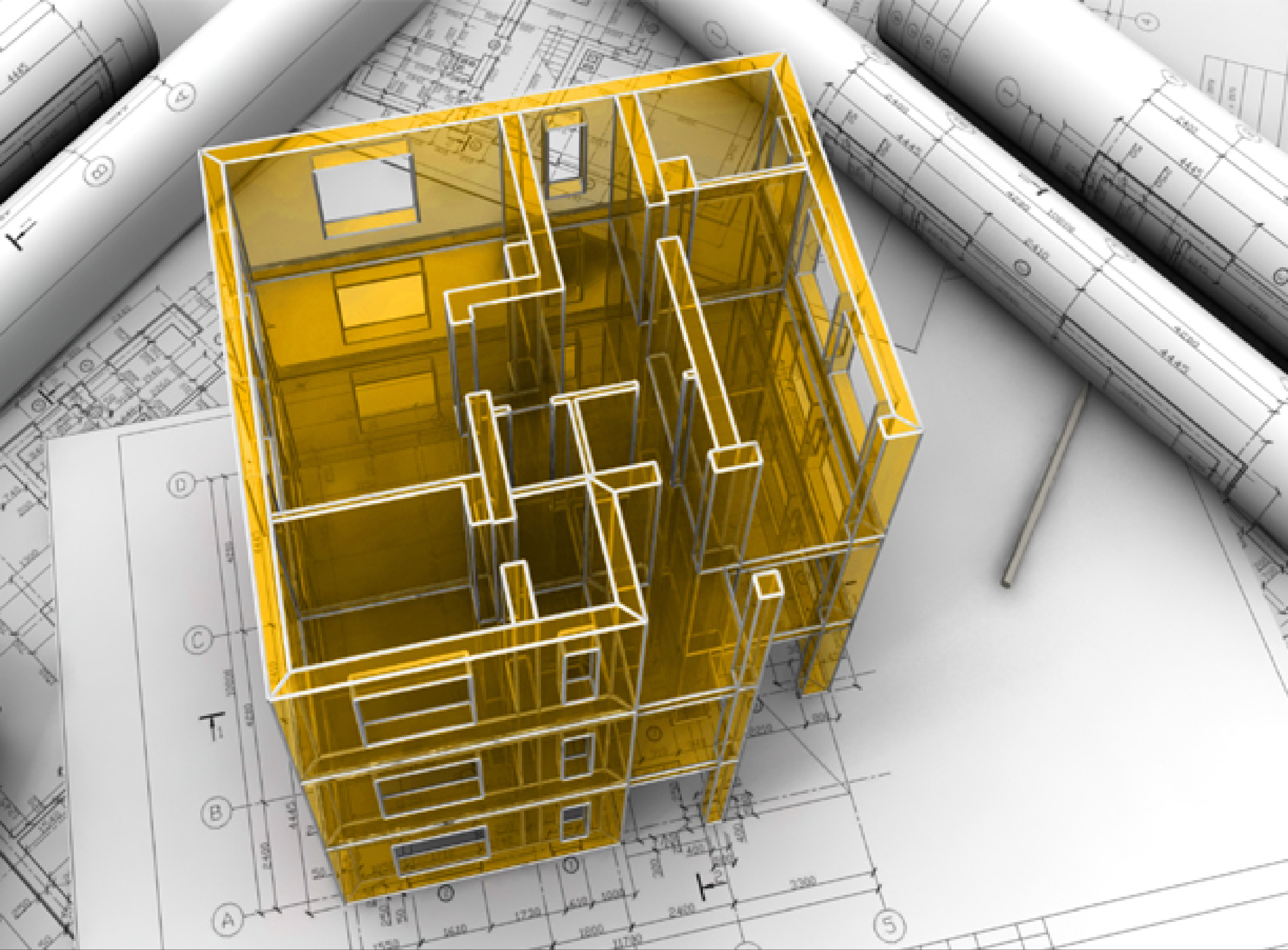

Structural Engineering

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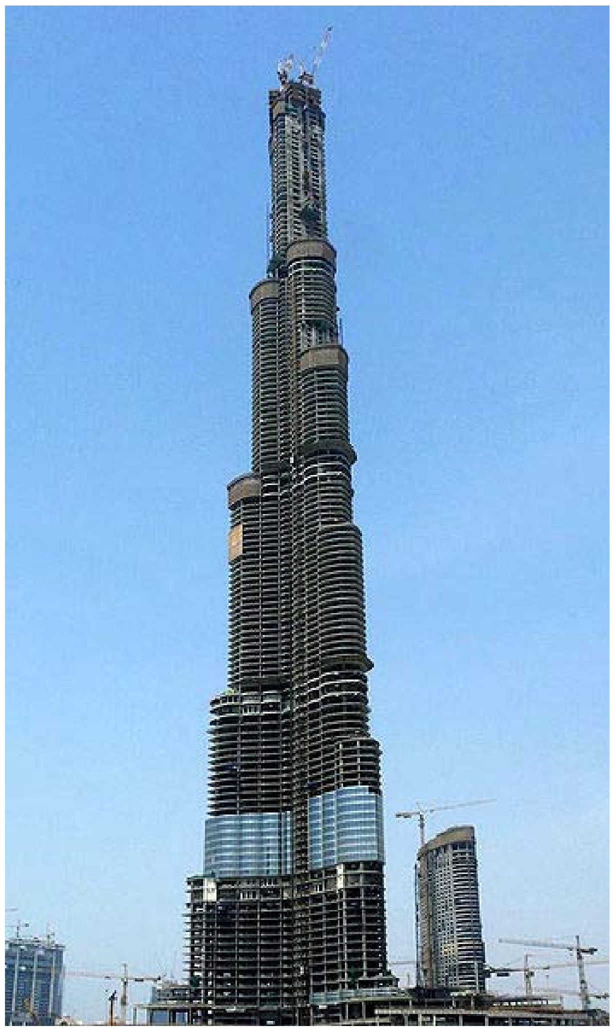
Structural Engineering



Burj Khalifa, in Dubai, the world's tallest building, shown under construction in 2007 (since completed)

Structural engineering is a field of engineering dealing with the analysis and design of structures that support or resist loads. Structural engineering is usually considered a specialty within civil engineering, but it can also be studied in its own right. Structural engineers are most commonly involved in the design of buildings and large nonbuilding structures but they can also be involved in the design of machinery, medical equipment, vehicles or any item where structural integrity affects the item's function or safety. Structural engineers must ensure their designs satisfy given design criteria, predicated on safety (e.g. structures must not collapse without due warning) or serviceability and performance (e.g. building sway must not cause discomfort to the occupants). Buildings are made to endure massive loads as well as changing climate and natural disasters.

Structural engineering theory is based upon physical laws and empirical knowledge of the structural performance of different landscapes and materials. Structural engineering design utilises a relatively small number of basic structural elements to build up structural systems that can be very complex. Structural engineers are responsible for making creative and efficient use of funds, structural elements and materials to achieve these goals.

Structural engineer

Structural engineers are responsible for engineering design and analysis. Entry-level structural engineers may design the individual structural elements of a structure, for example the beams, columns, and floors of a building. More experienced engineers would be responsible for the structural design and integrity of an entire system, such as a building.

Structural engineers often specialize in particular fields, such as bridge engineering, building engineering, pipeline engineering, industrial structures, or special mechanical structures such as vehicles or aircraft.

Structural engineering has existed since humans first started to construct their own structures. It became a more defined and formalised profession with the emergence of the architecture profession as distinct from the engineering profession during the industrial revolution in the late 19th Century. Until then, the architect and the structural engineer were usually one and the same - the master builder. Only with the development of specialised knowledge of structural theories that emerged during the 19th and early 20th centuries did the professional structural engineer come into existence.

The role of a structural engineer today involves a significant understanding of both static and dynamic loading, and the structures that are available to resist them. The complexity of modern structures often requires a great deal of creativity from the engineer in order to ensure the structures support and resist the loads they are subjected to. A structural engineer will typically have a four or five year undergraduate degree, followed by a minimum of three years of professional practice before being considered fully qualified.

Structural engineers are licensed or accredited by different learned societies and regulatory bodies around the world (for example, the Institution of Structural Engineers in the UK). Depending on the degree course they have studied and/or the jurisdiction they are seeking licensure in, they may be accredited (or licensed) as just structural engineers, or as civil engineers, or as both civil and structural engineers.

History of structural engineering

Pont du Gard, France, a Roman era aqueduct circa 19 BC

Structural engineering dates back to 2700 BC when the step pyramid for Pharaoh Djoser was built by Imhotep, the first engineer in history known by name. Pyramids were the most common major structures built by ancient civilizations because the structural form of a pyramid is inherently stable and can be almost infinitely scaled (as opposed to most other structural forms, which cannot be linearly increased in size in proportion to increased loads).

Throughout ancient and medieval history most architectural design and construction was carried out by artisans, such as stone masons and carpenters, rising to the role of master builder. No theory of structures existed, and understanding of how structures stood up was extremely limited, and based almost entirely on empirical evidence of 'what had worked before'. Knowledge was retained by guilds and seldom supplanted by advances. Structures were repetitive, and increases in scale were incremental.

No record exists of the first calculations of the strength of structural members or the behaviour of structural material, but the profession of structural engineer only really took shape with the industrial revolution and the re-invention of concrete. The physical sciences underlying structural engineering began to be understood in the Renaissance and have been developing ever since.

Structural failure

The history of structural engineering contains many collapses and failures. Sometimes this is due to obvious negligence, as in the case of the Pétionville school collapse, in which Rev. Fortin Augustin said that *"he constructed the building all by himself, saying he didn't need an engineer as he had good knowledge of construction"* following a partial collapse of the three-story schoolhouse that sent neighbors fleeing. The final collapse killed at least 362 people, mostly children.

In other cases structural failures require careful study, and the results of these inquiries have resulted in improved practices and greater understanding of the science of structural engineering. Some such studies are the result of Forensic engineering investigations where the original engineer seems to have done everything in accordance with the state of the profession and acceptable practice yet a failure still eventuated. A famous case of structural knowledge and practice being advanced in this manner can be found in a series of failures involving Box girders which collapsed in Australia during the 1970s.

Specializations

Building structures

Sydney Opera House, designed by Ove Arup & Partners, with the architect Jørn Utzon

Millennium Dome in London, UK, by Buro Happold and Richard Rogers

Structural building engineering includes all structural engineering related to the design of buildings. It is the branch of structural engineering that is close to architecture.

Structural building engineering is primarily driven by the creative manipulation of materials and forms and the underlying mathematical and scientific ideas to achieve an end which fulfills its functional requirements and is structurally safe when subjected to all the loads it could reasonably be expected to experience. This is subtly different from architectural design, which is driven by the creative manipulation of materials and forms, mass, space, volume, texture and light to achieve an end which is aesthetic, functional and often artistic.

The architect is usually the lead designer on buildings, with a structural engineer employed as a sub-consultant. The degree to which each discipline actually leads the design depends heavily on the type of structure. Many structures are structurally simple and led by architecture, such as multi-storey office buildings and housing, while other structures, such as tensile structures, shells and gridshells are heavily dependent on their form for their strength, and the engineer may have a more significant influence on the form, and hence much of the aesthetic, than the architect.

The structural design for a building must ensure that the building is able to stand up safely, able to function without excessive deflections or movements which may cause fatigue of structural elements, cracking or failure of fixtures, fittings or partitions, or discomfort for occupants. It must account for movements and forces due to temperature, creep, cracking and imposed loads. It must also ensure that the design is practically buildable within acceptable manufacturing tolerances of the materials. It must allow the architecture to work, and the building services to fit within the building and function (air conditioning, ventilation, smoke extract, electrics, lighting etc.). The structural design of a modern building can be extremely complex, and often requires a large team to complete.

Structural engineering specialties for buildings include:

- Earthquake engineering
- Façade engineering
- Fire engineering
- Roof engineering
- Tower engineering
- Wind engineering

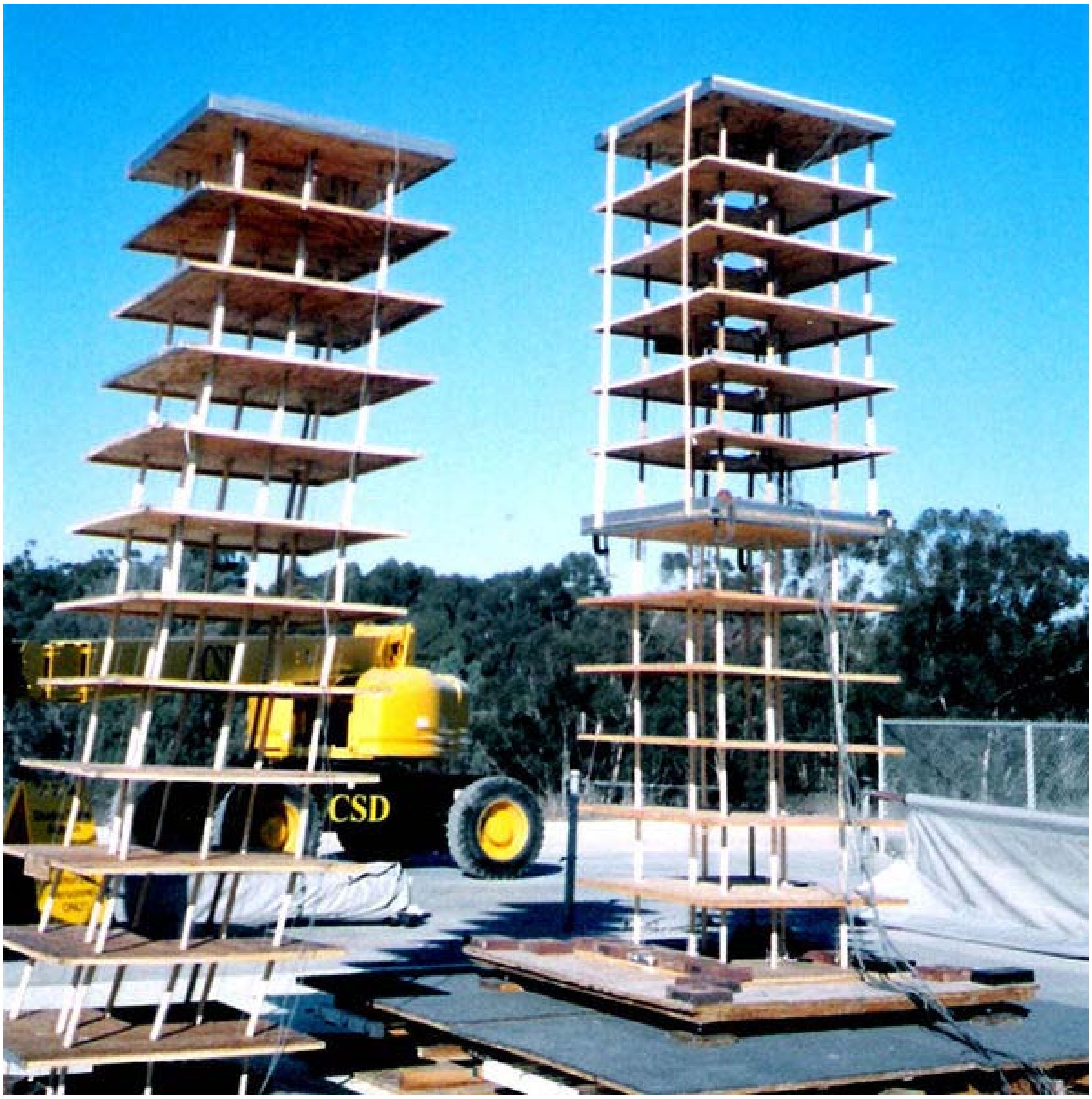
Earthquake engineering structures

Earthquake engineering structures are those engineered to withstand various types of hazardous earthquake exposures at the sites of their particular location.

Earthquake-proof and massive pyramid El Castillo, Chichen Itza

Earthquake engineering is treating its subject structures like defensive fortifications in military engineering but for the warfare on earthquakes. Both earthquake and military general design principles are similar: be ready to slow down or mitigate the advance of a possible attacker.

The main objectives of **earthquake engineering** are:



Snapshot from shake-table testing base-isolated (right) and regular (left) building model

- Understand interaction of structures with the shaky ground.
- Foresee the consequences of possible earthquakes.
- Design and construct the structures to perform while being exposed to an earthquake.

Earthquake engineering or **earthquake-proof structure** does not, necessarily, means *extremely strong* and *expensive* one like El Castillo pyramid at Chichen Itza shown above. In fact, many structures considered *strong* may in fact be actually *stiff,* which may result in poor seismic performance.

Now, the most *powerful* and *budgetary* tool of the earthquake engineering is base isolation which pertains to the passive structural vibration control technologies.

Civil engineering structures

Civil structural engineering includes all structural engineering related to the built environment. It includes:

- Bridges
- Dams
- Earthworks
- Foundations
- Offshore structures
- Pipelines
- Power stations
- Railways
- Retaining structures and walls
- Roads
- Tunnels
- Waterways
- Water and wastewater infrastructure

The structural engineer is the lead designer on these structures, and often the sole designer. In the design of structures such as these, structural safety is of paramount importance (in the UK, designs for dams, nuclear power stations and bridges must be signed off by a chartered engineer).

Civil engineering structures are often subjected to very extreme forces, such as large variations in temperature, dynamic loads such as waves or traffic, or high pressures from water or compressed gases. They are also often constructed in corrosive environments, such as at sea, in industrial facilities or below ground.

Mechanical structures

An Airbus A380, the world's largest passenger airliner

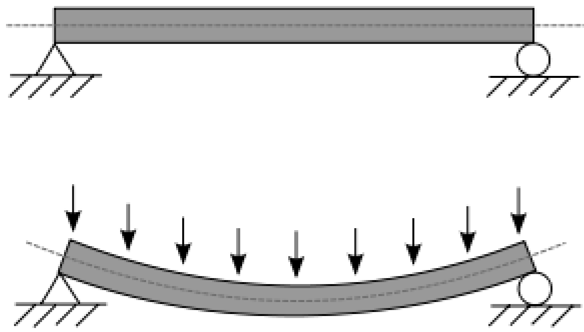
Principals of structural engineering are applied to variety of mechanical (moveable) structures. The design of static structures assumes they always have the same geometry (in fact, so-called static structures can move significantly, and structural engineering design must take this into account where necessary), but the design of moveable or moving structures must account for fatigue, variation in the method in which load is resisted and significant deflections of structures.

The forces which parts of a machine are subjected to can vary significantly, and can do so at a great rate. The forces which a boat or aircraft are subjected to vary enormously and will do so thousands of times over the structure's lifetime. The structural design must ensure that such structures are able to endure such loading for their entire design life without failing.

These works can require mechanical structural engineering:

- Airframes and fuselages
- Boilers and pressure vessels
- Coachworks and carriages
- Cranes
- Elevators
- Escalators
- Marine vessels and hulls

Structural elements



A statically determinate simply supported beam, bending under an evenly distributed load.

Any structure is essentially made up of only a small number of different types of elements:

- Columns
- Beams
- Plates
- Arches
- Shells
- Catenaries

Many of these elements can be classified according to form (straight, plane / curve) and dimensionality (one-dimensional / two-dimensional):

	One-dimensional		**Two-dimensional**	
	straight	**curve**	**plane**	**curve**
(predominantly) bending	beam	continuous arch	plate, concrete slab	lamina, dome
(predominant) tensile stress	rope	Catenary	shell	
(predominant) compression	pier, column		Load-bearing wall	

Columns

Columns are elements that carry only axial force - either tension or compression - or both axial force and bending (which is technically called a beam-column but practically, just a column). The design of a column must check the axial capacity of the element, and the buckling capacity.

The buckling capacity is the capacity of the element to withstand the propensity to buckle. Its capacity depends upon its geometry, material, and the effective length of the column, which depends upon the restraint conditions at the top and bottom of the column. The effective length is $K * l$ where l is the real length of the column.

The capacity of a column to carry axial load depends on the degree of bending it is subjected to, and vice versa. This is represented on an interaction chart and is a complex non-linear relationship.

Beams

A beam may be defined as an element in which one dimemsion is much greater than the other two and the applied loads are usually normal to the main axis of the element. Beams and columns are called line elements and are often represented by simple lines in structural modeling.

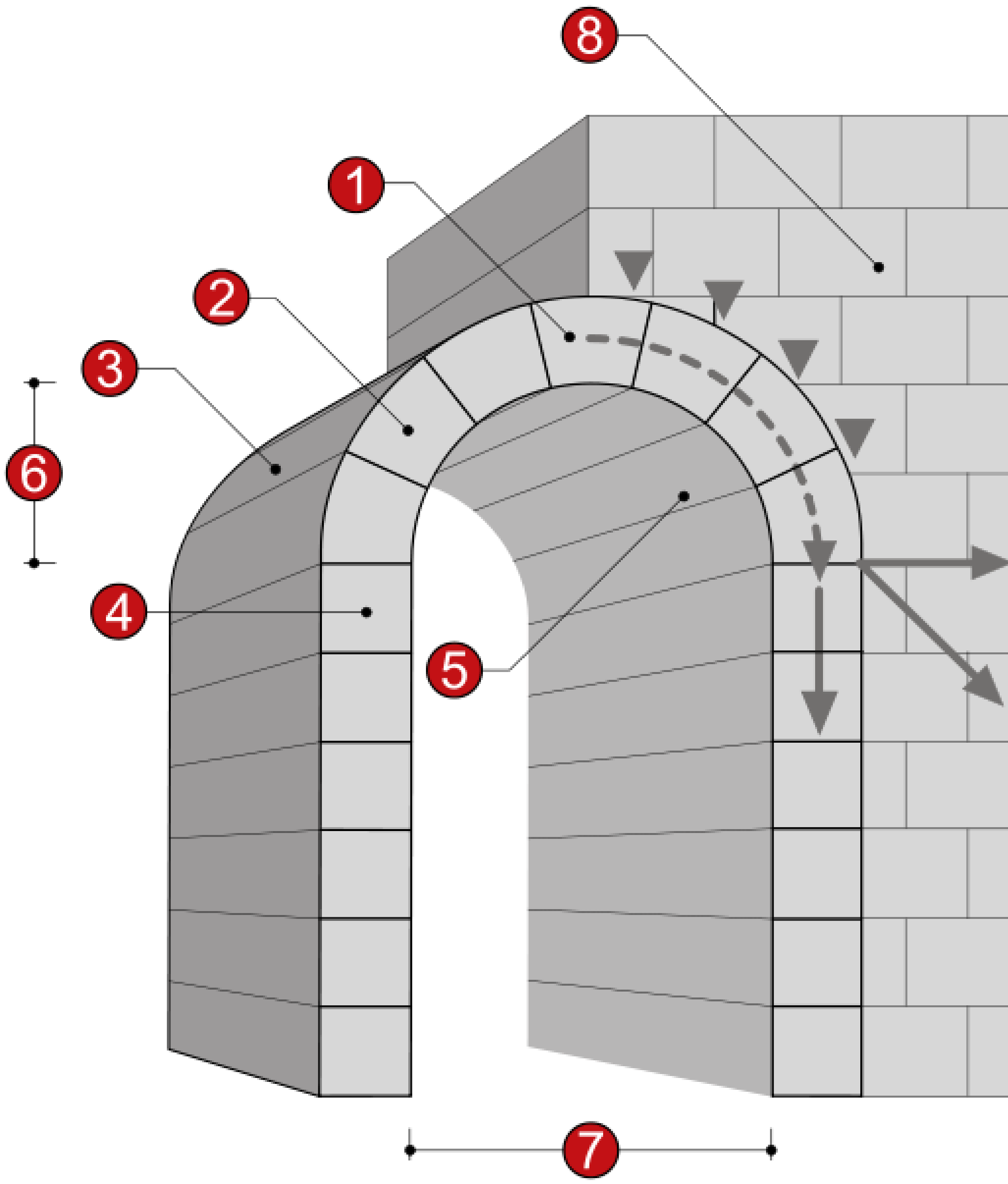
- cantilevered (supported at one end only with a fixed connection)
- simply supported (supported vertically at each end; horizontally on only one to withstand friction, and able to rotate at the supports)
- continuous (supported by three or more supports)
- a combination of the above (ex. supported at one end and in the middle)

Beams are elements which carry pure bending only. Bending causes one part of the section of a beam (divided along its length) to go into compression and the other part into tension. The compression part must be designed to resist buckling and crushing, while the tension part must be able to adequately resist the tension.

Struts and ties

Little Belt: a truss bridge in Denmark

The McDonnell Planetarium by Gyo Obata in St Louis, Missouri, USA, a concrete shell structure



A masonry arch
1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span
8. Abutment

A truss is a structure comprising two types of structural elements; compression members and tension members (i.e. struts and ties). Most trusses use gusset plates to connect intersecting elements. Gusset plates are relatively flexible and minimize bending moments at the connections, thus allowing the truss members to carry primarily tension or compression.

Trusses are usually utilised in span large distances, where it would be uneconomical to use solid beams.

Plates

Plates carry bending in two directions. A concrete flat slab is an example of a plate. Plates are understood by using continuum mechanics, but due to the complexity involved they are most often designed using a codified empirical approach, or computer analysis.

They can also be designed with yield line theory, where an assumed collapse mechanism is analysed to give an upper bound on the collapse load. This is rarely used in practice.

Shells

Shells derive their strength from their form, and carry forces in compression in two directions. A dome is an example of a shell. They can be designed by making a hanging-chain model, which will act as a catenary in pure tension, and inverting the form to achieve pure compression.

Arches

Arches carry forces in compression in one direction only, which is why it is appropriate to build arches out of masonry. They are designed by ensuring that the line of thrust of the force remains within the depth of the arch.

