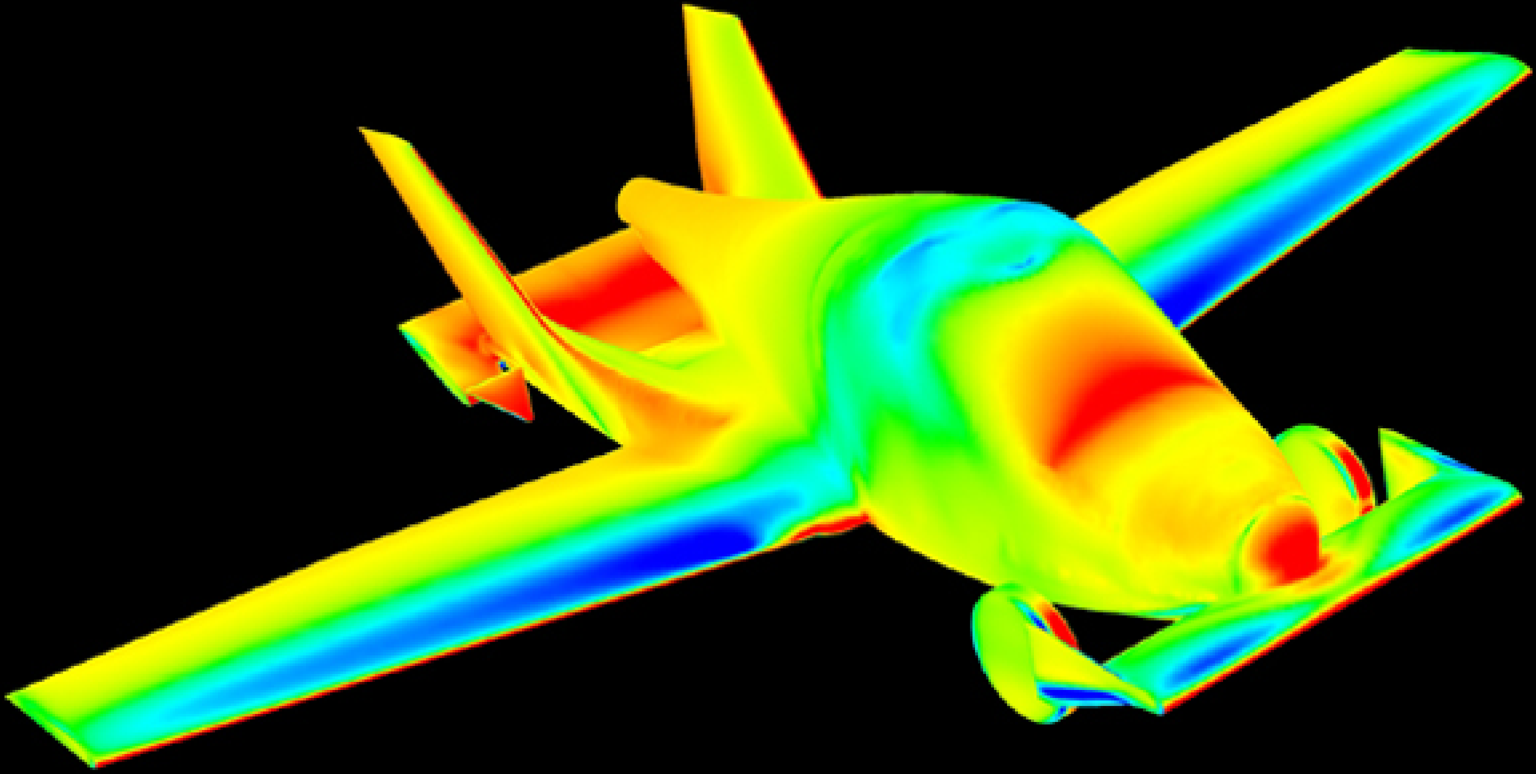
# Aeronautics & Aircraft Flight Dynamics



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# Chapter 1

# Aircraft Dynamic Modes and Aeronautical Chart

## Aircraft dynamic modes

The dynamic stability of an aircraft is how the motion of an aircraft behaves after it has been disturbed from steady non-oscillating flight.

### *Longitudinal modes*

Oscillating motions can be described by two parameters, the period of time required for one complete oscillation, and the time required to damp to half-amplitude, or the time to double the amplitude for a dynamically unstable motion. The longitudinal motion consists of two distinct oscillations, a long-period oscillation called a phugoid mode and a short-period oscillation referred to as the short-period mode.

### Phugoid (longer period) oscillations

The longer period mode, called the "phugoid mode" is the one in which there is a large-amplitude variation of air-speed, pitch angle, and altitude, but almost no angle-of-attack variation. The phugoid oscillation is really a slow interchange of kinetic energy (velocity) and potential energy (height) about some equilibrium energy level as the aircraft attempts to re-establish the equilibrium level-flight condition from which it had been disturbed. The motion is so slow that the effects of inertia forces and damping forces are very low. Although the damping is very weak, the period is so long that the pilot usually corrects for this motion without being aware that the oscillation even exists. Typically the period is 20–60 seconds.

### Short period oscillations

With no special name, the shorter period mode is called simply the "short-period mode". The short-period mode is a usually heavily damped oscillation with a period of only a few seconds. The motion is a rapid pitching of the aircraft about the center of gravity. The period is so short that the speed does not have time to change, so the oscillation is essentially an angle-of-attack variation. The time to damp the amplitude to one-half of its value is usually on the order of 1 second. Ability to quickly self damp when the stick is briefly displaced is one of the many criteria for general aircraft certification.

## *Lateral-directional modes*

"Lateral-directional" modes involve rolling motions and yawing motions. Motions in one of these axes almost always couples into the other so the modes are generally discussed as the "Lateral-Directional modes".

There are three types of possible lateral-directional dynamic motion: roll subsidence mode, Dutch roll mode, and spiral mode.

### Roll subsidence mode

Roll subsidence mode is simply the damping of rolling motion. There is no direct aerodynamic moment created tending to directly restore wings-level, i.e. there is no returning "spring force/moment" proportional to roll angle. However, there is a damping moment (proportional to roll *rate*) created by the slewing-about of long wings. This prevents large roll rates from building up when roll-control inputs are made or it damps the roll *rate* (not the angle) to zero when there are no roll-control inputs.

Roll mode can be improved by adding dihedral effects to the aircraft design, such as high wings, dihedral angles or sweep angles.

### Spiral mode

If a spirally unstable aircraft, through the action of a gust or other disturbance, gets a small initial roll angle to the right, for example, a gentle sideslip to the right is produced. The sideslip causes a yawing moment to the right. If the dihedral stability is low, and yaw damping is small, the directional stability keeps turning the aircraft while the continuing bank angle maintains the sideslip and the yaw angle. This spiral gets continuously steeper and tighter until finally, if the motion is not checked, a steep, high-speed spiral dive results. The motion develops so gradually, however that it is usually corrected unconsciously by the pilot, who may not be aware that spiral instability exists. If the pilot cannot see the horizon, for instance because of clouds, he might not notice that he is slowly going into the spiral dive, which can lead into the graveyard spiral.

To be spirally stable, an aircraft must have some combination of a sufficiently large dihedral, which increases roll stability, and a sufficiently long vertical tail arm, which increases yaw damping. Increasing the vertical tail area then magnifies the degree of stability or instability.

The spiral dive should not be confused with a spin.

## Detection

While descending turns are commonly performed by pilots as a standard flight manoeuvre, the spiral dive is differentiated from a descending turn owing to its feature of accelerating speed. It is therefore an unstable flight condition, and pilots are trained to recognise its onset and to implement recovery procedures safely and immediately. Without intervention by the pilot, acceleration of the aircraft will lead to structural failure of the airframe, either as a result of excess aerodynamic loading or flight into terrain. Spiral dive training therefore revolves around pilot recognition and recovery.

## Recovery

Spiral dive accidents are typically associated with visual flight (non-instrument flight) in conditions of poor visibility, where the pilot's reference to the visual natural horizon is effectively reduced, or prevented entirely, by such factors as cloud or darkness. The inherent danger of the spiral dive is that the condition, especially at onset, cannot be easily detected by the sensory mechanisms of the human body. The physical forces exerted on an aeroplane during a spiral dive are effectively balanced and the pilot cannot detect the banked attitude of the spiral descent. If the pilot detects acceleration, but fails to detect the banked attitude associated with the spiral descent, a mistaken attempt may be to recovery with mere backpressure (pitch-up inputs) on the control wheel. However, with the lift vector of the aircraft now directed to the centre of the spiral turn, this erred nose-up input simply tightens the spiral condition and increases the rate of acceleration and increases dangerous airframe loading. To successfully recover from a spiral dive, the lift vector must first be redirected upward (relative to the natural horizon) before backpressure is applied to the control column. Since the acceleration can be very rapid, recovery is dependent on the pilot's ability to quickly close the throttle (which is contributing to the acceleration), position the lift vector upward, relative to the Earth's surface before the dive recovery is implemented; any factor that would impede the pilot's external reference to the Earth's surface could delay or prevent recovery. The quick and efficient completion of these tasks is crucial as the aircraft can accelerate through maximum speed limits within only a few seconds, where the structural integrity of the airframe will be compromised.

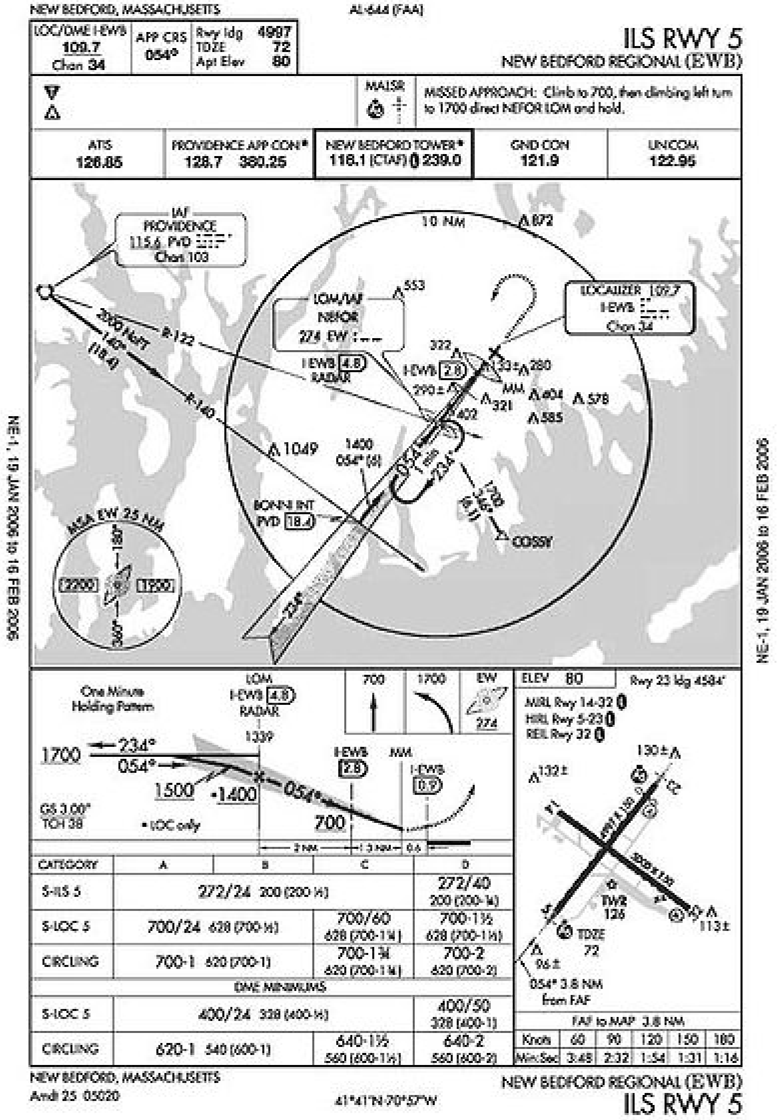
For the purpose of flight training, instructors typically establish the aircraft in a descending turn with initially slow but steadily accelerating airspeed – the initial slow speed facilitates the potentially slow and sometimes erred response of student pilots. The cockpit controls are released by the instructor and the student is instructed to recover. It is not uncommon for a spiral dive to result from an unsuccessful attempt to enter a spin, but the extreme nose-down attitude of the aircraft during the spin-spiral transition makes this method of entry ineffective for training purposes as there is little room to permit student error or delay.

All spiral dive recoveries entail the same recovery sequence: first, the throttle must be immediately closed; second, the aircraft is rolled level with co-ordinated use of ailerons and rudder; and third, backpressure is exerted smoothly on the control wheel to recover from the dive.

## Dutch roll

The second lateral motion is an oscillatory combined roll and yaw motion called Dutch roll, perhaps because of its similarity to an ice-skating motion of the same name made by Dutch skaters; the origin of the name is unclear. The Dutch roll may be described as a yaw and roll to the right, followed by a recovery towards the equilibrium condition, then an overshooting of this condition and a yaw and roll to the left, then back past the equilibrium attitude, and so on. The period is usually on the order of 3–15 seconds, but it can vary from a few seconds for light aircraft to a minute or more for airliners. Damping is increased by large directional stability and small dihedral and decreased by small directional stability and large dihedral. Although usually stable in a normal aircraft, the motion may be so slightly damped that the effect is very unpleasant and undesirable. In swept-back wing aircraft, the Dutch roll is solved by installing a yaw damper, in effect a special-purpose automatic pilot that damps out any yawing oscillation by applying rudder corrections. Some swept-wing aircraft have an unstable Dutch roll. If the Dutch roll is very lightly damped or unstable, the yaw damper becomes a safety requirement, rather than a pilot and passenger convenience. Dual yaw dampers are required and a failed yaw damper is cause for limiting flight to low altitudes, and possibly lower mach numbers, where the Dutch roll stability is improved.

# Aeronautical chart



Example of an Aeronautical chart

An **aeronautical chart** is a map designed to assist in navigation of aircraft, much as nautical charts do for watercraft, or a roadmap for drivers. Using these charts and other tools, pilots are able to determine their position, safe altitude, best route to a destination, navigation aids along the way, alternative landing areas in case of an in-flight emergency, and other useful information such as radio frequencies and airspace boundaries. There are charts for all land masses on Earth, and long-distance charts for trans-oceanic travel.

Specific charts are used for each phase of a flight and may vary from a map of a particular airport facility to an overview of the instrument routes covering an entire continent (e.g., global navigation charts), and many types in between.

## Charts for visual flight rules (VFR)

Under "visual flight rules", pilots are expected to see and avoid dangers along the way (obstacles, other aircraft, bad weather etc.), and to use pilotage and other means for navigating. VFR charts include a large amount of information describing the local topography, with an emphasis on elevation of the terrain. Standardized symbols are used to indicate land and water features such as mountains, shorelines and rivers. Roads, towns and other identifiable features, and aeronautical details such as airports, beacons and towers are included.

Visual flight charts are categorized according to their scale, which is proportional to the size of the area covered by one map. The amount of detail is necessarily reduced when larger areas are represented on a map.

- World aeronautical charts (WACs) have a scale of 1:1,000,000 and cover relatively large areas. Outside of WAC coverage, operational navigation charts (ONC) may be used. They use the same scale as WACs, but omit some useful information such as airspace restrictions.
- Sectional charts typically cover a total area of about 340x340 miles, printed on both sides of the map. The scale is 1:500,000.
- VFR Terminal area charts are created with a scale and coverage appropriate for the general vicinity of a large airport (1:250,000). They may depict preferred VFR flight routes within areas of congested airspace.

## Charts for instrument flight rules (IFR)

Instrument flight requires the use of external aids to navigation, under the control of an air traffic controller, usually based upon a flight plan. Charts used for IFR flights contain an abundance of information regarding locations of waypoints, known as "fixes", which are defined by measurements from electronic beacons of various types, as well as the routes connecting these waypoints. Only limited topographic information is found on IFR charts, although the minimum safe altitudes available on the routes are shown.

En-route low- and high-altitude charts are published with a scale that depends upon the density of navigation information required in the vicinity.

Information from IFR charts is often programmed into a flight management system or autopilot, which eases the task of following (or deviating from) a flight plan.

Terminal procedure publications such as Standard Terminal Arrival plates, Standard Instrument Departure plates and other documentation provide detailed information for arrival, departure and taxiing at each approved airport having instrument capabilities of some sort.

### Sources for charts

Aeronautical charts may be purchased at fixed base operators (FBOs), internet supply sources, or catalogs of aeronautical gear. They may also be viewed online from the FAA.

## Chapter 2

# Load Factor (Aeronautics) and Relative Wind

## Load factor

In aeronautics, the **load factor** is defined as the ratio of the lift of an aircraft to its weight and represents a global measure of the stress ("load") to which the structure of the aircraft is subjected:

$$n = \frac{L}{W}$$

where:

$n$ = Load factor
$L$ = Lift
$W$ = Weight

Since the load factor is the ratio of two forces, it is dimensionless. However, its units are traditionally referred to as **g**, because of the relation between load factor and apparent acceleration of gravity felt on board the aircraft. A load factor of one, or 1 g, represents conditions in straight and level flight, where the lift is equal to the weight. Load factors greater or less than one (or even negative) are the result of maneuvers or wind gusts.

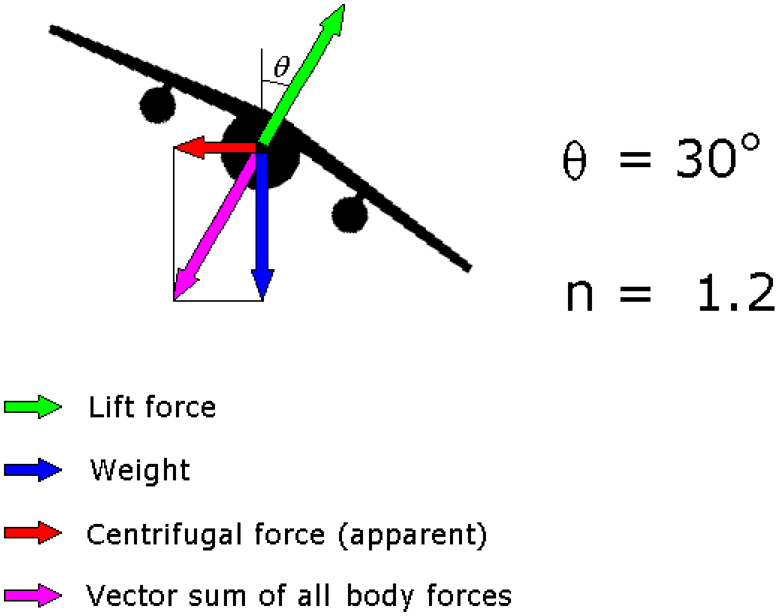
### *Load factor and g*

The fact that the load factor is commonly expressed in *g* units does not mean that it is dimensionally the same as the acceleration of gravity, also indicated with *g*. The load factor is strictly non-dimensional.

The use of *g* units refers to the fact that an observer on board an aircraft will experience an *apparent* acceleration of gravity (i.e. relative to his frame of reference) equal to load factor times the acceleration of gravity. For example, an observer on board an aircraft performing a turn with a load factor of 2 (i.e. a 2 g turn) will see objects falling to the floor at twice the normal acceleration of gravity.

In general, whenever the term *load factor* is used, it is formally correct to express it using numbers only, as in "a maximum load factor of 4". If the term *load factor* is omitted then *g* is used instead, as in "pulling a 3 g turn".

## *Positive and negative load factors*



Variation of the load factor *n* with the bank angle *θ*, during a coordinated turn.

The load factor, and in particular its sign, depends not only on the forces acting on the aircraft, but also on the orientation of its vertical axis.

During straight and level flight, the load factor is +1 if the aircraft is flown "the right way up", whereas it becomes -1 if the aircraft is flown "upside-down" (inverted). In both cases the lift vector is the same (as seen by an observer on the ground), but in the latter the vertical axis of the aircraft points downwards, making the lift vector's sign negative.

In turning flight the load factor is normally greater than +1. For example, in a turn with a 60° angle of bank the load factor is +2. Again, if the same turn is performed with the aircraft inverted, the load factor becomes -2. In general, in a balanced turn in which the angle of bank is *θ*, the load factor *n* is related to the cosine of *θ* by the formula:

$$n = \frac{1}{\cos\theta}$$

Another way to achieve load factors significantly higher than +1 is to pull on the elevator control at the bottom of a dive, whereas strongly pushing the stick forward during straight and level flight is likely to produce negative load factors, by causing the lift to act in the opposite direction to normal, i.e. downwards.

## *Load factor and lift*

In the definition of load factor, the lift is not simply that one generated by the aircraft's wing, instead it is the vector sum of the lift generated by the wing, by the fuselage and by the tailplane, or in other words it is the component perpendicular to the airflow of the sum of all aerodynamic forces acting on the aircraft.

The lift in the load factor is also intended as having a sign, which is positive if the lift vector points in the same direction, or close to, as the aircraft's vertical axis, or negative if it points in the opposite direction, or close to opposite, to the vertical axis.

## *Design standards*

Excessive load factors must be avoided because of the possibility of exceeding the structural strength of the aircraft.

Aviation authorities specify the load factor limits within which different classes of aircraft are required to operate without damage. For example, the US Federal Aviation Regulations prescribe the following limits (for the most restrictive case):

- For commercial transport airplanes, from -1 to +2.5 (or up to +3.8 depending on design takeoff weight)
- For light airplanes, from -1.5 to +3.8
- For aerobatic airplanes, from -3 to +6
- For helicopters, from -1 to +3.5

However, many aircraft types, in particular aerobatic airplanes, are designed so that they can tolerate load factors much higher than the minimum required. For example, the Sukhoi Su-26 family have load factors limits of -10 to +12.

The maximum load factors, both positive and negative, applicable to an aircraft are usually specified in the pilot's operating handbook.

## *Human perception of load factor*

When the load factor is +1, all occupants of the aircraft feel that their weight is normal. When the load factor is greater than +1 all occupants feel heavier than usual. For example, in a 2 g maneuver all occupants feel that their weight is twice normal. When the load factor is zero, or very small, all occupants feel weightless. When the load factor is negative, all occupants feel they are upside down.

Human beings have limited ability to withstand a load factor significantly greater than +1, both positive and negative. Unmanned aerial vehicles can be designed for much greater load factors, both positive and negative, than conventional aircraft because these vehicles can be used in maneuvers which would be incapacitating for a human pilot.

# Relative wind

In aeronautics, the **relative wind** is the direction of movement of the atmosphere relative to an aircraft or an airfoil. It is opposite to the direction of movement of the aircraft or airfoil relative to the atmosphere. Close to any point on the surface of an aircraft or airfoil, the air is moving parallel to the surface; but at a great distance from the aircraft or airfoil the movement of the air can be represented by a single vector. This vector is the relative wind or the *free stream velocity vector*.

The angle between the chord line of an airfoil and the relative wind defines the angle of attack. The relative wind is of great importance to pilots because exceeding the critical angle of attack will result in a stall, regardless of airspeed.

## *Relative wind in freefall*

Relative wind is also used to describe the airflow relative to an object in freefall through an atmosphere, such as that of a person's body during the freefall portion of a skydive or BASE jump. In a normal skydive the vertical descent of the skydiver creates an upward relative wind. The relative wind strength increases with increased descent rate.

The relative wind is directly opposite to the direction of travel.

Therefore, when a skydiver exits a forward-moving aircraft such as an aeroplane, the relative wind emanates from the direction the aeroplane is facing due to the skydiver's initial forward (horizontal) momentum. As aerodynamic drag gradually overcomes this forward momentum and, simultaneously, gravity attracts the skydiver downward, the relative wind alters proportionally into an upward (vertical) direction. This creates an arc of travel for the skydiver similar to water flowing from a low pressure hose held horizontally and creates a variation in the angle of the relative wind from horizontal to vertical.

When exiting from a forward-moving aircraft (as distinguished from a hovering aircraft, such as a balloon or a helicopter in hover mode) during a normal belly-to-earth skydive, the skydiver must arch his body in the direction of travel which is initially horizontal. If the skydiver continues to arch, his belly will gradually alter pitch until he is belly-to-earth. This section of the jump is commonly referred to as "the hill".

Relative wind differs from the wind in meteorology in that the object (*e.g.*, the skydiver) moves past the air, as opposed to the air moving past the object

## Chapter 3

# Hybrid Airship and Lift (Soaring)

## Hybrid airship

A **hybrid airship** is an aircraft that combines characteristics of heavier-than-air, (HTA), (fixed-wing aircraft or helicopter) and lighter than air (LTA), aerostat technology. Examples include helicopter/airship hybrids intended for heavy lift applications and dynamic lift airships intended for long-range cruising. No production vehicles have been built, but several manned and unmanned prototypes have flown and successfully demonstrated the concept.

The term has also been used to describe an airship combining elements of different types of airships.

### *Background*

Traditional airships have low operating costs but are limited in several ways, including low payload/volume ratios and low speeds. Additionally, ground handling of airships has historically presented great difficulty. When a purely LTA ship comes in for a landing, it is nearly neutrally buoyant and is very susceptible to wind buffeting. In even a slight breeze, a truck or many ground crew members are required to secure the ship to a mooring mast.

Heavier-than-air aircraft, while addressing these difficulties, require the use of power to generate lift, and airplanes also require runways, while helicopters need even more power to hover. Hybrid airship designs are intended to fill the middle ground between the low operating cost and low speeds of traditional airships and higher speed, but more expensive heavier-than-air aircraft. In addition, by combining dynamic and buoyant lift, hybrids may be able to provide otherwise unattainable air-cargo payload capacity and/or a hovering capability. Such a design is intended to be the "best of both worlds" combination: the high speed of aerodynamic craft and the lifting capacity of aerostatic craft. However, critics of the hybrid approach have labeled it as being the "worst of both worlds" declaring that such craft require a runway for take-off and landing, are difficult to control and protect on the ground, and have relatively poor aerodynamic performance.

Most modern airships, for instance the Zeppelin NT or Skyship 600 use some combination of vectored thrust and buoyancy. However, for these designs, almost all of the load is carried via buoyancy and vectored thrust is used primarily for maneuvering. To date, there is no formal distinction between hybrid airships and airships with vectored thrust.

### Concept

The idea behind the hybrid airship is to combine lift from a lighter-than-air gas such as helium with lift from aerodynamic forces. Such a craft is still heavier than air, which makes it similar in some ways to a regular aircraft. The rest of the lift is comes from vertical thrusters such as helicopter-like rotors, or a lift-producing shape (like a wing) combined with horizontal thrust, or a combination of the two. The aerodynamic approach is very similar to that of a conventional lifting body aircraft. The hybrid aircraft technology has a wide range of flight performance behaviors ranging from heavier than air to near buoyant characterizations. This uncommon dynamic flight range when coupled with an air cushion landing system has reinvigorated the LTA community and those seeking ultra heavy and affordable airlift transportation options.

### History

Alberto Santos-Dumont's combined aircraft/dirigible experiments of 1906

No hybrid aircraft design has ever been developed past the initial experimental stages despite many such designs having been proposed over the years, though recent advances may indicate that the technology has matured.

In 1905, Alberto Santos-Dumont made what is likely the first attempt at a hybrid aircraft. His *Number 14* combined an airship envelope with an airplane frame. At that time, Santos-Dumont was the world's most accomplished aviator. All of his previous flights had been made in purely aerostatically lifted airships. The Number 14 proved unworkable. Later, Santos-Dumont would remove the envelope and successfully use the recristened *14-bis* (meaning "14-again") to make the first public flight of any heavier-than-air aircraft in the world.

The 1986 Piasecki PA-97 Helistat combined four helicopters with a blimp in an attempt to create a heavy-lift vehicle for forestry work.

One hybrid aircraft design that flew was the Aereon 26; however, this was a small-scale prototype and derived all its lift aerodynamically, none from lighter-than-air gas. The development of this aircraft was documented in the book "The Deltoid Pumpkin Seed" by John McPhee.

The SkyCat or "Sky Catamaran" vehicular technology is a hybrid aircraft amalgamation; a scale version at 12 meters called "SkyKitten", built by the now defunct Advanced Technologies Group, flew in 2000.

The U.S. Defense Advanced Research Projects Agency, DARPA, initiated the WALRUS program in 2005, a technology development initiative focused on ultra heavy air lift technology explorations. The program was terminated in 2007.

In 2006, Lockheed Martin's P-791 manned flight test of the SkyCat technology indicated substantial progress of the technology, and presently several development efforts are underway.

## *Current and proposed designs*

The Hybrid Aircraft Corporation has trademarked the SkyCat and SkyFreighter (cargo variant) names for such vehicles and is involved in design and development efforts, and has a working prototype HAV-3.

The Aeroscraft, a design proposed by Worldwide Aeros Corp is also a hybrid airship that uses a lifting body shape, vectored thrust, as well as buoyancy control. Aeros was a beneficiary of the WALRUS program.

World SkyCat Ltd, in Britain, is also pursuing a design in the heritage of the SkyKitten.

The Millennium Airship Corporation has Patented their ITAMMS thrust management system and are currently developing a heavy hybrid lift system.

**Hybrid Air Vehicles** from Cranfield in England and partner Northrop Grumman won the 517 million dollar LEMV contract and are building three HAV 304's for the US Army, the first one will fly later this year. As a result of the development work that the HAV design team have undertaken over the past 25 years, combined with the very significant LEMV contract, they are now regarded as the leading company in this sector.

# Lift (soaring)

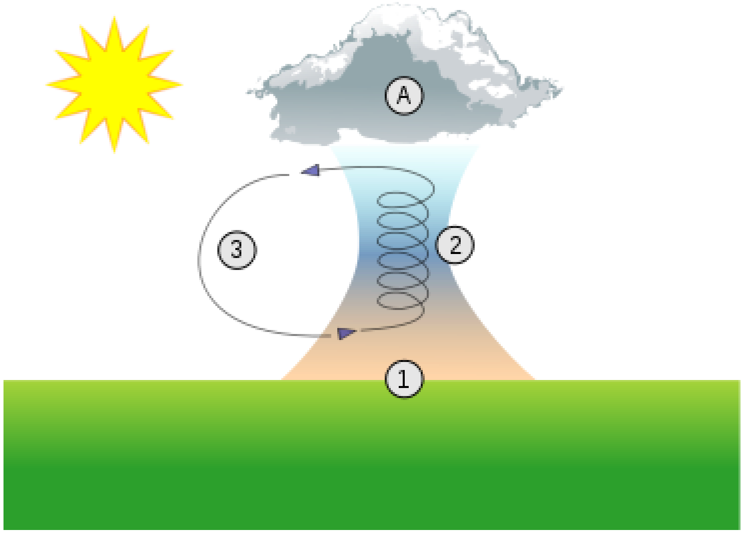
**Lift** is a meteorological phenomenon used as an energy source by soaring aircraft and soaring birds. The most common human application of lift is in sport and recreation. The three air sports that use soaring flight are: gliding, hang gliding and paragliding.

Energy can be gained by using rising air from four sources:

- Thermals (where air rises due to heat),
- Ridge lift, where air is forced upwards by a slope,
- Wave lift, where a mountain produces a standing wave,
- Convergence, where two air masses meet

In dynamic soaring it is also possible to gain energy, though this uses differences in wind speeds rather than rising air.

## *Thermals*



Example of a thermal column between the ground and a cumulus

Thermals are streams of rising air that are formed on the ground through the warming of the surface by sunlight. If the air contains enough moisture, the water will condense from the rising air and form cumulus clouds.

Thermal lift is often used by birds, such as raptors, vultures and storks. Although thermal lift was known to the Wright Brothers in 1901, it was not exploited by humans until 1921 by William Leusch at the Wasserkuppe in Germany. It was not until about 1930 that the use of thermals for soaring in gliders became commonplace.

Once a thermal is encountered, the pilot flies in circles to keep within the thermal, so gaining altitude before flying off to the next thermal and towards the destination. This is known as "thermalling". Climb rates depend on conditions, but rates of several meters per second are common. Thermals can also be formed in a line usually because of the wind or the terrain, creating cloud streets. These can allow flying straight while climbing in continuous lift.

When the air has little moisture or when an inversion stops the warm air from rising high enough for the moisture to condense, thermals do not create cumulus clouds. Typical locations to find thermals are over towns, freshly ploughed fields and asphalt roads, but thermals are often hard to associate with any feature on the ground. Occasionally thermals are caused by the exhaust gases from power stations or by fires.

As it requires rising heated air, thermalling is only effective in mid-latitudes from spring through into late summer. During winter the solar heat can only create weak thermals, but ridge and wave lift can still be used during this period.

A Scimitar glider ridge soaring in Lock Haven, Pennsylvania USA

## *Ridge lift*

Ridge lift, or Orographic lift, is caused by rising air on the windward side of a slope. Ridge lift is used extensively by sea birds and by aircraft. In places where a steady wind blows, a ridge may allow virtually unlimited time aloft.

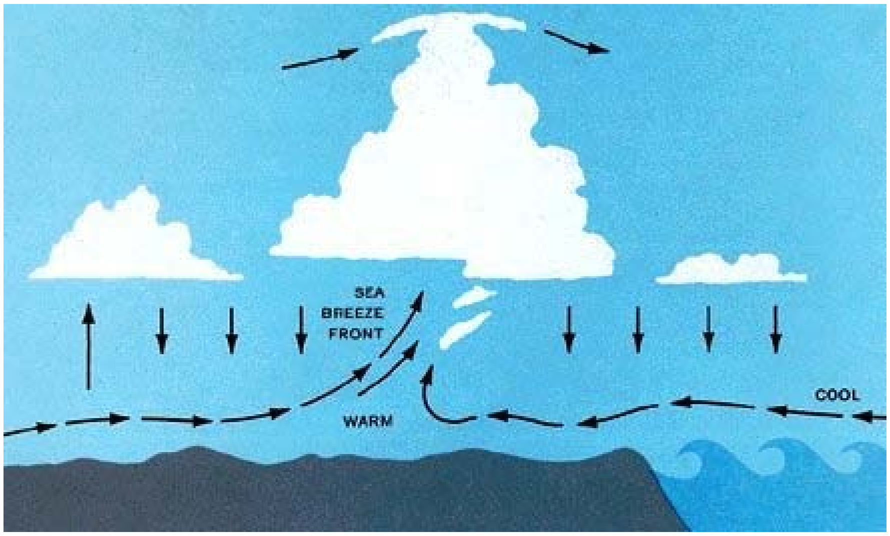
With winds of 20 to 25 knots (46 km/h), it is possible for aircraft to soar at an altitude up to two times the height of the obstacle. Ridge lift can also be augmented by thermals when the slopes also face the sun.

## *Wave lift*

A lenticular cloud produced by a mountain wave

Lee waves occur when a wind of 25 knots (46 km/h) blows over a mountain. Provided that there is a steady increase in wind strength with altitude without a significant change in direction, standing waves may be created. They were discovered by a glider pilot, Wolf Hirth, in 1933. These waves reach heights much greater than the original obstruction and so can permit gliders to climb to the stratosphere. Pilots use supplementary oxygen to avoid hypoxia because gliders do not have pressurized cockpits. This lift is often marked by long, stationary lenticular (lens-shaped) clouds lying perpendicular to the wind. Mountain wave was used to set the current altitude record of 50,699 feet (15,453 m) on August 29, 2006 over El Calafate, Argentina. The pilots were Steve Fossett and Einar Enevoldson, who were wearing pressure suits. The current world distance record of 3,008 km (1,869 statute miles) by Klaus Ohlmann (set on 21 January 2003) was also flown using mountain waves in South America.

A rare wave phenomenon is known as Morning Glory, a roll cloud producing strong lift. Pilots near Australia's Gulf of Carpentaria make use of it in springtime.



Schematic cross section through a sea breeze front. If the air inland is moist, cumulus often marks the front.

Birds have been observed using wave lift to cross mountainous regions.

