Structural and Earthquake Engineering

Kimiko Wooden

Melia Meek

First Edition, 2012

ISBN 978-81-323-0991-8

© All rights reserved.

Published by:
Academic Studio
4735/22 Prakashdeep Bldg,
Ansari Road, Darya Ganj,
Delhi - 110002
Email: info@wtbooks.com

Table of Contents

Chapter 1 - Structural Engineering

Chapter 2 - History of Structural Engineering

Chapter 3 - Earthquake Engineering Structures

Chapter 4 - Column (Structural Elements)

Chapter 5 - Catenary

Chapter 6 - Arch (Structural Element)

Chapter 7 - Other Structural Elements used in Structural Engineering

Chapter 8 - Truss

Chapter 9 - Introduction to Earthquake Engineering

Chapter 10 - Seismic Analysis

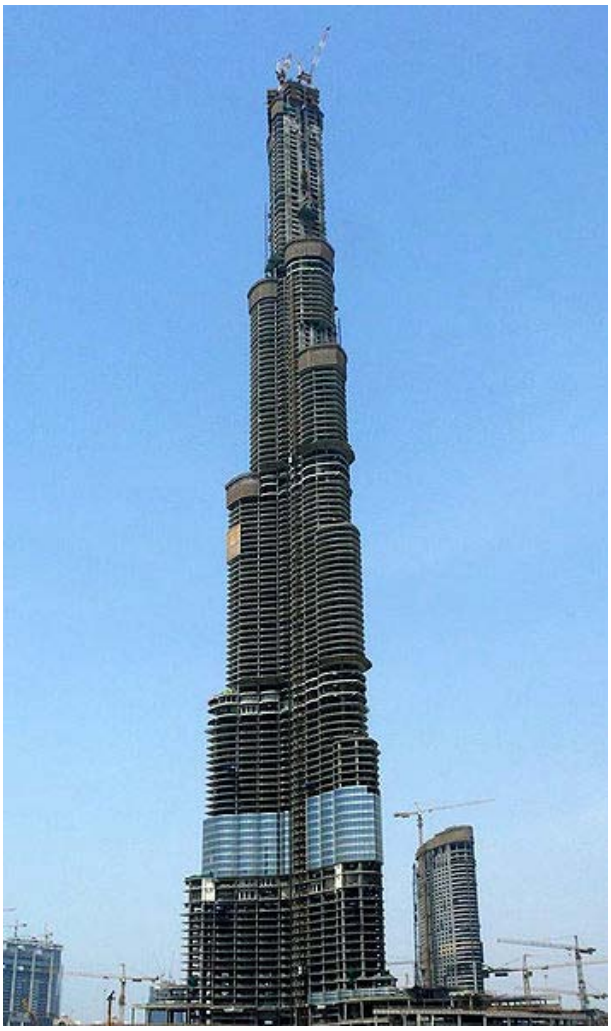
Chapter 11 - Tuned Mass Damper

Chapter 12 - Reinforced Concrete

Chapter 13 - Framing (Construction)

Chapter- 1

Structural Engineering



Burj Khalifa, in Dubai, the world's tallest building, shown under construction in 2007 (since completed)

Structural engineering is a field of engineering dealing with the analysis and design of structures that support or resist loads. Structural engineering is usually considered a specialty within civil engineering, but it can also be studied in its own right. Structural

engineers are most commonly involved in the design of buildings and large nonbuilding structures but they can also be involved in the design of machinery, medical equipment, vehicles or any item where structural integrity affects the item's function or safety. Structural engineers must ensure their designs satisfy given design criteria, predicated on safety (e.g. structures must not collapse without due warning) or serviceability and performance (e.g. building sway must not cause discomfort to the occupants). Buildings are made to endure massive loads as well as changing climate and natural disasters.

Structural engineering theory is based upon physical laws and empirical knowledge of the structural performance of different landscapes and materials. Structural engineering design utilises a relatively small number of basic structural elements to build up structural systems that can be very complex. Structural engineers are responsible for making creative and efficient use of funds, structural elements and materials to achieve these goals.

Structural engineer

Structural engineers are responsible for engineering design and analysis. Entry-level structural engineers may design the individual structural elements of a structure, for example the beams, columns, and floors of a building. More experienced engineers would be responsible for the structural design and integrity of an entire system, such as a building.

Structural engineers often specialize in particular fields, such as bridge engineering, building engineering, pipeline engineering, industrial structures, or special mechanical structures such as vehicles or aircraft.

Structural engineering has existed since humans first started to construct their own structures. It became a more defined and formalised profession with the emergence of the architecture profession as distinct from the engineering profession during the industrial revolution in the late 19th Century. Until then, the architect and the structural engineer were usually one and the same - the master builder. Only with the development of specialised knowledge of structural theories that emerged during the 19th and early 20th centuries did the professional structural engineer come into existence.

The role of a structural engineer today involves a significant understanding of both static and dynamic loading, and the structures that are available to resist them. The complexity of modern structures often requires a great deal of creativity from the engineer in order to ensure the structures support and resist the loads they are subjected to. A structural engineer will typically have a four or five year undergraduate degree, followed by a minimum of three years of professional practice before being considered fully qualified.

Structural engineers are licensed or accredited by different learned societies and regulatory bodies around the world (for example, the Institution of Structural Engineers in the UK). Depending on the degree course they have studied and/or the jurisdiction they

are seeking licensure in, they may be accredited (or licensed) as just structural engineers, or as civil engineers, or as both civil and structural engineers.

History of structural engineering

Pont du Gard, France, a Roman era aqueduct circa 19 BC

Structural engineering dates back to 2700 BC when the step pyramid for Pharaoh Djoser was built by Imhotep, the first engineer in history known by name. Pyramids were the most common major structures built by ancient civilizations because the structural form of a pyramid is inherently stable and can be almost infinitely scaled (as opposed to most other structural forms, which cannot be linearly increased in size in proportion to increased loads).

Throughout ancient and medieval history most architectural design and construction was carried out by artisans, such as stone masons and carpenters, rising to the role of master builder. No theory of structures existed, and understanding of how structures stood up was extremely limited, and based almost entirely on empirical evidence of 'what had worked before'. Knowledge was retained by guilds and seldom supplanted by advances. Structures were repetitive, and increases in scale were incremental.

No record exists of the first calculations of the strength of structural members or the behaviour of structural material, but the profession of structural engineer only really took

shape with the industrial revolution and the re-invention of concrete. The physical sciences underlying structural engineering began to be understood in the Renaissance and have been developing ever since.

Structural failure

The history of structural engineering contains many collapses and failures. Sometimes this is due to obvious negligence, as in the case of the Pétionville school collapse, in which Rev. Fortin Augustin said that *"he constructed the building all by himself, saying he didn't need an engineer as he had good knowledge of construction"* following a partial collapse of the three-story schoolhouse that sent neighbors fleeing. The final collapse killed at least 362 people, mostly children.

In other cases structural failures require careful study, and the results of these inquiries have resulted in improved practices and greater understanding of the science of structural engineering. Some such studies are the result of Forensic engineering investigations where the original engineer seems to have done everything in accordance with the state of the profession and acceptable practice yet a failure still eventuated. A famous case of structural knowledge and practice being advanced in this manner can be found in a series of failures involving Box girders which collapsed in Australia during the 1970s.

Specializations

Building structures

Sydney Opera House, designed by Ove Arup & Partners, with the architect Jørn Utzon

Millennium Dome in London, UK, by Buro Happold and Richard Rogers

Structural building engineering includes all structural engineering related to the design of buildings. It is the branch of structural engineering that is close to architecture.

Structural building engineering is primarily driven by the creative manipulation of materials and forms and the underlying mathematical and scientific ideas to achieve an end which fulfills its functional requirements and is structurally safe when subjected to all the loads it could reasonably be expected to experience. This is subtly different from architectural design, which is driven by the creative manipulation of materials and forms, mass, space, volume, texture and light to achieve an end which is aesthetic, functional and often artistic.

The architect is usually the lead designer on buildings, with a structural engineer employed as a sub-consultant. The degree to which each discipline actually leads the design depends heavily on the type of structure. Many structures are structurally simple and led by architecture, such as multi-storey office buildings and housing, while other structures, such as tensile structures, shells and gridshells are heavily dependent on their form for their strength, and the engineer may have a more significant influence on the form, and hence much of the aesthetic, than the architect.

The structural design for a building must ensure that the building is able to stand up safely, able to function without excessive deflections or movements which may cause fatigue of structural elements, cracking or failure of fixtures, fittings or partitions, or discomfort for occupants. It must account for movements and forces due to temperature, creep, cracking and imposed loads. It must also ensure that the design is practically buildable within acceptable manufacturing tolerances of the materials. It must allow the architecture to work, and the building services to fit within the building and function (air conditioning, ventilation, smoke extract, electrics, lighting etc.). The structural design of a modern building can be extremely complex, and often requires a large team to complete.

Structural engineering specialties for buildings include:

- Earthquake engineering
- Façade engineering
- Fire engineering
- Roof engineering
- Tower engineering
- Wind engineering

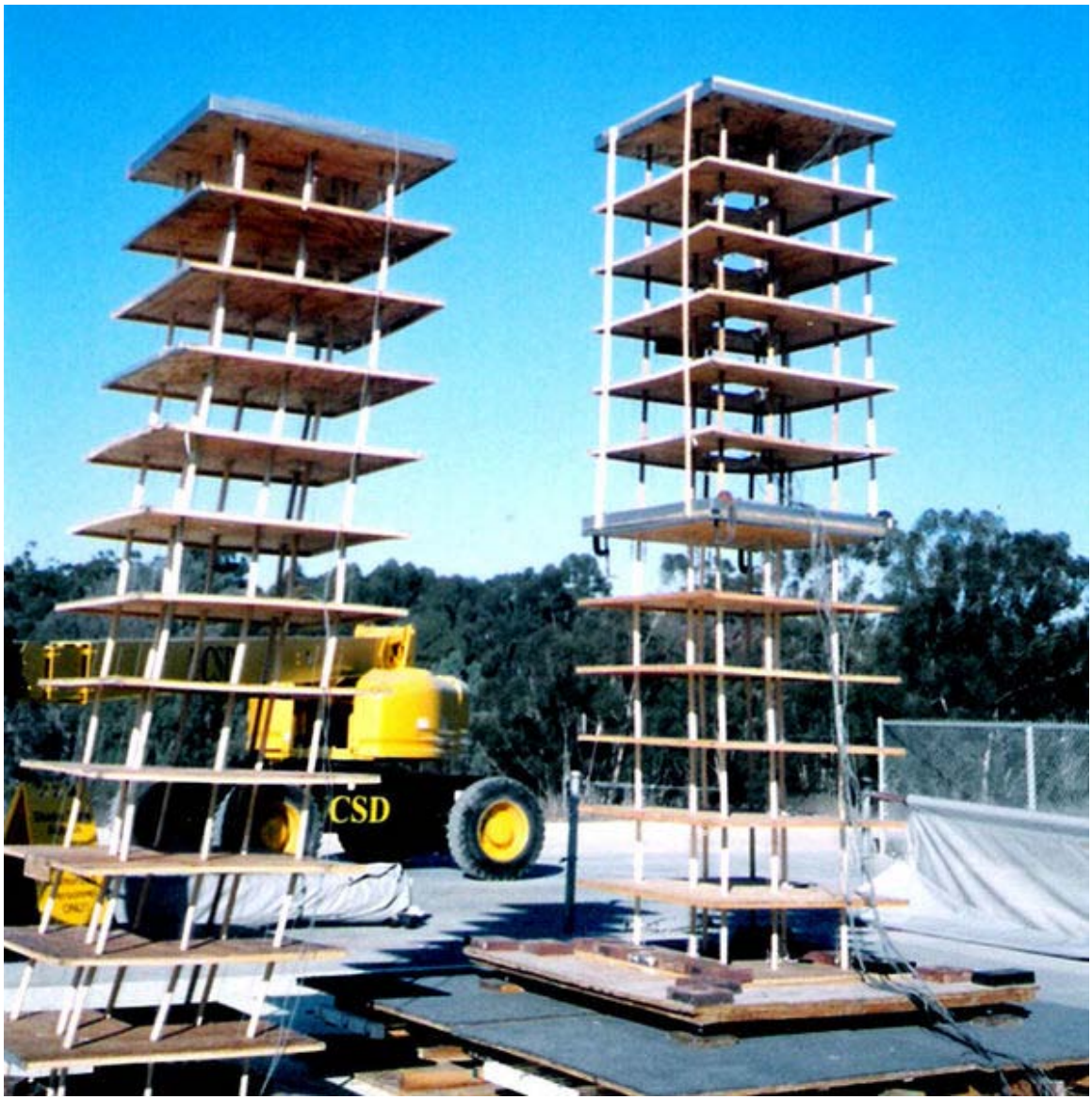
Earthquake engineering structures

Earthquake engineering structures are those engineered to withstand various types of hazardous earthquake exposures at the sites of their particular location.

Earthquake-proof and massive pyramid El Castillo, Chichen Itza

Earthquake engineering is treating its subject structures like defensive fortifications in military engineering but for the warfare on earthquakes. Both earthquake and military general design principles are similar: be ready to slow down or mitigate the advance of a possible attacker.

The main objectives of **earthquake engineering** are:



Snapshot from shake-table testing base-isolated (right) and regular (left) building model

- Understand interaction of structures with the shaky ground.
- Foresee the consequences of possible earthquakes.
- Design and construct the structures to perform while being exposed to an earthquake.

Earthquake engineering or **earthquake-proof structure** does not, necessarily, means *extremely strong* and *expensive* one like El Castillo pyramid at Chichen Itza shown

above. In fact, many structures considered *strong* may in fact be actually *stiff,* which may result in poor seismic performance.

Now, the most *powerful* and *budgetary* tool of the earthquake engineering is base isolation which pertains to the passive structural vibration control technologies.

Civil engineering structures

Civil structural engineering includes all structural engineering related to the built environment. It includes:

- Bridges
- Dams
- Earthworks
- Foundations
- Offshore structures
- Pipelines
- Power stations
- Railways
- Retaining structures and walls
- Roads
- Tunnels
- Waterways
- Water and wastewater infrastructure

The structural engineer is the lead designer on these structures, and often the sole designer. In the design of structures such as these, structural safety is of paramount importance (in the UK, designs for dams, nuclear power stations and bridges must be signed off by a chartered engineer).

Civil engineering structures are often subjected to very extreme forces, such as large variations in temperature, dynamic loads such as waves or traffic, or high pressures from water or compressed gases. They are also often constructed in corrosive environments, such as at sea, in industrial facilities or below ground.

Mechanical structures

An Airbus A380, the world's largest passenger airliner

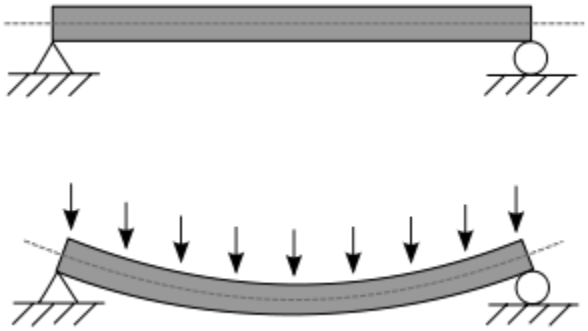
Principals of structural engineering are applied to variety of mechanical (moveable) structures. The design of static structures assumes they always have the same geometry (in fact, so-called static structures can move significantly, and structural engineering design must take this into account where necessary), but the design of moveable or moving structures must account for fatigue, variation in the method in which load is resisted and significant deflections of structures.

The forces which parts of a machine are subjected to can vary significantly, and can do so at a great rate. The forces which a boat or aircraft are subjected to vary enormously and will do so thousands of times over the structure's lifetime. The structural design must ensure that such structures are able to endure such loading for their entire design life without failing.

These works can require mechanical structural engineering:

- Airframes and fuselages
- Boilers and pressure vessels
- Coachworks and carriages
- Cranes
- Elevators
- Escalators
- Marine vessels and hulls

Structural elements



A statically determinate simply supported beam, bending under an evenly distributed load.

Any structure is essentially made up of only a small number of different types of elements:

- Columns
- Beams
- Plates
- Arches
- Shells
- Catenaries

Many of these elements can be classified according to form (straight, plane / curve) and dimensionality (one-dimensional / two-dimensional):

	One-dimensional		**Two-dimensional**	
	straight	**curve**	**plane**	**curve**
(predominantly) bending	beam	continuous arch	plate, concrete slab	lamina, dome
(predominant) tensile stress	rope	Catenary	shell	
(predominant) compression	pier, column		Load-bearing wall	

Columns

Columns are elements that carry only axial force - either tension or compression - or both axial force and bending (which is technically called a beam-column but practically, just a column). The design of a column must check the axial capacity of the element, and the buckling capacity.

The buckling capacity is the capacity of the element to withstand the propensity to buckle. Its capacity depends upon its geometry, material, and the effective length of the

column, which depends upon the restraint conditions at the top and bottom of the column. The effective length is $K * l$ where l is the real length of the column.

The capacity of a column to carry axial load depends on the degree of bending it is subjected to, and vice versa. This is represented on an interaction chart and is a complex non-linear relationship.

Beams

A beam may be defined as an element in which one dimemsion is much greater than the other two and the applied loads are usually normal to the main axis of the element. Beams and columns are called line elements and are often represented by simple lines in structural modeling.

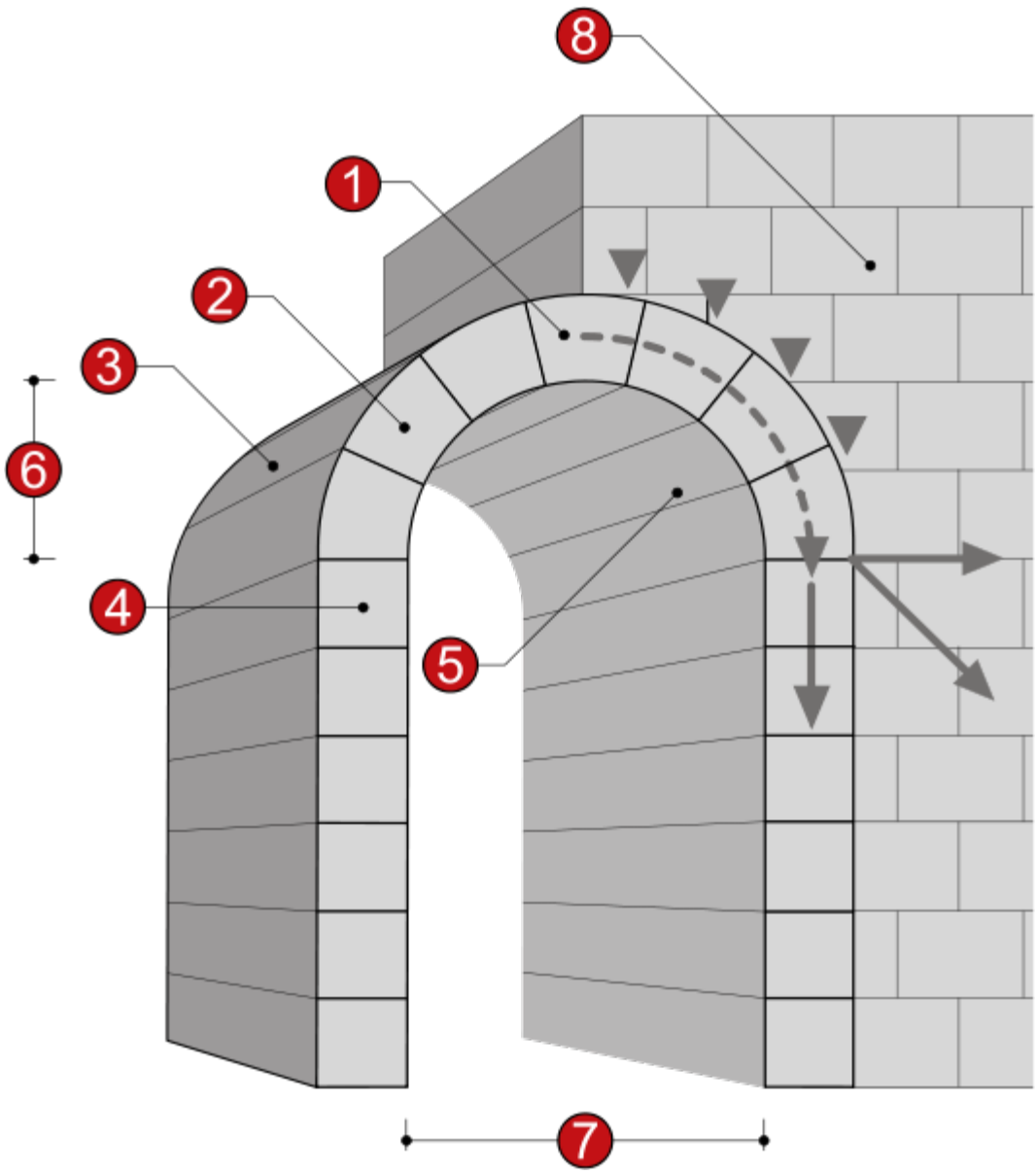
- cantilevered (supported at one end only with a fixed connection)
- simply supported (supported vertically at each end; horizontally on only one to withstand friction, and able to rotate at the supports)
- continuous (supported by three or more supports)
- a combination of the above (ex. supported at one end and in the middle)

Beams are elements which carry pure bending only. Bending causes one part of the section of a beam (divided along its length) to go into compression and the other part into tension. The compression part must be designed to resist buckling and crushing, while the tension part must be able to adequately resist the tension.

Struts and ties

Little Belt: a truss bridge in Denmark

The McDonnell Planetarium by Gyo Obata in St Louis, Missouri, USA, a concrete shell structure



A masonry arch
1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span
8. Abutment

A truss is a structure comprising two types of structural elements; compression members and tension members (i.e. struts and ties). Most trusses use gusset plates to connect intersecting elements. Gusset plates are relatively flexible and minimize bending moments at the connections, thus allowing the truss members to carry primarily tension or compression.

Trusses are usually utilised in span large distances, where it would be uneconomical to use solid beams.

Plates

Plates carry bending in two directions. A concrete flat slab is an example of a plate. Plates are understood by using continuum mechanics, but due to the complexity involved they are most often designed using a codified empirical approach, or computer analysis.

They can also be designed with yield line theory, where an assumed collapse mechanism is analysed to give an upper bound on the collapse load. This is rarely used in practice.

Shells

Shells derive their strength from their form, and carry forces in compression in two directions. A dome is an example of a shell. They can be designed by making a hanging-chain model, which will act as a catenary in pure tension, and inverting the form to achieve pure compression.

Arches

Arches carry forces in compression in one direction only, which is why it is appropriate to build arches out of masonry. They are designed by ensuring that the line of thrust of the force remains within the depth of the arch.

Catenaries

Catenaries derive their strength from their form, and carry transverse forces in pure tension by deflecting (just as a tightrope will sag when someone walks on it). They are almost always cable or fabric structures. A fabric structure acts as a catenary in two directions.

Structural engineering theory

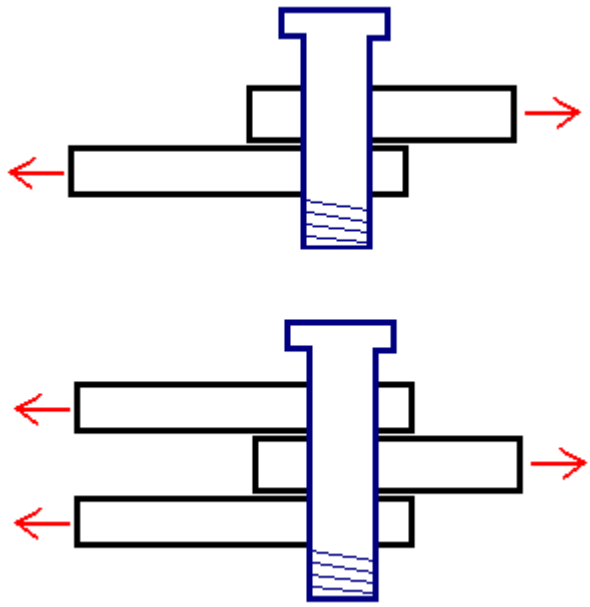


Figure of a bolt in shear stress. Top figure illustrates single shear, bottom figure illustrates double shear.

Structural engineering depends upon a detailed knowledge of loads, physics and materials to understand and predict how structures support and resist self-weight and imposed loads. To apply the knowledge successfully a structural engineer generally requires detailed knowledge of mathematics and relevant empirical and theoretical design codes. As well as, typically, some knowledge of the corrosion resistance of the materials and structures, especially when those structures are exposed to the external environment. Since the 1990s, specialist software has become available to aid in the design of structures, with the functionality to assist in both the drawing and designing of structures with maximum precision; examples include AutoCAD, StaadPro, Etab etc. Such software may also take into consideration environmental loads, such as from earthquakes and winds.

Materials

The 630 foot (192 m) high, stainless-clad (type 304) Gateway Arch in Saint Louis, Missouri

Structural engineering depends on the knowledge of materials and their properties, in order to understand how different materials support and resist loads.

Common structural materials are:

- Iron:
    - Wrought iron
    - Cast iron
    - Steel
    - Stainless steel
- Concrete:
    - Reinforced concrete
    - Prestressed concrete
- Aluminium
- Composites
- Alloy
- Masonry
- Timber
- Other structural materials:
    - Adobe

- Bamboo
- Carbon fibre
- Fiber reinforced plastic
- Mudbrick

Chapter- 2

History of Structural Engineering

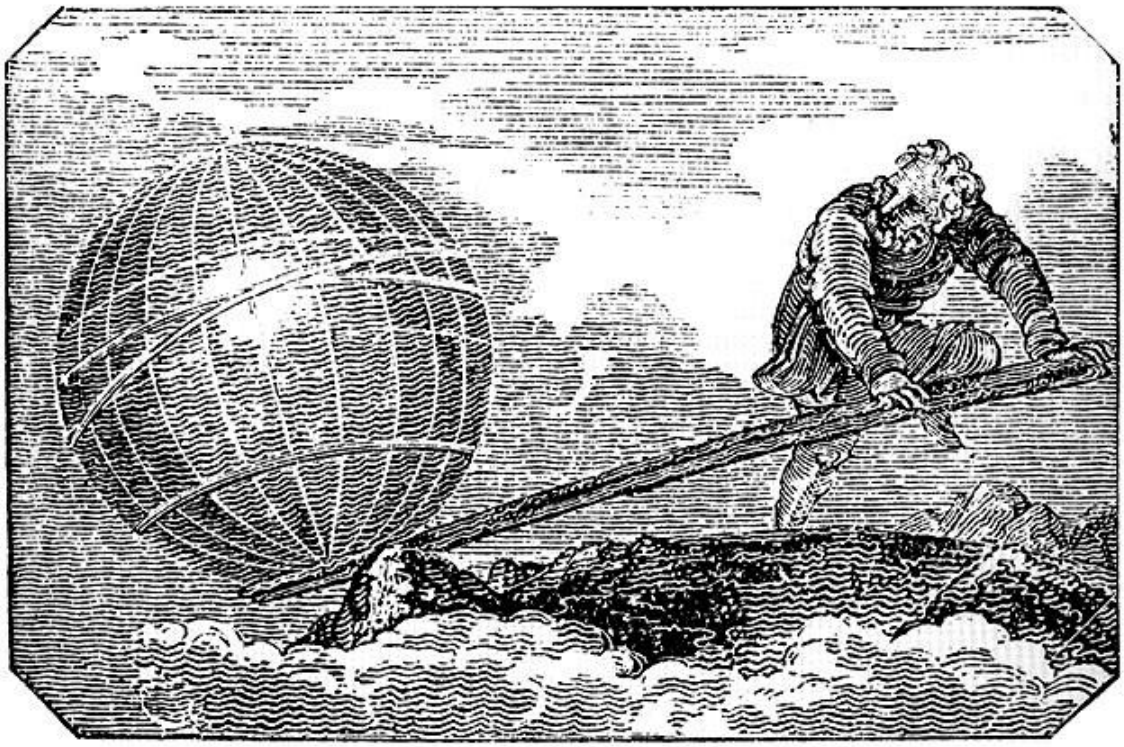
Statuette of Imhotep, in the Louvre, Paris, France

The **history of structural engineering** dates back to at least 2700 BC when the step pyramid for Pharaoh Djoser was built by Imhotep, the first engineer in history known by name. Pyramids were the most common major structures built by ancient civilisations because it is a structural form which is inherently stable and can be almost infinitely scaled (as opposed to most other structural forms, which cannot be linearly increased in size in proportion to increased loads).

Throughout ancient and medieval history most architectural design and construction was carried out by artisans, such as stone masons and carpenters, rising to the role of master builder. No theory of structures existed and understanding of how structures stood up was extremely limited, and based almost entirely on empirical evidence of 'what had worked before'. Knowledge was retained by guilds and seldom supplanted by advances. Structures were repetitive, and increases in scale were incremental.

No record exists of the first calculations of the strength of structural members or the behaviour of structural material, but the profession of structural engineer only really took shape with the industrial revolution and the re-invention of concrete. The physical sciences underlying structural engineering began to be understood in the Renaissance and have been developing ever since.

Early structural engineering developments



Archimedes is said to have remarked about the lever: "Give me a place to stand on, and I will move the Earth."

The recorded history of structural engineering starts with the ancient Egyptians. In the 27th century BC, Imhotep was the first structural engineer known by name and constructed the first known step pyramid in Egypt. In the 26th century BC, the Great Pyramid of Giza was constructed in Egypt. It remained the largest man-made structure for millennia and was considered an unsurpassed feat in architecture until the 19th century AD.

The understanding of the physical laws that underpin structural engineering dates back to the 3rd century BC, when Archimedes published his work *On the Equilibrium of Planes* in two volumes, in which he sets out the *Law of the Lever*, stating:

> “ Equal weights at equal distances are in equilibrium, and equal weights at unequal distances are not in equilibrium but incline towards the weight which is at the greater distance. ”

Archimedes used the principles derived to calculate the areas and centers of gravity of various geometric figures including triangles, paraboloids, and hemispheres. Archimedes work on this and his work on calculus and geometry, together with Euclidean geometry, underpin much of the mathematics and understanding of structures in modern structural engineering.

Pont du Gard, France, a Roman era aqueduct circa 19 BC.

The ancient Romans made great bounds in structural engineering, pioneering large structures in masonry and concrete, many of which are still standing today. They include aqueducts, thermae, columns, lighthouses, defensive walls and harbours. Their methods are recorded by Vitruvius in his De Architectura written in 25 BC, a manual of civil and structural engineering with extensive sections on materials and machines used in construction. One reason for their success is their accurate surveying techniques based on the dioptra, groma and chorobates.

Centuries later, in the 15th and 16th centuries and despite lacking beam theory and calculus, Leonardo da Vinci produced many engineering designs based on scientific observations and rigour, including a design for a bridge to span the Bosporus. Though dismissed at the time, the design has since been judged to be both feasible and structurally valid

Galileo Galilei. Portrait in crayon by Leoni

The foundations of modern structural engineering were laid in the 17th century by Galileo Galilei, Robert Hooke and Isaac Newton with the publication of three great scientific works. In 1638 Galileo published *Dialogues Relating to Two New Sciences*, outlining the sciences of the strength of materials and the motion of objects (essentially

defining gravity as a force giving rise to a constant acceleration). It was the first establishment of a scientific approach to structural engineering, including the first attempts to develop a theory for beams. This is also regarded as the beginning of structural analysis, the mathematical representation and design of building structures.

This was followed in 1676 by Robert Hooke's first statement of Hooke's Law, providing a scientific understanding of elasticity of materials and their behaviour under load.

Eleven years later, in 1687, Sir Isaac Newton published *Philosophiae Naturalis Principia Mathematica*, setting out his Laws of Motion, providing for the first time an understanding of the fundamental laws governing structures.

Also in the 17th century, Sir Isaac Newton and Gottfried Leibniz both independently developed the Fundamental theorem of calculus, providing one of the most important mathematical tools in engineering.

Leonhard Euler portrait by Johann Georg Brucker

Further advances in the mathematics needed to allow structural engineers to apply the understanding of structures gained through the work of Galileo, Hooke and Newton during the 17th century came in the 18th century when Leonhard Euler pioneered much of the mathematics and many of the methods which allow structural engineers to model and analyse structures. Specifically, he developed the Euler-Bernoulli beam equation with Daniel Bernoulli (1700–1782) circa 1750 - the fundamental theory underlying most structural engineering design.

Daniel Bernoulli, with Johann (Jean) Bernoulli (1667–1748), is also credited with formulating the theory of virtual work, providing a tool using equilibrium of forces and compatibility of geometry to solve structural problems. In 1717 Jean Bernoulli wrote to Pierre Varignon explaining the principle of virtual work, while in 1726 Daniel Bernoulli wrote of the "composition of forces".

In 1757 Leonhard Euler went on to derive the Euler buckling formula, greatly advancing the ability of engineers to design compression elements.

Modern developments in structural engineering

Bessemer converter, Kelham Island Museum, Sheffield, England (2002)

Belper North Mill

Eiffel Tower under construction in July 1888

The Forth Bridge

The Lattice shell structure of the Shukhov Tower in Moscow

Throughout the late 19th and early 20th centuries, materials science and structural analysis underwent development at a tremendous pace.

Though elasticity was understood in theory well before the 19th century, it was not until 1821 that Claude-Louis Navier formulated the general theory of elasticity in a mathematically usable form. In his *leçons* of 1826 he explored a great range of different structural theory, and was the first to highlight that the role of a structural engineer is not to understand the final, failed state of a structure, but to prevent that failure in the first place. In 1826 he also established the elastic modulus as a property of materials independent of the second moment of area, allowing engineers for the first time to both understand structural behaviour and structural materials.

Towards the end of the 19th century, in 1873, Carlo Alberto Castigliano presented his dissertation "Intorno ai sistemi elastici", which contains his theorem for computing displacement as partial derivative of the strain energy.

In 1824, Portland cement was patented by the engineer Joseph Aspdin as *"a superior cement resembling Portland Stone"*, British Patent no. 5022. Although different forms of cement already existed (Pozzolanic cement was used by the Romans as early as 100 B.C. and even earlier by the ancient Greek and Chinese civilizations) and were in common usage in Europe from the 1750s, the discovery made by Aspdin used commonly available, cheap materials, making concrete construction an economical possibility.

Developments in concrete continued with the construction in 1848 of a rowing boat built of ferrocement - the forerunner of modern reinforced concrete - by Joseph-Louis Lambot. He patented his system of mesh reinforcement and concrete in 1855, one year after W.B. Wilkinson also patented a similar system. This was followed in 1867 when a reinforced concrete planting tub was patented by Joseph Monier in Paris, using steel mesh reinforcement similar to that used by Lambot and Wilkinson. Monier took the idea forward, filing several patents for tubs, slabs and beams, leading eventually to the Monier system of reinforced structures, the first use of steel reinforcement bars located in areas of tension in the structure.

Steel construction was first made possible in the 1850s when Henry Bessemer developed the Bessemer process to produce steel. He gained patents for the process in 1855 and 1856 and successfully completed the conversion of cast iron into cast steel in 1858. Eventually mild steel would replace both wrought iron and cast iron as the preferred metal for construction.

During the late 19th century, great advancements were made in the use of cast iron, gradually replacing wrought iron as a material of choice. Ditherington Flax Mill in Shrewsbury, designed by Charles Bage, was the first building in the world with an interior iron frame. It was built in 1797. In 1792 William Strutt had attempted to build a fireproof mill at Belper in Derby (Belper West Mill), using cast iron columns and timber beams within the depths of brick arches that formed the floors. The exposed beam soffits were protected against fire by plaster. This mill at Belper was the world's first attempt to construct fireproof buildings, and is the first example of fire engineering. This was later improved upon with the construction of Belper North Mill, a collaboration between Strutt and Bage, which by using a full cast iron frame represented the world's first "fire proofed" building.