Catenaries

Catenaries derive their strength from their form, and carry transverse forces in pure tension by deflecting (just as a tightrope will sag when someone walks on it). They are almost always cable or fabric structures. A fabric structure acts as a catenary in two directions.

Structural engineering theory

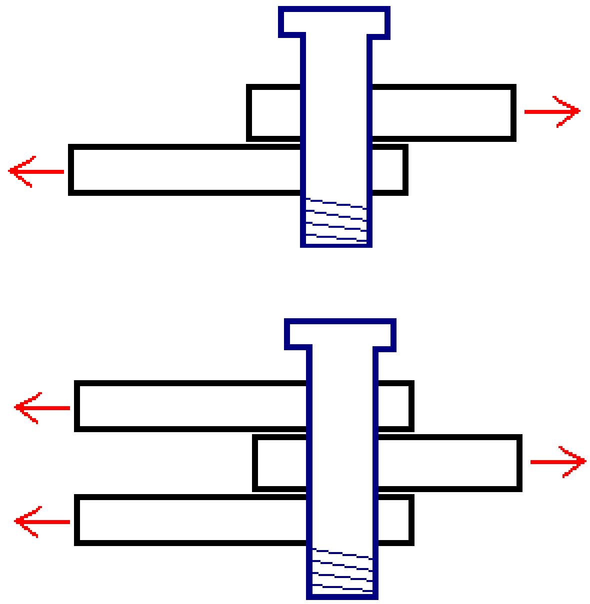


Figure of a bolt in shear stress. Top figure illustrates single shear, bottom figure illustrates double shear.

Structural engineering depends upon a detailed knowledge of loads, physics and materials to understand and predict how structures support and resist self-weight and imposed loads. To apply the knowledge successfully a structural engineer generally requires detailed knowledge of mathematics and relevant empirical and theoretical design codes. As well as, typically, some knowledge of the corrosion resistance of the materials and structures, especially when those structures are exposed to the external environment. Since the 1990s, specialist software has become available to aid in the design of structures, with the functionality to assist in both the drawing and designing of structures with maximum precision; examples include AutoCAD, StaadPro, Etab etc. Such software may also take into consideration environmental loads, such as from earthquakes and winds.

Materials

The 630 foot (192 m) high, stainless-clad (type 304) Gateway Arch in Saint Louis, Missouri

Structural engineering depends on the knowledge of materials and their properties, in order to understand how different materials support and resist loads.

Common structural materials are:

- Iron:
    - Wrought iron
    - Cast iron
    - Steel
    - Stainless steel
- Concrete:
    - Reinforced concrete
    - Prestressed concrete
- Aluminium
- Composites
- Alloy
- Masonry
- Timber
- Other structural materials:
    - Adobe

- Bamboo
- Carbon fibre
- Fiber reinforced plastic
- Mudbrick

Chapter- 2

History of Structural Engineering

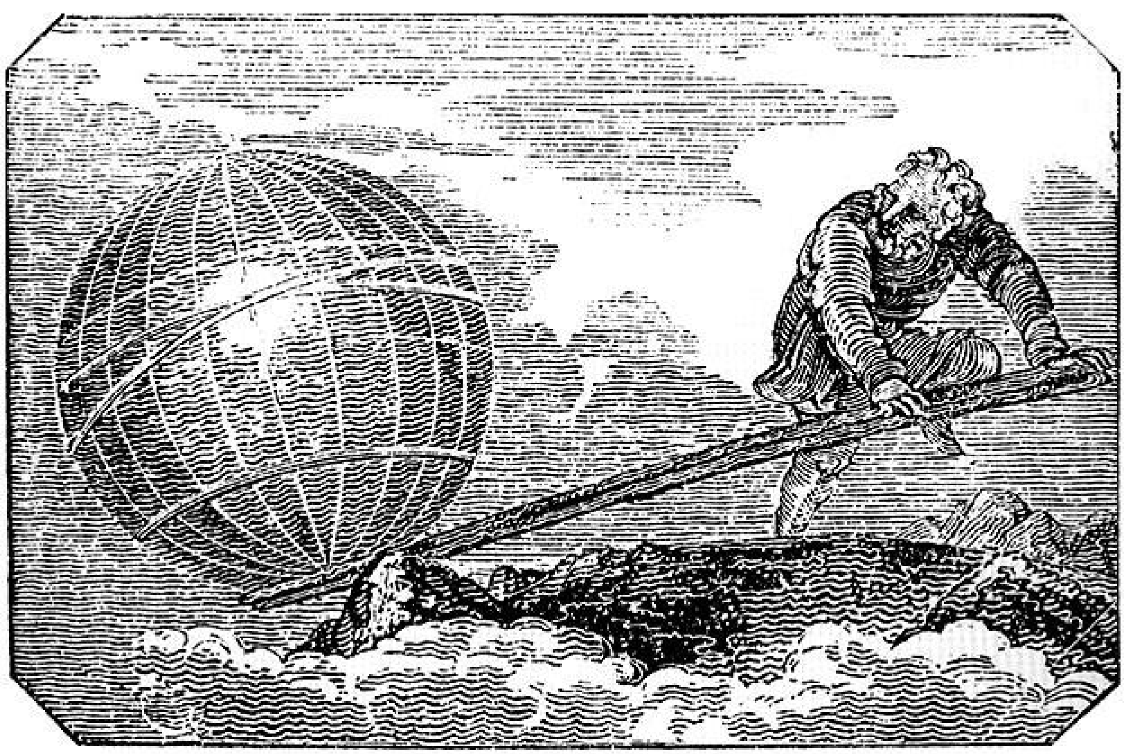
Statuette of Imhotep, in the Louvre, Paris, France

The **history of structural engineering** dates back to at least 2700 BC when the step pyramid for Pharaoh Djoser was built by Imhotep, the first engineer in history known by name. Pyramids were the most common major structures built by ancient civilisations because it is a structural form which is inherently stable and can be almost infinitely scaled (as opposed to most other structural forms, which cannot be linearly increased in size in proportion to increased loads).

Throughout ancient and medieval history most architectural design and construction was carried out by artisans, such as stone masons and carpenters, rising to the role of master builder. No theory of structures existed and understanding of how structures stood up was extremely limited, and based almost entirely on empirical evidence of 'what had worked before'. Knowledge was retained by guilds and seldom supplanted by advances. Structures were repetitive, and increases in scale were incremental.

No record exists of the first calculations of the strength of structural members or the behaviour of structural material, but the profession of structural engineer only really took shape with the industrial revolution and the re-invention of concrete. The physical sciences underlying structural engineering began to be understood in the Renaissance and have been developing ever since.

Early structural engineering developments



Archimedes is said to have remarked about the lever: "Give me a place to stand on, and I will move the Earth."

The recorded history of structural engineering starts with the ancient Egyptians. In the 27th century BC, Imhotep was the first structural engineer known by name and constructed the first known step pyramid in Egypt. In the 26th century BC, the Great Pyramid of Giza was constructed in Egypt. It remained the largest man-made structure for millennia and was considered an unsurpassed feat in architecture until the 19th century AD.

The understanding of the physical laws that underpin structural engineering dates back to the 3rd century BC, when Archimedes published his work *On the Equilibrium of Planes* in two volumes, in which he sets out the *Law of the Lever*, stating:

> “Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance.”

Archimedes used the principles derived to calculate the areas and centers of gravity of various geometric figures including triangles, paraboloids, and hemispheres. Archimedes work on this and his work on calculus and geometry, together with Euclidean geometry, underpin much of the mathematics and understanding of structures in modern structural engineering.

Pont du Gard, France, a Roman era aqueduct circa 19 BC.

The ancient Romans made great bounds in structural engineering, pioneering large structures in masonry and concrete, many of which are still standing today. They include aqueducts, thermae, columns, lighthouses, defensive walls and harbours. Their methods are recorded by Vitruvius in his De Architectura written in 25 BC, a manual of civil and structural engineering with extensive sections on materials and machines used in construction. One reason for their success is their accurate surveying techniques based on the dioptra, groma and chorobates.

Centuries later, in the 15th and 16th centuries and despite lacking beam theory and calculus, Leonardo da Vinci produced many engineering designs based on scientific observations and rigour, including a design for a bridge to span the Bosporus. Though dismissed at the time, the design has since been judged to be both feasible and structurally valid

Galileo Galilei. Portrait in crayon by Leoni

The foundations of modern structural engineering were laid in the 17th century by Galileo Galilei, Robert Hooke and Isaac Newton with the publication of three great scientific works. In 1638 Galileo published *Dialogues Relating to Two New Sciences*, outlining the sciences of the strength of materials and the motion of objects (essentially defining gravity as a force giving rise to a constant acceleration). It was the first establishment of a scientific approach to structural engineering, including the first attempts to develop a theory for beams. This is also regarded as the beginning of structural analysis, the mathematical representation and design of building structures.

This was followed in 1676 by Robert Hooke's first statement of Hooke's Law, providing a scientific understanding of elasticity of materials and their behaviour under load.

Eleven years later, in 1687, Sir Isaac Newton published *Philosophiae Naturalis Principia Mathematica*, setting out his Laws of Motion, providing for the first time an understanding of the fundamental laws governing structures.

Also in the 17th century, Sir Isaac Newton and Gottfried Leibniz both independently developed the Fundamental theorem of calculus, providing one of the most important mathematical tools in engineering.

Leonhard Euler portrait by Johann Georg Brucker

Further advances in the mathematics needed to allow structural engineers to apply the understanding of structures gained through the work of Galileo, Hooke and Newton during the 17th century came in the 18th century when Leonhard Euler pioneered much of the mathematics and many of the methods which allow structural engineers to model and analyse structures. Specifically, he developed the Euler-Bernoulli beam equation with Daniel Bernoulli (1700–1782) circa 1750 - the fundamental theory underlying most structural engineering design.

Daniel Bernoulli, with Johann (Jean) Bernoulli (1667–1748), is also credited with formulating the theory of virtual work, providing a tool using equilibrium of forces and compatibility of geometry to solve structural problems. In 1717 Jean Bernoulli wrote to Pierre Varignon explaining the principle of virtual work, while in 1726 Daniel Bernoulli wrote of the "composition of forces".

In 1757 Leonhard Euler went on to derive the Euler buckling formula, greatly advancing the ability of engineers to design compression elements.

Modern developments in structural engineering

Bessemer converter, Kelham Island Museum, Sheffield, England (2002)

Belper North Mill

Eiffel Tower under construction in July 1888

The Forth Bridge

The Lattice shell structure of the Shukhov Tower in Moscow

Throughout the late 19th and early 20th centuries, materials science and structural analysis underwent development at a tremendous pace.

Though elasticity was understood in theory well before the 19th century, it was not until 1821 that Claude-Louis Navier formulated the general theory of elasticity in a mathematically usable form. In his *leçons* of 1826 he explored a great range of different structural theory, and was the first to highlight that the role of a structural engineer is not to understand the final, failed state of a structure, but to prevent that failure in the first place. In 1826 he also established the elastic modulus as a property of materials independent of the second moment of area, allowing engineers for the first time to both understand structural behaviour and structural materials.

Towards the end of the 19th century, in 1873, Carlo Alberto Castigliano presented his dissertation "Intorno ai sistemi elastici", which contains his theorem for computing displacement as partial derivative of the strain energy.

In 1824, Portland cement was patented by the engineer Joseph Aspdin as *"a superior cement resembling Portland Stone"*, British Patent no. 5022. Although different forms of cement already existed (Pozzolanic cement was used by the Romans as early as 100 B.C. and even earlier by the ancient Greek and Chinese civilizations) and were in common usage in Europe from the 1750s, the discovery made by Aspdin used commonly available, cheap materials, making concrete construction an economical possibility.

Developments in concrete continued with the construction in 1848 of a rowing boat built of ferrocement - the forerunner of modern reinforced concrete - by Joseph-Louis Lambot. He patented his system of mesh reinforcement and concrete in 1855, one year after W.B. Wilkinson also patented a similar system. This was followed in 1867 when a reinforced concrete planting tub was patented by Joseph Monier in Paris, using steel mesh reinforcement similar to that used by Lambot and Wilkinson. Monier took the idea forward, filing several patents for tubs, slabs and beams, leading eventually to the Monier system of reinforced structures, the first use of steel reinforcement bars located in areas of tension in the structure.

Steel construction was first made possible in the 1850s when Henry Bessemer developed the Bessemer process to produce steel. He gained patents for the process in 1855 and 1856 and successfully completed the conversion of cast iron into cast steel in 1858. Eventually mild steel would replace both wrought iron and cast iron as the preferred metal for construction.

During the late 19th century, great advancements were made in the use of cast iron, gradually replacing wrought iron as a material of choice. Ditherington Flax Mill in Shrewsbury, designed by Charles Bage, was the first building in the world with an interior iron frame. It was built in 1797. In 1792 William Strutt had attempted to build a fireproof mill at Belper in Derby (Belper West Mill), using cast iron columns and timber beams within the depths of brick arches that formed the floors. The exposed beam soffits were protected against fire by plaster. This mill at Belper was the world's first attempt to construct fireproof buildings, and is the first example of fire engineering. This was later improved upon with the construction of Belper North Mill, a collaboration between Strutt and Bage, which by using a full cast iron frame represented the world's first "fire proofed" building.

The Forth Bridge was built by Benjamin Baker, Sir John Fowler and William Arrol in 1889, using steel, after the original design for the bridge by Thomas Bouch was rejected following the collapse of his Tay Rail Bridge. The Forth Bridge was one of the first major uses of steel, and a landmark in bridge design. Also in 1889, the wrought-iron Eiffel Tower was built by Gustave Eiffel and Maurice Koechlin, demonstrating the potential of construction using iron, despite the fact that steel construction was already being used elsewhere.

During the late 19th century, Russian structural engineer Vladimir Shukhov developed analysis methods for tensile structures, thin-shell structures, lattice shell structures and new structural geometries such as hyperboloid structures. Pipeline transport was pioneered by Vladimir Shukhov and the Branobel company in the late 19th century.

Again taking reinforced concrete design forwards, from 1892 onwards François Hennebique's firm used his patented reinforced concrete system to build thousands of structures throughout Europe. Thaddeus Hyatt in the US and Wayss & Freitag in Germany also patented systems. The firm *AG für Monierbauten* constructed 200 reinforced concrete bridges in Germany between 1890 and 1897. The great pioneering uses of reinforced concrete however came during the first third of the 20th century, with Robert Maillart and others furthering of the understanding of its behaviour. Maillart noticed that many concrete bridge structures were significantly cracked, and as a result left the cracked areas out of his next bridge design - correctly believing that if the concrete was cracked, it was not contributing to the strength. This resulted in the revolutionary Salginatobel Bridge design. Wilhelm Ritter formulated the truss theory for the shear design of reinforced concrete beams in 1899, and Emil Mörsch improved this in 1902. He went on to demonstrate that treating concrete in compression as a linear-elastic material was a conservative approximation of its behaviour. Concrete design and analysis has been progressing ever since, with the development of analysis methods such as yield line theory, based on plastic analysis of concrete (as opposed to linear-elastic), and many different variations on the model for stress distributions in concrete in compression

Prestressed concrete, pioneered by Eugène Freyssinet with a patent in 1928, gave a novel approach in overcoming the weakness of concrete structures in tension. Freyssinet constructed an experimental prestressed arch in 1908 and later used the technology in a limited form in the Plougastel Bridge in France in 1930. He went on to build six prestressed concrete bridges across the Marne River, firmly establishing the technology.

Structural engineering theory was again advanced in 1930 when Professor Hardy Cross developed his Moment distribution method, allowing the real stresses of many complex structures to be approximated quickly and accurately.

In the mid 20th century John Fleetwood Baker went on to develop the plasticity theory of structures, providing a powerful tool for the safe design of steel structures.

High-rise construction, though possible from the late 19th century onwards, was greatly advanced during the second half of the 20th century. Fazlur Khan designed structural systems that remain fundamental to many modern high rise constructions and which he employed in his structural designs for the John Hancock Center in 1969 and Sears Tower in 1973. Khan's central innovation in skyscraper design and construction was the idea of the "tube" and "bundled tube" structural systems for tall buildings. He defined the framed tube structure as "a three dimensional space structure composed of three, four, or possibly more frames, braced frames, or shear walls, joined at or near their edges to form a vertical tube-like structural system capable of resisting lateral forces in any direction by cantilevering from the foundation." Closely spaced interconnected exterior columns form the tube. Horizontal loads, for example wind, are supported by the structure as a whole. About half the exterior surface is available for windows. Framed tubes allow fewer interior columns, and so create more usable floor space. Where larger openings like garage doors are required, the tube frame must be interrupted, with transfer girders used to maintain structural integrity. The first building to apply the tube-frame construction was in the DeWitt-Chestnut Apartment Building which Khan designed in Chicago. This laid the foundations for the tube structures used in most later skyscraper constructions, including the construction of the World Trade Center.

Another innovation that Fazlur Khan developed was the concept of X-bracing, which reduced the lateral load on the building by transferring the load into the exterior columns. This allowed for a reduced need for interior columns thus creating more floor space, and can be seen in the John Hancock Center. The first sky lobby was also designed by Khan for the John Hancock Center in 1969. Later buildings with sky lobbies include the World Trade Center, Petronas Twin Towers and Taipei 101.

In 1987 Jörg Schlaich and Kurt Schafer published the culmination of almost ten years of work on the strut and tie method for concrete analysis - a tool to design structures with discontinuities such as corners and joints, providing another powerful tool for the analysis of complex concrete geometries.

In the late 20th and early 21st centuries the development of powerful computers has allowed finite element analysis to become a significant tool for structural analysis and design. The development of finite element programs has led to the ability to accurately predict the stresses in complex structures, and allowed great advances in structural engineering design and architecture. In the 1960s and 70s computational analysis was used in a significant way for the first time on the design of the Sydney Opera House roof. Many modern structures could not be understood and designed without the use of computational analysis.

Developments in the understanding of materials and structural behaviour in the latter part of the 20th century have been significant, with detailed understanding being developed of topics such as fracture mechanics, earthquake engineering, composite materials, temperature effects on materials, dynamics and vibration control, fatigue, creep and others. The depth and breadth of knowledge now available in structural engineering, and the increasing range of different structures and the increasing complexity of those structures has led to increasing specialisation of structural engineers.

Chapter- 3

Earthquake Engineering Structures

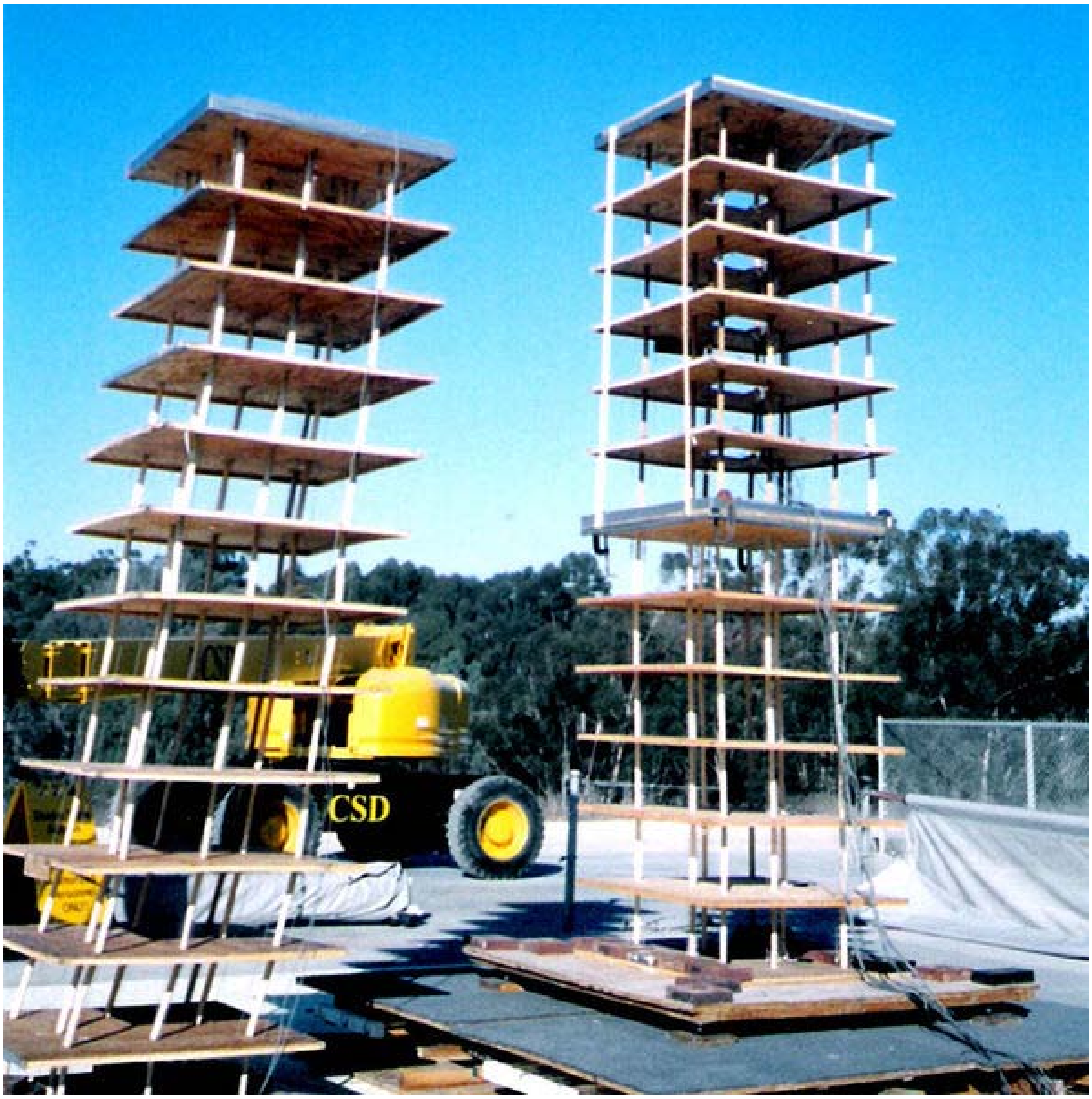
Earthquake-proof and massive pyramid El Castillo, Chichen Itza

Earthquake engineering structures are designed and constructed to withstand various types of hazardous earthquake exposures at the sites of their particular location.

Earthquake engineering is treating its subject structures like defensive fortifications in military engineering but for the warfare on earthquakes. Both earthquake and military general design principles are similar: be ready to slow down or mitigate the advance of a possible attacker.

According to building codes, earthquake engineering structures are meant to "withstand" the largest earthquake of a certain probability that is likely to occur at their location. This means the loss of life should be minimized by preventing collapse of the buildings.

Ancient architects believed that devastating earthquakes were a result of wrath of gods and therefore, could not be resisted by humans. Nowadays, the people's attitude has changed dramatically though the term "earthquake engineering structure" does not necessarily mean it is an extremely strong or expensive one like the El Castillo pyramid at Chichen Itza.



Snapshot from shake-table testing base-isolated (right) and regular (left) building model

Currently, the most powerful and cost-effective tool of earthquake engineering is base isolation which makes use of passive structural vibration control technologies, used for example in the Cathedral of Our Lady of the Angels.

Trends and projects

Some of the new state-of-the-art trends and/or projects in the field of earthquake engineering structures are presented below.

Proofed low cost earthquake building material

A leading German composite construction company developed a proofed earthquake-safe supported core material RexWall - based on internal beam/frame constructions. The same principle allows construction of hurricane-safe houses.

Earthquake shelter

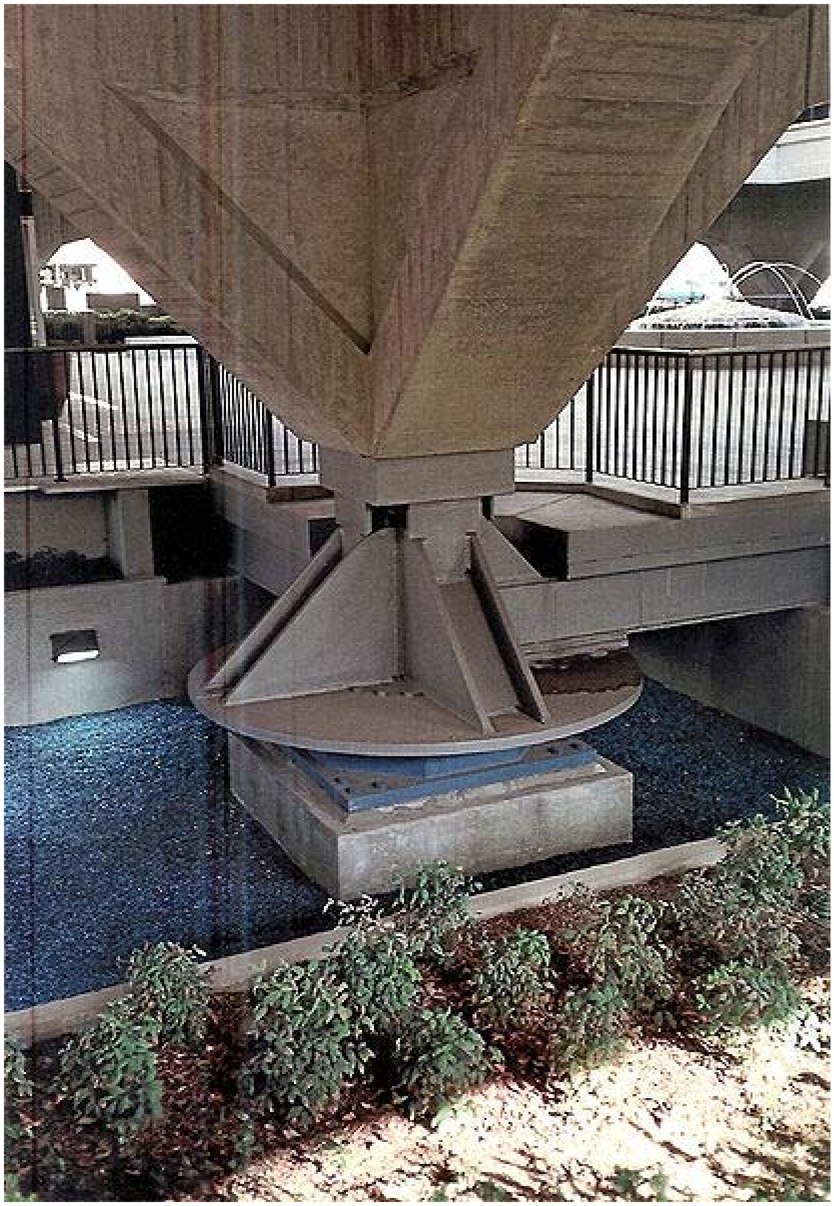
One Japanese construction company has developed a six-foot cubical shelter, presented as an alternative to earthquake-proofing an entire building.