## *Convergence zones*

The boundaries where two air masses meet are known as convergence zones. These can occur in sea breezes or in desert regions. A **sea-breeze** (or **onshore breeze**) is a wind from the sea that develops over land near coasts. In a sea-breeze front, cold air from the sea meets the warmer air from the land and creates a boundary like a shallow cold front along a shear line. This creates a narrow band of soarable lift with winds as light as 10 knots (19 km/h). These permit the gaining of altitude by flying along the intersection as if it were a ridge of land. Convergence may occur over considerable distances and so may permit virtually straight flight while climbing.

## *Dynamic soaring*

In dynamic soaring energy is gained by repeatedly crossing the boundary between air masses of different horizontal velocity rather than by rising air. Such zones of high "wind gradient" are usually too close to the ground to be used safely by gliders, but Albatrosses and model gliders use this phenomenon.

### *Illusions of lift*

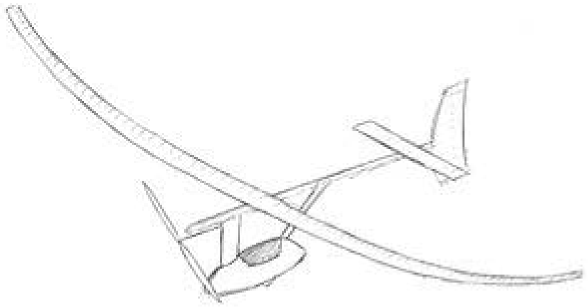
A pilot can create an indication of lift on certain uncompensated instruments by entering a climb by pulling back on the stick (hence “*stick* thermal”). This is not true lift, in that the energy to climb is being converted from decreasing airspeed, rather than being extracted from rising air. Inexperienced pilots can mistake this for actual lift.

## Chapter 4

# PSU Zephyrus and Rib (Aircraft)

## PSU Zephyrus

### PSU Zephyrus



An isometric concept view of the aircraft.

| | |
|---|---|
| **Role** | Human-powered aircraft |
| **Manufacturer** | Penn State |
| **Number built** | 0 (1 in progress) |
| **Unit cost** | US$25,000 |
| **Developed from** | Musculair II |

The **PSU Zephyrus** is a human-powered aircraft being constructed by the Penn State AERSP 404H team. It is a composite material, single-seat, single propeller, high-wing airplane. The Zephyrus is designed to compete in the Kremer prize sport competition.

### *Development*

The PSU Zephyrus was developed to compete in the Kremer's prize sport competition. The basic mission goal is to traverse an equilateral triangle with sides of 500 meters once in each direction in seven minutes. The competition specifies a minimum average wind speed of 5.0 m/s during the flight. In addition, for a flight to be considered official, the wind cannot drop below 5.0 m/s for a period of more than 20 seconds. The aircraft is also being developed and constructed as a fulfillment of the course requirements of Penn State's AERSP 404H course.

## *Design*

### Fuselage

The fuselage was sized based a dimension range of a 5’10”(1.78 m) pilot and the assumption that the pilot could output the necessary power to weight ratio to fly the aircraft will be no greater than 1.78 m. Constraints include minimum widths for pilot comfort and desired center of gravity of the aircraft. The shape of the pod was designed to be a low-drag body that will not generate lift regardless of angle of attack. The length of the shape was reduced to allow for aircraft maneuverability in crosswind. The internal structural members are designed to firmly hold the seat configuration in place, yet still provide a maximum field of vision for the pilot. Structural members attach to the main boom at a hard-point located behind the trailing edge of the wing.

### Propeller

To successfully complete the challenge, it was determined that the propeller would need to produce 27.5 N of thrust when cruising at 12.5 m/s and spinning at 135 rpm. To meet these requirements, the propeller design underwent many alterations, with the final design broken into two components.

### Wing

Starting from a parent aircraft approach, primarily using the Musculair 1 and 2, but also including the Monarch B, MIT Daedalus, and Velair models, a first iteration choice for an airfoil was made. A modified version of the FX-76MP, as used for the Musculair 2 was chosen. Taking characteristics from this airfoil, using an initial weight buildup, the wing planform size was determined. Then, assuming a take-off weight of 81 kg (27 kg empty weight), sea-level air density, and using a CL at cruise of 0.8.

### Control Surfaces

Ailerons were designed as a piano hinge attached near the upper surface, so the upper surface incorporates the leading edge radius, so that the upper surface maintains smooth flow. This design maximizes weight efficiency and construction ease.

## *Specifications*

### General characteristics

- **Crew:** 1
- **Length:** 13 ft 1 in (4 m)
- **Wingspan:** 57 ft 5 in (17.5 m)
- **Height:** 9 ft 4 in (3 m)
- **Max takeoff weight:** 206 lb (93.4kg)
- **Powerplant:** 1× Tractor, Propeller ()

**Performance**

- **Cruise speed:** 27.9 mph (12.5 m/s)

# Rib (aircraft)

Wing ribs of a de Havilland DH.60 Moth

In an aircraft, **ribs** are forming elements of the structure of a wing, especially in traditional construction.

By analogy with the anatomical definition of "rib", the ribs attach to the main spar, and by being repeated at frequent intervals, form a skeletal shape for the wing. Usually ribs incorporate the airfoil shape of the wing, and the skin adopts this shape when stretched over the ribs.

## *Type of ribs*

There are several types of **ribs**. Form-ribs, plate-type ribs, truss ribs, closed-ribs, forged ribs and milled ribs, where form-ribs are used for light to medium loading and milled ribs

are as strong as it can get.
Form-ribs are made from a sheet of metal bent into shape, such as a U-profile. This profile is place on the skin, just like a stringer, but then in the other direction.
Plate-type ribs consist of sheet-metal, which has upturned edges and (often has) weight-saving holes cut into it.
Truss ribs are built up out of profiles that are joined together. These joints require great attention during design and manufacture. The ribs may be light or heavy in design which make them suitable for a wide range of loads.
Closed-ribs are constructed from profiles and sheet metal and are suitable for closing off sections of the wing (e.g.: the fuel tank). Here too, particular care must be taken with the joints and this type of rib is also suitable for application in a variety of loading conditions.
Forged ribs are manufactured using heavy press-machinery. The result is fairly rough; for more refined parts, high-pressure presses are required, which are very expensive. Forged pieces (usually) have to undergo further treatment (for smoother edges and holes). Forged ribs are used for sections where very high loads apply - near the undercarriage for example.
Milled ribs are solid structures. They are manufactured by milling away excess material from a solid block of metal (usually using computer-controlled milling machines). The shape of these ribs is always accurately defined. Such ribs are used under similar conditions as those for forged ribs.

Ribs are made out of wood, metal, plastic, composites, foam. The wings of kites, hang gliders , paragliders , powered kites , powered hang gliders, ultralights, windmills  are aircraft that have versions that use ribs to form the wing shape.

For full size and flying model aircraft wing structures that are usually made of wood, ribs can either be in one piece (forming the airfoil at that rib's "station" in the wing), or be in a three-piece format, with the *rib web* being the part that the one-piece rib consisted of, with *capstrips* for the upper and lower edging of the rib, running from the leading edge to the trailing edge, being the other two component parts.

# Chapter 5

# Aircraft Flight Mechanics and Radio Direction Finder

## Aircraft flight mechanics

In aeronautics, **aircraft flight mechanics** is the study of the forces that act on an aircraft in flight, and the way the aircraft responds to those forces.

**Aircraft flight mechanics** are relevant to gliders, helicopters and aeroplanes.
An Aeroplane (Airplane in US usage), is defined as: *a power-driven heavier than air aircraft, deriving its lift chiefly from aerodynamic reactions on surface which remain fixed under given conditions of flight*. (ICAO Document 9110)

### *Straight and level flight of aircraft*

In flight, an aircraft can be considered as being acted on by four forces: lift, weight, thrust, and drag. Thrust is the force generated by the engine and acts along the engine's thrust vector. Lift acts perpendicular to the vector representing the aircraft's velocity relative to the atmosphere. Drag acts parallel to the aircraft's velocity vector, but in the opposite direction because drag resists motion through the air. Weight acts through the aircraft's centre of gravity, towards the centre of the Earth.

In straight and level flight, lift is approximately equal to weight. In addition, if the aircraft is not accelerating, thrust is approximately equal to drag.

In straight, climbing flight, lift is less than weight. At first, this seems incorrect because if an aircraft is climbing it seems lift must exceed weight. When an aircraft is climbing at constant speed it is its thrust that enables it to climb and gain extra potential energy. Lift acts perpendicular to the vector representing the velocity of the aircraft relative to the atmosphere, so lift is unable to alter the aircraft's potential energy or kinetic energy. This can be seen by considering an aerobatic aircraft in straight vertical flight - one that is climbing straight upwards (or descending straight downwards). Vertical flight requires no lift! When flying straight upwards the aircraft can reach zero airspeed before falling earthwards - the wing is generating no lift and so does not stall. In straight, climbing flight at constant airspeed, thrust exceeds drag.

In straight, descending flight, lift is less than weight. In addition, if the aircraft is not accelerating, thrust is less than drag. In turning flight, lift exceeds weight and produces a load factor greater than one, determined by the aircraft's angle of bank.

## *Aircraft control and movement*

There are three primary ways for an aircraft to change its orientation relative to the passing air. *Pitch* (movement of the nose up or down), *Roll* (rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) and *Yaw* (movement of the nose to left or right.) Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero.

## *Aircraft control surfaces*

*Yaw* is induced by a moveable rudder, attached to a vertical fin usually at the rear of the aircraft. Sometimes the entire fin is movable. Movement of the rudder changes the size and orientation of the force the vertical surface produces. Since the force is created a distance behind the centre of gravity this sideways force causes a yawing motion. On a large aircraft there may be several independent rudders on the single fin for both safety and to control the inter-linked yaw and roll actions.

Using yaw alone is not a very efficient way of executing a level turn in an aircraft and will result in some sideslip. A precise combination of bank and lift must be generated to cause the required centripetal forces without producing a sideslip.

*Pitch* is controlled by the rear part of the tailplane's horizontal stabilizer being hinged to create an elevator. By moving the elevator control backwards the pilot moves the elevator up (a position of negative camber) and the downwards force on the horizontal tail is increased. The angle of attack on the wings increased so the nose is pitched up and lift is generally increased. In micro-lights and hang gliders the pitch action is reversed - the pitch control system is much simpler so when the pilot moves the elevator control backwards it produces a nose-down pitch and the angle of attack on the wing is reduced.

The system of a fixed tail surface and moveable elevators is standard in subsonic aircraft. Craft capable of supersonic flight often have a stabilator, an all-moving tail surface. Pitch is changed in this case by moving the entire horizontal surface of the tail. This seemingly simple innovation was one of the key technologies that made supersonic flight possible. In early attempts, as pilots exceeded the critical Mach number, a strange phenomenon made their control surfaces useless, and their aircraft uncontrollable. It was determined that as an aircraft approaches the speed of sound, the air approaching the aircraft is compressed and shock waves begin to form at all the leading edges and around the hinge lines of the elevator. These shock waves caused movements of the elevator to cause no pressure change on the stabilizer upstream of the elevator. The problem was solved by changing the stabilizer and hinged elevator to an all-moving stabilizer - the entire horizontal surface of the tail became a one-piece control surface. Also, in supersonic flight the change in camber has less effect on lift and a stabilator produces less drag.

Aircraft that need control at extreme angles of attack are sometimes fitted with a canard configuration, in which pitching movement is created using a forward foreplane (roughly level with the cockpit). Such a system produces an immediate increase in lift and therefore a better response to pitch controls. This system is common in delta-wing aircraft (deltaplane), which use a stabilator-type canard foreplane. A disadvantage to a canard configuration compared to an aft tail is that the wing cannot use as much extension of flaps to increase wing lift at slow speeds due to stall performance. A combination tri-surface aircraft uses both a canard and an aft tail (in addition to the main wing) to achieve advantages of both configurations.

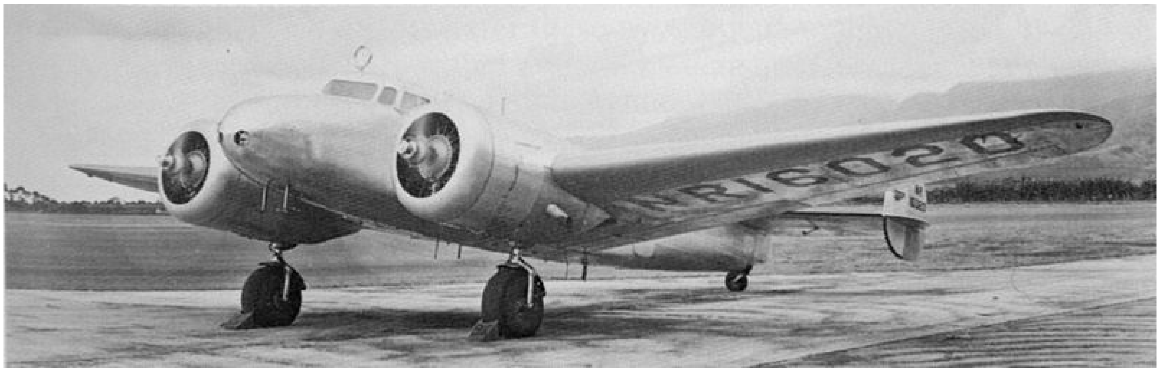
A further design of tailplane is the V-tail, so named because that instead of the standard inverted T or T-tail, there are two vertical fins angled away from each other in a V (if they're arranged like a V, at least one of them isn't vertical). To produce yaw like a rudder, the two trailing edge control surfaces move in the same direction. To produce pitch like an elevator, the surfaces move in opposite directions.

*Roll* is controlled by movable sections on the trailing edge of the wings called ailerons. The ailerons move differentially - one goes up as the other goes down. The difference in camber of the wing cause a difference in lift and thus a rolling movement. As well as ailerons, there are sometimes also spoilers - small hinged plates on the upper surface of the wing, originally used to produce drag to slow the aircraft down and to reduce lift when descending. On modern aircraft, which have the benefit of automation, they can be used in combination with the ailerons to provide roll control.

The earliest powered aircraft built by the Wright brothers did not have ailerons. The whole wing was warped using wires. Wing warping is efficient since there is no discontinuity in the wing geometry. But as speeds increased unintentional warping became a problem and so ailerons were developed.

# Radio direction finder

Civil Air Patrol members practice using a handheld radio direction finder to locate an emergency locator transmitter



Amelia Earhart's Lockheed Model 10 Electra with the circular *RDF* aerial visible above the cockpit

A **radio direction finder** (**RDF**) is a device for finding the direction to a radio source. Due to radio's ability to travel very long distances and "over the horizon", it makes a particularly good navigation system for ships, small boats, and aircraft that might be some distance from their destination.

## *History*

John Stone Stone patented the first direction finding system in 1902 (U.S. Patent 716,134). Alternate and improved direction finding systems were invented by Lee de Forest in 1904 (U.S. Patent 771,819), and by Italian engineers Ettore Bellini and Alessandro Tosi in 1909 (U.S. Patent 943,960). In 1919, British Army Officer Frank Adcock proposed an improved direction finding antenna design Adcock antenna (UK Patent 130,490).

The US Army Air Corps in 1931 tested a primitive radio compass that used commercial stations as the beacon.

### *Operation*

World War II US Navy high frequency radio direction finder

*Radio Direction Finding* works by comparing the signal strength of a directional antenna pointing in different directions. At first, this system was used by land and marine-based radio operators, using a simple rotatable loop antenna linked to a degree indicator. This system was later adopted for both ships and aircraft, and was widely used in the 1930s and 1940s. On pre-World War II aircraft, RDF antennas are easy to identify as the circular loops mounted above or below the fuselage. Later loop antenna designs were enclosed in a aerodynamic, teardrop-shaped fairing. In ships and small boats, RDF receivers first employed large metal loop antennae, similar to aircraft, but usually mounted atop a portable battery-powered receiver.

In use, the RDF operator would first tune the receiver to the correct frequency, then manually turn the loop, either listening or watching an S meter to determine the direction of the *null* (the direction at which a given signal is weakest) of a long wave (LW) or medium wave (AM) broadcast beacon or station (listening for the null is easier than listening for a peak signal, and normally produces a more accurate result). This null was symmetrical, and thus identified both the correct degree heading marked on the radio's compass rose as well as its 180-degree opposite. While this information provided a baseline from the station to the ship or aircraft, the navigator still needed to know beforehand if he was to the east or west of the station in order to avoid plotting a course 180-degrees in the wrong direction. By taking bearings to two or more broadcast stations and plotting the intersecting bearings, the navigator could locate the relative position of his ship or aircraft.

Later, RDF sets were equipped with rotatable ferrite loopstick antennae, which made the sets more portable and less bulky. Some were later partially automated by means of a motorized antenna (ADF). A key breakthrough was the introduction of a secondary vertical whip or 'sense' antenna that substantiated the correct bearing and allowed the navigator to avoid plotting a bearings 180 degrees opposite the actual heading. After World War II, there many small and large firms making direction finding equipment for mariners, including Apelco, Aqua Guide, Bendix, Gladding (and its marine division, Pearce-Simpson), Ray Jefferson, Raytheon, and Sperry. By the 1960s, many of these radios were actually made by Japanese electronics manufacturers, such as Panasonic, Fuji Onkyo, and Koden Electronics Co., Ltd. In aircraft equipment, Bendix and Sperry-Rand were two of the larger manufacturers of RDF radios and navigation instruments.

### *Usage in maritime and aircraft navigation*

The

KOLSTER RADIO COMPASS

*for*

YACHTS and SMALL CRAFT

**Reduction in Price**

HAS a normal range of 50 miles and is as accurate in thick or stormy weather as in clear weather. Suitable for vessels as small as 40-footers.

This radio compass is the last word in navigation aids. KOLSTER radio compasses are used throughout the world by many governments including 8 departments of the U. S. GOVT. and yacht owners everywhere.

*Dr. Kolster operating his yacht type Radio Compass*

Write for details, including prices and the many uses to which this compass can be put.

*Manufactured, sold and serviced by*

FEDERAL TELEGRAPH COMPANY

10700 Helena Ave.
CLEVELAND

625 Market Street
SAN FRANCISCO

Historic advertisement for Kolster radio compass

Radio transmitters for air and sea navigation are known as *beacons* and are the radio equivalent to a lighthouse. The transmitter sends a Morse Code transmission on a Long wave (150 - 400 Khz) or Medium wave (AM) (520 - 1720 Khz) frequency incorporating the station's identifier that is used to confirm the station and its operational status. Since these radio signals are broadcast in all directions (omnidirectional) during the day, the signal itself does not include direction information, and these beacons are therefore referred to as non-directional beacons, or **NDBs**.

As the commercial medium wave (AM) broadcast band lies within the frequency capability of most RDF units, these stations and their transmitters can also be used for navigational fixes. While these commercial radio stations can be useful due to their high power and location near major cities, there may be several miles between the location of the station and its transmitter, which can reduce the accuracy of the 'fix' when approaching the broadcast city. A second factor is that some AM radio stations are omnidirectional during the day, and switch to a reduced power, directional signal at night.

RDF was once the primary form of aircraft and marine navigation. Strings of beacons formed "airways" from airport to airport, while marine NDBs and commercial AM broadcast stations provided navigational assistance to small watercraft approaching a landfall. In the United States, commercial AM radio stations were required to broadcast their station identifier once per hour for use by pilots and mariners as an aid to navigation. In the 1950s, aviation NDBs were augmented by the VOR system, in which the direction to the beacon can be extracted from the signal itself, hence the distinction with non-directional beacons. Use of marine NDBs was largerly supplanted in North America by the development of LORAN in the 1970s.

Today many NDBs have been decommissioned in favor of faster and far more accurate GPS navigational systems. However the low cost of ADF and RDF systems, and the continued existence of AM broadcast stations (as well as navigational beacons in countries outside North America) has allowed these devices to continue to function, primarily for use in small boats, as an adjunct or backup to GPS.

## *Automatic direction finder (ADF)*

A typical aircraft ADF indicator

An **automatic direction finder** (**ADF**) is a marine or aircraft radio-navigation instrument that automatically and continuously displays the relative bearing from the ship or aircraft to a suitable radio station. ADF receivers are normally tuned to aviation or marine NDBs operating in the LW band between 190 – 535 kHz. Like RDF units, most ADF receivers can also receive medium wave (AM) broadcast stations, though as mentioned, these are less reliable for navigational purposes.

The operator tunes the ADF receiver to the correct frequency and verifies the identity of the beacon by listening to the Morse code signal transmitted by the NDB. On marine ADF receivers, the motorized ferrite-bar antenna atop the unit (or remotely mounted on the masthead) would rotate and lock when reaching the null of the desired station. A centerline on the antenna unit moving atop a compass rose indicated in degrees the bearing of the station. On aviation ADFs, the unit automatically moves a compass-like pointer (RMI) to show the direction of the beacon. The pilot may use this pointer to *home* directly towards the beacon, or may also use the magnetic compass and calculate the direction from the beacon (the *radial*) at which their aircraft is located.

Unlike the RDF, the ADF operates without direct intervention, and continuously displays the direction of the tuned beacon. Initially, all ADF receivers, both marine and aircraft versions, contained a rotating loop or ferrite loopstick aerial driven by a motor which was controlled by the receiver. Like the RDF, a sense antenna verified the correct direction from its 180-degree opposite.

More modern aviation ADFs contain a small array of fixed aerials and use electronic sensors to deduce the direction using the strength and phase of the signals from each aerial. The electronic sensors listen for the *trough* that occurs when the antenna is at right angles to the signal, and provide the heading to the station using a direction indicator. In flight, the ADF's RMI or direction indicator will always point to the broadcast station regardless of aircraft heading, however a banked attitude can have a slight affect on the reading, the needle will still generally indicate towards the beacon, however it suffers from DIP error where the needle dips down in the direction of the turn. Such receivers can be used to determine current position, track inbound and outbound flight path, and intercept a desired bearing. These procedures are also used to execute holding patterns and non-precision instrument approaches.

## Typical NDB services ranges

| Class of NDB | Transmission Power | Effective Range |
|---|---|---|
| Locator | below 25 watts | 15 NM |
| MH | below 50 watts | 25 NM |
| H | 50 to 1,999 watts | 50 NM |
| HH | 2,000+ watts | 75 NM |

## Station passage

As an aircraft nears an NDB station, the ADF becomes increasingly sensitive, small lateral deviations result in large deflections of the needle which sometimes shows erratic left/right oscillations. Ideally, as the aircraft overflies the beacon, the needle swings rapidly from directly-ahead to directly-behind. This indicates *station passage* and provides an accurate position fix for the navigator. Less accurate station passage, passing slightly to one side or another, is shown by slower (but still rapid) swinging of the needle. The time interval from the first indications of station proximity to positive station passage varies with altitude — a few moments at low levels to several minutes at high altitude.

## Homing

The ADF may be used to *home* in on a station. Homing is flying the aircraft on the heading required to keep the needle pointing directly to the 0° (straight ahead) position. To home into a station, tune the station, identify the Morse code signal, then turn the aircraft to bring the ADF azimuth needle to the 0° position. Turn to keep the ADF heading indicator pointing directly ahead. Homing is regarded as poor piloting technique because the aircraft may be blown significantly or dangerously off-course by a cross-wind, and will have to fly further and for longer than the direct track.

## Tracking

The ADF may also be used to *track* a desired course using a ADF and allowing for winds aloft, winds which may blow the aircraft off-course. Good pilotage technique has the pilot calculate a correction angle that exactly balances the expected crosswind. As the flight progresses, the pilot monitors the direction to or from the NDB using the ADF, adjusts the correction as required. A direct track will yield the shortest distance and time to the ADF location.

## Radio-magnetic indicator (RMI)

A **radio-magnetic indicator** (**RMI**) is an alternate ADF display providing more information than a standard ADF. While the ADF shows relative angle of the transmitter with respect to the aircraft, an RMI display incorporates a compass card, actuated by the aircraft's compass system, and permits the operator to read the magnetic bearing to or from the transmitting station, without resorting to arithmetic.

Most RMI incorporate two direction needles. Often one needle (thicker and/or double-barred) is connected to an ADF and the other (generally the thin, single-barred needle) is connected to a VOR. Using multiple indicators a navigator can accurately fix the position of their aircraft without requiring station passage. There is great variation between models and the operator must take care that their selection displays information from the appropriate ADF and VOR.

## Chapter 6

# Ultralight Trike

An **ultralight trike**, also known as a **flex-wing trike**, **weight-shift control aircraft**, **microlight trike** or **Motorized Deltaplane**, is a type of powered hang glider using a high performance Rogallo wing coupled to a propeller-powered three-wheeled undercarriage. While most powered aircraft have three-wheeled landing gear, the term "trike" refers specifically to the form of aircraft described here. The principles of this page can generally be applied to the single place ultralight trike and the two place weight-shift control light-sport aircraft.

Trike in the Top End of the Northern Territory in Australia

## *Control*

Flight control in a trike is by weight-shift . This is similar to controlling a hang glider, in which the aviator or pilot is suspended from the wing made from high-strength aluminium and fabric. The pilot controls the attitude of the wing by holding onto and operating a triangular control bar (or triangular control frame) (TCF) that is rigidly attached to the wing. Pushing, pulling, and turning the TCF causes a corresponding shift in the aircraft's center of gravity.

For instance, pushing the TCF's basebar forward causes the center of gravity to shift back. This, in turn, causes the nose of the aircraft to pitch up, causing the angle of attack to increase which causes the aircraft to fly more slowly. In contrast, pushing forward on the control stick of a traditional aircraft would cause that aircraft to dive.

Detail of a Mainair Blade ultralight trike (in 2009)

Turns are accomplished by rolling the wing in the direction of the intended turn. This is accomplished by moving the control bar to the left in order to enter a right hand turn. This causes the center of gravity--represented primarily by the weight of the undercarriage and pilot--to shift in the direction of the intended turn. This in itself does not cause the aircraft to turn, but it does cause the aircraft to bank, or tip, to the side. Some adverse yaw is also initially produced, which is soon damped by the natural yaw stability of the wing.

A banking maneuver becomes a turn because of the natural yaw stability of the wing. When a roll is applied, the aircraft begins to side slip towards the lower wing. Since the wing is yaw stable, a yaw is set up in the direction of the bank, thus coordinating the turn. A small anhedral effect may be built into the wing to aid roll response, where the side slip causes increased banking.

This is similar to the way in which a hang glider is controlled. In fact, trikes are essentially propeller-powered hang gliders with seats and wheels. Trikes have often employed wings designed for hang gliding; the Rogallo-winged trike Paresev 1B of NASA's 1960s experiments and Barry Hill Palmer's trike (Fleep inspired) modeled the wing that has evolved to contemporary trike wings. As weight and performance goals have increased purpose-built wings have become more commonplace. They are now long distance cross country machines as shown by record-breaking flights that echo the exploits of fixed-wing aviators in the 1920s and 1930s, e.g., the circumnavigation of the world.

## Stability and equilibrium

Varadero, Cuba.

Because trikes are most often used for recreational flying by part time pilots, a premium is placed on gentle behavior especially at the stall, natural pitch stability, and ease-of-operation.

Unlike a traditional aircraft with an extended fuselage and empennage for maintaining stability, trikes rely on the natural stability of their wings to return to equilibrium in yaw and pitch. Roll stability is generally set up to be near neutral. In calm air, a properly designed wing will maintain balanced trimmed flight though a slow spiral may build up in either direction.

In roll most trikes are set up with near-neutral roll due to side slip (some slightly negative, some slightly positive) and also near neutral spiral stability, often mildly unstable. Moderate negative roll due to side slip (anhedral effect) can be built in to improve roll response by weight shift.

The yaw axis, which represents the direction that the aircraft is facing relative to the wind, is stabilized through the sweep of the wings. Instead of having wings that extend almost straight out side-to-side as in many types of traditional light aircraft, trikes are provided with a swept back wing planform. The swept planform, when yawed out of the relative wind, creates more lift on the advancing wing and also more drag. The differential drag stabilizes the wing in yaw. The differential lift causes positive roll due to sideslip like dihedral would. Too much dihedral effect is undesirable because it opposes weight shift roll response; the aircraft will be too stable and won't manouever. The lateral and directional stability of the swept wing is proportional to angle of attack - at high speed, yaw and roll instability can become unacceptable, giving dutch roll or wing walking oscillations. This is the primary reason for over sized rear undercarriage spats and wing lets on recent high performance machines.

Thus, if one wing advances ahead of the other it presents more area to the wind and causes more drag on that side. This causes the advancing wing to go slower and to fall back. The wing is at equilibrium when the aircraft is traveling straight and both wings present the same amount of area to the wind.

The third axis, represented by pitch, is also stabilized by the sweep of the wings. A combination of high lift airfoils with moderate pitching moment such as the UI 1720 and washout (tip trailing edge upwards twist) caused by loading of the sail produces a positive pitching tendency in the wing where increasing airspeed causes increasing pitch-up. The wing centre of gravity is close to the trike hang point and is located forward of the mean aerodynamic center of the wing at a distance known as the static margin. Therefore at some speed, called the trim speed, the positive pitching of the wing is balanced by the nose down moment caused by the aircraft weight times the static margin. At the trim speed the wing will fly hands off and return to trim when disturbed. The weight shift control system only works when the wing is positively loaded. A combination of very steep nose-up pitch attitude and very low airspeed is very hazardous because of the probability of a tail slide and violent nose down pitch rotation into an irrecoverable tumble. This is the primary area of the flight envelope trike pilots must always avoid.

Pegasus Quantum 145-912 ultralight trike

When the lift load is removed from the sail the washout disappears and the aircraft would not recover from a vertical dive or may even tuck upside down. To maintain a minimum safe amount of washout when the wing is unloaded or even negatively loaded, positive pitching devices such as reflex lines or washout rods are employed. These systems are normally tested by a truck based aerodynamic test.

There is no "pendulum" wing stabilizing effect of the trike at the trim speed because the trike is freely suspended in the pitch and roll axes. To fly at other speeds, the pilot applies a pitching moment to the wing by levering the trike mass around using the control bar connected directly to the wing. The bar is pushed on to rotate the wing more nose-up and so fly slower, vice-versa for high speed. A properly designed trike will always require increasing pilot force to be applied each side of the trim speed.

The free suspension of the trike means that the center of gravity (CG) position of the trike only affects the trike attitude and control range, not the hands off trim speed. From the pilot's point of view only the load carried has to remain within the aircraft limitations, no complicated CG calculations are required and it is nearly impossible to mis-load the aircraft, adding to the simplicity of operation. One great advantage of weight shift pitch apart from simplicity is that the wing lifting performance is not compromised by up elevon deflection as required for an aerodynamically controlled tailless machine, hence a lower landing speed can be achieved. Additionally, with a pitch stable wing it is also nearly impossible to overspeed the aircraft because it will simply trim in pitch at a limited speed with the bar held fully back.

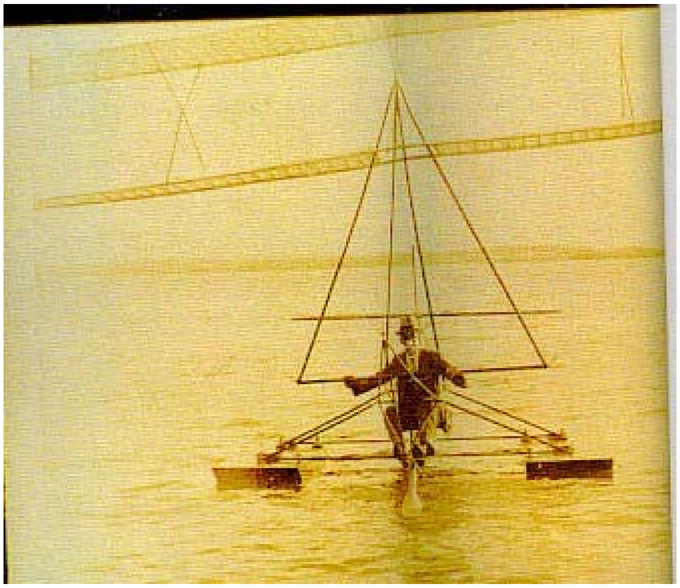
Pitch control response is very direct, but satisfactory weight shift roll response becomes more difficult to achieve as the sail is tightened to improve performance. In the roll axis, the pilot, using the wing control bar and reacting his input by the mass of the trike, applies a rolling moment directly to the wing. The wing is built to flex differentially across the span in response to the pilot applied roll moment. For example, under a right roll input, the right wing trailing edge flexes up more than the left, allowing the right wing to drop. Special features are built in such as a floating keel, four-bar control frame linkage to get a longer effective control frame height, keel pocket - all to ease roll response. Judicious use of anhedral improves roll response by converting the adverse yaw generated by the roll input into a pro-roll bank. Too much anhedral can cause instability in roll at high speeds.

Furthermore, the fact that the wing is designed to bend and flex in the wind provides favorable dynamics analogous to a spring suspension. This allows the wing to be less susceptible to turbulence and provides a gentler flying experience than a similarly sized rigid-winged aircraft.

## Engine placement

Because trikes do not require an empennage, the space directly behind the pilot is used to mount the engine and propeller. Engines range from between 25-40 hp for single-seaters and 50-100 hp for two-seaters. An aft engine placement allows exhaust to stay behind the pilot and enhances visibility. It also means that the turbulent vortex of air behind the propeller is not coming in contact with the wing.

### *History*



**Dr. George A. Spratt** towed his hang glider on floats using a motorboat. USA, 1929

First towing tests of NASA's Paresev glider (Para Wing Research Vehicle), March 1962

Barry Hill Palmer, 1961. First hang glider based on Rogallo's flexible wing.

**Richard Miller** flying his 'Bamboo Butterfly' hang glider. Vista Del Mar. California, 1966.

'Standard' flexible wing hang glider, based on variants of the Rogallo wing aircraft, 1975

Trikes are referred to as "microlights" in Europe and have been extremely popular since the 1980s. The history of the trike is traced back to the invention by Francis Rogallo's flexible wing and subsequent development by the Paresev engineering team's innovations and then others. On 1948, engineer Francis Rogallo invented a self-inflating wing which he patented on March 20, 1951 as the Flexible wing. It was on October 4, 1957 when the Russian satellite Sputnik shocked the United States and the space race caught the imagination of its government, causing major increases in U.S. government spending on scientific research, education and on the immediate creation of NASA. Rogallo was in position to seize the opportunity and released his patent to the government and with his help at the wind tunnels, NASA began a series of experiments testing Rogallo's wing - which was renamed **Para Wing**- in order to evaluate it as a recovery system for the Gemini space capsules and recovery of used Saturn rocket stages. F. Rogallo's team adapted and extended the totally flexible principle into semi-rigid variants. This mainly involved stabilizing the leading edges with compressed air beams or rigid structures like aluminum tubes. By 1960 NASA had already made test flights of a heavily framed cargo powered aircraft called the Ryan XV-8 or *Fleep* (short for 'Flying Jeep') and by March 1962, of a weight-shift experimental glider called Paresev. By 1967 all Para Wing projects were dropped by NASA in favor of using round parachutes without officially considering development of personal ultralight gliders, but the airfoil's simplicity of design and ease of construction, along with its capability of slow flight and its gentle landing characteristics, did not go unnoticed by hang glider enthusiasts. The challenge then, was to modify and fit a Rogallo flexible wing with an appropriate frame to allow it to be used as a hang glider.

## Some modern Rogallo flexible winged aircraft

A crucial development toward the trike was the severe mechanical innovations developed by the Paresev and the Fleep engineers; they proved the Rogallo wing for free-flight gliding, powered and unpowered, for safe landing.