The Forth Bridge was built by Benjamin Baker, Sir John Fowler and William Arrol in 1889, using steel, after the original design for the bridge by Thomas Bouch was rejected following the collapse of his Tay Rail Bridge. The Forth Bridge was one of the first major uses of steel, and a landmark in bridge design. Also in 1889, the wrought-iron Eiffel Tower was built by Gustave Eiffel and Maurice Koechlin, demonstrating the potential of construction using iron, despite the fact that steel construction was already being used elsewhere.

During the late 19th century, Russian structural engineer Vladimir Shukhov developed analysis methods for tensile structures, thin-shell structures, lattice shell structures and new structural geometries such as hyperboloid structures. Pipeline transport was pioneered by Vladimir Shukhov and the Branobel company in the late 19th century.

Again taking reinforced concrete design forwards, from 1892 onwards François Hennebique's firm used his patented reinforced concrete system to build thousands of structures throughout Europe. Thaddeus Hyatt in the US and Wayss & Freitag in Germany also patented systems. The firm *AG für Monierbauten* constructed 200 reinforced concrete bridges in Germany between 1890 and 1897. The great pioneering uses of reinforced concrete however came during the first third of the 20th century, with Robert Maillart and others furthering of the understanding of its behaviour. Maillart noticed that many concrete bridge structures were significantly cracked, and as a result left the cracked areas out of his next bridge design - correctly believing that if the concrete was cracked, it was not contributing to the strength. This resulted in the revolutionary Salginatobel Bridge design. Wilhelm Ritter formulated the truss theory for the shear design of reinforced concrete beams in 1899, and Emil Mörsch improved this in 1902. He went on to demonstrate that treating concrete in compression as a linear-elastic material was a conservative approximation of its behaviour. Concrete design and analysis has been progressing ever since, with the development of analysis methods such as yield line theory, based on plastic analysis of concrete (as opposed to linear-elastic), and many different variations on the model for stress distributions in concrete in compression

Prestressed concrete, pioneered by Eugène Freyssinet with a patent in 1928, gave a novel approach in overcoming the weakness of concrete structures in tension. Freyssinet constructed an experimental prestressed arch in 1908 and later used the technology in a limited form in the Plougastel Bridge in France in 1930. He went on to build six prestressed concrete bridges across the Marne River, firmly establishing the technology.

Structural engineering theory was again advanced in 1930 when Professor Hardy Cross developed his Moment distribution method, allowing the real stresses of many complex structures to be approximated quickly and accurately.

In the mid 20th century John Fleetwood Baker went on to develop the plasticity theory of structures, providing a powerful tool for the safe design of steel structures.

High-rise construction, though possible from the late 19th century onwards, was greatly advanced during the second half of the 20th century. Fazlur Khan designed structural systems that remain fundamental to many modern high rise constructions and which he employed in his structural designs for the John Hancock Center in 1969 and Sears Tower in 1973. Khan's central innovation in skyscraper design and construction was the idea of the "tube" and "bundled tube" structural systems for tall buildings. He defined the framed tube structure as "a three dimensional space structure composed of three, four, or possibly more frames, braced frames, or shear walls, joined at or near their edges to form a vertical tube-like structural system capable of resisting lateral forces in any direction by cantilevering from the foundation." Closely spaced interconnected exterior columns form

the tube. Horizontal loads, for example wind, are supported by the structure as a whole. About half the exterior surface is available for windows. Framed tubes allow fewer interior columns, and so create more usable floor space. Where larger openings like garage doors are required, the tube frame must be interrupted, with transfer girders used to maintain structural integrity. The first building to apply the tube-frame construction was in the DeWitt-Chestnut Apartment Building which Khan designed in Chicago. This laid the foundations for the tube structures used in most later skyscraper constructions, including the construction of the World Trade Center.

Another innovation that Fazlur Khan developed was the concept of X-bracing, which reduced the lateral load on the building by transferring the load into the exterior columns. This allowed for a reduced need for interior columns thus creating more floor space, and can be seen in the John Hancock Center. The first sky lobby was also designed by Khan for the John Hancock Center in 1969. Later buildings with sky lobbies include the World Trade Center, Petronas Twin Towers and Taipei 101.

In 1987 Jörg Schlaich and Kurt Schafer published the culmination of almost ten years of work on the strut and tie method for concrete analysis - a tool to design structures with discontinuities such as corners and joints, providing another powerful tool for the analysis of complex concrete geometries.

In the late 20th and early 21st centuries the development of powerful computers has allowed finite element analysis to become a significant tool for structural analysis and design. The development of finite element programs has led to the ability to accurately predict the stresses in complex structures, and allowed great advances in structural engineering design and architecture. In the 1960s and 70s computational analysis was used in a significant way for the first time on the design of the Sydney Opera House roof. Many modern structures could not be understood and designed without the use of computational analysis.

Developments in the understanding of materials and structural behaviour in the latter part of the 20th century have been significant, with detailed understanding being developed of topics such as fracture mechanics, earthquake engineering, composite materials, temperature effects on materials, dynamics and vibration control, fatigue, creep and others. The depth and breadth of knowledge now available in structural engineering, and the increasing range of different structures and the increasing complexity of those structures has led to increasing specialisation of structural engineers.

Chapter- 3

Earthquake Engineering Structures

Earthquake-proof and massive pyramid El Castillo, Chichen Itza

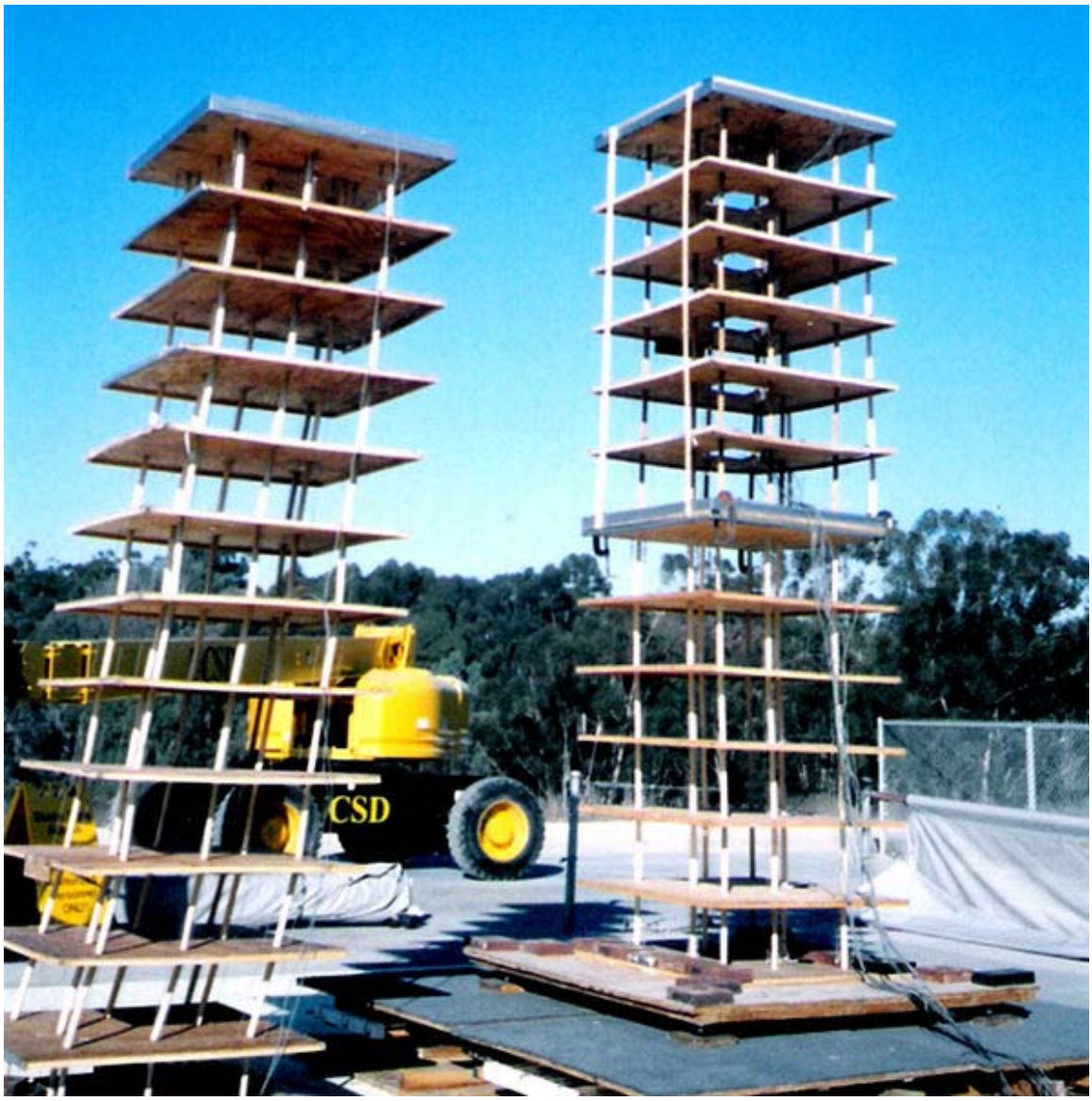
Earthquake engineering structures are designed and constructed to withstand various types of hazardous earthquake exposures at the sites of their particular location.

Earthquake engineering is treating its subject structures like defensive fortifications in military engineering but for the warfare on earthquakes. Both earthquake and military general design principles are similar: be ready to slow down or mitigate the advance of a possible attacker.

According to building codes, earthquake engineering structures are meant to "withstand" the largest earthquake of a certain probability that is likely to occur at their location. This means the loss of life should be minimized by preventing collapse of the buildings.

Ancient architects believed that devastating earthquakes were a result of wrath of gods and therefore, could not be resisted by humans. Nowadays, the people's attitude has changed dramatically though the term "earthquake engineering structure" does not

necessarily mean it is an extremely strong or expensive one like the El Castillo pyramid at Chichen Itza.



Snapshot from shake-table testing base-isolated (right) and regular (left) building model

Currently, the most powerful and cost-effective tool of earthquake engineering is base isolation which makes use of passive structural vibration control technologies, used for example in the Cathedral of Our Lady of the Angels.

Trends and projects

Some of the new state-of-the-art trends and/or projects in the field of earthquake engineering structures are presented below.

Proofed low cost earthquake building material

A leading German composite construction company developed a proofed earthquake-safe supported core material RexWall - based on internal beam/frame constructions. The same principle allows construction of hurricane-safe houses.

Earthquake shelter

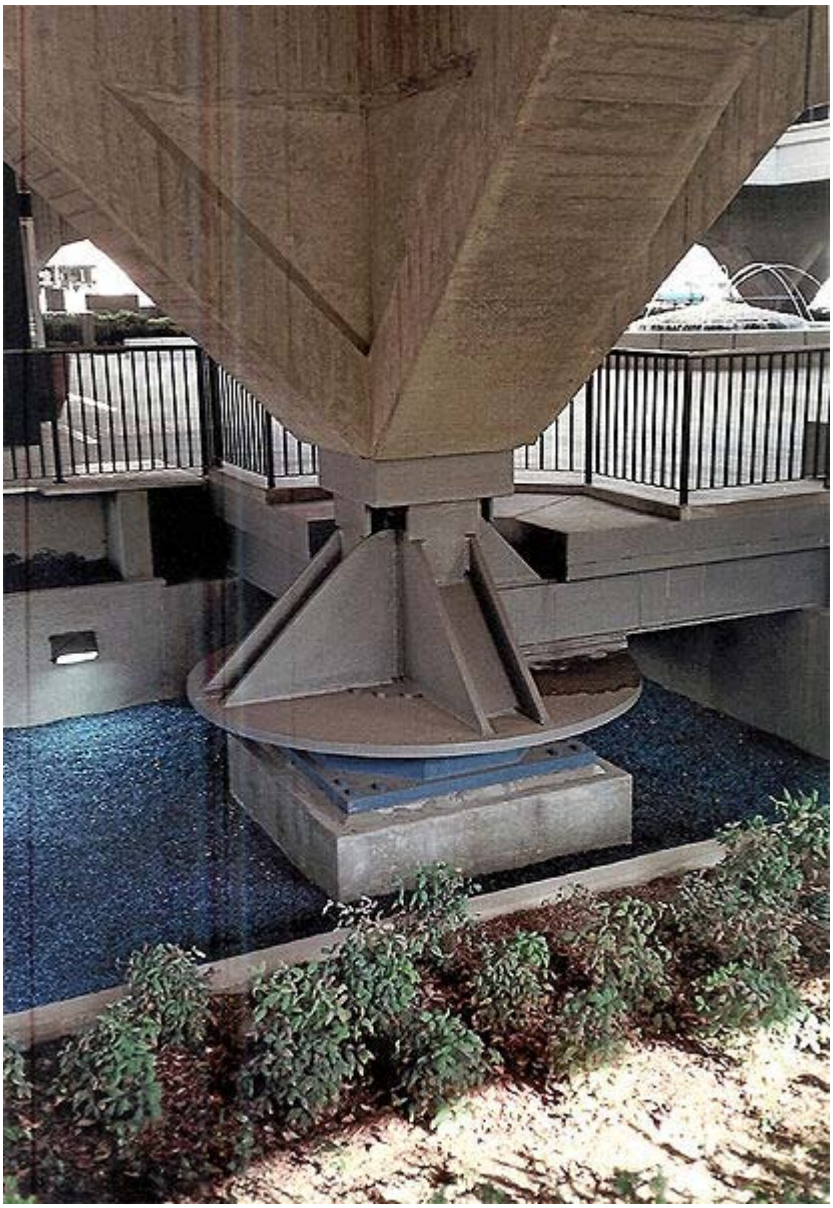
One Japanese construction company has developed a six-foot cubical shelter, presented as an alternative to earthquake-proofing an entire building.

Concurrent shake-table testing

Concurrent shake-table testing of two or more building models is a vivid, persuasive and effective way to validate earthquake engineering solutions experimentally.

Thus, two wooden houses built before adoption of the 1981 Japanese Building Code were moved to E-Defense for testing. The left house was reinforced to enhance its seismic resistance, while the other one was not. These two models were set on E-Defense platform and tested simultaneously.

Combined vibration control solution



Close-up of abutment of seismically retrofitted Municipal Services Building in Glendale, CA

Seismically retrofitted Municipal Services Building in Glendale, CA

Designed by architect Merrill W. Baird of Glendale, working in collaboration with A. C. Martin Architects of Los Angeles, the Municipal Services Building at 633 East Broadway, Glendale was completed in 1966. Prominently sited at the corner of East Broadway and Glendale Avenue, this civic building serves as a heraldic element of Glendale's civic center.

In October 2004 Architectural Resources Group (ARG) was contracted by Nabih Youssef & Associates, Structural Engineers, to provide services regarding a historic resource assessment of the building due to a proposed seismic retrofit.

In 2008, the Municipal Services Building of the City of Glendale, California was seismically retrofitted using an innovative combined vibration control solution: the existing elevated building foundation of the building was put on high damping rubber bearings.

Steel plate shear walls system

Coupled steel plate shear walls, Seattle

The Ritz-Carlton/JW Marriott hotel building engaging the advanced steel plate shear walls system, LA

A steel plate shear wall (SPSW) consists of steel infill plates bounded by a column-beam system. When such infill plates occupy each level within a framed bay of a structure, they constitute a SPSW system.

SPSW behavior is analogous to a vertical plate girder cantilevered from its base. Similar to plate girders, the SPSW system optimizes component performance by taking advantage of the post-buckling behavior of the steel infill panels.

The Ritz-Carlton/JW Marriott hotel building, a part of the LA Live development in Los Angeles, California, is the first building in Los Angeles that uses an advanced steel plate shear wall system to resist the lateral loads of strong earthquakes and winds.

Kashiwazaki-Kariwa Nuclear Power Plant is partially upgraded

Kashiwazaki-Kariwa Nuclear Power Plant: aerial view, Japan

The Kashiwazaki-Kariwa Nuclear Power Plant, the largest nuclear generating station in the world by net electrical power rating, happened to be near the epicenter of the strongest M_w 6.6 July 2007 Chūetsu offshore earthquake. This initiated an extended shutdown for structural inspection which indicated that a greater earthquake-proofing was needed before operation could be resumed.

On May 9, 2009, one unit (Unit 7) was restarted, after the seismic upgrades. The test run had to continue for 50 days. The plant had been completely shut down for almost 22 months following the earthquake.

Seismic Test of Seven-Story Building

Seven-story wooden structure prior to shake-table testing, Japan

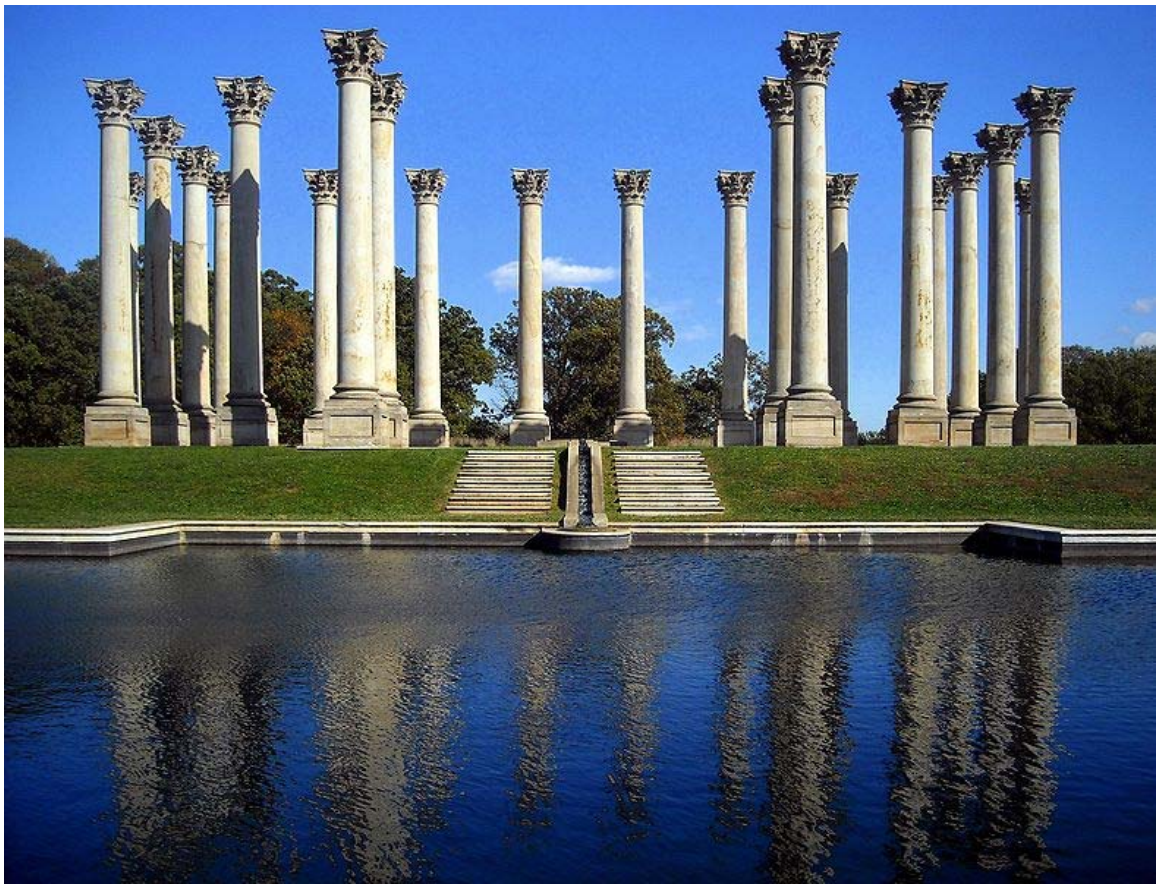
A destructive earthquake struck a lone, wooden condominium in Japan. The experiment was webcast live on July 14, 2009 to yield insight on how to make wooden structures stronger and better able to withstand major earthquakes.

The Miki shake at the Hyogo Earthquake Engineering Research Center is the capstone experiment of the four-year NEESWood project, which receives its primary support from the U.S. National Science Foundation Network for Earthquake Engineering Simulation (NEES) Program.

"NEESWood aims to develop a new seismic design philosophy that will provide the necessary mechanisms to safely increase the height of wood-frame structures in active seismic zones of the United States, as well as mitigate earthquake damage to low-rise wood-frame structures," said Rosowsky, Department of Civil Engineering at Texas A&M University. This philosophy is based on the application of seismic damping systems for wooden buildings. The systems, which can be installed inside the walls of most wooden buildings, include strong metal frame, bracing and dampers filled with viscous fluid.

Chapter- 4

Column (Structural Elements)



National Capitol Columns at the United States National Arboretum in Washington, D.C.

Marble columns with antique capitals in the Great Mosque of Kairouan also known as the Mosque of Uqba, city of Kairouan, Tunisia

A **column** in structural engineering is a vertical structural element that transmits, through compression, the weight of the structure above to other structural elements below. For the purpose of wind or earthquake engineering, columns may be designed to resist lateral forces. Other compression members are often termed "columns" because of the similar stress conditions. Columns are frequently used to support beams or arches on which the upper parts of walls or ceilings rest. In architecture "column" refers to such a structural element that also has certain proportional and decorative features. A column might also be a decorative or triumphant feature but need not be supporting any structure e.g. a statue on top.

History

Columns found at the Temple of Apollo in Delphi

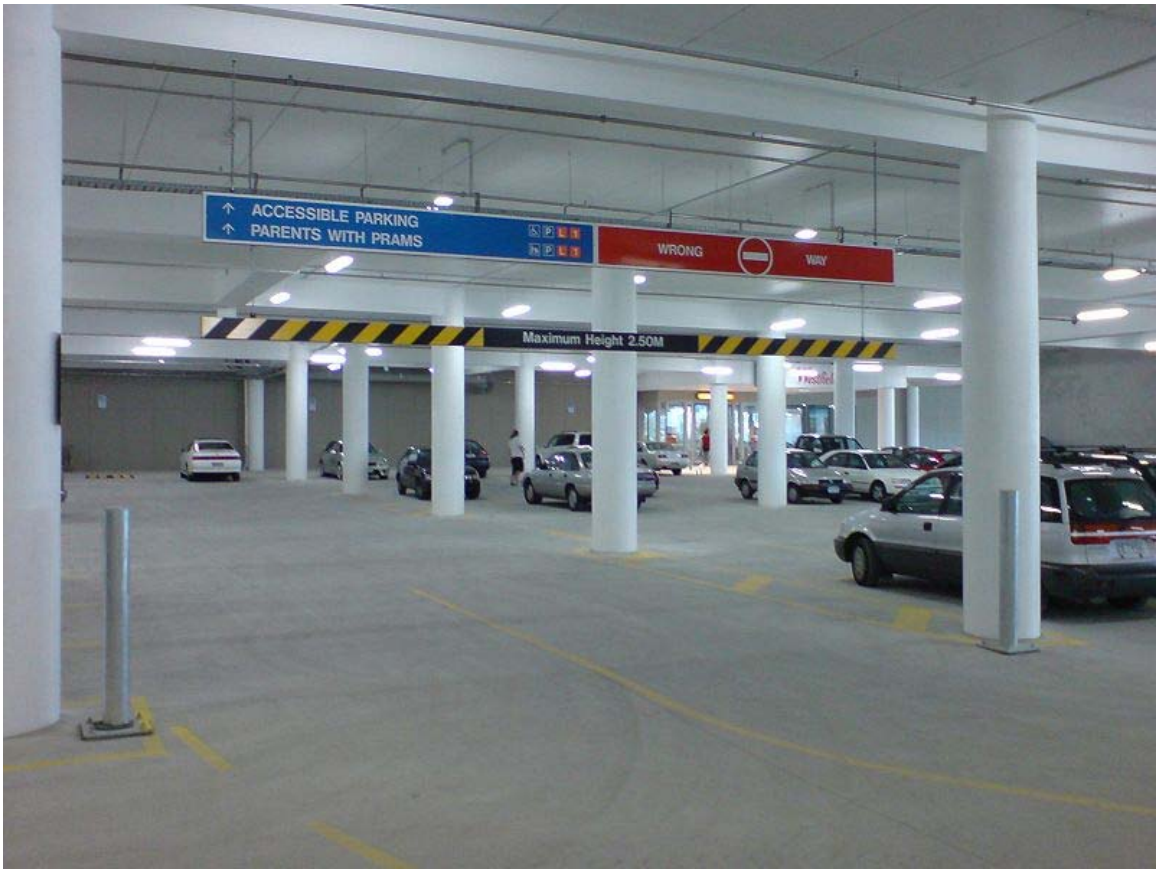
In the architecture of ancient Egypt as early as 2600 BC the architect Imhotep made use of stone columns whose surface was carved to reflect the organic form of bundled reeds; in later Egyptian architecture faceted cylinders were also common.

Some of the most elaborate columns in the ancient world were those of Persia especially the massive stone columns erected in Persepolis. They included double-bull structures in their capitals. The Hall of Hundred Columns at Persepolis, measuring 70 × 70 meters was built by the Achaemenid king Darius I (524–486 BC). Many of the ancient Persian columns are standing, some being more than 30 meters tall.

The Greeks pioneered the use of the classical orders (Doric, Ionic, Corinthian) which was expanded by the Romans to include the Tuscan and Composite styles.

The impost (or pier) is the topmost member of a column. The bottom-most part of the arch, called the springing, rests on the impost.

Structure

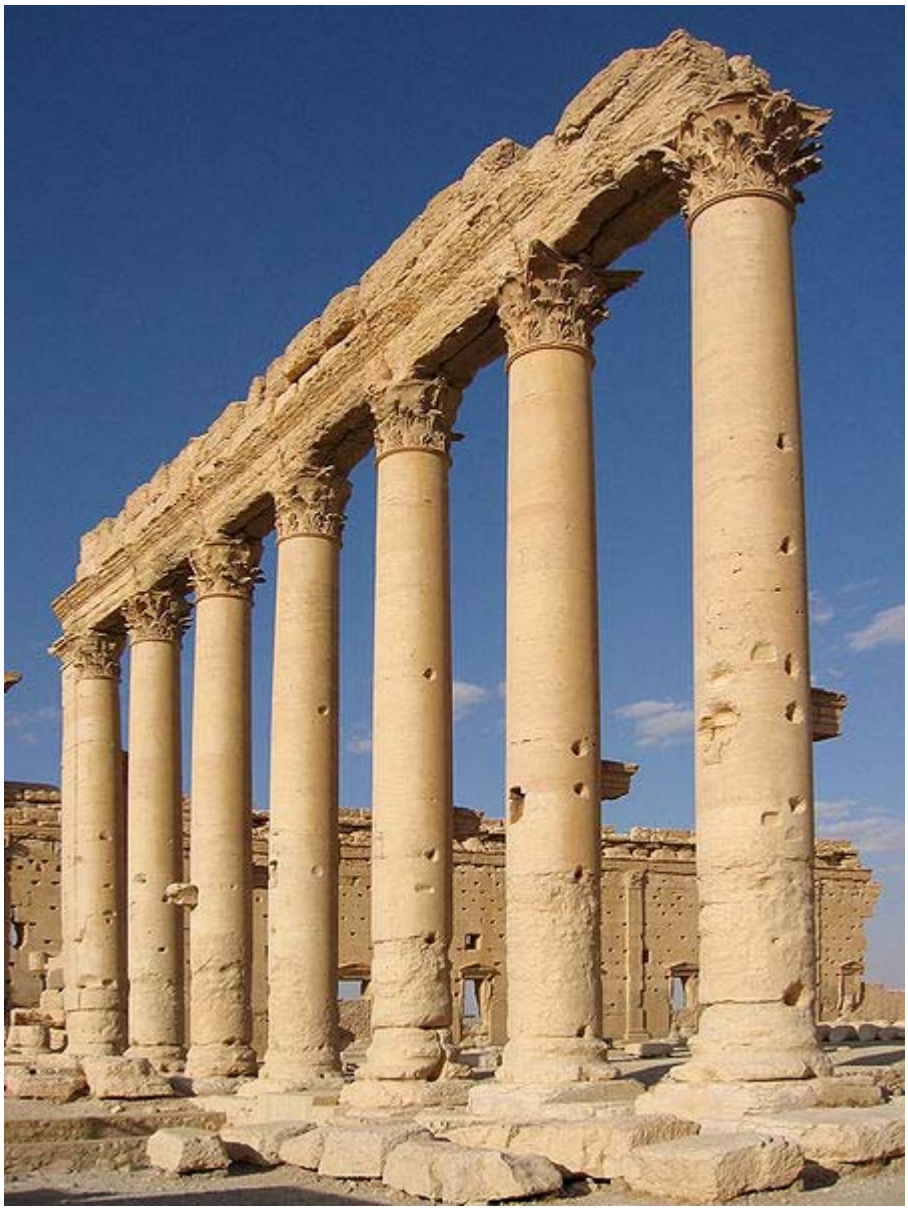


Modern column grid in a car park

Early columns were constructed of stone, some out of a single piece of stone, usually by turning on a lathe-like apparatus. Single-piece columns are among the heaviest stones used in architecture. Other stone columns are created out of multiple sections of stone, mortared or dry-fit together. In many classical sites, sectioned columns were carved with a center hole or depression so that they could be pegged together, using stone or metal pins. The design of most classical columns incorporates entasis (the inclusion of a slight outward curve in the sides) plus a reduction in diameter along the height of the column, so that the top is as little as 83% of the bottom diameter. This reduction mimics the parallax effects which the eye expects to see, and tends to make columns look taller and straighter than they are while entasis adds to that effect.

Modern columns are constructed out of steel, poured or precast concrete, or brick. They may then be clad in an architectural covering (or veneer), or left bare.

Equilibrium, instability, and loads



These are composed of stacked segments and finished in the Corinthian style (Temple of Bel, Syria)

As the axial load on a perfectly straight slender column with elastic material properties is increased in magnitude, this ideal column passes through three states: stable equilibrium, neutral equilibrium, and instability. The straight column under load is in stable equilibrium if a lateral force, applied between the two ends of the column, produces a small lateral deflection which disappears and the column returns to its straight form when the lateral force is removed. If the column load is gradually increased, a condition is reached in which the straight form of equilibrium becomes so-called neutral equilibrium, and a small lateral force will produce a deflection that does not disappear and the column remains in this slightly bent form when the lateral force is removed. The load at which

neutral equilibrium of a column is reached is called the critical or buckling load. The state of instability is reached when a slight increase of the column load causes uncontrollably growing lateral deflections leading to complete collapse.

For an axially loaded straight column with any end support conditions, the equation of static equilibrium, in the form of a differential equation, can be solved for the deflected shape and critical load of the column. With hinged, fixed or free end support conditions the deflected shape in neutral equilibrium of an initially straight column with uniform cross section throughout its length always follows a partial or composite sinusoidal curve shape, and the critical load is given by

$$f_{cr} \equiv \frac{\pi^2 E I_{min}}{L^2} \qquad (1)$$

where E = elastic modulus of the material, I_{min} = the minimal moment of inertia of the cross section, and L = actual length of the column between its two end supports. A variant of (1) is given by

$$f_{cr} \equiv \frac{\pi^2 E_T}{(\frac{KL}{r})^2} \qquad (2)$$

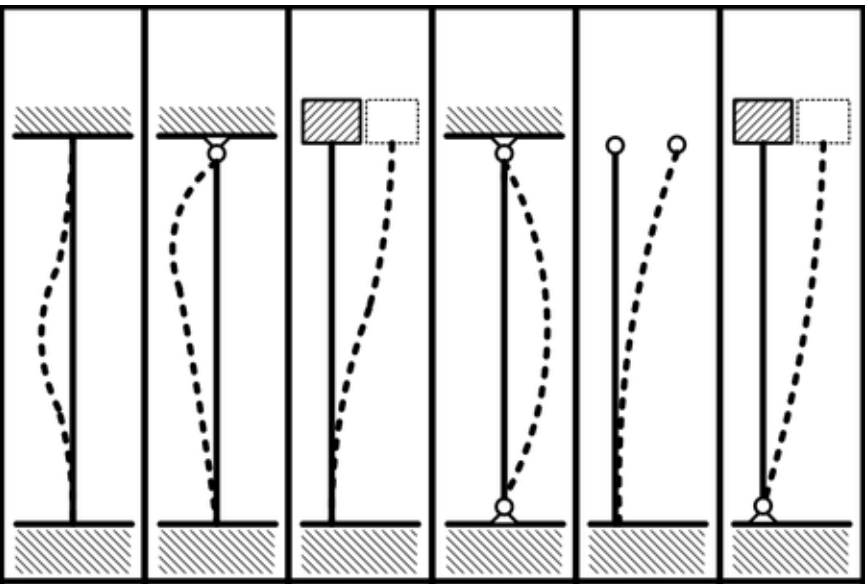
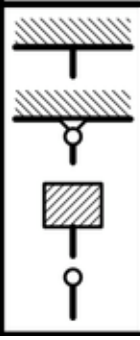
Buckled shape of column shown by dashed line						
Theoretical K value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value K	0.65	0.80	1.2	1.0	2.10	2.0
End condition key		Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free				

Table showing values of K for structural columns of various end conditions (adapted from Manual of Steel Construction, 8th edition, American Institute of Steel Construction, Table C1.8.1)

where r = radius of gyration of [column]cross-section which is equal to the square root of (I/A), K = ratio of the longest half sine wave to the actual column length, and KL = effective length (length of an equivalent hinged-hinged column). From Equation (2) it can be noted that the buckling strength of a column is inversely proportional to the square of its length.

When the critical stress, F_{cr} ($F_{cr} = P_{cr}/A$, where A = cross-sectional area of the column), is greater than the proportional limit of the material, the column is experiencing inelastic buckling. Since at this stress the slope of the material's stress-strain curve, E_t (called the

tangent modulus), is smaller than that below the proportional limit, the critical load at inelastic buckling is reduced. More complex formulas and procedures apply for such cases, but in its simplest form the critical buckling load formula is given as Equation (3),

$$f_{cr} \equiv F_y - \frac{F_y^2}{4\pi^2 E}\left(\frac{KL}{r^2}\right) \qquad (3)$$

where E_t = tangent modulus at the stress F_{cr}

A column with a cross section that lacks symmetry may suffer torsional buckling (sudden twisting) before, or in combination with, lateral buckling. The presence of the twisting deformations renders both theoretical analyses and practical designs rather complex.

Eccentricity of the load, or imperfections such as initial crookedness, decreases column strength. If the axial load on the column is not concentric, that is, its line of action is not precisely coincident with the centroidal axis of the column, the column is characterized as eccentrically loaded. The eccentricity of the load, or an initial curvature, subjects the column to immediate bending. The increased stresses due to the combined axial-plus-flexural stresses result in a reduced load-carrying ability.

Column elements are considered to be massive if minimal side dimension is equal or more than 400 mm. Massive columns have ability to increase concrete strength during long time period (even during exploitation period). Taking into account possible loads onto structure increase in future (and even threat of progressive failure - terroristic attacks, explosions etc.) - massive columns have advantage comparing with not ones. A little economy today has no sense as usual for future. Moreover relatively small sections are not technological for reinforced structures during their production. Balance between economy, mass of structures and so called "sustainable" construction is necessary.