Concurrent shake-table testing

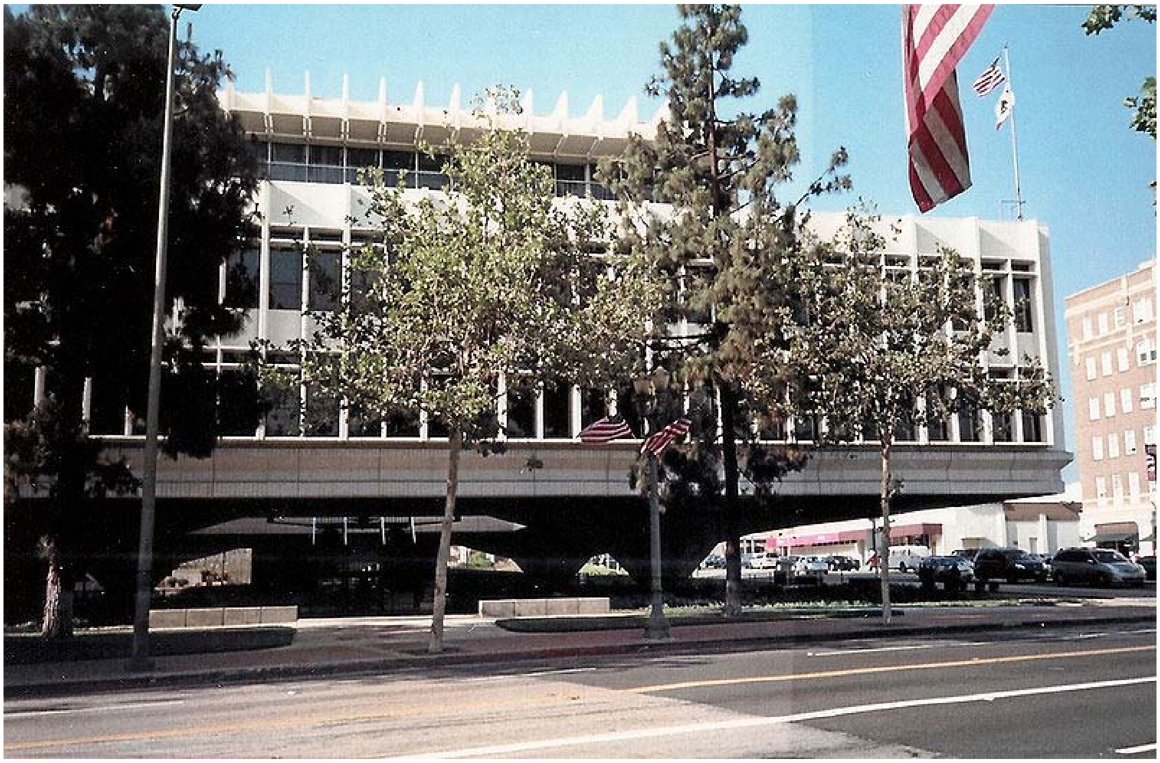
Concurrent shake-table testing of two or more building models is a vivid, persuasive and effective way to validate earthquake engineering solutions experimentally.

Thus, two wooden houses built before adoption of the 1981 Japanese Building Code were moved to E-Defense for testing. The left house was reinforced to enhance its seismic resistance, while the other one was not. These two models were set on E-Defense platform and tested simultaneously.

Combined vibration control solution



Close-up of abutment of seismically retrofitted Municipal Services Building in Glendale, CA



Seismically retrofitted Municipal Services Building in Glendale, CA

Designed by architect Merrill W. Baird of Glendale, working in collaboration with A. C. Martin Architects of Los Angeles, the Municipal Services Building at 633 East Broadway, Glendale was completed in 1966. Prominently sited at the corner of East Broadway and Glendale Avenue, this civic building serves as a heraldic element of Glendale's civic center.

In October 2004 Architectural Resources Group (ARG) was contracted by Nabih Youssef & Associates, Structural Engineers, to provide services regarding a historic resource assessment of the building due to a proposed seismic retrofit.

In 2008, the Municipal Services Building of the City of Glendale, California was seismically retrofitted using an innovative combined vibration control solution: the existing elevated building foundation of the building was put on high damping rubber bearings.

Steel plate shear walls system

Coupled steel plate shear walls, Seattle

The Ritz-Carlton/JW Marriott hotel building engaging the advanced steel plate shear walls system, LA

A steel plate shear wall (SPSW) consists of steel infill plates bounded by a column-beam system. When such infill plates occupy each level within a framed bay of a structure, they constitute a SPSW system.

SPSW behavior is analogous to a vertical plate girder cantilevered from its base. Similar to plate girders, the SPSW system optimizes component performance by taking advantage of the post-buckling behavior of the steel infill panels.

The Ritz-Carlton/JW Marriott hotel building, a part of the LA Live development in Los Angeles, California, is the first building in Los Angeles that uses an advanced steel plate shear wall system to resist the lateral loads of strong earthquakes and winds.

Kashiwazaki-Kariwa Nuclear Power Plant is partially upgraded

Kashiwazaki-Kariwa Nuclear Power Plant: aerial view, Japan

The Kashiwazaki-Kariwa Nuclear Power Plant, the largest nuclear generating station in the world by net electrical power rating, happened to be near the epicenter of the strongest M_w 6.6 July 2007 Chūetsu offshore earthquake. This initiated an extended shutdown for structural inspection which indicated that a greater earthquake-proofing was needed before operation could be resumed.

On May 9, 2009, one unit (Unit 7) was restarted, after the seismic upgrades. The test run had to continue for 50 days. The plant had been completely shut down for almost 22 months following the earthquake.

Seismic Test of Seven-Story Building

Seven-story wooden structure prior to shake-table testing, Japan

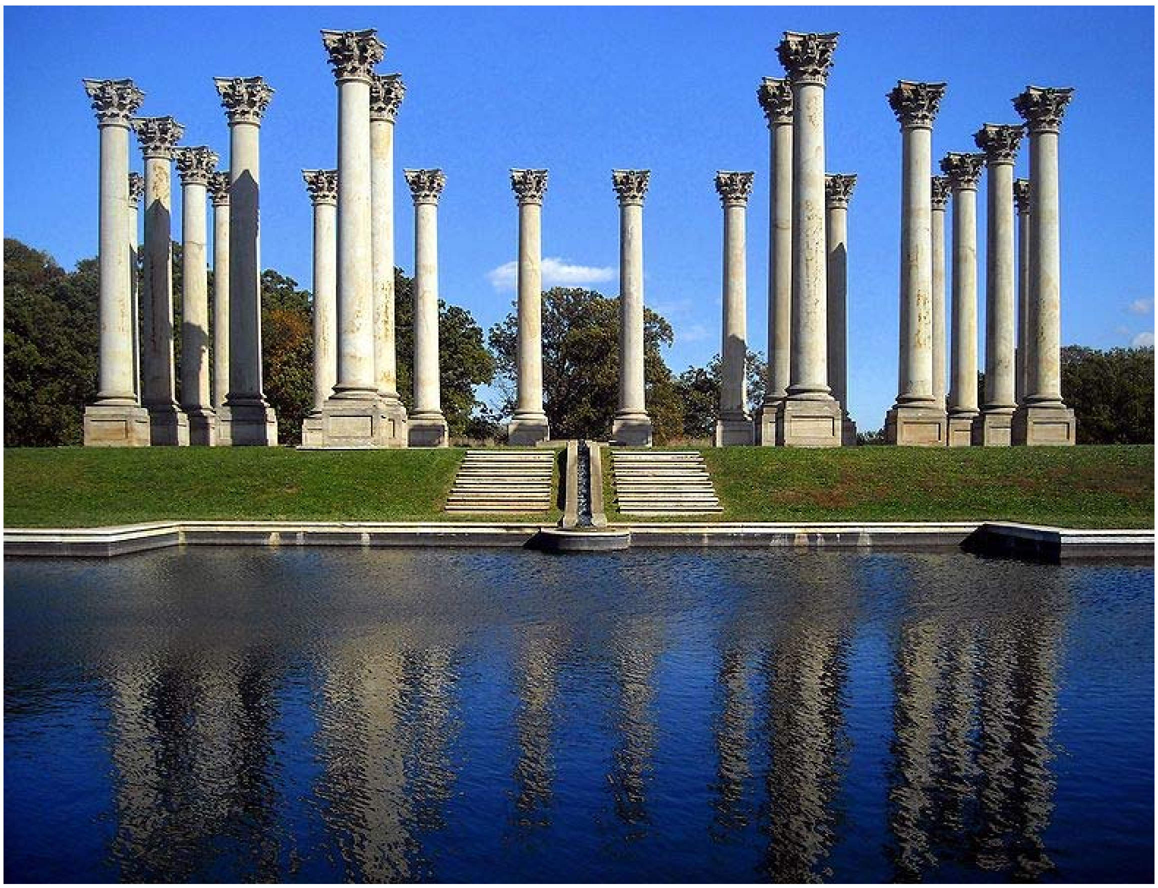
A destructive earthquake struck a lone, wooden condominium in Japan. The experiment was webcast live on July 14, 2009 to yield insight on how to make wooden structures stronger and better able to withstand major earthquakes.

The Miki shake at the Hyogo Earthquake Engineering Research Center is the capstone experiment of the four-year NEESWood project, which receives its primary support from the U.S. National Science Foundation Network for Earthquake Engineering Simulation (NEES) Program.

“NEESWood aims to develop a new seismic design philosophy that will provide the necessary mechanisms to safely increase the height of wood-frame structures in active seismic zones of the United States, as well as mitigate earthquake damage to low-rise wood-frame structures,” said Rosowsky, Department of Civil Engineering at Texas A&M University. This philosophy is based on the application of seismic damping systems for wooden buildings. The systems, which can be installed inside the walls of most wooden buildings, include strong metal frame, bracing and dampers filled with viscous fluid.

Chapter- 4

Column (Structural Elements)



National Capitol Columns at the United States National Arboretum in Washington, D.C.

Marble columns with antique capitals in the Great Mosque of Kairouan also known as the Mosque of Uqba, city of Kairouan, Tunisia

A **column** in structural engineering is a vertical structural element that transmits, through compression, the weight of the structure above to other structural elements below. For the purpose of wind or earthquake engineering, columns may be designed to resist lateral forces. Other compression members are often termed "columns" because of the similar stress conditions. Columns are frequently used to support beams or arches on which the upper parts of walls or ceilings rest. In architecture "column" refers to such a structural element that also has certain proportional and decorative features. A column might also be a decorative or triumphant feature but need not be supporting any structure e.g. a statue on top.

History

Columns found at the Temple of Apollo in Delphi

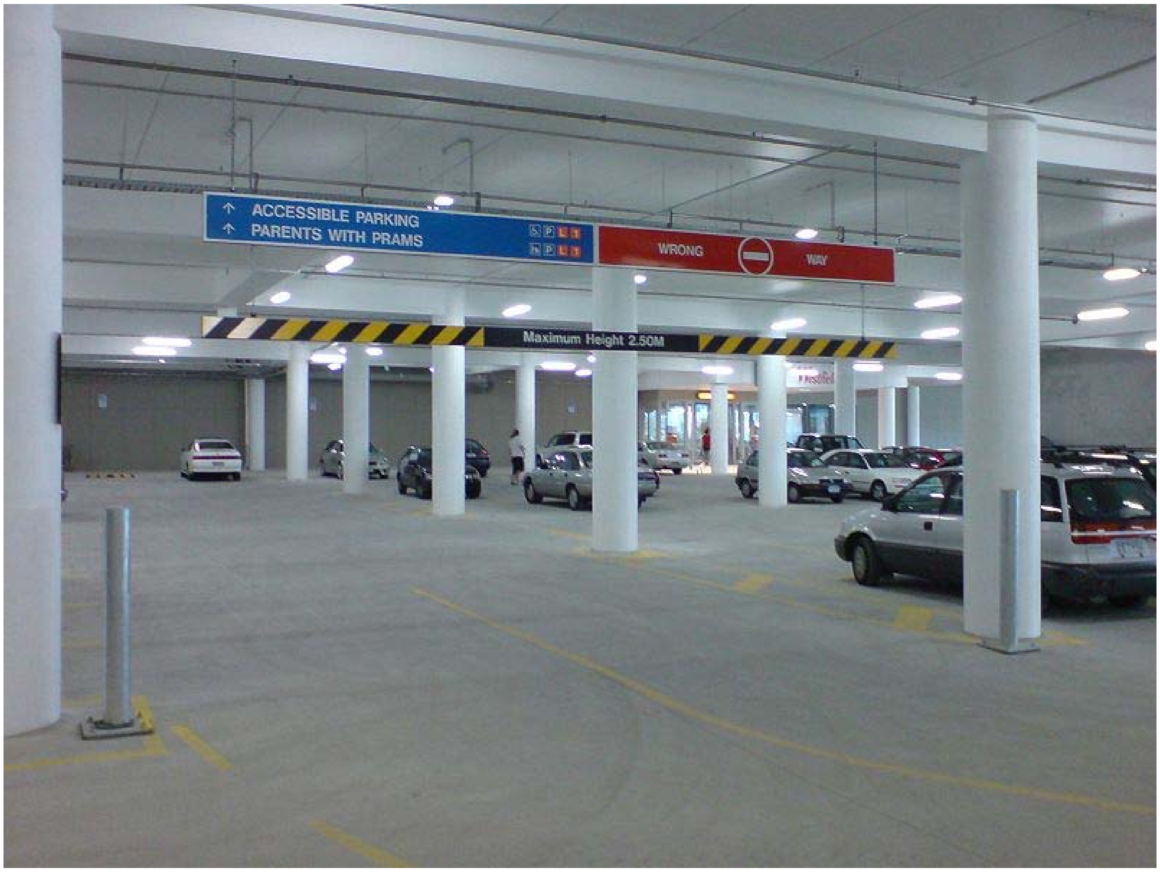
In the architecture of ancient Egypt as early as 2600 BC the architect Imhotep made use of stone columns whose surface was carved to reflect the organic form of bundled reeds; in later Egyptian architecture faceted cylinders were also common.

Some of the most elaborate columns in the ancient world were those of Persia especially the massive stone columns erected in Persepolis. They included double-bull structures in their capitals. The Hall of Hundred Columns at Persepolis, measuring 70 × 70 meters was built by the Achaemenid king Darius I (524–486 BC). Many of the ancient Persian columns are standing, some being more than 30 meters tall.

The Greeks pioneered the use of the classical orders (Doric, Ionic, Corinthian) which was expanded by the Romans to include the Tuscan and Composite styles.

The impost (or pier) is the topmost member of a column. The bottom-most part of the arch, called the springing, rests on the impost.

Structure

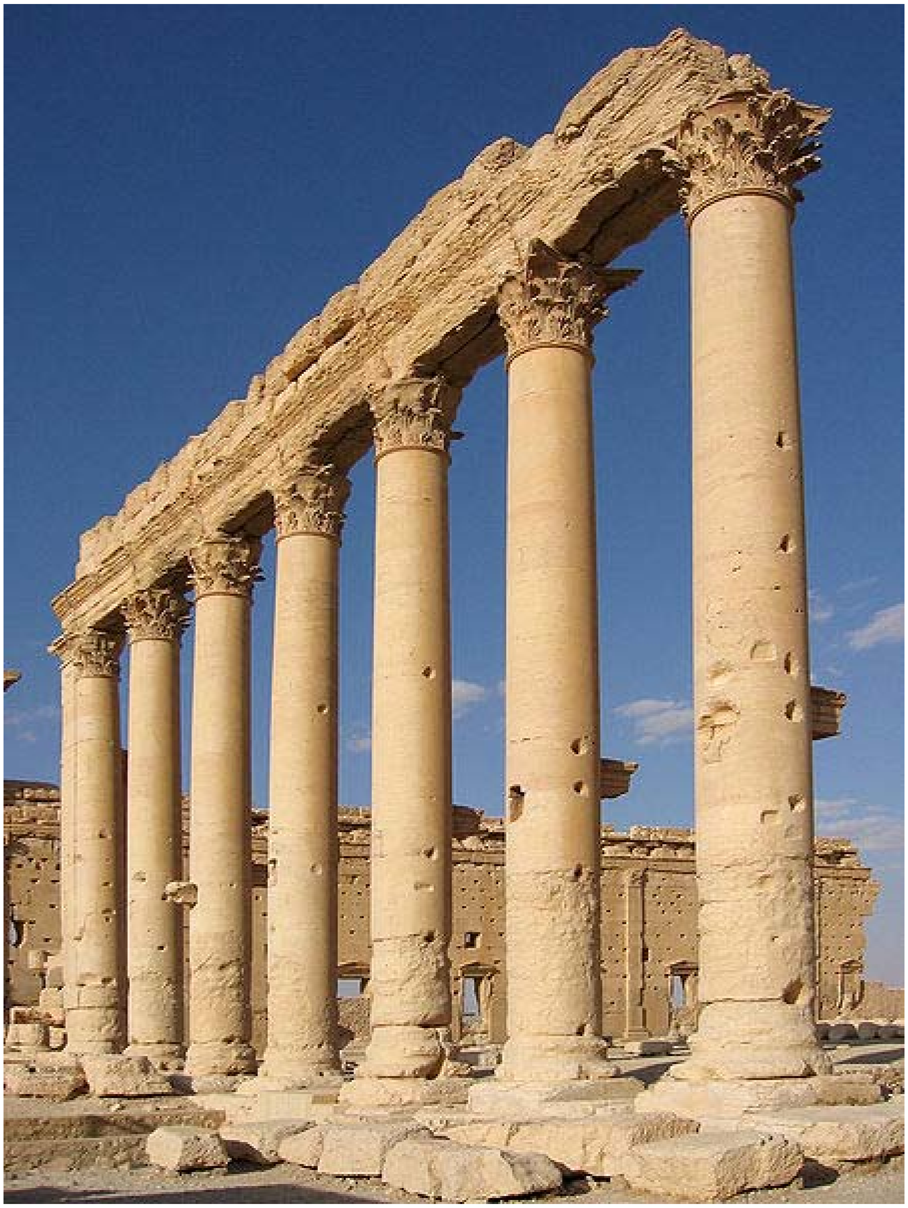


Modern column grid in a car park

Early columns were constructed of stone, some out of a single piece of stone, usually by turning on a lathe-like apparatus. Single-piece columns are among the heaviest stones used in architecture. Other stone columns are created out of multiple sections of stone, mortared or dry-fit together. In many classical sites, sectioned columns were carved with a center hole or depression so that they could be pegged together, using stone or metal pins. The design of most classical columns incorporates entasis (the inclusion of a slight outward curve in the sides) plus a reduction in diameter along the height of the column, so that the top is as little as 83% of the bottom diameter. This reduction mimics the parallax effects which the eye expects to see, and tends to make columns look taller and straighter than they are while entasis adds to that effect.

Modern columns are constructed out of steel, poured or precast concrete, or brick. They may then be clad in an architectural covering (or veneer), or left bare.

Equilibrium, instability, and loads



These are composed of stacked segments and finished in the Corinthian style (Temple of Bel, Syria)

As the axial load on a perfectly straight slender column with elastic material properties is increased in magnitude, this ideal column passes through three states: stable equilibrium, neutral equilibrium, and instability. The straight column under load is in stable equilibrium if a lateral force, applied between the two ends of the column, produces a small lateral deflection which disappears and the column returns to its straight form when the lateral force is removed. If the column load is gradually increased, a condition is reached in which the straight form of equilibrium becomes so-called neutral equilibrium, and a small lateral force will produce a deflection that does not disappear and the column remains in this slightly bent form when the lateral force is removed. The load at which neutral equilibrium of a column is reached is called the critical or buckling load. The state of instability is reached when a slight increase of the column load causes uncontrollably growing lateral deflections leading to complete collapse.

For an axially loaded straight column with any end support conditions, the equation of static equilibrium, in the form of a differential equation, can be solved for the deflected shape and critical load of the column. With hinged, fixed or free end support conditions the deflected shape in neutral equilibrium of an initially straight column with uniform cross section throughout its length always follows a partial or composite sinusoidal curve shape, and the critical load is given by

$$f_{cr} \equiv \frac{\pi^2 E I_{min}}{L^2} \qquad (1)$$

where E = elastic modulus of the material, I_{min} = the minimal moment of inertia of the cross section, and L = actual length of the column between its two end supports. A variant of (1) is given by

$$f_{cr} \equiv \frac{\pi^2 E_T}{(\frac{KL}{r})^2} \qquad (2)$$

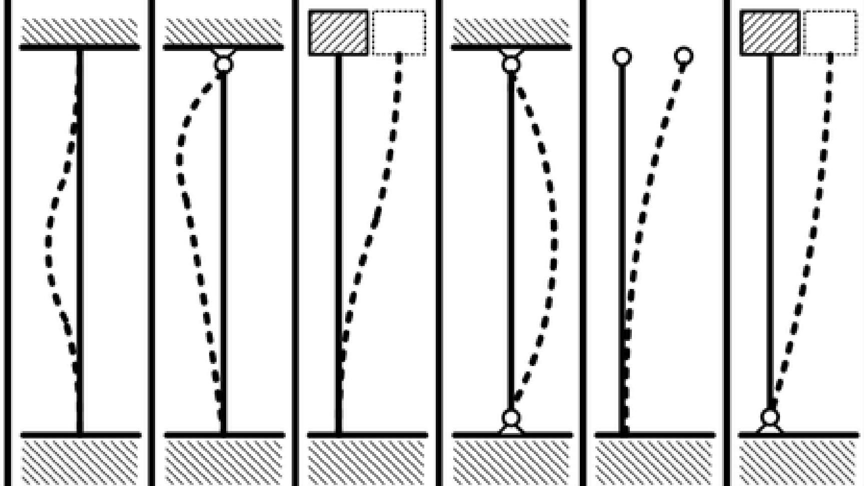
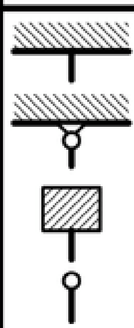
Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0
End condition key		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

Table showing values of K for structural columns of various end conditions (adapted from Manual of Steel Construction, 8th edition, American Institute of Steel Construction, Table C1.8.1)

where r = radius of gyration of [column]cross-section which is equal to the square root of (I/A), K = ratio of the longest half sine wave to the actual column length, and KL = effective length (length of an equivalent hinged-hinged column). From Equation (2) it can be noted that the buckling strength of a column is inversely proportional to the square of its length.

When the critical stress, F_{cr} ($F_{cr} = P_{cr}/A$, where A = cross-sectional area of the column), is greater than the proportional limit of the material, the column is experiencing inelastic buckling. Since at this stress the slope of the material's stress-strain curve, E_t (called the tangent modulus), is smaller than that below the proportional limit, the critical load at inelastic buckling is reduced. More complex formulas and procedures apply for such cases, but in its simplest form the critical buckling load formula is given as Equation (3),

$$f_{cr} \equiv F_y - \frac{F_y^2}{4\pi^2 E}\left(\frac{KL}{r^2}\right) \qquad (3)$$

where E_t = tangent modulus at the stress F_{cr}

A column with a cross section that lacks symmetry may suffer torsional buckling (sudden twisting) before, or in combination with, lateral buckling. The presence of the twisting deformations renders both theoretical analyses and practical designs rather complex.

Eccentricity of the load, or imperfections such as initial crookedness, decreases column strength. If the axial load on the column is not concentric, that is, its line of action is not precisely coincident with the centroidal axis of the column, the column is characterized as eccentrically loaded. The eccentricity of the load, or an initial curvature, subjects the column to immediate bending. The increased stresses due to the combined axial-plus-flexural stresses result in a reduced load-carrying ability.

Column elements are considered to be massive if minimal side dimension is equal or more than 400 mm. Massive columns have ability to increase concrete strength during long time period (even during exploitation period). Taking into account possible loads onto structure increase in future (and even threat of progressive failure - terroristic attacks, explosions etc.) - massive columns have advantage comparing with not ones. A little economy today has no sense as usual for future. Moreover relatively small sections are not technological for reinforced structures during their production. Balance between economy, mass of structures and so called "sustainable" construction is necessary.

Extensions

When a column is too long to be built or transported in one piece, it has to be extended or spliced at the construction site. A reinforced concrete column is extended by having the steel reinforcing bars protrude a few inches or feet above the top of the concrete, then placing the next level of reinforcing bars to overlap, and pouring the concrete of the next level. A steel column is extended by welding or bolting splice plates on the flanges and webs or walls of the columns to provide a few inches or feet of load transfer from the upper to the lower column section. A timber column is usually extended by the use of a steel tube or wrapped-around sheet-metal plate bolted onto the two connecting timber sections.