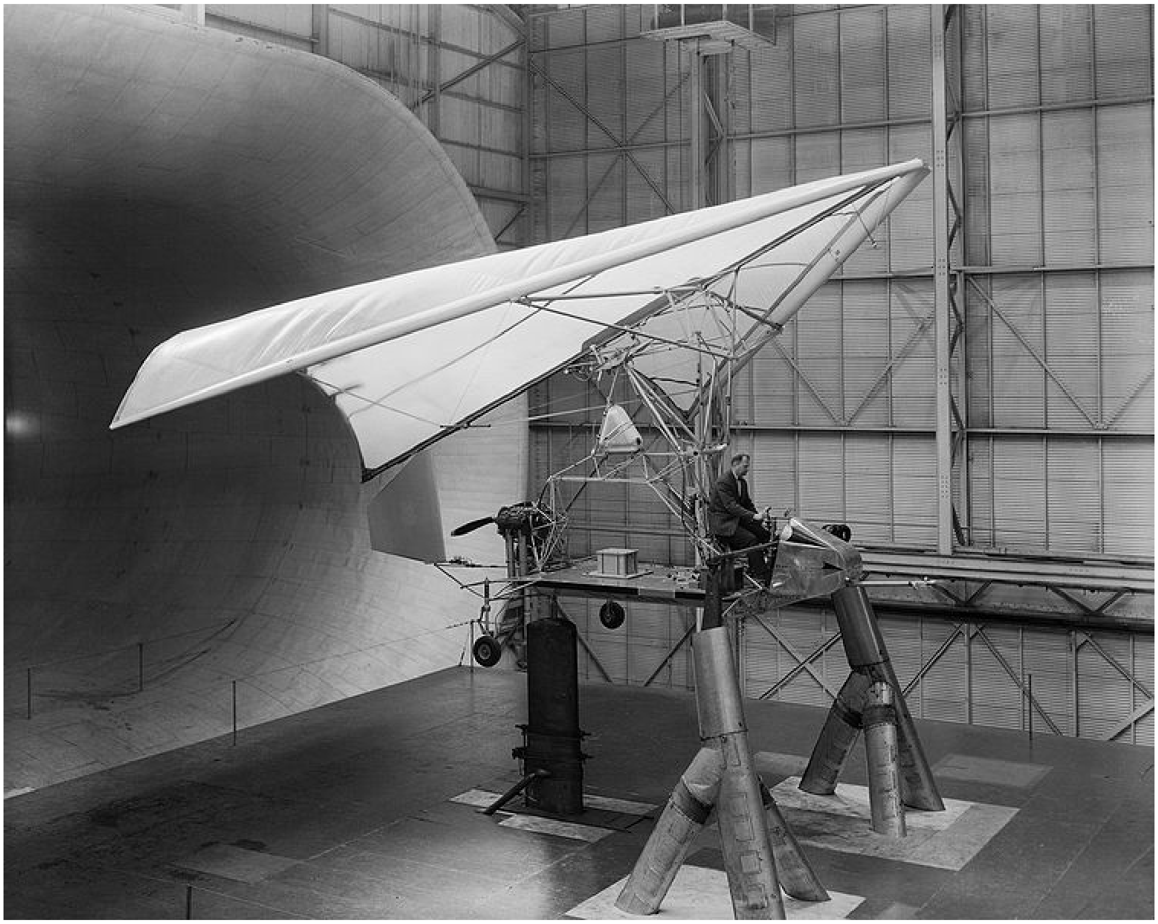
Publicity from the Fleep and the Paresev tests sparked interest in the design among several tinkerers, first through Barry Palmer. An engineer, Mike Burns of Australia, developed and used the boat-towed Rogallo airfoiled SkiPlane from 1962 through the 1960s. A fellow countryman of Mike Burns, John W. Dickenson, made ski-kites and eventually partnered with Mike Burns to improve the ski-kite; he formatted a ski-kite that used what could be found in the 1929 George A. Spratt simple triangle control bar or A-frame with single-point pendulum weight-shift control.

An influence through John Dickenson's duplication of his device, who named his flexible wing ski-kite the **Ski Wing**. Dickenson fashioned a water ski kite airframe to fit on a Rogallo airfoil where the pilot sat on a swinging seat while the control frame and wire bracing distributes the load to the wing as well as giving a frame to push/pull for weight-shift control. Dickenson's Ski Wing turned out to be stable and controllable under tow, unlike the flat manned kites used at water ski shows. The Ski Wing kite was first kited in public at the 'Grafton Jacaranda Festival' in September 1963 by **Rod Fuller** while towed behind a motorboat. Australian manufacturers like **Bill Bennett** and **Bill Moyes**, actively developed and marketed Dickenson's innovations to the world, which significantly fueled the hang glider revolution.

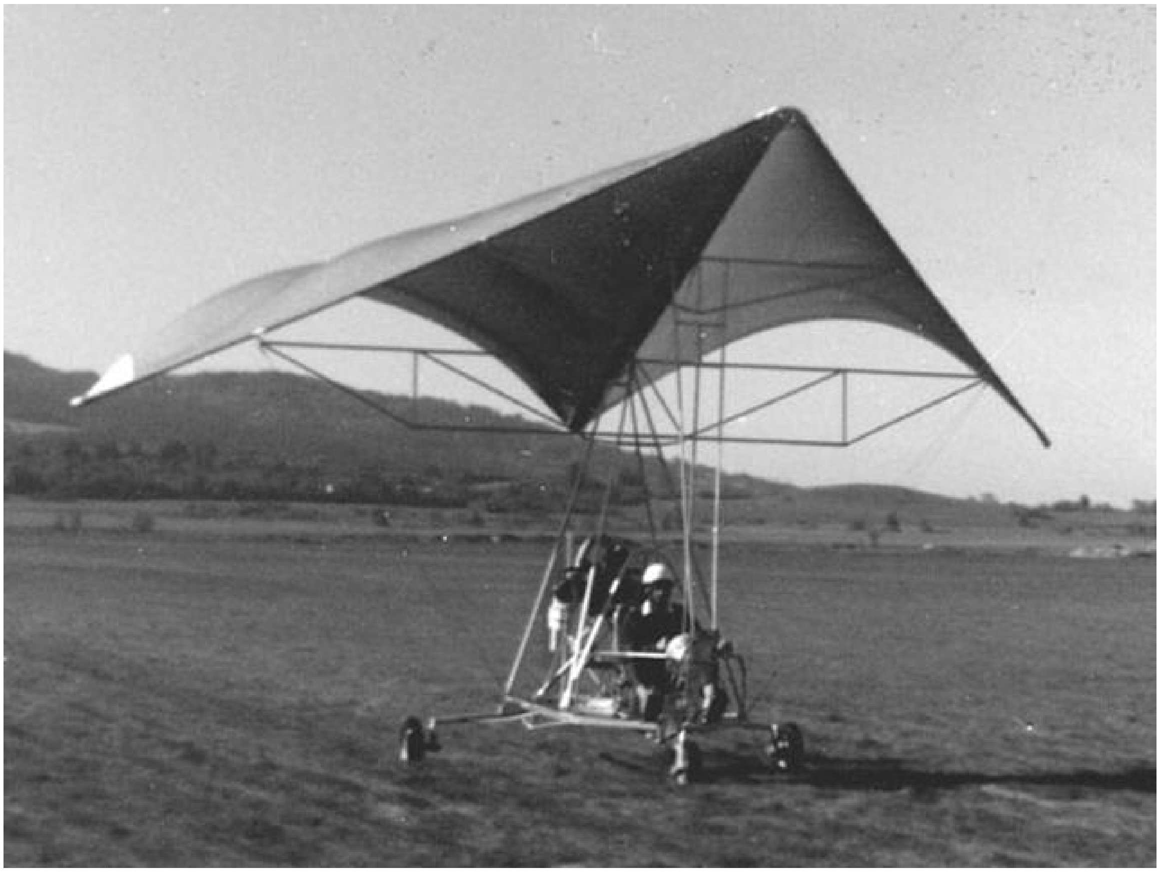
Although by the early 1970s many rigid wings were developed, none sold well, while dozens of flexible wing hang glider companies were springing up world-wide, building variants of Dickenson's Ski Wing. In 1972, Popular Mechanics and Popular Science magazines published articles on hang gliding which further increased its popularity, as the *Sky Raiders* hang gliding movie released in 1975.

Francis Rogallo, Barry Palmer, John Dickenson, and others never made any money out of their innovations. Profit to manufacturers of hang gliders and Rogallo winged hang gliders came once organized and insured sporting events grew in popularity. Dickenson's adaptation and innovations eventually produced a foldable hang glider that dramatically reduced difficulty in control, storage, transport, assembly and repair. In addition, the flexible wing lends itself open to design changes for possible improvements. The crucial developments put together by the Paresev engineers, Barry Palmer, John Dickenson, Bill Bennett, Bill Moyes, Richard Miller, and then hundreds of other innovators gave success to the flexible wing hang glider.

**First trikes**



Ryan XV-8 'Fleep' flown in the Full Scale Tunnel at Langley, 1962



**Pierre Aubert**, Switzerland, 1964

In 1961, Engineer **Thomas Purcell** built a towable Rogallo wing glider with an aluminum frame, wheels, a seat and basic control rods; soon he replaced the wheels for floats and motorized the aircraft. In 1964, Swiss inventor **Pierre Aubert** saw a photo of NASA's Fleep and completed construction of a similar trike. Like with the Fleep, his Rogallo wing was fixed and did not allow for pendulum weight-shift control.

In March 1967 aeronautical engineer Barry Palmer completed the earliest example of a true weight-shift powered trike: the **Paraplane**; it was controlled by a single vertical control bar as the Paresev experimental glider that inspired him. The Paraplane used two *West Bend-Chrysler* 820 engines of 8hp at 6000rpm, reduced to 4700rpm for about 6.5hp each, for a total of 13hp. Each engine had a direct drive to a 27in diameter two-blade propeller made of polyester and fiberglass. On March 24, 1967 Palmer registered the trike at the American FAA as the *Palmer Parawing D-6*, serial 1A, N7144; no restrictions were noted. The second Palmer trike, **Skyhook** (FAA registered N4411) in spite of its early date of origin, had most of the attributes of a modern ultralight, except it used a single cylinder snowmobile engine, as the two-stroke twin cylinders were not available yet. It was powered by a 17hp at 5000rpm single cylinder JLO L297 two stroke engine, driving a composite propeller designed and built by Palmer himself and driven by a 2.1/1 reduction gearbox. The engine had electric start and the craft had fiberglass composite spring landing gear. Airframe construction was bolted 6061-T6 aluminum thin wall tube, with 6061 T-6 extruded angle. The craft took off, flew, and landed at around 30mph. Palmer's trikes were not developed further and remained in obscurity.

The commercial availability of Dikenson's hang glider made the Rogallo wing very popular, and prompted several builders during the 1970s to attempt motorization of their flexible wing aircraft but unlike Barry Palmer -who placed the center of gravity well below the keel- most builders were mounting the engine to the wing, where a fine balance existed between applying too much power, causing the aircraft to overtake the pilot or not enough power for flight. It was not until **Roland Magallon** took a long look at the *Motodelta* ultralight (a hybrid Rogallo wing designed by Jean-Marc Geiser had a 'fuselage' and rudder) and Magallon decided replace the Motodelta's 'fuselage' with a simple tubular framework pendulum and dispensing with the rudder. Magallon is thus generally thought to have invented the trike because it was he who first marketed it. He called the first version 'Mosquito' and marketed it from October 1979 through 1981. The prototype had flown with a McCulloch MC-101A motor of 125 cc, delivering 10 hp at 8000 rpm to a direct-drive prop with ground adjustable pitch. Later he offered it with a Solo 210 engine which produced 15 hp (11 kW) at much less frantic RPM.

The "trike", as it soon became known, quickly became popular in the UK and France where it had been reborn. Trike technology still shows its hang gliding origins, though the wings are no longer converted hang gliders, but are designed for power. In fact, none of the commercially available trike wings can be used as foot-launched hang gliders as they are too heavy and too fast.

## *Regulation*

In the United States, trikes are often referred to as "ultralight trikes" and are designed to operate under the designation of the Federal Aviation Regulations (FAR 103) that define an ultralight as a single seat vehicle with under 5 US gallons (19 L) of fuel capacity, an empty weight of less than 254 pounds (115 kg), a top speed of 55 knots (102 km/h), and a maximum stall speed that does not exceed 24 knots (45 km/h). Ultralights are only allowed to operate during daylight hours. FAR 103 makes further weight allowances for two-seat trainers(in which both persons are able to control the craft and thus definable as pilots), amphibious landing gear, and ballistic parachute systems.

A light sport aircraft (LSA) certification code has been produced for heavier and higher performance machines. This is an airworthiness code based on a consensus of industry experts, drawing from many years experience including the British BCAR-S requirements. The LSA initiative also includes similar consensus-based pilot licencing and maintenance requirements. It is proving popular, enabling more people to fly modern designs safely. Trike pilots may also obtain the private pilot certificate which permits them to fly at night, above 10,000 feet, and in other venues not permissible for ultralight or sport pilots.

## Popularity

Due to their relatively low cost, low fuel consumption, light weight, ability to take off and land in very short distances, and ability to fly in somewhat turbulent conditions, trikes have become popular with recreational pilots. In particular, trikes have been used to fly across oceans, frozen tundra, barren deserts, and even into backyard landing strips. As with all types of aviation, regulations in the United States dictate where and how these aircraft are allowed to fly, especially with regard to avoiding other air traffic and populated areas. Notwithstanding, trikes continue to grow in popularity with those living in urban areas due to their portable nature and rapid assembly time. Most trikes may be transported on a very small trailer, stored in a standard garage, and assembled for flight in less than thirty minutes.

## Manufacturers

There are around 56 manufacturers world-wide that build trikes. Some started out making hang glider wings and now provide both wing and undercarriage. Many produce only the undercarriage and procure their wings elsewhere. The majority of these companies are found in Europe with a number appearing in the United States. Manufacturers can also be found in India, South Africa, Australia, and elsewhere. Manufacturers often sell their ultralight trikes at a price of around $5000-10000.

## Records

On January 19, 2008 Mark Jackson from Altrincham, UK, flew over Kilimanjaro. In doing so he broke the record for the highest altitude attained in a microlight (24,262 feet) and the fastest climb to 20,000 feet (25 minutes). He also broke the British record for the fastest climb to 10,000 feet (19 minutes). He did the flight with Eve Jackson.

## Chapter 7

# Ultralight Aviation

Huntair Pathfinder Mark 1 ultralight

The term "**ultralight aviation**" refers to light-weight, 1- or 2-person airplanes. During the late 1970s and early 1980s, many people sought to fly affordably. As a result, many aviation authorities set up definitions of lightweight, slow-flying aeroplanes that could be subject to minimum regulation. The resulting aeroplanes are commonly called **ultralight** or **microlight**, although the weight and speed limits differ from country to country.

There is also an allowance of another 10% on Maximum Take Off Weight for seaplanes and amphibians, and some countries (such as Germany and France) also allow another 5% for installation of a ballistic parachute.

The safety regulations used to approve microlights vary between countries, the strictest being in the United Kingdom, Italy, Sweden and Germany, while they are almost non-existent in France and the United States. The disparity between regulations can be a barrier to international trade and overflight in strict regions, as is the fact that these regulations are invariably sub-ICAO, which means that they are not internationally recognised.

In most affluent countries, microlights or ultralights now account for a significant portion of the civil aircraft fleet. For instance in Canada in October 2010, the ultralight fleet made up 19% of the total civil aircraft registered. In other countries that do not register ultralights, like the United States, it is unknown what proportion of the total fleet they make up.

In countries where there is no specific extra regulation, ultralights are considered regular aircraft and subject to certification requirements for both aircraft and pilot.

Ultralight aircraft are generally called *microlight aircraft* in the UK, India and New Zealand, and *ULMs* in France and Italy. Some countries differentiate between weight shift and 3-axis aircraft, calling the former *microlight* and the latter *ultralight*.

The U.S. light-sport aircraft is similar to the UK and NZ *Microlight* in definition and licensing requirement, the U.S. 'Ultralight' being in a class of its own.

## *Definitions*

Pegasus Quantum 145-912 ultralight trike

Flight Design CTSW

A US-made Pterodactyl Ascender ultralight on a camping flight

A powered paraglider

Canadian Lazair ultralight covered in clear Mylar

Aeroprakt A-22 Foxbat 3-axis ultralight

Ikarus C42, a German ultralight

A weight-shift ultralight, the Air Creation Tanarg

Phantom - MKI

FM250 Vampire

K-10 Swift – MKI

Quicksilver MXII

A foot-launched powered hang glider

Weight Shift Ultralight ("Trike")

P and M Aviation Quik GT450 ultralight

Pipistrel Sinus 912

Rans S-6 Coyote II, classified as an ultralight aircraft in Belgium

Australian Ultralight Industries Bunyip, 3-axis ultralight

## Australia

In Australia Recreational Aircraft fall under many categories, but the most common category imposes:

- a maximum take off weight (MTOW) of 544 kg (1,199 lb) or less (614 kg (1,354 lb) for a seaplane);
- a stalling speed under 45 knots in landing configuration and
- a maximum of two seats.

A new certification category for Light Sport Aircraft came into effect on 7 January 2006. This category does not replace the previous categories, but creates a new category with the following characteristics:

- A maximum takeoff weight of 600 kg (1,323 lb) or 650 kg (1,433 lb) for an aircraft intended and configured for operation on water or 560 kg (1,235 lb) for a lighter-than-air aircraft.
- A maximum stall speed in the landing configuration (Vso) of 45 kn (83 km/h) CAS.
- Maximum of two occupants, including the pilot.
- A fixed landing gear. A glider may have retractable landing gear. (For an aircraft intended for operation on water, a fixed or repositionable landing gear)
- A single, non-turbine engine fitted with a propeller.
- A non-pressurised cabin.
- If the aircraft is a glider a maximum never exceed speed (Vne) of 135 kn (250 km/h) CAS

In either of the above categories, there are distinctions between factory manufactured and home built aircraft.

In Australia, microlight aircraft are defined as one or two seat weight-shift aircraft, with a maximum takeoff weight of 450 kg (992 lb), as set out by the Civil Aviation Safety Authority. In Australia microlights are also referred to as trikes and are distinguished from three-axis aircraft, of which the smallest are known as ultralights.

In Australia, microlight aircraft and their pilots can either be registered with the Hang Gliding Federation of Australia (HGFA) or Recreational Aviation Australia (RA Aus). In all cases, except for privately built single-place ultralight aeroplanes, microlight aircraft or trikes are regulated by the Civil Aviation Regulations.

## Brazil

The Brazilian Aviation Regulation (RBHA 103A) defines an ultralight plane as: a very light manned experimental aircraft used mainly, or intended for, sports or recreation, during daylight, in visual conditions, with a maximum capacity of 2 people and with the following characteristics:

- Single internal combustion engine and one propeller;
- Maximum take-off weight equal or less than 750 kg (1,653 lb); and
- Calibrated stall speed (CAS), power off, in landing configuration (Vso) equal or less than 45 kn (83 km/h).

## Canada

The Canadian Aviation Regulations define two types of ultralight aeroplanes: basic ultra-light aeroplanes (BULA), and advanced ultra-light aeroplanes (AULA). The US light sport aircraft is similar to, and was based upon, the Canadian AULA. AULAs may operate at a controlled airport without prior arrangement. Operating either class of ultralight in Canada requires an Ultralight Pilot Permit which requires both ground school, dual and solo supervised flights. The ultralight may be operated from land or water, but may only carry a passenger if the pilot has an Ultralight Aeroplane Passenger Carrying Rating and the aircraft is an AULA.

## Europe

The definition of a microlight according to the Joint Aviation Authorities document JAR-1 is an aeroplane having no more than two seats, maximum stall speed ($V_{S0}$) of 35 knots (65 km/h) CAS, and a maximum take-off mass of no more than:

- 300 kg (661 lb) for a landplane, single seater; or
- 450 kg (992 lb) for a landplane, two-seater; or
- 330 kg (728 lb) for an amphibian or floatplane, single seater; or
- 495 kg (1,091 lb) for an amphibian or floatplane, two-seater, provided that a microlight capable of operating as both a floatplane and a landplane falls below both MTOM limits, as appropriate.

## India

In India a microlight is an aircraft that has the following characteristics:

- two seater aircraft having an all up weight of not more than 450 kg (992 lb) without parachute and 472 kg (1,041 lb) with parachute
- a stall speed of less than 80 km/h (43 kn)
- a maximum level speed of less than 220 km/h (119 kn)
- 1 or 2 seats
- a single engine, reciprocating, rotary or diesel
- a fixed or ground adjustable propeller
- un-pressurized cabin
- wing area more than 10 square metres
- a fixed landing gear, except for operation on water or as a glider

Indian ultralights require aircraft registration, periodic condition inspections and a current permit to fly which has to be renewed annually.

## Italy

In Italy, the category for this class of aircraft is Microlight.

- Requires flying with a helmet.
- Maximum weight requirements excludes seat belts, parachute and instruments.
- Single-seat maximum weight of 300 kg (661 lb), and 330 kg (728 lb) for amphibious, stall speed must not exceed 65 km/h (35 kn).
- Two-seat maximum weight of 450 kg (992 lb), and 500 kg (1,102 lb) for amphibious, stall speed must not exceed 65 km/h (35 kn). Aircraft may be used for instruction or flown by pilots with a valid private license, and at least 30 hours flight time.
- Intended for use at private fields. Use at civil airports requires prior permission.
- Airspace restrictions - Must remain within the territory of the state (the flight limit of 4 km (2.2 nmi) from the border of another state was abolished by the law 24 April 1998, n. 128 "Disposizioni per l'adempimento di obblighi derivanti dall'appartenenza dell'Italia alle Comunità Europee" - communitary law 1995/97-art.22 comma 20-, published on the Gazzetta Ufficiale n.88/L of May 7, 1998). It is forbidden to fly over cities.
- All aircraft must have a metal plate with the identification number issued by the AeCI (Aero Club Italia). The same number must be fixed onto the underneath of the wing with letters that measure a minimum of 30×15 cm (12 X 6 inches), in contrasting colour.
- From dawn till sunset, flight must be below 500 ft (152 m)
- On Saturday and holidays flight must be below 1,000 ft (305 m) with 5 km (2.7 nmi) separation from airports not located within ATZ .
- Microlight operation requires a certificate exam, insurance and a medical examination.

## New Zealand

In New Zealand microlight aircraft are separated into two classes, basically single and two seat aircraft. All microlights are required to have a prescribed endurance testing period when they are first flown, and all microlights must have a minimum set of instrumentation to show airspeed (except powered parachutes), altitude and magnetic heading.

NZ Class 1

Single seat aircraft with a design gross weight of 544 kg (1,199 lb) (landplanes) or 579 kg (1,276 lb) (seaplanes or amphibians), or less, and a stall speed in the landing configuration of 45 knots (83 km/h) or less. Requires aircraft registration, and annual condition inspections, but does not require a permit to fly.

NZ Class 2

Two seat aircraft with a design gross weight of 544 kg (1,199 lb) (landplanes) or 614 kg (1,354 lb) (seaplanes or amphibians), or less, and a stall speed of 45 knots (83 km/h) or less in the landing configuration. Must meet minimum type acceptance standards which may be foreign standards which have been deemed acceptable, or via a temporary permit to fly and flight testing regime. Requires aircraft registration, annual condition inspections, and a current permit to fly.

## Philippines

The Civil Aviation Regulations define "non-type certified aircraft", under which ultralights and microlights fall, as:

An aircraft that does not possess an aircraft type certificate issued by any country/state. It is, of simple design and constriction, either a homebuilt or a kit built variety and for recreational and sport use, day VFR condition only.

A class of non-type certificated aircraft is applicable to all classifications, including powered parachutes, gyrocopter, fixed wing aircraft and helicopters.

## United Kingdom

The UK regulations describe a microlight aeroplane as limited to two people, with a Maximum Total Weight Authorised (MTWA) not exceeding:

- 300 kg (661 lb) for a single seat landplane.
- 390 kg (860 lb) for a single seat landplane for which a UK Permit to Fly or Certificate of Airworthiness was in force prior to 1 January 2003
- 450 kg (992 lb) for a two seat landplane
- 330 kg (728 lb) for a single seat amphibian or floatplane
- 495 kg (1,091 lb) for a two seat amphibian or floatplane

A microlight must also have either a wing loading at the maximum weight authorised not exceeding 25 kg per square metre or a stalling speed at the maximum weight authorised not exceeding 35 kn (65 km/h) calibrated speed. All UK registered aeroplanes (3-axis or flex-wing) falling within these parameters are Microlight aircraft.

A sub-category of microlights (SSDR) was introduced which allows owners more freedom to modify and experiment with their aircraft. Single Seat De-Regulated microlights must weigh less than 115 kg (254 lb) without fuel and pilot and the wing loading must not be more than 10 kg per sq m. There is no airworthiness requirement or annual inspection regime for SSDR microlights although pilots who fly them must have a normal microlight licence, and must observe the rules of the air.

A license is required to fly a microlight in the UK.

## United States

The United States FAA's definition of an ultralight is significantly different from that in most other countries and can lead to some confusion when discussing the topic. The governing regulation in the United States is FAR 103 Ultralight Vehicles, which specifies a powered "ultralight" as a single seat vehicle of less than 5 US gallons (19 L) fuel capacity, empty weight of less than 254 pounds (115 kg), a top speed of 55 knots (102 km/h or 64 mph), and a maximum stall speed not exceeding 24 knots (45 km/h or 27.6 mph). Restrictions include flying only during daylight hours and over unpopulated areas. Unpowered "ultralights" (hang gliders, paragliders, etc.) are limited to a weight of 155 lb (70 kg) with extra weight allowed for amphibious landing gear and ballistic parachute systems.

In 2004 the FAA introduced the "Light-sport aircraft" category, which resembles some other countries' microlight categories.

In the United States no license or training is required by law for ultralights, but training is highly advisable. For light-sport aircraft a sport pilot certificate is required.

Ultralight aviation is represented by the United States Ultralight Association (USUA), which represents the US portion of the sport to the world through its affiliation with the FAI.

## *Types of aircraft*

While ultralight-type planes date back to the early 1900s (such as the Santos-Dumont Demoiselle), there have been three generations of modern, fixed-wing ultralight aircraft designs, which are generally classed by the type of structure.

The first generation of modern ultralights were actually hang gliders with small engines added to them, to create powered hang gliders. The wings on these were flexible, braced by wires, and steered by shifting the pilot's weight under the wing.

The second generation ultralights began to arrive in the mid-1970s. These were designed as powered aircraft, but still used wire bracing and usually single-surface wings. Most of these have "2-Axis" control systems, operated by stick or yoke, which control the elevators (pitch) and the rudder (yaw) -- there are no ailerons, so may be no direct control of banking (roll). A few 2-Axis designs use spoilers on the top of the wings, and pedals for rudder control. Examples of 2-Axis ultralights are the "Pterodactyl" and the "Quicksilver MX".

The third generation ultralights, arriving in the early 1980s, have strut-braced wings and airframe structure. Nearly all use 3-Axis control systems, as used on standard airplanes, and these are the most popular. Third generation designs include the CGS Hawk, Kolb Ultrastar and Quad City Challenger.

There are several types of aircraft which qualify as ultralights, but which do not have fixed-wing designs. These include:

- **Weight-shift control trike** - while the first generation ultralights were also controlled by weight shift, most of the current weight shift ultralights use a hang glider-style wing, below which is suspended a three wheeled carriage which carries the engine and aviators. These aircraft are controlled by pushing against a horizontal control bar in roughly the same way as a hang glider pilot flies. Trikes generally have impressive climb rates and are ideal for rough field operation, but are slower than other types of fixed-wing ultralights.

- **Powered parachutes** - cart mounted engines with parafoil wings, which are wheeled aircraft.

- **Powered paragliding** - backpack engines with parafoil wings, which are foot-launched.

- **Powered hang glider** - motorized foot-launched hang glider harness.

- **Autogyro** - rotary wing with cart mounted engine, a gyrocopter is different from a helicopter in that the rotating wing is not powered, the engine provides forward thrust and the airflow through the rotary blades causes them to *autorotate* or "spin up" to create lift. Most of these use a design based on the Bensen B-8 gyrocopter.

- **Helicopter** - there are a number of single-seat and two-place helicopters which fall under the microlight categories in countries such as New Zealand. However, few helicopter designs fall within the more restrictive ultralight category defined in the United States of America. One example that does is the experimental Martin Jetpack.

- **Hot air balloon** - there are numerous ultralight hot air balloons in the US, and several more have been built and flown in France and Australia in recent years. Some ultralight hot air balloons are hopper balloons, while others are regular hot air balloons that carry passengers in a basket.

## Electric powered ultralights

Research has been conducted in recent years to replace gasoline engines in ultralights with electric motors powered by batteries to produce electric aircraft. This has now resulted in practical production electric power systems for some ultralight applications. These developments have been motivated by cost as well as environmental concerns. In many ways ultralights are a good application for electric power as some models are capable of flying with low power, which allows longer duration flights on battery power.

In 2007 ElectraFlyer began offering engine kits to convert ultralight weight shift trikes to electric power. The 18 hp motor weighs 26 lb (12 kg) and an efficiency of 90% is claimed by designer Randall Fishman. The battery consists of a lithium-polymer battery pack of 5.6kwh which provides 1.5 hours of flying in the trike application. The power system for a trike costs USD $8285. to $11285. The company claims a flight recharge cost of 60 cents.

## Safety

Historically, ultralights have had a poor safety reputation. Most of the early designs were fragile or unstable, and this resulted in a number of accidents.

As designs matured, pilot error was shown to be the cause of the vast majority of incidents involving ultralights. As a result, most countries now require an Ultralight Pilot's license/certificate, often regulated by one or more officially-delegated pilots' organizations. The United States does not require any training for ultralight pilots; however, experienced ultralighters are nearly unanimous in recommending that no one solo before receiving dual training. Instruction may be given in two-place light-sport versions of the ultralight. An instructor must be certified by the FAA to give dual instruction in a light-sport aircraft.

The build quality and airworthiness of ultralight aircraft (and homebuilt light-sport aircraft in the USA) can now equal that of Certified light aircraft. Some types satisfy both sets of requirements and are available for registration to either Ultralight or Certified status. When registered as an ultralight (or Experimental), the pilot is permitted to do more of the simple maintenance tasks, resulting in a lower cost of operation, although this comes at the cost of restrictions such as avoiding densely populated urban areas, bad weather, or night. Many older pilots are willing to trade these operational restrictions for a lower drain on their retirement incomes, and as a result many ultralights are now flown by experienced General Aviation (GA) pilots or ex-commercial pilots. One other reason for this increase in acceptance is that any pilot is "only one medical away from being an ultralight pilot" -- a reference to the requirement that most other pilots must pass periodic physical examinations, but not to fly ultralights.

## The future

Ultralight/microlight aircraft were once regarded as "flying clotheslines", since early aircraft were typically completely open, wire, tube and rag aircraft – these aircraft were seldom used for anything more than local area flying.

However, ultralights are rapidly transforming into high performance aircraft, capable of very respectable speed and range. In recent years there has been a dramatic rise in the number of General Aviation pilots flying high performance ultralights due to the cost benefits.

These aircraft are now often referred to as recreational aircraft.

A rapidly growing area of the class is scale-replica "warbirds", such as the offerings from Titan Aircraft and Loehle Aircraft.

# Chapter 8

# Stability Derivatives

$$C_{M_\alpha}$$

**A Stability Derivative.** This is an example of a common shorthand notation for stability derivatives. The "M" indicates it is a measure of *pitching* moment changes. The α indicates the changes are in response to changes in Angle of Attack. This stability derivative is pronounced "see-em-alpha". It is one measure of how strongly an aircraft wants to fly "nose first", which is clearly very important.

***Stability Derivatives***, and also ***Control Derivatives***, are measures of how particular forces and moments on an aircraft change as other parameters related to stability change (parameters such as airspeed, altitude, Angle of attack, etc.). For a defined "trim" flight condition, changes and oscillations occur in these parameters. *Equations of motion* are used to analyze these changes and oscillations. Stability and control derivatives are used to linearize (simplify) these equations of motion so the stability of the vehicle can be more readily analyzed.

Stability and control derivatives change as flight conditions change. The collection of stability and control derivatives as they change over a range of flight conditions is called an **Aero Model**. Aero models are used in engineering **flight simulators** to analyze stability, and in real-time flight simulators for training and entertainment.

## Stability *derivative vs.* Control *derivative*

*Stability* derivatives and *Control* derivatives are related because they both are measures of forces and moments on a vehicle as other parameters change. Often the words are used together and abbreviated in the term "S&C derivatives". They differ in that stability derivatives measure the effects of changes in flight conditions while control derivatives measure effects of changes in the control surface positions:

- A ***stability*** **derivative** measures how much change occurs in a force or moment acting on the vehicle when there is a small change in a ***flight condition parameter*** such as angle of attack, airspeed, altitude, etc. (Such parameters are called "states".)

- A ***control*** **derivative** measures how much change occurs in a force or moment acting on the vehicle when there is a small change in the ***deflection of a control surface*** such as the ailerons, elevator, and rudder.

## *Uses*

### Linearization (simplification) of stability analysis

Stability and control derivatives change as flight conditions change. That is, the forces and moments on the vehicle are seldom simple (linear) functions of its states. Because of this, the dynamics of atmospheric flight vehicles can be difficult to analyze. The following are two methods used to tackle this complexity.

> **Small oscillations about otherwise steady flight conditions**
> One way to simplify analysis is to consider only small oscillations about otherwise steady flight conditions. The set of flight conditions (such as altitude, airspeed, angle of attack) are called "trim" conditions when they are steady and not changing. When flight conditions are steady, stability and control derivatives are constant and can be more easily analyzed mathematically. The analysis at a single set of flight conditions is then applied to a range of different flight conditions.
> **Application in simulators for stability analysis**
> In a flight simulator, it is possible to "look up" new values for stability and control derivatives as conditions change. And so, the "linear approximations" aren't as great and stability can be assessed in maneuvers that span a greater range of flight conditions. Flight simulators used for analysis such as this are called "engineering simulators". The set of values for stability and control derivatives (as they change over various flight conditions) is called an **Aero Model**.

### Use in flight simulators

In addition to engineering simulators, aero models are often used in *real time flight simulators* for home use and professional flight training.

## *Names for the axes of vehicles*

Air vehicles use a coordinate system of axes to help name important parameters used in the analysis of stability. All the axes run through the center of gravity (called the "CG"):

- "X" or "x" axis runs from back to front along the body, called the *Roll Axis*.
- "Y" or "y" axis runs left to right along the wing, called the *Pitch Axis*.
- "Z" or "z" runs from top to bottom, called the *Yaw Axis*.

Two slightly different alignments of these axes are used depending on the situation: "Body-fixed Axes", and "Stability Axes".

### Body-fixed Axes

Body-fixed axes, or "Body Axes", are defined and fixed relative to the body of the vehicle.:

- X body axis is aligned along the vehicle body and is usually positive toward the normal direction of motion.
- Y body axis is at a right angle to the x body axis and is oriented along the wings of the vehicle. If there are no wings (as with a missile), a "horizontal" direction is defined in a way that is useful. The Y body axis is usually taken to be positive to right side of the vehicle.
- Z body axis is perpendicular to wing-body (XY) plane and usually points downward.

### Stability Axes

Aircraft (usually not missiles) operate at a nominally constant "trim" angle of attack. The angle of the nose (the X Axis) does not align with the direction of the oncoming air. The difference in these directions *is* the *angle of attack*. So, for many purposes, parameters are defined in terms of a slightly modified axis system called "stability axes". The stability axis system is used to get the X axis aligned with the oncoming flow direction. Essentially, the body axis system is rotated about the Y body axis by the trim angle of attack and then "re-fixed" to the body of the aircraft:

- X stability axis is aligned into the direction of the oncoming air in *steady* flight. (It is projected into the plane made by the X and Z body axes if there is sideslip).
- Y stability axis is the *same* as the Y body-fixed axis.
- Z stability axis is perpendicular to the plane made by the X stability axis and the Y *body* axis.