Extensions

When a column is too long to be built or transported in one piece, it has to be extended or spliced at the construction site. A reinforced concrete column is extended by having the steel reinforcing bars protrude a few inches or feet above the top of the concrete, then placing the next level of reinforcing bars to overlap, and pouring the concrete of the next level. A steel column is extended by welding or bolting splice plates on the flanges and webs or walls of the columns to provide a few inches or feet of load transfer from the upper to the lower column section. A timber column is usually extended by the use of a steel tube or wrapped-around sheet-metal plate bolted onto the two connecting timber sections.

Foundations

A column that carries the load down to a foundation must have means to transfer the load without overstressing the foundation material. Reinforced concrete and masonry columns are generally built directly on top of concrete foundations. A steel column, when seated

on a concrete foundation, must have a base plate to spread the load over a larger area and thereby reduce the bearing pressure. The base plate is a thick rectangular steel plate usually welded to the bottom end of the column.

Classical orders

Church of San Prospero, Reggio Emilia, Italy

The Roman author Vitruvius, relying on the writings (now lost) of Greek authors, tells us that the ancient Greeks believed that their Doric order developed from techniques for building in wood in which the earlier smoothed tree trunk was replaced by a stone cylinder.

Doric order

The Doric order is the oldest and simplest of the classical orders. It is composed of a vertical cylinder that is wider at the bottom. It generally has neither a base nor a detailed capital. It is instead often topped with an inverted frustum of a shallow cone or a cylindrical band of carvings. It is often referred to as the masculine order because it is represented in the bottom level of the Colosseum and the Parthenon, and was therefore considered to be able to hold more weight. The height-to-thickness ratio is about 8:1. The shaft of a Doric Column is always fluted.

The Greek Doric, developed in the western Dorian region of Greece, is the heaviest and most massive of the orders. It rises from the stylobate without any base; it is from four to six times as tall as its diameter; it has twenty broad flutes; the capital consists simply of a banded necking swelling out into a smooth echinus, which carries a flat square abacus; the Doric entablature is also the heaviest, being about one-fourth the height column. The Greek Doric order was not used after c. 100 B.C. until its "rediscovery" in the mid-eighteenth century.

Tuscan order

The Tuscan order, also known as Roman Doric, is also a simple design, the base and capital both being series of cylindrical disks of alternating diameter. The shaft is almost never fluted. The proportions vary, but are generally similar to Doric columns. Height to width ratio is about 7:1.

Ionic order

The Ionic column is considerably more complex than the Doric or Tuscan. It usually has a base and the shaft is often fluted (it has grooves carved up its length). On the top is a capital in the characteristic shape of a scroll, called a volute, or scroll, at the four corners. The height-to-thickness ratio is around 9:1. Due to the more refined proportions and scroll capitals, the Ionic column is sometimes associated with academic buildings.

Ionic capital

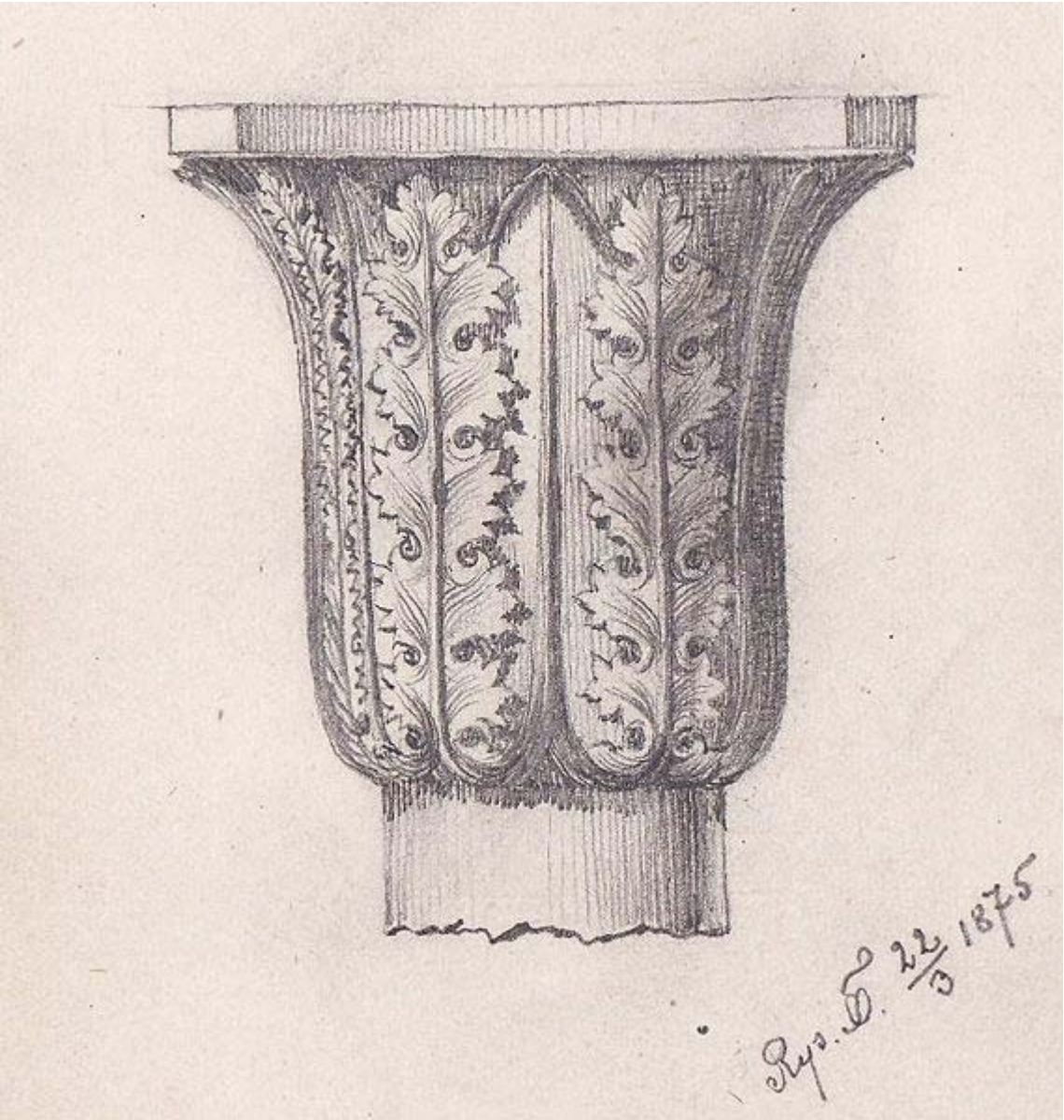
Corinthian order

The Corinthian order is named for the Greek city-state of Corinth, to which it was connected in the period. However, according to the architectural historian Vitruvius, the column was created by the sculptor Callimachus, probably an Athenian, who drew acanthus leaves growing around a votive basket. In fact, the oldest known Corinthian capital was found in Bassae, dated at 427 BC. It is sometimes called the feminine order because it is on the top level of the Colosseum and holding up the least weight, and also has the slenderest ratio of thickness to height. Height to width ratio is about 10:1.

Composite order

The Composite order draws its name from the capital being a composite of the Ionic and Corinthian capitals. The acanthus of the Corinthian column already has a scroll-like element, so the distinction is sometimes subtle. Generally the Composite is similar to the Corinthian in proportion and employment, often in the upper tiers of colonnades. Height to width ratio is about 11:1 or 12:1.

Solomonic



Capital of Solomonic Column

Solomonic columns were inventions of Baroque architects in Europe. They were not used in antiquity, but were called “Solomonic” by baroque architects because they were based on a description of columns in the great temple of King Solomon in the Old Testament. A Solomonic column begins on a base and ends in a capital, just like a classical column, but the shaft twists around the usual parameters of a column, producing a dramatic, serpentine effect of movement. The most famous use of Solomonic columns is in the baldacchino designed by Bernini for Saint Peter’s Basilica in the Vatican City.

Chapter- 5

Catenary

The silk on a spider's web forming multiple elastic catenaries

In physics and geometry, the **catenary** is the curve that an idealised hanging chain or cable assumes when supported at its ends and acted on only by its own weight. The curve is the graph of the hyperbolic cosine function, and has a U-like shape, superficially similar in appearance to a parabola (though mathematically quite different). Its surface of revolution, the catenoid, is a minimal surface and is the shape assumed by a soap film bounded by two parallel circular rings.

History

The word *catenary* is derived from the Latin word *catena,* which means "chain". Huygens first used the term *catenaria* in a letter to Leibniz in 1690. However, Thomas Jefferson is usually credited with the English word *catenary*. The curve is also called the "alysoid", "chainette", or, particularly in the material sciences, "funicular".

It is often stated that Galileo thought that the curve followed by a hanging chain is a parabola. A careful reading of his book *Two new sciences* shows this to be an oversimplification. Galileo discusses the catenary in two places; in the dialog of the Second Day he states that a hanging chain resembles a parabola. But later, in the dialog of the Fourth Day, he gives more details, and states that a hanging cord is approximated by a parabola, correctly observing that this approximation improves as the curvature gets smaller and is almost exact when the elevation is less than 45°. That the curve followed by a chain is not a parabola was proven by Joachim Jungius (1587–1657) and published posthumously in 1669.

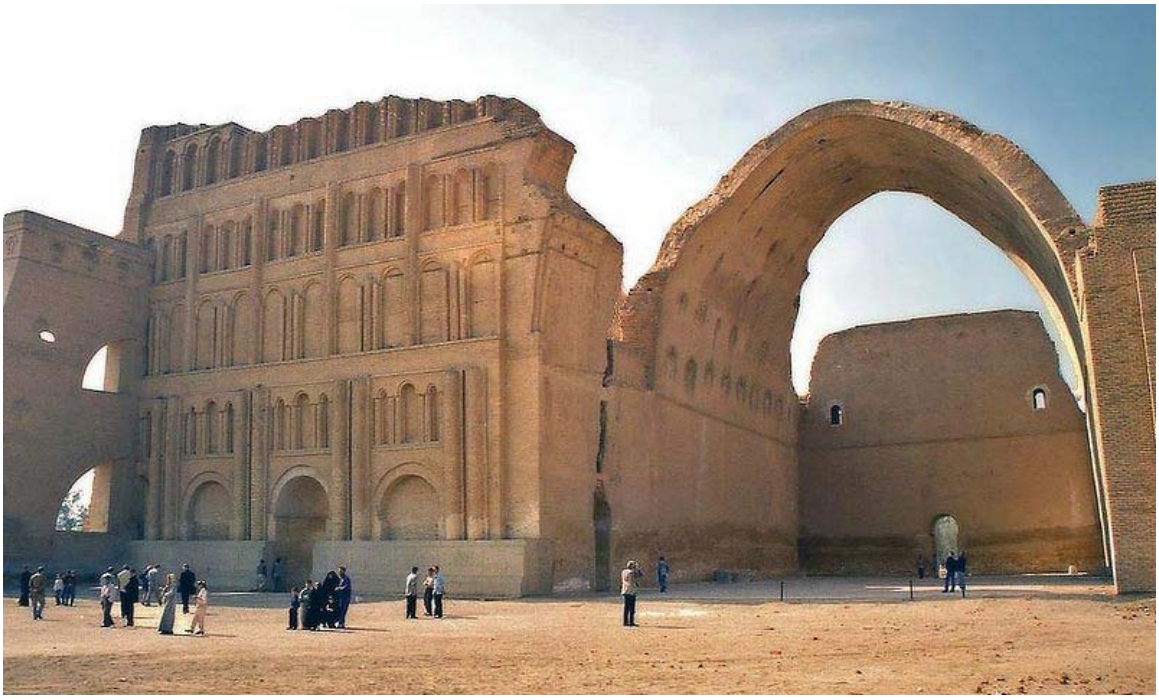
The application of the catenary to the construction of arches is due to Robert Hooke, who discovered it in the context of the rebuilding of St Paul's Cathedral, possibly having seen Huygens' work on the catenary. (Some much older arches are also approximate catenaries.)

In 1671, Hooke announced to the Royal Society that he had solved the problem of the optimal shape of an arch, and in 1675 published an encrypted solution as a Latin anagram in an appendix to his *Description of Helioscopes,* where he wrote that he had found "a true mathematical and mechanical form of all manner of Arches for Building." He did not publish the solution of this anagram in his lifetime, but in 1705 his executor provided it as *Ut pendet continuum flexile, sic stabit contiguum rigidum inversum,* meaning "As hangs a flexible cable so, inverted, stand the touching pieces of an arch."

In 1691 Gottfried Leibniz, Christiaan Huygens, and Johann Bernoulli derived the equation in response to a challenge by Jakob Bernoulli. David Gregory wrote a treatise on the catenary in 1697.

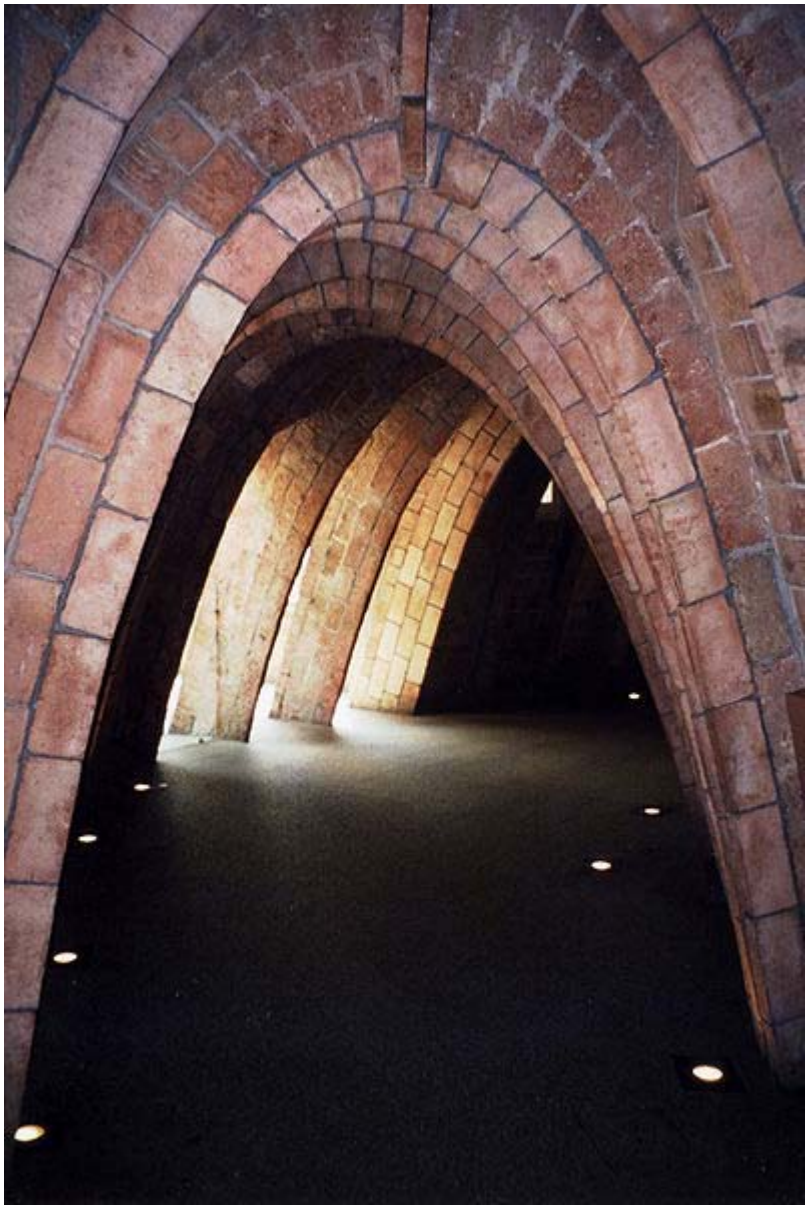
Euler proved in 1744 that the catenary is the curve which, when rotated about the x-axis, gives the surface of minimum surface area (the catenoid) for the given bounding circle.

The inverted catenary arch



Arch of Taq-i Kisra in Ctesiphon as seen today is roughly but not exactly a catenary

Gaudi's catenary model at Casa Milà



Arches under the roof of Gaudí's *Casa Milà*, Barcelona, Spain that are close to catenaries

Hooke discovered that the catenary is the ideal curve for an arch of uniform density and thickness which supports only its own weight. When the centerline of an arch is made to follow the curve of an up-side-down (i.e. inverted) catenary, the arch endures almost pure compression, in which no significant bending moment occurs inside the material.

The Sheffield Winter Garden is enclosed by a series of catenary arches

Catenary arches are often used in the construction of kilns. In this construction technique, the shape of a hanging chain of the desired dimensions is transferred to a form which is then used as a guide for the placement of bricks or other building material.

However the conditions for a catenary to be the ideal arch are almost never fulfilled: arches usually support more than their own weight, and on the rare occasions when they are freestanding they are sometimes not of uniform thickness.

The Gateway Arch (looking East) is a flattened catenary

Catenary arch kiln under construction over temporary form

The Gateway Arch in St. Louis, Missouri, United States is sometimes said to be an (inverted) catenary, but this is incorrect. It is close to a more general curve called a flattened catenary, with equation $y=A\cosh(Bx)$. (A catenary would have $AB=1$.) While a catenary is the ideal shape for a freestanding arch of constant thickness, the Gateway Arch is narrower near the top. According to the U.S. National Historic Landmark nomination for the arch, it is a "weighted catenary" instead. Its shape corresponds to the shape that a weighted chain, having lighter links in the middle, would form.

Simple suspension bridges

In simple suspension bridges such as the Capilano Suspension Bridge, where the weight runs parallel to the cables, the cables follow a catenary curve.

Free-hanging chains follow the catenary curve, but suspension bridge chains or cables do not hang freely since they support the weight of the bridge. In most cases the weight of the cable is negligible compared with the weight being supported. When the force exerted is uniform with respect to the length of the chain, as in a simple suspension bridge, the result is a catenary.

When the force exerted is uniform with respect to horizontal distance, as in a suspension bridge, the result is a parabola.

When suspension bridges are constructed, the suspension cables initially sag as the catenary curve, before being tied to the deck below, and then gradually assume a parabolic curve as additional connecting cables are tied to connect the main suspension cables with the bridge deck below.

Golden Gate Bridge, San Francisco, California. Most suspension bridge cables follow a parabolic, not catenary curve.

Anchoring of marine objects

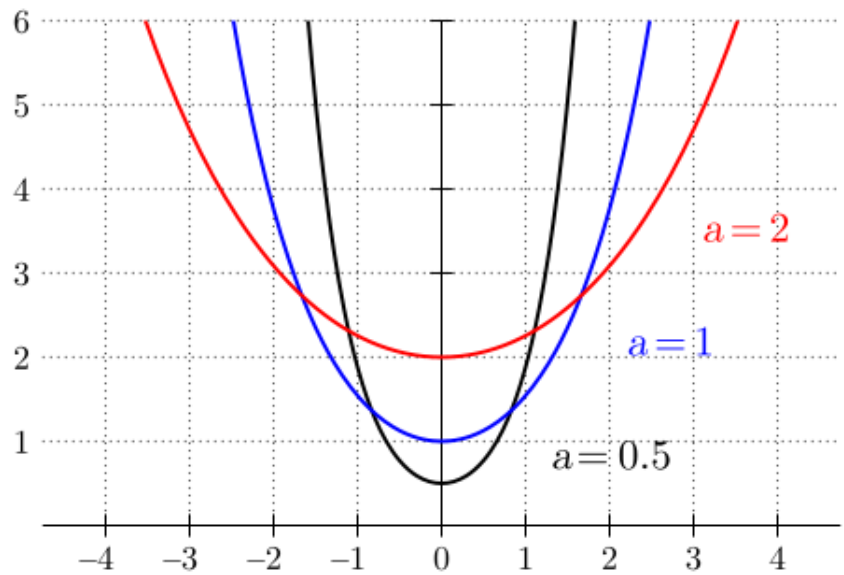
The catenary form given by gravity is taken advantage of in its presence in heavy anchor rodes. An anchor rode (or anchor line) usually consists of chain and/or cable. Anchor rodes are used by ships, oilrigs, docks, wind turbines and other marine assets which must be anchored to the seabed.

Particularly with larger vessels, the catenary curve given by the weight of the rode presents a lower angle of pull on the anchor or mooring device. This assists the performance of the anchor and raises the level of force it will resist before dragging. With smaller vessels and in shallow water it is less effective.

The catenary curve in this context is only fully present in the anchoring system when the rode has been lifted clear of the seabed by the vessel's pull, as the seabed obviously affects its shape while it supports the chain or cable. There is also typically a section of rode above the water and thus unaffected by buoyancy, creating a slightly more complicated curve.

Mathematical description

Equation



Catenaries for different values of *a*

The equation of a catenary in Cartesian coordinates has the form

$$y = a\,\cosh\left(\frac{x}{a}\right) = \frac{a}{2}\left(e^{x/a} + e^{-x/a}\right),$$

where cosh is the hyperbolic cosine function.

The Whewell equation for the catenary is

$$\tan\varphi = \frac{s}{a}.$$

Differentiating gives

$$\frac{d\varphi}{ds} = \frac{\cos^2\varphi}{a}$$

and eliminating φ gives the Cesàro equation:

$$\kappa = \frac{a}{s^2 + a^2}.$$

Other properties

All catenary curves are similar to each other. Changing the parameter *a* is equivalent to a uniform scaling of the curve.

A parabola rolled along a straight line traces out a catenary with its focus.

Square wheels can roll perfectly smoothly if the road has evenly spaced bumps in the shape of a series of inverted catenary curves. The wheels can be any regular polygon except a triangle, but the catenary must have parameters corresponding to the shape and dimensions of the wheels.

A charge in a uniform electric field moves along a catenary (which tends to a parabola if the charge velocity is much less than the speed of light *c*).

The surface of revolution with fixed radii at either end that has minimum surface area is a catenary revolved about the x-axis.

Over any horizontal interval [a,b], the ratio of the area under the caternary to its length equals a, independent of the interval selected. The catenary is the only plane curve other than a horizontal line with this property. Also, the geometric centroid of the area under a stretch of catenary is the midpoint of the perpendicular segment connecting the centroid of the curve itself and the x-axis.

Analysis

We assume that the path followed by the chain is given parametrically by $\mathbf{r} = (x, y) = (x(s),\ y(s))$ where *s* represents arc length and $\mathbf{r}$ is the position vector.

This is the natural parameterization and has the property that $\frac{d\mathbf{r}}{ds}$ is the unit tangent vector,

$\mathbf{u}$. The derivation of the curve for an optimal arch is similar except that the forces of tension become forces of compression and everything is inverted.

It is now possible to derive two equations which together define the shape of the curve and the tension of the chain at each point. This is done by a careful inspection of the various forces acting on a small segment of the chain and using the fact that these forces must be in balance if the chain is in static equilibrium.

First, let $\mathbf{T} = \mathbf{T}(s)$ be the force of tension as a function of *s*. The chain is flexible so it can only exert a force parallel to itself. Since tension is defined as the force that the chain exerts on itself, $\mathbf{T}$ must be parallel to the chain. In other words,

$$(1) \qquad \mathbf{T} = T\mathbf{u}$$

where *T* is the magnitude of $\mathbf{T}$, a positive scalar function of *s*.

Second, let $\mathbf{G} = \mathbf{G}(s)$ be the external force per unit length acting on a small segment of a chain as a function of *s*. The forces acting on the segment of the chain between *s* and *s* + Δ*s* are the force of tension $\mathbf{T}(s + \Delta s)$ at one end of the segment, the nearly opposite force $-\mathbf{T}(s)$ at the other end, and the external force acting on the segment which is approximately $\mathbf{G}\Delta s$. These forces must balance so

$$\mathbf{T}(s + \Delta s) - \mathbf{T}(s) + \mathbf{G}\Delta s \approx \mathbf{0}.$$

Divide by Δ*s* and take the limit as $\Delta s \to 0$ to obtain

$$(2) \qquad \frac{d\mathbf{T}}{ds} + \mathbf{G} = \mathbf{0}.$$

Note that, up till now, no assumptions have been made regarding the force $\mathbf{G}$, so equations (1) and (2) can be used as the starting point in the analysis of a flexible chain acting under any external force. The next step is to put in the specific expression for $\mathbf{G}$ and solve the resulting equations.

In this case, $\mathbf{G} = (0, -\lambda g)$ where the chain has constant mass per unit length λ and the only external force acting on the chain is that of a uniform gravitational field $\mathbf{g} = (0, -g)$. So we have

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

Integrating we get,

$$\mathbf{T} = (c, \lambda g s + d).$$

Note that at the minimum the curve is horizontal and $\mathbf{T} = T\mathbf{u} = T(1,0) = (T,0) = (c, \lambda g s + d)$. So c is the tension of the chain at its lowest point this point occurs at $s = -d / \lambda g$. The point from which s is measured is arbitrary, so pick this point to be the minimum, giving $d = 0$. The equation becomes

$$\mathbf{T} = (c, \lambda g s).$$

Note that the horizontal component of the tension is a constant.

From here, we can continue the derivation in two ways.

Alternative 1

If φ is the tangential angle of the curve then $\mathbf{T}$ is parallel to $(1, \tan\varphi)$ so

$$\tan\varphi = \frac{\lambda g s}{c}.$$

Write $a = \dfrac{c}{\lambda g}$ to combine constants and obtain the Whewell equation for the curve,

$$\tan\varphi = \frac{s}{a}.$$

In general, parametric equations can be obtained from a Whewell equation by integrating:

$$x = \int \cos\varphi \, ds$$
$$y = \int \sin\varphi \, ds$$

To find these integrals, make the substitution $\tan\varphi = \sinh u$ (or $\varphi = \mathrm{gd}u$ where gd is the Gudermannian function).

Then $s = a \sinh u$ and

$$x = \int \cos\varphi ds = \int \operatorname{sech} u \, a \cosh u du = a \int du = au + \alpha$$
$$y = \int \sin\varphi ds = \int \tanh u \, a \cosh u du = a \int \sinh u du = a \cosh u + \beta.$$

We can eliminate u to obtain

$$y = a\cosh\frac{x-\alpha}{a} + \beta$$

where α and β are constants to be determined, along with *a*, by the boundary conditions of the problem. Usually these conditions include two points from which the chain is being suspended and the length of the chain.

Alternative 2

From

$$\mathbf{T} = T\mathbf{u} = T\left(\frac{dx}{ds}, \frac{dy}{ds}\right) = (c, \lambda g s),$$

$$\frac{dy}{dx} = \frac{dy}{ds}\Big/\frac{dx}{ds} = \frac{\lambda g s}{c} = \frac{s}{a},$$

where $a = \dfrac{c}{\lambda g}$, same as before. Then

$$\frac{dx}{ds} = 1\Big/\frac{ds}{dx} = \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \frac{a}{\sqrt{a^2+s^2}}$$

and

$$\frac{dy}{ds} = \frac{dy}{dx}\cdot\frac{dx}{ds} = \frac{s}{\sqrt{a^2+s^2}}.$$

The integrals of the right hand sides of these equations can be found using standard techniques giving

$$x = a\ \mathrm{arcsinh}(s/a) + \alpha,\ \ y = \sqrt{a^2+s^2} + \beta.$$

Isolating *s* in the first equation and using the result to substitute *s* in the second equation gives

$$y = a\cosh\frac{x-\alpha}{a} + \beta$$

as before, α and β are constants to be determined, along with *a*, by the boundary conditions of the problem, which is exact the same result as that obtained with Alternative 1.

Variations

Elastic catenary

In an elastic catenary, the cable replaced by a spring and is no longer assumed to be of fixed density, but is allowed to stretch in accordance with Hooke's Law. In this case, the mass per unit length is no longer constant but can be given as

$$\lambda = \frac{\lambda_0}{1 + \epsilon T}$$

where λ_0 is the mass per unit length for the chain in its relaxed state and ε is the spring constant. As in the earlier derivation,

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

So the horizontal component of $\mathbf{T}$, $T\cos\varphi$ is a constant c. Putting this into the equation for density produces

$$\lambda = \frac{\lambda_0}{1 + c\epsilon \sec\varphi}.$$

Then the equation for the vertical component of $\mathbf{T}$ is

$$\frac{d}{ds}(T\sin\varphi) = c\frac{d}{ds}(\tan\varphi) = \frac{g\lambda_0}{1 + c\epsilon\sec\varphi},$$

or, combining constants,

$$\frac{d}{ds}(\tan\varphi) = \frac{1}{a + b\sec\varphi}.$$

Using the substitution $\tan\varphi = \sinh u$ gives

$$\frac{d}{ds}(\sinh u) = \cosh u\frac{du}{ds} = \frac{1}{a + b\cosh u}$$

or

$$\cosh u(a + b\cosh u) = \frac{ds}{du}.$$

Parametric equations can be obtained by integrating:

$$x = \int \cos\varphi\, ds = \int \operatorname{sech} u \cosh u(a+b\cosh u)du = \int (a+b\cosh u)du = au+b\sinh u+\alpha,$$
$$y = \int \sin\varphi\, ds = \int \tanh u \cosh u(a+b\cosh u)du = \int \sinh u(a+b\cosh u)du = a\cosh u+\tfrac{b}{2}\sinh^2 u+\beta.$$

When $b = 0$, corresponding to a completely inelastic cable, this is simply the catenary. When $a = 0$, corresponding to the case there the cable essentially has length 0 in its relaxed state, similar to a Slinky, this is a parabola. When a and b are both >0 then the curve is intermediate between a catenary and a parabola.

Equal resistance catenary

In an equal resistance catenary, cable is strengthened according to the magnitude of the tension at each point, so its resistance to breaking is constant along its length. Assuming that the strength of the cable is proportional to its density, the mass per unit length can be given as

$$\lambda = \lambda_r T$$

where λ_r is the mass per unit length per unit of tension force required for the chain to resist breaking. As in the earlier derivation,

$$\frac{d\mathbf{T}}{ds} = (0, \lambda g).$$

So the horizontal component of $\mathbf{T}$, $T\cos\varphi$ is a constant c. Putting this into the equation for density produces

$$\lambda = \lambda_r c \sec\varphi.$$

Then the equation for the vertical component of $\mathbf{T}$ is

$$\frac{d}{ds}(T\sin\varphi) = c\frac{d}{ds}(\tan\varphi) = g\lambda_r c\sec\varphi,$$

or, combining constants,

$$\frac{d}{ds}(\tan\varphi) = \frac{1}{a}\sec\varphi$$

or

$$\frac{d}{ds}\left(\frac{dy}{dx}\right) = \frac{1}{a}\frac{ds}{dx}.$$

Multiplying both sides by *ds* / *dx* gives

$$\frac{d^2y}{dx^2} = \frac{1}{a}\left(\frac{ds}{dx}\right)^2 = \frac{1}{a}\left[1 + \left(\frac{dy}{dx}\right)^2\right].$$

This can be reduced to a differential equation of degree one using separation of variables to obtain

$$\arctan\frac{dy}{dx} = \frac{x}{a} - \alpha$$

or

$$\frac{dy}{dx} = \tan\left(\frac{x}{a} - \alpha\right).$$

Another integration produces

$$y = -a\ln\cos\left(\frac{x}{a} - \alpha\right) + \beta.$$

Towed cables

Instead of gravity, we assume we have a cylindrical cable that is acted on by drag forces due to the movement of some surrounding fluid (e.g. air or water). The velocity relative to the cable is assumed to be a constant $\mathbf{v} = (0, -v)$. (Velocity is assumed to be vertical here to preserve similarities with the gravitational case.) To compute the force due to drag, write $\mathbf{v} = \mathbf{v}_u + \mathbf{v}_n$ where $\mathbf{v}_u$ and $\mathbf{v}_n$ respectively are the components parallel to and orthogonal to the cable. The cable is assumed to be smooth so the force on the cable due to $\mathbf{v}_u$ is taken to be negligible. The force acting on the cable, following the Drag equation is

$$\mathbf{G} = -c|\mathbf{v}_n|^2\mathbf{n}$$

where *c* is a constant depending on the density of the fluid, the diameter of the cable, and the Drag coefficient. If $\mathbf{n} = (-\sin\varphi,\ \cos\varphi)$ denotes the unit normal vector, then

$$\mathbf{v}_n = \mathbf{v} \cdot \mathbf{n}\ \mathbf{n} = -\cos\varphi\ \mathbf{n}.$$

So

$$\mathbf{G} = -c\cos^2\varphi\ \mathbf{n}.$$

From equations (1) and (2) above,

$$\frac{d\mathbf{T}}{ds} = \frac{dT}{ds}\mathbf{u} + T\frac{d\mathbf{u}}{ds} = \frac{dT}{ds}\mathbf{u} + T\frac{d\varphi}{ds}\mathbf{n} = c\cos^2\varphi\ \mathbf{n}.$$

Setting the coefficients of $\mathbf{u}$ and $\mathbf{n}$ equal produces

$$\frac{dT}{ds} = 0,\ T\frac{d\varphi}{ds} = c\cos^2\varphi.$$

So T is a constant in this case and combining constants in the second equation gives

$$\frac{d\varphi}{ds} = \frac{\cos^2\varphi}{a}$$

which is one of the equations for the catenary given above. This is a case where a different expression for the force acting on the chain/cable produce the same curve but a different expression for tension.

In applications, the force of gravity and additional terms in the force due to drag may be added to the expression for force, yielding equations that must be solved numerically.

Alternative analysis

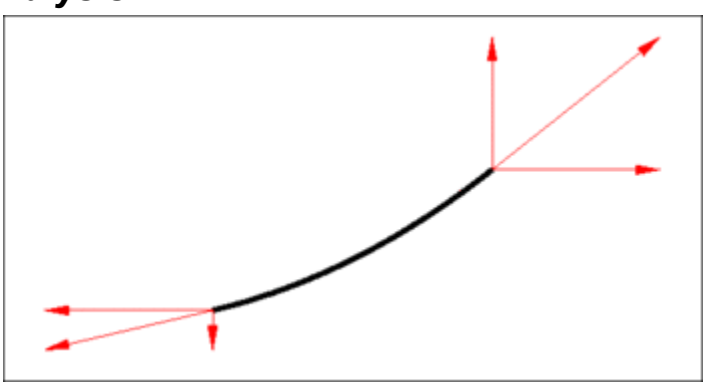


Figure 1: The forces acting on the two extremes of a segment of a catenary decomposed into horizontal and vertical components

The forces acting on a segment of catenary curve are shown in the figure at right.

The vector sum of the forces acting on the segment from the two extremities and from the gravitational force must be zero. As the gravitational force is directed downwards the horizontal components of the forces acting on the extremes must have the same magnitude. As this is true for any segment of the catenary this is a fixed constant for the whole of the catenary. Denoting this constant with *f* one gets that the vertical component of the force at the left extreme x_1 is $-f\ y'(x_1)$ and at the right extreme x_2 is $f\ y'(x_2)$ The path length of the curve representing a function $y(x)$ with x varying from x_1 to x_2 is

$$\int_{x_1}^{x_2} \sqrt{1+y'^2}dx$$

If *g* is the gravitational constant and ρ is the mass per length unit of the chain the gravitational force acting on the arc from x_1 to x_2 is $g\rho \int_{x_1}^{x_2} \sqrt{1+y'^2}dx$

This force must be compensated by the vertical components of the forces acting on the two extremes of the arc, i.e.

$$f\ (y'(x_2) - y'(x_1)) = g\rho \int_{x_1}^{x_2} \sqrt{1+y'^2}dx \quad \textbf{(1)}$$

Denoting the constant ratio $\frac{f}{g\rho}$ with *a* and taking the derivative of equation (**1**) with respect to the upper limit of the integral, i.e. with respect to x_2, one gets

$$y'' = \frac{1}{a}\sqrt{1+y'^2}$$

Denoting y' with *z* this equation takes the form

$$z' = \frac{dz}{dx} = \frac{1}{a}\sqrt{1+z^2}$$

what means that for the inverse function $x(z)$ one has

$$\frac{dx}{dz} = \frac{a}{\sqrt{1+z^2}}$$

which is integrate to

$$x = a\ \sinh^{-1}(z) + x_0$$

where x_0 is the constant of integration or equivalently

$$z = y' = \sinh(\frac{x - x_0}{a})$$

Again integrating with respect to x one gets

$$y = a\ \cosh(\frac{x - x_0}{a})\ +\ y_0 \quad \textbf{(2)}$$

where y_0 is the second constant of integration

The lowest point of this curve has the coordinates $(\ x_0\ ,\ y_0 + a)$

The length of the curve given by (**2**) from $x = x_1$ to $x = x_2$ is

$$l = \int_{x_1}^{x_2} \sqrt{1 + y'^2} dx = \left[a \sinh \frac{x - x_0}{a} \right]_{x_1}^{x_2} \quad \textbf{(3)}$$

This family of solutions is parametrized with the 3 parameters $a,\ x_0,\ y_0$. For any concrete case these 3 parameters must be computed to fit the boundary value conditions. In a typical case the form of a chain having a given length l and being attached in two fixed point with the coordinates $x_1,\ y_1$and $x_2,\ y_2$relative a vertical coordinate system should be computed.