Foundations

A column that carries the load down to a foundation must have means to transfer the load without overstressing the foundation material. Reinforced concrete and masonry columns are generally built directly on top of concrete foundations. A steel column, when seated on a concrete foundation, must have a base plate to spread the load over a larger area and thereby reduce the bearing pressure. The base plate is a thick rectangular steel plate usually welded to the bottom end of the column.

Classical orders

Church of San Prospero, Reggio Emilia, Italy

The Roman author Vitruvius, relying on the writings (now lost) of Greek authors, tells us that the ancient Greeks believed that their Doric order developed from techniques for building in wood in which the earlier smoothed tree trunk was replaced by a stone cylinder.

Doric order

The Doric order is the oldest and simplest of the classical orders. It is composed of a vertical cylinder that is wider at the bottom. It generally has neither a base nor a detailed capital. It is instead often topped with an inverted frustum of a shallow cone or a cylindrical band of carvings. It is often referred to as the masculine order because it is represented in the bottom level of the Colosseum and the Parthenon, and was therefore considered to be able to hold more weight. The height-to-thickness ratio is about 8:1. The shaft of a Doric Column is always fluted.

The Greek Doric, developed in the western Dorian region of Greece, is the heaviest and most massive of the orders. It rises from the stylobate without any base; it is from four to six times as tall as its diameter; it has twenty broad flutes; the capital consists simply of a banded necking swelling out into a smooth echinus, which carries a flat square abacus; the Doric entablature is also the heaviest, being about one-fourth the height column. The Greek Doric order was not used after c. 100 B.C. until its "rediscovery" in the mid-eighteenth century.

Tuscan order

The Tuscan order, also known as Roman Doric, is also a simple design, the base and capital both being series of cylindrical disks of alternating diameter. The shaft is almost never fluted. The proportions vary, but are generally similar to Doric columns. Height to width ratio is about 7:1.

Ionic order

The Ionic column is considerably more complex than the Doric or Tuscan. It usually has a base and the shaft is often fluted (it has grooves carved up its length). On the top is a capital in the characteristic shape of a scroll, called a volute, or scroll, at the four corners. The height-to-thickness ratio is around 9:1. Due to the more refined proportions and scroll capitals, the Ionic column is sometimes associated with academic buildings.

Ionic capital

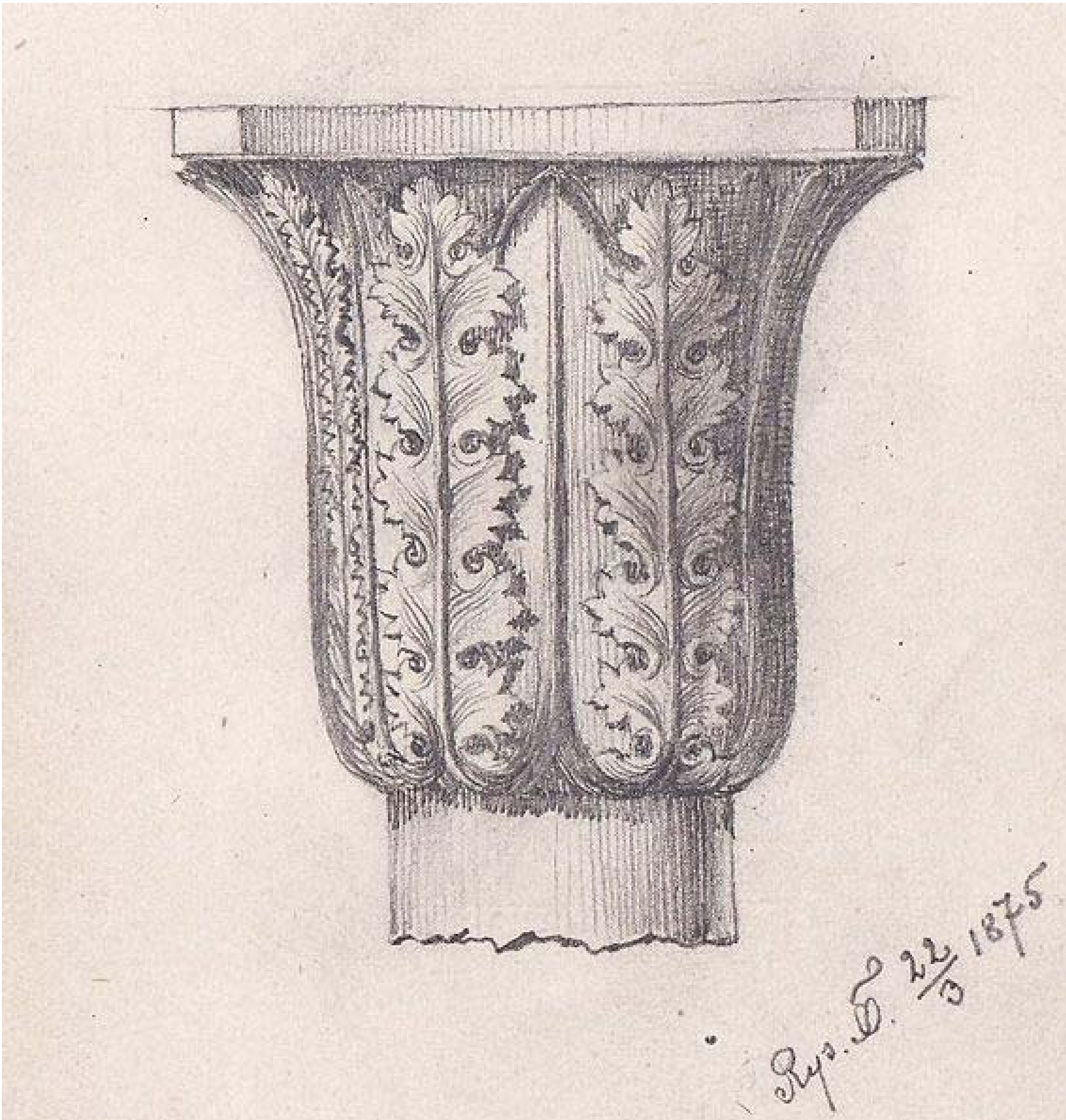
Corinthian order

The Corinthian order is named for the Greek city-state of Corinth, to which it was connected in the period. However, according to the architectural historian Vitruvius, the column was created by the sculptor Callimachus, probably an Athenian, who drew acanthus leaves growing around a votive basket. In fact, the oldest known Corinthian capital was found in Bassae, dated at 427 BC. It is sometimes called the feminine order because it is on the top level of the Colosseum and holding up the least weight, and also has the slenderest ratio of thickness to height. Height to width ratio is about 10:1.

Composite order

The Composite order draws its name from the capital being a composite of the Ionic and Corinthian capitals. The acanthus of the Corinthian column already has a scroll-like element, so the distinction is sometimes subtle. Generally the Composite is similar to the Corinthian in proportion and employment, often in the upper tiers of colonnades. Height to width ratio is about 11:1 or 12:1.

Solomonic



Capital of Solomonic Column

Solomonic columns were inventions of Baroque architects in Europe. They were not used in antiquity, but were called “Solomonic” by baroque architects because they were based on a description of columns in the great temple of King Solomon in the Old Testament. A Solomonic column begins on a base and ends in a capital, just like a classical column, but the shaft twists around the usual parameters of a column, producing a dramatic, serpentine effect of movement. The most famous use of Solomonic columns is in the baldacchino designed by Bernini for Saint Peter’s Basilica in the Vatican City.

Chapter- 5

Catenary

The silk on a spider's web forming multiple elastic catenaries

In physics and geometry, the **catenary** is the curve that an idealised hanging chain or cable assumes when supported at its ends and acted on only by its own weight. The curve is the graph of the hyperbolic cosine function, and has a U-like shape, superficially similar in appearance to a parabola (though mathematically quite different). Its surface of revolution, the catenoid, is a minimal surface and is the shape assumed by a soap film bounded by two parallel circular rings.

History

The word *catenary* is derived from the Latin word *catena,* which means "chain". Huygens first used the term *catenaria* in a letter to Leibniz in 1690. However, Thomas Jefferson is usually credited with the English word *catenary*. The curve is also called the "alysoid", "chainette", or, particularly in the material sciences, "funicular".

It is often stated that Galileo thought that the curve followed by a hanging chain is a parabola. A careful reading of his book *Two new sciences* shows this to be an oversimplification. Galileo discusses the catenary in two places; in the dialog of the Second Day he states that a hanging chain resembles a parabola. But later, in the dialog of the Fourth Day, he gives more details, and states that a hanging cord is approximated by a parabola, correctly observing that this approximation improves as the curvature gets smaller and is almost exact when the elevation is less than 45^{o}. That the curve followed by a chain is not a parabola was proven by Joachim Jungius (1587–1657) and published posthumously in 1669.

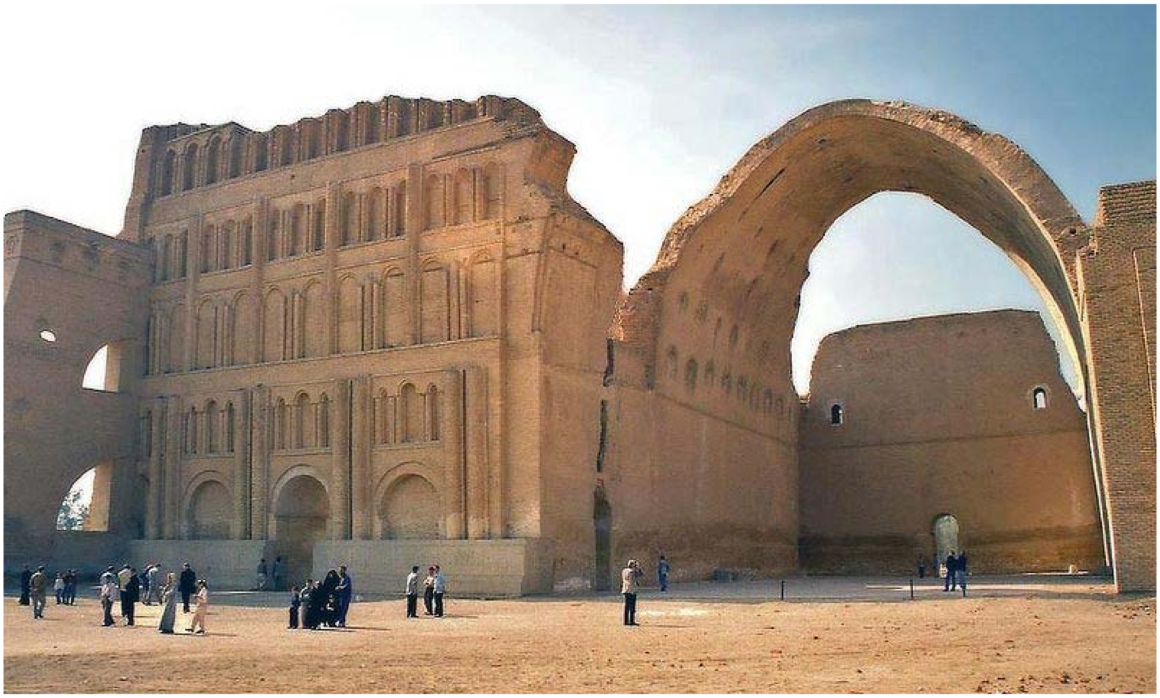
The application of the catenary to the construction of arches is due to Robert Hooke, who discovered it in the context of the rebuilding of St Paul's Cathedral, possibly having seen Huygens' work on the catenary. (Some much older arches are also approximate catenaries.)

In 1671, Hooke announced to the Royal Society that he had solved the problem of the optimal shape of an arch, and in 1675 published an encrypted solution as a Latin anagram in an appendix to his *Description of Helioscopes,* where he wrote that he had found "a true mathematical and mechanical form of all manner of Arches for Building." He did not publish the solution of this anagram in his lifetime, but in 1705 his executor provided it as *Ut pendet continuum flexile, sic stabit contiguum rigidum inversum,* meaning "As hangs a flexible cable so, inverted, stand the touching pieces of an arch."

In 1691 Gottfried Leibniz, Christiaan Huygens, and Johann Bernoulli derived the equation in response to a challenge by Jakob Bernoulli. David Gregory wrote a treatise on the catenary in 1697.

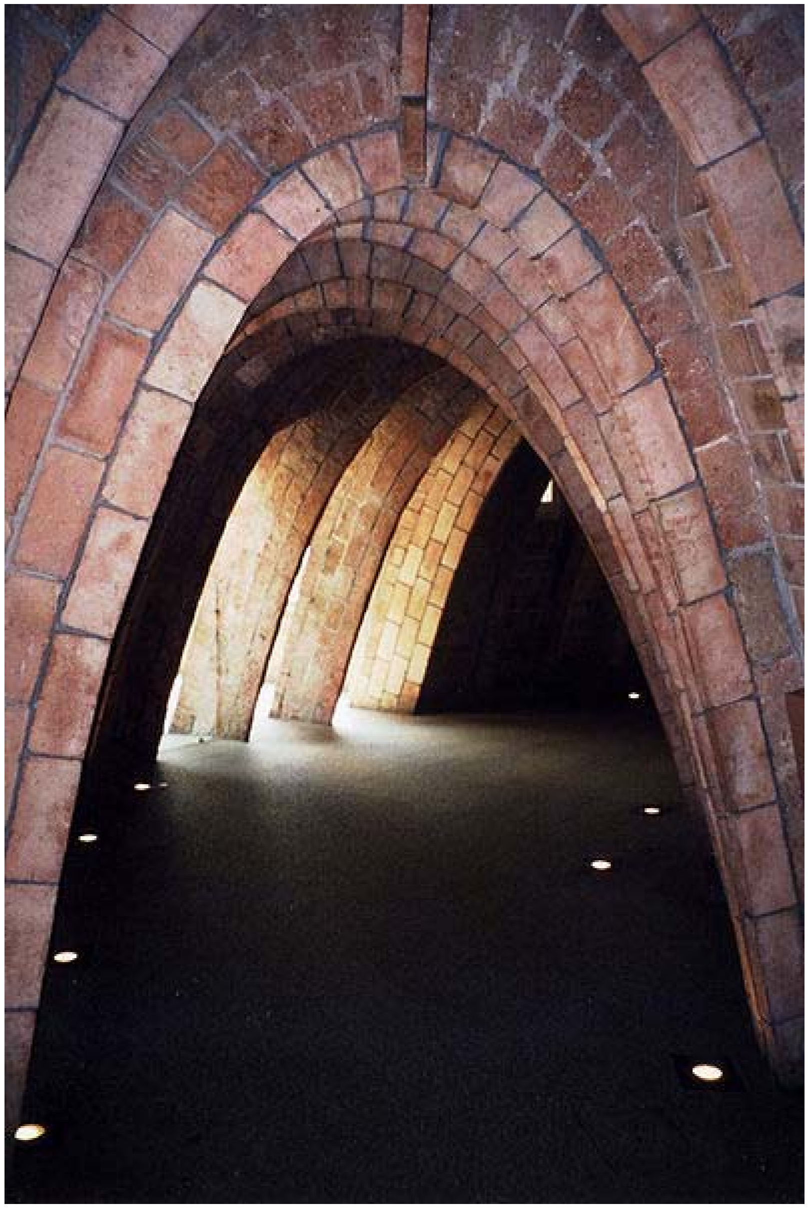
Euler proved in 1744 that the catenary is the curve which, when rotated about the *x*-axis, gives the surface of minimum surface area (the catenoid) for the given bounding circle.

The inverted catenary arch



Arch of Taq-i Kisra in Ctesiphon as seen today is roughly but not exactly a catenary

Gaudi's catenary model at Casa Milà



Arches under the roof of Gaudí's *Casa Milà*, Barcelona, Spain that are close to catenaries

Hooke discovered that the catenary is the ideal curve for an arch of uniform density and thickness which supports only its own weight. When the centerline of an arch is made to follow the curve of an up-side-down (i.e. inverted) catenary, the arch endures almost pure compression, in which no significant bending moment occurs inside the material.

The Sheffield Winter Garden is enclosed by a series of catenary arches

Catenary arches are often used in the construction of kilns. In this construction technique, the shape of a hanging chain of the desired dimensions is transferred to a form which is then used as a guide for the placement of bricks or other building material.

However the conditions for a catenary to be the ideal arch are almost never fulfilled: arches usually support more than their own weight, and on the rare occasions when they are freestanding they are sometimes not of uniform thickness.

The Gateway Arch (looking East) is a flattened catenary

Catenary arch kiln under construction over temporary form

The Gateway Arch in St. Louis, Missouri, United States is sometimes said to be an (inverted) catenary, but this is incorrect. It is close to a more general curve called a flattened catenary, with equation $y=A\cosh(Bx)$. (A catenary would have $AB=1$.) While a catenary is the ideal shape for a freestanding arch of constant thickness, the Gateway Arch is narrower near the top. According to the U.S. National Historic Landmark nomination for the arch, it is a "weighted catenary" instead. Its shape corresponds to the shape that a weighted chain, having lighter links in the middle, would form.

Simple suspension bridges

In simple suspension bridges such as the Capilano Suspension Bridge, where the weight runs parallel to the cables, the cables follow a catenary curve.

Free-hanging chains follow the catenary curve, but suspension bridge chains or cables do not hang freely since they support the weight of the bridge. In most cases the weight of the cable is negligible compared with the weight being supported. When the force exerted is uniform with respect to the length of the chain, as in a simple suspension bridge, the result is a catenary.

When the force exerted is uniform with respect to horizontal distance, as in a suspension bridge, the result is a parabola.

When suspension bridges are constructed, the suspension cables initially sag as the catenary curve, before being tied to the deck below, and then gradually assume a parabolic curve as additional connecting cables are tied to connect the main suspension cables with the bridge deck below.

Golden Gate Bridge, San Francisco, California. Most suspension bridge cables follow a parabolic, not catenary curve.

Anchoring of marine objects

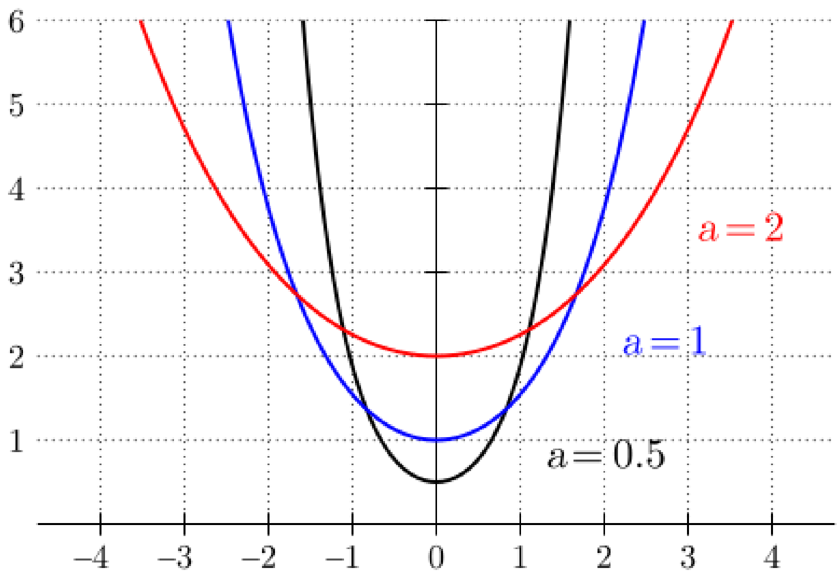
The catenary form given by gravity is taken advantage of in its presence in heavy anchor rodes. An anchor rode (or anchor line) usually consists of chain and/or cable. Anchor rodes are used by ships, oilrigs, docks, wind turbines and other marine assets which must be anchored to the seabed.

Particularly with larger vessels, the catenary curve given by the weight of the rode presents a lower angle of pull on the anchor or mooring device. This assists the performance of the anchor and raises the level of force it will resist before dragging. With smaller vessels and in shallow water it is less effective.

The catenary curve in this context is only fully present in the anchoring system when the rode has been lifted clear of the seabed by the vessel's pull, as the seabed obviously affects its shape while it supports the chain or cable. There is also typically a section of rode above the water and thus unaffected by buoyancy, creating a slightly more complicated curve.

Mathematical description

Equation



Catenaries for different values of *a*

The equation of a catenary in Cartesian coordinates has the form

$$y = a\cosh\left(\frac{x}{a}\right) = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right),$$

where cosh is the hyperbolic cosine function.

The Whewell equation for the catenary is

$$\tan\varphi = \frac{s}{a}.$$

Differentiating gives

$$\frac{d\varphi}{ds} = \frac{\cos^2\varphi}{a}$$

and eliminating φ gives the Cesàro equation:

$$\kappa = \frac{a}{s^2 + a^2}.$$

Other properties

All catenary curves are similar to each other. Changing the parameter *a* is equivalent to a uniform scaling of the curve.

A parabola rolled along a straight line traces out a catenary with its focus.

Square wheels can roll perfectly smoothly if the road has evenly spaced bumps in the shape of a series of inverted catenary curves. The wheels can be any regular polygon except a triangle, but the catenary must have parameters corresponding to the shape and dimensions of the wheels.

A charge in a uniform electric field moves along a catenary (which tends to a parabola if the charge velocity is much less than the speed of light *c*).

The surface of revolution with fixed radii at either end that has minimum surface area is a catenary revolved about the x-axis.

Over any horizontal interval [a,b], the ratio of the area under the caternary to its length equals a, independent of the interval selected. The catenary is the only plane curve other than a horizontal line with this property. Also, the geometric centroid of the area under a stretch of catenary is the midpoint of the perpendicular segment connecting the centroid of the curve itself and the x-axis.

Analysis

We assume that the path followed by the chain is given parametrically by $\mathbf{r} = (x, y) = (x(s),\ y(s))$ where *s* represents arc length and $\mathbf{r}$ is the position vector.

This is the natural parameterization and has the property that $\frac{d\mathbf{r}}{ds}$ is the unit tangent vector,

$\mathbf{u}$. The derivation of the curve for an optimal arch is similar except that the forces of tension become forces of compression and everything is inverted.

It is now possible to derive two equations which together define the shape of the curve and the tension of the chain at each point. This is done by a careful inspection of the various forces acting on a small segment of the chain and using the fact that these forces must be in balance if the chain is in static equilibrium.

First, let $\mathbf{T} = \mathbf{T}(s)$ be the force of tension as a function of *s*. The chain is flexible so it can only exert a force parallel to itself. Since tension is defined as the force that the chain exerts on itself, $\mathbf{T}$ must be parallel to the chain. In other words,

$$(1) \qquad \mathbf{T} = T\mathbf{u}$$

where *T* is the magnitude of $\mathbf{T}$, a positive scalar function of *s*.

Second, let $\mathbf{G} = \mathbf{G}(s)$ be the external force per unit length acting on a small segment of a chain as a function of *s*. The forces acting on the segment of the chain between *s* and *s* + Δ*s* are the force of tension $\mathbf{T}(s + \Delta s)$ at one end of the segment, the nearly opposite force $-\mathbf{T}(s)$ at the other end, and the external force acting on the segment which is approximately $\mathbf{G}\Delta s$. These forces must balance so

$$\mathbf{T}(s + \Delta s) - \mathbf{T}(s) + \mathbf{G}\Delta s \approx \mathbf{0}.$$

Divide by Δ*s* and take the limit as $\Delta s \to 0$ to obtain

$$(2) \qquad \frac{d\mathbf{T}}{ds} + \mathbf{G} = \mathbf{0}.$$

Note that, up till now, no assumptions have been made regarding the force $\mathbf{G}$, so equations (1) and (2) can be used as the starting point in the analysis of a flexible chain acting under any external force. The next step is to put in the specific expression for $\mathbf{G}$ and solve the resulting equations.

In this case, $\mathbf{G} = (0, -\lambda g)$ where the chain has constant mass per unit length λ and the only external force acting on the chain is that of a uniform gravitational field $\mathbf{g} = (0, -g)$. So we have

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

Integrating we get,

$$\mathbf{T} = (c, \lambda g s + d).$$

Note that at the minimum the curve is horizontal and $\mathbf{T} = T\mathbf{u} = T(1,0) = (T,0) = (c, \lambda g s + d)$. So *c* is the tension of the chain at its lowest point this point occurs at *s* = − *d* / λ*g*. The point from which *s* is measured is arbitrary, so pick this point to be the minimum, giving *d* = 0. The equation becomes

$$\mathbf{T} = (c, \lambda g s).$$

Note that the horizontal component of the tension is a constant.

From here, we can continue the derivation in two ways.

Alternative 1

If φ is the tangential angle of the curve then $\mathbf{T}$ is parallel to $(1, \tan\varphi)$ so

$$\tan\varphi = \frac{\lambda g s}{c}.$$

Write $a = \frac{c}{\lambda g}$ to combine constants and obtain the Whewell equation for the curve,

$$\tan\varphi = \frac{s}{a}.$$

In general, parametric equations can be obtained from a Whewell equation by integrating:

$$x = \int \cos\varphi\, ds$$
$$y = \int \sin\varphi\, ds$$

To find these integrals, make the substitution $\tan\varphi = \sinh u$ (or $\varphi = \operatorname{gd} u$ where gd is the Gudermannian function).

Then $s = a \sinh u$ and

$$x = \int \cos\varphi ds = \int \operatorname{sech} u\, a \cosh u du = a \int du = au + \alpha$$
$$y = \int \sin\varphi ds = \int \tanh u\, a \cosh u du = a \int \sinh u du = a \cosh u + \beta.$$

We can eliminate u to obtain

$$y = a\cosh\frac{x-\alpha}{a} + \beta$$

where α and β are constants to be determined, along with *a*, by the boundary conditions of the problem. Usually these conditions include two points from which the chain is being suspended and the length of the chain.