## *Names for Forces, Moments, and Velocities*

### Forces and velocities along each of the axes

Forces on the vehicle along the body axes are called "Body-axis Forces":

- X, or $F_X$, is used to indicate forces on the vehicle along the X axis
- Y, or $F_Y$, is used to indicate forces on the vehicle along the Y axis
- Z, or $F_Z$, is used to indicate forces on the vehicle along the Z axis

- u (lower case) is used for speed of the oncoming flow along the X body axis
- v (lower case) is used for speed of the oncoming flow along the Y body axis
- w (lower case) is used for speed of the oncoming flow along the Z body axis

  It is helpful to think of these speeds as projections of the relative wind vector on to the three body axes, rather than in terms of the translational motion of the vehicle relative to the fluid. As the body rotates relative to direction of the relative wind, these components change, even when there is no net change in speed.

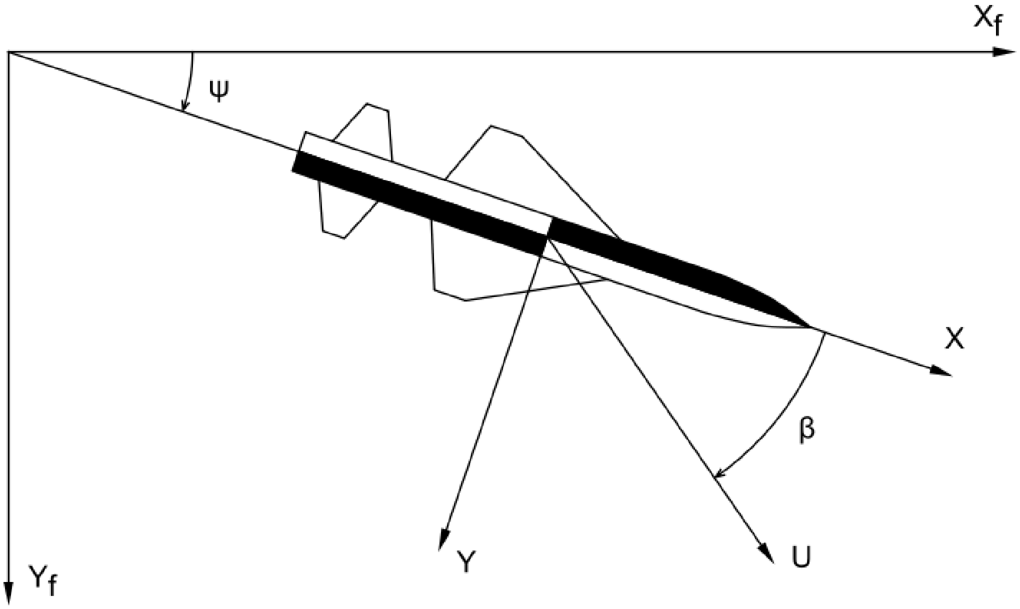
### Moments and angular rates around each of the axes

- L is used to indicate the "rolling moment", which is around the X axis. Whether it is around the X body axis or the X stability axis depends on context (such as a subscript).
- M is used to indicate the name of the "pitching moment", which is around the Y axis.
- N is used to indicate the name of the "yawing moment", which is around the Z axis. Whether it is around the Z body axis or the Z stability axis depends on context (such as a subscript).

- "P" or "p" is used for angular rate about the X axis ("Roll rate about the roll axis"). Whether it is around the X body axis or the X stability axis depends on context (such as a subscript).
- "Q" or "q" is used for angular rate about the Y axis ("Pitch rate about the pitch axis").
- "R" or "r" is used for angular rate about the Z axis ("Yaw rate about the yaw axis"). Whether it is around the Z body axis or the Z stability axis depends on context (such as a subscript).

## *Equations of Motion*

The use of stability derivatives is most conveniently demonstrated with missile or rocket configurations, because these exhibit greater symmetry than aeroplanes, and the equations of motion are correspondingly simpler. If it is assumed that the vehicle is roll-controlled, the pitch and yaw motions may be treated in isolation. It is common practice to consider the yaw plane, so that only 2D motion need be considered. Furthermore, it is assumed that thrust equals drag, and the longitudinal equation of motion may be ignored.



Derivation of Equations of Motion

The body is oriented at angle ψ (psi) with respect to inertial axes. The body is oriented at an angle β (beta) with respect to the velocity vector, so that the components of velocity in body axes are:

$$u = U\cos\beta$$
$$v = U\sin\beta$$

where U is the speed.

The aerodynamic forces are generated with respect to body axes, which is not an inertial frame. In order to calculate the motion, the forces must be referred to inertial axes. This requires the body components of velocity to be resolved through the heading angle (β) into inertial axes.

Resolving into fixed (inertial) axes:

$$u_f = U\cos(\beta)\cos(\psi) - U\sin(\beta)\sin(\psi) = U\cos(\beta + \psi)$$
$$v_f = U\sin(\beta)\cos(\psi) + U\cos(\beta)\sin(\psi) = U\sin(\beta + \psi)$$

The acceleration with respect to inertial axes is found by differentiating these components of velocity with respect to time:

$$\frac{du_f}{dt} = \frac{dU}{dt}\cos(\beta+\psi) - U\frac{d(\beta+\psi)}{dt}\sin(\beta+\psi)$$
$$\frac{dv_f}{dt} = \frac{dU}{dt}\sin(\beta+\psi) + U\frac{d(\beta+\psi)}{dt}\cos(\beta+\psi)$$

From Newton's Second Law, this is equal to the force acting divided by the mass. Now forces arise from the pressure distribution over the body, and hence are generated in body axes, and not in inertial axes, so the body forces must be resolved to inertial axes, as Newton's Second Law does not apply in its simplest form to an accelerating frame of reference.

Resolving the body forces:

$X_f = X\cos(\psi) - Y\sin(\psi)$
$Y_f = Y\cos(\psi) + X\sin(\psi)$

Newton's Second Law, assuming constant mass:

$$X_f = m\frac{du_f}{dt}$$
$$Y_f = m\frac{dv_f}{dt}$$

where $m$ is the mass. Equating the inertial values of acceleration and force, and resolving back into body axes, yields the equations of motion:

$$X = m\frac{dU}{dt}\cos(\beta) - mU\frac{d(\beta+\psi)}{dt}\sin(\beta)$$
$$Y = m\frac{dU}{dt}\sin(\beta) + mU\frac{d(\beta+\psi)}{dt}\cos(\beta)$$

The sideslip, $\beta$, is a small quantity, so the small perturbation equations of motion become:

$$X = m\frac{dU}{dt}$$
$$Y = mU\frac{d(\beta+\psi)}{dt}$$

The first resembles the usual expression of Newton's Second Law, whilst the second is essentially the centrifugal acceleration. The equation of motion governing the rotation of the body is derived from the time derivative of angular momentum:

$$N = C\frac{d^2\psi}{dt^2}$$

where C is the moment of inertia about the yaw axis. Assuming constant speed, there are only two state variables; β and $\frac{d\psi}{dt}$, which will be written more compactly as the yaw rate r. There is one force and one moment, which for a given flight condition will each be functions of β, r and their time derivatives. For typical missile configurations the forces and moments depend, in the short term, on β and r. The forces may be expressed in the form:

$$Y = Y_0 + \frac{\partial Y}{\partial \beta}\beta + \frac{\partial Y}{\partial r}r$$

where $Y_0$ is the force corresponding to the equilibrium condition (usually called the trim) whose stability is being investigated. It is common practice to employ a shorthand:

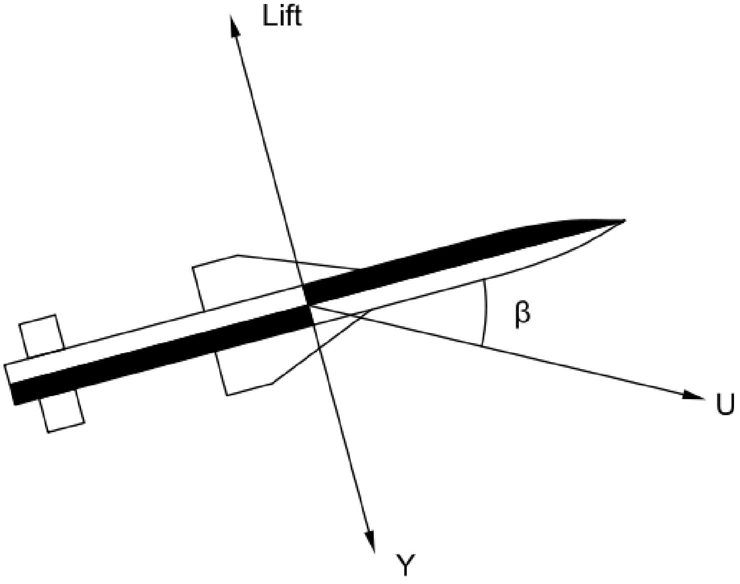
$$\frac{\partial Y}{\partial \beta} = Y_\beta$$

The partial derivative $\frac{\partial Y}{\partial \beta}$ and all similar terms characterising the increments in forces and moments due to increments in the state variables are called stability derivatives. Typically, $\frac{\partial Y}{\partial r}$ is insignificant for missile configurations, so the equations of motion reduce to:

$$\frac{d\beta}{dt} = \frac{Y_\beta}{mU}\beta - r$$
$$\frac{dr}{dt} = \frac{N_\beta}{C}\beta + \frac{N_r}{C}r$$

## *Stability Derivative Contributions*

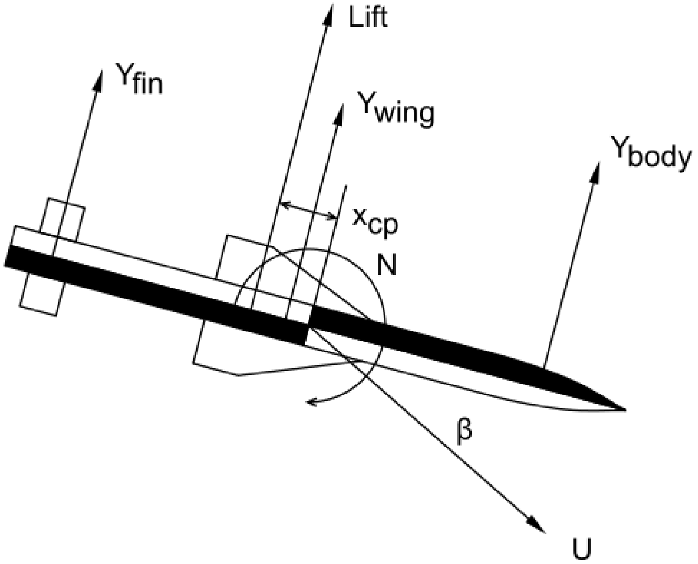
Each stability derivative is determined by the position, size, shape and orientation of the missile components. In aircraft, the directional stability determines such features as dihedral of the main planes, size of fin and area of tailplane, but the large number of important stability derivatives involved precludes a detailed discussion. The missile is characterised by only three stability derivatives, and hence provides a useful introduction to the more complex aeroplane dynamics.



Sign of $Y_\beta$

This diagram shows *lift* as perpendicular to the longitudinal body axis. In most technical usage, lift is perpendicular to the oncoming flow. That is, perpendicular to the longitudinal *stability* axis.

Consider first $Y_\beta$, a body at an angle of attack β generates a lift force in the opposite direction to the motion of the body. For this reason $Y_\beta$ is always negative.

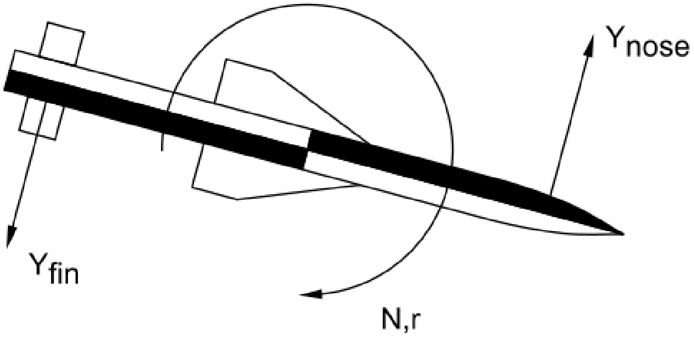


Static Stability and $N_\beta$

This diagram shows *lift* as perpendicular to the longitudinal body axis. In most technical usage, lift is perpendicular to the oncoming flow. That is, perpendicular to the longitudinal *stability* axis.

At low angles of attack, the lift is generated primarily by the wings, fins and the nose region of the body. The total lift acts at a distance $x_{cp}$ ahead of the centre of gravity (it has a negative value in the figure), this, in missile parlance, is the centre of pressure . If the lift acts ahead of the centre of gravity, the yawing moment will be negative, and will tend to increase the angle of attack, increasing both the lift and the moment further. It follows that the centre of pressure must lie aft of the centre of gravity for static stability. $x_{cp}$ is the static margin and must be negative for longitudinal static stability. Alternatively, positive angle of attack must generate positive yawing moment on a statically stable missile, i.e. $N_\beta$ must be positive. It is common practice to design manoeuvrable missiles with near zero static margin (i.e. neutral static stability).

The need for positive $N_\beta$ explains why arrows and darts have flights and unguided rockets have fins.



Yaw Dumping ($N_r$)

The effect of angular velocity is mainly to decrease the nose lift and increase the tail lift, both of which act in a sense to oppose the rotation. $N_r$ is therefore always negative. There is a contribution from the wing, but since missiles tend to have small static margins (typically less than a calibre), this is usually small. Also the fin contribution is greater than that of the nose, so there is a net force $Y_r$, but this is usually insignificant compared with $Y_\beta$ and is usually ignored.

## Response

Manipulation of the equations of motion yields a second order homogeneous linear differential equation in the angle of attack β:

$$\frac{d^2\beta}{dt^2} - \left(\frac{Y_\beta}{mU} + \frac{N_r}{C}\right)\frac{d\beta}{dt} + \left(\frac{N_\beta}{C} + \frac{Y_\beta}{mU}\frac{N_r}{C}\right)\beta = 0$$

The qualitative behavior of this equation is considered in directional stability. Since $Y_\beta$ and $N_r$ are both negative, the damping is positive. The stiffness does not only depend on the static stability term $N_\beta$, it also contains a term which effectively determines the angle of attack due to the body rotation. The distance of the center of lift, including this term, ahead of the centre of gravity is called the maneuver margin. It must be negative for stability.

This damped oscillation in angle of attack and yaw rate, following a disturbance, is called the 'weathercock' mode, after the tendency of a weathercock to point into wind.

## Comments

The state variables were chosen to be the angle of attack β and the yaw rate r, and have omitted the speed perturbation u, together with the associated derivatives e.g. $Y_u$. This may appear arbitrary. However, since the timescale of the speed variation is much greater than that of the variation in angle of attack, its effects are negligible as far as the directional stability of the vehicle is concerned. Similarly, the effect of roll on yawing motion was also ignored, because missiles generally have low aspect ratio configurations and the roll inertia is much less than the yaw inertia, consequently the roll loop is expected to be much faster than the yaw response, and is ignored. These simplifications of the problem based on *a priori* knowledge, represent an engineer's approach. Mathematicians prefer to keep the problem as general as possible and only simplify it at the end of the analysis, if at all.

Aircraft dynamics is more complex than missile dynamics, mainly because the simplifications, such as separation of fast and slow modes, and the similarity between pitch and yaw motions, are not obvious from the equations of motion, and are consequently deferred until a late stage of the analysis. Subsonic transport aircraft have high aspect ratio configurations, so that yaw and roll cannot be treated as decoupled. However, this is merely a matter of degree; the basic ideas needed to understand aircraft dynamics are covered in this simpler analysis of missile motion.

## Control Derivatives

Deflection of control surfaces modifies the pressure distribution over the vehicle, and these are dealt with by including perturbations in forces and moments due to control deflection. The fin deflection is normally denoted ζ (zeta). Including these terms, the equations of motion become:

$$\frac{d\beta}{dt} = \frac{Y_\beta}{mU}\beta - r + \frac{Y_\zeta}{mU}\zeta$$
$$\frac{dr}{dt} = \frac{N_\beta}{C}\beta + \frac{N_r}{C}r + \frac{N_\zeta}{C}\zeta$$

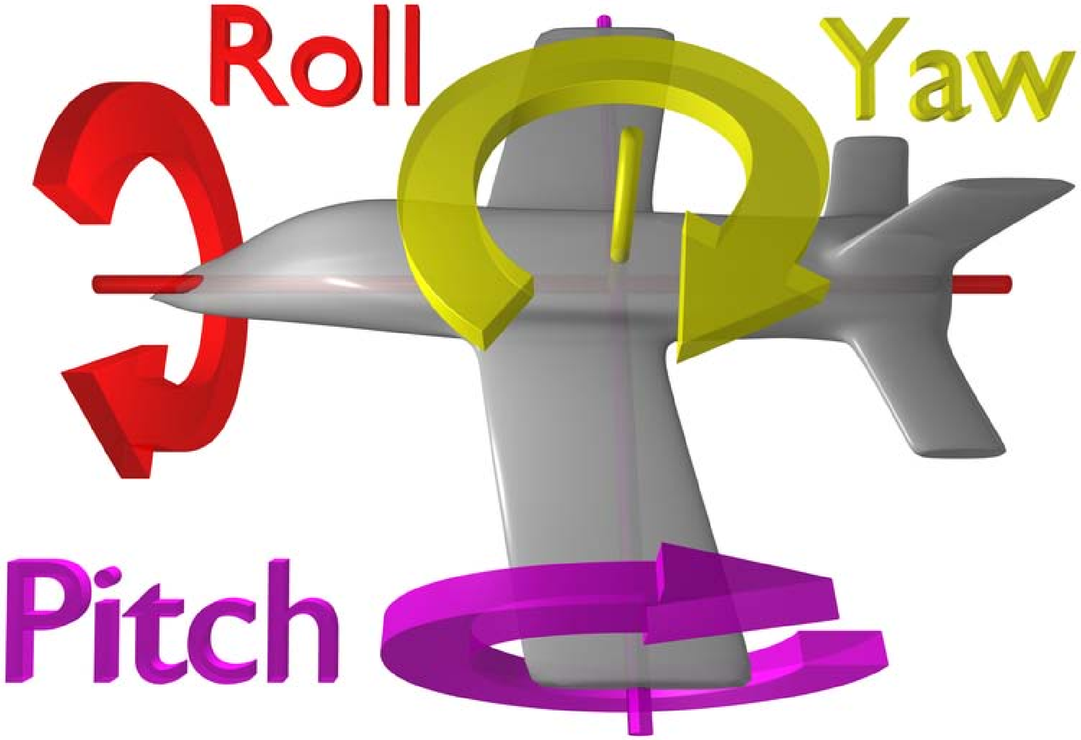
Including the control derivatives enables the response of the vehicle to be studied, and the equations of motion used to design the autopilot.

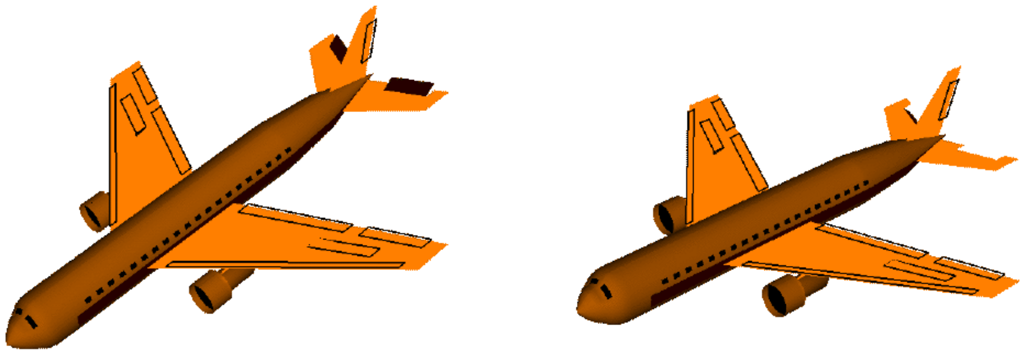
### *Examples*

- $\mathbf{C_{L\beta}}$ , called *Dihedral Effect*, is a stability derivative that measures changes in *rolling moment* as Angle of sideslip changes. The "L" indicates *rolling* moment and the β indicates sideslip angle.

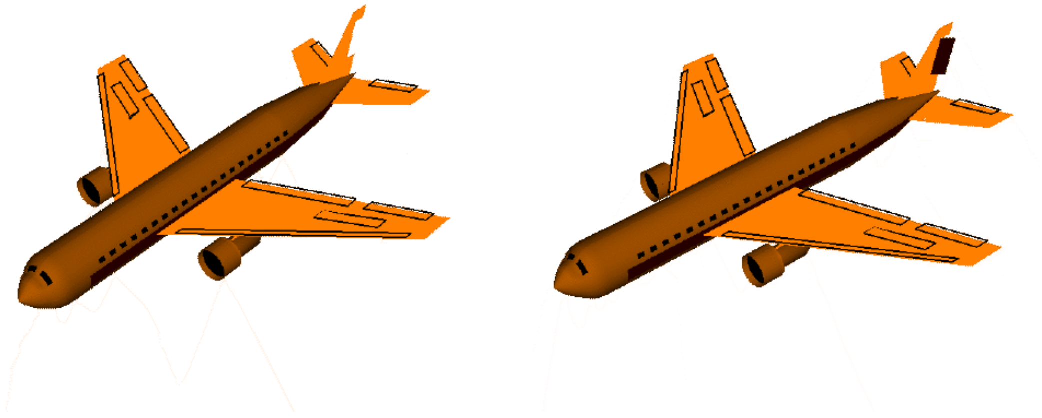
# Chapter 9

# Flight Dynamics

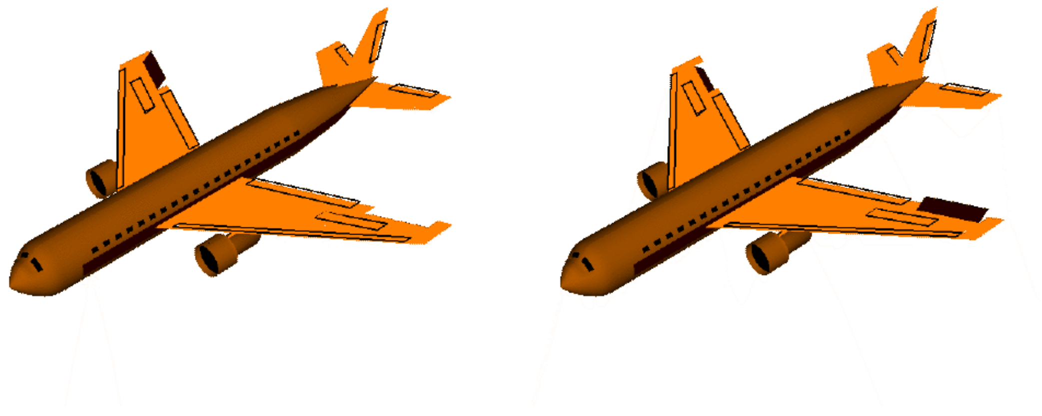




Pitch



Yaw



Roll

**Flight dynamics** is the science of air vehicle orientation and control in three dimensions. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as *pitch*, *roll* and *yaw* (quite different from their use as Tait-Bryan angles).

Aerospace engineers develop control systems for a vehicle's orientation (attitude) about its center of mass. The control systems include actuators, which exert forces in various directions, and generate rotational forces or moments about the aerodynamic center of the aircraft, and thus rotate the aircraft in pitch, roll, or yaw. For example, a pitching moment is a vertical force applied at a distance forward or aft from the aerodynamic center of the aircraft, causing the aircraft to pitch up or down.

Roll, pitch and yaw refer to rotations about the respective axes starting from a defined equilibrium state. The equilibrium roll angle is known as wings level or zero bank angle, equivalent to a level heeling angle on a ship. Yaw is known as "heading". The equilibrium pitch angle in submarine and airship parlance is known as "trim", but in aircraft, this usually refers to angle of attack, rather than orientation. However, common usage ignores this distinction between equilibrium and dynamic cases.

The most common aeronautical convention defines the roll as acting about the longitudinal axis, positive with the starboard (right) wing down. The yaw is about the vertical body axis, positive with the nose to starboard. Pitch is about an axis perpendicular to the longitudinal plane of symmetry, positive nose up.

A fixed-wing aircraft increases or decreases the lift generated by the wings when it pitches nose up or down by increasing or decreasing the angle of attack (AOA). The roll angle is also known as bank angle on a fixed wing aircraft, which usually "banks" to change the horizontal direction of flight. An aircraft is usually streamlined from nose to tail to reduce drag making it typically advantageous to keep the sideslip angle near zero, though there are instances when an aircraft may be deliberately "sideslipped" for example a slip in a fixed wing aircraft.

## *Introduction*

### Basic coordinate systems

The position (and hence motion) of an aircraft is generally defined relative to one of 3 sets of co-ordinate systems:

- Wind axes
    - X axis - positive in the direction of the oncoming air (relative wind)
    - Y axis - positive to right of X axis, perpendicular to X axis
    - Z axis - positive downwards, perpendicular to X-Y plane

- Inertial axes (or body axes) - based about aircraft CG
    - X axis - positive forward, through nose of aircraft

  - Y axis - positive to right of X axis, perpendicular to X axis
  - Z axis - positive downwards, perpendicular to X-Y plane

- Earth Axes
  - X axis - positive in the direction of north
  - Y axis - positive in the direction of east (perpendicular to X axis)
  - Z axis - positive towards the center of Earth (perpendicular to X-Y plane)

For flight dynamics applications the earth axes are generally of minimal use, and hence will be ignored. The motions relevant to dynamic stability are usually too short in duration for the motion of the Earth itself to be considered relevant for aircraft.

In flight dynamics, pitch, roll and yaw angles measure both the absolute attitude angles (relative to the horizon/North) and *changes* in attitude angles, relative to the equilibrium orientation of the vehicle. These are defined as:

- Pitch - angle of X body axis (nose) relative to horizon. Also a positive (nose up) rotation about Y body axis
- Roll - angle of Y body axis (wing) relative to horizon. Also a positive (right wing down) rotation about X body axis
- Yaw - angle of X body axis (nose) relative to North. Also a positive (nose right) rotation about Z body axis

In analyzing the dynamics, we are concerned both with rotation and translation of this axis set with respect to a fixed inertial frame. For all practical purposes a local Earth axis set is used, this has X and Y axis in the local horizontal plane, usually with the x-axis coinciding with the projection of the velocity vector at the start of the motion, on to this plane. The z axis is vertical, pointing generally towards the Earth's center, completing an orthogonal set.

In general, the body axes are not aligned with the Earth axes. The body orientation may be defined by three Euler angles, the Tait-Bryan rotations, a quaternion, or a direction cosine matrix (rotation matrix). A rotation matrix is particularly convenient for converting velocity, force, angular velocity, and torque vectors between body and Earth coordinate frames.

Body axes tend to be used with missile and rocket configurations. Aircraft stability uses wind axes in which the x-axis points along the velocity vector. For straight and level flight this is found from body axes by rotating nose down through the angle of attack.

Stability deals with small perturbations in angular displacements about the orientation at the start of the motion. This consists of two components; rotation about each axis, and angular displacements due change in orientation of each axis. The latter term is of second order for the purpose of stability analysis, and is ignored.

### Design cases

In analyzing the stability of an aircraft, it is usual to consider perturbations about a nominal equilibrium position. So the analysis would be applied, for example, assuming:

Straight and level flight
Turn at constant speed
Approach and landing
Takeoff

The speed, height and trim angle of attack are different for each flight condition, in addition, the aircraft will be configured differently, e.g. at low speed flaps may be deployed and the undercarriage may be down.

Except for asymmetric designs (or symmetric designs at significant sideslip), the longitudinal equations of motion (involving pitch and lift forces) may be treated independently of the lateral motion (involving roll and yaw).

The following considers perturbations about a nominal straight and level flight path.

To keep the analysis (relatively) simple, the control surfaces are assumed fixed throughout the motion, this is stick-fixed stability. Stick-free analysis requires the further complication of taking the motion of the control surfaces into account.

Furthermore, the flight is assumed to take place in still air, and the aircraft is treated as a rigid body.

## *Aerodynamic and propulsive forces*

### Aerodynamic forces

### Components of the aerodynamic force

The expression to calculate the aerodynamic force is:

$$\mathbf{F}_A = \int_{\Sigma} \left(-\Delta p \mathbf{n} + \mathbf{f}\right) d\sigma$$

where:

$\Delta p \equiv$ Difference between static pressure and free current pressure
$\mathbf{n} \equiv$ outer normal vector of the element of area
$\mathbf{f} \equiv$ tangential stress vector practised by the air on the body
$\Sigma \equiv$ adequate reference surface

projected on wind axes we obtain:

$$\mathbf{F}_A = -(\mathbf{i}_w D + \mathbf{j}_w Q + \mathbf{k}_w L)$$

where:

$D$ ≡Drag
$Q$ ≡Lateral force
$L$ ≡Lift

## Aerodynamic coefficients

Dynamic pressure of the free current $\equiv q = \frac{1}{2}\,\rho V^2$

Proper reference surface (wing surface, in case of planes) $\equiv S$

Pressure coefficient $\equiv C_p = \dfrac{p - p_\infty}{q}$

Friction coefficient $\equiv C_f = \dfrac{f}{q}$

Drag coefficient $\equiv C_d = \dfrac{D}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{i_w} + C_f \mathbf{t} \bullet \mathbf{i_w}]\, d\sigma$

Lateral force coefficient $\equiv C_Q = \dfrac{Q}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{j_w} + C_f \mathbf{t} \bullet \mathbf{j_w}]\, d\sigma$

Lift coefficient $\equiv C_L = \dfrac{L}{qS} = -\dfrac{1}{S}\displaystyle\int_\Sigma [(-C_p)\mathbf{n} \bullet \mathbf{k_w} + C_f \mathbf{t} \bullet \mathbf{k_w}]\, d\sigma$

It is necessary to know $C_p$ and $C_f$ in every point on the considered surface.

## Dimensionless parameters and aerodynamic regimes

In absence of thermal effects, there are three remarkable dimensionless numbers:

- Compressibility of the flow:

  Mach number $\equiv M = \dfrac{V}{a}$

- Viscosity of the flow:

Reynolds number $\equiv Re = \frac{\rho V l}{\mu}$

- Rarefaction of the flow:

Knudsen number $\equiv Kn = \frac{\lambda}{l}$

where:

$a = \sqrt{kR\theta} \equiv$speed of sound
$R \equiv$gas constant by mass unity
$\theta \equiv$absolute temperature
$\lambda = \frac{\mu}{\rho}\sqrt{\frac{\pi}{2R\theta}} = \frac{M}{Re}\sqrt{\frac{k\pi}{2}} \equiv$mean free path

According to λ there are three possible rarefaction grades and their corresponding motions are called:

- Continuum current (negligible rarefaction): $\frac{M}{Re} \ll 1$
- Transition current (moderate rarefaction): $\frac{M}{Re} \approx 1$
- Free molecular current (high rarefaction): $\frac{M}{Re} \gg 1$

The motion of a body through a flow is considered, in flight dynamics, as continuum current. In the outer layer of the space that surrounds the body viscosity will be negligible. However viscosity effects will have to be considered when analysing the flow in the nearness of the boundary layer.

Depending on the compressibility of the flow, different kinds of currents can be considered:

- Incompressible subsonic current: $0 < M < 0.5$
- Compressible subsonic current: $0.5 < M < 0.8$
- Transonic current: $0.8 < M < 1.2$
- Supersonic current: $1.2 < M < 5$
- Hypersonic current: $5 < M$

## Drag coefficient equation and aerodynamic efficiency

If the geometry of the body is fixed and in case of symmetric flight ($\beta=0$ and $Q=0$), pressure and friction coefficients are functions depending on:

$$C_p = C_p(\alpha, M, Re, P)$$
$$C_f = C_f(\alpha, M, Re, P)$$

where:

$\alpha$ ≡angle of attack
$P$ ≡considered point of the surface

Under these conditions, drag and lift coefficient are functions depending exclusively on the angle of attack of the body and Mach and Reynolds numbers. Aerodynamic efficiency, defined as the relation between lift and drag coefficients, will depend on those parameters as well.

$$\begin{cases} C_D = C_D(\alpha, M, Re) \\ C_L = C_L(\alpha, M, Re) \\ E = E(\alpha, M, Re) = \dfrac{C_L}{C_D} \end{cases}$$

It is also possible to get the dependency of the drag coefficient respect to the lift coefficient. This relation is konwn as the drag coefficient equation:

$C_D = C_D(C_L, M, Re) \equiv$drag coefficient equation

The aerodynamic efficiency has a maximum value, $E_{max}$, respect to $C_L$ where the tangent line from the coordinate origin touches the drag coefficient equation plot.

The drag coefficient, $C_D$, can be decomposed in two ways. First typical decomposition separates pressure and friction effects:

$$C_D = C_{Df} + C_{Dp} \begin{cases} C_{Df} = \dfrac{D}{qS} = -\dfrac{1}{S} \int_\Sigma C_f \mathbf{t} \bullet \mathbf{i_w} \, d\sigma \\ C_{Dp} = \dfrac{D}{qS} = -\dfrac{1}{S} \int_\Sigma (-C_p) \mathbf{n} \bullet \mathbf{i_w} \, d\sigma \end{cases}$$

There´s a second typical decomposition taking into account the definition of the drag coefficient ecuation. This decomposition separates the effect of the lift coefficient in the equation, obtaining two terms $C_{D0}$ and $C_{Di}$. $C_{D0}$ is known as the parasitic drag coefficient and it is the base draft coefficient at zero lift. $C_{Di}$ is known as the induced drag coefficient and it is produced by the body lift.

$$C_D = C_{D0} + C_{Di} \begin{cases} C_{D0} = (C_D)_{C_L=0} \\ C_{Di} \end{cases}$$

## Parabolic and generic drag coefficient

A good attempt for the induced drag coefficient is to assume a parabolic dependency of the lift

$$C_{Di} = kC_L^2 \Rightarrow C_D = C_{D0} + kC_L^2$$

Aerodynamic efficiency is now calculated as:

$$E = \frac{C_L}{C_{D0} + kC_L^2} \Rightarrow \begin{cases} E_{max} = \dfrac{1}{2\sqrt{kC_{D0}}} \\ (C_L)_{Emax} = \sqrt{\dfrac{C_{D0}}{k}} \\ (C_{Di})_{Emax} = C_{D0} \end{cases}$$

If the configuration of the pane is symmetrical respect to the XY plane, minimum drag coefficient equals to the parasitic drag of the plane.

$$C_{Dmin} = (C_D)_{CL=0} = C_{D0}$$

In case the configuration is asymmetrical respect to the XY plane, however, minimum flag difers from the parasitic drag. On these cases, a new approximate parabolic drag equation can be traced leaving the minimum drag value at zero lift value.

$$C_{Dmin} = C_{DM} \neq (C_D)_{CL=0}$$
$$C_D = C_{DM} + k(C_L - C_{LM})^2$$

## *Dynamic stability and control*

### Longitudinal modes

It is common practice to derive a fourth order characteristic equation to describe the longitudinal motion, and then factorize it approximately into a high frequency mode and a low frequency mode. This requires a level of algebraic manipulation which most readers will doubtless find tedious, and adds little to the understanding of aircraft dynamics. The approach adopted here is to use our qualitative knowledge of aircraft behavior to simplify the equations from the outset, reaching the same result by a more accessible route.