This means that $a,\ x_0,\ y_0$have to be determined such that

$$y_1 = a\ \cosh(\frac{x_1 - x_0}{a})\ +\ y_0 \quad \textbf{(4)}$$

$$y_2 = a\ \cosh(\frac{x_2 - x_0}{a})\ +\ y_0 \quad \textbf{(5)}$$

$$l = a\left(\sinh \frac{x_2 - x_0}{a}\ -\ \sinh \frac{x_1 - x_0}{a}\right) \quad \textbf{(6)}$$

Setting

$$x_m = \frac{x_2 + x_1}{2}$$

$$\Delta x = \frac{x_2 - x_1}{2}$$

subtracting (**4**) from (**5**) and then dividing with a one gets

$$\frac{y_2 - y_1}{a} = \cosh\left(\frac{x_m - x_0}{a} + \frac{\Delta x}{a}\right) - \cosh\left(\frac{x_m - x_0}{a} - \frac{\Delta x}{a}\right) = 2\sinh\frac{x_m - x_0}{a}\sinh\frac{\Delta x}{a} \quad \textbf{(7)}$$

For any given values $x_1,\ y_1,\ x_2,\ y_2,\ a$ one can determine $\sinh\frac{x_m - x_0}{a}$ from (**7**)

When $\sinh\frac{x_m - x_0}{a}$ has been determined $\frac{x_m - x_0}{a}$ is computed by solving a quadratic equation.

In case $y_1 = y_2$, i.e. in the case that the two attachment points are at the same heigth, one has that $x_0 = x_m$ and that the length is $l = 2\,a\left(\sinh\frac{\Delta x}{a}\right)$

With x_0 known (**4**) or (**5**) can subsequently be used to determine y_0.

Having determined x_0 with the algorithm just described the curve length l corresponding to the selected a value can be computed from (**6**). With an iterative algorithm the a value that corresponds to a certain curve length l can finally be derived.

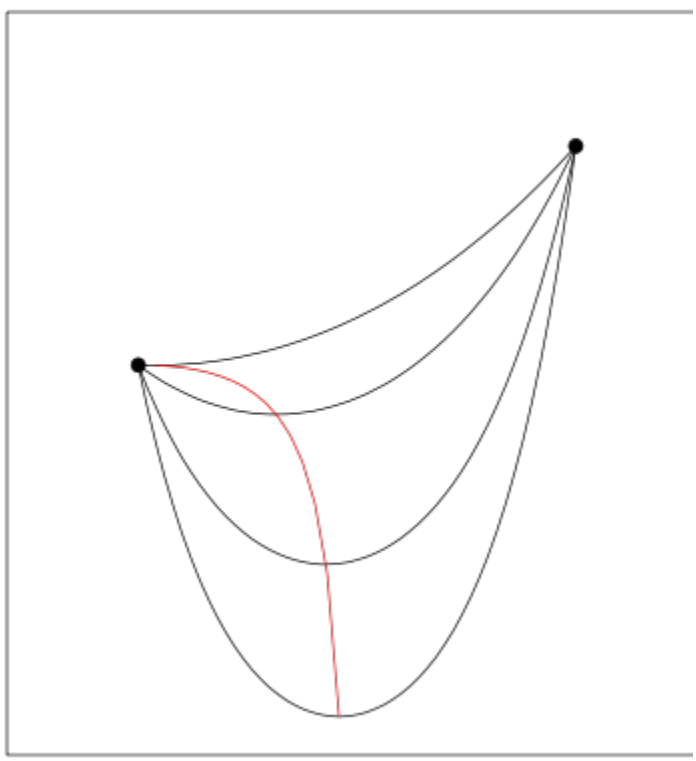


Figure 2:The red line corresponds to parameters X_0 and Y_0 + a determined with the algorithm described above for different values of a

From figure 1 it is further clear that the tension of the chain at any point $(x\ ,\ y)$ is $f\sqrt{1+y'^2} = f\cosh\frac{x-x_0}{a} = f\ \frac{y-y_0}{a}$ where $f = a\ g\ \rho$ is the magnitude of the constant horizontal force component

If the mass density ρ is not constant but varies depending on some law the resulting differential equation will in most cases not have a closed form analytic solution. But the resulting curve can still be determined with arbitrary accuracy by the numerical integration of the differential equations

$$\begin{aligned} y' &= z \\ z' &= \frac{g}{f}\ \ \rho(z)\sqrt{1+z^2} \end{aligned}$$

Given any initial values for $y(x1)$ and $z(x1)$ and any value for the parameter f these differential equations can be propagated to $x = x_2$ with ρ specified as any function of the state variable z. The free parameters to be iteratively adjusted to fit the boundary constraints are now $z(x1)$ and f. They can for example be adjusted iteratively such that $y(x2) = y_2$ where $(x_2\ ,\ y_2)$ is the second attachment point. This leaves an additional degree of freedom for the two parameters that can be used to get the correct length of the curve.

An example is the "elastic catenary" for which the force

$$f\sqrt{1+y'^2} = f\sqrt{1+z^2}$$

stretches the material with a factor

$$1\ +\ \epsilon f\ \sqrt{1+z^2}$$

where ε is an elasticity coefficient and that therefore the mass density (mass per unit length) is

$$\rho = \frac{\rho_0}{1\ +\ \epsilon f\ \sqrt{1+z^2}}$$

where ρ_0 is the mass density of the material in the absence of stress.

A case where a closed form mathematical solution is possible is the case of "the equal resistance catenary" where the mass density (mass per unit length) is proportional to the force $f\sqrt{1+y'^2} = f\sqrt{1+z^2}$, i.e.

$$\rho = \rho_0\sqrt{1+z^2}$$

where ρ_0 is the density at the lowest point

Setting $a = \dfrac{f}{g\rho_0}$ the differential equations now take the form

$$y' = z$$
$$z' = \frac{1}{a}\left(1 + z^2\right)$$

what means that for the inverse function $x(z)$ one has

$$\frac{dx}{dz} = \frac{a}{1 + z^2}$$

which is integrate to

$$x = a \ \arctan(z) + x_0$$

where x_0 is the constant of integration or equivalently

$$z = y' = \tan\left(\frac{x - x_0}{a}\right)$$

where x is constraint to an interval $x_0 - a\,\dfrac{\pi}{2} < x < x_0 + a\,\dfrac{\pi}{2}$

Again integrating with respect to x one gets

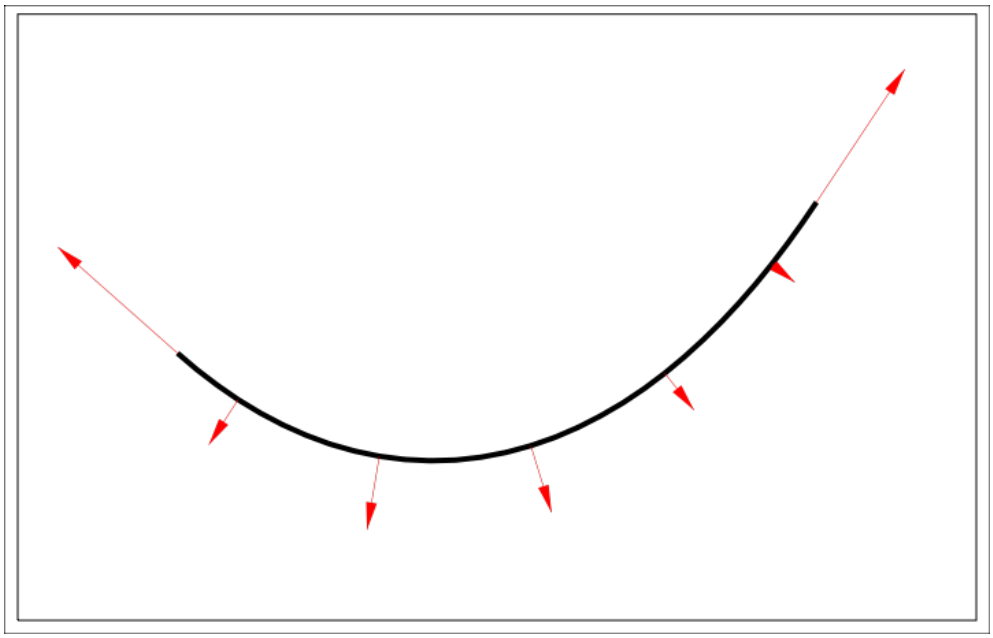
$$y = y_0 \ - a \ln\left(\cos\left(\frac{x - x_0}{a}\right)\right)$$

where y_0 is the second constant of integration.

As $a \ \sinh\dfrac{C}{a} \to \infty$ when $a \ \to 0$ for any constant C it follows from (**6**) that by making a catenary that is fixed at two points sufficiently long the constant horizontal force component f can be made arbitrarily small. For this generalized "catenary of equal resistance" this is no more true, as a must be larger then $\dfrac{|x - x_0|}{\frac{\pi}{2}}$ for any x between x_1 and x_2 the positions of the two attachment points and the density ρ_0 at the lowest point impose a lower limit for the fixed horizontal force component f

Alternative analysis "towed cables"

The following figure illustrates a segment of a cable that is fixed in both ends and exposed to drag.



The forces acting on a cable subject to drag. The medium causing the drag is moving downwards. The drag force is orthogonal to the cable and the forces acting on the two extremities of the segment compensate the net drag force on the segment

The velocity relative to the cable is assumed to be constant and the coordinate system is selected such that this velocity is in the -y direction, i.e. $\mathbf{v} = (0, -v)$. To compute the force due to drag, write $\mathbf{v} = \mathbf{v}_u + \mathbf{v}_n$

where $\mathbf{v}_u$and $\mathbf{v}_n$respectively are the components parallel to and orthogonal to the cable. The cable is assumed to be smooth so the force on the cable due to $\mathbf{v}_u$is taken to be negligible. The force acting on the cable, per unit length, following the Drag equation is therefore

$$\mathbf{G} = f(x)\mathbf{n}$$

with

$$f(x) = c\ v^2\ {n_y}^2 \quad (1)$$

where *c* is a constant depending on the density of the fluid, the diameter of the cable, and the Drag coefficient and $\mathbf{n} = (n_x\ ,\ n_y)$denotes the unit normal vector.

For any curve $y(x)$ the tangent (unit vector) is

$$(t_x\ ,\ t_y) = \frac{(1\ ,\ y')}{\sqrt{1+y'^2}} \quad \textbf{(2)}$$

and the normal (unit vector) is

$$(n_x\ ,\ n_y) = \frac{(y'\ ,\ -1)}{\sqrt{1+y'^2}} \quad \textbf{(3)}$$

From (**1**) and (**3**) follows that

$$f(x) = c\ v^2\ \frac{1}{1+y'^2} \quad \textbf{(4)}$$

From (**3**) and (**4**) follows that the x-component of the total force on the segment of the curve from $x = x_1$ to $x = x_2$ is

$$F_x = \int_{x_1}^{x_2} f(x)\ n_x\ \sqrt{1+y'^2}dx = c\ v^2 \int_{x_1}^{x_2} \frac{y'}{1+y'^2}\ dx \quad \textbf{(5)}$$

and the component in the y-direction is

$$F_y = \int_{x_1}^{x_2} f(x)\ n_y\ \sqrt{1+y'^2}dx = c\ v^2 \int_{x_1}^{x_2} \frac{-1}{1+y'^2}\ dx \quad \textbf{(6)}$$

If now

$$y' = \sinh(\frac{x-x_0}{a})$$

one has that

$$1+y'^2 = \cosh^2(\frac{x-x_0}{a})$$

and from (**2**),(**5**) and (**6**) that

$$F_x = c\ v^2 \int_{x_1}^{x_2} \frac{\sinh(\frac{x-x_0}{a})}{\cosh^2(\frac{x-x_0}{a})}\ dx = -\left[\frac{c\ v^2\ a}{\cosh(\frac{x-x_0}{a})}\right]_{x_1}^{x_2} =$$

$$-\left[\frac{c\ v^2\ a}{\sqrt{1+y'^2}}\right]_{x_1}^{x_2} = -c\ v^2\ a\ (t_x(x_2) - t_x(x_1)) \qquad \textbf{(7)}$$

$$F_y = c\ v^2 \int_{x_1}^{x_2} \frac{-1}{\cosh^2(\frac{x-x_0}{a})}\ dx = -\left[\frac{c\ v^2\ a\ \sinh(\frac{x-x_0}{a})}{\cosh(\frac{x-x_0}{a})}\right]_{x_1}^{x_2} =$$

$$-\left[\frac{c\ v^2\ a\ y'}{\sqrt{1+y'^2}}\right]_{x_1}^{x_2} = -c\ v^2\ a\ (t_y(x_2) - t_y(x_1)) \qquad \textbf{(8)}$$

If the now the force in the cable is

$$F\ = c\ v^2\ a$$

the force at the right extreme of the cable segment is

$$c\ v^2\ a\ (t_x(x_2)\ ,\ t_y(x_2))$$

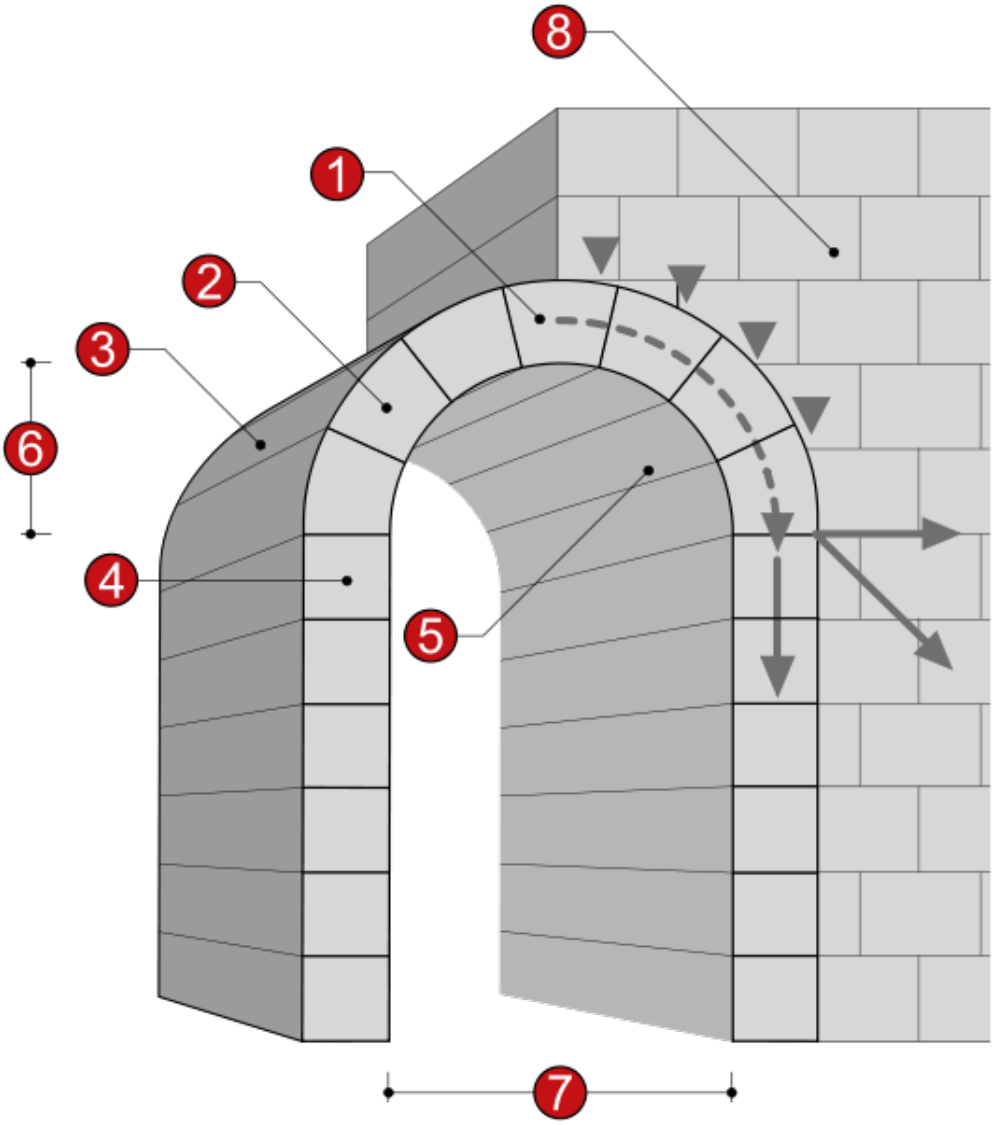
and at the left extreme

$$-c\ v^2\ a\ (t_x(x_1)\ ,\ t_y(x_1))$$

From (**7**) and (**8**) follows that the vector sum of these forces is precisely the force needed to counter act the forces on the segment caused by the drag

Chapter- 6

Arch (Structural Element)



A masonry arch
1. Keystone 2. Voussoir 3. Extrados 4. Impost 5. Intrados 6. Rise 7. Clear span
8. Abutment

An **arch** is a structure that spans a space while supporting weight (e.g. a doorway in a stone wall). Arches appeared as early as the 2nd millennium BC in Mesopotamian brick architecture and their systematic use started with the Ancient Romans who were the first to apply the technique to a wide range of structures.

The semicircular arch was followed in Europe by the pointed Gothic arch or ogive whose centreline more closely followed the forces of compression and which was therefore stronger. The semicircular arch can be flattened to make an elliptical arch as in the Ponte Santa Trinita. The parabolic and catenary arches are now known to be the theoretically strongest forms. Parabolic arches were introduced in construction by the Spanish architect Antoni Gaudí, who admired the structural system of Gothic style, but for the buttresses, which he termed "architectural crutches". The catenary and parabolic arches carry all horizontal thrust to the foundation and so do not need additional elements.

Arch of Constantine, Rome, Italy commemorating a victory by Constantine I in 312 AD

The horseshoe arch is based on the semicircular arch, but its lower ends are extended further round the circle until they start to converge. The first known built horseshoe arches are known from Aksum (modern day Ethiopia and Eritrea) from around the 3rd–4th century, around the same time as the earliest contemporary examples in Roman Syria, suggesting either an Aksumite or Syrian origin for the type of arch.

History

Roman arch architecture in Ostia Antica, Italy

The Arc de Triomphe, Paris; a 19th-century triumphal arch modeled on the classical Roman design

True arches, as opposed to corbel arches, were known by a number of civilizations in the Ancient Near East, the Levant, and Mexico, but their use was infrequent and mostly confined to underground structures such as drains where the problem of lateral thrust is greatly diminished. A rare exception is the bronze age arched city gate of Ashkelon, Israel, dating to ca. 1850 BC. An early example of a voussoir arch appears in the Greek Rhodes Footbridge. In 2010, a robot discovered a long arch-roofed passageway underneath the Pyramid of Quetzalcoatl which stands in the ancient city of Teotihuacan north of Mexico City, dated to around 200 AD.

The ancient Romans learned the arch from the Greeks and Etruscans, refined it and were the first builders to tap its full potential for above ground buildings:

The Romans were the first builders in Europe, perhaps the first in the world, fully to appreciate the advantages of the arch, the vault and the dome.

Arches in the nave of the church in monastery of Alcobaça, Portugal

Throughout the Roman empire, their engineers erected arch structures such as bridges, aqueducts, and gates. They also introduced the triumphal arch as a military monument. Vaults began to be used for roofing large interior spaces such as halls and temples, a function which was also assumed by domed structures from the 1st century BC onwards.

The segmental arch was first built by the Romans who realized that an arch in a bridge did not have to be a semicircle, such as in Alconétar Bridge or Ponte San Lorenzo.

Catenary arches inside Casa Milà in Barcelona, Spain by Antoni Gaudí

Construction

An arch requires all of its elements to hold it together, raising the question of how an arch is constructed. One answer is to build a frame (historically, of wood) which exactly follows the form of the underside of the arch. This is known as a centre or centring. The voussoirs are laid on it until the arch is complete and self-supporting. For an arch higher than head height, scaffolding would in any case be required by the builders, so the scaffolding can be combined with the arch support. Occasionally arches would fall down when the frame was removed if construction or planning had been incorrect. (The A85 bridge at Dalmally, Scotland suffered this fate on its first attempt, in the 1940s). The interior and lower line or curve of an arch is known as the *intrados*.

Old arches sometimes need reinforcement due to decay of the keystones, forming what is known as **bald arch**.

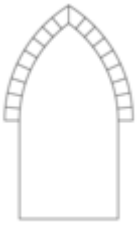
The gallery shows arch forms displayed in roughly the order in which they were developed.

Triangular arch

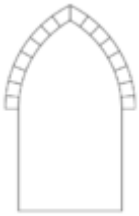
Round arch or Semi-circular arch

Segmental arch or arch that is less than a semicircle

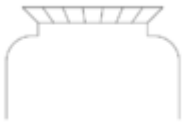
Unequal round arch or Rampant round arch



Lancet arch

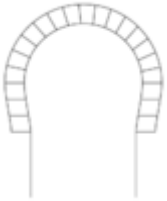


Equilateral pointed arch

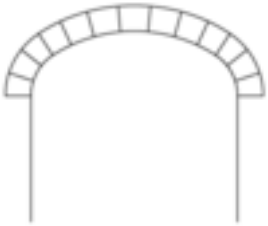


Shouldered flat arch

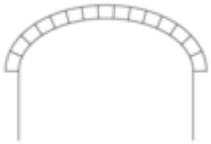
Three-foiled cusped arch



Horseshoe arch



Three-centered arch



Elliptical arch

Inflexed arch

Ogee arch

Reverse ogee arch

Tudor arch

Catenary or Parabolic arch

Technical aspects

Roman Pont-Saint-Martin

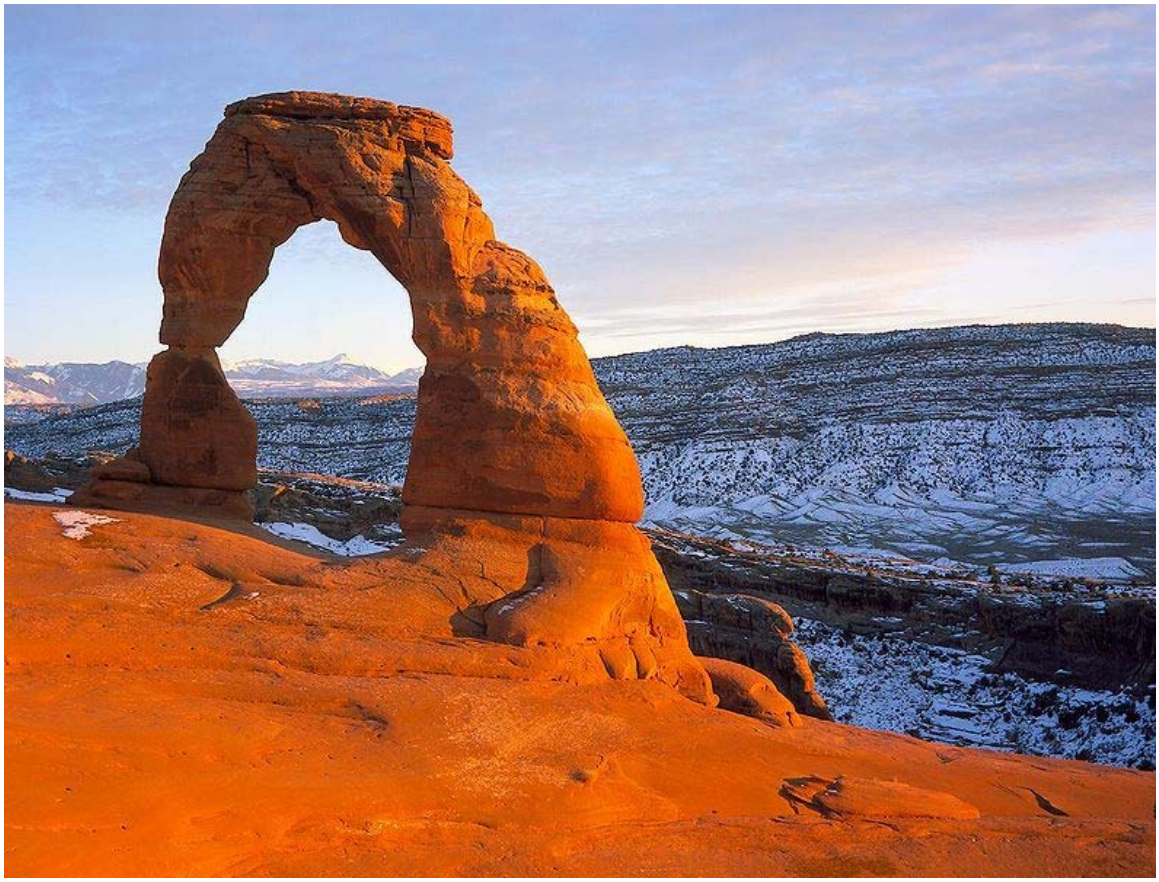
Arches in the Armenian monastery of Geghard

The arch is significant because, in theory at least, it provides a structure which eliminates tensile stresses in spanning an open space. All the forces are resolved into compressive stresses. This is useful because several of the available building materials such as stone, cast iron and concrete can strongly resist compression but are very weak when tension, shear or torsional stress is applied to them. By using the arch configuration, significant spans can be achieved. This is because all the compressive forces hold it together in a state of equilibrium. This even applies to frictionless surfaces. However, one downside is that an arch pushes outward at the base, and this needs to be restrained in some way, either with heavy sides and friction or angled cuts into bedrock or similar.

This same principle holds when the force acting on the arch is not vertical such as in spanning a doorway, but horizontal, such as in arched retaining walls or dams.

Even when using concrete, where the structure may be monolithic, the principle of the arch is used so as to benefit from the concrete's strength in resisting compressive stress. Where any other form of stress is raised, it has to be resisted by carefully placed reinforcement rods or fibres.

Other types



The Delicate Arch, a natural arch in Moab, Utah

A blind arch is an arch infilled with solid construction so it cannot function as a window, door, or passageway.

A dome is an arch rotated 360 degrees about its vertical axis.

Natural rock formations may also be referred to as arches. These natural arches are formed by erosion rather than being carved or constructed by man.

A special form of the arch is the triumphal arch, usually built to celebrate a victory in war. A famous example is the Arc de Triomphe in Paris, France.

A vault is an arch extended along the axis perpendicular to the its plane; the groin vault is the intersection of two vaults.

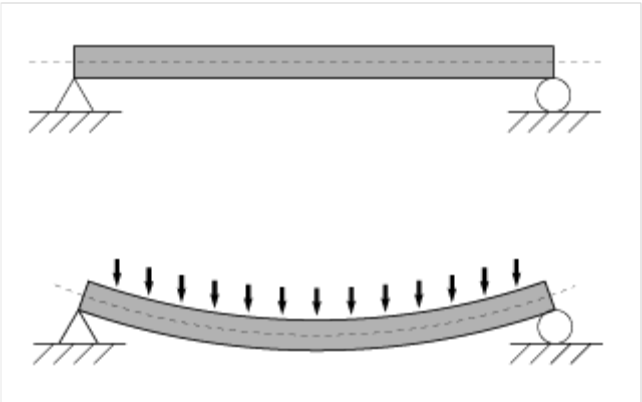
The Gateway Arch in Saint Louis, Missouri; a sculpture based on a catenary arch

Several arches at the Casa Simón Bolívar in Havana, Cuba

Chapter- 7

Other Structural Elements used in Structural Engineering

Beam



A statically determinate beam, bending under an evenly distributed load

A **beam** is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

Overview

Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e., loads due to an earthquake or wind). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. In light frame construction the joists rest on the beam.

Beams are characterized by their profile (the shape of their cross-section), their length, and their material. In contemporary construction, beams are typically made of steel, reinforced concrete, or wood. One of the most common types of steel beam is the I-beam or wide-flange beam (also known as a "universal beam" or, for stouter sections, a "universal column"). This is commonly used in steel-frame buildings and bridges. Other

common beam profiles are the C-channel, the hollow structural section beam, the pipe, and the angle.

Structural characteristics

Moment of inertia

The moment of inertia of an object about a given axis describes how difficult it is to change its angular motion about that axis. Therefore, it encompasses not just how much mass the object has overall, but how far each bit of mass is from the axis. The farther out the object's mass is, the more rotational inertia the object has, and the more force is required to change its rotation rate.

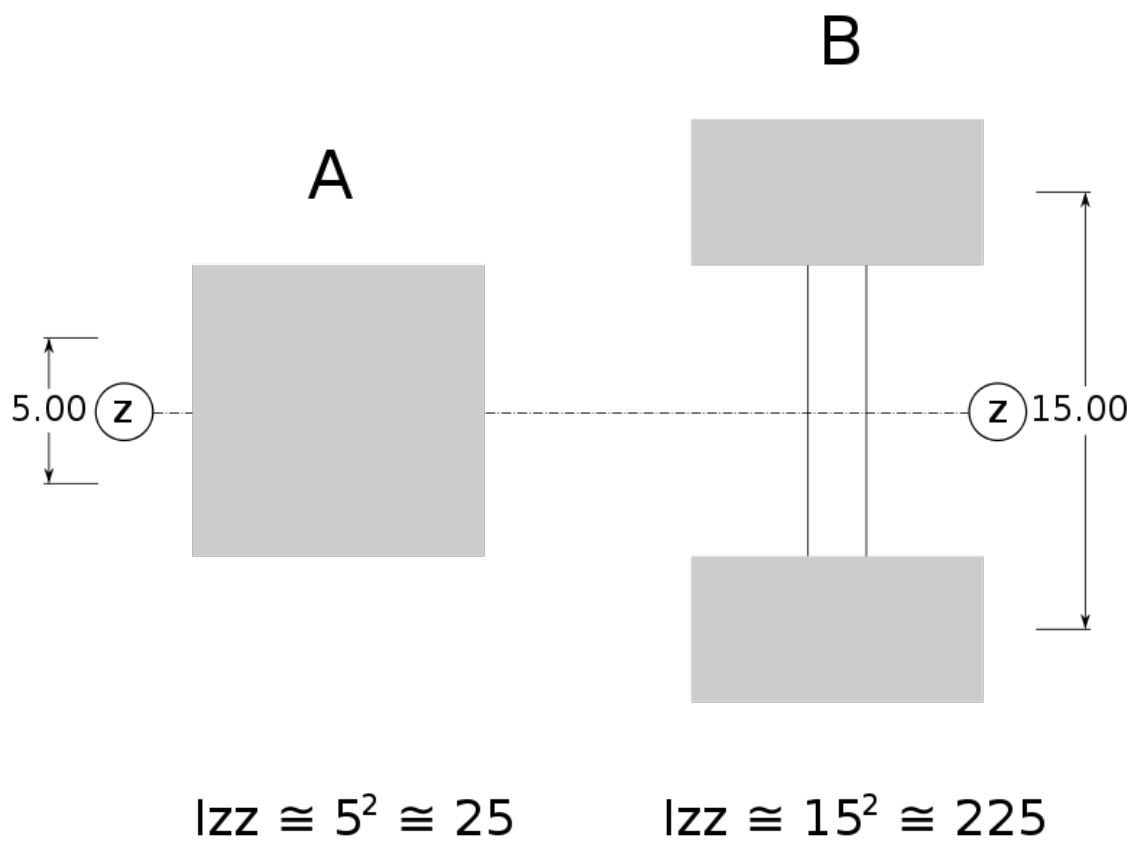


Diagram of stiffness of a simple square beam (A) and universal beam (B). The universal beam flange sections are three times further apart than the solid beam's upper and lower halves. The second moment of inertia of the universal beam is nine times that of the square beam of equal cross section (universal beam web ignored for simplification)

stress in beams

Internally, beams experience compressive, tensile and shear stresses as a result of the loads applied to them. Typically, under gravity loads, the original length of the beam is slightly reduced to enclose a smaller radius arc at the top of the beam, resulting in compression, while the same original beam length at the bottom of the beam is slightly stretched to enclose a larger radius arc, and so is under tension. The same original length of the middle of the beam, generally halfway between the top and bottom, is the same as the radial arc of bending, and so it is under neither compression nor tension, and defines the neutral axis (dotted line in the beam figure). Above the supports, the beam is exposed to shear stress. There are some reinforced concrete beams in which the concrete is entirely in compression with tensile forces taken by steel tendons. These beams are known as prestressed concrete beams, and are fabricated to produce a compression more than the expected tension under loading conditions. High strength steel tendons are stretched while the beam is cast over them. Then, when the concrete has cured, the tendons are slowly released and the beam is immediately under eccentric axial loads. This eccentric loading creates an internal moment, and, in turn, increases the moment carrying capacity of the beam. They are commonly used on highway bridges.

The primary tool for structural analysis of beams is the Euler–Bernoulli beam equation. Other mathematical methods for determining the deflection of beams include "method of virtual work" and the "slope deflection method". Engineers are interested in determining deflections because the beam may be in direct contact with a brittle material such as glass. Beam deflections are also minimized for aesthetic reasons. A visibly sagging beam, even if structurally safe, is unsightly and to be avoided. A stiffer beam (high modulus of elasticity and high second moment of area) produces less deflection.

Mathematical methods for determining the beam forces (internal forces of the beam and the forces that are imposed on the beam support) include the "moment distribution method", the force or flexibility method and the direct stiffness method.

General shapes

Most beams in reinforced concrete buildings have rectangular cross sections, but the most efficient cross section for a simply supported beam is an I or H section. Because of the parallel axis theorem and the fact that most of the material is away from the neutral axis, the second moment of area of the beam increases, which in turn increases the stiffness.

An I-beam is only the most efficient shape in one direction of bending: up and down looking at the profile as an I. If the beam is bent side to side, it functions as an H where it is less efficient. The most efficient shape for both directions in 2D is a box (a square shell) however the most efficient shape for bending in any direction is a cylindrical shell or tube. But, for unidirectional bending, the I or wide flange beam is superior.

Efficiency means that for the same cross sectional area (volume of beam per length) subjected to the same loading conditions, the beam deflects less.