Alternative 2

From

$$\mathbf{T} = T\mathbf{u} = T\left(\frac{dx}{ds}, \frac{dy}{ds}\right) = (c, \lambda g s),$$

$$\frac{dy}{dx} = \frac{dy}{ds}\bigg/\frac{dx}{ds} = \frac{\lambda g s}{c} = \frac{s}{a},$$

where $a = \frac{c}{\lambda g},$ same as before. Then

$$\frac{dx}{ds} = 1\bigg/\frac{ds}{dx} = \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \frac{a}{\sqrt{a^2+s^2}}$$

and

$$\frac{dy}{ds} = \frac{dy}{dx}\cdot\frac{dx}{ds} = \frac{s}{\sqrt{a^2+s^2}}.$$

The integrals of the right hand sides of these equations can be found using standard techniques giving

$$x = a\ \operatorname{arcsinh}(s/a) + \alpha,\ \ y = \sqrt{a^2+s^2} + \beta.$$

Isolating *s* in the first equation and using the result to substitute *s* in the second equation gives

$$y = a\cosh\frac{x-\alpha}{a} + \beta$$

as before, α and β are constants to be determined, along with *a*, by the boundary conditions of the problem, which is exact the same result as that obtained with Alternative 1.

Variations

Elastic catenary

In an elastic catenary, the cable replaced by a spring and is no longer assumed to be of fixed density, but is allowed to stretch in accordance with Hooke's Law. In this case, the mass per unit length is no longer constant but can be given as

$$\lambda = \frac{\lambda_0}{1 + \epsilon T}$$

where λ_0 is the mass per unit length for the chain in its relaxed state and ε is the spring constant. As in the earlier derivation,

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

So the horizontal component of $\mathbf{T}$, $T\cos\varphi$ is a constant c. Putting this into the equation for density produces

$$\lambda = \frac{\lambda_0}{1 + c\epsilon\sec\varphi}.$$

Then the equation for the vertical component of $\mathbf{T}$ is

$$\frac{d}{ds}(T\sin\varphi) = c\frac{d}{ds}(\tan\varphi) = \frac{g\lambda_0}{1 + c\epsilon\sec\varphi},$$

or, combining constants,

$$\frac{d}{ds}(\tan\varphi) = \frac{1}{a + b\sec\varphi}.$$

Using the substitution $\tan\varphi = \sinh u$ gives

$$\frac{d}{ds}(\sinh u) = \cosh u\frac{du}{ds} = \frac{1}{a + b\cosh u}$$

or

$$\cosh u(a + b\cosh u) = \frac{ds}{du}.$$

Parametric equations can be obtained by integrating:

$$x = \int \cos\varphi\, ds = \int \operatorname{sech} u \cosh u(a+b\cosh u)du = \int (a+b\cosh u)du = au+b\sinh u+\alpha,$$
$$y = \int \sin\varphi\, ds = \int \tanh u \cosh u(a+b\cosh u)du = \int \sinh u(a+b\cosh u)du = a\cosh u+\tfrac{b}{2}\sinh^2 u+\beta.$$

When $b = 0$, corresponding to a completely inelastic cable, this is simply the catenary. When $a = 0$, corresponding to the case there the cable essentially has length 0 in its relaxed state, similar to a Slinky, this is a parabola. When a and b are both >0 then the curve is intermediate between a catenary and a parabola.

Equal resistance catenary

In an equal resistance catenary, cable is strengthened according to the magnitude of the tension at each point, so its resistance to breaking is constant along its length. Assuming that the strength of the cable is proportional to its density, the mass per unit length can be given as

$$\lambda = \lambda_r T$$

where λ_r is the mass per unit length per unit of tension force required for the chain to resist breaking. As in the earlier derivation,

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

So the horizontal component of $\mathbf{T}$, $T\cos\varphi$ is a constant c. Putting this into the equation for density produces

$$\lambda = \lambda_r c \sec\varphi.$$

Then the equation for the vertical component of $\mathbf{T}$ is

$$\frac{d}{ds}(T\sin\varphi) = c\frac{d}{ds}(\tan\varphi) = g\lambda_r c\sec\varphi,$$

or, combining constants,

$$\frac{d}{ds}(\tan\varphi) = \frac{1}{a}\sec\varphi$$

or

$$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \frac{1}{a}\frac{ds}{dx}.$$

Multiplying both sides by *ds* / *dx* gives

$$\frac{d^2y}{dx^2} = \frac{1}{a}\left(\frac{ds}{dx}\right)^2 = \frac{1}{a}\left[1 + \left(\frac{dy}{dx}\right)^2\right].$$

This can be reduced to a differential equation of degree one using separation of variables to obtain

$$\arctan\frac{dy}{dx} = \frac{x}{a} - \alpha$$

or

$$\frac{dy}{dx} = \tan\left(\frac{x}{a} - \alpha\right).$$

Another integration produces

$$y = -a\ln\cos\left(\frac{x}{a} - \alpha\right) + \beta.$$

Towed cables

Instead of gravity, we assume we have a cylindrical cable that is acted on by drag forces due to the movement of some surrounding fluid (e.g. air or water). The velocity relative to the cable is assumed to be a constant $\mathbf{v} = (0, -v)$. (Velocity is assumed to be vertical here to preserve similarities with the gravitational case.) To compute the force due to drag, write $\mathbf{v} = \mathbf{v}_u + \mathbf{v}_n$where $\mathbf{v}_u$and $\mathbf{v}_n$respectively are the components parallel to and orthogonal to the cable. The cable is assumed to be smooth so the force on the cable due to $\mathbf{v}_u$is taken to be negligible. The force acting on the cable, following the Drag equation is

$$\mathbf{G} = -c|\mathbf{v}_n|^2\mathbf{n}$$

where *c* is a constant depending on the density of the fluid, the diameter of the cable, and the Drag coefficient. If $\mathbf{n} = (-\sin\varphi,\ \cos\varphi)$denotes the unit normal vector, then

$$\mathbf{v}_n = \mathbf{v} \cdot \mathbf{n}\ \mathbf{n} = -\cos\varphi\ \mathbf{n}.$$

So

$$\mathbf{G} = -c\cos^2\varphi\ \mathbf{n}.$$

From equations (1) and (2) above,

$$\frac{d\mathbf{T}}{ds} = \frac{dT}{ds}\mathbf{u} + T\frac{d\mathbf{u}}{ds} = \frac{dT}{ds}\mathbf{u} + T\frac{d\varphi}{ds}\mathbf{n} = c\cos^2\varphi\ \mathbf{n}.$$

Setting the coefficients of $\mathbf{u}$and $\mathbf{n}$equal produces

$$\frac{dT}{ds} = 0,\ T\frac{d\varphi}{ds} = c\cos^2\varphi.$$

So T is a constant in this case and combining constants in the second equation gives

$$\frac{d\varphi}{ds} = \frac{\cos^2\varphi}{a}$$

which is one of the equations for the catenary given above. This is a case where a different expression for the force acting on the chain/cable produce the same curve but a different expression for tension.

In applications, the force of gravity and additional terms in the force due to drag may be added to the expression for force, yielding equations that must be solved numerically.

Alternative analysis

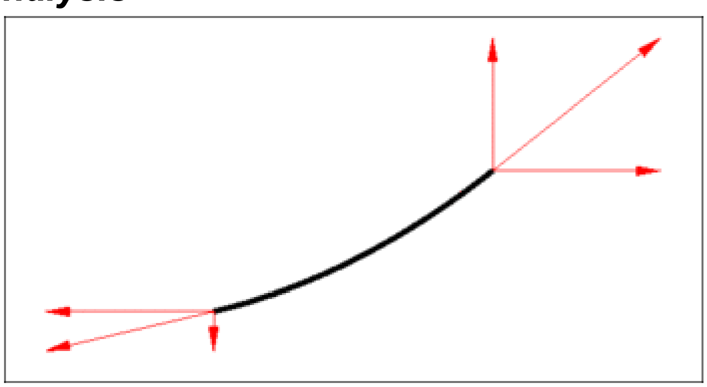


Figure 1: The forces acting on the two extremes of a segment of a catenary decomposed into horizontal and vertical components

The forces acting on a segment of catenary curve are shown in the figure at right.

The vector sum of the forces acting on the segment from the two extremities and from the gravitational force must be zero. As the gravitational force is directed downwards the horizontal components of the forces acting on the extremes must have the same magnitude. As this is true for any segment of the catenary this is a fixed constant for the whole of the catenary. Denoting this constant with f one gets that the vertical component of the force at the left extreme x_1 is $-f\ y'(x_1)$ and at the right extreme x_2 is $f\ y'(x_2)$ The path length of the curve representing a function $y(x)$ with x varying from x_1 to x_2 is

$$\int_{x1}^{x2} \sqrt{1+y'^2}dx$$

If g is the gravitational constant and ρ is the mass per length unit of the chain the gravitational force acting on the arc from x_1 to x_2 is $g\rho \int_{x1}^{x2} \sqrt{1+y'^2}dx$

This force must be compensated by the vertical components of the forces acting on the two extremes of the arc, i.e.

$$f\ (y'(x_2) - y'(x_1))\ = g\rho \int_{x1}^{x2} \sqrt{1+y'^2}dx \quad (1)$$

Denoting the constant ratio $\frac{f}{g\rho}$ with a and taking the derivative of equation (**1**) with respect to the upper limit of the integral, i.e. with respect to x_2, one gets

$$y'' = \frac{1}{a}\sqrt{1+y'^2}$$

Denoting y' with z this equation takes the form

$$z' = \frac{dz}{dx} = \frac{1}{a}\sqrt{1+z^2}$$

what means that for the inverse function $x(z)$ one has

$$\frac{dx}{dz} = \frac{a}{\sqrt{1+z^2}}$$

which is integrate to

$$x = a\ \sinh^{-1}(z) + x_0$$

where x_0 is the constant of integration or equivalently

$$z = y' = \sinh(\frac{x - x_0}{a})$$

Again integrating with respect to *x* one gets

$$y = a\ \cosh(\frac{x - x_0}{a})\ +\ y_0 \quad \textbf{(2)}$$

where y_0 is the second constant of integration

The lowest point of this curve has the coordinates $(\ x_0\ ,\ y_0 + a)$

The length of the curve given by (**2**) from $x = x_1$ to $x = x_2$ is

$$l = \int_{x1}^{x2} \sqrt{1 + y'^2} dx = \left[a \sinh \frac{x - x_0}{a}\right]_{x1}^{x2} \quad \textbf{(3)}$$

This family of solutions is parametrized with the 3 parameters $a,\ x_0,\ y_0$. For any concrete case these 3 parameters must be computed to fit the boundary value conditions. In a typical case the form of a chain having a given length l and being attached in two fixed point with the coordinates $x_1,\ y_1$ and $x_2,\ y_2$ relative a vertical coordinate system should be computed.

This means that $a,\ x_0,\ y_0$ have to be determined such that

$$y_1 = a\ \cosh(\frac{x_1 - x_0}{a})\ +\ y_0 \quad \textbf{(4)}$$

$$y_2 = a\ \cosh(\frac{x_2 - x_0}{a})\ +\ y_0 \quad \textbf{(5)}$$

$$l = a\left(\sinh \frac{x_2 - x_0}{a}\ -\ \sinh \frac{x_1 - x_0}{a}\right) \quad \textbf{(6)}$$

Setting

$$x_m = \frac{x_2 + x_1}{2}$$

$$\Delta x = \frac{x_2 - x_1}{2}$$

subtracting (**4**) from (**5**) and then dividing with *a* one gets

$$\frac{y_2 - y_1}{a} = \cosh\left(\frac{x_m - x_0}{a} + \frac{\Delta x}{a}\right) - \cosh\left(\frac{x_m - x_0}{a} - \frac{\Delta x}{a}\right) = 2\sinh\frac{x_m - x_0}{a}\sinh\frac{\Delta x}{a} \quad \textbf{(7)}$$

For any given values $x_1,\ y_1,\ x_2,\ y_2,\ a$ one can determine $\sinh\frac{x_m - x_0}{a}$ from (**7**)

When $\sinh\frac{x_m - x_0}{a}$ has been determined $\frac{x_m - x_0}{a}$ is computed by solving a quadratic equation.

In case $y_1 = y_2$, i.e. in the case that the two attachment points are at the same heigth, one has that $x_0 = x_m$ and that the length is $l = 2\ a\left(\sinh\frac{\Delta x}{a}\right)$

With x_0 known (**4**) or (**5**) can subsequently be used to determine y_0.

Having determined x_0 with the algorithm just described the curve length l corresponding to the selected a value can be computed from (**6**). With an iterative algorithm the a value that corresponds to a certain curve length l can finally be derived.

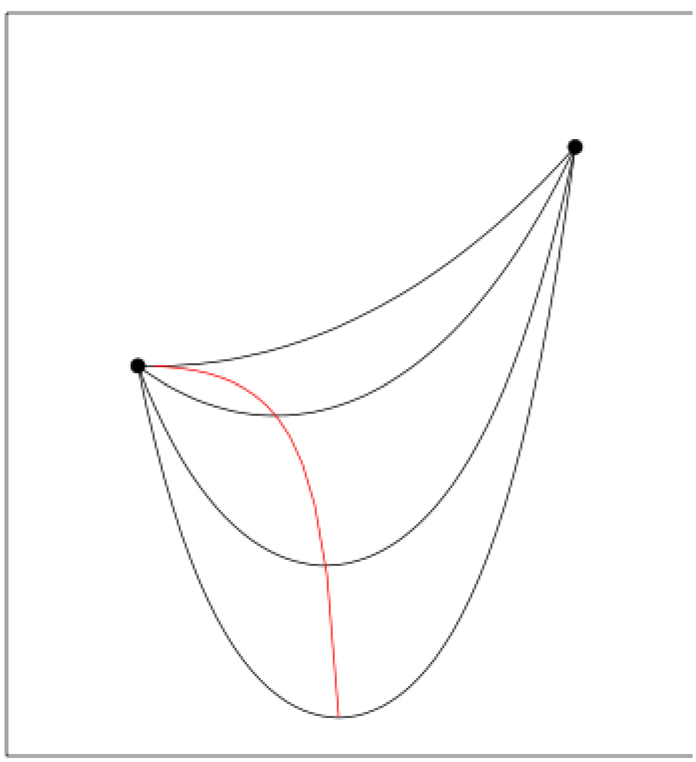


Figure 2:The red line corresponds to parameters X_0 and Y_0 + a determined with the algorithm described above for different values of a

From figure 1 it is further clear that the tension of the chain at any point $(x\ ,\ y)$ is $f\sqrt{1+y'^2} = f\cosh\frac{x-x_0}{a} = f\ \frac{y-y_0}{a}$ where $f = a\ g\ \rho$ is the magnitude of the constant horizontal force component

If the mass density ρ is not constant but varies depending on some law the resulting differential equation will in most cases not have a closed form analytic solution. But the resulting curve can still be determined with arbitrary accuracy by the numerical integration of the differential equations

$$\begin{aligned} y' &= z \\ z' &= \frac{g}{f}\ \rho(z)\sqrt{1+z^2} \end{aligned}$$

Given any initial values for $y(x1)$ and $z(x1)$ and any value for the parameter f these differential equations can be propagated to $x = x_2$ with ρ specified as any function of the state variable z. The free parameters to be iteratively adjusted to fit the boundary constraints are now $z(x1)$ and f. They can for example be adjusted iteratively such that $y(x2) = y_2$ where $(x_2\ ,\ y_2)$ is the second attachment point. This leaves an additional degree of freedom for the two parameters that can be used to get the correct length of the curve.

An example is the "elastic catenary" for which the force

$$f\sqrt{1+y'^2} = f\sqrt{1+z^2}$$

stretches the material with a factor

$$1\ +\ \epsilon f\ \sqrt{1+z^2}$$

where ε is an elasticity coefficient and that therefore the mass density (mass per unit length) is

$$\rho = \frac{\rho_0}{1\ +\ \epsilon f\ \sqrt{1+z^2}}$$

where ρ_0 is the mass density of the material in the absence of stress.

A case where a closed form mathematical solution is possible is the case of "the equal resistance catenary" where the mass density (mass per unit length) is proportional to the force $f\sqrt{1+y'^2} = f\sqrt{1+z^2}$, i.e.

$$\rho = \rho_0\sqrt{1+z^2}$$

where ρ_0 is the density at the lowest point

Setting $a = \frac{f}{g\rho_0}$ the differential equations now take the form

$$y' = z$$
$$z' = \frac{1}{a}\left(1 + z^2\right)$$

what means that for the inverse function $x(z)$ one has

$$\frac{dx}{dz} = \frac{a}{1 + z^2}$$

which is integrate to

$$x = a\ \arctan(z) + x_0$$

where x_0 is the constant of integration or equivalently

$$z = y' = \tan(\frac{x - x_0}{a})$$

where x is constraint to an interval $x_0 - a\ \frac{\pi}{2} < x < x_0 + a\ \frac{\pi}{2}$

Again integrating with respect to x one gets

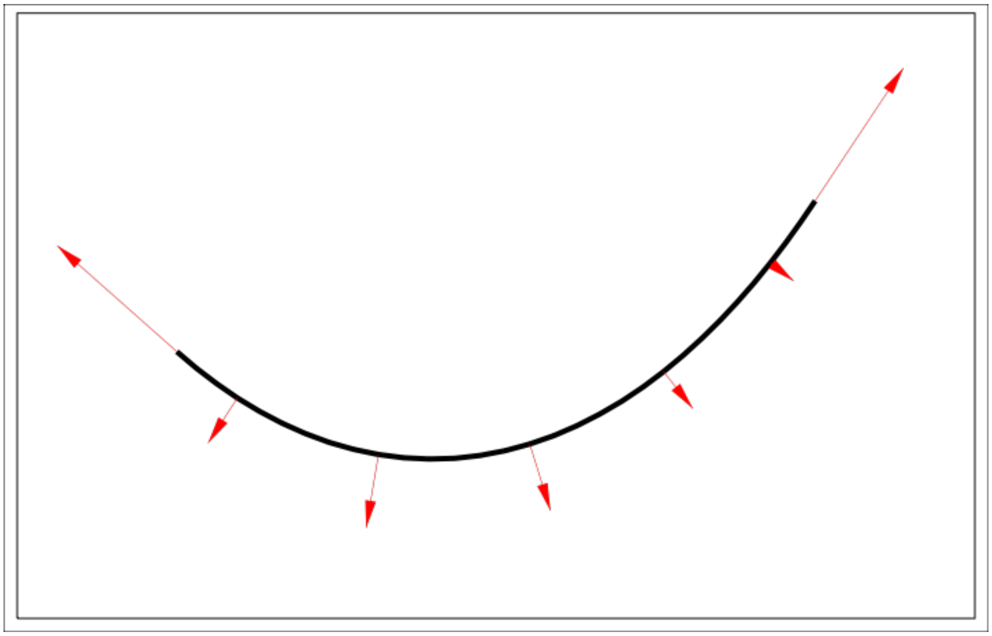
$$y = y_0 \ - a \ln\left(\cos\left(\frac{x - x_0}{a}\right)\right)$$

where y_0 is the second constant of integration.

As $a\ \sinh\frac{C}{a} \rightarrow \infty$ when $a \ \rightarrow 0$for any constant C it follows from (**6**) that by making a catenary that is fixed at two points sufficiently long the constant horizontal force component f can be made arbitrarily small. For this generalized "catenary of equal resistance" this is no more true, as a must be larger then $\frac{|x - x_0|}{\frac{\pi}{2}}$ for any x between x_1 and x_2 the positions of the two attachment points and the density ρ_0 at the lowest point impose a lower limit for the fixed horizontal force component f

Alternative analysis "towed cables"

The following figure illustrates a segment of a cable that is fixed in both ends and exposed to drag.



The forces acting on a cable subject to drag. The medium causing the drag is moving downwards. The drag force is orthogonal to the cable and the forces acting on the two extremities of the segment compensate the net drag force on the segment

The velocity relative to the cable is assumed to be constant and the coordinate system is selected such that this velocity is in the -y direction, i.e. $\mathbf{v} = (0, -v)$. To compute the force due to drag, write $\mathbf{v} = \mathbf{v}_u + \mathbf{v}_n$

where $\mathbf{v}_u$and $\mathbf{v}_n$respectively are the components parallel to and orthogonal to the cable. The cable is assumed to be smooth so the force on the cable due to $\mathbf{v}_u$is taken to be negligible. The force acting on the cable, per unit length, following the Drag equation is therefore

$$\mathbf{G} = f(x)\mathbf{n}$$

with

$$f(x) = c\, v^2\, {n_y}^2 \quad (1)$$

where *c* is a constant depending on the density of the fluid, the diameter of the cable, and the Drag coefficient and $\mathbf{n} = (n_x\ ,\ n_y)$denotes the unit normal vector.

For any curve $y(x)$ the tangent (unit vector) is

$$(t_x\ ,\ t_y) = \frac{(1\ ,\ y')}{\sqrt{1+y'^2}}\ \textbf{(2)}$$

and the normal (unit vector) is

$$(n_x\ ,\ n_y) = \frac{(y'\ ,\ -1)}{\sqrt{1+y'^2}}\ \textbf{(3)}$$

From (**1**) and (**3**) follows that

$$f(x) = c\ v^2\ \frac{1}{1+y'^2}\ \textbf{(4)}$$

From (**3**) and (**4**) follows that the x-component of the total force on the segment of the curve from $x = x_1$ to $x = x_2$ is

$$F_x = \int_{x1}^{x2} f(x)\ n_x\ \sqrt{1+y'^2}dx = c\ v^2 \int_{x1}^{x2} \frac{y'}{1+y'^2}\ dx\ \textbf{(5)}$$

and the component in the y-direction is

$$F_y = \int_{x1}^{x2} f(x)\ n_y\ \sqrt{1+y'^2}dx = c\ v^2 \int_{x1}^{x2} \frac{-1}{1+y'^2}\ dx\ \textbf{(6)}$$

If now

$$y' = \sinh(\frac{x-x_0}{a})$$

one has that

$$1+y'^2 = \cosh^2(\frac{x-x_0}{a})$$

and from (**2**),(**5**) and (**6**) that

$$F_x = c\ v^2 \int_{x_1}^{x_2} \frac{\sinh(\frac{x-x_0}{a})}{\cosh^2(\frac{x-x_0}{a})}\ dx = -\left[\frac{c\ v^2\ a}{\cosh(\frac{x-x_0}{a})}\right]_{x_1}^{x_2} =$$

$$-\left[\frac{c\ v^2\ a}{\sqrt{1+y'^2}}\right]_{x_1}^{x_2} = -c\ v^2\ a\ (t_x(x_2) - t_x(x_1)) \qquad (7)$$

$$F_y = c\ v^2 \int_{x_1}^{x_2} \frac{-1}{\cosh^2(\frac{x-x_0}{a})}\ dx = -\left[\frac{c\ v^2\ a\ \sinh(\frac{x-x_0}{a})}{\cosh(\frac{x-x_0}{a})}\right]_{x_1}^{x_2} =$$

$$-\left[\frac{c\ v^2\ a\ y'}{\sqrt{1+y'^2}}\right]_{x_1}^{x_2} = -c\ v^2\ a\ (t_y(x_2) - t_y(x_1)) \qquad (8)$$

If the now the force in the cable is

$$F = c\ v^2\ a$$

the force at the right extreme of the cable segment is

$$c\ v^2\ a\ (t_x(x_2)\ ,\ t_y(x_2))$$

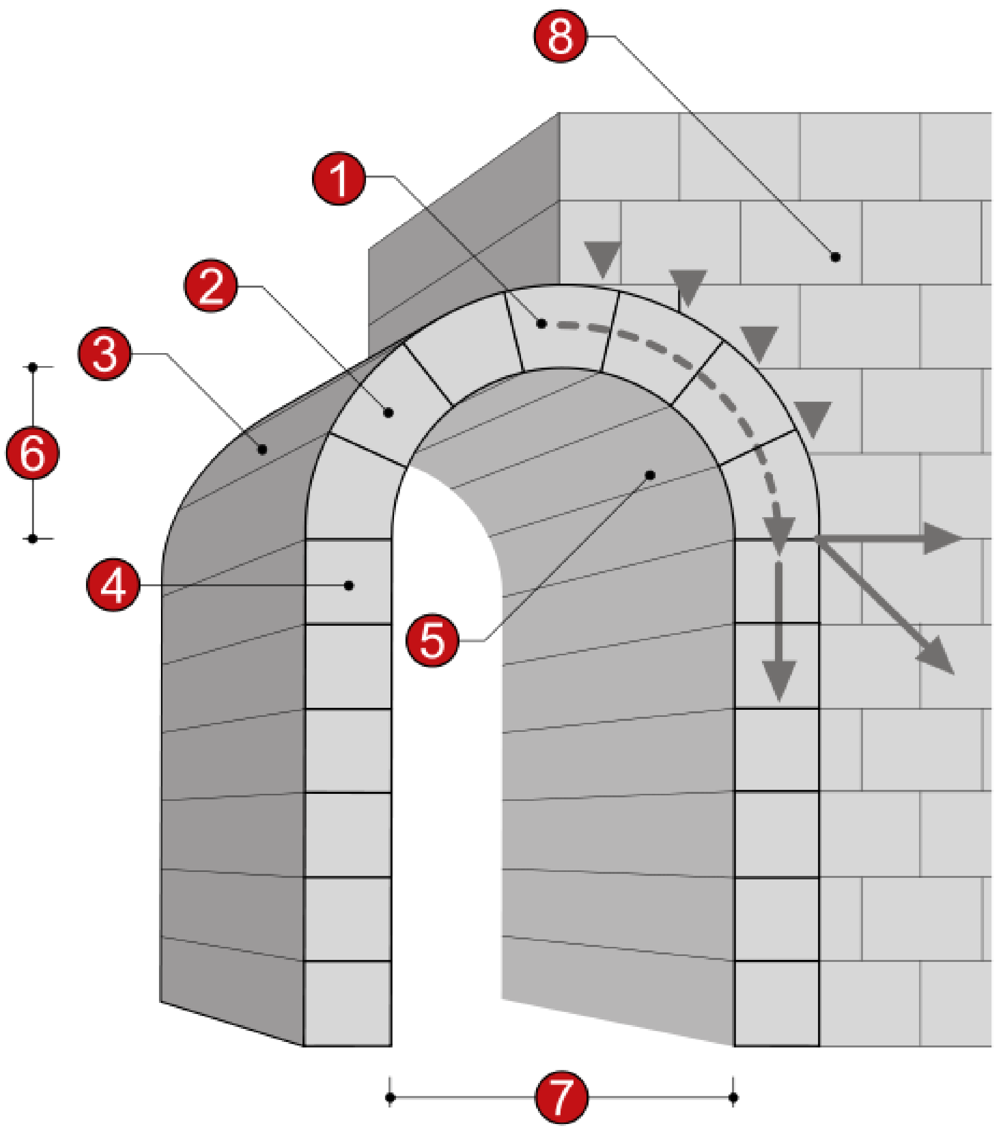
and at the left extreme

$$-c\ v^2\ a\ (t_x(x_1)\ ,\ t_y(x_1))$$

From (**7**) and (**8**) follows that the vector sum of these forces is precisely the force needed to counter act the forces on the segment caused by the drag

Chapter- 6

Arch (Structural Element)



A masonry arch
1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span
8. Abutment

An **arch** is a structure that spans a space while supporting weight (e.g. a doorway in a stone wall). Arches appeared as early as the 2nd millennium BC in Mesopotamian brick architecture and their systematic use started with the Ancient Romans who were the first to apply the technique to a wide range of structures.