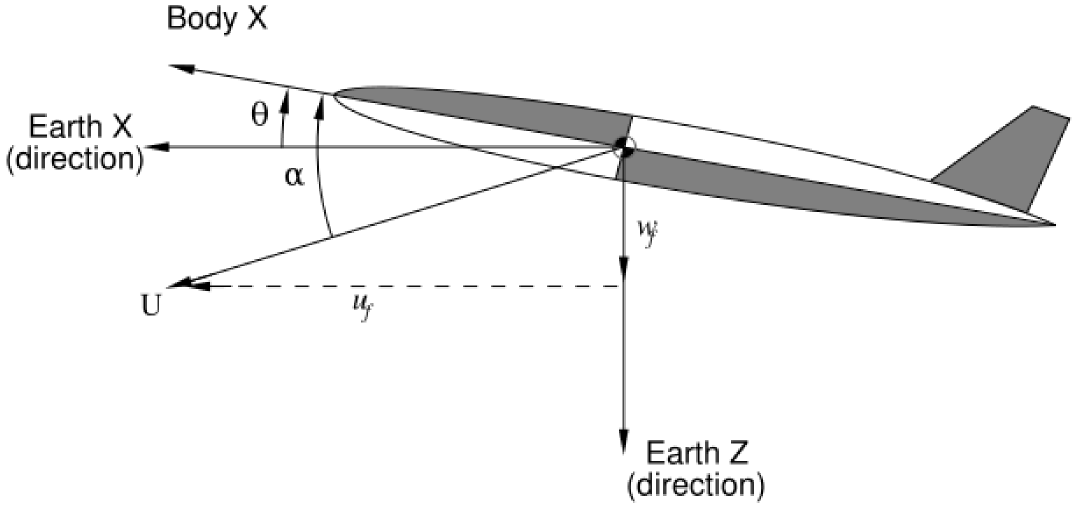
The two longitudinal motions (modes) are called the short period pitch oscillation (SPPO), and the phugoid.

## Short-period pitch oscillation

A short input (in control systems terminology an impulse) in pitch (generally via the elevator in a standard configuration fixed wing aircraft) will generally lead to overshoots about the trimmed condition. The transition is characterized by a damped simple harmonic motion about the new trim. There is very little change in the trajectory over the time it takes for the oscillation to damp out.

Generally this oscillation is high frequency (hence short period) and is damped over a period of a few seconds. A real-world example would involve a pilot selecting a new climb attitude, for example 5º nose up from the original attitude. A short, sharp pull back on the control column may be used, and will generally lead to oscillations about the new trim condition. If the oscillations are poorly damped the aircraft will take a long period of time to settle at the new condition, potentially leading to Pilot-induced oscillation. If the short period mode is unstable it will generally be impossible for the pilot to safely control the aircraft for any period of time.

This damped harmonic motion is called the short period pitch oscillation, it arises from the tendency of a stable aircraft to point in the general direction of flight. It is very similar in nature to the weathercock mode of missile or rocket configurations. The motion involves mainly the pitch attitude θ (theta) and incidence α (alpha). The direction of the velocity vector, relative to inertial axes is θ − α. The velocity vector is:



Longitudinal Equations of Motion

$u_f = U\cos(\theta - \alpha)$
$w_f = U\sin(\theta - \alpha)$

where $u_f, w_f$ are the inertial axes components of velocity. According to Newton's Second Law, the accelerations are proportional to the forces, so the forces in inertial axes are:

$$X_f = m\frac{du_f}{dt} = m\frac{dU}{dt}\cos(\theta - \alpha) - mU\frac{d(\theta - \alpha)}{dt}\sin(\theta - \alpha)$$
$$Z_f = m\frac{dw_f}{dt} = m\frac{dU}{dt}\sin(\theta - \alpha) + mU\frac{d(\theta - \alpha)}{dt}\cos(\theta - \alpha)$$

where $m$ is the mass. By the nature of the motion, the speed variation $m\frac{dU}{dt}$ is negligible over the period of the oscillation, so:

$$X_f = -mU\frac{d(\theta - \alpha)}{dt}\sin(\theta - \alpha)$$
$$Z_f = mU\frac{d(\theta - \alpha)}{dt}\cos(\theta - \alpha)$$

But the forces are generated by the pressure distribution on the body, and are referred to the velocity vector. But the velocity (wind) axes set is not an inertial frame so we must resolve the fixed axes forces into wind axes. Also, we are only concerned with the force along the z-axis:

$Z = -Z_f\cos(\theta - \alpha) + X_f\sin(\theta - \alpha)$

Or:

$$Z = -mU\frac{d(\theta - \alpha)}{dt}$$

In words, the wind axes force is equal to the centripetal acceleration.

The moment equation is the time derivative of the angular momentum:

$$M = B\frac{d^2\theta}{dt^2}$$

where M is the pitching moment, and B is the moment of inertia about the pitch axis. Let: $\frac{d\theta}{dt} = q$, the pitch rate. The equations of motion, with all forces and moments referred to wind axes are, therefore:

$$\frac{d\alpha}{dt} = q + \frac{Z}{mU}$$
$$\frac{dq}{dt} = \frac{M}{B}$$

We are only concerned with perturbations in forces and moments, due to perturbations in the states α and q, and their time derivatives. These are characterized by stability derivatives determined from the flight condition. The possible stability derivatives are:

> $Z_\alpha$ Lift due to incidence, this is negative because the z-axis is downwards whilst positive incidence causes an upwards force.
> $Z_q$ Lift due to pitch rate, arises from the increase in tail incidence, hence is also negative, but small compared with $Z_\alpha$.
> $M_\alpha$ Pitching moment due to incidence - the static stability term. Static stability requires this to be negative.
> $M_q$ Pitching moment due to pitch rate - the pitch damping term, this is always negative.

Since the tail is operating in the flowfield of the wing, changes in the wing incidence cause changes in the downwash, but there is a delay for the change in wing flowfield to affect the tail lift, this is represented as a moment proportional to the rate of change of incidence:

$$M_{\dot\alpha}$$

Increasing the wing incidence without increasing the tail incidence produces a nose up moment, so $M_{\dot\alpha}$is expected to be positive.

The equations of motion, with small perturbation forces and moments become:

$$\frac{d\alpha}{dt} = \left(1 + \frac{Z_q}{mU}\right) q + \frac{Z_\alpha}{mU}\alpha$$
$$\frac{dq}{dt} = \frac{M_q}{B}q + \frac{M_\alpha}{B}\alpha + \frac{M_{\dot\alpha}}{B}\dot\alpha$$

These may be manipulated to yield as second order linear differential equation in α:

$$\frac{d^2\alpha}{dt^2} - \left(\frac{Z_\alpha}{mU} + \frac{M_q}{B} + (1 + \frac{Z_q}{mU})\frac{M_{\dot\alpha}}{B}\right)\frac{d\alpha}{dt} + \left(\frac{Z_\alpha}{mU}\frac{M_q}{B} - \frac{M_\alpha}{B}(1 + \frac{Z_q}{mU})\right)\alpha = 0$$

This represents a damped simple harmonic motion.

We should expect $\frac{Z_q}{mU}$ to be small compared with unity, so the coefficient of α (the 'stiffness' term) will be positive, provided $M_\alpha < \frac{Z_\alpha}{mU} M_q$. This expression is dominated by $M_\alpha$, which defines the longitudinal static stability of the aircraft, it must be negative for stability. The damping term is reduced by the downwash effect, and it is difficult to design an aircraft with both rapid natural response and heavy damping. Usually, the response is underdamped but stable.

## Phugoid

If the stick is held fixed, the aircraft will not maintain straight and level flight, but will start to dive, level out and climb again. It will repeat this cycle until the pilot intervenes. This long period oscillation in speed and height is called the phugoid mode. This is analyzed by assuming that the SSPO performs its proper function and maintains the angle of attack near its nominal value. The two states which are mainly affected are the climb angle γ (gamma) and speed. The small perturbation equations of motion are:

$$mU\frac{d\gamma}{dt} = -Z$$

which means the centripetal force is equal to the perturbation in lift force.

For the speed, resolving along the trajectory:

$$m\frac{du}{dt} = X - mg\gamma$$

where g is the acceleration due to gravity at the earths surface. The acceleration along the trajectory is equal to the net x-wise force minus the component of weight. We should not expect significant aerodynamic derivatives to depend on the climb angle, so only $X_u$ and $Z_u$ need be considered. $X_u$ is the drag increment with increased speed, it is negative, likewise $Z_u$ is the lift increment due to speed increment, it is also negative because lift acts in the opposite sense to the z-axis.

The equations of motion become:

$$mU\frac{d\gamma}{dt} = -Z_u u$$
$$m\frac{du}{dt} = X_u u - mg\gamma$$

These may be expressed as a second order equation in climb angle or speed perturbation:

$$\frac{d^2u}{dt^2} - \frac{X_u}{m}\frac{du}{dt} - \frac{Z_u g}{mU}u = 0$$

Now lift is very nearly equal to weight:

$$Z = \frac{1}{2}\rho U^2 c_L S_w = W$$

where ρ is the air density, $S_w$ is the wing area, W the weight and $c_L$ is the lift coefficient (assumed constant because the incidence is constant), we have, approximately:

$$Z_u = \frac{2W}{U} = \frac{2mg}{U}$$

The period of the phugoid, T, is obtained from the coefficient of u:

$$\frac{2\pi}{T} = \sqrt{\frac{2g^2}{U^2}}$$

Or:

$$T = \frac{2\pi U}{\sqrt{2}g}$$

Since the lift is very much greater than the drag, the phugoid is at best lightly damped. A propeller with fixed speed would help. Heavy damping of the pitch rotation or a large rotational inertia increase the coupling between short period and phugoid modes, so that these will modify the phugoid.

## Lateral modes

With a symmetrical rocket or missile, the directional stability in yaw is the same as the pitch stability; it resembles the short period pitch oscillation, with yaw plane equivalents to the pitch plane stability derivatives. For this reason pitch and yaw directional stability are collectively known as the "weathercock" stability of the missile.

Aircraft lack the symmetry between pitch and yaw, so that directional stability in yaw is derived from a different set of stability derivatives. The yaw plane equivalent to the short period pitch oscillation, which describes yaw plane directional stability is called Dutch roll. Unlike pitch plane motions, the lateral modes involve both roll and yaw motion.

## Dutch roll

It is customary to derive the equations of motion by formal manipulation in what, to the engineer, amounts to a piece of mathematical sleight of hand. The current approach follows the pitch plane analysis in formulating the equations in terms of concepts which are reasonably familiar.

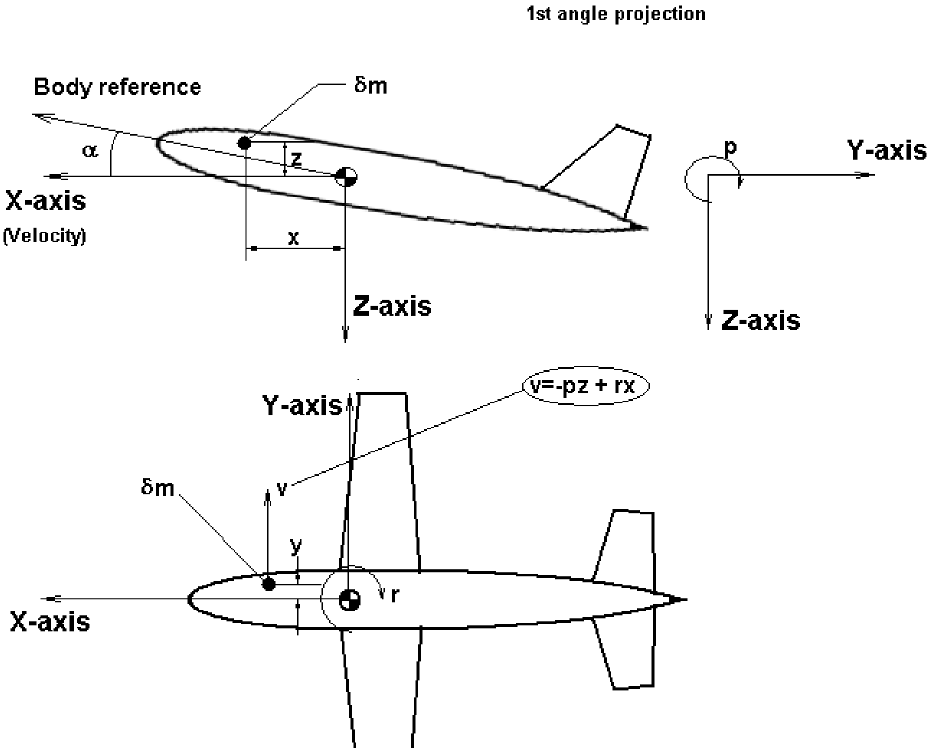
Applying an impulse via the rudder pedals should induce Dutch roll, which is the oscillation in roll and yaw, with the roll motion lagging yaw by a quarter cycle, so that the wing tips follow elliptical paths with respect to the aircraft.

The yaw plane translational equation, as in the pitch plane, equates the centripetal acceleration to the side force.

$$\frac{d\beta}{dt} = \frac{Y}{mU} - r$$

where β (beta) is the sideslip angle, Y the side force and r the yaw rate.

The moment equations are a bit trickier. The trim condition is with the aircraft at an angle of attack with respect to the airflow. The body x-axis does not align with the velocity vector, which is the reference direction for wind axes. In other words, wind axes are not principal axes (the mass is not distributed symmetrically about the yaw and roll axes). Consider the motion of an element of mass in position -z,x in the direction of the y-axis, i.e. into the plane of the paper.



Lateral Equations Product of Inertia

If the roll rate is p, the velocity of the particle is:

$$v = -pz + xr$$

Made up of two terms, the force on this particle is first the proportional to rate of v change, the second is due to the change in direction of this component of velocity as the body moves. The latter terms gives rise to cross products of small quantities (pq,pr,qr), which are later discarded. In this analysis, they are discarded from the outset for the sake of clarity. In effect, we assume that the direction of the velocity of the particle due to the simultaneous roll and yaw rates does not change significantly throughout the motion. With this simplifying assumption, the acceleration of the particle becomes:

$$\frac{dv}{dt} = -\frac{dp}{dt}z + \frac{dr}{dt}x$$

The yawing moment is given by:

$$\delta mx\frac{dv}{dt} = -\frac{dp}{dt}xz\delta m + \frac{dr}{dt}x^2\delta m$$

There is an additional yawing moment due to the offset of the particle in the y direction: $\frac{dr}{dt}y^2\delta m$

The yawing moment is found by summing over all particles of the body:

$$N = -\frac{dp}{dt}\int xzdm + \frac{dr}{dt}\int x^2 + y^2 dm = -E\frac{dp}{dt} + C\frac{dr}{dt}$$

where N is the yawing moment, E is a product of inertia, and C is the moment of inertia about the yaw axis. A similar reasoning yields the roll equation:

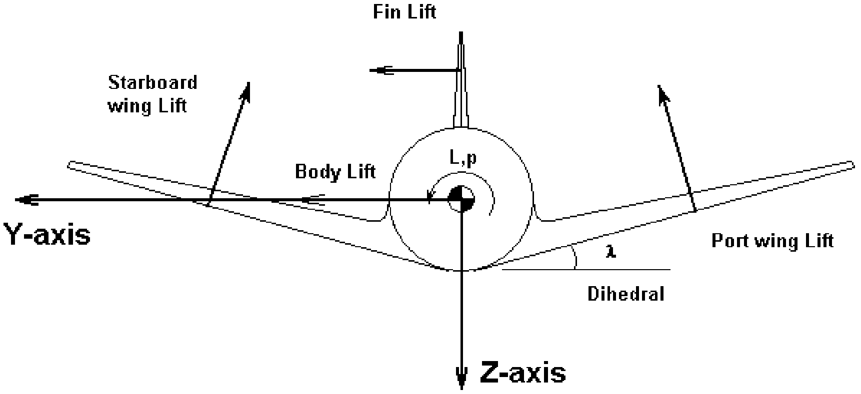
$$L = A\frac{dp}{dt} - E\frac{dr}{dt}$$

where L is the rolling moment and A the roll moment of inertia.

## Lateral and longitudinal stability derivatives

The states are β (sideslip), r (yaw rate) and p (roll rate), with moments N (yaw) and L (roll), and force Y (sideways). There are nine stability derivatives relevant to this motion, the following explains how they originate. However a better intuitive understanding is to be gained by simply playing with a model airplane, and considering how the forces on each component are affected by changes in sideslip and angular velocity:

**View from nose (negative X direction)**



**Lateral Stability - Main Sources of Stabilising Forces and Moments**

$Y_\beta$ Side force due to side slip (in absence of yaw).

Sideslip generates a sideforce from the fin and the fuselage. In addition, if the wing has dihedral, side slip at a positive roll angle increases incidence on the starboard wing and reduces it on the port side, resulting in a net force component directly opposite to the sideslip direction. Sweep back of the wings has the same effect on incidence, but since the wings are not inclined in the vertical plane, backsweep alone does not affect $Y_\beta$. However, anhedral may be used with high backsweep angles in high performance aircraft to offset the wing incidence effects of sideslip. Oddly enough this does not reverse the sign of the wing configuration's contribution to $Y_\beta$ (compared to the dihedral case).

$Y_p$ Side force due to roll rate.

Roll rate causes incidence at the fin, which generates a corresponding side force. Also, positive roll (starboard wing down) increases the lift on the starboard wing and reduces it on the port. If the wing has dihedral, this will result in a side force momentarily opposing the resultant sideslip tendency. Anhedral wing and or stabilizer configurations can cause the sign of the side force to invert if the fin effect is swamped.

$Y_r$ Side force due to yaw rate.

Yawing generates side forces due to incidence at the rudder, fin and fuselage.

$N_\beta$ Yawing moment due to sideslip forces.

Sideslip in the absence of rudder input causes incidence on the fuselage and empennage, thus creating a yawing moment counteracted only by the directional stiffness which would tend to point the aircraft's nose back into the wind in horizontal flight conditions. Under sideslip conditions at a given roll angle $N_\beta$ will tend to point the nose into the sideslip direction even without rudder input, causing a downward spiraling flight.

$N_p$ Yawing moment due to roll rate.

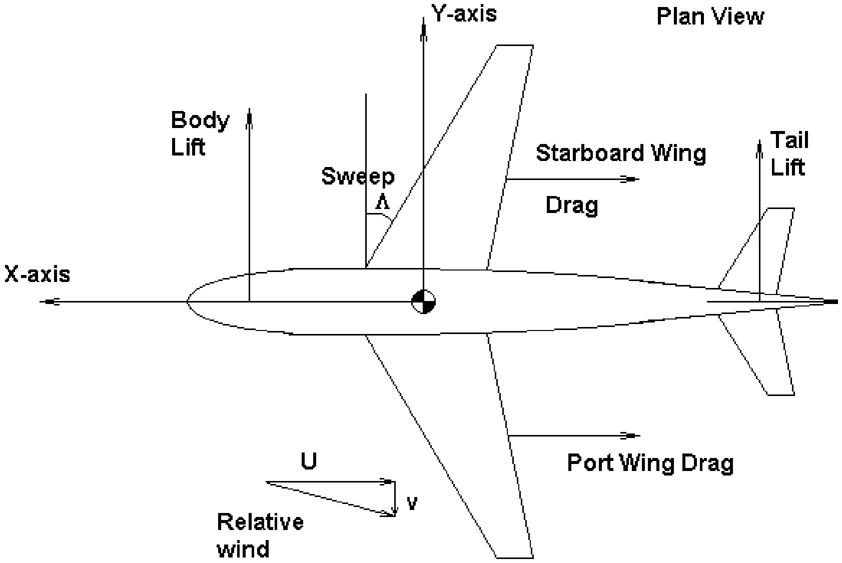
Roll rate generates fin lift causing a yawing moment and also differentially alters the lift on the wings, thus affecting the induced drag contribution of each wing, causing a (small) yawing moment contribution. Positive roll generally causes positive $N_p$ values unless the empennage is anhedral or fin is below the roll axis. Lateral force components resulting from dihedral or anhedral wing lift differences has little effect on $N_p$ because the wing axis is normally closely aligned with the center of gravity.

$N_r$ Yawing moment due to yaw rate.

Yaw rate input at any roll angle generates rudder, fin and fuselage force vectors which dominate the resultant yawing moment. Yawing also increases the speed of the outboard wing whilst slowing down the inboard wing, with corresponding changes in drag causing a (small) opposing yaw moment. $N_r$ opposes the inherent directional stiffness which tends to point the aircraft's nose back into the wind and always matches the sign of the yaw rate input.

$L_\beta$ Rolling moment due to sideslip.

A positive sideslip angle generates empennage incidence which can cause positive or negative roll moment depending on its configuration. For any non-zero sideslip angle dihedral wings causes a rolling moment which tends to return the aircraft to the horizontal, as does back swept wings. With highly swept wings the resultant rolling moment may be excessive for all stability requirements and anhedral could be used to offset the effect of wing sweep induced rolling moment.



Note:
sideslip increases velocity normal to starboard wing leading edge, but reduces it for port wing - similar effect to dihedral

Additional Contributions to Lateral Stability

$L_r$ Rolling moment due to yaw rate.

Yaw increases the speed of the outboard wing whilst reducing speed of the inboard one, causing a rolling moment to the inboard side. The contribution of the fin normally supports this inward rolling effect unless offset by anhedral stabilizer above the roll axis (or dihedral below the roll axis).

$L_p$ Rolling moment due to roll rate.

Roll creates counter rotational forces on both starboard and port wings whilst also generating such forces at the empennage. These opposing rolling moment effects have to be overcome by the aileron input in order to sustain the roll rate. If the roll is stopped at a non-zero roll angle the $L_\beta$ *upward* rolling moment induced by the ensueing sideslip should return the aircraft to the horizontal unless exceeded in turn by the *downward* $L_r$ rolling moment resulting from sideslip induced yaw rate. Longitudinal stability could be ensured or improved by minimizing the latter effect.

## Equations of motion

Since Dutch roll is a handling mode, analogous to the short period pitch oscillation, any effect it might have on the trajectory may be ignored. The body rate *r* is made up of the rate of change of sideslip angle and the rate of turn. Taking the latter as zero, assuming no effect on the trajectory, for the limited purpose of studying the Dutch roll:

$$\frac{d\beta}{dt} = -r$$

The yaw and roll equations, with the stability derivatives become:

$$C\frac{dr}{dt} - E\frac{dp}{dt} = N_\beta\beta - N_r\frac{d\beta}{dt} + N_p p \text{ (yaw)}$$
$$A\frac{dp}{dt} - E\frac{dr}{dt} = L_\beta\beta - L_r\frac{d\beta}{dt} + L_p p \text{ (roll)}$$

The inertial moment due to the roll acceleration is considered small compared with the aerodynamic terms, so the equations become:

$$-C\frac{d^2\beta}{dt^2} = N_\beta\beta - N_r\frac{d\beta}{dt} + N_p p$$
$$E\frac{d^2\beta}{dt^2} = L_\beta\beta - L_r\frac{d\beta}{dt} + L_p p$$

This becomes a second order equation governing either roll rate or sideslip:

$$\left(\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}\right)\frac{d^2\beta}{dt^2} + \left(\frac{L_p}{A}\frac{N_r}{C} - \frac{N_p}{C}\frac{L_r}{A}\right)\frac{d\beta}{dt} - \left(\frac{L_p}{A}\frac{N_\beta}{C} - \frac{L_\beta}{A}\frac{N_p}{C}\right)\beta = 0$$

The equation for roll rate is identical. But the roll angle, $\varphi$ (phi) is given by:

$$\frac{d\phi}{dt} = p$$

If $p$ is a damped simple harmonic motion, so is $\varphi$, but the roll must be in quadrature with the roll rate, and hence also with the sideslip. The motion consists of oscillations in roll and yaw, with the roll motion lagging 90 degrees behind the yaw. The wing tips trace out elliptical paths.

Stability requires the "stiffness" and "damping" terms to be positive. These are:

$$\frac{\frac{L_p}{A}\frac{N_r}{C} - \frac{N_p}{C}\frac{L_r}{A}}{\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}} \text{ (damping)}$$
$$\frac{\frac{L_\beta}{A}\frac{N_p}{C} - \frac{L_p}{A}\frac{N_\beta}{C}}{\frac{N_p}{C}\frac{E}{A} - \frac{L_p}{A}} \text{ (stiffness)}$$

The denominator is dominated by $L_p$, the roll damping derivative, which is always negative, so the denominators of these two expressions will be positive.

Considering the "stiffness" term: $-L_pN_\beta$ will be positive because $L_p$ is always negative and $N_\beta$ is positive by design. $L_\beta$ is usually negative, whilst $N_p$ is positive. Excessive dihedral can destabilize the Dutch roll, so configurations with highly swept wings require anhedral to offset the wing sweep contribution to $L_\beta$.

The damping term is dominated by the product of the roll damping and the yaw damping derivatives, these are both negative, so their product is positive. The Dutch roll should therefore be damped.

The motion is accompanied by slight lateral motion of the center of gravity and a more "exact" analysis will introduce terms in $Y_\beta$ etc.

## Roll subsidence

Jerking the stick sideways and returning it to center causes a net change in roll orientation.

The roll motion is characterized by an absence of natural stability, there are no stability derivatives which generate moments in response to the inertial roll angle. A roll disturbance induces a roll rate which is only canceled by pilot or autopilot intervention. This takes place with insignificant changes in sideslip or yaw rate, so the equation of motion reduces to:

$$A\frac{dp}{dt} = L_p p.$$

$L_p$ is negative, so the roll rate will decay with time. The roll rate reduces to zero, but there is no direct control over the roll angle.

## Spiral mode

Simply holding the stick still, when starting with the wings near level, an aircraft will usually have a tendency to gradually veer off to one side of the straight flightpath. This is the (slightly unstable) **spiral mode**. The opposite holds for a stable spiral mode. The spiral mode is so-named because when it is slightly unstable, and the controls are not moved, the aircraft will tend to increase its bank angle slowly at first, then ever faster. The resulting path through the air is a continuously tightening and ever more rapidly descending *spiral*. An unstable spiral mode is common to most aircraft. It is not dangerous because the times to double the bank angle are large compared to the pilot's ability to respond and correct errors with aileron inputs.

When the spiral mode is stable, it behaves in a way opposite to the exponential divergence of the unstable mode. The stable spiral mode, when starting with the wings at a moderate bank angle, will return to near wings level, first quickly, then more slowly. When the spiral mode is stable and starting at a moderate bank angle, the spiral nature of the flight path is not as obvious. This is because usually only a fraction of a turn is made while the wings are not fully level. The turning starts out (relatively) tight, then becomes less and less so as the wings become more level.

The divergence rate of the *unstable* spiral mode will be roughly proportional to the roll angle itself (i.e. roughly exponential growth). The *con*vergence rate of the *stable* spiral mode will be roughly proportional to the roll angle itself (i.e. roughly exponential *decay*).

## *Spiral mode trajectory*

In studying the trajectory, it is the direction of the velocity vector, rather than that of the body, which is of interest. The direction of the velocity vector when projected on to the horizontal will be called the track, denoted $\mu$ (mu). The body orientation is called the heading, denoted $\psi$ (psi). The force equation of motion includes a component of weight:

$$\frac{d\mu}{dt} = \frac{Y}{mU} + \frac{g}{U}\phi$$

where $g$ is the gravitational acceleration, and $U$ is the speed.

Including the stability derivatives:

$$\frac{d\mu}{dt} = \frac{Y_\beta}{mU}\beta + \frac{Y_r}{mU}r + \frac{Y_p}{mU}p + \frac{g}{U}\phi$$

Roll rates and yaw rates are expected to be small, so the contributions of $Y_r$ and $Y_p$ will be ignored.

The sideslip and roll rate vary gradually, so their time derivatives are ignored. The yaw and roll equations reduce to:

$$N_\beta\beta + N_r\frac{d\mu}{dt} + N_p p = 0 \quad \text{(yaw)}$$

$$L_\beta\beta + L_r\frac{d\mu}{dt} + L_p p = 0 \quad \text{(roll)}$$

Solving for $\beta$ and $p$:

$$\beta = \frac{(L_r N_p - L_p N_r)}{(L_p N_\beta - N_p L_\beta)}\frac{d\mu}{dt}$$

$$p = \frac{(L_\beta N_r - L_r N_\beta)}{(L_p N_\beta - N_p L_\beta)}\frac{d\mu}{dt}$$

Substituting for sideslip and roll rate in the force equation results in a first order equation in roll angle:

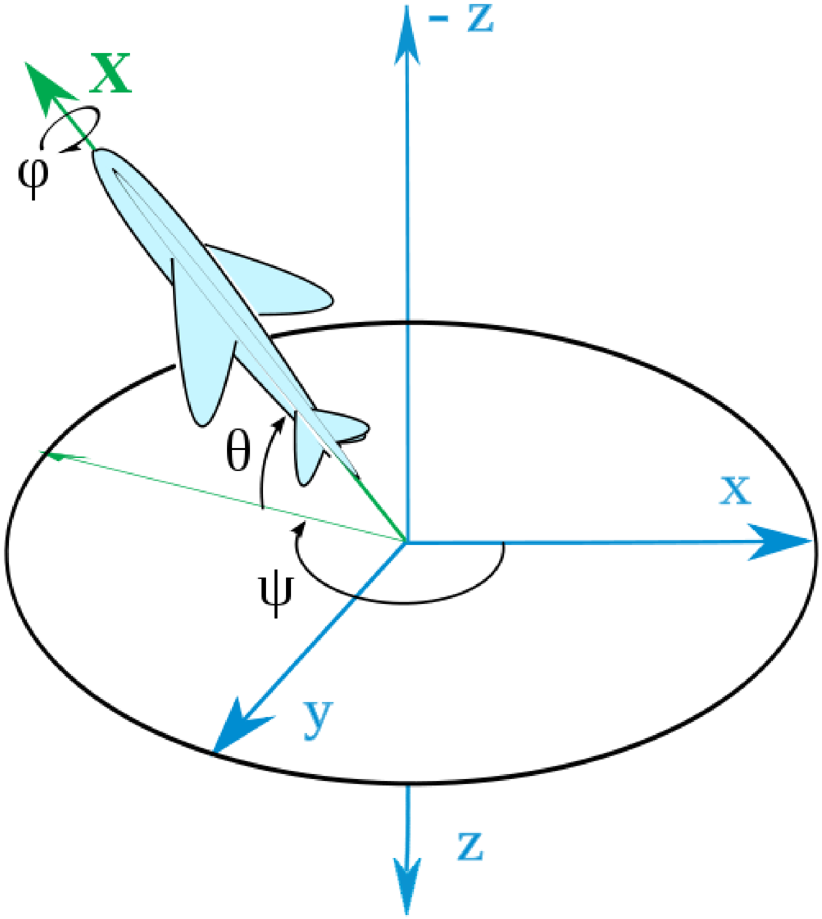
$$\frac{d\phi}{dt} = mg \frac{(L_\beta N_r - N_\beta L_r)}{mU(L_p N_\beta - N_p L_\beta) - Y_\beta (L_r N_p - L_p N_r)} \phi$$

This is an exponential growth or decay, depending on whether the coefficient of $\varphi$ is positive or negative. The denominator is usually negative, which requires $L_\beta N_r > N_\beta L_r$ (both products are positive). This is in direct conflict with the Dutch roll stability requirement, and it is difficult to design an aircraft for which both the Dutch roll and spiral mode are inherently stable.

Since the spiral mode has a long time constant, the pilot can intervene to effectively stabilize it, but an aircraft with an unstable Dutch roll would be difficult to fly. It is usual to design the aircraft with a stable Dutch roll mode, but slightly unstable spiral mode.

## Chapter 10

# Axes Conventions



Heading, elevation and bank angles (Z-Y’-X’’) for an aircraft. The aircraft's pitch and yaw axes Y and Z are not shown, and its fixed reference frame xyz has been shifted backwards from its center of gravity (preserving angles) for clarity. Axes named according to the air norm DIN 9300

The orientation of a vehicle is normally referred to as *attitude*. It is described normally by the orientation of a frame fixed in the body relative to a fixed reference frame. The attitude is described by *attitude coordinates*, and consists of at least three coordinates.

While from a geometrical point of view the different methods to describe orientations are defined using only some reference frames, in engineering applications it is important also to describe how these frames are attached to the lab and the body in motion.

Due to the special importance of international conventions in air vehicles, several organizations have published standards to be followed. For example, German DIN has published the DIN 9300 norm for aircrafts (adopted by ISO as ISO 1151–2:1985).

## *Frames conventions*

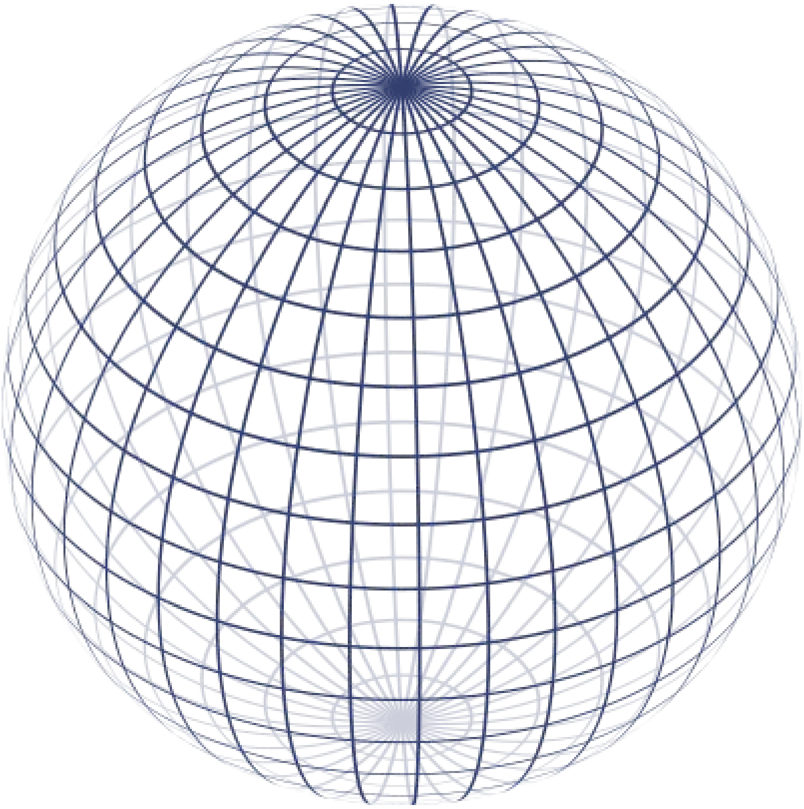
To stablish a standard convention to describe attitudes, it is required to establish at least the axes of the reference system and the axes of the rigid body or vehicle. When an ambiguous notation system is used (such as Euler angles) also the used convention should be stated. Nevertheless most used notations (matrices and quaternions) are unambiguous.

Tait–Bryan angles are often used to describe a vehicle's attitude with respect to a chosen reference frame, though any other notation can be used. The positive *x*-axis in vehicles points always in the direction of movement. For positive *y*- and *z*-axis, we have to face two different conventions:

- In case of land vehicles like cars, tanks etc., which use the ENU-system (East-North-Up) as external reference (*world frame*), the vehicle's positive *y*- or pitch axis always points to its left, and the positive *z*- or yaw axis always points up.

- By contrast, in case of air and sea vehicles like submarines, ships, airplanes etc., which use the NED-system (North-East-Down) as external reference (*world frame*), the vehicle's positive *y*- or pitch axis always points to its right, and its positive *z*- or yaw axis always points down.