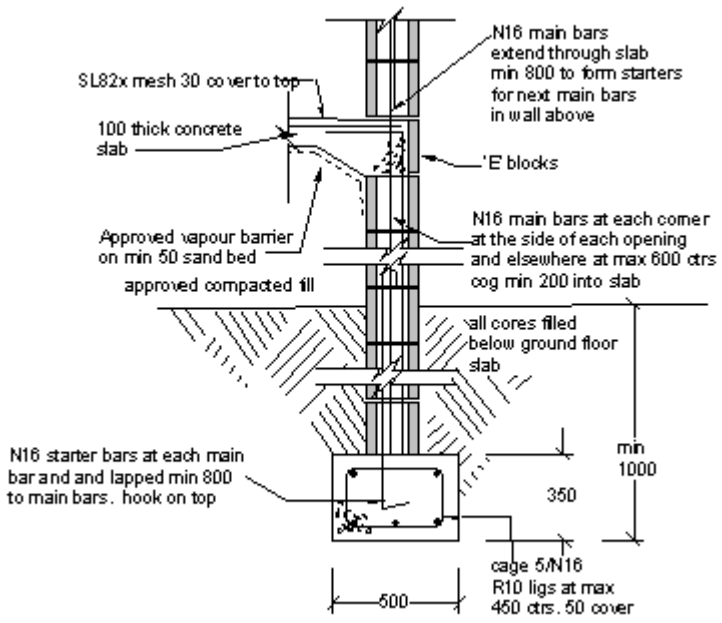
Other shapes, like L (angles), C (channels) or tubes, are also used in construction when there are special requirements.

Shallow Foundation

A **shallow foundation** is a type of foundation which transfers building loads to the earth very near the surface, rather than to a subsurface layer or a range of depths as does a deep foundation. Shallow foundations include **spread footing foundations**, **mat-slab foundations**, **slab-on-grade foundations**, **rubble trench foundations**, and **earthbag foundations**.

Spread footing foundation

In ground reinforced concrete foundation in cyclonic area, Northern Australia



In ground reinforced concrete foundation in cyclonic area, Northern Australia

Spread footing foundations consists of strips or pads of concrete (or other materials) which transfer the loads from walls and columns to the soil or bedrock. Embedment of spread footings is controlled by several factors, including development of lateral capacity, penetration of soft near-surface layers, and penetration through near-surface layers likely to change volume due to frost heave or shrink-swell.

These foundations are common in residential construction that includes a basement, and in many commercial structures. But for high rise building it is not sufficient.

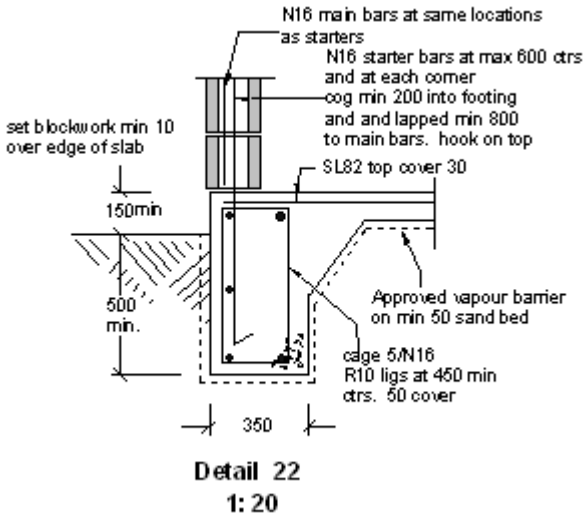
Mat-slab foundations

Mat-slab foundations are used to distribute heavy column and wall loads across the entire building area, to lower the contact pressure compared to conventional spread footings. Mat-slab foundations can be constructed near the ground surface, or at the bottom of basements. In high-rise buildings, mat-slab foundations can be several meters thick, with extensive reinforcing to ensure relatively uniform load transfer.

Slab-on-grade foundation

Example of slab on grade foundation

Raft slab house foundation in cyclonic area, Northern Australia



Raft slab house foundation in cyclonic area, Northern Australia

Slab-on-grade foundations are a structural engineering practice whereby the concrete slab that is to serve as the foundation for the structure is formed from a mold set into the

ground. The concrete is then placed into the mold, leaving no space between the ground and the structure. This type of construction is most often seen in warmer climates, where ground freezing and thawing is less of a concern and where there is no need for heat ducting underneath the floor.

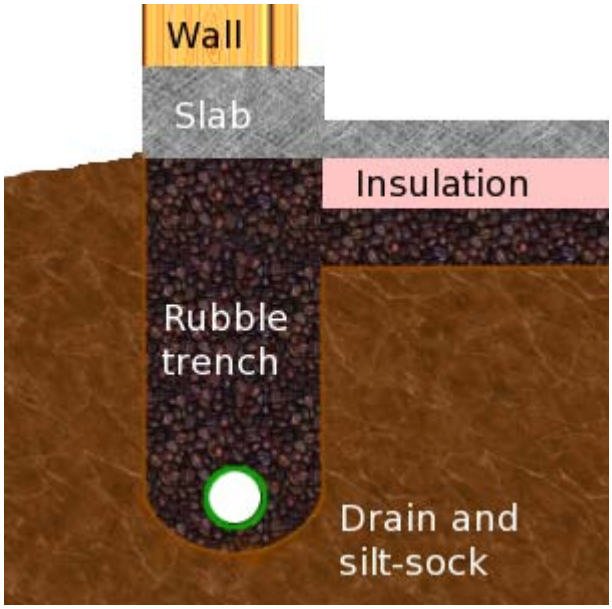
The advantages of the slab technique are that it is cheap and sturdy, and is considered less vulnerable to termite infestation because there are no hollow spaces or wood channels leading from the ground to the structure (assuming wood siding, etc., is not carried all the way to the ground on the outer walls).

The disadvantages are the lack of access from below for utility lines, the potential for large heat losses where ground temperatures fall significantly below the interior temperature, and a very low elevation that exposes the building to flood damage in even moderate rains. Remodeling or extending such a structure may also be more difficult. Over the long term, ground settling (or subsidence) may be a problem, as a slab foundation cannot be readily jacked up to compensate; proper soil compaction prior to pour can minimize this. The slab can be decoupled from ground temperatures by insulation, with the concrete poured directly over insulation (for example, Styrofoam panels), or heating provisions (such as hydronic heating) can be built into the slab (an expensive installation, with associated running expenses).

Slab-on-grade foundations are commonly used in areas with expansive clay soil, particularly in California and Texas. While elevated structural slabs actually perform better on expansive clays, it is generally accepted by the engineering community that slab-on-grade foundations offer the greatest cost-to-performance ratio for tract homes. Elevated structural slabs are generally only found on custom homes or homes with basements.

Care must be taken with the provision of services through the slab. Copper piping, commonly used to carry natural gas and water, reacts with concrete over a long period, slowly degrading until the pipe fails. Copper pipes must be *lagged* (that is, *insulated*) or run through a conduit or plumbed into the building above the slab. Electrical conduits through the slab need to be water-tight, as they extend below ground level and can potentially expose the wiring to groundwater.

Rubble Trench foundation



A cross section view of a rubble trench foundation

The **rubble trench foundation**, a construction approach popularized by architect Frank Lloyd Wright, is a type of foundation that uses loose stone or rubble to minimize the use of concrete and improve drainage. It is considered more environmentally friendly than other types of foundation because cement manufacturing requires the use of enormous amounts of energy. However, some soil environments (such as particularly expansive or poor load-bearing (< 1 ton/sf) soils) are not suitable for this kind of foundation.

A foundation must bear the structural loads imposed upon it and allow proper drainage of ground water to prevent expansion or weakening of soils and frost heaving. While the far more common concrete foundation requires separate measures to ensure good soil drainage, the rubble trench foundation serves both foundation functions at once.

To construct a rubble trench foundation a narrow trench is dug down below the frost line. The bottom of the trench would ideally be gently sloped to an outlet. Drainage tile, graded 1":8' to daylight, is then placed at the bottom of the trench in a bed of washed stone protected by filter fabric. The trench is then filled with either screened stone (typically 1-1/2") or recycled rubble. A steel-reinforced concrete grade beam is poured at the surface to provide ground clearance for the structure.

If an insulated slab is to be poured inside the grade beam, then the outer surface of the grade beam and the rubble trench should be insulated with rigid XPS foam board, which must be protected above grade from mechanical and UV degradation.

The rubble-trench foundation is a relatively simple, low-cost, and environmentally-friendly alternative to a conventional foundation, but may require an engineer's approval if building officials are not familiar with it. Frank Lloyd Wright used them successfully

for more than 50 years in the first half of the 20th century, and there is a revival of this style of foundation with the increased interest in green building.

Earthbag foundation

The basic construction method begins by digging a trench down to undisturbed mineral subsoil. Rows of woven bags (or tubes) are filled with available material, placed into this trench, compacted with a pounder to around 1/3 thickness of pre-pounded thickness, and form a foundation. Each successive layer will have one or more strands of barbed wire placed on top. This digs into the bag's weave and prevents slippage of subsequent layers, and also resists any tendency for the outward expansion of walls. The next row of bags is offset by half a bag's width to form a staggered pattern. These are either pre-filled with material and delivered, or filled in place (often the case with Superadobe). The weight of this earth-filled bag pushes down on the barbed wire strands, locking the bag in place on the row below. The same process continues layer upon layer, forming walls. A roof can be formed by gradually sloping the walls inward to construct a dome. Traditional types of roof can also be made.

Thin-shell Structure

Shell structure of the TWA Flight Center Building by Eero Saarinen, John F. Kennedy International Airport, New York

The Montreal Biosphère by Buckminster Fuller, 1967

Lattice Shell of the Shukhov Hyperboloid Tower. Currently under threat of demolition.

Great Court, with a lattice thin-shell roof by Buro Happold with Norman Foster, British Museum, London

Thin-shell structures are light weight constructions using shell elements. These elements are typically curved and are assembled to large structures. Typical applications are fuselages of aeroplanes, boat hulls and roof structures in some buildings.

A thin shell is defined as a shell with a thickness which is small compared to its other dimensions and in which deformations are not large compared to thickness. A primary difference between a shell structure and a plate structure is that, in the unstressed state, the shell structure has curvature as opposed to plates structures which are flat. Membrane action in a shell is primarily caused by in-plane forces (plane stress), though there may be secondary forces resulting from flexural deformations. Where a flat plate acts similar to a

beam with bending and shear stresses, shells are analogous to a cable which resists loads through tensile stresses. Though the ideal thin shell must be capable of developing both tension and compression.

Chapter- 8

Truss

In architecture and structural engineering, a **truss** is a structure comprising one or more triangular units constructed with straight members whose ends are connected at joints referred to as nodes. External forces and reactions to those forces are considered to act only at the nodes and result in forces in the members which are either tensile or compressive forces. Moments (torques) are explicitly excluded because, and only because, all the joints in a truss are treated as revolutes.

A planar truss is one where all the members and nodes lie within a two dimensional plane, while a space truss has members and nodes extending into three dimensions.

Truss bridge for a single track railway, converted to pedestrian use and pipeline support

Characteristics of trusses

A truss is composed of triangles because of the structural stability of that shape and design. A triangle is the simplest geometric figure that will not change shape when the lengths of the sides are fixed. In comparison, both the angles and the lengths of a four-sided figure must be fixed for it to retain its shape.

Planar truss

Planar roof trusses

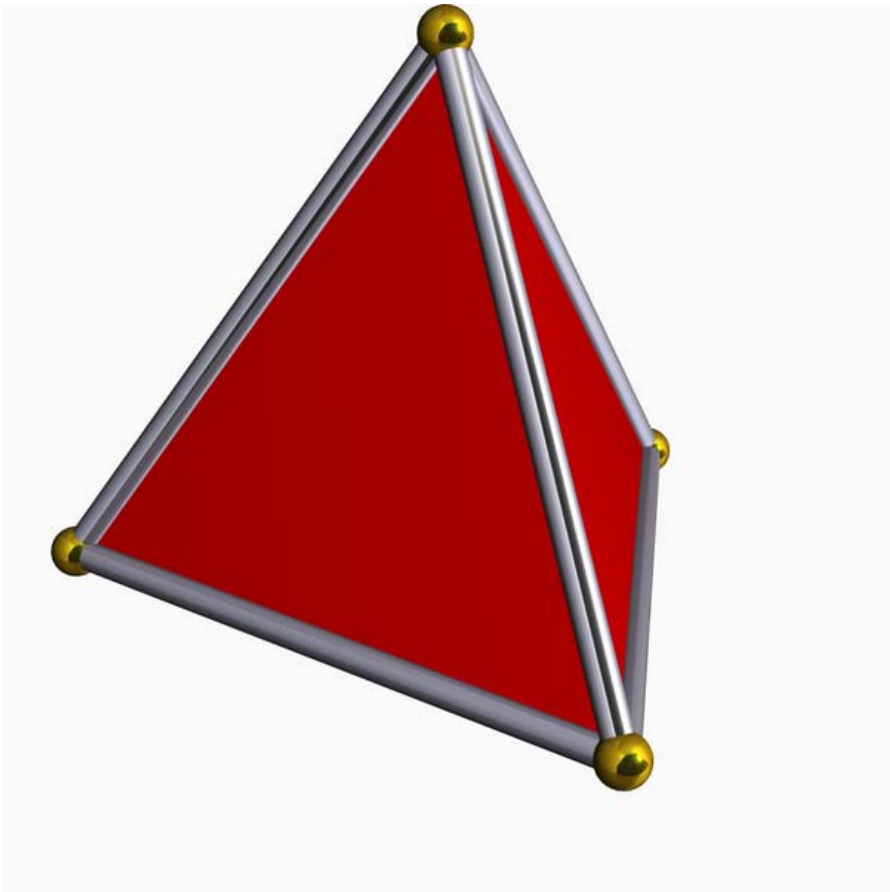
The simplest form of a truss is one single triangle. This type of truss is seen in a framed roof consisting of rafters and a ceiling joist. Because of the stability of this shape and the methods of analysis used to calculate the forces within it, a truss composed entirely of triangles is known as a simple truss.

A planar truss lies in a single plane. Planar trusses are typically used in parallel to form roofs and bridges.

The depth of a truss, or the height between the upper and lower chords, is what makes it an efficient structural form. A solid girder or beam of equal strength would have substantial weight and material cost as compared to a truss. For a given span length, a deeper truss will require less material in the chords and greater material in the verticals and diagonals. An optimum depth of the truss will maximize the efficiency.

Space frame truss

A space frame truss is a three-dimensional framework of members pinned at their ends. A tetrahedron shape is the simplest space truss, consisting of six members which meet at four joints. Large planar structures may be composed from tetrahedrons with common edges and they are also employed in the base structures of large free-standing power line pylons



Simple tetrahedron

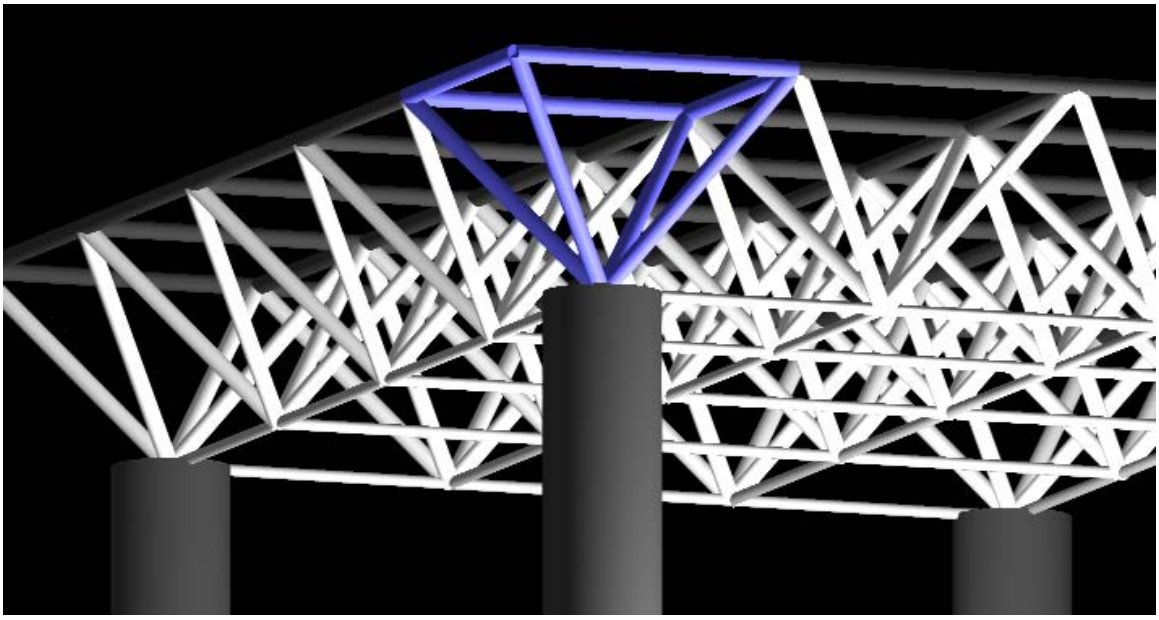


Diagram of a planar space frame such as used for a roof

Four tetrahedons form each of the two lower base structures of this power pylon

Truss types

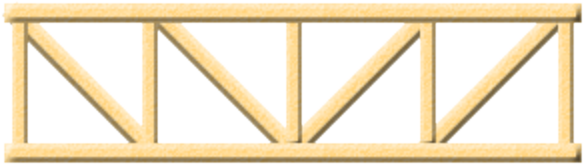
A large timber Howe truss in a commercial building

There are two basic types of truss:

- The *pitched truss*, or *common truss*, is characterized by its triangular shape. It is most often used for roof construction. Some common trusses are named according to their *web configuration*. The chord size and web configuration are determined by *span*, *load* and *spacing*.
- The *parallel chord truss*, or *flat truss*, gets its name from its parallel top and bottom chords. It is often used for floor construction.

A combination of the two is a truncated truss, used in hip roof construction. A metal plate-connected wood truss is a roof or floor truss whose wood members are connected with metal connector plates.

Pratt truss



The **Pratt truss** was patented in 1844 by two Boston railway engineers, Caleb Pratt and his son Thomas Willis Pratt. The design uses vertical members for compression and horizontal members to respond to tension. What is remarkable about this style is that it remained popular even as wood gave way to iron, and even still as iron gave way to steel.. The continued popularity of the Pratt truss is probably due to the fact that the configuration of the members means that longer diagonal members are only in tension for gravity load effects. This allows these members to be used more efficiently, as slenderness effects related to buckling under compression loads (which are compounded by the length of the member) will typically not control the design. Therefore, for given planar truss with a fixed depth, the Pratt configuration is usually the most efficient under static, vertical loading.

The Southern Pacific Railroad bridge in Tempe, Arizona is a 393 meter (1,291 foot) long truss bridge built in 1912. The structure is composed of nine Pratt truss spans of varying lengths. The bridge is still in use today.

Bowstring truss

Named for their shape, bowstring trusses were first used for arched truss bridges, known as tied-arch bridges.

Thousands of bowstring trusses were used during World War II for holding up the curved roofs of aircraft hangars and other military buildings. Many variations exist in the arrangements of the members connecting the nodes of the upper arc with those of the lower, straight sequence of members, from nearly isosceles triangles to a variant of the Platt truss.

King post truss

One of the simplest truss styles to implement, the **king post** consists of two angled supports leaning into a common vertical support.

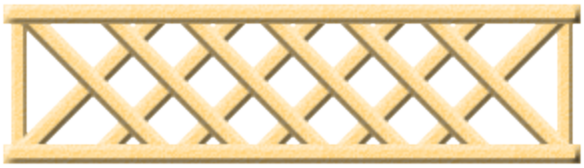
The **queen post** truss, sometimes *queenpost* or *queenspost*, is similar to a king post truss in that the outer supports are angled towards the center of the structure. The primary difference is the horizontal extension at the centre which relies on beam action to provide mechanical stability. This truss style is only suitable for relatively short spans.

Lenticular truss

The Waterville Bridge in Swatara State Park in Pennsylvania is a lenticular truss

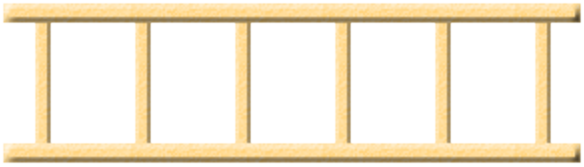
Lenticular trusses, patented in 1878 by William Douglas, have the top and bottom chords of the truss arched, forming a lens shape. A lenticular pony truss bridge is a bridge design that involves a lenticular truss extending above and below the roadbed.

Town's lattice truss



American architect Ithiel Town designed **Town's Lattice Truss** as an alternative to heavy-timber bridges. His design, patented in 1820 and 1835, uses easy-to-handle planks arranged diagonally with short spaces in between them.

Vierendeel truss



A Vierendeel bridge; note the lack of diagonal elements in the primary structure and the way bending loads are carried between elements

The **Vierendeel truss** is a truss where the members are not triangulated but form rectangular openings, and is a frame with fixed joints that are capable of transferring and resisting bending moments. Regular trusses comprise members that are commonly assumed to have pinned joints, with the implication that no moments exist at the jointed ends. This style of truss was named after the Belgian engineer Arthur Vierendeel, who developed the design in 1896. Its use for bridges is rare due to higher costs compared to a triangulated truss.

The utility of this type of truss in buildings is that a large amount of the exterior envelope remains unobstructed and can be used for fenestration and door openings. This is preferable to a braced-frame system, which would leave some areas obstructed by the diagonal braces.

Statics of trusses

A truss that is assumed to comprise members that are connected by means of pin joints, and which is supported at both ends by means of hinged joints or rollers, is described as being statically determinate. Newton's Laws apply to the structure as a whole, as well as to each node or joint. In order for any node that may be subject to an external load or force to remain static in space, the following conditions must hold: the sums of all (horizontal and vertical) forces, as well as all moments acting about the node equal zero. Analysis of these conditions at each node yields the magnitude of the forces in each member of the truss. These may be compression or tension forces.

Trusses that are supported at more than two positions are said to be statically indeterminate, and the application of Newton's Laws alone is not sufficient to determine the member forces.

In order for a truss with pin-connected members to be stable, it must be entirely composed of triangles. In mathematical terms, we have the following necessary condition for stability:

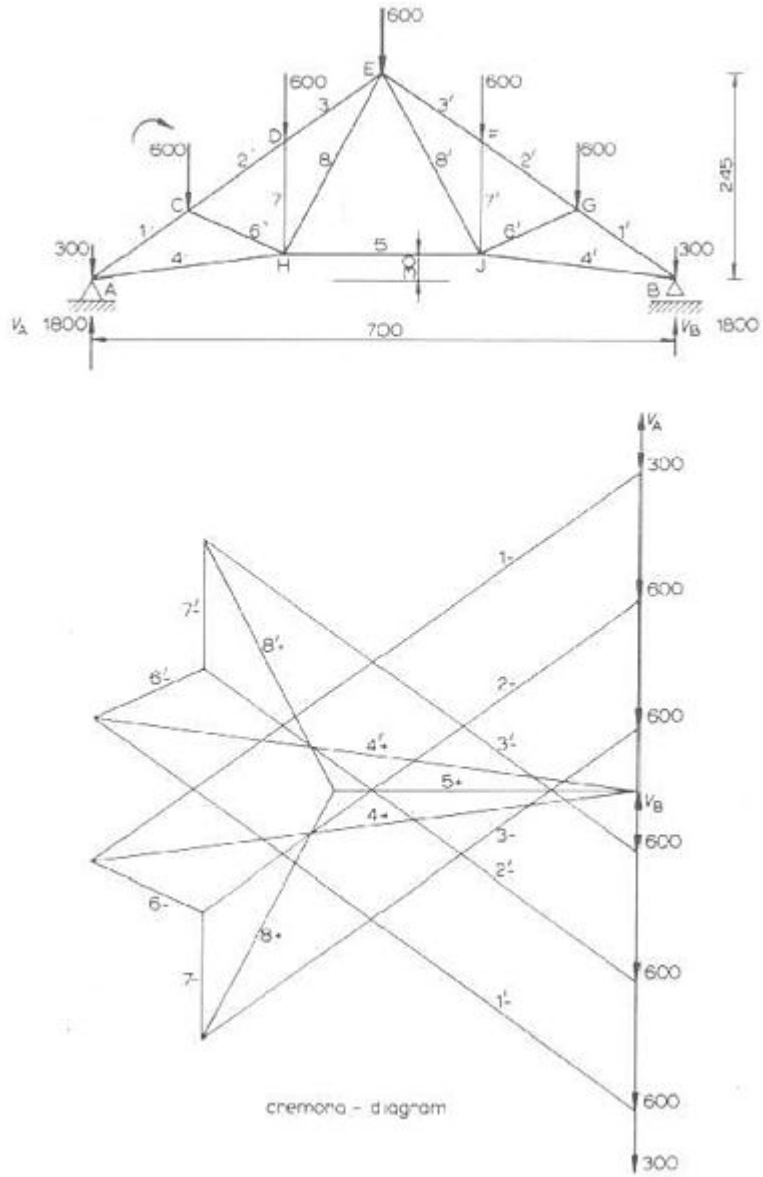
$$m \geq 2j - r \qquad \text{(a)}$$

where m is the total number of truss members, j is the total number of joints and r is the number of reactions (equal to 3 generally) in a 2-dimensional structure.

When $m = 2j - 3$, the truss is said to be *statically determinate*, because the (m+3) internal member forces and support reactions can then be completely determined by $2j$ equilibrium equations, once we know the external loads and the geometry of the truss. Given a certain number of joints, this is the minimum number of members, in the sense that if any member is taken out (or fails), then the truss as a whole fails. While the relation (a) is necessary, it is not sufficient for stability, which also depends on the truss geometry, support conditions and the load carrying capacity of the members.

Some structures are built with more than this minimum number of truss members. Those structures may survive even when some of the members fail. Their member forces depend on the relative stiffness of the members, in addition to the equilibrium condition described.

Analysis of trusses



Cremona diagram for a plane truss

Because the forces in each of its two main girders are essentially planar, a truss is usually modelled as a two-dimensional plane frame. If there are significant out-of-plane forces, the structure must be modelled as a three-dimensional space.

The analysis of trusses often assumes that loads are applied to joints only and not at intermediate points along the members. The weight of the members is often insignificant compared to the applied loads and so is often omitted. If required, half of the weight of each member may be applied to its two end joints. Provided the members are long and slender, the moments transmitted through the joints are negligible and they can be treated as "hinges" or 'pin-joints'. Every member of the truss is then in pure compression or pure tension – shear, bending moment, and other more complex stresses are all practically zero. This makes trusses easier to analyze. This also makes trusses physically stronger than other ways of arranging material – because nearly every material can hold a much larger load in tension and compression than in shear, bending, torsion, or other kinds of force.

Structural analysis of trusses of any type can readily be carried out using a matrix method such as the direct stiffness method, the flexibility method or the finite element method.

Forces in members

On the right is a simple, statically determinate flat truss with 9 joints and $(2 \times 9) - 3 = 15$ members. External loads are concentrated in the outer joints. Since this is a symmetrical truss with symmetrical vertical loads, it is clear to see that the reactions at A and B are equal, vertical and half the total load.

The internal forces in the members of the truss can be calculated in a variety of ways including the graphical methods:

- *Cremona diagram*
- *Culmann diagram*
- *the analytical* Ritter *method (method of sections).*

Design of members

A truss can be thought of as a beam where the web consists of a series of separate members instead of a continuous plate. In the truss, the lower horizontal member (the **bottom chord**) and the upper horizontal member (the **top chord**) carry tension and compression, fulfilling the same function as the flanges of an I-beam. Which chord carries tension and which carries compression depends on the overall direction of bending. In the truss pictured above right, the bottom chord is in tension, and the top chord in compression.

The diagonal and vertical members form the **truss web**, and carry the shear force. Individually, they are also in tension and compression, the exact arrangement of forces is depending on the type of truss and again on the direction of bending. In the truss shown above right, the vertical members are in tension, and the diagonals are in compression.

A building under construction in Shanghai. The truss sections stabilize the building and will house mechanical floors.

In addition to carrying the static forces, the members serve additional functions of stabilizing each other, preventing buckling. In the picture to the right, the top chord is prevented from buckling by the presence of bracing and by the stiffness of the web members.

The inclusion of the elements shown is largely an engineering decision based upon economics, being a balance between the costs of raw materials, off-site fabrication, component transportation, on-site erection, the availability of machinery and the cost of labor. In other cases the appearance of the structure may take on greater importance and so influence the design decisions beyond mere matters of economics. Modern materials such as prestressed concrete and fabrication methods, such as automated welding, have significantly influenced the design of modern bridges.

Once the force on each member is known, the next step is to determine the cross section of the individual truss members. For members under tension the cross-sectional area A can be found using $A = F \times \gamma / \sigma_y$, where F is the force in the member, γ is a safety factor (typically 1.5 but depending on building codes) and σ_y is the yield tensile strength of the steel used.
The members under compression also have to be designed to be safe against buckling.

The weight of a truss member depends directly on its cross section—that weight partially determines how strong the other members of the truss need to be. Giving one member a larger cross section than on a previous iteration requires giving other members a larger cross section as well, to hold the greater weight of the first member—one needs to go through another iteration to find exactly how much greater the other members need to be. Sometimes the designer goes through several iterations of the design process to converge on the "right" cross section for each member. On the other hand, reducing the size of one member from the previous iteration merely makes the other members have a larger (and more expensive) safety factor than is technically necessary, but doesn't *require* another iteration to find a buildable truss.

The effect of the weight of the individual truss members in a large truss, such as a bridge, is usually insignificant compared to the force of the external loads.

Design of joints

After determining the minimum cross section of the members, the last step in the design of a truss would be detailing of the bolted joints, e.g., involving shear of the bolt connections used in the joints.

Applications

Post frame structures

Component connections are critical to the structural integrity of a framing system. In buildings with large, clearspan wood trusses, the most critical connections are those between the truss and its supports. In addition to gravity-induced forces (a.k.a. bearing loads), these connections must resist shear forces acting perpendicular to the plane of the truss and uplift forces due to wind. Depending upon overall building design, the connections may also be required to transfer bending moment.

Wood posts enable the fabrication of strong, direct, yet inexpensive connections between large trusses and walls. Exact details for post-to-truss connections vary from designer to designer, and may be influenced by post type. Solid-sawn timber and glulam posts are generally notched to form a truss bearing surface. The truss is rested on the notches and bolted into place. A special plate/bracket may be added to increase connection load transfer capabilities. With mechanically-laminated posts, the truss may rest on a shortened outer-ply or on a shortened inner-ply. The later scenario places the bolts in double shear and is a very effective connection.

The HSBC Main Building, Hong Kong has an externally visible truss structure

Support structure under the Auckland Harbour Bridge

Little Belt: a truss bridge in Denmark

Chapter- 9

Introduction to Earthquake Engineering

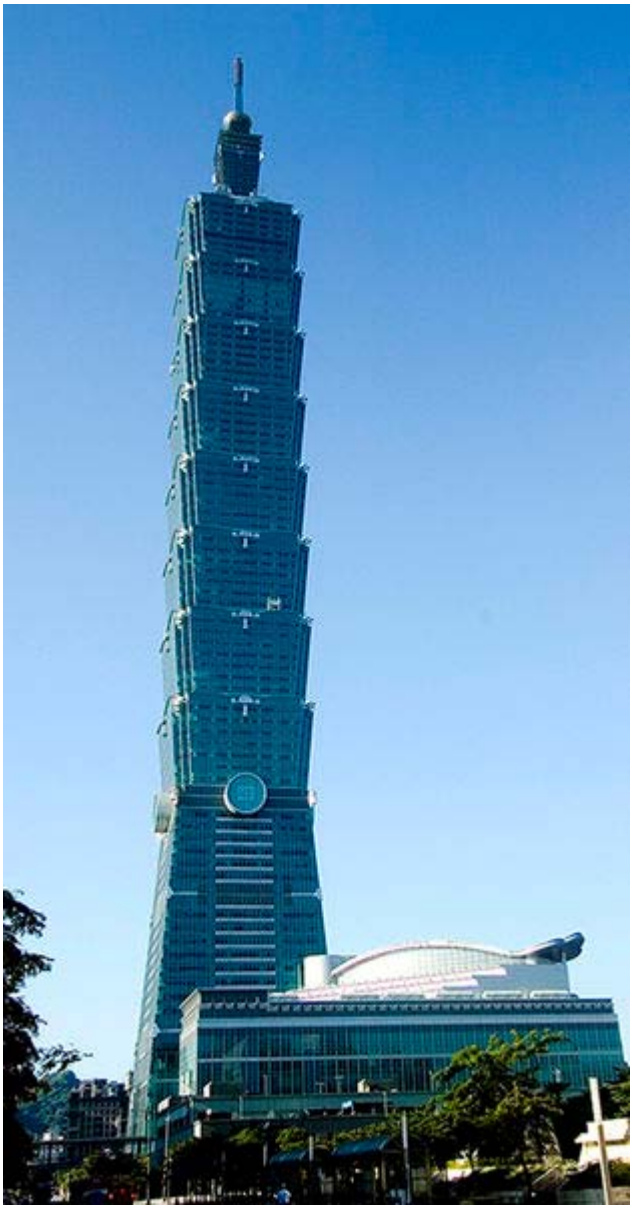
Earthquake engineering is the study of the behavior of buildings and structures subject to seismic loading. It is a subset of both structural and civil engineering.

The main objectives of earthquake engineering are:

- Understand the interaction between buildings or civil infrastructure and the ground.
- Foresee the potential consequences of strong earthquakes on urban areas and civil infrastructure.
- Design, construct and maintain structures to perform at earthquake exposure up to the expectations and in compliance with building codes.

A properly engineered structure does not necessarily have to be extremely strong or expensive.

Shake-table crash testing of a regular building model (left) and a base-isolated building model (right) at UCSD



Taipei 101, equipped with a tuned mass damper, is the world's second tallest skyscraper.

The most powerful and budgetary tools of earthquake engineering are vibration control technologies and, in particular, base isolation.

Seismic loadings

Seismic loading means application of an earthquake-generated agitation to a structure. It happens at contact surfaces of a structure either with the ground, or with adjacent structures, or with gravity waves from tsunami. Seismic loading depends, primarily, on:

The Last Day of Pompeii

- Anticipated earthquake's parameters at the site
- Geotechnical parameters of the site
- Structure's parameters
- Characteristics of the anticipated gravity waves from tsunami (if applicable).

Ancient builders believed that earthquakes were a result of wrath of gods (in Greek mythology, e.g., the main "Earth-Shaker" was Poseidon) and, therefore, could not be resisted by humans. Nowadays, the people's attitude has changed dramatically though the seismic loads, sometimes, exceed ability of a structure to resist them without being broken, partially or completely.

Due to their mutual interaction, seismic loading and seismic performance of a structure are intimately related.

Seismic performance evaluation

Engineers need to know the quantified level of an actual or anticipated seismic performance associated with the direct damage to an individual building subject to a specified ground shaking.

The best way to do it is to put the structure on a shake-table that simulates the earth shaking and watch what may happen next. Such kinds of experiments were performed still more than a century ago

Shake-table of a 6-story non-ductile concrete building destructive testing

Another way is to evaluate the earthquake performance analytically.

Seismic performance analysis

Seismic performance analysis or, simply, seismic analysis is a major intellectual tool of earthquake engineering which breaks the complex topic into smaller parts to gain a better understanding of seismic performance of building and non-building structures. The technique as a formal concept is a relatively recent development.