The semicircular arch was followed in Europe by the pointed Gothic arch or ogive whose centreline more closely followed the forces of compression and which was therefore stronger. The semicircular arch can be flattened to make an elliptical arch as in the Ponte Santa Trinita. The parabolic and catenary arches are now known to be the theoretically strongest forms. Parabolic arches were introduced in construction by the Spanish architect Antoni Gaudí, who admired the structural system of Gothic style, but for the buttresses, which he termed “architectural crutches”. The catenary and parabolic arches carry all horizontal thrust to the foundation and so do not need additional elements.

Arch of Constantine, Rome, Italy commemorating a victory by Constantine I in 312 AD

The horseshoe arch is based on the semicircular arch, but its lower ends are extended further round the circle until they start to converge. The first known built horseshoe arches are known from Aksum (modern day Ethiopia and Eritrea) from around the 3rd–4th century, around the same time as the earliest contemporary examples in Roman Syria, suggesting either an Aksumite or Syrian origin for the type of arch.

History

Roman arch architecture in Ostia Antica, Italy

The Arc de Triomphe, Paris; a 19th-century triumphal arch modeled on the classical Roman design

True arches, as opposed to corbel arches, were known by a number of civilizations in the Ancient Near East, the Levant, and Mexico, but their use was infrequent and mostly confined to underground structures such as drains where the problem of lateral thrust is greatly diminished. A rare exception is the bronze age arched city gate of Ashkelon, Israel, dating to ca. 1850 BC. An early example of a voussoir arch appears in the Greek Rhodes Footbridge. In 2010, a robot discovered a long arch-roofed passageway underneath the Pyramid of Quetzalcoatl which stands in the ancient city of Teotihuacan north of Mexico City, dated to around 200 AD.

The ancient Romans learned the arch from the Greeks and Etruscans, refined it and were the first builders to tap its full potential for above ground buildings:

The Romans were the first builders in Europe, perhaps the first in the world, fully to appreciate the advantages of the arch, the vault and the dome.

Arches in the nave of the church in monastery of Alcobaça, Portugal

Throughout the Roman empire, their engineers erected arch structures such as bridges, aqueducts, and gates. They also introduced the triumphal arch as a military monument. Vaults began to be used for roofing large interior spaces such as halls and temples, a function which was also assumed by domed structures from the 1st century BC onwards.

The segmental arch was first built by the Romans who realized that an arch in a bridge did not have to be a semicircle, such as in Alconétar Bridge or Ponte San Lorenzo. They were also routinely used in house construction as in Ostia Antica (see picture).

Catenary arches inside Casa Milà in Barcelona, Spain by Antoni Gaudí

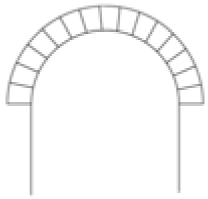
Construction

An arch requires all of its elements to hold it together, raising the question of how an arch is constructed. One answer is to build a frame (historically, of wood) which exactly follows the form of the underside of the arch. This is known as a centre or centring. The voussoirs are laid on it until the arch is complete and self-supporting. For an arch higher than head height, scaffolding would in any case be required by the builders, so the scaffolding can be combined with the arch support. Occasionally arches would fall down when the frame was removed if construction or planning had been incorrect. (The A85 bridge at Dalmally, Scotland suffered this fate on its first attempt, in the 1940s). The interior and lower line or curve of an arch is known as the *intrados*.

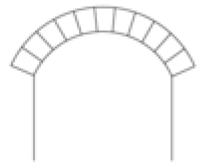
Old arches sometimes need reinforcement due to decay of the keystones, forming what is known as **bald arch**.

The gallery shows arch forms displayed in roughly the order in which they were developed.

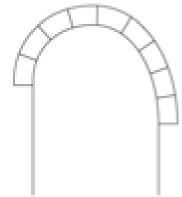
Triangular arch



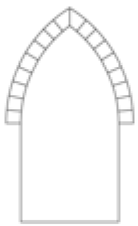
Round arch or Semi-circular arch



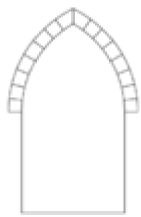
Segmental arch or arch that is less than a semicircle



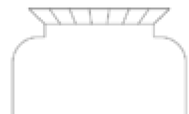
Unequal round arch or Rampant round arch



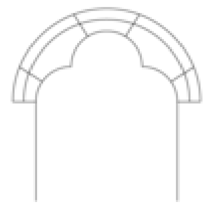
Lancet arch



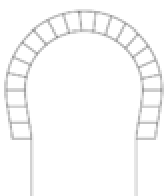
Equilateral pointed arch



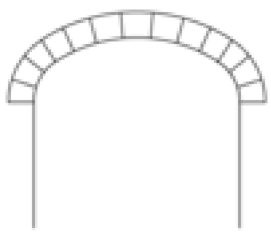
Shouldered flat arch



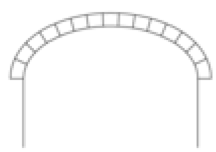
Three-foiled cusped arch



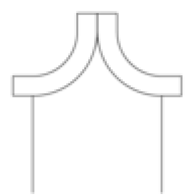
Horseshoe arch



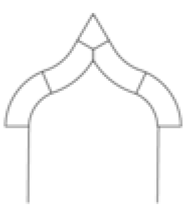
Three-centered arch



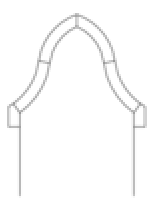
Elliptical arch



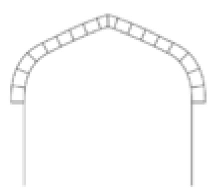
Inflexed arch



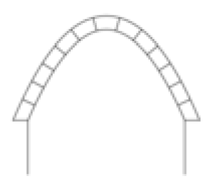
Ogee arch



Reverse ogee arch

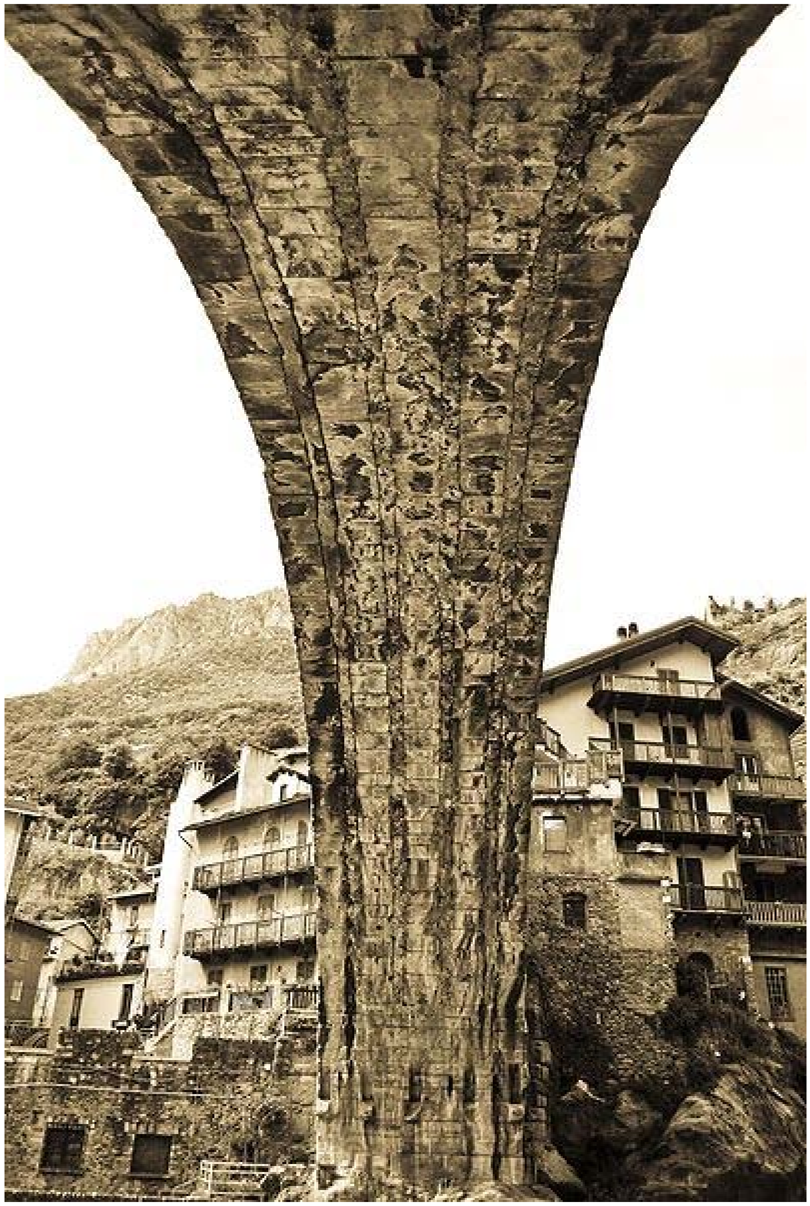


Tudor arch



Catenary or Parabolic arch

Technical aspects



Roman Pont-Saint-Martin

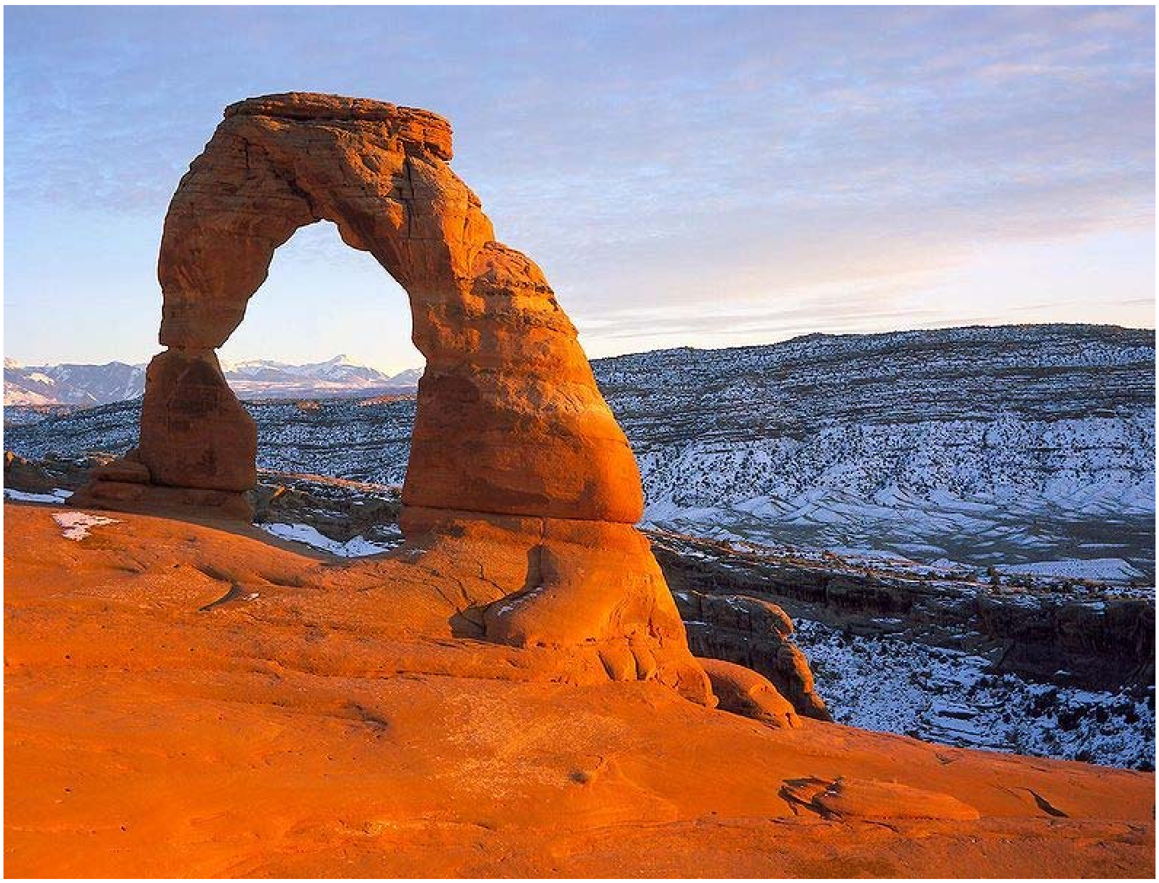
Arches in the Armenian monastery of Geghard

The arch is significant because, in theory at least, it provides a structure which eliminates tensile stresses in spanning an open space. All the forces are resolved into compressive stresses. This is useful because several of the available building materials such as stone, cast iron and concrete can strongly resist compression but are very weak when tension, shear or torsional stress is applied to them. By using the arch configuration, significant spans can be achieved. This is because all the compressive forces hold it together in a state of equilibrium. This even applies to frictionless surfaces. However, one downside is that an arch pushes outward at the base, and this needs to be restrained in some way, either with heavy sides and friction or angled cuts into bedrock or similar.

This same principle holds when the force acting on the arch is not vertical such as in spanning a doorway, but horizontal, such as in arched retaining walls or dams.

Even when using concrete, where the structure may be monolithic, the principle of the arch is used so as to benefit from the concrete's strength in resisting compressive stress. Where any other form of stress is raised, it has to be resisted by carefully placed reinforcement rods or fibres.

Other types



The Delicate Arch, a natural arch in Moab, Utah

A blind arch is an arch infilled with solid construction so it cannot function as a window, door, or passageway.

A dome is an arch rotated 360 degrees about its vertical axis.

Natural rock formations may also be referred to as arches. These natural arches are formed by erosion rather than being carved or constructed by man.

A special form of the arch is the triumphal arch, usually built to celebrate a victory in war. A famous example is the Arc de Triomphe in Paris, France.

A vault is an arch extended along the axis perpendicular to the its plane; the groin vault is the intersection of two vaults.

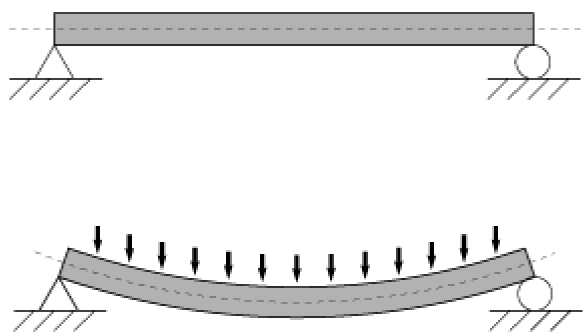
The Gateway Arch in Saint Louis, Missouri; a sculpture based on a catenary arch

Several arches at the Casa Simón Bolívar in Havana, Cuba

Chapter- 7

Other Structural Elements used in Structural Engineering

Beam



A statically determinate beam, bending under an evenly distributed load

A **beam** is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

Overview

Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e., loads due to an earthquake or wind). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. In light frame construction the joists rest on the beam.

Beams are characterized by their profile (the shape of their cross-section), their length, and their material. In contemporary construction, beams are typically made of steel, reinforced concrete, or wood. One of the most common types of steel beam is the I-beam or wide-flange beam (also known as a "universal beam" or, for stouter sections, a "universal column"). This is commonly used in steel-frame buildings and bridges. Other common beam profiles are the C-channel, the hollow structural section beam, the pipe, and the angle.

Structural characteristics

Moment of inertia

The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis. Therefore, it encompasses not just how much mass the object has overall, but how far each bit of mass is from the axis. The farther out the object's mass is, the more rotational inertia the object has, and the more force is required to change its rotation rate.

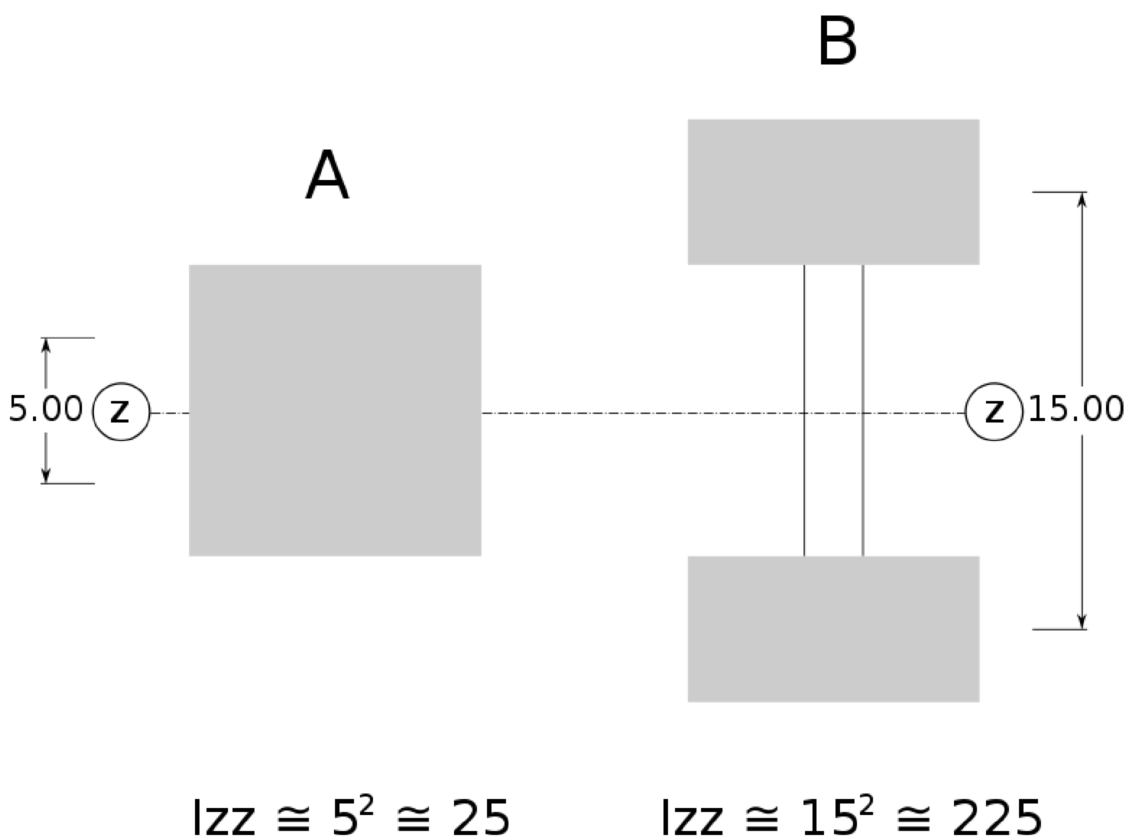


Diagram of stiffness of a simple square beam (A) and universal beam (B). The universal beam flange sections are three times further apart than the solid beam's upper and lower halves. The second moment of inertia of the universal beam is nine times that of the square beam of equal cross section (universal beam web ignored for simplification)

stress in beams

Internally, beams experience compressive, tensile and shear stresses as a result of the loads applied to them. Typically, under gravity loads, the original length of the beam is slightly reduced to enclose a smaller radius arc at the top of the beam, resulting in compression, while the same original beam length at the bottom of the beam is slightly stretched to enclose a larger radius arc, and so is under tension. The same original length of the middle of the beam, generally halfway between the top and bottom, is the same as the radial arc of bending, and so it is under neither compression nor tension, and defines the neutral axis (dotted line in the beam figure). Above the supports, the beam is exposed to shear stress. There are some reinforced concrete beams in which the concrete is entirely in compression with tensile forces taken by steel tendons. These beams are known as prestressed concrete beams, and are fabricated to produce a compression more than the expected tension under loading conditions. High strength steel tendons are stretched while the beam is cast over them. Then, when the concrete has cured, the tendons are slowly released and the beam is immediately under eccentric axial loads. This eccentric loading creates an internal moment, and, in turn, increases the moment carrying capacity of the beam. They are commonly used on highway bridges.

The primary tool for structural analysis of beams is the Euler–Bernoulli beam equation. Other mathematical methods for determining the deflection of beams include "method of virtual work" and the "slope deflection method". Engineers are interested in determining deflections because the beam may be in direct contact with a brittle material such as glass. Beam deflections are also minimized for aesthetic reasons. A visibly sagging beam, even if structurally safe, is unsightly and to be avoided. A stiffer beam (high modulus of elasticity and high second moment of area) produces less deflection.

Mathematical methods for determining the beam forces (internal forces of the beam and the forces that are imposed on the beam support) include the "moment distribution method", the force or flexibility method and the direct stiffness method.

General shapes

Most beams in reinforced concrete buildings have rectangular cross sections, but the most efficient cross section for a simply supported beam is an I or H section. Because of the parallel axis theorem and the fact that most of the material is away from the neutral axis, the second moment of area of the beam increases, which in turn increases the stiffness.

An I-beam is only the most efficient shape in one direction of bending: up and down looking at the profile as an I. If the beam is bent side to side, it functions as an H where it is less efficient. The most efficient shape for both directions in 2D is a box (a square shell) however the most efficient shape for bending in any direction is a cylindrical shell or tube. But, for unidirectional bending, the I or wide flange beam is superior.

Efficiency means that for the same cross sectional area (volume of beam per length) subjected to the same loading conditions, the beam deflects less.

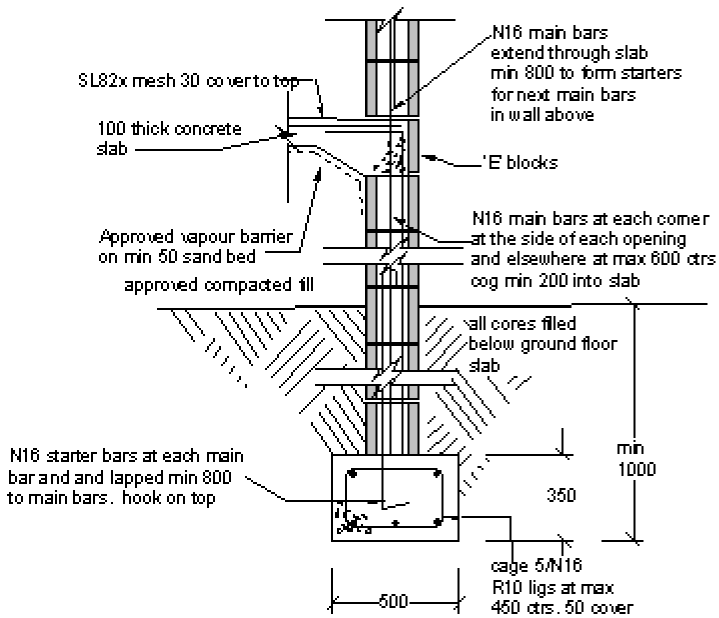
Other shapes, like L (angles), C (channels) or tubes, are also used in construction when there are special requirements.

Shallow Foundation

A **shallow foundation** is a type of foundation which transfers building loads to the earth very near the surface, rather than to a subsurface layer or a range of depths as does a deep foundation. Shallow foundations include **spread footing foundations**, **mat-slab foundations**, **slab-on-grade foundations**, **rubble trench foundations**, and **earthbag foundations**.

Spread footing foundation

In ground reinforced concrete foundation in cyclonic area, Northern Australia



In ground reinforced concrete foundation in cyclonic area, Northern Australia

Spread footing foundations consists of strips or pads of concrete (or other materials) which transfer the loads from walls and columns to the soil or bedrock. Embedment of spread footings is controlled by several factors, including development of lateral capacity, penetration of soft near-surface layers, and penetration through near-surface layers likely to change volume due to frost heave or shrink-swell.

These foundations are common in residential construction that includes a basement, and in many commercial structures. But for high rise building it is not sufficient.

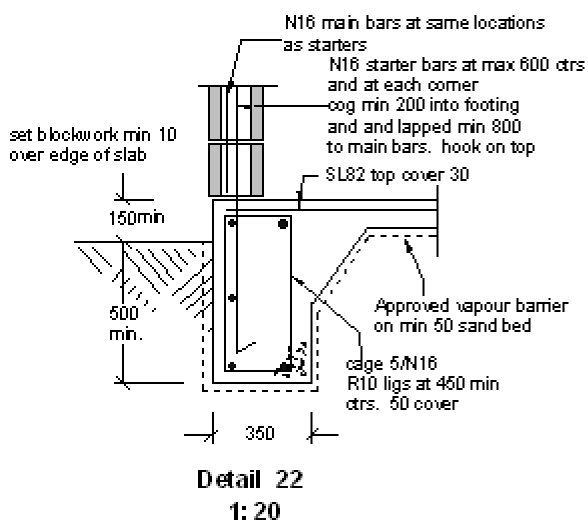
Mat-slab foundations

Mat-slab foundations are used to distribute heavy column and wall loads across the entire building area, to lower the contact pressure compared to conventional spread footings. Mat-slab foundations can be constructed near the ground surface, or at the bottom of basements. In high-rise buildings, mat-slab foundations can be several meters thick, with extensive reinforcing to ensure relatively uniform load transfer.