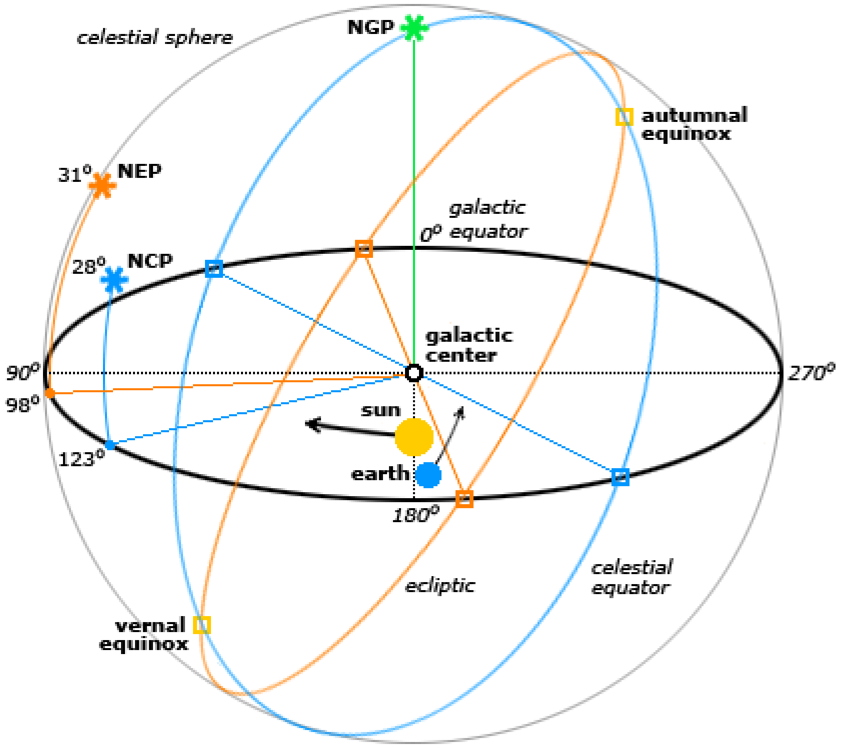
- Finally, in case of space vehicles like space shuttles etc., a modification of the latter convention is used, where the vehicle's positive *y*- or pitch axis again always points to its right, and its positive z- or yaw axis always points down, but “down” now may have two different meanings: If a so-called *local frame* is used as external reference, its positive z-axis points “down” to the center of the earth as it does in case of the earlier mentioned NED-system, but if the *inertial frame* is used as reference, its positive z-axis will point now to the North Celestial Pole, and its positive x-axis to the Vernal Equinox or some other reference meridian.



Representation of the earth with parallels and meridians

## *World axes conventions*

### Earth reference frames conventions: ENU and NED



Different reference systems for coordinates in the space

Basically, as lab frame or reference frame, there are two kinds of conventions for the frames:

- East, North, Up, referred as ENU
- North, East, Down, referred as NED, used specially in aerospace

These frames are location dependent. For movements around the globe, like air or see navigation, the frames are defined as tangent to the lines of coordinates.

- East-West tangent to parallels,
- North-South tangent to meridians, and
- Up-Down in the direction to the center of the earth (when using a spherical Earth simplification), or in the direction normal to the local tangent plane (using an oblate spheroidal or geodetic ellipsoidal model of the earth) which does not generally pass through the center of the Earth.

### Frames for space study

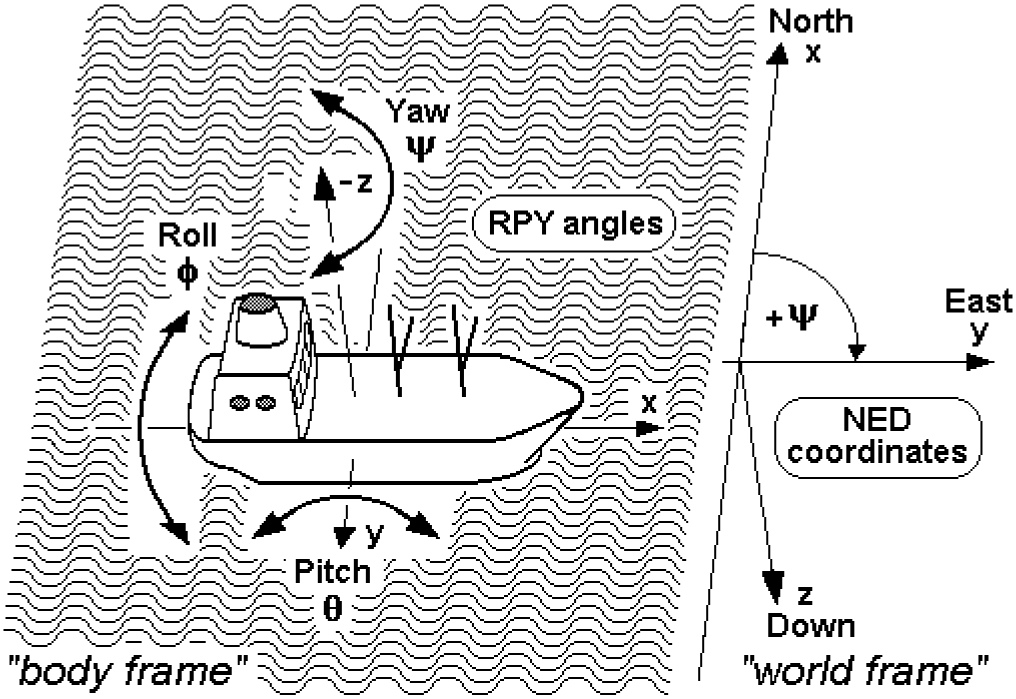
For satellites orbiting the earth it is normal to use the Equatorial coordinate system. The projection of the Earth's equator onto the celestial sphere is called the celestial equator.

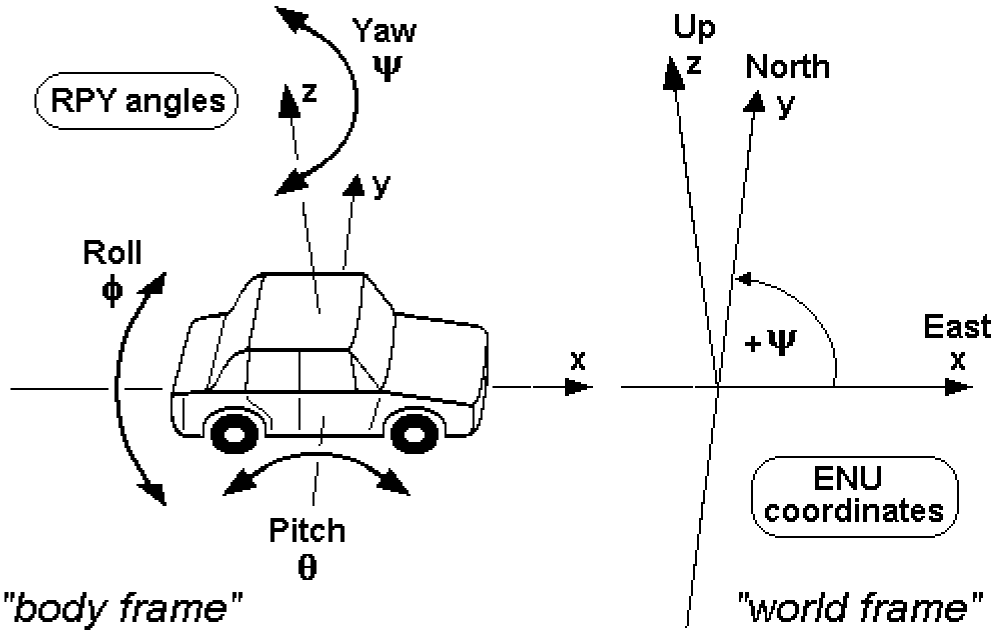
Similarly, the projections of the Earth's north and south geographic poles become the north and south celestial poles, respectively.

Deep space satellites use other Celestial coordinate system, like the Ecliptic coordinate system.

## *Vehicles conventions*

### Conventions for ships and land vehicles



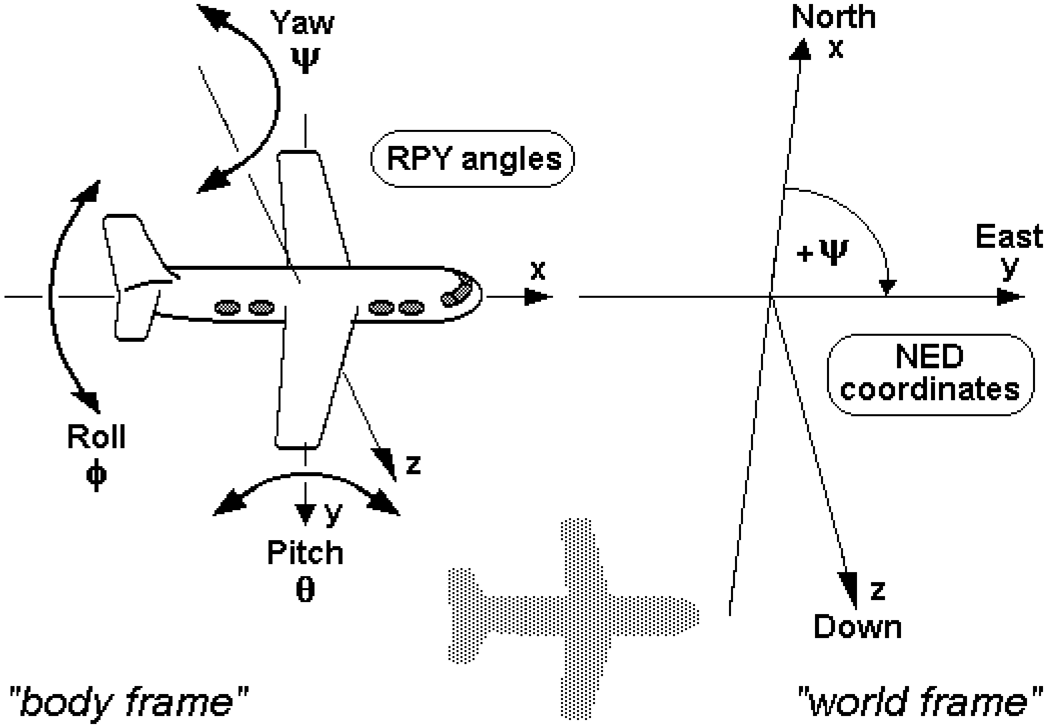


RPY angles of ships, cars and other sea and land vehicles

For land vehicles is rare to describe their complete orientation, except when speaking about electronic stability control. In this case, the convention is normally the one of the adjacent drawing.

As well as aircraft, the same terminology is used for the motion of ships and boats. It is interesting to note that some words commonly used were introduced in maritime navigation. For example, the *yaw* angle or heading, has a nautical origin, with the meaning of "bending out of the course". Etymologically, it is related with the verb 'to go'. It is related to the concept of bearing. It is typically assigned the shorthand notation ψ.

## Frames conventions for aircraft attitudes



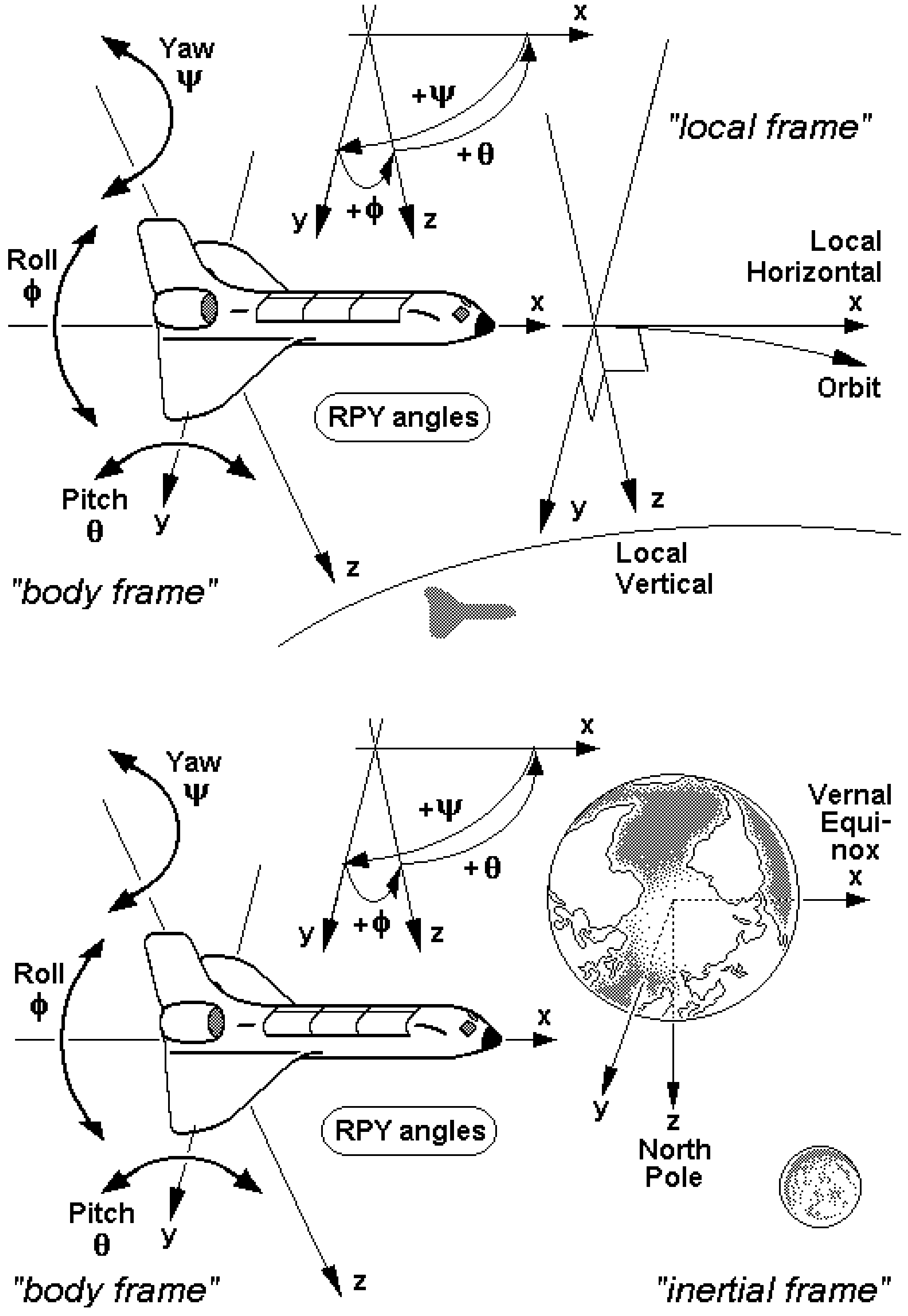
RPY angles of airplanes and other air vehicles

Yaw, pitch and roll are names used in aerospace for a vehicle-fixed axis system (*body frame*), but also as names for the rotations about those axes and also as names for the Tait–Bryan angles of the vehicle respect a *world frame*, which in the context of an aircraft sometimes are called its Heading, Elevation and Bank.

These axes are normally taken so that X axis is the longitudinal axis pointing ahead, Z axis is the vertical axis pointing downwards, and the Y axis is the lateral one, pointing in such a way that the frame is right handed.

The *motion* of an aircraft is often described in terms of rotation about these axes, so rotation about the *X*-axis is called rolling, rotation about the *Y*-axis is called pitching, and rotation about the *Z*-axis is called yawing.

### Frames conventions for space ships



RPY angles of space shuttles and other space vehicles, first using a local frame as reference and second using an inertial frame as reference.

If the goal is to keep the shuttle during its orbits in a constant attitude with respect to the sky, e.g. in order to perform certain astronomical observations, the preferred reference is the *inertial frame*, and the RPY angle vector (0|0|0) describes an attitude then, where the shuttle's wings are kept permanently parallel to the earth's equator, its nose points permanently to the vernal equinox, and its belly towards the Northern polar star. (Note that rockets and missiles more commonly follow the conventions for aircraft where the RPY angle vector (0|0|0) points north, rather than toward the vernal equinox).

On the other hand, if it's the goal to keep the shuttle during its orbits in an constant attitude with respect to the surface of the earth, the preferred reference will be the *local frame*, with the RPY angle vector (0|0|0) describing an attitude, where the shuttle's wings are parallel to the earth's surface, its nose points to its heading, and its belly down towards the centre of the earth.

# Chapter 11

# Aerodynamics

A vortex is created by the passage of an aircraft wing, revealed by smoke. Vortices are one of the many phenomena associated to the study of aerodynamics. The equations of aerodynamics show that the vortex is created by the difference in pressure between the upper and lower surface of the wing. At the end of the wing, the lower surface effectively tries to 'reach over' to the low pressure side, creating rotation and the vortex.

**Aerodynamics** is a branch of dynamics concerned with studying the motion of air, particularly when it interacts with a moving object. Aerodynamics is a subfield of fluid dynamics and gas dynamics, with much theory shared between them. Aerodynamics is often used synonymously with gas dynamics, with the difference being that gas dynamics applies to all gases. Understanding the motion of air (often called a flow field) around an object enables the calculation of forces and moments acting on the object. Typical properties calculated for a flow field include velocity, pressure, density and temperature as a function of position and time. By defining a control volume around the flow field, equations for the conservation of mass, momentum, and energy can be defined and used to solve for the properties. The use of aerodynamics through mathematical analysis, empirical approximations, wind tunnel experimentation, and computer simulations form the scientific basis for heavier-than-air flight.

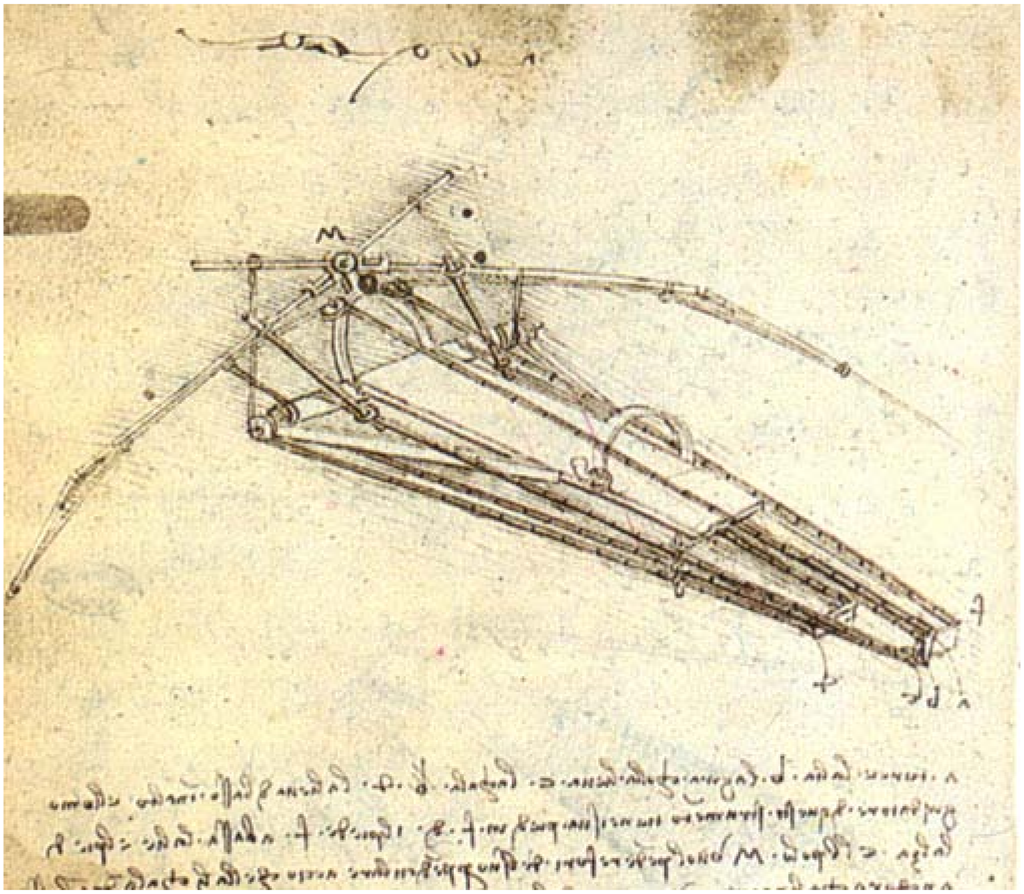
Aerodynamic problems can be identified in a number of ways. The flow environment defines the first classification criterion. *External* aerodynamics is the study of flow around solid objects of various shapes. Evaluating the lift and drag on an airplane or the shock waves that form in front of the nose of a rocket are examples of external aerodynamics. *Internal* aerodynamics is the study of flow through passages in solid objects. For instance, internal aerodynamics encompasses the study of the airflow through a jet engine or through an air conditioning pipe.

The ratio of the problem's characteristic flow speed to the speed of sound comprises a second classification of aerodynamic problems. A problem is called subsonic if all the speeds in the problem are less than the speed of sound, transonic if speeds both below and above the speed of sound are present (normally when the characteristic speed is approximately the speed of sound), supersonic when the characteristic flow speed is greater than the speed of sound, and hypersonic when the flow speed is much greater than the speed of sound. Aerodynamicists disagree over the precise definition of hypersonic flow; minimum Mach numbers for hypersonic flow range from 3 to 12.

The influence of viscosity in the flow dictates a third classification. Some problems may encounter only very small viscous effects on the solution, in which case viscosity can be considered to be negligible. The approximations to these problems are called inviscid flows. Flows for which viscosity cannot be neglected are called viscous flows.

## *History*

### Early ideas - ancient times to the 17th century



A drawing of a design for a flying machine by Leonardo da Vinci (c. 1488). This machine was an ornithopter, with flapping wings similar to a bird, first presented in his Codex on the Flight of Birds in 1505.

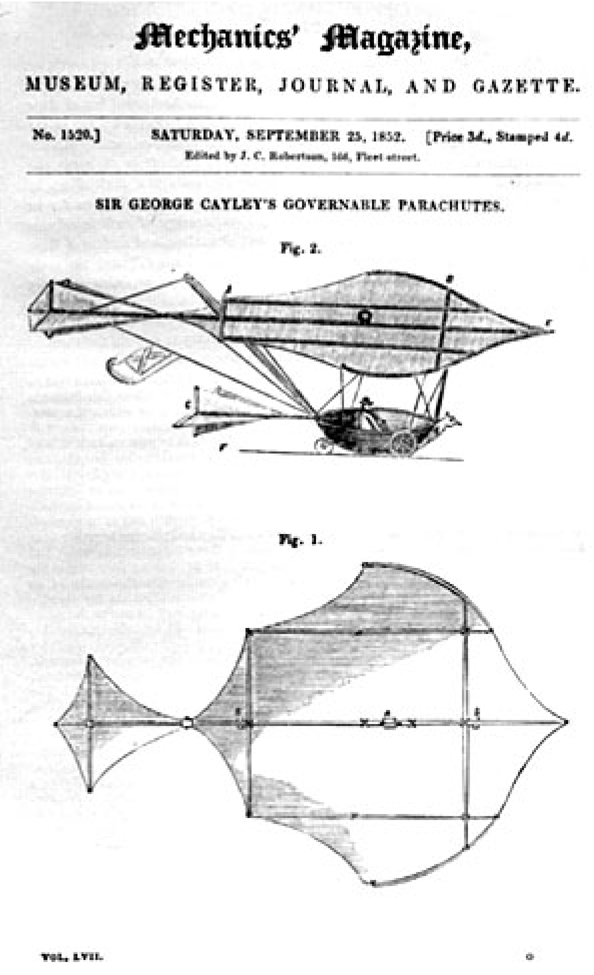
Images and stories of flight have appeared throughout recorded history, such as the legendary story of Icarus and Daedalus. Although observations of some aerodynamic effects like wind resistance (a.k.a. drag) were recorded by the likes of Aristotle, Leonardo da Vinci and Galileo Galilei, very little effort was made to develop governing laws for understanding the nature of flight prior to the 17th century.

In 1505, Leonardo da Vinci wrote the *Codex on the Flight of Birds*, one of the earliest treatises on aerodynamics. He notes for the first time that the center of gravity of a flying bird does not coincide with its center of pressure, and he describes the construction of an ornithopter, with flapping wings similar to a bird's.

Sir Isaac Newton was the first person to develop a theory of air resistance, making him one of the first aerodynamicists. As part of that theory, Newton believed that drag was due to the dimensions of a body, the density of the fluid, and the velocity raised to the second power. These beliefs all turned out to be correct for low flow speeds. Newton also developed a law for the drag force on a flat plate inclined towards the direction of the fluid flow. Using $F$ for the drag force, $\rho$ for the density, $S$ for the area of the flat plate, $V$ for the flow velocity, and $\theta$ for the inclination angle, his law was expressed as $F = \rho S V^2 \sin^2(\theta)$

Unfortunately, this equation is incorrect for the calculation of drag in most cases. Drag on a flat plate is closer to being linear with the angle of inclination as opposed to acting quadratically at low angles. The Newton formula can lead one to believe that flight is more difficult than it actually is, and it may have contributed to a delay in human flight. However, it is correct for a very slender plate when the angle becomes large and flow separation occurs, or if the flow speed is supersonic.

## Modern beginnings - 18th to 19th century



Mechanics' Magazine,

MUSEUM, REGISTER, JOURNAL, AND GAZETTE.

No. 1520.] SATURDAY, SEPTEMBER 25, 1852. [Price 3d., Stamped 4d.

Edited by J. C. Robertson, 166, Fleet street.

SIR GEORGE CAYLEY'S GOVERNABLE PARACHUTES.

VOL. LVII.

A drawing of a glider by Sir George Cayley, one of the early attempts at creating an aerodynamic shape.

In 1738 The Dutch-Swiss mathematician Daniel Bernoulli published his book *Hydrodynamica*, in which he first set out the principle, named after him, by which aerodynamic lift may be derived.

Sir George Cayley is credited as the first person to identify the four aerodynamic forces of flight—weight, lift, drag, and thrust—and the relationship between them. Cayley believed that the drag on a flying machine must be counteracted by a means of propulsion in order for level flight to occur. Cayley also looked to nature for aerodynamic shapes with low drag. Among the shapes he investigated were the cross-sections of trout. This may appear counterintuitive, however, the bodies of fish are shaped to produce very low resistance as they travel through water. Their cross-sections are sometimes very close to that of modern low drag airfoils.

Air resistance experiments were carried out by investigators throughout the 18th and 19th centuries. Drag theories were developed by Jean le Rond d'Alembert, Gustav Kirchhoff, and Lord Rayleigh. Equations for fluid flow with friction were developed by Claude-Louis Navier and George Gabriel Stokes. To simulate fluid flow, many experiments involved immersing objects in streams of water or simply dropping them off the top of a tall building. Towards the end of this time period Gustave Eiffel used his Eiffel Tower to assist in the drop testing of flat plates.

Of course, a more precise way to measure resistance is to place an object within an artificial, uniform stream of air where the velocity is known. The first person to experiment in this fashion was Francis Herbert Wenham, who in doing so constructed the first wind tunnel in 1871. Wenham was also a member of the first professional organization dedicated to aeronautics, the Royal Aeronautical Society of the United Kingdom. Objects placed in wind tunnel models are almost always smaller than in practice, so a method was needed to relate small scale models to their real-life counterparts. This was achieved with the invention of the dimensionless Reynolds number by Osborne Reynolds. Reynolds also experimented with laminar to turbulent flow transition in 1883.

By the late 19th century, two problems were identified before heavier-than-air flight could be realized. The first was the creation of low-drag, high-lift aerodynamic wings. The second problem was how to determine the power needed for sustained flight. During this time, the groundwork was laid down for modern day fluid dynamics and aerodynamics, with other less scientifically inclined enthusiasts testing various flying machines with little success.

A replica of the Wright Brothers' wind tunnel is on display at the Virginia Air and Space Center. Wind tunnels were key in the development and validation of the laws of aerodynamics.

In 1889, Charles Renard, a French aeronautical engineer, became the first person to reasonably predict the power needed for sustained flight. Renard and German physicist Hermann von Helmholtz explored the wing loading of birds, eventually concluding that humans could not fly under their own power by attaching wings onto their arms. Otto Lilienthal, following the work of Sir George Cayley, was the first person to become highly successful with glider flights. Lilienthal believed that thin, curved airfoils would produce high lift and low drag.

Octave Chanute provided a great service to those interested in aerodynamics and flying machines by publishing a book outlining all of the research conducted around the world up to 1893.

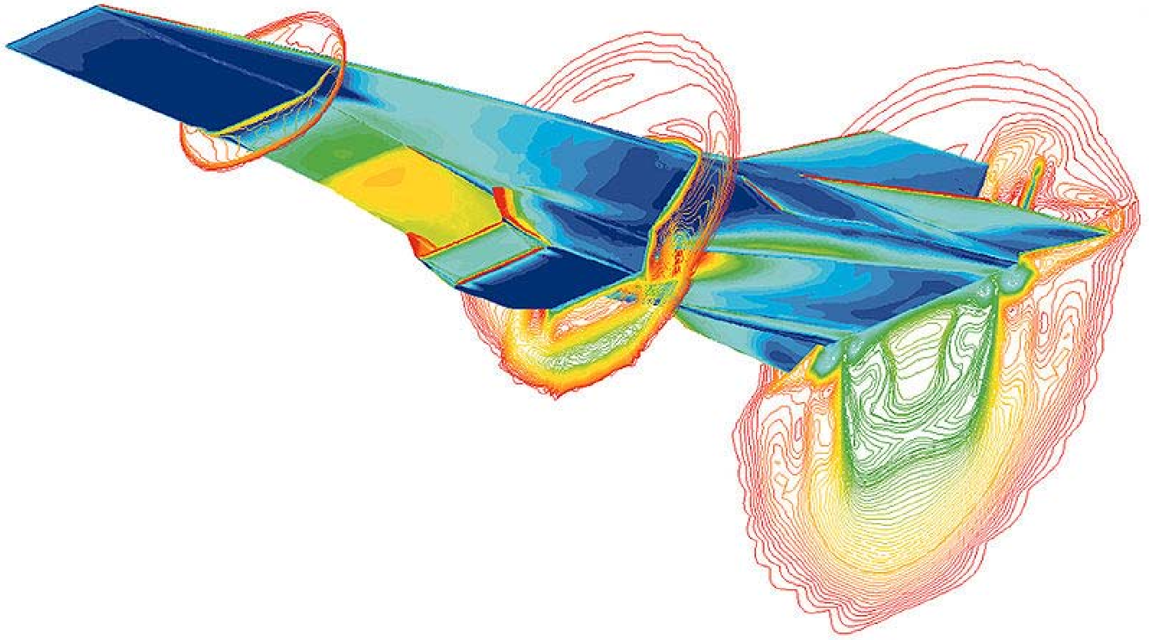
## Practical flight - early 20th century

With the information contained in Chanute's book, the personal assistance of Chanute himself, and research carried out in their own wind tunnel, the Wright brothers gained just enough knowledge of aerodynamics to fly the first powered aircraft on December 17, 1903, just in time to beat the efforts of Samuel Pierpont Langley. The Wright brothers' flight confirmed or disproved a number of aerodynamics theories. Newton's drag force theory was finally proved incorrect. This first widely-publicised flight led to a more organized effort between aviators and scientists, leading the way to modern aerodynamics.

During the time of the first flights, Frederick W. Lanchester, Martin Wilhelm Kutta, and Nikolai Zhukovsky independently created theories that connected circulation of a fluid flow to lift. Kutta and Zhukovsky went on to develop a two-dimensional wing theory. Expanding upon the work of Lanchester, Ludwig Prandtl is credited with developing the mathematics behind thin-airfoil and lifting-line theories as well as work with boundary layers. Prandtl, a professor at the University of Göttingen, instructed many students who would play important roles in the development of aerodynamics like Theodore von Kármán and Max Munk.

## Faster than sound - latter 20th century

As aircraft began to travel faster, aerodynamicists realized that the density of air began to change as it came into contact with an object, leading to a division of fluid flow into the incompressible and compressible regimes. In compressible aerodynamics, density and pressure both change, which is the basis for calculating the speed of sound. Newton was the first to develop a mathematical model for calculating the speed of sound, but it was not correct until Pierre-Simon Laplace accounted for the molecular behavior of gases and introduced the heat capacity ratio. The ratio of the flow speed to the speed of sound was named the Mach number after Ernst Mach, who was one of the first to investigate the properties of supersonic flow which included Schlieren photography techniques to visualize the changes in density. William John Macquorn Rankine and Pierre Henri Hugoniot independently developed the theory for flow properties before and after a shock wave. Jakob Ackeret led the initial work on calculating the lift and drag on a supersonic airfoil. Theodore von Kármán and Hugh Latimer Dryden introduced the term transonic to describe flow speeds around Mach 1 where drag increases rapidly. Because of the increase in drag approaching Mach 1, aerodynamicists and aviators disagreed on whether supersonic flight was achievable.



A computer generated model of NASA's X-43A hypersonic research vehicle flying at Mach 7 using a computational fluid dynamics code.

On September 30, 1935 an exclusive conference was held in Rome with the topic of high velocity flight and the possibility of breaking the sound barrier. Participants included Theodore von Kármán, Ludwig Prandtl, Jakob Ackeret, Eastman Jacobs, Adolf Busemann, Geoffrey Ingram Taylor, Gaetano Arturo Crocco, and Enrico Pistolesi. The new research presented was impressive. Ackeret presented a design for a supersonic wind tunnel. Busemann gave perhaps the best presentation on the need for aircraft with swept wings for high speed flight. Eastman Jacobs, working for NACA, presented his optimized airfoils for high subsonic speeds which led to some of the high performance American aircraft during World War II. Supersonic propulsion was also discussed. The sound barrier was broken using the Bell X-1 aircraft twelve years later, thanks in part to those individuals.

By the time the sound barrier was broken, much of the subsonic and low supersonic aerodynamics knowledge had matured. The Cold War fueled an ever evolving line of high performance aircraft. Computational fluid dynamics was started as an effort to solve for flow properties around complex objects and has rapidly grown to the point where entire aircraft can be designed using a computer.

With some exceptions, the knowledge of hypersonic aerodynamics has matured between the 1960s and the present decade. Therefore, the goals of an aerodynamicist have shifted from understanding the behavior of fluid flow to understanding how to engineer a vehicle to interact appropriately with the fluid flow. For example, while the behavior of hypersonic flow is understood, building a scramjet aircraft to fly at hypersonic speeds has seen very limited success. Along with building a successful scramjet aircraft, the desire to improve the aerodynamic efficiency of current aircraft and propulsion systems will continue to fuel new research in aerodynamics.

### Introductory terminology

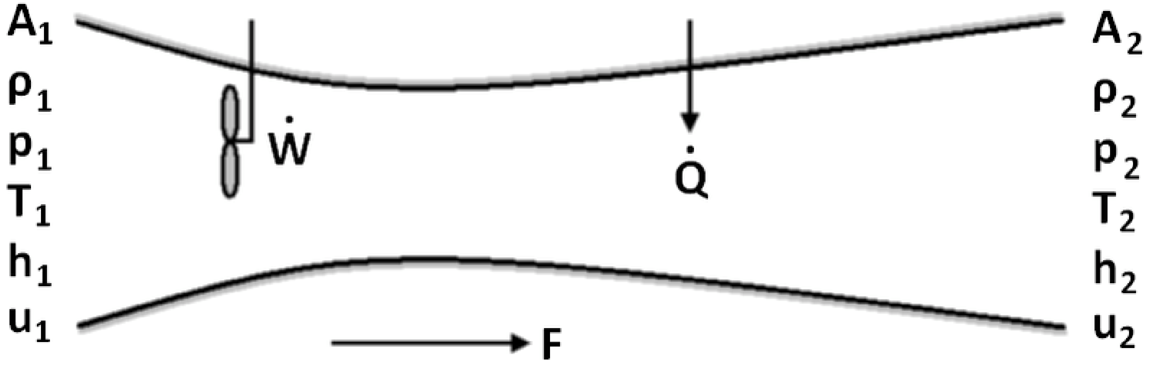
- Lift
- Drag
- Reynolds number
- Mach number

### Continuity assumption

Gases are composed of molecules which collide with one another and solid objects. If density and velocity are taken to be well-defined at infinitely small points, and are assumed to vary continuously from one point to another, the discrete molecular nature of a gas is ignored.