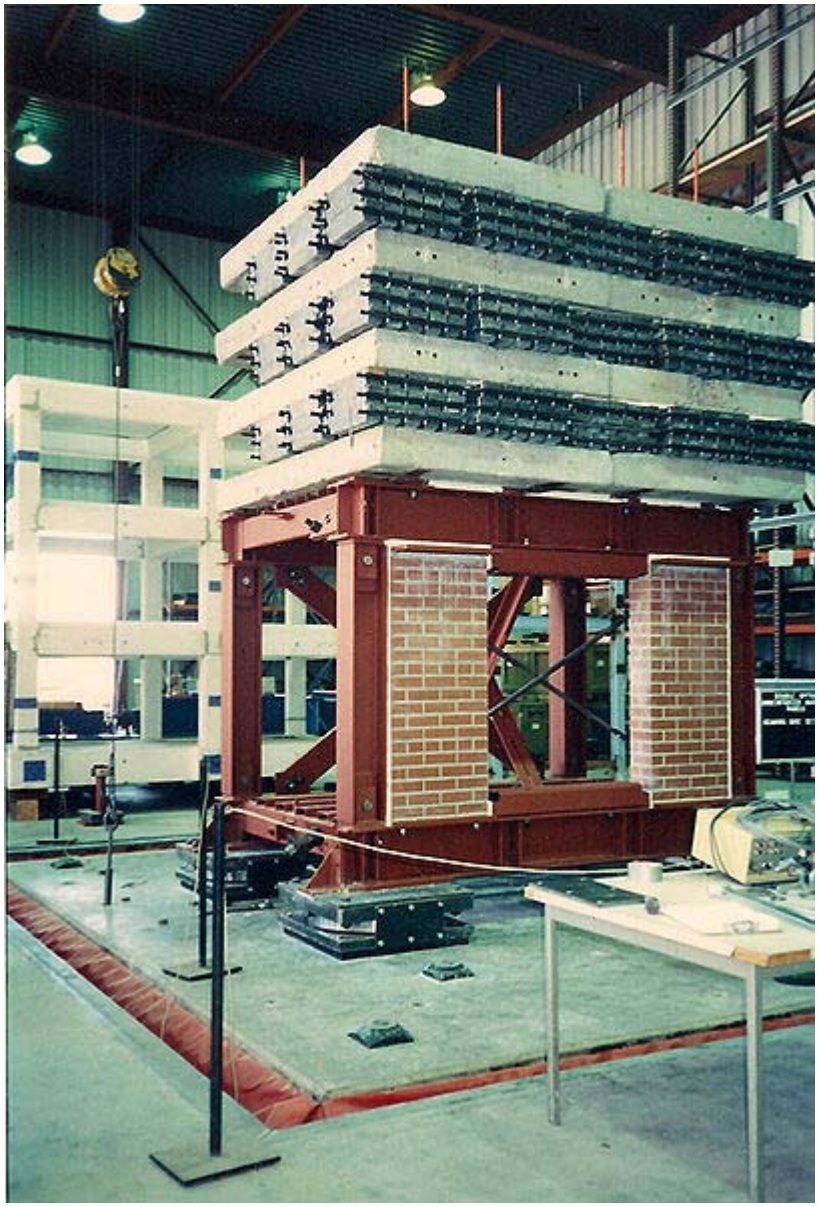
In general, seismic analysis is based on the methods of structural dynamics. For decades, the most prominent instrument of seismic analysis has been the earthquake response spectrum method which, also, contributed to the proposed building code's concept of today.

However, those spectra are good, mostly, for single-degree-of-freedom systems. Numerical *step-by-step integration* proved to be a more effective method of analysis for multi-degree-of-freedom structural systems with several non-linearity under a substantially transient process of kinematic excitation.

Basically, numerical analysis is conducted in order to evaluate the seismic performance of buildings. Performance evaluations is generally carried out by using nonlinear static pushover analysis or full direct nonlinear time history analysis. In these analysis process, first nonlinear modeling of building components such as beams, columns, beam-column joints, shear walls etc. are required. It is essential that nonlinear response of these components should be validated by the experimental results in order to ensure the accuracy of the analysis. After validating all components of buildings with experimental results, these components are assemble to create a full nonlinear model of the structure. Thus created model are analyzed to evaluate the performance of buildings.

One of the important thing in seismic evaluation of the building is also the computational platform. There are several commercially available Finite Element Analysis software's such as CSI-SAP2000/CSI-PERFORM-3D / CSI-ETABS which can be used to for seismic performance evaluation of buildings. Moreover, there are research based finite element analysis software such as OpenSees(Open Source)/RUAUMOKO/DRAIN-3D for the seismic evaluation of the buildings.

Research for earthquake engineering



Shake-table testing of Friction Pendulum Bearings at EERC

Research for earthquake engineering means both field and analytical investigation or experimentation intended for discovery and scientific explanation of earthquake engineering related facts, revision of conventional concepts in the light of new findings, and practical application of the developed theories. The National Science Foundation (NSF) is the main United States government agency that supports fundamental research and education in all fields of earthquake engineering. In particular, it focuses on experimental, analytical, and computational research on design and performance enhancement of structural systems.

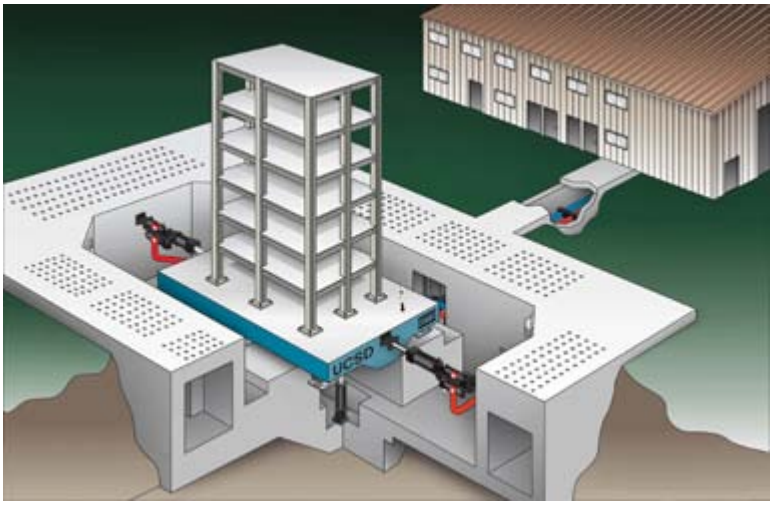
E-Defense Shake Table

The Earthquake Engineering Research Institute (EERI) is a leader in dissemination of earthquake engineering research related information both in the U.S. and globally.

A definitive list of earthquake engineering research related shaking tables around the world may be found in Experimental Facilities for Earthquake Engineering Simulation Worldwide. The most prominent of them is now E-Defense Shake Table in Japan.

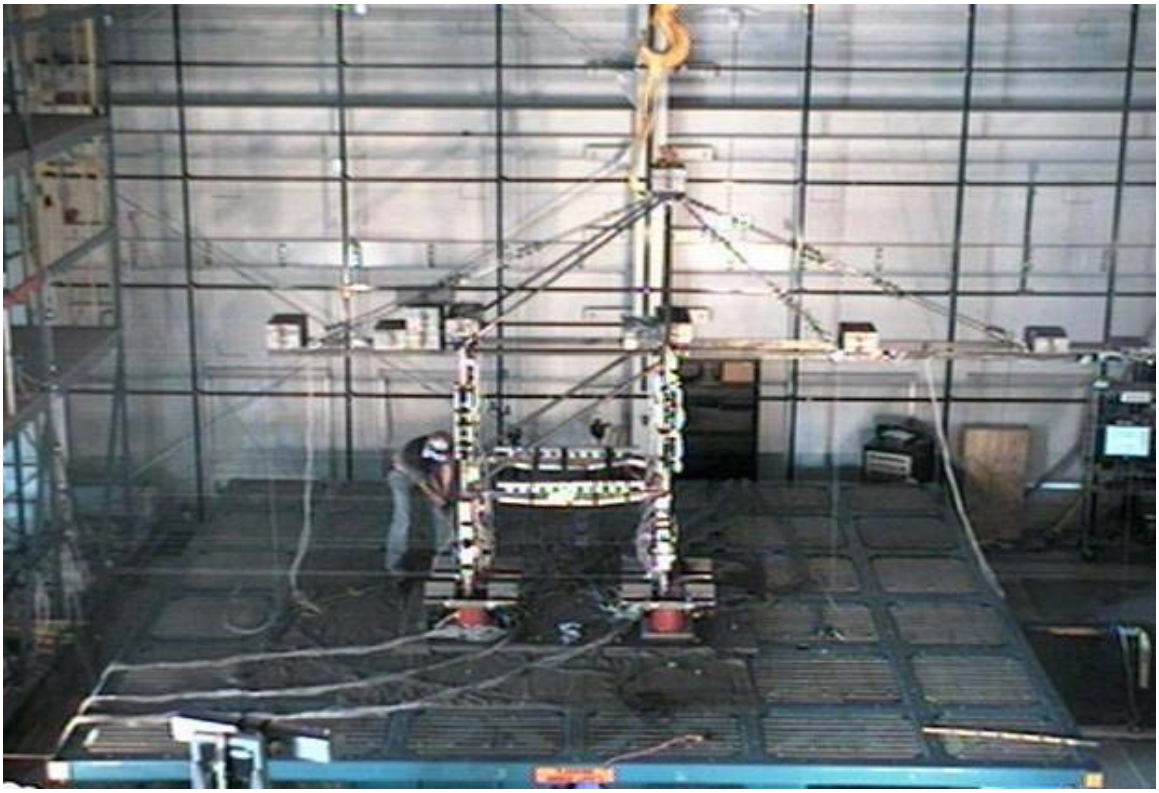
Major U.S. research programs

The NSF Hazard Mitigation and Structural Engineering program (HMSE) supports research on new technologies for improving the behavior and response of structural systems subject to earthquake hazards; fundamental research on safety and reliability of constructed systems; innovative developments in analysis and model based simulation of structural behavior and response including soil-structure interaction; design concepts that improve structure performance and flexibility; and application of new control techniques for structural systems.



Large High Performance Outdoor Shake Table, UCSD, NEES network

NSF also supports George E. Brown, Jr. Network for Earthquake Engineering Simulation (NEES) that advances knowledge discovery and innovation for earthquakes and tsunami loss reduction of the nation's civil infrastructure, and new experimental simulation techniques and instrumentation.



NEES@Buffalo testing facility

The NEES network features 14 geographically-distributed, shared-use laboratories that support several types of experimental work: geotechnical centrifuge research, shake-table tests, large-scale structural testing, tsunami wave basin experiments, and field site research. Participating universities include: Cornell University; Lehigh University; Oregon State University; Rensselaer Polytechnic Institute; University at Buffalo, SUNY; University of California, Berkeley; University of California, Davis; University of California, Los Angeles; University of California, San Diego; University of California, Santa Barbara; University of Illinois, Urbana-Champaign; University of Minnesota; University of Nevada, Reno; and the University of Texas, Austin.

The equipment sites (labs) and a central data repository are connected to the global earthquake engineering community via the NEEShub, a website that it more than a website. The NEES website is powered by HUBzero software developed at Purdue University specifically to help the scientific community share resources and collaborate. The cyberinfrastructure, connected via Internet2, provides interactive simulation tools, a simulation tool development area, a curated central data repository, animated presentations, user support, telepresence, mechanism for uploading and sharing resources and statistics about users, and usage patterns.

This cyberinfrastructure allows researchers to: securely store, organize and share data within a standardized framework in a central location; remotely observe and participate in experiments through the use of synchronized real-time data and video; collaborate with colleagues to facilitate the planning, performance, analysis, and publication of research experiments; and conduct computational and hybrid simulations that may combine the results of multiple distributed experiments and link physical experiments with computer simulations to enable the investigation of overall system performance.

These resources jointly provide the means for collaboration and discovery to improve the seismic design and performance of civil and mechanical infrastructure systems.

Earthquake simulation

The very first **earthquake simulations** were performed by statically applying some *horizontal inertia forces* based on scaled peak ground accelerations to a mathematical model of a building. With the further development of computational technologies, static approaches began to give way to dynamic ones.

Dynamic experiments on building and non-building structures may be physical, like shake-table testing, or virtual ones. In both cases, to verify a structure's expected seismic performance, some researchers prefer to deal with so called "real time-histories" though the last cannot be "real" for a hypothetical earthquake specified by either a building code or by some particular research requirements. Therefore, there is a strong incentive to engage an earthquake simulation which is the seismic input that possesses only essential features of a real event.

Sometimes, earthquake simulation is understood as a re-creation of local effects of a strong earth shaking.

Structure simulation

Theoretical or experimental evaluation of anticipated seismic performance mostly requires a **structure simulation** which is based on the concept of structural likeness or similarity. Similarity is some degree of analogy or resemblance between two or more objects. The notion of similarity rests either on exact or approximate repetitions of patterns in the compared items.

Concurrent experiments with two building models which are *kinematically equivalent* to a real prototype.

In general, a building model is said to have similarity with the real object if the two share *geometric similarity*, *kinematic similarity* and *dynamic similarity*. The most vivid and effective type of similarity is the *kinematic* one. *Kinematic similarity* exists when the paths and velocities of moving particles of a model and its prototype are similar.

The ultimate level of *kinematic similarity* is *kinematic equivalence* when, in the case of earthquake engineering, time-histories of each story lateral displacements of the model and its prototype would be the same.

Seismic vibration control

Seismic vibration control is a set of technical means aimed to mitigate seismic impacts in building and non-building structures. All seismic vibration control devices may be classified as *passive*, *active* or *hybrid* where:

- *passive control devices* have no feedback capability between them, structural elements and the ground;
- *active control devices* incorporate real-time recording instrumentation on the ground integrated with earthquake input processing equipment and actuators within the structure;
- *hybrid control devices* have combined features of active and passive control systems.

When ground seismic waves reach up and start to penetrate a base of a building, their energy flow density, due to reflections, reduces dramatically: usually, up to 90%. However, the remaining portions of the incident waves during a major earthquake still bear a huge devastating potential.

After the seismic waves enter a superstructure, there are a number of ways to control them in order to soothe their damaging effect and improve the building's seismic performance, for instance:

- to dissipate the wave energy inside a superstructure with properly engineered dampers;

- to disperse the wave energy between a wider range of frequencies;

- to absorb the resonant portions of the whole wave frequencies band with the help of so called *mass dampers*.

Mausoleum of Cyrus, the oldest base-isolated structure in the world

Devices of the last kind, abbreviated correspondingly as TMD for the tuned (*passive*), as AMD for the *active*, and as HMD for the *hybrid mass dampers*, have been studied and installed in high-rise buildings, predominantly in Japan, for a quarter of a century.

However, there is quite another approach: partial suppression of the seismic energy flow into the superstructure known as seismic or base isolation.

For this, some pads are inserted into or under all major load-carrying elements in the base of the building which should substantially decouple a superstructure from its substructure resting on a shaking ground.

The first evidence of earthquake protection by using the principle of base isolation was discovered in Pasargadae, a city in ancient Persia, now Iran: it goes back to 6th century BCE. Below, there are some samples of seismic vibration control technologies of today.

Dry-stone walls control

Dry-stone walls of Machu Picchu Temple of the Sun, Peru

People of Inca civilization were masters of the polished **dry-stone walls**, called ashlar, where blocks of stone were cut to fit together tightly without any mortar. The Incas were among the best stone masons the world has ever seen and many junctions in their masonry were so perfect that even blades of grass could not fit between the stones.

Peru is a highly seismic land, and for centuries the mortar-free construction proved to be apparently more earthquake-resistant than using mortar. The stones of the dry-stone walls built by the Incas could move slightly and resettle without the walls collapsing which should be recognized as an ingenious passive structural control technique employing both the principle of energy dissipation and that of suppressing resonant amplifications.

Lead Rubber Bearing

LRB being tested at the UCSD Caltrans-SRMD facility

Lead Rubber Bearing or LRB is a type of base isolation employing a heavy damping. It was invented by Bill Robinson, a New Zealander.

Heavy damping mechanism incorporated in vibration control technologies and, particularly, in base isolation devices, is often considered a valuable source of suppressing vibrations thus enhancing a building's seismic performance. However, for the rather pliant systems such as base isolated structures, with a relatively low bearing stiffness but with a high damping, the so-called "damping force" may turn out the main pushing force at a strong earthquake. The video shows a Lead Rubber Bearing being tested at the UCSD Caltrans-SRMD facility. The bearing is made of rubber with a lead core. It was a uniaxial test in which the bearing was also under a full structure load. Many buildings and bridges, both in New Zealand and elsewhere, are protected with lead dampers and lead and rubber bearings. Te Papa Tongarewa, the national museum of New

Zealand, and the New Zealand Parliament Buildings have been fitted with the bearings. Both are in Wellington, which sits on an active earthquake fault.

Friction pendulum bearing

FPB shake-table testing

Friction Pendulum Bearing (FPB) is another name of **Friction Pendulum System** (FPS). It is based on three pillars:

- articulated friction slider;
- spherical concave sliding surface;
- enclosing cylinder for lateral displacement restraint.

Snapshot with the link to video clip of a shake-table testing of FPB system supporting a rigid building model is presented at the right.

Building elevation control

Transamerica Pyramid building

Building elevation control is a valuable source of vibration control of seismic loading. Pyramid-shaped skyscrapers continue to attract attention of architects and engineers because such structures promise a better stability against earthquakes and winds. The elevation configuration can prevent buildings' resonant amplifications because a properly configured building disperses the shear wave energy between a wide range of frequencies.

Earthquake or wind quieting ability of the elevation configuration is provided by a specific pattern of multiple reflections and transmissions of vertically propagating shear waves, which are generated by breakdowns into homogeneity of story layers, and a taper.

Any abrupt changes of the propagating waves velocity result in a considerable dispersion of the wave energy between a wide ranges of frequencies thus preventing the resonant displacement amplifications in the building.

A tapered profile of a building is not a compulsory feature of this method of structural control. A similar resonance preventing effect can be also obtained by a proper *tapering* of other characteristics of a building structure, namely, its mass and stiffness. As a result, the building elevation configuration techniques permit an architectural design that may be both attractive and functional (see, e.g., Pyramid).

Simple roller bearing

Simple roller bearing is a base isolation device which is intended for protection of various building and non-building structures against potentially damaging lateral impacts of strong earthquakes.

This metallic bearing support may be adapted, with certain precautions, as a seismic isolator to skyscrapers and buildings on soft ground. Recently, it has been employed under the name of *Metallic Roller Bearing* for a housing complex (17 stories) in Tokyo, Japan.

Springs-with-damper base isolator

Springs-with-damper close-up

Springs-with-damper base isolator installed under a three-story town-house, Santa Monica, California is shown on the photo taken prior to the 1994 Northridge earthquake exposure. It is a base isolation device conceptually similar to *Lead Rubber Bearing*.

One of two three-story town-houses like this, which was well instrumented for recording of both vertical and horizontal accelerations on its floors and the ground, has survived a severe shaking during the Northridge earthquake and left valuable recorded information for further study.

Hysteretic damper

Hysteretic damper is intended to provide better and more reliable seismic performance than that of a conventional structure at the expense of the seismic input energy dissipation. There are four major groups of hysteretic dampers used for the purpose, namely:

- Fluid viscous dampers (FVDs)
- Metallic yielding dampers (MYDs)
- Viscoelastic dampers (VEDs)
- Friction dampers (FDs)

Each group of dampers has specific characteristics, advantages and disadvantages for structural applications.

Seismic design

Seismic design is based on authorized engineering procedures, principles and criteria meant to design or retrofit structures subject to earthquake exposure. Those criteria are consistent just with the contemporary state of the knowledge about earthquake engineering structures. Therefore, the building design which blindly follows some seismic code regulations does not guarantee safety against collapse or serious damage.

The price of poor seismic design may be enormous. Nevertheless, seismic design has always been a trial and error process no matter it was based upon physical laws or empirical knowledge of the structural performance of different shapes and materials.

Ruin of the $7,000,000 poorly designed San Francisco City Hall by 1906 earthquake and fire

To practice seismic design, seismic analysis or seismic evaluation of new and existing civil engineering projects, an engineer should, normally, pass examination on *Seismic Principles* which, e.g. in the State of California, include:

- Seismic Data and Seismic Design Criteria
- Seismic Characteristics of Engineered Systems
- Seismic Forces
- Seismic Analysis Procedures
- Seismic Detailing and Construction Quality Control

San Francisco after the 1906 earthquake and fire

To build up complex structural systems, seismic design utilizes, mostly, the same relatively small number of basic structural elements (to say nothing of vibration control devices) as any non-seismic design project.

Normally, according to building codes, structures are designed to "withstand" the largest earthquake of a certain probability that is likely to occur at their location. This means the loss of life should be minimized by preventing collapse of the buildings.

Seismic design is carried out by understanding the possible failure modes of a structure and providing the structure with appropriate strength, stiffness, ductility, and configuration to ensure those modes cannot occur.

Seismic design requirements

Seismic design requirements depend on the type of the structure, locality of the project and its authorities which stipulate applicable seismic design codes and criteria. For instance, California Department of Transportation's requirements called *The Seismic Design Criteria* (SDC) and aimed at the design of new bridges in California incorporate an innovative seismic performance based approach.

Metsamor, Armenia nuclear power plant was closed after the 1988 destructive earthquake

The most significant feature in the SDC design philosophy is a shift from a *force-based assessment* of seismic demand to a *displacement-based assessment* of demand and capacity. Thus, the newly adopted displacement approach is based on comparing the *elastic displacement* demand to the *inelastic displacement* capacity of the primary structural components while ensuring a minimum level of inelastic capacity at all potential plastic hinge locations.

In addition to the designed structure itself, seismic design requirements may include a *ground stabilization* underneath the structure: sometimes, heavily shaken ground breaks up which leads to collapse of the structure sitting upon it. The following topics should be of primary concerns: liquefaction; dynamic lateral earth pressures on retaining walls; seismic slope stability; earthquake-induced settlement.

Nuclear facilities should not jeopardise their safety in case of earthquakes or other hostile external events. Therefore, their seismic design is based on criteria far more stringent than those applying to non-nuclear facilities.

Failure modes

Failure mode is the manner by which an earthquake induced failure is observed. It, generally, describes the way the failure occurs. Though costly and time consuming, learning from each real earthquake failure remains a routine recipe for advancement in *seismic design* methods. Below, some typical modes of earthquake-generated failures are presented.

Typical damage to unreinforced masonry buildings at earthquakes

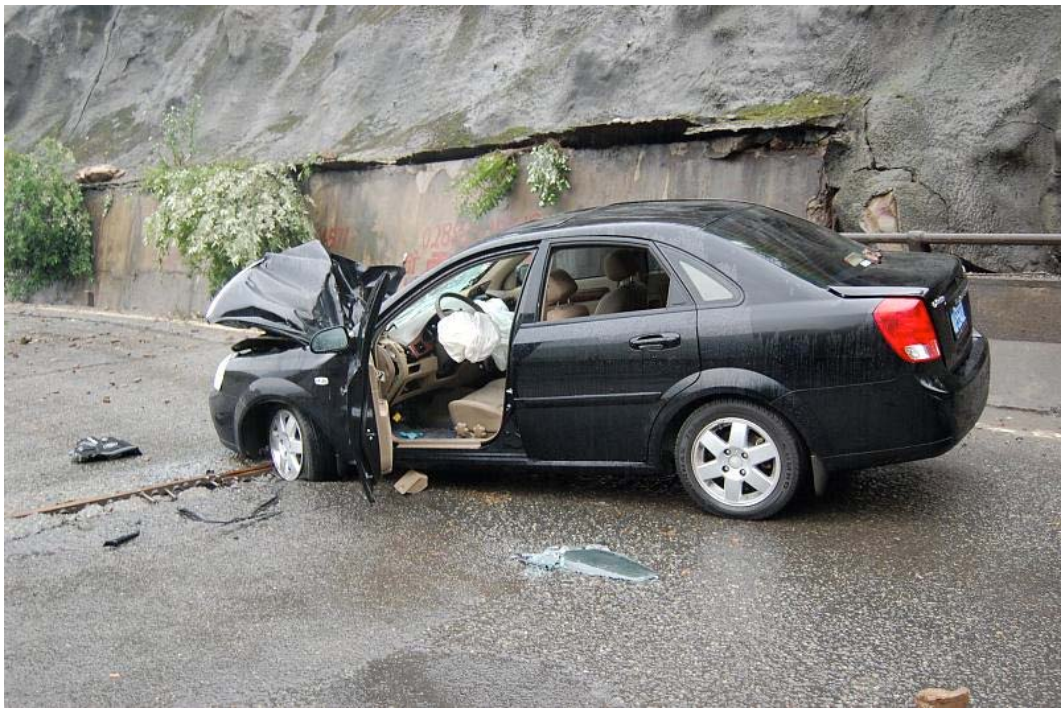
The lack of reinforcement coupled with poor mortar and inadequate roof-to-wall ties can result in substantial damage to a **unreinforced masonry building**. Severely cracked or leaning walls are some of the most common earthquake damage. Also hazardous is the damage that may occur between the walls and roof or floor diaphragms. Separation between the framing and the walls can jeopardize the vertical support of roof and floor systems.

Soft story collapse due to inadequate shear strength at ground level, Loma Prieta earthquake

Soft story effect. Absence of adequate shear walls on the ground level caused damage to this structure. A close examination of the image reveals that the rough board siding, once covered by a brick veneer, has been completely dismantled from the studwall. Only the rigidity of the floor above combined with the support on the two hidden sides by continuous walls, not penetrated with large doors as on the street sides, is preventing full collapse of the structure.

Effects of soil liquefaction during the 1964 Niigata earthquake

Soil liquefaction. In the cases where the soil consists of loose granular deposited materials with the tendency to develop excessive hydrostatic pore water pressure of sufficient magnitude and compact, liquefaction of those loose saturated deposits may result in non-uniform settlements and tilting of structures. This caused major damage to thousands of buildings in Niigata, Japan during the 1964 earthquake.



Car smashed by landslide rock, 2008 Sichuan earthquake

Landslide rock fall. A landslide is a geological phenomenon which includes a wide range of ground movement, including **rock falls**. Typically, the action of gravity is the primary driving force for a landslide to occur though in this case there was another contributing factor which affected the original slope stability: the landslide required an *earthquake trigger* before being released.

Effects of pounding against adjacent building, Loma Prieta

Pounding against adjacent building. This is a photograph of the collapsed five-story tower, St. Joseph's Seminary, Los Altos, California which resulted in one fatality. During Loma Prieta earthquake, the tower pounded against the independently vibrating adjacent building behind. A possibility of pounding depends on both buildings' lateral displacements which should be accurately estimated and accounted for.

Effects of completely shattered joints of concrete frame, Northridge

At Northridge earthquake, the Kaiser Permanente concrete frame office building had joints completely shattered, revealing **inadequate confinement steel**, which resulted in the second story collapse. In the transverse direction, composite end shear walls, consisting of two wythes of brick and a layer of shotcrete that carried the lateral load, peeled apart because of **inadequate through-ties** and failed.

7-story reinforced concrete **buildings on steep slope collapse** due to the following:

- Improper construction site on a foothill.
- Poor detailing of the reinforcement (lack of concrete confinement in the columns and at the beam-column joints, inadequate splice length).
- Seismically weak soft story at the first floor.
- Long cantilevers with heavy dead load.

Shifting from foundation, Whittier

Sliding off foundations effect of a relatively rigid residential building structure during 1987 Whittier Narrows earthquake. The magnitude 5.9 earthquake pounded the Garvey West Apartment building in Monterey Park, California and shifted its superstructure about 10 inches to the east on its foundation.

Earthquake damage in Pichilemu.

If a superstructure is not mounted on a base isolation system, its shifting on the basement should be prevented.

Insufficient shear reinforcement let main rebars to buckle, Northridge

Reinforced concrete column burst at Northridge earthquake due to **insufficient shear reinforcement mode** which allows main reinforcement to buckle outwards. The deck unseated at the hinge and failed in shear. As a result, the La Cienega-Venice underpass section of the 10 Freeway collapsed.

Support-columns and upper deck failure, Loma Prieta earthquake

Loma Prieta earthquake: side view of reinforced concrete **support-columns failure** which trigged **the upper deck collapse onto the lower deck** of the two-level Cypress viaduct of Interstate Highway 880, Oakland, CA.

Failure of retaining wall due to ground movement, Loma Prieta

Retaining wall failure at Loma Prieta earthquake in Santa Cruz Mountains area: prominent northwest-trending extensional cracks up to 12 cm (4.7 in) wide in the concrete spillway to Austrian Dam, the north abutment.

Lateral spreading mode of ground failure, Loma Prieta

Ground shaking triggered soil liquefaction in a subsurface layer of sand, producing differential lateral and vertical movement in an overlying carapace of unliquified sand and silt. This **mode of ground failure**, termed **lateral spreading**, is a principal cause of liquefaction-related earthquake damage.

Beams and pier columns diagonal cracking, 2008 Sichuan earthquake

Severely damaged building of Agriculture Development Bank of China after 2008 Sichuan earthquake: most of the **beams and pier columns are sheared**. Large diagonal cracks in masonry and veneer are due to in-plane loads while abrupt settlement of the right end of the building should be attributed to a landfill which may be hazardous even without any earthquake.

Tsunami strikes Ao Nang,

Twofold tsunami impact: sea waves hydraulic pressure and inundation. Thus, 2004 Indian Ocean earthquake of December 26, 2004, with the epicenter off the west coast of Sumatra, Indonesia, triggered a series of devastating tsunamis, killing more than 225,000 people in eleven countries by **inundating surrounding coastal communities with huge waves** up to 30 meters (100 feet) high.

Earthquake construction

Earthquake construction means implementation of **seismic design** to enable building and non-building structures to live through the anticipated earthquake exposure up to the expectations and in compliance with the applicable building codes.

Construction of Pearl River Tower X-bracing to resist lateral forces of earthquakes and winds

Design and construction are intimately related. To achieve a good workmanship, detailing of the members and their connections should be, possibly, simple. As any construction in general, earthquake construction is a process that consists of the building, retrofitting or assembling of infrastructure given the construction materials available.

The destabilizing action of an earthquake on constructions may be *direct* (seismic motion of the ground) or *indirect* (earthquake-induced landslides, soil liquefaction and waves of tsunami).

A structure might have all the appearances of stability, yet offer nothing but danger when an earthquake occurs. The crucial fact is that, for safety, earthquake-resistant construction techniques are as important as quality control and using correct materials. *Earthquake contractor* should be registered in the state of the project location, bonded and insured.

To minimize possible losses, construction process should be organized with keeping in mind that earthquake may strike any time prior to the end of construction.

Each construction project requires a qualified team of professionals who understand the basic features of seismic performance of different structures as well as construction management.

Adobe structures

Partially collapsed adobe building in Westmorland, California

Around thirty percent of the world's population lives or works in earth-made construction. Adobe type of mud bricks is one of the oldest and most widely used building materials. The use of adobe is very common in some of the world's most hazard-prone regions, traditionally across Latin America, Africa, Indian subcontinent and other parts of Asia, Middle East and Southern Europe.

Adobe buildings are considered very vulnerable at strong quakes. However, multiple ways of seismic strengthening of new and existing adobe buildings are available, see, e.g.

Key factors for the improved seismic performance of adobe construction are:

- Quality of construction.
- Compact, box-type layout.
- Seismic reinforcement.

Limestone and sandstone structures

Base-isolated City and County Building, Salt Lake City, Utah

Limestone is very common in architecture, especially in North America and Europe. Many landmarks across the world, including the pyramids in Egypt, are made of limestone. Many medieval churches and castles in Europe are made of limestone and sandstone masonry. They are the long-lasting materials but their rather heavy weight is not beneficial for adequate seismic performance.

Application of modern technology to seismic retrofitting can enhance the survivability of unreinforced masonry structures. As an example, from 1973 to 1989, the Salt Lake City and County Building in Utah was exhaustively renovated and repaired with an emphasis on preserving historical accuracy in appearance. This was done in concert with a seismic upgrade that placed the weak sandstone structure on base isolation foundation to better protect it from earthquake damage.

Timber frame structures

Half-timbered museum buildings, Denmark, date from 1560

Timber framing dates back thousands of years, and has been used in many parts of the world during various periods such as ancient Japan, Europe and medieval England in localities where timber was in good supply and building stone and the skills to work it were not.

The use of timber framing in buildings provides their complete skeletal framing which offers some structural benefits as the timber frame, if properly engineered, lends itself to better *seismic survivability*.

Light-frame structures

A two-story wooden-frame for a residential building structure

Light-frame structures usually gain seismic resistance from rigid plywood shear walls and wood structural panel diaphragms. Special provisions for seismic load-resisting systems for all engineered wood structures requires consideration of diaphragm ratios, horizontal and vertical diaphragm shears, and connector/fastener values. In addition, collectors, or drag struts, to distribute shear along a diaphragm length are required.

Reinforced masonry structures

Reinforced hollow masonry wall

A construction system where steel reinforcement is embedded in the mortar joints of masonry or placed in holes and after filled with concrete or grout is called **reinforced masonry**.

Devastating 1933 Long Beach earthquake revealed that masonry construction should be improved immediately. Then, the California State Code made the reinforced masonry mandatory.

There are various practices and techniques to achieve reinforced masonry. The most common type is the reinforced hollow unit masonry. The effectiveness of both vertical

and horizontal reinforcement strongly depends on the type and quality of the masonry, i.e. masonry units and mortar.

To achieve a ductile behavior of masonry, it is necessary that the shear strength of the wall is greater than the tensile strength of reinforcement to ensure a kind of bending failure.

Reinforced concrete structures

Stressed Ribbon pedestrian bridge over the Rogue River, Grants Pass, Oregon

Reinforced concrete is concrete in which steel reinforcement bars (rebars) or fibers have been incorporated to strengthen a material that would otherwise be brittle. It can be used to produce beams, columns, floors or bridges.

Prestressed concrete is a kind of reinforced concrete used for overcoming concrete's natural weakness in tension. It can be applied to beams, floors or bridges with a longer span than is practical with ordinary reinforced concrete. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load which produces a compressive stress that offsets the tensile stress that the concrete compression member would, otherwise, experience due to a bending load.

To prevent catastrophic collapse in response earth shaking (in the interest of life safety), a traditional reinforced concrete frame should have ductile joints. Depending upon the methods used and the imposed seismic forces, such buildings may be immediately usable, require extensive repair, or may have to be demolished.