Slab-on-grade foundation

Example of slab on grade foundation

Raft slab house foundation in cyclonic area, Northern Australia



Raft slab house foundation in cyclonic area, Northern Australia

Slab-on-grade foundations are a structural engineering practice whereby the concrete slab that is to serve as the foundation for the structure is formed from a mold set into the ground. The concrete is then placed into the mold, leaving no space between the ground and the structure. This type of construction is most often seen in warmer climates, where ground freezing and thawing is less of a concern and where there is no need for heat ducting underneath the floor.

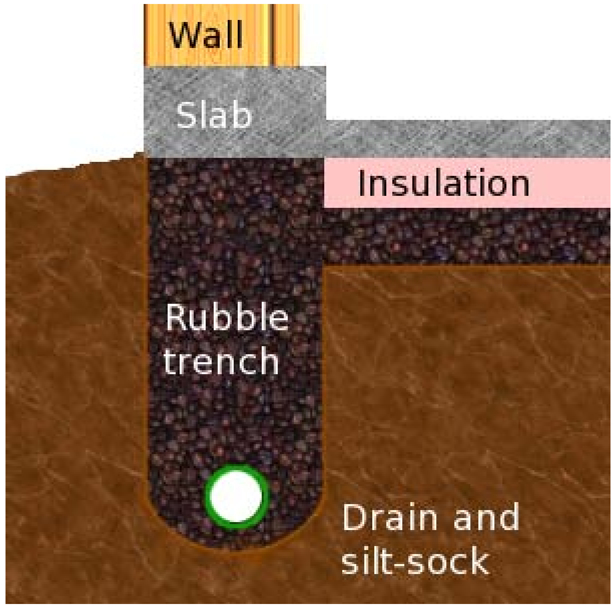
The advantages of the slab technique are that it is cheap and sturdy, and is considered less vulnerable to termite infestation because there are no hollow spaces or wood channels leading from the ground to the structure (assuming wood siding, etc., is not carried all the way to the ground on the outer walls).

The disadvantages are the lack of access from below for utility lines, the potential for large heat losses where ground temperatures fall significantly below the interior temperature, and a very low elevation that exposes the building to flood damage in even moderate rains. Remodeling or extending such a structure may also be more difficult. Over the long term, ground settling (or subsidence) may be a problem, as a slab foundation cannot be readily jacked up to compensate; proper soil compaction prior to pour can minimize this. The slab can be decoupled from ground temperatures by insulation, with the concrete poured directly over insulation (for example, Styrofoam panels), or heating provisions (such as hydronic heating) can be built into the slab (an expensive installation, with associated running expenses).

Slab-on-grade foundations are commonly used in areas with expansive clay soil, particularly in California and Texas. While elevated structural slabs actually perform better on expansive clays, it is generally accepted by the engineering community that slab-on-grade foundations offer the greatest cost-to-performance ratio for tract homes. Elevated structural slabs are generally only found on custom homes or homes with basements.

Care must be taken with the provision of services through the slab. Copper piping, commonly used to carry natural gas and water, reacts with concrete over a long period, slowly degrading until the pipe fails. Copper pipes must be *lagged* (that is, *insulated*) or run through a conduit or plumbed into the building above the slab. Electrical conduits through the slab need to be water-tight, as they extend below ground level and can potentially expose the wiring to groundwater.

Rubble Trench foundation



A cross section view of a rubble trench foundation

The **rubble trench foundation**, a construction approach popularized by architect Frank Lloyd Wright, is a type of foundation that uses loose stone or rubble to minimize the use of concrete and improve drainage. It is considered more environmentally friendly than other types of foundation because cement manufacturing requires the use of enormous amounts of energy. However, some soil environments (such as particularly expansive or poor load-bearing (< 1 ton/sf) soils) are not suitable for this kind of foundation.

A foundation must bear the structural loads imposed upon it and allow proper drainage of ground water to prevent expansion or weakening of soils and frost heaving. While the far more common concrete foundation requires separate measures to ensure good soil drainage, the rubble trench foundation serves both foundation functions at once.

To construct a rubble trench foundation a narrow trench is dug down below the frost line. The bottom of the trench would ideally be gently sloped to an outlet. Drainage tile, graded 1":8' to daylight, is then placed at the bottom of the trench in a bed of washed stone protected by filter fabric. The trench is then filled with either screened stone (typically 1-1/2") or recycled rubble. A steel-reinforced concrete grade beam is poured at the surface to provide ground clearance for the structure.

If an insulated slab is to be poured inside the grade beam, then the outer surface of the grade beam and the rubble trench should be insulated with rigid XPS foam board, which must be protected above grade from mechanical and UV degradation.

The rubble-trench foundation is a relatively simple, low-cost, and environmentally-friendly alternative to a conventional foundation, but may require an engineer's approval if building officials are not familiar with it. Frank Lloyd Wright used them successfully for more than 50 years in the first half of the 20th century, and there is a revival of this style of foundation with the increased interest in green building.

Earthbag foundation

The basic construction method begins by digging a trench down to undisturbed mineral subsoil. Rows of woven bags (or tubes) are filled with available material, placed into this trench, compacted with a pounder to around 1/3 thickness of pre-pounded thickness, and form a foundation. Each successive layer will have one or more strands of barbed wire placed on top. This digs into the bag's weave and prevents slippage of subsequent layers, and also resists any tendency for the outward expansion of walls. The next row of bags is offset by half a bag's width to form a staggered pattern. These are either pre-filled with material and delivered, or filled in place (often the case with Superadobe). The weight of this earth-filled bag pushes down on the barbed wire strands, locking the bag in place on the row below. The same process continues layer upon layer, forming walls. A roof can be formed by gradually sloping the walls inward to construct a dome. Traditional types of roof can also be made.

Thin-shell Structure

Shell structure of the TWA Flight Center Building by Eero Saarinen, John F. Kennedy International Airport, New York

The Montreal Biosphère by Buckminster Fuller, 1967

Lattice Shell of the Shukhov Hyperboloid Tower. Currently under threat of demolition.

Great Court, with a lattice thin-shell roof by Buro Happold with Norman Foster, British Museum, London

Thin-shell structures are light weight constructions using shell elements. These elements are typically curved and are assembled to large structures. Typical applications are fuselages of aeroplanes, boat hulls and roof structures in some buildings.

A thin shell is defined as a shell with a thickness which is small compared to its other dimensions and in which deformations are not large compared to thickness. A primary difference between a shell structure and a plate structure is that, in the unstressed state, the shell structure has curvature as opposed to plates structures which are flat. Membrane action in a shell is primarily caused by in-plane forces (plane stress), though there may be secondary forces resulting from flexural deformations. Where a flat plate acts similar to a beam with bending and shear stresses, shells are analogous to a cable which resists loads through tensile stresses. Though the ideal thin shell must be capable of developing both tension and compression.

Chapter- 8

Truss

In architecture and structural engineering, a **truss** is a structure comprising one or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes. External forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile or compressive forces. Moments (torques) are explicitly excluded because, and only because, all the joints in a truss are treated as revolutes.

A planar truss is one where all the members and nodes lie within a two dimensional plane, while a space truss has members and nodes extending into three dimensions.

Truss bridge for a single track railway, converted to pedestrian use and pipeline support

Characteristics of trusses

A truss is composed of triangles because of the structural stability of that shape and design. A triangle is the simplest geometric figure that will not change shape when the lengths of the sides are fixed. In comparison, both the angles and the lengths of a four-sided figure must be fixed for it to retain its shape.

Planar truss

Planar roof trusses

The simplest form of a truss is one single triangle. This type of truss is seen in a framed roof consisting of rafters and a ceiling joist. Because of the stability of this shape and the methods of analysis used to calculate the forces within it, a truss composed entirely of triangles is known as a simple truss.

A planar truss lies in a single plane. Planar trusses are typically used in parallel to form roofs and bridges.

The depth of a truss, or the height between the upper and lower chords, is what makes it an efficient structural form. A solid girder or beam of equal strength would have substantial weight and material cost as compared to a truss. For a given span length, a deeper truss will require less material in the chords and greater material in the verticals and diagonals. An optimum depth of the truss will maximize the efficiency.

Space frame truss

A space frame truss is a three-dimensional framework of members pinned at their ends. A tetrahedron shape is the simplest space truss, consisting of six members which meet at four joints. Large planar structures may be composed from tetrahedrons with common edges and they are also employed in the base structures of large free-standing power line pylons

Simple tetrahedron

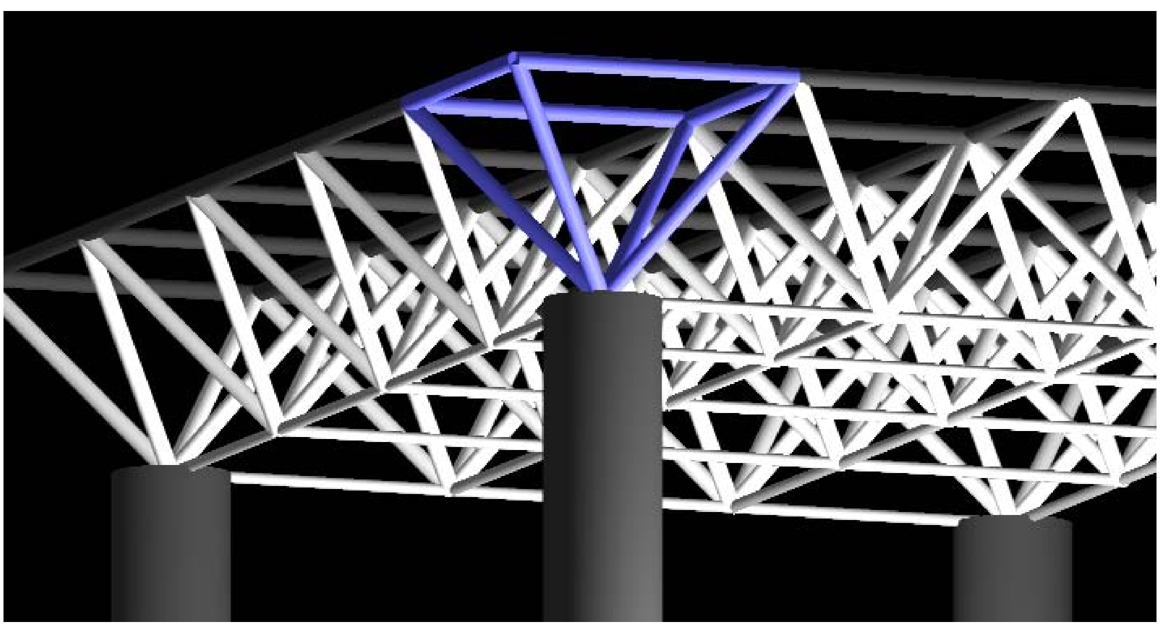


Diagram of a planar space frame such as used for a roof

Four tetrahedons form each of the two lower base structures of this power pylon

Truss types

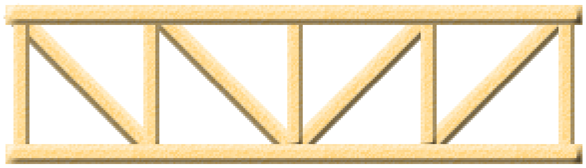
A large timber Howe truss in a commercial building

There are two basic types of truss:

- The *pitched truss*, or *common truss*, is characterized by its triangular shape. It is most often used for roof construction. Some common trusses are named according to their *web configuration*. The chord size and web configuration are determined by *span*, *load* and *spacing*.
- The *parallel chord truss*, or *flat truss*, gets its name from its parallel top and bottom chords. It is often used for floor construction.

A combination of the two is a truncated truss, used in hip roof construction. A metal plate-connected wood truss is a roof or floor truss whose wood members are connected with metal connector plates.

Pratt truss



The **Pratt truss** was patented in 1844 by two Boston railway engineers, Caleb Pratt and his son Thomas Willis Pratt. The design uses vertical members for compression and horizontal members to respond to tension. What is remarkable about this style is that it remained popular even as wood gave way to iron, and even still as iron gave way to steel.. The continued popularity of the Pratt truss is probably due to the fact that the configuration of the members means that longer diagonal members are only in tension for gravity load effects. This allows these members to be used more efficiently, as slenderness effects related to buckling under compression loads (which are compounded by the length of the member) will typically not control the design. Therefore, for given planar truss with a fixed depth, the Pratt configuration is usually the most efficient under static, vertical loading.

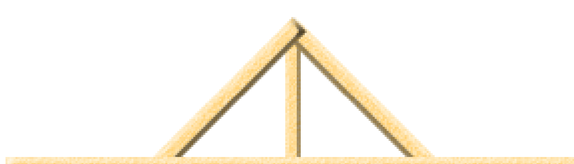
The Southern Pacific Railroad bridge in Tempe, Arizona is a 393 meter (1,291 foot) long truss bridge built in 1912. The structure is composed of nine Pratt truss spans of varying lengths. The bridge is still in use today.

Bowstring truss

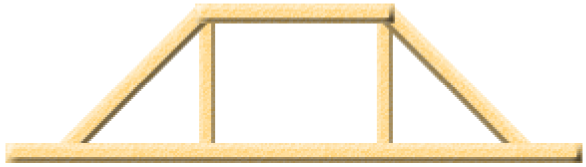
Named for their shape, bowstring trusses were first used for arched truss bridges, known as tied-arch bridges.

Thousands of bowstring trusses were used during World War II for holding up the curved roofs of aircraft hangars and other military buildings. Many variations exist in the arrangements of the members connecting the nodes of the upper arc with those of the lower, straight sequence of members, from nearly isosceles triangles to a variant of the Platt truss.

King post truss



One of the simplest truss styles to implement, the **king post** consists of two angled supports leaning into a common vertical support.



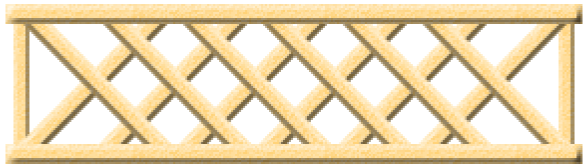
The **queen post** truss, sometimes *queenpost* or *queenspost*, is similar to a king post truss in that the outer supports are angled towards the center of the structure. The primary difference is the horizontal extension at the centre which relies on beam action to provide mechanical stability. This truss style is only suitable for relatively short spans.

Lenticular truss

The Waterville Bridge in Swatara State Park in Pennsylvania is a lenticular truss

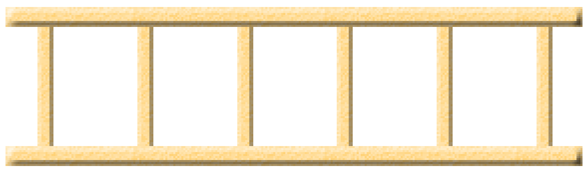
Lenticular trusses, patented in 1878 by William Douglas, have the top and bottom chords of the truss arched, forming a lens shape. A lenticular pony truss bridge is a bridge design that involves a lenticular truss extending above and below the roadbed.

Town's lattice truss



American architect Ithiel Town designed **Town's Lattice Truss** as an alternative to heavy-timber bridges. His design, patented in 1820 and 1835, uses easy-to-handle planks arranged diagonally with short spaces in between them.

Vierendeel truss



A Vierendeel bridge; note the lack of diagonal elements in the primary structure and the way bending loads are carried between elements

The **Vierendeel truss** is a truss where the members are not triangulated but form rectangular openings, and is a frame with fixed joints that are capable of transferring and resisting bending moments. Regular trusses comprise members that are commonly assumed to have pinned joints, with the implication that no moments exist at the jointed ends. This style of truss was named after the Belgian engineer Arthur Vierendeel, who developed the design in 1896. Its use for bridges is rare due to higher costs compared to a triangulated truss.

The utility of this type of truss in buildings is that a large amount of the exterior envelope remains unobstructed and can be used for fenestration and door openings. This is preferable to a braced-frame system, which would leave some areas obstructed by the diagonal braces.

Statics of trusses

A truss that is assumed to comprise members that are connected by means of pin joints, and which is supported at both ends by means of hinged joints or rollers, is described as being statically determinate. Newton's Laws apply to the structure as a whole, as well as to each node or joint. In order for any node that may be subject to an external load or force to remain static in space, the following conditions must hold: the sums of all (horizontal and vertical) forces, as well as all moments acting about the node equal zero. Analysis of these conditions at each node yields the magnitude of the forces in each member of the truss. These may be compression or tension forces.

Trusses that are supported at more than two positions are said to be statically indeterminate, and the application of Newton's Laws alone is not sufficient to determine the member forces.

In order for a truss with pin-connected members to be stable, it must be entirely composed of triangles. In mathematical terms, we have the following necessary condition for stability:

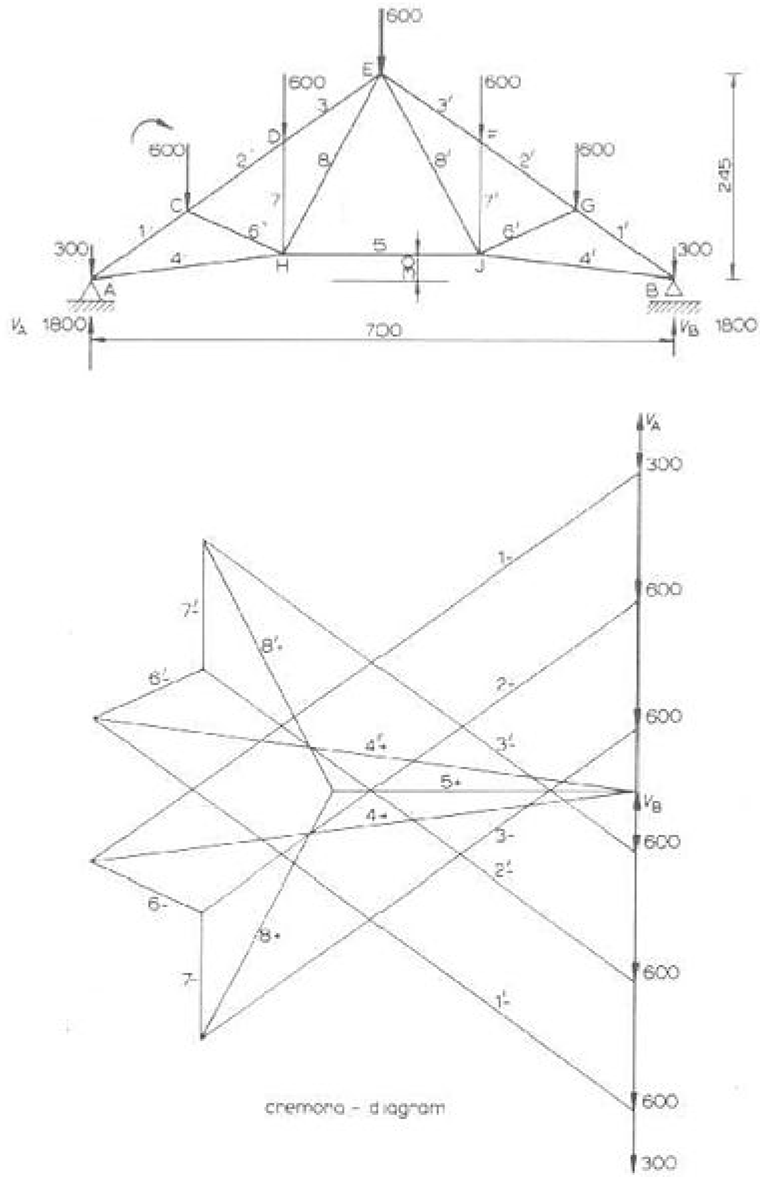
$$m \geq 2j - r \qquad \text{(a)}$$

where m is the total number of truss members, j is the total number of joints and r is the number of reactions (equal to 3 generally) in a 2-dimensional structure.

When $m = 2j - 3$, the truss is said to be *statically determinate*, because the $(m+3)$ internal member forces and support reactions can then be completely determined by $2j$ equilibrium equations, once we know the external loads and the geometry of the truss. Given a certain number of joints, this is the minimum number of members, in the sense that if any member is taken out (or fails), then the truss as a whole fails. While the relation (a) is necessary, it is not sufficient for stability, which also depends on the truss geometry, support conditions and the load carrying capacity of the members.

Some structures are built with more than this minimum number of truss members. Those structures may survive even when some of the members fail. Their member forces depend on the relative stiffness of the members, in addition to the equilibrium condition described.

Analysis of trusses



Cremona diagram for a plane truss

Because the forces in each of its two main girders are essentially planar, a truss is usually modelled as a two-dimensional plane frame. If there are significant out-of-plane forces, the structure must be modelled as a three-dimensional space.

The analysis of trusses often assumes that loads are applied to joints only and not at intermediate points along the members. The weight of the members is often insignificant compared to the applied loads and so is often omitted. If required, half of the weight of each member may be applied to its two end joints. Provided the members are long and slender, the moments transmitted through the joints are negligible and they can be treated as "hinges" or 'pin-joints'. Every member of the truss is then in pure compression or pure tension – shear, bending moment, and other more complex stresses are all practically zero. This makes trusses easier to analyze. This also makes trusses physically stronger than other ways of arranging material – because nearly every material can hold a much larger load in tension and compression than in shear, bending, torsion, or other kinds of force.

Structural analysis of trusses of any type can readily be carried out using a matrix method such as the direct stiffness method, the flexibility method or the finite element method.

Forces in members

On the right is a simple, statically determinate flat truss with 9 joints and (2 x 9) − 3 = 15 members. External loads are concentrated in the outer joints. Since this is a symmetrical truss with symmetrical vertical loads, it is clear to see that the reactions at A and B are equal, vertical and half the total load.

The internal forces in the members of the truss can be calculated in a variety of ways including the graphical methods:

- *Cremona diagram*
- *Culmann diagram*
- *the analytical* Ritter *method (method of sections).*

Design of members

A truss can be thought of as a beam where the web consists of a series of separate members instead of a continuous plate. In the truss, the lower horizontal member (the **bottom chord**) and the upper horizontal member (the **top chord**) carry tension and compression, fulfilling the same function as the flanges of an I-beam. Which chord carries tension and which carries compression depends on the overall direction of bending. In the truss pictured above right, the bottom chord is in tension, and the top chord in compression.

The diagonal and vertical members form the **truss web**, and carry the shear force. Individually, they are also in tension and compression, the exact arrangement of forces is depending on the type of truss and again on the direction of bending. In the truss shown above right, the vertical members are in tension, and the diagonals are in compression.

A building under construction in Shanghai. The truss sections stabilize the building and will house mechanical floors.

In addition to carrying the static forces, the members serve additional functions of stabilizing each other, preventing buckling. In the picture to the right, the top chord is prevented from buckling by the presence of bracing and by the stiffness of the web members.

The inclusion of the elements shown is largely an engineering decision based upon economics, being a balance between the costs of raw materials, off-site fabrication, component transportation, on-site erection, the availability of machinery and the cost of labor. In other cases the appearance of the structure may take on greater importance and so influence the design decisions beyond mere matters of economics. Modern materials such as prestressed concrete and fabrication methods, such as automated welding, have significantly influenced the design of modern bridges.

Once the force on each member is known, the next step is to determine the cross section of the individual truss members. For members under tension the cross-sectional area A can be found using $A = F \times \gamma / \sigma_y$, where F is the force in the member, γ is a safety factor (typically 1.5 but depending on building codes) and σ_y is the yield tensile strength of the steel used.
The members under compression also have to be designed to be safe against buckling.

The weight of a truss member depends directly on its cross section—that weight partially determines how strong the other members of the truss need to be. Giving one member a larger cross section than on a previous iteration requires giving other members a larger cross section as well, to hold the greater weight of the first member—one needs to go through another iteration to find exactly how much greater the other members need to be. Sometimes the designer goes through several iterations of the design process to converge on the "right" cross section for each member. On the other hand, reducing the size of one member from the previous iteration merely makes the other members have a larger (and more expensive) safety factor than is technically necessary, but doesn't *require* another iteration to find a buildable truss.

The effect of the weight of the individual truss members in a large truss, such as a bridge, is usually insignificant compared to the force of the external loads.

Design of joints

After determining the minimum cross section of the members, the last step in the design of a truss would be detailing of the bolted joints, e.g., involving shear of the bolt connections used in the joints.

Applications

Post frame structures

Component connections are critical to the structural integrity of a framing system. In buildings with large, clearspan wood trusses, the most critical connections are those between the truss and its supports. In addition to gravity-induced forces (a.k.a. bearing loads), these connections must resist shear forces acting perpendicular to the plane of the truss and uplift forces due to wind. Depending upon overall building design, the connections may also be required to transfer bending moment.

Wood posts enable the fabrication of strong, direct, yet inexpensive connections between large trusses and walls. Exact details for post-to-truss connections vary from designer to designer, and may be influenced by post type. Solid-sawn timber and glulam posts are generally notched to form a truss bearing surface. The truss is rested on the notches and bolted into place. A special plate/bracket may be added to increase connection load transfer capabilities. With mechanically-laminated posts, the truss may rest on a shortened outer-ply or on a shortened inner-ply. The later scenario places the bolts in double shear and is a very effective connection.