The continuity assumption becomes less valid as a gas becomes more rarefied. In these cases, statistical mechanics is a more valid method of solving the problem than continuous aerodynamics. The Knudsen number can be used to guide the choice between statistical mechanics and the continuous formulation of aerodynamics.

### Laws of conservation



Control volume schematic of internal flow with one inlet and exit including an axial force, work, and heat transfer. State 1 is the inlet and state 2 is the exit.

Aerodynamics problems are often solved using conservation laws as applied to a fluid continuum. The conservation laws can be written in integral or differential form. In many basic problems, three conservation principles are used:

- Continuity: If a certain mass of fluid enters a volume, it must either exit the volume or change the mass inside the volume. In fluid dynamics, the continuity equation is analogous to Kirchhoff's Current Law in electric circuits. The differential form of the continuity equation is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

Above, ρ is the fluid density, **u** is a velocity vector, and *t* is time. Physically, the equation also shows that mass is neither created nor destroyed in the control volume. For a steady state process, the rate at which mass enters the volume is equal to the rate at which it leaves the volume. Consequently, the first term on the left is then equal to zero. For flow through a tube with one inlet (state 1) and exit (state 2) as shown in the figure in this section, the continuity equation may be written and solved as:

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

Above, *A* is the variable cross-section area of the tube at the inlet and exit. For incompressible flows, density remains constant.

- Conservation of Momentum: This equation applies Newton's second law of motion to a continuum, whereby force is equal to the time derivative of momentum. Both surface and body forces are accounted for in this equation. For instance, *F* could be expanded into an expression for the frictional force acting on an internal flow.

$$\frac{D\mathbf{u}}{Dt} = \mathbf{F} - \frac{\nabla p}{\rho}$$

For the same figure, a control volume analysis yields:

$$p_1 A_1 + \rho_1 A_1 u_1^2 + F = p_2 A_2 + \rho_2 A_2 u_2^2$$

Above, the force *F* is placed on the left side of the equation, assuming it acts with the flow moving in a left-to-right direction. Depending on the other properties of the flow, the resulting force could be negative which means it acts in the opposite direction as depicted in the figure.

- Conservation of Energy: Although energy can be converted from one form to another, the total energy in a given system remains constant.

$$\rho \frac{Dh}{Dt} = \frac{Dp}{Dt} + \nabla \cdot (k \nabla T) + \Phi$$

Above, *h* is enthalpy, *k* is the thermal conductivity of the fluid, *T* is temperature, and Φ is the viscous dissipation function. The viscous dissipation function governs the rate at which mechanical energy of the flow is converted to heat. The expression on the left side is a material derivative. The term is always positive since, according to the second law of thermodynamics, viscosity cannot add energy to the control volume. Again using the figure, the energy equation in terms of the control volume may be written as:

$$\rho_1 u_1 A_1 \left(h_1 + \frac{u_1^2}{2}\right) + \dot{W} + \dot{Q} = \rho_2 u_2 A_2 \left(h_2 + \frac{u_2^2}{2}\right)$$

Above, the shaft work and heat transfer are assumed to be acting on the flow. They may be positive (to the flow from the surroundings) or negative (to the surroundings from the flow) depending on the problem. The ideal gas law or another equation of state is often used in conjunction with these equations to form a system to solve for the unknown variables.

## *Incompressible aerodynamics*

An incompressible flow is characterized by a constant density despite flowing over surfaces or inside ducts. A flow can be considered incompressible as long as its speed is low. For higher speeds, the flow will begin to compress as it comes into contact with surfaces. The Mach number is used to distinguish between incompressible and compressible flows.

### Subsonic flow

Subsonic (or low-speed) aerodynamics is the study of fluid motion which is everywhere much slower than the speed of sound through the fluid or gas. There are several branches of subsonic flow but one special case arises when the flow is inviscid, incompressible and irrotational. This case is called Potential flow and allows the differential equations used to be a simplified version of the governing equations of fluid dynamics, thus making available to the aerodynamicist a range of quick and easy solutions. It is a special case of Subsonic aerodynamics.

In solving a subsonic problem, one decision to be made by the aerodynamicist is whether to incorporate the effects of compressibility. Compressibility is a description of the amount of change of density in the problem. When the effects of compressibility on the solution are small, the aerodynamicist may choose to assume that density is constant. The problem is then an incompressible low-speed aerodynamics problem. When the density is allowed to vary, the problem is called a compressible problem. In air, compressibility effects are usually ignored when the Mach number in the flow does not exceed 0.3 (about 335 feet (102m) per second or 228 miles (366 km) per hour at 60°F). Above 0.3, the problem should be solved by using compressible aerodynamics.

## *Compressible aerodynamics*

According to the theory of aerodynamics, a flow is considered to be compressible if its change in density with respect to pressure is non-zero along a streamline. This means that

- unlike incompressible flow - changes in density must be considered. In general, this is the case where the Mach number in part or all of the flow exceeds 0.3. The Mach .3 value is rather arbitrary, but it is used because gas flows with a Mach number below that value demonstrate changes in density with respect to the change in pressure of less than 5%. Furthermore, that maximum 5% density change occurs at the stagnation point of an object immersed in the gas flow and the density changes around the rest of the object will be significantly lower. Transonic, supersonic, and hypersonic flows are all compressible.

## Transonic flow

The term Transonic refers to a range of velocities just below and above the local speed of sound (generally taken as Mach 0.8–1.2). It is defined as the range of speeds between the critical Mach number, when some parts of the airflow over an aircraft become supersonic, and a higher speed, typically near Mach 1.2, when all of the airflow is supersonic. Between these speeds some of the airflow is supersonic, and some is not.

## Supersonic flow

Supersonic aerodynamic problems are those involving flow speeds greater than the speed of sound. Calculating the lift on the Concorde during cruise can be an example of a supersonic aerodynamic problem.

Supersonic flow behaves very differently from subsonic flow. Fluids react to differences in pressure; pressure changes are how a fluid is "told" to respond to its environment. Therefore, since sound is in fact an infinitesimal pressure difference propagating through a fluid, the speed of sound in that fluid can be considered the fastest speed that "information" can travel in the flow. This difference most obviously manifests itself in the case of a fluid striking an object. In front of that object, the fluid builds up a stagnation pressure as impact with the object brings the moving fluid to rest. In fluid traveling at subsonic speed, this pressure disturbance can propagate upstream, changing the flow pattern ahead of the object and giving the impression that the fluid "knows" the object is there and is avoiding it. However, in a supersonic flow, the pressure disturbance cannot propagate upstream. Thus, when the fluid finally does strike the object, it is forced to change its properties -- temperature, density, pressure, and Mach number -- in an extremely violent and irreversible fashion called a shock wave. The presence of shock waves, along with the compressibility effects of high-velocity fluids, is the central difference between supersonic and subsonic aerodynamics problems.

## Hypersonic flow

In aerodynamics, hypersonic speeds are speeds that are highly supersonic. In the 1970s, the term generally came to refer to speeds of Mach 5 (5 times the speed of sound) and above. The hypersonic regime is a subset of the supersonic regime. Hypersonic flow is characterized by high temperature flow behind a shock wave, viscous interaction, and chemical dissociation of gas.

## *Associated terminology*

The incompressible and compressible flow regimes produce many associated phenomena, such as boundary layers and turbulence.

### Boundary layers

The concept of a boundary layer is important in many aerodynamic problems. The viscosity and fluid friction in the air is approximated as being significant only in this thin layer. This principle makes aerodynamics much more tractable mathematically.

### Turbulence

In aerodynamics, turbulence is characterized by chaotic, stochastic property changes in the flow. This includes low momentum diffusion, high momentum convection, and rapid variation of pressure and velocity in space and time. Flow that is not turbulent is called laminar flow.

## *Aerodynamics in other fields*

Aerodynamics is important in a number of applications other than aerospace engineering. It is a significant factor in any type of vehicle design, including automobiles. It is important in the prediction of forces and moments in sailing. It is used in the design of large components such as hard drive heads. Structural engineers also use aerodynamics, and particularly aeroelasticity, to calculate wind loads in the design of large buildings and bridges. Urban aerodynamics seeks to help town planners and designers improve comfort in outdoor spaces, create urban microclimates and reduce the effects of urban pollution. The field of environmental aerodynamics studies the ways atmospheric circulation and flight mechanics affect ecosystems. The aerodynamics of internal passages is important in heating/ventilation, gas piping, and in automotive engines where detailed flow patterns strongly affect the performance of the engine.

## Chapter 12

# Propeller

The feathered propellers of an RAF Hercules C.4

**Aircraft propellers** convert rotary motion from piston engines or turboprops to provide propulsive force. They may be fixed or variable pitch. Early aircraft propellers were carved by hand from solid or laminated wood with later propellers being constructed from metal. The most modern propeller designs use high-technology composite materials.

The propeller is usually attached to the crankshaft of a piston engine, either directly or through a reduction unit. Light aircraft engines often do not require the complexity of gearing but on larger engines and turboprop aircraft it is essential.

## History

The twisted airfoil (aerofoil) shape of modern aircraft propellers was pioneered by the Wright brothers. They found that a propeller is essentially the same as a wing and so were able to use data collated from their earlier wind tunnel experiments on wings. They also found that the relative angle of attack from the forward movement of the aircraft was different for all points along the length of the blade, thus it was necessary to introduce a twist along its length. Their original propeller blades were only about 5% less efficient than the modern equivalent, some 100 years later.

Alberto Santos Dumont was another early pioneer, having designed propellers before the Wright Brothers (albeit not as efficient) for his airships. He applied the knowledge he gained from experiences with airships to make a propeller with a steel shaft and aluminium blades for his 14 bis biplane. Some of his designs used a bent aluminium sheet for blades, thus creating an airfoil shape. These are heavily undercambered because of this and combined with the lack of a lengthwise twist made them less efficient than the Wright propellers. Even so, this was perhaps the first use of aluminium in the construction of an airscrew.

## Theory and design of aircraft propellers

A well-designed propeller typically has an efficiency of around 80% when operating in the best regime.Changes to a propeller's efficiency are produced by a number of factors, notably adjustments to the helix angle($\theta$), the angle between the resultant relative velocity and the blade rotation direction, and to blade pitch (where $\theta = \Phi + \alpha$) . Very small pitch and helix angles give a good performance against resistance but provide little thrust, while larger angles have the opposite effect. The best helix angle is when the blade is acting as a wing producing much more lift than drag.

A propeller's efficiency is determined by

$$\eta = \frac{\text{propulsive power out}}{\text{shaft power in}} = \frac{\text{thrust} \cdot \text{axial speed}}{\text{resistance torque} \cdot \text{rotational speed}}.$$

Propellers are similar in aerofoil section to a low drag wing and as such are poor in operation when at other than their optimum angle of attack. Control systems are required to counter the need for accurate matching of pitch to flight speed and engine speed.

The three-bladed propeller of a light aircraft: the Vans RV-7A

A further consideration is the number and the shape of the blades used. Increasing the aspect ratio of the blades reduces drag but the amount of thrust produced depends on blade area, so using high aspect blades can lead to the need for a propeller diameter which is unusable. A further balance is that using a smaller number of blades reduces interference effects between the blades, but to have sufficient blade area to transmit the available power within a set diameter means a compromise is needed. Increasing the number of blades also decreases the amount of work each blade is required to perform, limiting the local Mach number - a significant performance limit on propellers.

A propeller's performance suffers as the blade speed exceeds the speed of sound. As the relative air speed at the blade is rotation speed plus axial speed, a propeller blade tip will reach sonic speed sometime before the rest of the aircraft (with a theoretical blade the maximum aircraft speed is about 845 km/h (Mach 0.7) at sea-level, in reality it is rather lower). When a blade tip becomes supersonic, drag and torque resistance increase suddenly and shock waves form creating a sharp increase in noise. Aircraft with conventional propellers, therefore, do not usually fly faster than Mach 0.6. There are certain propeller-driven aircraft, usually military, which do operate at Mach 0.8 or higher, although there is considerable fall off in efficiency.

There have been efforts to develop propellers for aircraft at high subsonic speeds. The 'fix' is similar to that of transonic wing design. The maximum relative velocity is kept as low as possible by careful control of pitch to allow the blades to have large helix angles;

thin blade sections are used and the blades are swept back in a scimitar shape (Scimitar propeller); a large number of blades are used to reduce work per blade and so circulation strength; contra-rotation is used. The propellers designed are more efficient than turbofans and their cruising speed (Mach 0.7–0.85) is suitable for airliners, but the noise generated is tremendous.

## Forces acting on a propeller

Five forces act on the blades of an aircraft propeller in motion, they are:

Thrust bending force

Thrust loads on the blades act to bend them forwards, opposite to the direction of flight.

Centrifugal twisting force

Acts to twist the blades to a low or fine pitch angle.

Aerodynamic twisting force

As the centre of pressure of a propeller blade is forward of its centreline the blade is twisted towards a coarse pitch position.

Centrifugal force

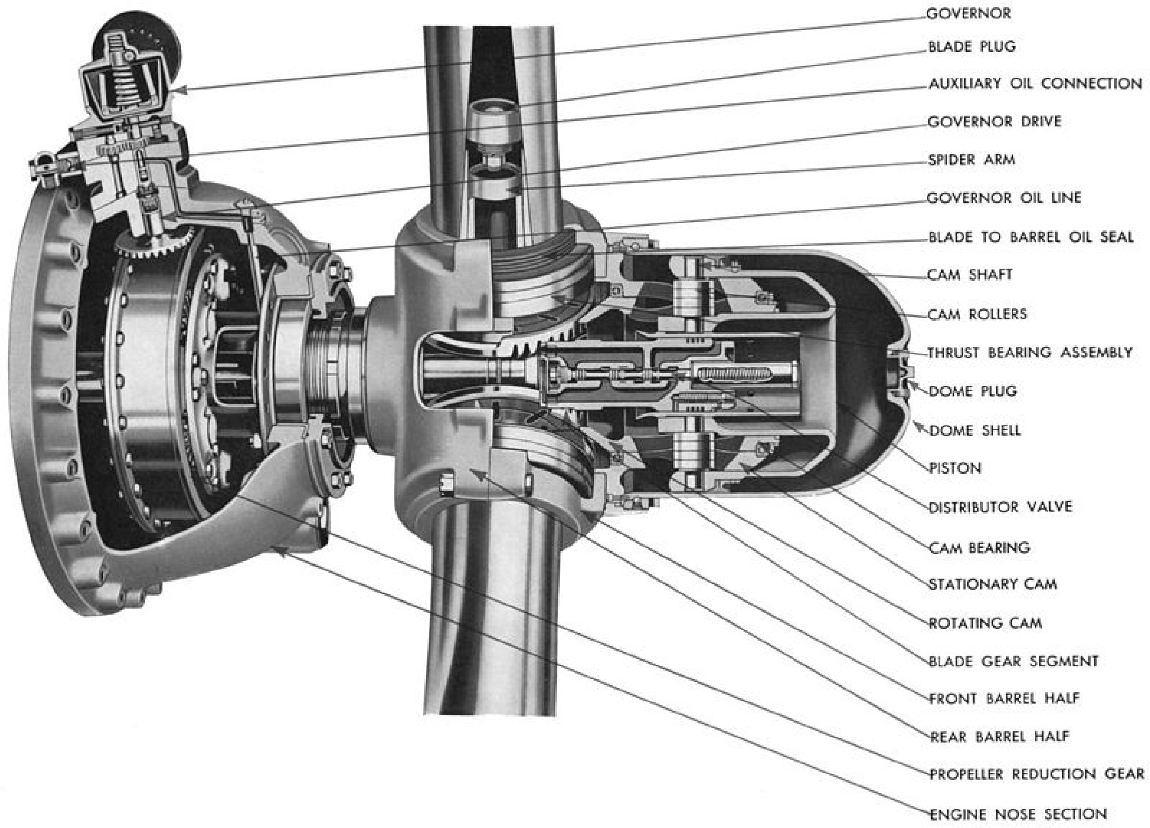
The force felt by the blades acting to pull them away from the hub when turning.

Torque bending force

Air resistance acting against the blades, combined with inertial effects causes propeller blades to bend away from the direction of rotation.

## *Propeller control*

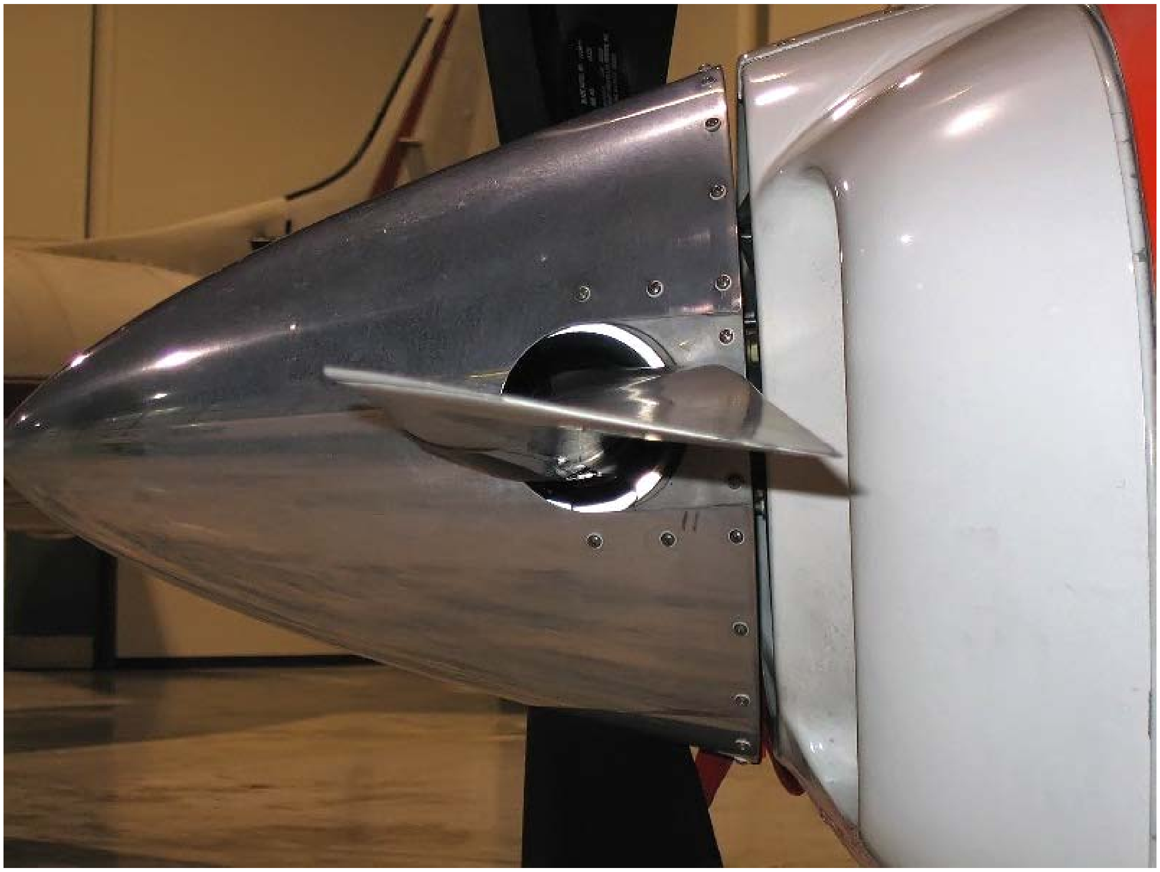
### Variable pitch



Cut away view of a Hamilton Standard propeller. The engineers at Hamilton came up with a propeller design that was very reliable and maintenance friendly. With an auxiliary oil connection to feather the propeller in flight, gave multi-engined aircraft a huge safety factor. This type of propeller was used on most American fighters, bombers and transport aircraft of WWII.

The purpose of varying pitch angle with a variable pitch propeller is to maintain an optimal angle of attack (maximum lift to drag ratio) on the propeller blades as aircraft speed varies. Early pitch control settings were pilot operated, either two-position or manually variable. Following World War II, automatic propellers were developed to maintain an optimum angle of attack. This was done by balancing the centripetal twisting moment on the blades and a set of counterweights against a spring and the aerodynamic forces on the blade. Automatic props had the advantage of being simple, lightweight, and requiring no external control, but a particular propeller's performance was difficult to match with that of the aircraft's powerplant. An improvement on the automatic type was the constant-speed propeller. Constant speed propellers allow the pilot to select a rotational speed for maximum engine power or maximum efficiency, and a propeller governor acts as a closed-loop controller to vary propeller pitch angle as required to maintain the RPM commanded by the pilot. In most aircraft this system is hydraulic, with engine oil serving as the hydraulic fluid. However, electrically controlled propellers were developed during World War II and saw extensive use on military aircraft, and have recently seen a revival in use on homebuilt aircraft.

## Feathering



A propeller blade in feathered position

On some variable-pitch propellers, the blades can be rotated parallel to the airflow to reduce drag in case of an engine failure. This is called *feathering*. Feathering propellers were developed for military fighter aircraft prior to World War II, as a fighter is more likely to experience an engine failure due to the inherent danger of combat. On single-engined aircraft, whether a powered glider or turbine powered aircraft, the effect is to increase the gliding distance. On a multi-engine aircraft, feathering the propeller on a failed engine reduces drag.

Most feathering systems for reciprocating engines sense a drop in oil pressure and move the blades toward the feather position, and require the pilot to pull the propeller control back to disengage the high-pitch stop pins before the engine reaches idle RPM. Turboprop control systems usually utilize a *negative torque sensor* in the reduction gearbox which moves the blades toward feather when the engine is no longer providing power to the propeller. Depending on design, the pilot may have to push a button to override the high-pitch stops and complete the feathering process, or the feathering process may be totally automatic.

### Reverse pitch

Contra-rotating propellers of a modified
P-51 Mustang fitted with a Rolls-Royce Griffon.

In some aircraft (e.g., the C-130 Hercules), the pilot can manually override the constant speed mechanism to reverse the blade pitch angle, and thus the thrust of the engine (although the rotation of the engine itself does not reverse). This is used to help slow the plane down after landing in order to save wear on the brakes and tires, but in some cases also allows the aircraft to back up on its own. This is known as "Beta Range" operation.

## *Contra-rotating propellers*

Contra-rotating propellers use a second propeller rotating in the opposite direction immediately 'downstream' of the main propeller so as to recover energy lost in the swirling motion of the air in the propeller slipstream. Contra-rotation also increases power without increasing propeller diameter and provides a counter to the torque effect of high-power piston engine as well as the gyroscopic precession effects, and of the slipstream swirl. However on small aircraft the added cost, complexity, weight and noise of the system rarely make it worthwhile.

## *Counter-rotating propellers*

Counter-rotating propellers, are found on twin-, and multi-engine, propeller-driven aircraft and have propellers that spin in opposite directions. Generally, the propellers on both engines of most conventional twin-engined aircraft spin clockwise (as viewed from the rear of the aircraft). Counter-rotating propellers generally spin clockwise on the left engine, and counter-clockwise on the right. The advantage of counter-rotating propellers is to balance out the effects of torque and p-factor, eliminating the problem of the critical engine.

## *Aircraft fans*

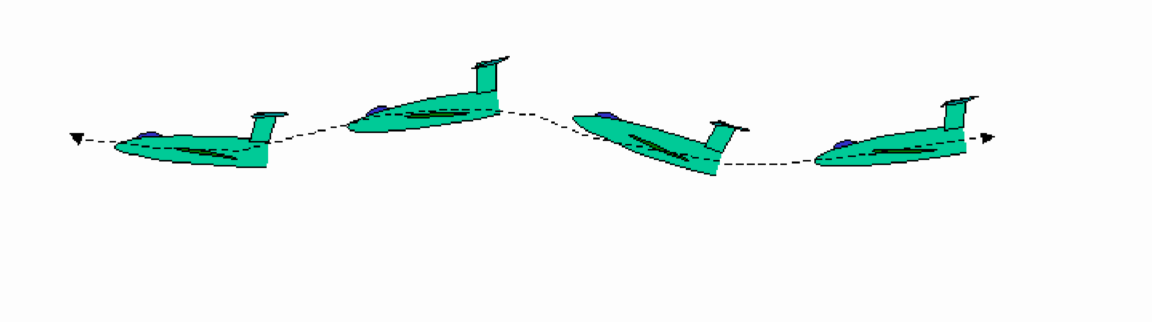
A fan is a propeller with a large number of blades. A fan therefore produces a lot of thrust for a given diameter but the closeness of the blades means that each strongly affects the flow around the others. If the flow is supersonic, this interference can be beneficial if the flow can be compressed through a series of shock waves rather than one. By placing the fan within a shaped duct, specific flow patterns can be created depending on flight speed and engine performance. As air enters the duct, its speed is reduced while its pressure and temperature increases. If the aircraft is at a high subsonic speed this creates two advantages: the air enters the fan at a lower Mach speed; and the higher temperature increases the local speed of sound. While there is a loss in efficiency as the fan is drawing on a smaller area of the free stream and so using less air, this is balanced by the ducted fan retaining efficiency at higher speeds where conventional propeller efficiency would be poor. A ducted fan or propeller also has certain benefits at lower speeds but the duct needs to be shaped in a different manner than one for higher speed flight. More air is taken in and the fan therefore operates at an efficiency equivalent to a larger un-ducted propeller. Noise is also reduced by the ducting and should a blade become detached the duct would contain the damage. However the duct adds weight, cost, complexity and (to a certain degree) drag.

# Chapter 13

# Phugoid and Dutch Roll

## Phugoid

A **phugoid** is an aircraft motion where the vehicle pitches up and climbs, and then pitches down and descends, accompanied by speeding up and slowing down as it goes "uphill" and "downhill." This is one of the basic flight dynamics modes of an aircraft (others include short period, dutch roll, and spiral divergence), and a classic example of a positive feedback system.



A diagrammatic representation of a fixed-wing airplane in phugoid

### *Detailed description*

The phugoid has a nearly constant angle of attack but varying pitch, caused by a repeated exchange of airspeed and altitude. It can be excited by an elevator singlet (a short, sharp deflection followed by a return to the centered position) resulting in a pitch increase with no change in trim from the cruise condition. As speed decays, the nose will drop below the horizon. Speed will increase, and the nose will climb above the horizon. Periods can vary from under 30 seconds for light aircraft to minutes for larger aircraft. Microlight aircraft typically show a phugoid period of 15–25 seconds, and it has been suggested that birds and model airplanes show convergence between the phugoid and short period modes. A classical model for the phugoid period can be simplified to about (0.85 × speed in knots) seconds, but this only really works for larger aircraft.

Phugoids are often demonstrated to student pilots as an example of the speed stability of the aircraft and the importance of proper trimming. When it occurs, it is considered a nuisance, and in lighter airplanes (typically showing a shorter period) it can be a cause of pilot-induced oscillation.

The phugoid, for moderate amplitude, occurs at an effectively constant angle of attack, although in practice the angle of attack actually varies by a few tenths of a degree. This means that the stalling angle of attack is never exceeded, and it is possible (in the <1g section of the cycle) to fly at speeds below the known stalling speed. Free flight models with badly unstable phugoid typically stall or loop, depending on thrust.

An unstable or divergent phugoid is caused, mainly, by a large difference between the incidence angles of the wing and tail. A stable, decreasing phugoid can be attained by building a smaller stabilizer on a longer tail, or, at the expense of pitch and yaw "static" stability, by shifting the center of gravity to the rear.

The term "phugoid" was coined by Frederick W. Lanchester, the British aerodynamicist who first characterized the phenomenon. He derived the word from the Greek words φυγή and εἶδος to mean "flight-like" but recognized the diminished appropriateness of the derivation given that φυγή meant flight in the sense of "escape" rather than vehicle flight.

## *Aviation incidents*

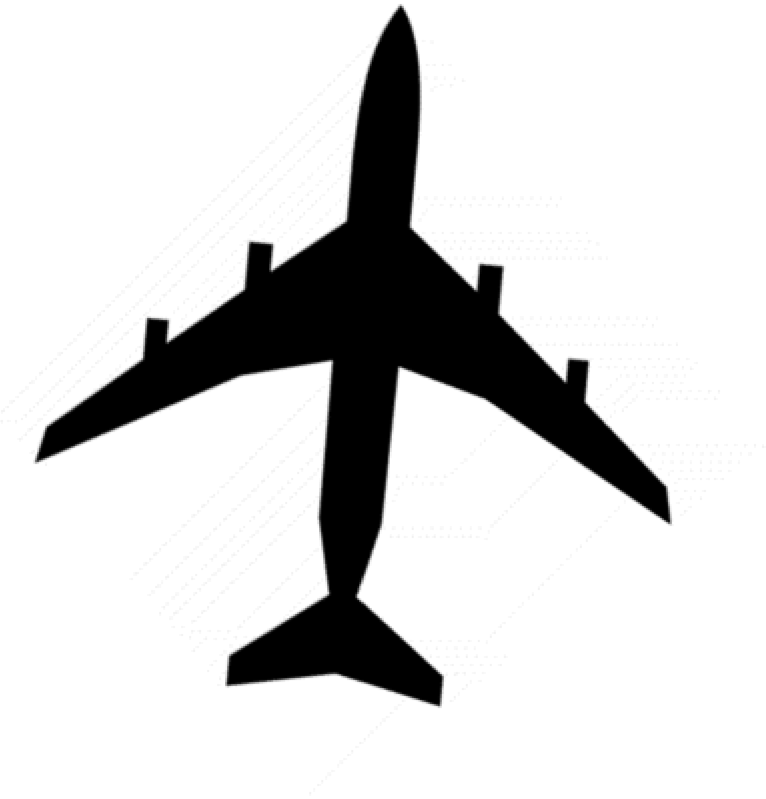
Japan Airlines Flight 123 lost all hydraulic controls and its vertical stabiliser, and went into a phugoid before crashing into a mountain. With 520 deaths it remains the deadliest single-aircraft disaster in history.

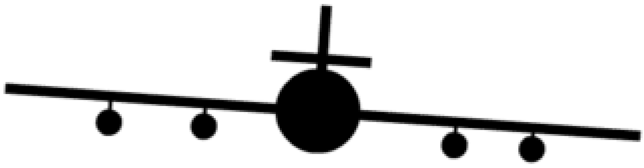
United Airlines Flight 232 suffered an engine failure which caused total hydraulic system failure. The crew flew the aircraft with throttle only. Suppressing the phugoid tendency was particularly difficult. The pilots were able to reach Sioux Gateway Airport but crashed during the landing attempt. The pilots and a majority of the passengers survived.

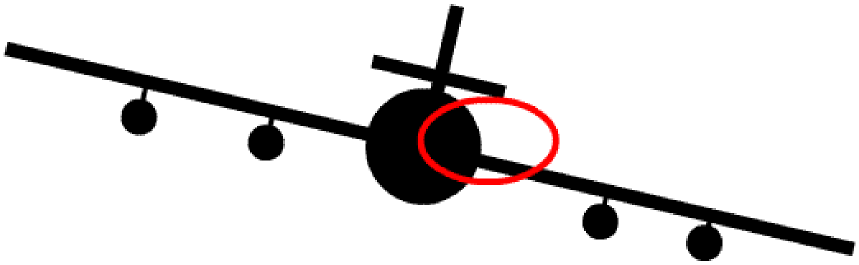
Another aircraft that lost all hydraulics was a DHL operated Airbus A300B4 that was hit by a surface-to-air missile fired by the Iraqi resistance in the 2003 Baghdad DHL attempted shootdown incident. This was the first time that a crew was able to land an air transport aircraft safely only adjusting engine thrust.

The crash of the Helios solar powered aircraft was precipitated by reacting to an inappropriately diagnosed phugoid oscillation the results of which ultimately resulted in the aircraft structure exceeding design loads.

## Dutch roll







**Dutch roll** is a type of aircraft motion, consisting of an out-of-phase combination of "tail-wagging" and rocking from side to side. This yaw-roll coupling is one of the basic flight dynamic modes (others include phugoid, short period, and spiral divergence). This motion is normally well damped in most light aircraft, though some aircraft with well-damped Dutch roll modes can experience a degradation in damping as airspeed decreases and altitude increases. Dutch roll stability can be artificially increased by the installation of a yaw damper. Wings placed well above the center of mass, sweepback (swept wings) and dihedral wings tend to increase the roll restoring force, and therefore increase the Dutch roll tendencies; this is why high-winged aircraft often are slightly anhedral, and transport category swept wing aircraft are equipped with yaw dampers.

In aircraft design, Dutch roll results from relatively weaker positive directional stability as opposed to positive lateral stability. When an aircraft rolls around the longitudinal axis, a sideslip is introduced into the relative wind in the direction of the rolling motion. Strong lateral stability begins to restore the aircraft to level flight. At the same time, somewhat weaker directional stability attempts to correct the sideslip by aligning the aircraft with the perceived relative wind. Since directional stability is weaker than lateral stability for the particular aircraft, the restoring yaw motion lags significantly behind the restoring roll motion. As such, the aircraft passes through level flight as the yawing motion is continuing in the direction of the original roll. At that point, the sideslip is introduced in the opposite direction and the process is reversed.

The Dutch roll mode can be excited by any use of aileron or rudder, but for flight test purposes it is usually excited with a rudder singlet (short, sharp motions of the rudder to a specified angle, and then back to the centered position) or doublet (a pair of such motions in opposite directions). Some larger aircraft are better excited with aileron inputs. Periods can range from a few seconds for light aircraft to a minute or more for airliners.

Dutch roll is also the name (considered by professionals to be a misnomer) given to a coordination maneuver generally taught to student pilots to help them improve their "stick-and-rudder" technique. The aircraft is alternately rolled as much as 60 degrees left and right while rudder is applied to keep the nose of the aircraft pointed at a fixed point.

This coordination technique is better referred to as "rolling on a heading", where the aircraft is rolled in such a way as to maintain an accurate heading without the nose moving from side-to-side (or yawing). The yaw motion is induced through the use of ailerons alone due to aileron drag where the lifting wing (aileron down) is doing more work than the descending wing (aileron up) and therefore creates more drag, forcing the lifting wing back, yawing the aircraft toward it. This yawing effect produced by rolling motion is known as adverse yaw. This has to be countered precisely by application of rudder *in the same direction* as the aileron control (left stick, left rudder - right stick, right rudder). This is known as synchronised controls when done properly, and is difficult to learn and apply well. As each aircraft is different, learning the correct amount of rudder to apply with aileron is different for each aircraft.

The origin of the name Dutch roll is uncertain. However, it is likely that this term, describing a lateral asymmetric motion of an airplane, was borrowed from a reference to similar appearing motion in ice skating. In 1916, aeronautical engineer Jerome C. Hunsaker published the following quote: “Dutch roll – the third element in the [lateral] motion [of an airplane] is a yawing to the right and left, combined with rolling. The motion is oscillatory of period for 5 to 12 seconds which may or may not be damped. The analogy to ‘Dutch Roll’ or ‘Outer Edge’ in ice skating is obvious.” In 1916, Dutch Roll was the term used for skating repetitively to right and left (by analogy to the motion described for the aircraft) on the outer edge of one’s skates. By 1916, the term had been imported from skating to aeronautical engineering, perhaps by Hunsaker himself. 1916 was only five years after G.H. Bryan did the first mathematics of lateral motion of aircraft in 1911.

