insulating dielectric ionizes and begins to allow undesirable current.

Dielectric resonator

A *dielectric resonator oscillator* (DRO) is an electronic component that exhibits resonance for a narrow range of frequencies, generally in the microwave band. It consists of a "puck" of ceramic that has a large dielectric constant and a low dissipation factor. Such resonators are often used to provide a frequency reference in an oscillator circuit. An unshielded dielectric resonator can be used as a Dielectric Resonator Antenna (DRA).

Some practical dielectrics

Dielectric materials can be solids, liquids, or gases. In addition, a high vacuum can also be a useful, lossless dielectric even though its relative dielectric constant is only unity.

Solid dielectrics are perhaps the most commonly used dielectrics in electrical engineering, and many solids are very good insulators. Some examples include porcelain, glass, and most plastics. Air, nitrogen and sulfur hexafluoride are the three most commonly used gaseous dielectrics.

- Industrial coatings such as parylene provide a dielectric barrier between the substrate and its environment.
- Mineral oil is used extensively inside electrical transformers as a fluid dielectric and to assist in cooling. Dielectric fluids with higher dielectric constants, such as electrical grade castor oil, are often used in high voltage capacitors to help prevent corona discharge and increase capacitance.
- Because dielectrics resist the flow of electricity, the surface of a dielectric may retain *stranded* excess electrical charges. This may occur accidentally when the dielectric is rubbed (the triboelectric effect). This can be useful, as in a Van de Graaff generator or electrophorus, or it can be potentially destructive as in the case of electrostatic discharge.
- Specially processed dielectrics, called electrets (also known as ferroelectrics), may retain excess internal charge or "frozen in" polarization. Electrets have a semipermanent external electric field, and are the electrostatic equivalent to magnets. Electrets have numerous practical applications in the home and industry.

- Some dielectrics can generate a potential difference when subjected to mechanical stress, or change physical shape if an external voltage is applied across the material. This property is called piezoelectricity. Piezoelectric materials are another class of very useful dielectrics.
- Some ionic crystals and polymer dielectrics exhibit a spontaneous dipole moment which can be reversed by an externally applied electric field. This behavior is called the ferroelectric effect. These materials are analogous to the way ferromagnetic materials behave within an externally applied magnetic field. Ferroelectric materials often have very high dielectric constants, making them quite useful for capacitors.

Principles

Steady-state

For a redox reaction R $\leftrightarrow$O + e, without mass-transfer limitation, the relationship between the current density and the electrode overpotential is given by the Butler-Volmer equation:

$$j_t = j_0 \left(\exp(\alpha_o f \eta) - \exp(-\alpha_r f \eta)\right)$$

with

$\eta = E - E_{eq},\ f = F/(RT),\ \alpha_o + \alpha_r = 1$. j_0 is the exchange current density and α_o and α_r are the symmetry factors.

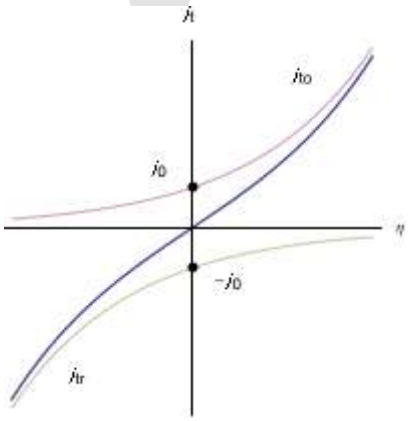


Fig. 1: Steady-state current density vs. overpotential for a redox reaction.

The curve $j_t \ vs. \ E$ is not a straight line (Fig. 1), therefore a redox reaction is not a linear system.

Dynamic behavior

Faradaic impedance

Let us suppose that the Butler-Volmer relationship correctly describes the dynamic behavior of the redox reaction:

$$j_t(t) = j_t(\eta(t)) = j_0 \ (\exp(\alpha_o f \eta(t)) - \exp(-\alpha_r f \eta(t)))$$

Dynamic behavior of the redox reaction is characterized by the so-called charge transfer resistance defined by:

$$R_{ct} = \frac{1}{\partial j_t / \partial \eta} = \frac{1}{f j_0 \ (\alpha_o \exp(\alpha_o f \eta) + \alpha_r \exp(-\alpha_r f \eta))}$$

The value of the charge transfer resistance changes with the overpotential. For this simplest example the Faradaic impedance is reduced to a resistance. It is worthwhile to notice that:

$$R_{ct} = \frac{1}{f j_0}$$

for $\eta = 0$.

Double layer capacitance

An electrode | electrolyte interface behaves like a capacitance called electrochemical double-layer capacitance C_{dl}. The equivalent electrical circuit for the redox reaction taking account of the double-layer capacitance is shown in Fig. 2. Another analog circuit commonly used to model the electrochemical double-layer is called a constant phase element.

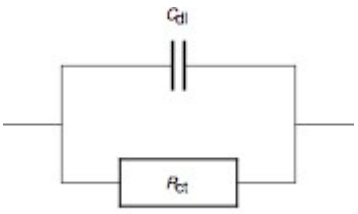


Fig. 2: Equivalent circuit for a redox reaction without mass-transfer limitation

The electrical impedance of this circuit is easily obtained remembering the impedance of a capacitance which is given by:

$$Z_{\mathrm{dl}}(\omega) = \frac{1}{\mathrm{i}\,\omega\,C_{\mathrm{dl}}}$$

where ω is the angular frequency of a sinusoidal signal (rd/s), and $\mathrm{i}=\sqrt{-1}$. It is obtained:

$$Z(\omega) = \frac{R_{\mathrm{t}}}{1 + R_{\mathrm{t}}\,C_{\mathrm{dl}}\,\mathrm{i}\,\omega}$$

Nyquist diagram of the impedance of the circuit shown in Fig. 2 is a semicircle with a diameter $R_{\mathrm{t}}$and an angular frequency at the apex equal to $1/(R_{\mathrm{t}}\,C_{\mathrm{dc}})$(Fig. 3). Others representations, Bode or Black plans can be used.

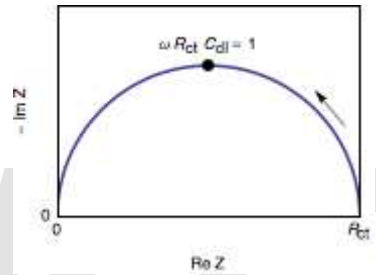


Fig. 3: Electrochemists Nyquist diagram of a RC parallel circuit. The arrow indicates increasing angular frequencies.

Ohmic resistance

The ohmic resistance R_{Ω} appears in series with the electrode impedance of the reaction and the Nyquist diagram is translated to the right.

Measurement of the impedance parameters

Plotting the Nyquist diagram with a potentiostat and an impedance analyzer, most often included in modern potentiostats, allows the user to determine charge transfer resistance, double layer capacitance and ohmic resistance. The exchange current density j_0 can be easily determined measuring the impedance of a redox reaction for η = 0.

Nyquist diagrams are made of several arcs for reaction more complex than redox reaction and with mass-transfer limitation.