The HSBC Main Building, Hong Kong has an externally visible truss structure

Support structure under the Auckland Harbour Bridge

Little Belt: a truss bridge in Denmark

Chapter- 9

Structural Engineering Theory

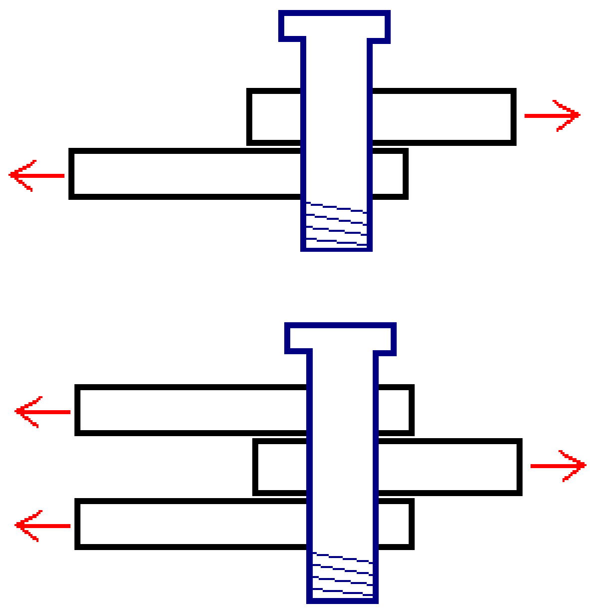


Figure of a bolt in shear. Top figure illustrates single shear, bottom figure illustrates double shear.

Structural engineering depends upon a detailed knowledge of loads, physics and materials to understand and predict how structures support and resist self-weight and imposed loads. To apply the knowledge successfully a structural engineer will need a detailed knowledge of mathematics and of relevant empirical and theoretical design codes. He will also need to know about the corrosion resistance of the materials and structures, especially when those structures are exposed to the external environment.

The criteria which govern the design of a structure are either serviceability (criteria which define whether the structure is able to adequately fulfill its function) or strength (criteria which define whether a structure is able to safely support and resist its design loads). A structural engineer designs a structure to have sufficient strength and stiffness to meet these criteria.

Loads imposed on structures are supported by means of forces transmitted through structural elements. These forces can manifest themselves as:

- tension (axial force)
- compression (axial force)
- shear
- bending, or flexure (a bending moment is a force multiplied by a distance, or lever arm, hence producing a turning effect or torque)

Strength

Strength depends upon material properties. The strength of a material depends on its capacity to withstand axial stress, shear stress, bending, and torsion. The strength of a material is measured in force per unit area (newtons per square millimetre or N/mm^2, or the equivalent megapascals or MPa in the SI system and often pounds per square inch psi in the United States Customary Units system).

A structure fails the strength criterion when the stress (force divided by area of material) induced by the loading is greater than the capacity of the structural material to resist the load without breaking, or when the strain (percentage extension) is so great that the element no longer fulfills its function (yield).

Stiffness

Stiffness depends upon material properties and geometry. The stiffness of a structural element of a given material is the product of the material's Young's modulus and the element's second moment of area. Stiffness is measured in force per unit length (newtons per millimetre or N/mm), and is equivalent to the 'force constant' in Hooke's Law.

The deflection of a structure under loading is dependent on its stiffness. The dynamic response of a structure to dynamic loads (the natural frequency of a structure) is also dependent on its stiffness.

In a structure made up of multiple structural elements where the surface distributing the forces to the elements is rigid, the elements will carry loads in proportion to their relative stiffness - the stiffer an element, the more load it will attract. This means that load/stiffness ratio, which is deflection, remains same in two connected (jointed) elements. In a structure where the surface distributing the forces to the elements is flexible (like a wood framed structure), the elements will carry loads in proportion to their relative tributary areas.

A structure is considered to fail the chosen serviceability criteria if it is insufficiently stiff to have acceptably small deflection or dynamic response under loading.

The inverse of stiffness is flexibility.

Safety factors

The safe design of structures requires a design approach which takes account of the statistical likelihood of the failure of the structure. Structural design codes are based upon the assumption that both the loads and the material strengths vary with a normal distribution.

The job of the structural engineer is to ensure that the chance of overlap between the distribution of loads on a structure and the distribution of material strength of a structure is acceptably small (it is impossible to reduce that chance to zero).

It is normal to apply a *partial safety factor* to the loads and to the material strengths, to design using 95th percentiles (two standard deviations from the mean). The safety factor applied to the load will typically ensure that in 95% of times the actual load will be smaller than the design load, while the factor applied to the strength ensures that 95% of times the actual strength will be higher than the design strength.

The safety factors for material strength vary depending on the material and the use it is being put to and on the design codes applicable in the country or region.

Load cases

A **load case** is a combination of different types of loads with safety factors applied to them. A structure is checked for strength and serviceability against all the load cases it is likely to experience during its lifetime.

Typical load cases for design for strength (ultimate load cases; ULS) are:

> *1.2 x Dead Load + 1.6 x Live Load*
> *1.2 x Dead Load + 1.2 x Live Load + 1.2 x Wind Load*

A typical load case for design for serviceability (characteristic load cases; SLS) is:

> *1.0 x Dead Load + 1.0 x Live Load*

Different load cases would be used for different loading conditions. For example, in the case of design for fire a load case of *1.0 x Dead Load + 0.8 x Live Load* may be used, as it is reasonable to assume everyone has left the building if there is a fire.

In multi-story buildings it is normal to reduce the total live load depending on the number of stories being supported, as the probability of maximum load being applied to all floors simultaneously is negligibly small.

It is not uncommon for large buildings to require hundreds of different load cases to be considered in the design.

Newton's laws of motion

The most important natural laws for structural engineering are Newton's Laws of Motion

Newton's first law states that *every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.*

Newton's second law states that *the rate of change of momentum of a body is proportional to the resultant force acting on the body and is in the same direction.* Mathematically, F=ma (force = mass x acceleration).

Newton's third law states that *all forces occur in pairs, and these two forces are equal in magnitude and opposite in direction.*

With these laws it is possible to understand the forces on a structure and how that structure will resist them. The Third Law requires that for a structure to be stable all the internal and external forces must be in equilibrium. This means that the sum of all internal and external forces on a *free-body diagram* must be zero:

- $\sum \vec{F} = 0$: the vectorial sum of the forces acting on the body equals zero. This translates to

  $\Sigma H = 0$: the sum of the horizontal components of the forces equals zero;
  $\Sigma V = 0$: the sum of the vertical components of forces equals zero;

- $\sum \vec{M} = 0$: the sum of the moments (about an arbitrary point) of all forces equals zero.

Statical determinacy

A structural engineer must understand the internal and external forces of a structural system consisting of structural elements and nodes at their intersections.

A statically determinate structure can be fully analysed using only consideration of equilibrium, from Newton's Laws of Motion.

A statically indeterminate structure has more unknowns than equilibrium considerations can supply equations for. Such a system can be solved using consideration of equations of *compatibility* between geometry and deflections in addition to equilibrium equations, or by using virtual work.

If a system is made up of *b* bars, *j* pin joints and *r* support reactions, then it cannot be statically determinate if the following relationship does not hold:

$r + b = 2j$

It should be noted that even if this relationship does hold, a structure can be arranged in such a way as to be statically indeterminate.

Elasticity

Much engineering design is based on the assumption that materials behave elastically. For most materials this assumption is incorrect, but empirical evidence has shown that design using this assumption can be safe. Materials that are elastic obey Hooke's Law, and plasticity does not occur.

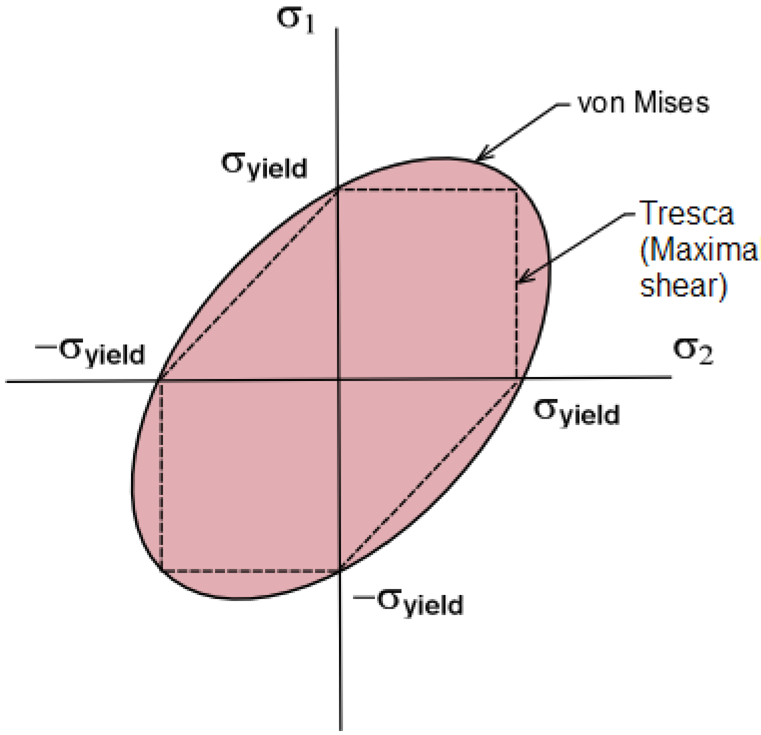
For systems that obey Hooke's Law, the extension produced is directly proportional to the load:

$$\vec{\mathbf{F}} = -k\vec{\mathbf{x}}$$

where

> x is the distance that the spring has been stretched or compressed away from the equilibrium position, which is the position where the spring would naturally come to rest [usually in meters],
> F is the restoring force exerted by the material [usually in newtons], and
> k is the **force constant** (or **spring constant**). This is the stiffness of the spring. The constant has units of force per unit length (usually in newtons per metre)

Plasticity



Comparison of Tresca and Von Mises Criteria

Some design is based on the assumption that materials will behave plastically. A plastic material is one which does not obey Hooke's Law, and therefore deformation is not proportional to the applied load. Plastic materials are ductile materials. Plasticity theory can be used for some reinforced concrete structures assuming they are underreinforced, meaning that the steel reinforcement fails before the concrete does.

Plasticity theory states that the point at which a structure collapses (reaches yield) lies between an upper and a lower bound on the load, defined as follows:

- If, for a given external load, it is possible to find a distribution of moments that satisfies equilibrium requirements, with the moment not exceeding the yield moment at any location, and if the boundary conditions are satisfied, then the given load is a **lower bound** on the collapse load.

- If, for a small increment of displacement, the internal work done by the structure, assuming that the moment at every plastic hinge is equal to the yield moment and that the boundary conditions are satisfied, is equal to the external work done by the given load for that same small increment of displacement, then that load is an **upper bound** on the collapse load.

If the correct collapse load is found, the two methods will give the same result for the collapse load.

Plasticity theory depends upon a correct understanding of when yield will occur. A number of different models for stress distribution and approximations to the yield surface of plastic materials exist:

- Mohr's circle
- Von Mises yield criterion
- Henri Tresca

The Euler-Bernoulli beam equation

The Euler-Bernoulli beam equation defines the behaviour of a beam element (see below). It is based on five assumptions:

(1) continuum mechanics is valid for a bending beam
(2) the stress at a cross section varies linearly in the direction of bending, and is zero at the centroid of every cross section.
(3) the bending moment at a particular cross section varies linearly with the second derivative of the deflected shape at that location.
(4) the beam is composed of an isotropic material
(5) the applied load is orthogonal to the beam's neutral axis and acts in a unique plane.

A simplified version of Euler-Bernoulli beam equation is:

$$EI\frac{d^4u}{dx^4} = w(x).$$

Here *u* is the deflection and *w*(*x*) is a load per unit length. *E* is the elastic modulus and *I* is the second moment of area, the product of these giving the stiffness of the beam.

This equation is very common in engineering practice: it describes the deflection of a uniform, static beam.

Successive derivatives of u have important meaning:

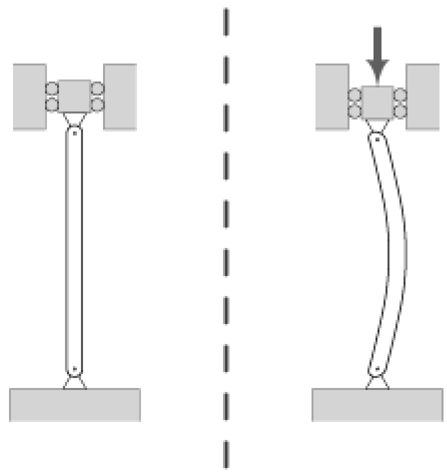
- u is the deflection.
- $\frac{\partial u}{\partial x}$ is the slope of the beam.
- $EI\frac{\partial^2 u}{\partial x^2}$ is the bending moment in the beam.
- $-\frac{\partial}{\partial x}\left(EI\frac{\partial^2 u}{\partial x^2}\right)$ is the shear force in the beam.

A bending moment manifests itself as a tension and a compression force, acting as a couple in a beam. The stresses caused by these forces can be represented by:

$$\sigma = \frac{My}{I} = Ey\frac{\partial^2 u}{\partial x^2}$$

where σ is the stress, *M* is the bending moment, *y* is the distance from the neutral axis of the beam to the point under consideration and *I* is the second moment of area. Often the equation is simplified to the moment divided by the section modulus (S), which is I/y. This equation allows a structural engineer to assess the stress in a structural element when subjected to a bending moment.

Buckling



A column under a centric axial load exhibiting the characteristic deformation of buckling

Lateral-torsional buckling of an aluminium alloy plate girder designed and built by students at Imperial College London.

When subjected to compressive forces it is possible for structural elements to deform significantly due to the destabilising effect of that load. The effect can be initiated or exacerbated by possible inaccuracies in manufacture or construction.

The Euler buckling formula defines the axial compression force which will cause a strut (or column) to fail in buckling.

$$F = \frac{\pi^2 EI}{(Kl)^2}$$

where

F = maximum or critical force (vertical load on column),
E = modulus of elasticity,
I = area moment of inertia, or second moment of area
l = unsupported length of column,
K = column effective length factor, whose value depends on the conditions of end support of the column, as follows.

For both ends pinned (hinged, free to rotate), $K = 1.0$.
For both ends fixed, $K = 0.50$.
For one end fixed and the other end pinned, $K \approx 0.70$.
For one end fixed and the other end free to move laterally, $K = 2.0$.

This value is sometimes expressed for design purposes as a critical buckling stress.

$$\sigma = \frac{\pi^2 E}{(\frac{Kl}{r})^2}$$

where

σ = maximum or critical stress
r = the least radius of gyration of the cross section

Other forms of buckling include lateral torsional buckling, where the compression flange of a beam in bending will buckle, and buckling of plate elements in plate girders due to compression in the plane of the plate.

Chapter- 10

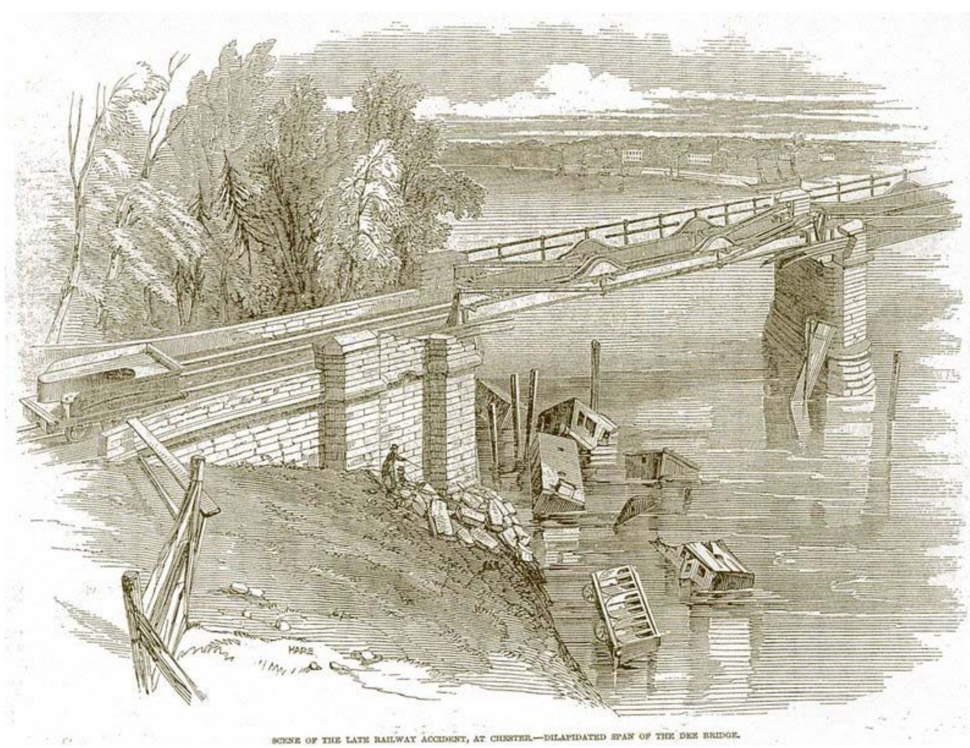
Structural Failure

Structural failure refers to loss of the load-carrying capacity of a component or member within a structure or of the structure itself. Structural failure is initiated when the material is stressed to its strength limit, thus causing fracture or excessive deformations. In a well-designed system, a localized failure should not cause immediate or even progressive collapse of the entire structure. Ultimate failure strength is one of the limit states that must be accounted for in structural engineering and structural design.

Notable failures

Bridges

Dee bridge



The Dee bridge after its collapse

On 24 May 1847 the new railway bridge over the river Dee collapsed as a train passed over it, with the loss of 5 lives. It was designed by Robert Stephenson, using cast iron girders reinforced with wrought iron struts. The bridge collapse was the subject of one of the first formal inquiries into a structural failure. The result of the inquiry was that the design of the structure was fundamentally flawed, as the wrought iron did not reinforce the cast iron at all, and due to repeated flexing it suffered a brittle failure due to fatigue.

First Tay Rail Bridge

The Dee bridge disaster was followed by a number of cast iron bridge collapses, including the collapse of the first **Tay Rail Bridge** on 28 December 1879. Like the Dee bridge, the Tay collapsed when a train passed over it causing 75 people to lose their lives. The bridge failed because of poorly made cast iron, and the failure of the designer Thomas Bouch to consider wind loading on the bridge. The collapse resulted in cast iron largely being replaced by steel construction, and a complete redesign in 1890 of the Forth Railway Bridge. As a result, the Forth Bridge was the first entirely steel bridge in the world.

First Tacoma Narrows Bridge

Tacoma Narrows Bridge collapsing

The 1940 collapse of Tacoma Narrows Bridge (1940), as the original Tacoma Narrows Bridge is known, is sometimes characterized in physics textbooks as a classical example of resonance; although, this description is misleading. The catastrophic vibrations that destroyed the bridge were not due to simple mechanical resonance, but to a more complicated oscillation between the bridge and winds passing through it, known as aeroelastic flutter. Robert H. Scanlan, father of the field of bridge aerodynamics, wrote an article about this misunderstanding. This collapse, and the research that followed, led to an increased understanding of wind/structure interactions. Several bridges were altered following the collapse to prevent a similar event occurring again. The only fatality was 'Tubby' the dog.

I-35W Bridge

The I-35W Mississippi River bridge (officially known simply as Bridge 9340) was an eight-lane steel truss arch bridge that carried Interstate 35W across the Mississippi River in Minneapolis, Minnesota, United States. The bridge was completed in 1967, and its maintenance was performed by the Minnesota Department of Transportation. The bridge was Minnesota's fifth–busiest, carrying 140,000 vehicles daily. The bridge catastrophically failed during the evening rush hour on 1 August 2007, collapsing to the river and riverbanks beneath. Thirteen people were killed and 145 were injured. Following the collapse The Federal Highway Administration (FHWA)advised states to inspect the 700 U.S. bridges of similar construction after a possible design flaw in the bridge was discovered, related to large steel sheets called gusset plates which were used to connect girders together in the truss structure. Officials expressed concern about many other bridges in the United States sharing the same design and raised questions as to why such a flaw would not have been discovered in over 40 years of inspections.

Buildings

Ronan Point

On 16 May 1968 the 22 storey residential tower Ronan Point in the London Borough of Newham collapsed when a relatively small gas explosion on the 18th floor caused a structural wall panel to be blown away from the building. The tower was constructed of precast concrete, and the failure of the single panel caused one entire corner of the building to collapse. The panel was able to be blown out because there was insufficient reinforcement steel passing between the panels. This also meant that the loads carried by the panel could not be redistributed to other adjacent panels, because there was no route for the forces to follow. As a result of the collapse, building regulations were overhauled to prevent "disproportionate collapse" and the understanding of precast concrete detailing was greatly advanced. Many similar buildings were altered or demolished as a result of the collapse.

Oklahoma City bombing

On 19 April 1995, the nine story concrete framed Alfred P. Murrah Federal Building in Oklahoma was struck by a huge car bomb causing partial collapse, resulting in the deaths of 168 people. The bomb, though large, caused a significantly disproportionate collapse of the structure. The bomb blew all the glass off the front of the building and completely shattered a ground floor reinforced concrete column. At second story level a wider column spacing existed, and loads from upper story columns were transferred into fewer columns below by girders at second floor level. The removal of one of the lower story columns caused neighbouring columns to fail due to the extra load, eventually leading to the complete collapse of the central portion of the building. The bombing was one of the first to highlight the extreme forces that blast loading from terrorism can exert on buildings, and led to increased consideration of terrorism in structural design of buildings.

9/11

In the September 11 attacks, two commercial airliners were deliberately crashed into the Twin Towers of the World Trade Center in New York City. The impact and resulting fires caused both towers to collapse within two hours. After the impacts had severed exterior columns and damaged core columns, the loads on these columns were redistributed. The hat trusses at the top of each building played a significant role in this redistribution of the loads in the structure. The impacts dislodged some of the fireproofing from the steel, increasing its exposure to the heat of the fires. Temperatures became high enough to weaken the core columns to the point of creep and plastic deformation under the weight of higher floors. Perimeter columns and floors were also weakened by the heat of the fires, causing the floors to sag and exerting an inward force on exterior walls of the building.

Aircraft

A 1964 B-52 Stratofortress test demonstrated the same failure that caused the 1963 Elephant Mountain & 1964 Savage Mountain crashes.

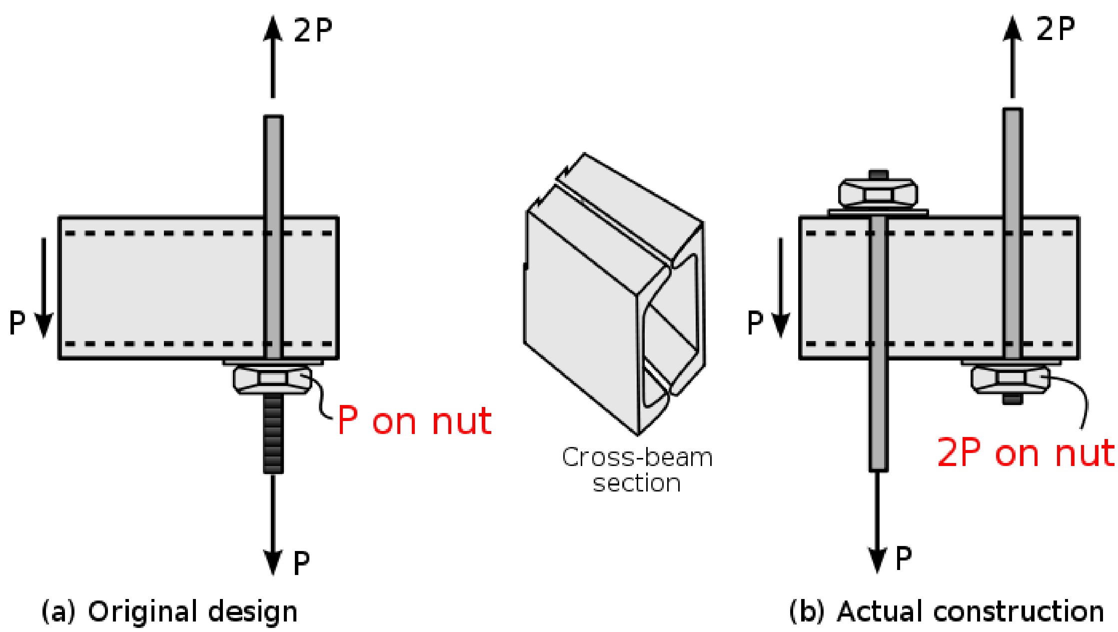
Repeated structural failures of aircraft types occurred in 1954, when 2 de Havilland Comet C1 jet airliners crashed due to decompression caused by metal fatigue, and in 1963-4, when the vertical stabilizer on 4 Boeing B-52 bombers broke off in mid-air.

Other

Warsaw Radio Mast

On 8 August 1991 at 16:00 UTC Warsaw radio mast, the tallest man-made object ever built before the erection of Burj Khalifa collapsed as consequence of an error in exchanging the guy-wires on the highest stock. The mast first bent and then snapped at roughly half its height. It destroyed at its collapse a small mobile crane of Mostostal Zabrze. As all workers left the mast before the exchange procedures, there were no fatalities, in contrast to the similar collapse of WLBT Tower in 1997.

Hyatt Regency walkway



Design change on the Hyatt Regency walkways

On 17 July 1981, two suspended walkways through the lobby of the Hyatt Regency in Kansas City, Missouri, collapsed, killing 115 people at a tea dance. The collapse was due to a late change in design, altering the method in which the rods supporting the walkways were connected to them, and inadvertently doubling the forces on the connection. The failure highlighted the need for good communication between design engineers and contractors, and rigorous checks on designs and especially on contractor-proposed design changes. The failure is a standard case study on engineering courses around the world, and is used to teach the importance of ethics in engineering.