## Chapter 14

# Mach Number

An F/A-18 Hornet at transonic speed and displaying the Prandtl–Glauert singularity just before reaching the speed of sound

**Mach number** (Ma or *M*) is the speed of an object moving through air, or any other fluid substance, divided by the speed of sound as it is in that substance for its particular physical conditions, including those of temperature and pressure. It is commonly used to represent the speed of an object when it is traveling close to or above the speed of sound.

$$M = \frac{V}{a}$$

where

$M$ is the Mach number
$V$ is the relative velocity of the source to the medium and
$a$ is the speed of sound in the medium

The Mach number is named after Austrian physicist and philosopher Ernst Mach, a designation proposed by aeronautical engineer Jakob Ackeret. Because the Mach number is often viewed as a dimensionless quantity rather than a unit of measure, with Mach, the number comes *after* the unit; the second Mach number is "Mach 2" instead of "2 Mach" (or Machs). This is somewhat reminiscent of the early modern ocean sounding unit "mark" (a synonym for fathom), which was also unit-first, and may have influenced the use of the term Mach. In the decade preceding faster-than-sound human flight, aeronautical engineers referred to the speed of sound as *Mach's number*, never "Mach 1."

In French, the Mach number is sometimes called the "nombre de Sarrau" ("Sarrau number") after Émile Sarrau, researching on explosions in the 1870s and 1880s.

## Overview

The Mach number is commonly used both with objects traveling at high speed in a fluid, and with high-speed fluid flows inside channels such as nozzles, diffusers or wind tunnels. As it is defined as a ratio of two speeds, it is a dimensionless number. At Standard Sea Level conditions (corresponding to a temperature of 15 degrees Celsius), the speed of sound is 340.3 m/s (1225 km/h, or 761.2 mph, or 661.5 knots, or 1116 ft/s) in the Earth's atmosphere. The speed represented by Mach 1 is not a constant; for example, it is mostly dependent on temperature and atmospheric composition and largely independent of pressure. In the stratosphere, where the temperatures are constant, it does not vary with altitude even though the air pressure changes significantly with altitude.

Since the speed of sound increases as the temperature increases, the actual speed of an object traveling at Mach 1 will depend on the fluid temperature around it. Mach number is useful because the fluid behaves in a similar way at the same Mach number. So, an aircraft traveling at Mach 1 at 20°C or 68°F will experience shock waves in much the same manner as when it is traveling at Mach 1 at 11,000 m (36,000 ft) at -50°C or -58F, even though it is traveling at only 86% of its speed at higher temperature like 20°C or 68°F.

## High-speed flow around objects

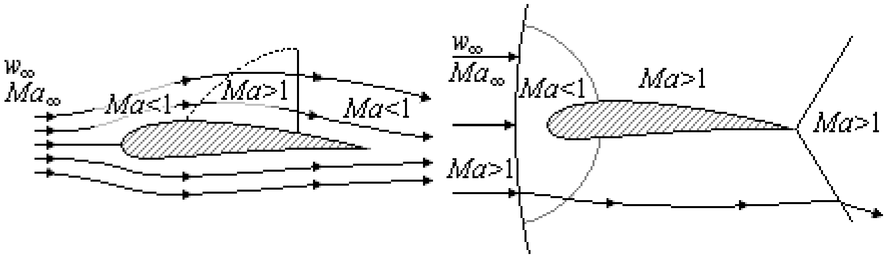
Flight can be roughly classified in six categories:

| Regime | Subsonic | Transonic | Sonic | Supersonic | Hypersonic | High-hypersonic |
|---|---|---|---|---|---|---|
| **Mach** | <0.75 | 0.75–1.2 | 1.0 | 1.2–5.0 | 5.0–10.0 | >10.0 |

For comparison: the required speed for low Earth orbit is approximately 7.5 km/s = Mach 25.4 in air at high altitudes. The speed of light in a vacuum corresponds to a Mach number of approximately 881,000 (relative to air at sea level).

At transonic speeds, the flow field around the object includes both sub- and supersonic parts. The transonic period begins when first zones of $M>1$ flow appear around the object. In case of an airfoil (such as an aircraft's wing), this typically happens above the wing. Supersonic flow can decelerate back to subsonic only in a normal shock; this typically happens before the trailing edge. (Fig.1a)

As the speed increases, the zone of $M>1$ flow increases towards both leading and trailing edges. As $M=1$ is reached and passed, the normal shock reaches the trailing edge and becomes a weak oblique shock: the flow decelerates over the shock, but remains supersonic. A normal shock is created ahead of the object, and the only subsonic zone in the flow field is a small area around the object's leading edge. (Fig.1b)



(a) (b)

**Fig. 1.** Mach number in transonic airflow around an airfoil; $M<1$ (a) and $M>1$ (b)

When an aircraft exceeds Mach 1 (i.e. the sound barrier) a large pressure difference is created just in front of the aircraft. This abrupt pressure difference, called a shock wave, spreads backward and outward from the aircraft in a cone shape (a so-called Mach cone). It is this shock wave that causes the sonic boom heard as a fast moving aircraft travels overhead. A person inside the aircraft will not hear this. The higher the speed, the more narrow the cone; at just over $M=1$ it is hardly a cone at all, but closer to a slightly concave plane.

At fully supersonic speed, the shock wave starts to take its cone shape and flow is either completely supersonic, or (in case of a blunt object), only a very small subsonic flow area remains between the object's nose and the shock wave it creates ahead of itself. (In the case of a sharp object, there is no air between the nose and the shock wave: the shock wave starts from the nose.)

As the Mach number increases, so does the strength of the shock wave and the Mach cone becomes increasingly narrow. As the fluid flow crosses the shock wave, its speed is reduced and temperature, pressure, and density increase. The stronger the shock, the greater the changes. At high enough Mach numbers the temperature increases so much over the shock that ionization and dissociation of gas molecules behind the shock wave begin. Such flows are called hypersonic.

It is clear that any object traveling at hypersonic speeds will likewise be exposed to the same extreme temperatures as the gas behind the nose shock wave, and hence choice of heat-resistant materials becomes important.

## High-speed flow in a channel

As a flow in a channel crosses $M$=1 becomes supersonic, one significant change takes place. The conservation of mass flow rate leads one to expect that contracting the flow channel would increase the flow speed (i.e. making the channel narrower results in faster air flow) and at subsonic speeds this holds true. However, once the flow becomes supersonic, the relationship of flow area and speed is reversed: expanding the channel actually increases the speed.

The obvious result is that in order to accelerate a flow to supersonic, one needs a convergent-divergent nozzle, where the converging section accelerates the flow to $M$=1, sonic speeds, and the diverging section continues the acceleration. Such nozzles are called de Laval nozzles and in extreme cases they are able to reach incredible, hypersonic speeds (Mach 13 at 20°C).

An aircraft Machmeter or electronic flight information system (EFIS) can display Mach number derived from stagnation pressure (pitot tube) and static pressure.

## Calculating Mach Number

Assuming air to be an ideal gas, the formula to compute Mach number in a subsonic compressible flow is derived from Bernoulli's equation for $M$<1:

$$M = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{q_c}{p} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

where:

$M$ is Mach number
$q_c$ is impact pressure and
$p$ is static pressure
$\gamma$ is the ratio of specific heat of a gas at a constant pressure to heat at a constant volume (1.4 for air).

The formula to compute Mach number in a supersonic compressible flow is derived from the Rayleigh Supersonic Pitot equation:

$$\frac{q_c}{p} = \left[\left(\frac{\gamma+1}{2}\right)M^2\right]^{\left(\frac{\gamma}{\gamma-1}\right)} \cdot \left[\frac{\gamma+1}{(1-\gamma+2\gamma\cdot M^2)^{\left(\frac{1}{\gamma-1}\right)}}\right]^{\left(\frac{1}{\gamma-1}\right)} - 1$$

or for air, a simplfied formula:

$$M = 0.88128485\sqrt{\left[\left(\frac{q_c}{p}+1\right)\left(1-\frac{1}{[7M^2]}\right)^{2.5}\right]}$$

where:

$q_c$ is now impact pressure measured behind a normal shock.

The Mach number at which an aircraft is flying at can be calculated by

$$M = \frac{V}{a}$$

where:

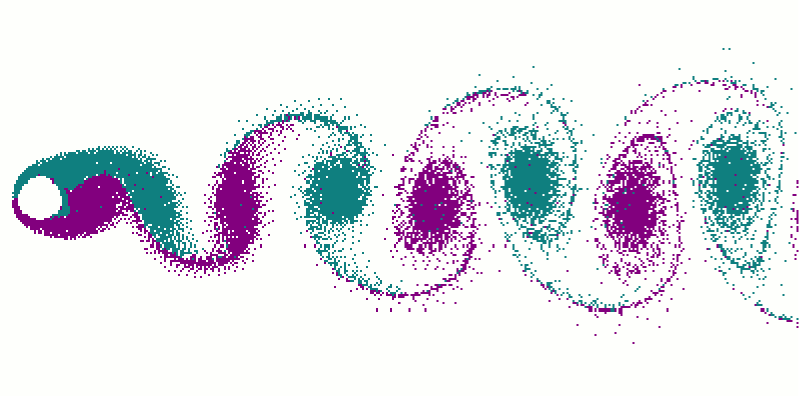
$M$ is Mach number
$V$ is velocity of the moving aircraft and
$a$ is the speed of sound at the given altitude

Note that the dynamic pressure can be found as:

$$q = \frac{\gamma}{2}pM^2$$

## Chapter 15

# Reynolds Number



A vortex street around a cylinder. This occurs around cylinders, independently of the fluid, the cylinder size and the fluid speed, provided that there is a Reynolds number of between 250 and 200,000.

In fluid mechanics, the **Reynolds number** Re is a dimensionless number that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions. The concept was introduced by George Gabriel Stokes in 1851, but the Reynolds number is named after Osborne Reynolds (1842–1912), who popularized its use in 1883.

Reynolds numbers frequently arise when performing dimensional analysis of fluid dynamics problems, and as such can be used to determine dynamic similitude between different experimental cases. They are also used to characterize different flow regimes, such as laminar or turbulent flow: laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion, while turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

## *Definition*

Reynolds number can be defined for a number of different situations where a fluid is in relative motion to a surface (the definition of the Reynolds number is not to be confused with the Reynolds Equation or lubrication equation). These definitions generally include the fluid properties of density and viscosity, plus a velocity and a *characteristic length* or *characteristic dimension*. This dimension is a matter of convention - for example a radius or diameter are equally valid for spheres or circles, but one is chosen by convention. For aircraft or ships, the length or width can be used. For flow in a pipe or a sphere moving in a fluid the internal diameter is generally used today. Other shapes (such as rectangular pipes or non-spherical objects) have an *equivalent diameter* defined. For fluids of variable density (e.g. compressible gases) or variable viscosity (non-Newtonian fluids) special rules apply. The velocity may also be a matter of convention in some circumstances, notably stirred vessels.

$$\mathrm{Re} = \frac{\rho \mathrm{V} L}{\mu} = \frac{\mathrm{V} L}{\nu}$$

where:

- $\mathrm{V}$ is the mean velocity of the object relative to the fluid (SI units: m/s)
- $L$ is a characteristic linear dimension, (travelled length of the fluid; hydraulic diameter when dealing with river systems) (m)
- $\mu$ is the dynamic viscosity of the fluid (Pa·s or N·s/m² or kg/(m·s))
- $\nu$ is the kinematic viscosity ($\nu = \mu / \rho$) (m²/s)
- $\rho$ is the density of the fluid (kg/m³)

Note that multiplying the Reynolds number, $\frac{\rho V L}{\mu}$ by $\frac{VL}{VL}$ yields $\frac{\rho V^2 L^2}{\mu V L}$ which is the ratio, $\frac{inertia force (drag)}{viscous force}$.

## Significance

$$\mathrm{Re} = \frac{\text{total momentum transfer}}{\text{molecular momentum transfer}}$$

## Flow in Pipe

For flow in a pipe or tube, the Reynolds number is generally defined as:

$$\mathrm{Re} = \frac{\rho \mathrm{V} D_H}{\mu} = \frac{\mathrm{V} D_H}{\nu} = \frac{\mathrm{Q} D_H}{\nu A}$$

where:

- $D_H$ is the hydraulic diameter of the pipe (m).
- Q is the volumetric flow rate (m$^3$/s)
- $A$ is the pipe *cross-sectional* area (m²).

## Flow in a non-circular duct (annulus)

For shapes such as squares, rectangular or annular ducts (where the height and width are comparable) the characteristic dimension for internal flow situations is taken to be the *hydraulic diameter*, $D_H$, defined as 4 times the cross-sectional area (of the fluid), divided by the **wetted perimeter**. The wetted perimeter for a channel is the total perimeter of all channel walls that are in contact with the flow. This means the length of the water exposed to air is NOT included in the wetted perimeter

$$D_H = \frac{4A}{P}.$$

For a circular pipe, the hydraulic diameter is exactly equal to the inside pipe diameter, as can be shown mathematically.

For an annular duct, such as the outer channel in a tube-in-tube heat exchanger, the hydraulic diameter can be shown algebraically to reduce to

$$D_{H,\text{annulus}} = D_o - D_i$$

where

$D_o$ is the inside diameter of the outside pipe, and
$D_i$ is the outside diameter of the inside pipe.

For calculations involving flow in non-circular ducts, the hydraulic diameter can be substituted for the diameter of a circular duct, with reasonable accuracy.

## Flow in a Wide Duct

For a fluid moving between two plane parallel surfaces (where the width is much greater than the space between the plates) then the characteristic dimension is twice the distance between the plates.

## Flow in an Open Channel

For flow of liquid with a free surface, the *hydraulic radius* must be determined. This is the cross-sectional area of the channel divided by the wetted perimeter. For a semi-circular channel, it is half the radius. For a rectangular channel, the hydraulic radius is the cross-sectional area divided by the wetted perimeter. Some texts then use a characteristic dimension that is 4 times the hydraulic radius (chosen because it gives the same value of Re for the onset of turbulence as in pipe flow), while others the hydraulic radius as the characteristic length-scale with consequently different values of Re for transition and turbulent flow.

## Object in a fluid

The Reynolds number for an object in a fluid, called the particle Reynolds number and often denoted $\mathbf{Re_p}$, is important when considering the nature of flow around that grain, whether or not vortex shedding will occur, and its fall velocity.

### Sphere in a fluid

For a sphere in a fluid, the characteristic length-scale is the diameter of the sphere and the characteristic velocity is that of the sphere relative to the fluid some distance away from the sphere (such that the motion of the sphere does not disturb that reference parcel of fluid). The density and viscosity are those belonging to the fluid. Note that purely laminar flow only exists up to Re = 0.1 under this definition.

Under the condition of low Re, the relationship between force and speed of motion is given by Stokes' law.

### Oblong object in a fluid

The equation for an oblong object is identical to that of a sphere, with the object being approximated as an ellipsoid and the axis of length being chosen as the characteristic length scale. Such considerations are important in natural streams, for example, where there are few perfectly spherical grains. For grains in which measurement of each axis is impractical (e.g., because they are too small), sieve diameters are used instead as the characteristic particle length-scale. Both approximations alter the values of the critical Reynolds number.

### Fall velocity

The particle Reynolds number is important in determining the fall velocity of a particle. When the particle Reynolds number indicates laminar flow, Stokes' law can be used to calculate its fall velocity. When the particle Reynolds number indicates turbulent flow, a turbulent drag law must be constructed to model the appropriate settling velocity.

## Packed Bed

For flow of fluid through a bed of approximately spherical particles of diameter ***D*** in contact, if the voidage (fraction of the bed not filled with particles) is **ε** and the superficial velocity **V** (i.e. the velocity through the bed as if the particles were not there - the actual velocity will be higher) then a Reynolds number can be defined as:

$$\mathrm{Re} = \frac{\rho V D}{\mu(1-\epsilon)}.$$

Laminar conditions apply up to Re = 10, fully turbulent from 2000.

### Stirred Vessel

In a cylindrical vessel stirred by a central rotating paddle, turbine or propellor, the characteristic dimension is the diameter of the agitator *D*. The velocity is *ND* where *N* is the rotational speed (revolutions per second). Then the Reynolds number is:

$$\mathrm{Re} = \frac{\rho N D^2}{\mu}.$$

The system is fully turbulent for values of Re above 10 000.

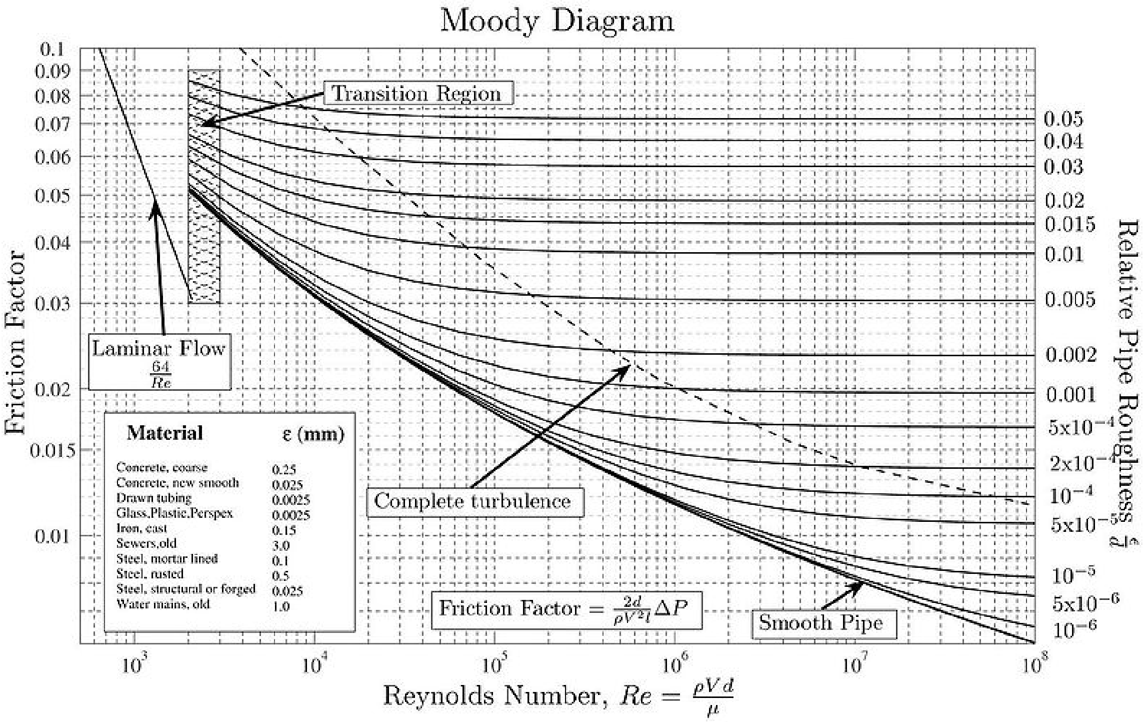
## *Transition Reynolds number*

In boundary layer flow over a flat plate, experiments can confirm that, after a certain length of flow, a laminar boundary layer will become unstable and become turbulent. This instability occurs across different scales and with different fluids, usually when $\mathrm{Re}_x \approx 5 \times 10^5$, where *x* is the distance from the leading edge of the flat plate, and the flow velocity is the freestream velocity of the fluid outside the boundary layer.

For flow in a pipe of diameter *D*, experimental observations show that for 'fully developed' flow (Note:), laminar flow occurs when $\mathrm{Re}_D < 2300$ and turbulent flow occurs when $\mathrm{Re}_D > 4000$. In the interval between 2300 and 4000, laminar and turbulent flows are possible ('transition' flows), depending on other factors, such as pipe roughness and flow uniformity). This result is generalised to non-circular channels using the hydraulic diameter, allowing a transition Reynolds number to be calculated for other shapes of channel.

These transition Reynolds numbers are also called *critical Reynolds numbers*, and were studied by Osborne Reynolds around 1895.

## Reynolds number in pipe friction



| **Material** | **ε (mm)** |
|---|---|
| Concrete, coarse | 0.25 |
| Concrete, new smooth | 0.025 |
| Drawn tubing | 0.0025 |
| Glass,Plastic,Perspex | 0.0025 |
| Iron, cast | 0.15 |
| Sewers,old | 3.0 |
| Steel, mortar lined | 0.1 |
| Steel, rusted | 0.5 |
| Steel, structural or forged | 0.025 |
| Water mains, old | 1.0 |

Pressure drops seen for fully-developed flow of fluids through pipes can be predicted using the Moody diagram which plots the Darcy–Weisbach friction factor *f* against Reynolds number Re and relative roughness ε / *D*. The diagram clearly shows the laminar, transition, and turbulent flow regimes as Reynolds number increases. The nature of pipe flow is strongly dependent on whether the flow is laminar or turbulent

## The similarity of flows

In order for two flows to be similar they must have the same geometry, and have equal Reynolds numbers and Euler numbers. When comparing fluid behaviour at corresponding points in a model and a full-scale flow, the following holds:

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re} \\
\mathrm{Eu}_m &= \mathrm{Eu} \qquad \text{i.e.} \quad \frac{p_m}{\varrho_m {v_m}^2} = \frac{p}{\varrho v^2} \, ,
\end{aligned}
$$

quantities marked with 'm' concern the flow around the model and the others the actual flow. This allows engineers to perform experiments with reduced models in water channels or wind tunnels, and correlate the data to the actual flows, saving on costs during experimentation and on lab time. Note that true dynamic similitude may require matching other dimensionless numbers as well, such as the Mach number used in compressible flows, or the Froude number that governs open-channel flows. Some flows involve more dimensionless parameters than can be practically satisfied with the available apparatus and fluids (for example air or water), so one is forced to decide which parameters are most important. For experimental flow modeling to be useful, it requires a fair amount of experience and judgement of the engineer.

## Typical values of Reynolds number

- Ciliate ~ 1 x $10^{-1}$
- Smallest Fish ~ 1

- Blood flow in brain ~ 1 × $10^2$
- Blood flow in aorta ~ 1 × $10^3$

**Onset of turbulent flow** ~ 2.3 × $10^3$ to 5.0 × $10^4$ for pipe flow to $10^6$ for boundary layers

- Typical pitch in Major League Baseball ~ 2 × $10^5$
- Person swimming ~ 4 × $10^6$
- Fastest Fish ~ 1 x $10^6$
- Blue Whale ~ 3 × $10^8$
- A large ship (RMS Queen Elizabeth 2) ~ 5 × $10^9$

## *Reynolds number sets the smallest scales of turbulent motion*

In a turbulent flow, there is a range of scales of the time-varying fluid motion. The size of the largest scales of fluid motion (sometimes called eddies) are set by the overall geometry of the flow. For instance, in an industrial smoke stack, the largest scales of fluid motion are as big as the diameter of the stack itself. The size of the smallest scales is set by the Reynolds number. As the Reynolds number increases, smaller and smaller scales of the flow are visible. In a smoke stack, the smoke may appear to have many very small velocity perturbations or eddies, in addition to large bulky eddies. In this sense, the Reynolds number is an indicator of the range of scales in the flow. The higher the Reynolds number, the greater the range of scales. The largest eddies will always be the same size; the smallest eddies are determined by the Reynolds number.

What is the explanation for this phenomenon? A large Reynolds number indicates that viscous forces are not important at large scales of the flow. With a strong predominance of inertial forces over viscous forces, the largest scales of fluid motion are undamped—there is not enough viscosity to dissipate their motions. The kinetic energy must "cascade" from these large scales to progressively smaller scales until a level is reached for which the scale is small enough for viscosity to become important (that is, viscous forces become of the order of inertial ones). It is at these small scales where the dissipation of energy by viscous action finally takes place. The Reynolds number indicates at what scale this viscous dissipation occurs. Therefore, since the largest eddies are dictated by the flow geometry and the smallest scales are dictated by the viscosity, the Reynolds number can be understood as the ratio of the largest scales of the turbulent motion to the smallest scales.

## Example of the importance of the Reynolds number

If an airplane wing needs testing, one can make a scaled down model of the wing and test it in a wind tunnel using the same Reynolds number that the actual airplane is subjected to. If for example the scale model has linear dimensions one quarter of full size, the flow velocity of the model would have to be multiplied by a factor of 4 to obtain similar flow behavior.

Alternatively, tests could be conducted in a water tank instead of in air (provided the compressibility effects of air are not significant). As the kinematic viscosity of water is around 13 times less than that of air at 15 °C, in this case the scale model would need to be about one thirteenth the size in all dimensions to maintain the same Reynolds number, assuming the full-scale flow velocity was used.

The results of the laboratory model will be similar to those of the actual plane wing results. Thus there is no need to bring a full scale plane into the lab and actually test it. This is an example of "dynamic similarity".

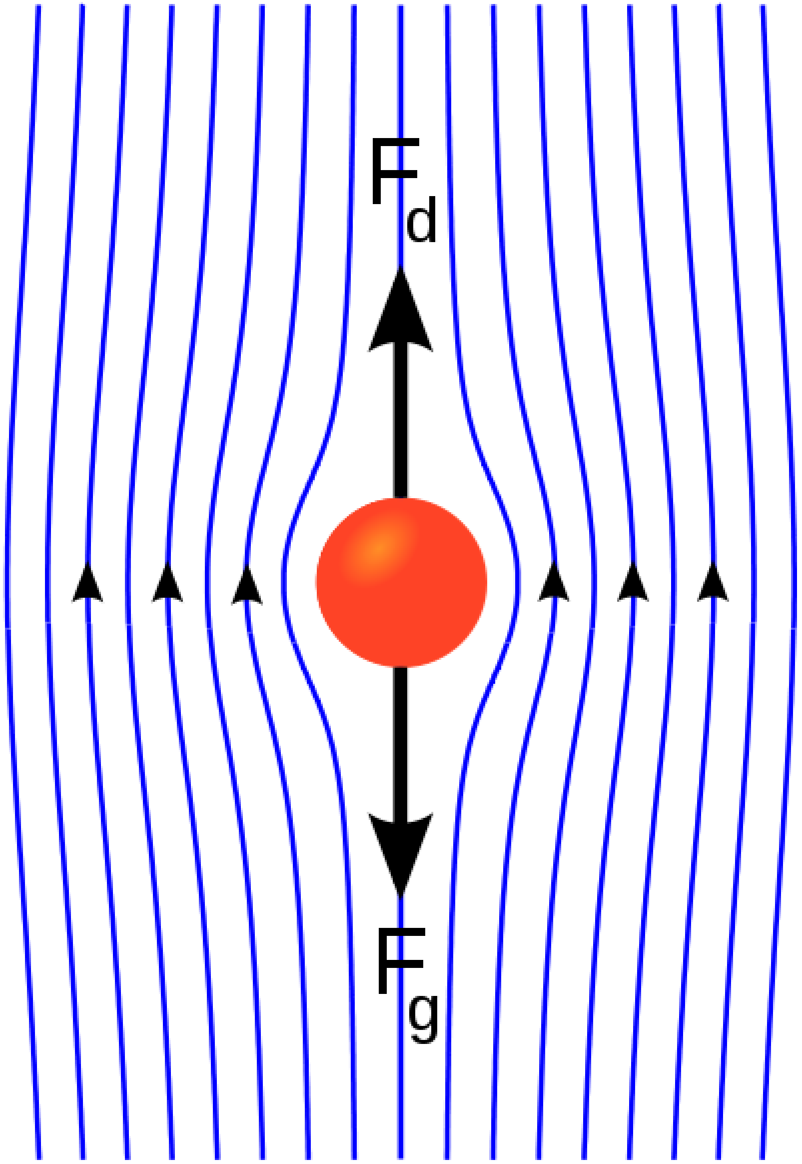
Reynolds number is important in the calculation of a body's drag characteristics. A notable example is that of the flow around a cylinder. Above roughly $3\times10^6$ Re the drag coefficient drops considerably. This is important when calculating the optimal cruise speeds for low drag (and therefore long range) profiles for airplanes.

## Reynolds number in physiology

Poiseuille's law on blood circulation in the body is dependent on laminar flow. In turbulent flow the flow rate is proportional to the square root of the pressure gradient, as opposed to its direct proportionality to pressure gradient in laminar flow.

Using the definition of the Reynolds number we can see that a large diameter with rapid flow, where the density of the blood is high, tends towards turbulence. Rapid changes in vessel diameter may lead to turbulent flow, for instance when a narrower vessel widens to a larger one. Furthermore, an atheroma may be the cause of turbulent flow, and as such detecting turbulence with a stethoscope may be a sign of such a condition.

### *Reynolds number in viscous fluids*



Creeping flow past a sphere: streamlines, drag force $F_d$ and force by gravity $F_g$

Where the viscosity is naturally high, such as polymer solutions and polymer melts, flow is normally laminar. The Reynolds number is very small and Stokes' Law can be used to measure the viscosity of the fluid. Spheres are allowed to fall through the fluid and they reach the terminal velocity quickly, from which the viscosity can be determined.

The laminar flow of polymer solutions is exploited by animals such as fish and dolphins, who exude viscous solutions from their skin to aid flow over their bodies while swimming. It has been used in yacht racing by owners who want to gain a speed advantage by pumping a polymer solution such as low molecular weight polyoxyethylene in water, over the wetted surface of the hull. It is however, a problem for mixing of polymers, because turbulence is needed to distribute fine filler (for example) through the material. Inventions such as the "cavity transfer mixer" have been developed to produce multiple folds into a moving melt so as to improve mixing efficiency. The device can be fitted onto extruders to aid mixing.

### Where does it come from?

The Reynolds number can be obtained when one uses the nondimensional form of the incompressible Navier-Stokes equations:

$$\varrho\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+\mu\nabla^2\mathbf{v}+\mathbf{f}.$$

Each term in the above equation has the units of a volume force or, equivalently, an acceleration times a density. Each term is thus dependent on the exact measurements of a flow. When one renders the equation nondimensional, that is that we multiply it by a factor with inverse units of the base equation, we obtain a form which does not depend directly on the physical sizes. One possible way to obtain a nondimensional equation is to multiply the whole equation by the following factor:

$$\frac{D}{\varrho V^2}$$

where the symbols are the same as those used in the definition of the Reynolds number. If we now set:

$$\mathbf{v}'=\frac{\mathbf{v}}{V},\ p'=p\frac{1}{\varrho V^2},\ \mathbf{f}'=\mathbf{f}\frac{D}{\varrho V^2},\ \frac{\partial}{\partial t'}=\frac{D}{V}\frac{\partial}{\partial t},\ \nabla'=D\nabla$$

we can rewrite the Navier-Stokes equation without dimensions:

$$\frac{\partial \mathbf{v}'}{\partial t'}+\mathbf{v}'\cdot\nabla'\mathbf{v}'=-\nabla'p'+\frac{\mu}{\varrho DV}\nabla'^2\mathbf{v}'+\mathbf{f}'$$

where the term : $\frac{\mu}{\varrho DV}=\frac{1}{\mathrm{Re}}.$

Finally, dropping the primes for ease of reading:

$$\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}=-\nabla p+\frac{1}{\mathrm{Re}}\nabla^2\mathbf{v}+\mathbf{f}.$$

This is why mathematically all flows with the same Reynolds number are comparable. Notice also, in the above equation, as :$\mathrm{Re} \to \infty$ the viscous terms vanish. Thus, high Reynolds number flows are approximately inviscid in the free-stream.

# Chapter 16

# Knudsen Number

The **Knudsen number** (***Kn***) is a dimensionless number defined as the ratio of the molecular mean free path length to a representative physical length scale. This length scale could be, for example, the radius of a body in a fluid. The number is named after Danish physicist Martin Knudsen (1871–1949).

## *Definition*

The Knudsen number is a dimensionless number defined as:

$$Kn = \frac{\lambda}{L}$$

where

- $\lambda$ = mean free path [$L^1$]
- $L$ = representative physical length scale [$L^1$].

For an ideal gas, the mean free path may be readily calculated so that:

$$Kn = \frac{k_B T}{\sqrt{2}\pi\sigma^2 p L}$$

where

- $k_B$ is the Boltzmann constant ($1.3806504(24) \times 10^{-23}$ J/K in SI units), [$M^1 L^2 T^{-2} \theta^{-1}$]
- $T$ is the thermodynamic temperature, [$\theta^1$]
- $\sigma$ is the particle hard shell diameter, [$L^1$]
- $p$ is the total pressure, [$M^1 L^{-1} T^{-2}$].

For particle dynamics in the atmosphere, and assuming standard temperature and pressure, i.e. 25 °C and 1 atm, we have $\lambda \approx 8 \times 10^{-8}$ m, or approximately $2.6 \times 10^{-9}$ ft.

### Relationship to Mach and Reynolds numbers

The Knudsen number can be related to the Mach number and the Reynolds number:

Noting the following:

Dynamic viscosity,

$$\mu = \frac{1}{2}\rho\bar{c}\lambda.$$

Average molecule speed (from Maxwell-Boltzmann distribution),

$$\bar{c} = \sqrt{\frac{8k_BT}{\pi m}}$$

thus the mean free path,

$$\lambda = \frac{\mu}{\rho}\sqrt{\frac{\pi m}{2k_BT}}$$

dividing through by $L$ (some characteristic length) the Knudsen number is obtained:

$$\frac{\lambda}{L} = \frac{\mu}{\rho L}\sqrt{\frac{\pi m}{2k_BT}}$$

where

- $\bar{c}$ is the average molecular speed from the Maxwell–Boltzmann distribution, [$L^1$ $T^{-1}$]
- $T$ is the thermodynamic temperature, [$\theta^1$]
- $\mu$ is the dynamic viscosity, [$M^1$ $L^{-1}$ $T^{-1}$]
- $m$ is the molecular mass, [$M^1$]
- $k_B$ is the Boltzmann constant, [$M^1$ $L^2$ $T^{-2}$ $\theta^{-1}$]
- $\rho$ is the density, [$M^1$ $L^{-3}$].

The dimensionless Mach number can be written:

$$Ma = \frac{U_\infty}{c_s}$$

where the speed of sound is given by

$$c_s = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma k_B T}{m}}$$

where

- $U_\infty$ is the freestream speed, [$L^1$ $T^{-1}$]
- $R$ is the Universal gas constant, (in SI, 8.314 47215 J $K^{-1}$ $mol^{-1}$), [$M^1$ $L^2$ $T^{-2}$ $\theta^{-1}$ $'mol'^{-1}$]
- $M$ is the molar mass, [$M^1$ $'mol'^{-1}$]
- $\gamma$ is the ratio of specific heats, and is dimensionless.

The dimensionless Reynolds number can be written:

$$Re = \frac{\rho U_\infty L}{\mu}.$$

Dividing the Mach number by the Reynolds number,

$$\frac{Ma}{Re} = \frac{U_\infty \div c_s}{\rho U_\infty L \div \mu} = \frac{\mu}{\rho L c_s} = \frac{\mu}{\rho L \sqrt{\frac{\gamma k_B T}{m}}} = \frac{\mu}{\rho L}\sqrt{\frac{m}{\gamma k_B T}}$$

and by multiplying by $\sqrt{\frac{\gamma\pi}{2}}$,

$$\frac{\mu}{\rho L}\sqrt{\frac{m}{\gamma k_B T}}\sqrt{\frac{\gamma\pi}{2}} = \frac{\mu}{\rho L}\sqrt{\frac{\pi m}{2 k_B T}}$$

the Knudsen number is obtained.

The Mach, Reynolds and Knudsen numbers are therefore related by:

$$Kn = \frac{Ma}{Re}\sqrt{\frac{\gamma\pi}{2}}.$$

## *Application*

The Knudsen number is useful for determining whether statistical mechanics or the continuum mechanics formulation of fluid dynamics should be used: If the Knudsen number is near or greater than one, the mean free path of a molecule is comparable to a length scale of the problem, and the continuum assumption of fluid mechanics is no longer a good approximation. In this case statistical methods must be used.

Problems with high Knudsen numbers include the calculation of the motion of a dust particle through the lower atmosphere, or the motion of a satellite through the exosphere. One of the most widely used applications for the Knudsen number is in microfluidics and MEMS device design. The solution of the flow around an aircraft has a low Knudsen number, making it firmly in the realm of continuum mechanics. Using the Knudsen number an adjustment for Stokes' Law can be used in the Cunningham correction factor, this is a drag force correction due to slip in small particles (i.e. $d_p < 5$ µm).