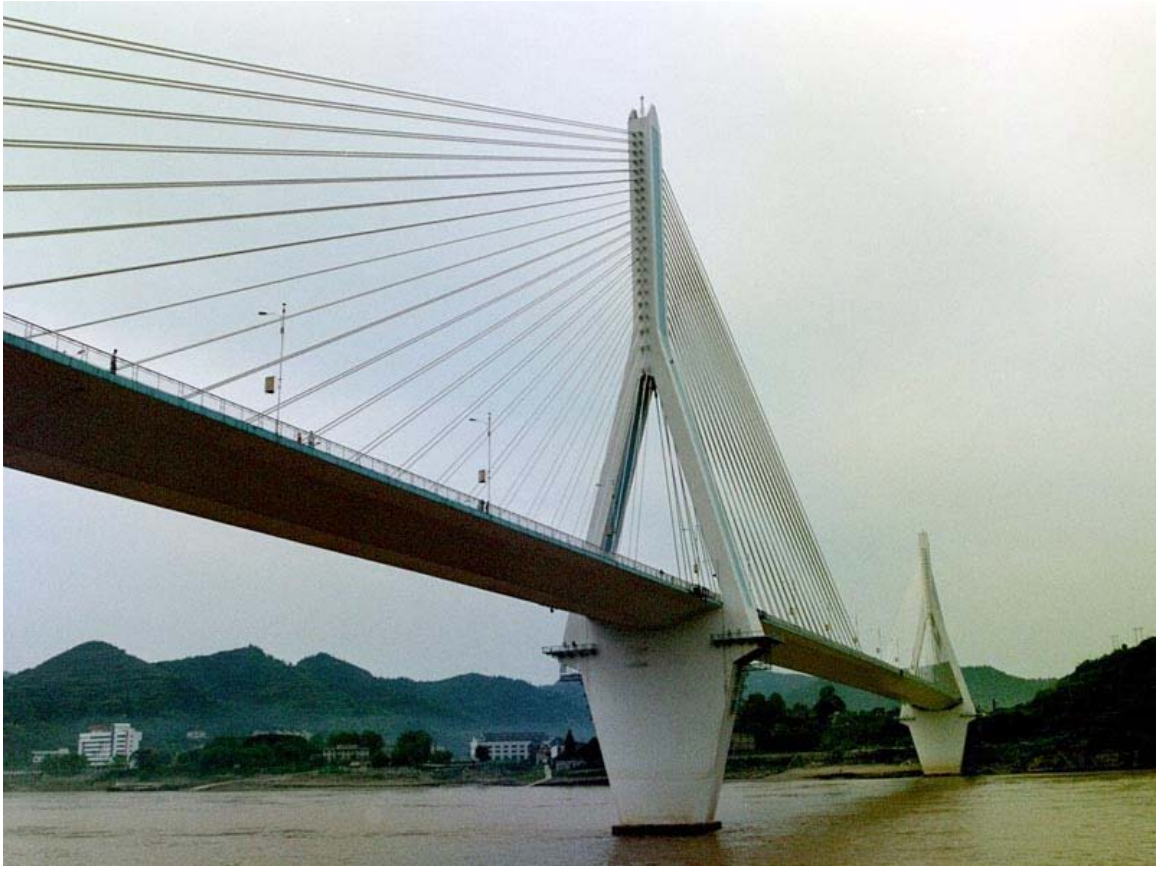
Prestressed structures

Prestressed structure is the one whose overall integrity, stability and security depend, primarily, on a *prestressing*. *Prestressing* means the intentional creation of permanent stresses in a structure for the purpose of improving its performance under various service conditions.

Naturally pre-compressed exterior wall of Colosseum, Rome

There are the following basic types of prestressing:

- Pre-compression (mostly, with the own weight of a structure)
- Pretensioning with high-strength embedded tendons
- Post-tensioning with high-strength bonded or unbonded tendons



Prestressed concrete cable-stayed bridge over Yangtze river

Today, the concept of prestressed structure is widely engaged in design of buildings, underground structures, TV towers, power stations, floating storage and offshore facilities, nuclear reactor vessels, and numerous kinds of bridge systems.

A beneficial idea of *prestressing* was, apparently, familiar to the ancient Rome architects; look, e.g., at the tall attic wall of Colosseum working as a press for the wall piers beneath.

Steel structures

Collapsed section of the San Francisco – Oakland Bay Bridge in response to Loma Prieta earthquake

Steel structures are considered mostly earthquake resistant but their resistance should never be taken for granted. A great number of welded steel moment frame buildings, which looked earthquake-proof, surprisingly experienced brittle behavior and were hazardously damaged in the 1994 Northridge earthquake. After that, the Federal Emergency Management Agency (FEMA) initiated development of repair techniques and new design approaches to minimize damage to steel moment frame buildings in future earthquakes.

For structural steel seismic design based on Load and Resistance Factor Design (LRFD) approach, it is very important to assess ability of a structure to develop and maintain its bearing resistance in the inelastic range. A measure of this ability is ductility, which may be observed in a *material itself*, in a *structural element*, or to a *whole structure*.

As a consequence of Northridge earthquake experience, all pre-qualified connection details and design methods contained in the building codes of that time have been rescinded. The new provisions stipulated that new designs be substantiated by testing or by use of test-verified calculations.

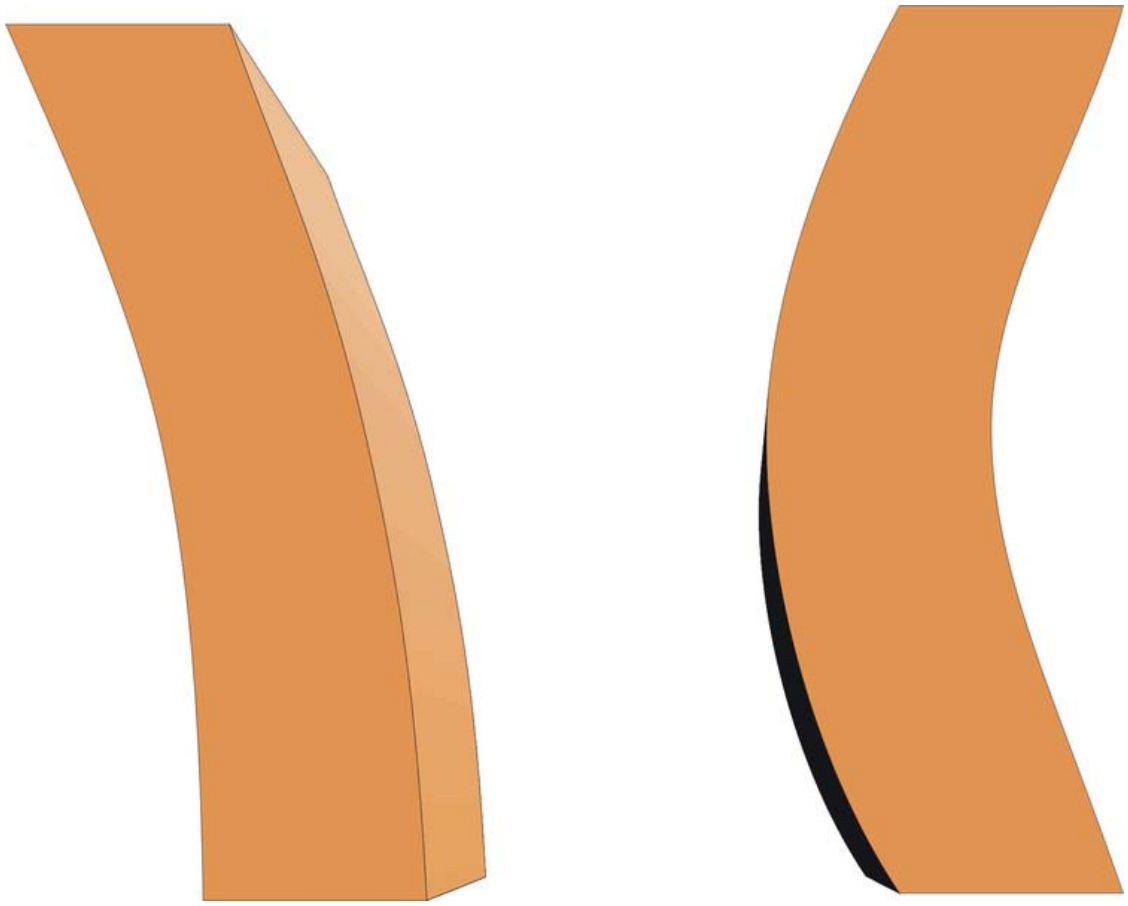
Prediction of earthquake losses

Earthquake loss estimation is usually defined as a *Damage Ratio* (**DR**) which is a ratio of the earthquake damage repair cost to the total value of a building. *Probable Maximum Loss* (**PML**) is a common term used for earthquake loss estimation, but it lacks a precise definition. In 1999, ASTM E2026 'Standard Guide for the Estimation of Building Damageability in Earthquakes' was produced in order to standardize the nomenclature for seismic loss estimation, as well as establish guidelines as to the review process and qualifications of the reviewer.

Earthquake loss estimations are also referred to as *Seismic Risk Assessments*. The risk assessment process generally involves determining the probability of various ground motions coupled with the vulnerability or damage of the building under those ground motions. The results are defined as a percent of building replacement value.

Chapter- 10

Seismic Analysis



First and second modes of building seismic response

Seismic Analysis is a subset of structural analysis and is the calculation of the response of a building (or nonbuilding) structure to earthquakes. It is part of the process of structural design, earthquake engineering or structural assessment and retrofit in regions where earthquakes are prevalent.

As seen in the figure, a building has the potential to 'wave' back and forth during an earthquake (or even a severe wind storm). This is called the 'fundamental mode', and is the lowest frequency of building response. Most buildings, however, have higher modes

of response, which are uniquely activated during earthquakes. The figure just shows the second mode, but there are higher ‘shimmy’ (abnormal vibration) modes. Nevertheless, the first and second modes tend to cause the most damage in most cases.

The earliest provisions for seismic resistance were the requirement to design for a lateral force equal to a proportion of the building weight (applied at each floor level). This approach was adopted in the appendix of the 1927 Uniform Building Code (UBC), which was used on the west coast of the USA. It later became clear that the dynamic properties of the structure affected the loads generated during an earthquake. In the Los Angeles County Building Code of 1943 a provision to vary the load based on the number of floor levels was adopted (based on research carried out at Caltech in collaboration with Stanford University and the U.S. Coast and Geodetic Survey, which started in 1937). The concept of "response spectra" was developed in the 1930s, but it wasn't until 1952 that a joint committee of the San Francisco Section of the ASCE and the Structural Engineers Association of Northern California (SEAONC) proposed using the building period (the inverse of the frequency) to determine lateral forces.

The University of California, Berkeley was an early base for computer-based seismic analysis of structures, led by Professor Ray Clough (who coined the term finite element). Students included Ed Wilson, who went on to write the program SAP in 1970, an early "Finite Element Analysis" program.

Earthquake engineering has developed a lot since the early days, and some of the more complex designs now use special earthquake protective elements either just in the foundation (base isolation) or distributed throughout the structure. Analyzing these types of structures requires specialized explicit finite element computer code, which divides time into very small slices and models the actual physics, much like common video games often have "physics engines". Very large and complex buildings can be modeled in this way (such as the Osaka Internation Convention Center).

Structural analysis methods can be divided into the following five categories.

Equivalent Static Analysis

This approach defines a series of forces acting on a building to represent the effect of earthquake ground motion, typically defined by a seismic design response spectrum. It assumes that the building responds in its fundamental mode. For this to be true, the building must be low-rise and must not twist significantly when the ground moves. The response is read from a design response spectrum, given the natural frequency of the building (either calculated or defined by the building code). The applicability of this method is extended in many building codes by applying factors to account for higher buildings with some higher modes, and for low levels of twisting. To account for effects due to "yielding" of the structure, many codes apply modification factors that reduce the design forces (e.g. force reduction factors).

Response Spectrum Analysis

This approach permits the multiple modes of response of a building to be taken into account (in the frequency domain). This is required in many building codes for all except for very simple or very complex structures. The response of a structure can be defined as a combination of many special shapes (modes) that in a vibrating string correspond to the "harmonics". Computer analysis can be used to determine these modes for a structure. For each mode, a response is read from the design spectrum, based on the modal frequency and the modal mass, and they are then combined to provide an estimate of the total response of the structure. Combination methods include the following:

- absolute - peak values are added together
- square root of the sum of the squares (SRSS)
- complete quadratic combination (CQC) - a method that is an improvement on SRSS for closely spaced modes

The result of a response spectrum analysis using the response spectrum from a ground motion is typically different from that which would be calculated directly from a linear dynamic analysis using that ground motion directly, since phase information is lost in the process of generating the response spectrum.

In cases where structures are either too irregular, too tall or of significance to a community in disaster response, the response spectrum approach is no longer appropriate, and more complex analysis is often required, such as non-linear static or dynamic analysis.

A sample response spectrum analysis.

Linear Dynamic Analysis

Static procedures are appropriate when higher mode effects are not significant. This is generally true for short, regular buildings. Therefore, for tall buildings, buildings with torsional irregularities, or non-orthogonal systems, a dynamic procedure is required. In the linear dynamic procedure, the building is modelled as a multi-degree-of-freedom (MDOF) system with a linear elastic stiffness matrix and an equivalent viscous damping matrix.

The seismic input is modelled using either modal spectral analysis or time history analysis but in both cases, the corresponding internal forces and displacements are determined using linear elastic analysis. The advantage of these linear dynamic procedures with respect to linear static procedures is that higher modes can be considered. However, they are based on linear elastic response and hence the applicability decreases with increasing nonlinear behaviour, which is approximated by global force reduction factors.

In linear dynamic analysis, the response of the structure to ground motion is calculated in the time domain, and all phase information is therefore maintained. Only linear properties are assumed. The analytical method can use modal decomposition as a means of reducing the degrees of freedom in the analysis.

Non-linear Static Analysis

In general, linear procedures are applicable when the structure is expected to remain nearly elastic for the level of ground motion or when the design results in nearly uniform distribution of nonlinear response throughout the structure. As the performance objective of the structure implies greater inelastic demands, the uncertainty with linear procedures increases to a point that requires a high level of conservatism in demand assumptions and acceptability criteria to avoid unintended performance. Therefore, procedures incorporating inelastic analysis can reduce the uncertainty and conservatism.

This approach is also known as "pushover" analysis. A pattern of forces is applied to a structural model that includes non-linear properties (such as steel yield), and the total force is plotted against a reference displacement to define a capacity curve. This can then be combined with a demand curve (typically in the form of an acceleration-displacement response spectrum (ADRS)). This essentially reduces the problem to a single degree of freedom system.

Nonlinear static procedures use equivalent SDOF structural models and represent seismic ground motion with response spectra. Story drifts and component actions are related subsequently to the global demand parameter by the pushover or capacity curves that are the basis of the non-linear static procedures.

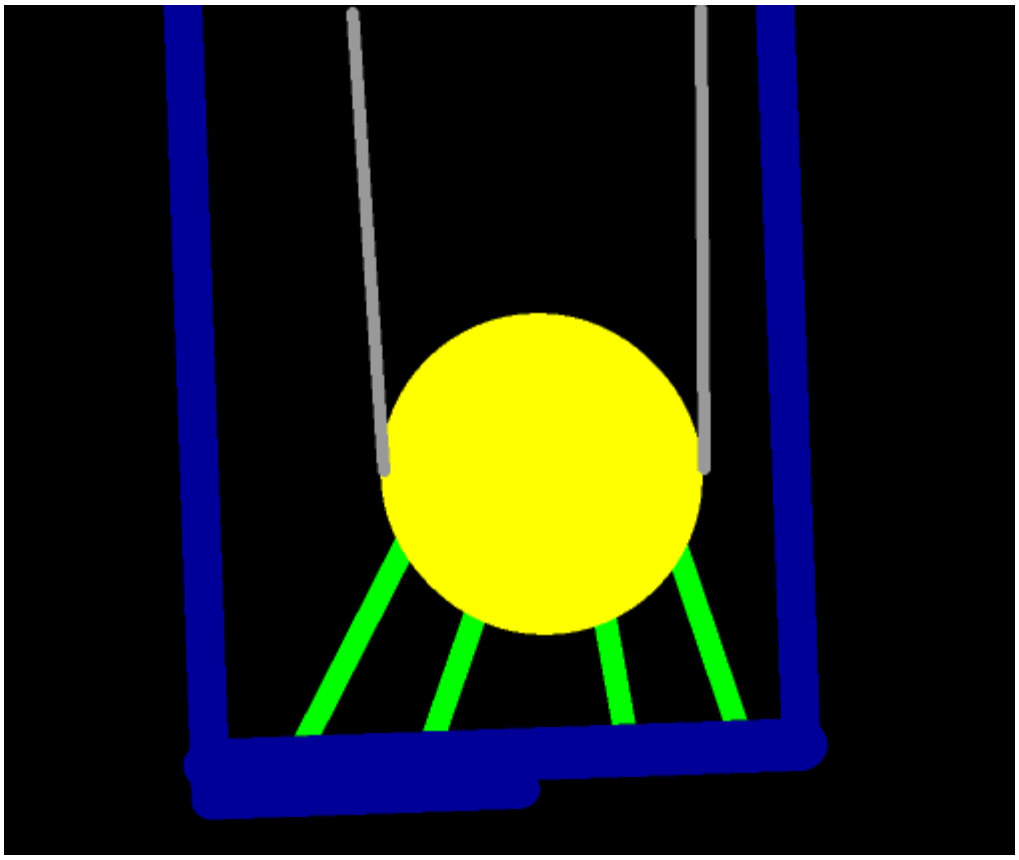
Non-linear Dynamic Analysis

Nonlinear dynamic analysis utilizes the combination of ground motion records with a detailed structural model, therefore is capable of producing results with relatively low uncertainty. In nonlinear dynamic analyses, the detailed structural model subjected to a ground-motion record produces estimates of component deformations for each degree of freedom in the model and the modal responses are combined using schemes such as the square-root-sum-of-squares.

In non-linear dynamic analysis, the non-linear properties of the structure are considered as part of a time domain analysis. This approach is the most rigorous, and is required by some building codes for buildings of unusual configuration or of special importance. However, the calculated response can be very sensitive to the characteristics of the individual ground motion used as seismic input; therefore, several analyses are required using different ground motion records.

Chapter- 11

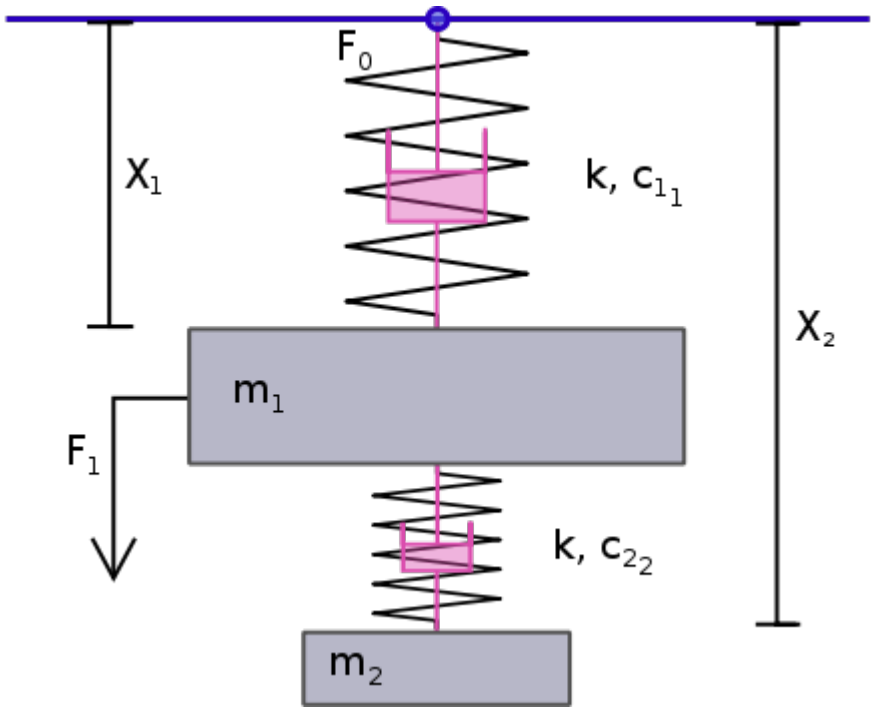
Tuned Mass Damper



An image showing the movement of a skyscraper versus the mass damper. The green indicates the hydraulic cylinders used to damp the motion of the skyscraper.

A **tuned mass damper**, also known as an **active mass damper** (**AMD**) or **harmonic absorber**, is a device mounted in structures to reduce the amplitude of mechanical vibrations. Their application can prevent discomfort, damage, or outright structural failure. They are frequently used in power transmission, automobiles, and buildings.

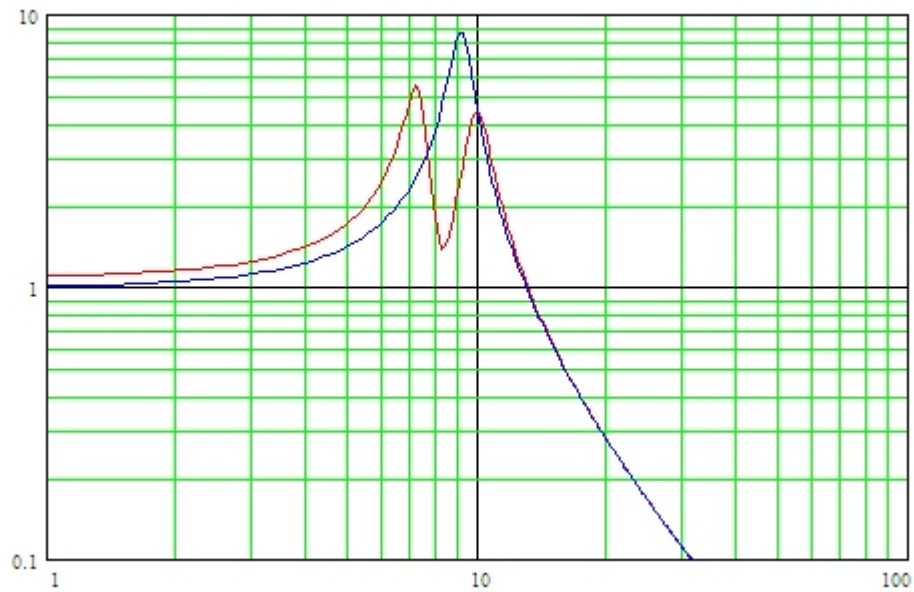
Principle



A schematic of a simple spring–mass–damper system used to demonstrate the tuned mass damper system.

Tuned mass dampers stabilize against violent motion caused by harmonic vibration. A tuned damper reduces the vibration of a system with a comparatively lightweight component so that the worst-case vibrations are less intense. Roughly speaking practical systems are tuned to either move the main mode away from a troubling excitation frequency, or to add damping to a resonance that is difficult or expensive to damp directly. An example of the latter is a crankshaft torsional damper. Mass dampers are frequently implemented with a frictional or hydraulic component that turns mechanical kinetic energy into heat, like an automotive shock absorber. An electrical analogue is a LCR circuit.

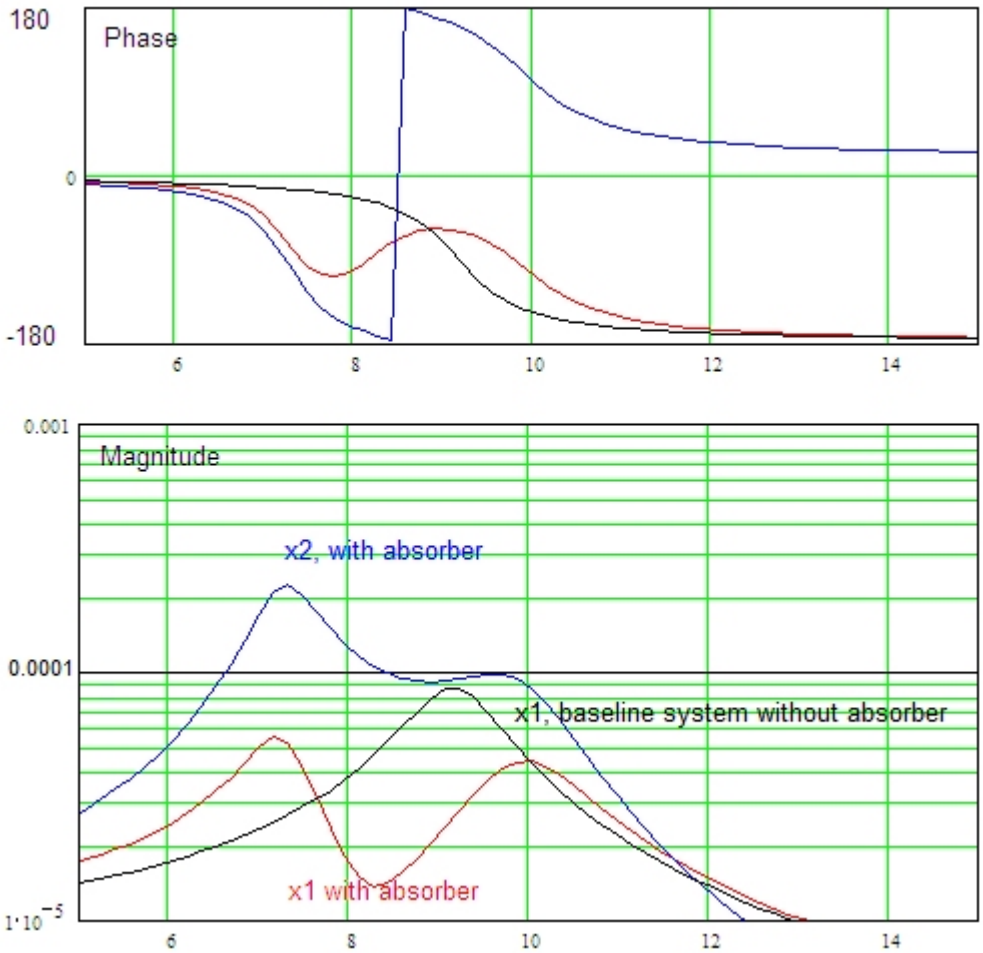
Given a motor with mass m_1 attached via motor mounts to the ground, the motor vibrates as it operates and the soft motor mounts act as a parallel spring and damper, k_1 and c_1. The force on the motor mounts is F_0. In order to reduce the maximum force on the motor mounts as the motor operates over a range of speeds, a smaller mass, m_2, is connected to m_1 by a spring and a damper, k_2 and c_2. F_1 is the effective force on the motor due to its operation.



Response of the system excited by one unit of force, with (red) and without (blue) the 10% tuned mass. The peak response is reduced from 9 units down to 5.5 units. While the maximum response force is reduced, there are some operating frequencies for which the response force is increased.

The graph shows the effect of a tuned mass damper on a simple spring–mass–damper system, excited by vibrations with an amplitude of one unit of force applied to the main mass, m_1. An important measure of performance is the ratio of the force on the motor mounts to the force vibrating the motor, F_0 / F_1. This assumes that the system is linear, so if the force on the motor were to double, so would the force on the motor mounts. The blue line represents the baseline system, with a maximum response of 9 units of force at around 9 units of frequency. The red line shows the effect of adding a tuned mass of 10% of the baseline mass. It has a maximum response of 5.5, at a frequency of 7. As a side effect, it also has a second normal mode and will vibrate somewhat more than the baseline system at frequencies below about 6 and above about 10.

The heights of the two peaks can be adjusted by changing the stiffness of the spring in the tuned mass damper. Changing the damping also changes the height of the peaks, in a complex fashion. The split between the two peaks can be changed by altering the mass of the damper (m_2).



A Bode plot of displacements in the system with (red) and without (blue) the 10% tuned mass.

The Bode plot is more complex, showing the phase and magnitude of the motion of each mass, for the two cases, relative to F1.

In the plots at right, the black line shows the baseline response ($m_2 = 0$). Now considering $m_2 = m_1 / 10$, the blue line shows the motion of the damping mass and the red line shows the motion of the primary mass. The amplitude plot shows that at low frequencies, the damping mass resonates much more than the primary mass. The phase plot shows that at low frequencies, the two masses are in phase. As the frequency increases m_2 moves out of phase with m_1 until at around 9.5 Hz it is 180° out of phase with m_1, maximizing the damping effect by maximizing the amplitude of $x_2 - x_1$, this maximizes the energy dissipated into c_2 and simultaneously pulls on the primary mass in the same direction as the motor mounts.

Mass dampers in automobiles

Motorsport

The tuned mass damper was introduced as part of the suspension system by Renault, on its 2005 F1 car (the R25), at the 2005 Brazilian Grand Prix. It was deemed to be legal at first, and it was in use up to the 2006 German Grand Prix.

At Hockenheim, the mass damper was deemed illegal by the FIA, since the mass wasn't rigidly attached to the chassis and, due to the influence it had on the pitch attitude of the car, which in turn significantly affected the gap under the car and hence the ground effects of the car, to be a movable aerodynamic device and hence as a consequence, to be illegally influencing the performance of the aerodynamics.

The Stewards of the meeting deemed it legal, but the FIA appealed against that decision. Two weeks later, the FIA International Court of Appeal deemed the mass damper illegal.

Production cars

Tuned mass dampers are widely used in production cars, typically on the crankshaft pulley to control torsional vibration and bending modes of the crankshaft, on the driveline for gearwhine, and other noises. They are also used on the exhaust, on the body and on the suspension. Almost all cars will have one mass damper, some may have 10 or more.

Mass dampers in spacecraft

One proposal to reduce vibration on NASA's Ares solid fuel booster is to use 16 tuned mass dampers as part of a design strategy to reduce peak loads from 6g to 0.25 g, the TMDs being responsible for the reduction from 1 g to 0.25 g, the rest being done by conventional vibration isolators between the upper stages and the booster.

Dampers in power transmission lines

Stockbridge dampers on power lines.

High-tension lines often have small barbell-shaped Stockbridge dampers hanging from the wires to reduce the high-frequency, low-amplitude oscillation termed flutter.

Dampers in buildings and related structures

Tuned mass damper atop Taipei 101.

Typically, the dampers are huge concrete blocks or steel bodies mounted in skyscrapers or other structures, and moved in opposition to the resonance frequency oscillations of the structure by means of springs, fluid or pendulums.

Sources of vibration and resonance

Unwanted vibration may be caused by environmental forces acting on a structure, such as wind or earthquake, or by a seemingly innocuous vibration source causing resonance that may be destructive, unpleasant or simply inconvenient.

Earthquakes

The seismic waves caused by an earthquake will make buildings sway and oscillate in various ways depending on the frequency and direction of ground motion, and the height and construction of the building. Seismic activity can cause excessive oscillations of the building which may lead to structural failure. To enhance the building's seismic performance, a proper building design is performed engaging various seismic vibration control technologies.

Mechanical human sources

Dampers on the Millennium Bridge in London. The white disk is not part of the damper.

Masses of people walking up and down stairs at once, or great numbers of people stomping in unison, can cause serious problems in large structures like stadiums if those structures lack damping measures. Vibration caused by heavy industrial machinery, generators and diesel engines can also pose problems to structural integrity, especially if mounted on a steel structure or floor. Large ocean going vessels may employ tuned mass dampers to isolate the vessel from its engine vibration.

Wind

The force of wind against tall buildings can cause the top of skyscrapers to move more than a metre. This motion can be in the form of swaying or twisting, and can cause the upper floors of such buildings to move. Certain angles of wind and aerodynamic properties of a building can accentuate the movement and cause motion sickness in people.

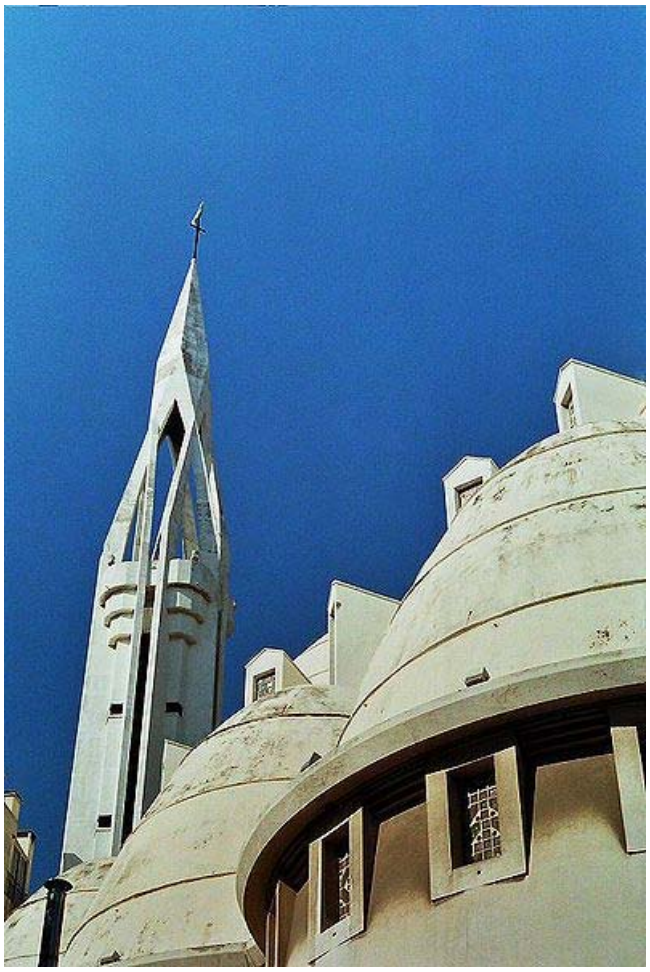
Examples of buildings and structures with tuned mass dampers

- Bally's to Bellagio, Bally's to Caesars Palace, and Treasure Island to The Venetian Pedestrian Bridges in Las Vegas
- Berlin Television Tower (Fernsehturm) — tuned mass damper located in the spire.

- Bloomberg Tower/731 Lexington in New York
- Burj al-Arab in Dubai — 11 tuned mass dampers.
- Citigroup Center in New York City — Designed by William LeMessurier and completed in 1977, it was one of the first skyscrapers to use a tuned mass damper to reduce sway. Uses a concrete version.
- Comcast Center in Philadelphia, PA — Contains the largest Tuned Liquid Column Damper (TLCD) in the world at 1,300 tons.
- Dublin Spire in Dublin, Ireland — This narrow slender structure was designed with a tuned mass damper to ensure aerodynamic stability during a wind storm.
- Grand Canyon Skywalk
- John Hancock Tower in Boston — A tuned mass damper was added to it after it was built.
- London Millennium Bridge — 'The Wobbly Bridge'
- One Rincon Hill South Tower — First building in California to have a liquid tuned mass damper
- One Wall Centre in Vancouver — It employs tuned liquid column dampers, at the time of its installation, a unique form of tuned mass damper.
- Park Tower in Chicago — The first building in the United States to be designed with a tuned mass damper from the outset.
- Random House Tower Uses two liquid filled dampers in New York City
- Sakhalin-I — An offshore drilling platform
- Shanghai World Financial Center in Shanghai, China
- Taipei 101 skyscraper — Contains one of the world's largest tuned mass dampers, at 730 tons.
- Theme Building at Los Angeles International Airport
- Trump World Tower in New York
- Yokohama Landmark Tower

Chapter- 12

Reinforced Concrete



Reinforced concrete at Sainte Jeanne d'Arc Church (Nice, France): architect Jacques Dror, 1926–1933

Reinforced concrete is concrete in which reinforcement bars ("rebars"), reinforcement grids, plates or fibers have been incorporated to strengthen the concrete in tension. The term Ferro Concrete refers only to concrete that is reinforced with iron or steel. Other materials used to reinforce concrete can be organic and inorganic fibres as well as composites in different forms. Concrete is strong in compression, but weak in tension,

thus adding reinforcement increases the strength in tension. In addition, the failure strain of concrete in tension is so low that the reinforcement has to hold the cracked sections together. For a strong, ductile and durable construction the reinforcement shall have the following properties:

- High strength
- High tensile strain
- Good bond to the concrete
- Thermal compatibility
- Durability in the concrete environment

In most cases reinforced concrete uses steel rebars that have been inserted to add strength.

Use in construction

Rebars of Sagrada Familia's roof in construction (2009)

Concrete is reinforced to give it extra tensile strength; without reinforcement, many concrete buildings would not have been possible.

Reinforced concrete can encompass many types of structures and components, including slabs, walls, beams, columns, foundations, frames and more.

Reinforced concrete can be classified as precast or cast in-situ concrete.

Much of the focus on reinforcing concrete is placed on floor systems. Designing and implementing the most efficient floor system is key to creating optimal building structures. Small changes in the design of a floor system can have significant impact on material costs, construction schedule, ultimate strength, operating costs, occupancy levels and end use of a building.

Behavior of reinforced concrete

Materials

Concrete is a mixture of cement (usually Portland cement) and stone aggregate. When mixed with a small amount of water, the cement hydrates form microscopic opaque crystal lattices encapsulating and locking the aggregate into a rigid structure. Typical concrete mixes have high resistance to compressive stresses (about 4,000 psi (28 MPa)); however, any appreciable tension (*e.g.,* due to bending) will break the microscopic rigid lattice, resulting in cracking and separation of the concrete. For this reason, typical non-reinforced concrete must be well supported to prevent the development of tension.

If a material with high strength in tension, such as steel, is placed in concrete, then the composite material, **reinforced concrete**, resists not only compression but also bending and other direct tensile actions. A reinforced concrete section where the concrete resists the compression and steel resists the tension can be made into almost any shape and size for the construction industry.

Key characteristics

Three physical characteristics give reinforced concrete its special properties.

First, the coefficient of thermal expansion of concrete is similar to that of steel, eliminating large internal stresses due to differences in thermal expansion or contraction.

Second, when the cement paste within the concrete hardens this conforms to the surface details of the steel, permitting any stress to be transmitted efficiently between the different materials. Usually steel bars are roughened or corrugated to further improve the bond or cohesion between the concrete and steel.

Third, the alkaline chemical environment provided by the alkali reserve (KOH, NaOH) and the portlandite (calcium hydroxide) contained in the hardened cement paste causes a passivating film to form on the surface of the steel, making it much more resistant to corrosion than it would be in neutral or acidic conditions. When the cement paste exposed to the air and meteoric water reacts with the atmospheric CO_2, portlandite and the Calcium Silicate Hydrate (CSH) of the hardened cement paste become progressively carbonated and the high pH gradually decreases from 13.5 – 12.5 to 8.5, the pH of water in equilibrium with calcite (calcium carbonate) and the steel is no longer passivated.

As a rule of thumb, only to give an idea on orders of magnitude, steel is protected at pH above ~11 but starts to corrode below ~10 depending on steel characteristics and local physico-chemical conditions when concrete becomes carbonated. Carbonation of concrete along with chloride ingress are amongst the chief reasons for the failure of reinforcement bars in concrete.

The relative cross-sectional area of steel required for typical reinforced concrete is usually quite small and varies from 1% for most beams and slabs to 6% for some columns. Reinforcing bars are normally round in cross-section and vary in diameter. Reinforced concrete structures sometimes have provisions such as ventilated hollow cores to control their moisture & humidity.

Distribution of concrete (in spite of reinforcement) strength characteristics along the cross-section of vertical reinforced concrete elements is inhomogeneous article "Concrete Inhomogeneity of Vertical Cast-In-Situ Elements In Frame-Type Buildings".

Anti-corrosion measures

In wet and cold climates, reinforced concrete for roads, bridges, parking structures and other structures that may be exposed to deicing salt may benefit from use of epoxy-coated, hot dip galvanised or stainless steel rebar, although good design and a well-chosen cement mix may provide sufficient protection for many applications. Epoxy coated rebar can easily be identified by the light green colour of its epoxy coating. Hot dip galvanized rebar may be bright or dull grey depending on length of exposure, and stainless rebar exhibits a typical white metallic sheen that is readily distinguishable from carbon steel reinforcing bar. Reference ASTM standard specifications **A767** Standard Specification for Hot Dip Galvanised Reinforcing Bars, **A775** Standard Specification for Epoxy Coated Steel Reinforcing Bars and **A955** Standard Specification for Deformed and Plain Stainless Bars for Concrete Reinforcement.

Another, cheaper way or protecting rebars is coating them with zinc phosphate. Zinc phosphate slowly reacts with calcium cations and the hydroxyl anions present in the cement pore water and forms a stable hydroxyapatite layer.

Penetrating sealants typically must be applied some time after curing. Sealants include paint, plastic foams, films and aluminum foil, felts or fabric mats sealed with tar, and layers of bentonite clay, sometimes used to seal roadbeds.

Corrosion inhibitors, such as calcium nitrite [$Ca(NO_2)_2$], can also be added to the water mix before pouring concrete. Generally, 1–2 wt. % of [$Ca(NO_2)_2$] with respect to cement weight is needed to prevent corrosion of the rebars. The nitrite anion is a mild oxidizer that oxidizes the soluble and mobile ferrous ions (Fe^{2+}) present at the surface of the corroding steel and causes it to precipitate as an insoluble ferric hydroxide ($Fe(OH)_3$). This causes the passivation of steel at the anodic oxidation sites. Nitrite is a much more active corrosion inhibitor than nitrate, a less powerful oxidizer of the divalent iron.

Reinforcement and terminology of Beams

A beam bends under bending moment, resulting in a small curvature. At the outer face (**tensile face**) of the curvature the concrete experiences tensile stress, while at the inner face (**compressive face**) it experiences compressive stress.

A **singly-reinforced** beam is one in which the concrete element is only reinforced near the tensile face and the reinforcement, called tension steel, is designed to resist the tension.

A **doubly-reinforced** beam is one in which besides the tensile reinforcement the concrete element is also reinforced near the compressive face to help the concrete resist compression. The latter reinforcement is called compression steel. When the compression zone of a concrete is inadequate to resist the compressive Moment(positive moment), extra reinforcement has to be provided if the architect limits the dimensions of the section.

An **under-reinforced** beam is one in which the tension capacity of the tensile reinforcement is **smaller** than the combined compression capacity of the concrete and the compression steel (under-reinforced at tensile face). When the reinforced concrete element is subject to increasing bending moment, the tension steel yields while the concrete does not reach its ultimate failure condition. As the tension steel yields and stretches, an "under-reinforced" concrete also yields in a ductile manner, exhibiting a large deformation and warning before its ultimate failure. In this case the yield stress of the steel governs the design.

An **over-reinforced** beam is one in which the tension capacity of the tension steel is **greater** than the combined compression capacity of the concrete and the compression steel (over-reinforced at tensile face). So the "over-reinforced concrete" beam fails by crushing of the compressive-zone concrete and before the tension zone steel yields, which does not provide any warning before failure as the failure is instantaneous.

A **balanced-reinforced** beam is one in which both the compressive and tensile zones reach yielding at the same imposed load on the beam, and the concrete will crush and the tensile steel will yield at the same time. This design criterios is however as risky as over-reinforced concrete, because failure is sudden as the concrete crushes at the same time of the tensile steel yields, which gives a very little warning of distress in tension failure.

Steel-reinforced concrete moment-carrying elements should normally be designed to be under-reinforced so that users of the structure will receive warning of impending collapse.

The **characteristic strength** is the strength of a material where less than 5% of the specimen shows lower strength.

The **design strength** or **nominal strength** is the strength of a material, including a material-safety factor. The value of the safety factor generally ranges from 0.75 to 0.85 in Allowable Stress Design.

The **ultimate limit state** is the theoretical failure point with a certain probability. It is stated under factored loads and factored resistances.

Prestressed Concrete

Prestressed concrete is a technique that greatly increases loadbearing strengh of concrete beams. The reinforcing steel in the bottom part of the beam, which will be subjected to tensile forces when in service, is placed in tension prior to the concrete being poured around it. Once the concrete has hardened, the tension on the reinforcing steel is released, placing a built-in compressive force on the concrete. When loads are applied, the reinforcing steel takes on more stress and the compressive force in the concrete is reduced, but does not become a tensile force. Since the concrete is always under compression, it is less subject to cracking and failure.

Common failure modes of steel reinforced concrete

Reinforced concrete can fail due to inadequate strength, leading to mechanical failure, or due to a reduction in its durability. Corrosion and freeze/thaw cycles may damage poorly designed or constructed reinforced concrete. When rebar corrodes, the oxidation products (rust) expand and tends to flake, cracking the concrete and unbonding the rebar from the concrete. Typical mechanisms leading to durability problems are discussed below.

Mechanical failure

Cracking of the concrete section can not be prevented; however, the size of and location of the cracks can be limited and controlled by reinforcement, placement of control joints, the curing methodology and the mix design of the concrete. Cracking defects can allow moisture to penetrate and corrode the reinforcement. This is a serviceability failure in limit state design. Cracking is normally the result of an inadequate quantity of rebar, or rebar spaced at too great a distance. The concrete then cracks either under excess loading, or due to internal effects such as **early thermal shrinkage** when it cures.

Ultimate failure leading to collapse can be caused by crushing of the concrete, when compressive stresses exceed its strength; by yielding or failure of the rebar, when bending or shear stresses exceed the strength of the reinforcement; or by bond failure between the concrete and the rebar.

Carbonation

Rebar for foundations and walls of sewage pump station.

Carbonation, or neutralisation, is a chemical reaction between carbon dioxide in the air with calcium hydroxide and hydrated calcium silicate in the concrete. The water in the pores of Portland cement concrete is normally alkaline with a pH in the range of 12.5 to 13.5. This highly alkaline environment is one in which the embedded steel is passivated and is protected from corrosion. According to the Pourbaix diagram for iron, the metal is passive when the pH is above 9.5. The carbon dioxide in the air reacts with the alkali in the cement and makes the pore water more acidic, thus lowering the pH. Carbon dioxide will start to carbonate the cement in the concrete from the moment the object is made. This **carbonation** process will start at the surface, then slowly move deeper and deeper into the concrete. The rate of carbonation is dependent on the relative humidity of the concrete - a 50% relative humidity being optimal. If the object is cracked, the carbon dioxide in the air will be better able to penetrate into the concrete. When designing a concrete structure, it is normal to state the concrete cover for the rebar (the depth within the object that the rebar will be). The minimum concrete cover is normally regulated by design or building codes. If the reinforcement is too close to the surface, early failure due to corrosion may occur. The concrete cover depth can be measured with a cover meter. However, carbonated concrete only becomes a durability problem when there is also sufficient moisture and oxygen to cause electro-potential corrosion of the reinforcing steel.

One method of testing a structure for carbonation is to drill a fresh hole in the surface and then treat the cut surface with phenolphthalein indicator solution. This solution will turn [pink] when in contact with alkaline concrete, making it possible to see the depth of carbonation. An existing hole is no good because the exposed surface will already be carbonated.

Chlorides

The Paulins Kill Viaduct, Hainesburg, New Jersey, is 115 feet (35 m) tall and 1,100 feet (335 m) long, and was heralded as the largest reinforced concrete structure in the world when it was completed in 1910 as part of the Lackawanna Cut-Off rail line project. The Lackawanna Railroad was a pioneer in the use of reinforced concrete.

Chlorides, including sodium chloride, can promote the corrosion of embedded steel rebar if present in sufficienty high concentration. Chloride anions induce both localized corrosion (pitting corrosion) and generalized corrosion of steel reinforcements. For this reason, one should only use fresh raw water or potable water for mixing concrete, insure that the coarse and fine aggregates do not contain chlorides, and not use admixtures that contain chlorides.

It was once common for calcium chloride to be used as an admixture to promote rapid set-up of the concrete. It was also mistakenly believed that it would prevent freezing.

However, this practice has fallen into disfavor once the deleterious effects of chlorides became known. It should be avoided when ever possible.

The use of de-icing salts on roadways, used to reduce the freezing point of water, is probably one of the primary causes of premature failure of reinforced or prestressed concrete bridge decks, roadways, and parking garages. The use of epoxy-coated reinforcing bars and the application of cathodic protection has mitigated this problem to some extent. Also FRP rebars are known to be less susceptible to chlorides. Properly designed concrete mixtures that have been allowed to cure properly are effectively impervious to the effects of deicers. (One common problem today is that contractors allow concrete to "dry" (out) for two to three days by before it cures and thus ultimately develops less than 10% of its design strength).

Another important source of chloride ions is from sea water. Sea water contains by weight approximately 3.5 wt.% salts. These salts include sodium chloride, magnesium sulfate, calcium sulfate, and bicarbonates. In water these salts dissociate in free ions (Na^+, Mg^{2+}, Cl^-, SO_4^{2-}, HCO_3^-) and migrate with the water into the capillaries of the concrete. Chloride ions are particularly aggressive for the corrosion of the carbon steel reinforcement bars and make up about 50% of these ions.

Alkali Silica Reaction

Characteristic crack pattern associated to the alkali-silica reaction affecting a concrete step barrier on an US motorway (photograph courtesy of the Federal Highway Administration, US Department of Transportation).

The **Alkali-Silica Reaction** (ASR) is a reaction which occurs over time in concrete between the highly alkaline cement paste and reactive non-crystalline (amorphous) silica, which is found in many common aggregates.

The ASR reaction is the same as the Pozzolanic reaction which is a simple acid-base reaction between calcium hydroxide, also known as Portlandite, or ($Ca(OH)_2$), and silicic acid (H_4SiO_4, or $Si(OH)_4$). For the sake of simplicity, this reaction can be schematically represented as following:

$$Ca(OH)_2 + H_4SiO_4 \rightarrow Ca^{2+} + H_2SiO_4^{2-} + 2\ H_2O \rightarrow CaH_2SiO_4 \cdot 2\ H_2O$$

This reaction causes the expansion of the altered aggregate by the formation of a swelling gel of Calcium Silicate Hydrate (CSH). This gel increases in volume with water and exerts an expansive pressure inside the material, causing spalling and loss of strength of the concrete, finally leading to its failure.

So, ASR can cause serious expansion and cracking in concrete, resulting in critical structural problems that can even force the demolition of a particular structure.

The mechanism of ASR causing the deterioration of concrete can be described in four steps as follows:

1. The alkaline solution attacks the siliceous aggregate to convert it to viscous alkali silicate gel.
2. Consumption of alkali by the reaction induces the dissolution of Ca^{2+} ions into the cement pore water. Calcium ions then react with the gel to convert it to hard calcium silicate hydrate.
3. The penetrated alkaline solution converts the remaining siliceous minerals into bulky alkali silicate gel. The resultant expansive pressure is stored in the aggregate.
4. The accumulated pressure cracks the aggregate and the surrounding cement paste when the pressure exceeds the tolerance of the aggregate.

Mitigation

ASR can be mitigated in concrete by three complementary approaches:

1. **Limit the alkali metal content of the cement**. Many standards impose limits on the "Equivalent Na_2O" content of cement.
2. **Limit the reactive silica content of the aggregate**. Certain volcanic rocks are particularly susceptible to ASR because they contain volcanic glass (obsidian) and should not be used as aggregate. The use of calcium carbonate aggregates is sometimes envisaged as an ultimate solution to avoid any problem. However, while it may be considered as a necessary condition, it is not a sufficient one. In principle, limestone ($CaCO_3$) is not expected to contain high level of silica, but it actually depends on its purity. Indeed, some siliceous limestones (a.o., Kieselkalk found in Switzerland) may be cemented by amorphous or poorly crystalline silica and can be very sensitive to the ASR reaction as observed with some siliceous limestones exploited in quarries in the area of Tournai in Belgium. So, the use of limestone as aggregate is not a guarantee against ASR in itself. The silica content of the limestone and its reactivity must remain below a threshold value that has to be carefully experimentally assessed by the aggregate producer.
3. **Add very fine siliceous materials** to neutralize the excessive alkalinity of cement with silicic acid by voluntary provoking a controlled pozzolanic reaction at the early stage of the cement setting. Convenient pozzolanic materials to add to the mix may be, *e.g.*, pozzolan, silica fume, fly ashs, or metakaolin. These react preferentially with the cement alkalis without formation of an expansive pressure, because siliceous minerals in fine particles convert to alkali silicate and then to calcium silicate without formation of semipermeable reaction rims.

In other words, as it is sometimes possible *to fight the fire by the fire*, it is also feasible to combat the ASR reaction by itself. A prompt reaction initiated at the early stage of concrete hardening on very fine silica particles will help to suppress a slow and delayed reaction with large siliceous aggregates on the long term. Following the same principle,

the fabrication of low-pH cement also implies the addition of finely-divided pozzolanic materials rich in silicic acid to the concrete mix to decrease its alkalinity.

ASR test

The concrete microbar test was proposed by Grattan-Bellew *et al.* (2003) as a universal accelerated test for alkali-aggregate reaction.

Conversion of high alumina cement

Resistant to weak acids and especially sulfates, this cement cures quickly and reaches very high durability and strength. It was greatly used after World War II for making precast concrete objects. However, it can lose strength with heat or time (conversion), especially when not properly cured. With the collapse of three roofs made of prestressed concrete beams using high alumina cement, this cement was banned in the UK in 1976. Subsequent inquiries into the matter showed that the beams were improperly manufactured, but the ban remained.

Sulfates

Sulfates (SO_4) in the soil or in groundwater, in sufficient concentration, can react with the Portland cement in concrete causing the formation of expansive products, e.g. ettringite or thaumasite, which can lead to early failure of the structure. The most typical attack of this type is on concrete slabs and foundation walls at grade where the sulfate ion, via alternate wetting and drying, can increase in concentration. As the concentration increases, the attack on the Portland cement can begin. For buried structures such as pipe, this type of attack is much rarer especially in the Eastern half of the United States. The sulfate ion concentration increases much slower in the soil mass and is especially dependent upon the initial amount of sulfates in the native soil. The chemical analysis of soil borings should be done during the design phase of any project involving concrete in contact with the native soil to check for the presence of sulfates. If the concentrations are found to be aggressive, various protective coatings can be used. Also, in the US ASTM C150 Type 5 Portland cement can be used in the mix. This type of cement is designed to be particularly resistant to a sulfate attack.

Steel plate construction

In steel plate construction, stringers join parallel steel plates. The plate assemblies are fabricated off site, and welded together on-site to form steel walls connected by stringers. The walls become the form into which concrete is poured. Steel plate construction speeds reinforced concrete construction by cutting out the time consuming on-site manual steps of tying rebar and building forms. The method has excellent strength because the steel is on the outside, where tensile forces are often greatest.

Fiber reinforced concrete

Fibre reinforced concrete (FRC) is concrete containing fibrous material which increases its structural integrity. It contains short discrete fibres that are uniformly distributed and randomly oriented. Fibres include steel fibres, glass fibres, synthetic fibres and natural fibres. Within these different fibres that character of fibre reinforced concrete changes with varying concretes, fibre materials, geometries, distribution, orientation and densities.

Historical perspective

The concept of using fibres as reinforcement is not new. Fibres have been used as reinforcement since ancient times. Historically, horsehair was used in mortar and straw in mud bricks. In the early 1900s, asbestos fibres were used in concrete, and in the 1950s the concept of composite materials came into being and fibre reinforced concrete was one of the topics of interest. There was a need to find a replacement for the asbestos used in concrete and other building materials once the health risks associated with the substance were discovered. By the 1960s, steel, glass (GFRC), and synthetic fibres such as polypropylene fibres were used in concrete, and research into new fibre reinforced concretes continues today.

Effect of fibres in concrete

Fibers are usually used in concrete to control cracking due to both plastic shrinkage and drying shrinkage. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibres produce greater impact, abrasion and shatter resistance in concrete. Generally fibres do not increase the flexural strength of concrete, and so cannot replace moment resisting or structural steel reinforcement. Indeed, some fibres actually reduce the strength of concrete. The amount of fibres added to a concrete mix is expressed as a percentage of the total volume of the composite (concrete and fibres), termed volume fraction (V_f). V_f typically ranges from 0.1 to 3%. Aspect ratio (l/d) is calculated by dividing fibre length (l) by its diameter (d). Fibres with a non-circular cross section use an equivalent diameter for the calculation of aspect ratio. If the modulus of elasticity of the fibre is higher than the matrix (concrete or mortar binder), they help to carry the load by increasing the tensile strength of the material. Increase in the aspect ratio of the fibre usually segments the flexural strength and toughness of the matrix. However, fibres which are too long tend to "ball" in the mix and create workability problems.

Some recent research indicated that using fibres in concrete has limited effect on the impact resistance of the materials[1 & 2]. This finding is very important since traditionally, people think that ductility increases when concrete is reinforced with fibres. The results also indicated out that the use of micro fibres offers better impact resistance compared with the longer fibres.

The High Speed 1 tunnel linings incorporated concrete containing 1 kg/m³ of polypropylene fibres, of diameter 18 & 32 µm, giving the benefits noted below.

Benefits

Polypropylene fibres can:

- Improve mix cohesion, improving pumpability over long distances
- Improve freeze-thaw resistance
- Improve resistance to explosive spalling in case of a severe fire
- Improve impact resistance
- Increase resistance to plastic shrinkage during curing

Steel fibres can:

- Improve structural strength
- Reduce steel reinforcement requirements
- Improve ductility
- Reduce crack widths
- Improve impact & abrasion resistance
- Improve freeze-thaw resistance

Blends of both steel and polymeric fibres are often used in construction projects in order to combine the benefits of both products; structural improvements provided by steel fibres and the resistance to explosive spalling and plastic shrinkage improvements provided by polymeric fibres.

In certain specific circumstances, steel fibre can entirely replace traditional steel reinforcement bar in reinforced concrete. This is most common in industrial flooring but also in some other precasting applications. Typically, these are corroborated with laboratory testing to confirm performance requirements are met. Care should be taken to ensure that local design code requirements are also met which may impose minimum quantities of steel reinforcement within the concrete. There are increasing numbers of tunnelling projects using precast lining segments reinforced only with steel fibres.

Useful standards:

- EN 14889-1:2006 Fibres for Concrete. Steel Fibres. Definitions, specifications & conformity
- EN 14889-2:2006 Fibres for Concrete. Polymer Fibres. Definitions, specifications & conformity
- EN 14845-1:2007 Test methods for fibres in concrete
- ASTM A820-06 Standard Specification for fibres in Fibre Reinforced Concrete
- ASTM C1018-07 Standard test methods for flexural toughness & first crack strength

Some developments in fibre reinforced concrete

The newly developed FRC named Engineered Cementitious Composite (ECC) is 500 times more resistant to cracking and 40 percent lighter than traditional concrete. ECC can sustain strain-hardening up to several percent strain, resulting in a material ductility of at least two orders of magnitude higher when compared to normal concrete or standard fibre reinforced concrete. ECC also has unique cracking behaviour. When loaded to beyond the elastic range, ECC maintains crack width to below 100 μm, even when deformed to several percent tensile strains.

Recent studies performed on a high-performance fibre-reinforced concrete in a bridge deck found that adding fibres provided residual strength and controlled cracking. There were fewer and narrower cracks in the FRC even though the FRC had more shrinkage than the control. Residual strength is directly proportional to the fiber content.

A new kind of natural fibre reinforced concrete (NFRC) made of cellulose fibres processed from genetically modified slash pine trees is giving good results. The cellulose fibres are longer and greater in diameter than other timber sources. Some studies were performed using waste carpet fibers in concrete as an environmentally friendly use of recycled carpet waste. A carpet typically consists of two layers of backing (usually fabric from polypropylene tape yarns), joined by $CaCO_3$ filled styrene-butadiene latex rubber (SBR), and face fibres (majority being nylon 6 and nylon 66 textured yarns). Such nylon and polypropylene fibres can be used for concrete reinforcement.

Non-steel reinforcement

Some construction cannot tolerate the use of steel. For example, MRI machines have huge magnets, and require nonmagnetic buildings. Another example are toll-booths that read radio tags, and need reinforced concrete that is transparent to radio.

In some instances, the lifetime of the concrete structure is more important than its initial costs. Since corrosion is the main cause of failure of reinforced concrete, a corrosion-proof reinforcement can extend a structure's life substantially.

For these purposes some structures have been constructed using fiber-reinforced plastic (FRP) rebar, grids or fibers. The "plastic" reinforcement can be as strong as steel. Because it resists corrosion, it does not need a protective concrete cover of 30 to 50 mm or more as steel reinforcement does. This means that FRP-reinforced structures can be lighter, have longer lifetime and for some applications be price-competitive to steel-reinforced concrete.

Although FRP and concrete are brittle materials, in general structures with internal FRP reinforcement show a great elastic deformability which is comparable to the plastic deformability (ductility) of steel reinforced structures.

Existing structures can be reinforced with external reinforcement as carbon fibre. In this case especially the strength can be increased.

One drawback to use of FRP reinforcement is the limited fire resistance. Structures employing FRP have to ensure the strength and the anchoring of the forces at elevated temperatures. This can be achieved by defining a suitable concrete cover or protective cladding.

Another problem is the effectiveness of shear reinforcement. Stirrups made by (before hardening) bended FRP show generally reduced strength through waved fibres. Exposed to strain and slip the transition between the straight and bent part is loaded by the superposition of a strong bending stress, shear and the longitudinal stress. Alternative shear reinforcement elements can solve this problem.

However, the addition of short monofilament polypropylene fibers to the concrete during mixing may have the beneficial effect of reducing spalling during a fire. In a severe fire, such as the Channel Tunnel fire of 1996, conventionally reinforced concrete can suffer severe spalling leading to failure. This is in part due to the pore water remaining within the concrete boiling explosively; the steam pressure then causes the spalling. The action of fibers within the concrete is due to their ability to melt, forming pathways out through the concrete, allowing the steam pressure to dissipate.

Chapter- 13

Framing (Construction)

A two-story wooden-frame house under construction—the location of the upper floor platform is readily discerned by the wide joists between the floors, and the upper structure rests on this platform.

Framing, in construction known as **light-frame construction**, is a building technique based around structural members, usually called studs, which provide a stable frame to which interior and exterior wall coverings are attached, and covered by a roof comprising horizontal ceiling joists and sloping rafters (together forming a truss structure) or manufactured pre-fabricated roof trusses—all of which are covered by various sheathing materials to give weather resistance.

Modern light-frame structures usually gain strength from rigid panels (plywood and other plywood-like composites such as oriented strand board (OSB) used to form all or part of wall sections, but until recently carpenters employed various forms of diagonal bracing (called *wind braces*) to stabilize walls. Diagonal bracing remains a vital interior part of many roof systems, and in-wall wind braces are required by building codes in many municipalities or by individual state laws in the United States.

Light frame construction using standardized dimensional lumber has become the dominant construction method in North America and Australia because of its economy. Use of minimal structural materials allows builders to enclose a large area with minimal cost, while achieving a wide variety of architectural styles. The ubiquitous platform framing and the older balloon framing are the two different light frame construction systems used in North America.

Walls

Wall framing in house construction includes the vertical and horizontal members of exterior walls and interior partitions, both of bearing walls and non-bearing walls. These *stick* members, referred to as studs, wall plates and lintels (*headers*), serve as a nailing base for all covering material and support the upper floor platforms, which provide the lateral strength along a wall. The platforms may be the boxed structure of a ceiling and roof, or the ceiling and floor joists of the story above. The technique is variously referred to colloquially in the building trades as *stick and frame*, *stick and platform*, or *stick and box* as the sticks (studs) give the structure its vertical support, and the box shaped floor sections with joists contained within length-long post and lintels (more commonly called *headers*), supports the weight of whatever is above, including the next wall up and the roof above the top story. The platform, also provides the lateral support against wind and holds the stick walls true and square. Any lower platform supports the weight of the platforms and walls above the level of its component headers and joists.

Framing lumber should be grade-stamped, and have a moisture content not exceeding 19%.

There are three historically common methods of framing a house.

- Post and Beam, which is now used predominately in barn construction.
- Balloon framing using a technique suspending floors from the walls was common until the late 1940s, but since that time, platform framing has become the predominant form of house construction.
- Platform framing often forms wall sections horizontally on the sub-floor prior to erection, easing positioning of studs and increasing accuracy while cutting the necessary manpower. The top and bottom plates are end-nailed to each stud with two nails at least 3.25 in (83 mm) in length (*16d* or *16 penny* nails). Studs are at least doubled (creating posts) at openings, the jack stud being cut to receive the lintels(headers) that are placed and end-nailed through the outer studs.

Wall sheathing, usually a plywood or other laminate, is usually applied to the framing prior to erection, thus eliminating the need to scaffold, and again increasing speed and cutting manpower needs and expenses. Some types of exterior sheathing, such as asphalt-impregnated fibreboard, plywood, oriented strand board and waferboard, will provide adequate bracing to resist lateral loads and keep the wall square, but construction codes in most jurisdictions will require a stiff plywood sheathing. Others, such as rigid glass-fibre, asphalt-coated fibreboard, polystyrene or polyurethane board, will not. In this latter case,

the wall should be reinforced with a diagonal wood or metal bracing inset into the studs. In jurisdictions subject to strong wind storms (hurricane countries, tornado alleys) local codes or state law will generally require both the diagonal wind braces and the stiff exterior sheathing regardless of the type and kind of outer weather resistant coverings.

Corners

A multiple-stud post made up of at least three studs, or the equivalent, is generally used at exterior corners and intersections to secure a good tie between adjoining walls and to provide nailing support for the interior finish and exterior sheathing. Corners and intersections, however, must be framed with at least two studs.

Nailing support for the edges of the ceiling is required at the junction of the wall and ceiling where partitions run parallel to the ceiling joists. This material is commonly referred to as 'dead wood' or backing.

Exterior wall studs

Wall framing in house construction includes the vertical and horizontal members of exterior walls and interior partitions. These members, referred to as studs, wall plates and lintels, serve as a nailing base for all covering material and support the upper floors, ceiling and roof.

Exterior wall studs are the vertical members to which the wall sheathing and cladding are attached. They are supported on a bottom plate or foundation sill and in turn support the top plate. Studs usually consist of 2 × 4 in (51 × 100 mm) or 2 × 6 in (51 × 150 mm) lumber and are commonly spaced at 16 in (410 mm) on centre. This spacing may be changed to 12 in (300 mm) or 24 in (610 mm) on centre depending on the load and the limitations imposed by the type and thickness of the wall covering used. Wider 2 × 6 in (51 × 150 mm) studs may be used to provide space for more insulation. Insulation beyond that which can be accommodated within a 3.5 in (89 mm) stud space can also be provided by other means, such as rigid or semi-rigid insulation or batts between 2 × 2 in (51 × 51 mm) horizontal furring strips, or rigid or semi-rigid insulation sheathing to the outside of the studs. The studs are attached to horizontal top and bottom wall plates of 2 in (nominal) (38 mm) lumber that are the same width as the studs.

Interior partitions

Interior partitions supporting floor, ceiling or roof loads are called loadbearing walls; others are called non-loadbearing or simply partitions. Interior loadbearing walls are framed in the same way as exterior walls. Studs are usually 2 × 4 in (51 × 100 mm) lumber spaced at 16 in (410 mm) on centre. This spacing may be changed to 12 in (300 mm) or 24 in (610 mm) depending on the loads supported and the type and thickness of the wall finish used.

Partitions can be built with 2 × 3 in (51 × 76 mm) or 2 × 4 in (51 × 100 mm) studs spaced at 16 or 24 in (400 or 600 mm) on center depending on the type and thickness of the wall finish used. Where a partition does not contain a swinging door, 2 × 4 in (51 × 100 mm) studs at 16 in (410 mm) on centre are sometimes used with the wide face of the stud parallel to the wall. This is usually done only for partitions enclosing clothes closets or cupboards to save space. Since there is no vertical load to be supported by partitions, single studs may be used at door openings. The top of the opening may be bridged with a single piece of 2 in (nominal) (38 mm) lumber the same width as the studs. These members provide a nailing support for wall finish, door frames and trim.

Lintels (headers)

Lintels (or, headers) are the horizontal members placed over window, door and other openings to carry loads to the adjoining studs. Lintels are usually constructed of two pieces of 2 in (nominal) (38 mm) lumber separated with spacers to the width of the studs and nailed together to form a single unit. The preferable spacer material is rigid insulation. The depth of a lintel is determined by the width of the opening and vertical loads supported.

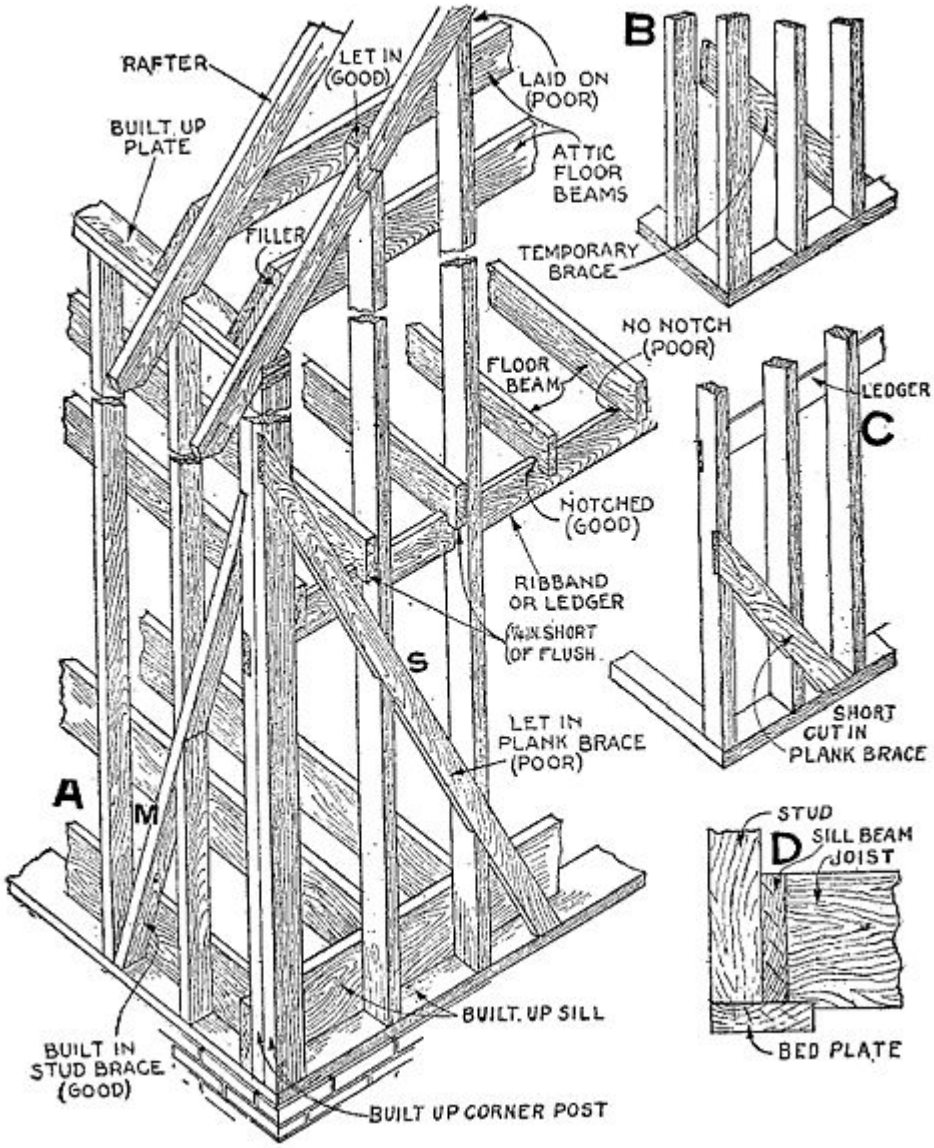
Wall Sections

The complete wall sections are then raised and put in place, temporary braces added and the bottom plates nailed through the subfloor to the floor framing members. The braces should have their larger dimension on the vertical and should permit adjustment of the vertical position of the wall.

Once the assembled sections are plumbed, they are nailed together at the corners and intersections. A strip of polyethylene is often placed between the interior walls and the exterior wall, and above the first top plate of interior walls before the second top plate is applied to attain continuity of the air barrier when polyethylene is serving this function.

A second top plate, with joints offset at least one stud space away from the joints in the plate beneath, is then added. This second top plate usually laps the first plate at the corners and partition intersections and, when nailed in place, provides an additional tie to the framed walls. Where the second top plate does not lap the plate immediately underneath at corner and partition intersections, these may be tied with 0.036 in (0.91 mm) galvanized steel plates at least 3 in (76 mm) wide and 6 in (150 mm) long, nailed with at least three 2.5 in (64 mm) nails to each wall.

Balloon framing



Balloon framing is a method of wood construction used primarily in Scandinavia, Canada and the United States (up until the mid-1950s). It utilizes long continuous framing members (studs) that run from sill plate to eave line with intermediate floor structures nailed to them, with the heights of window sills, headers and next floor height marked out on the studs with a storey pole. Once popular when long lumber was plentiful, balloon framing has been largely replaced by *platform framing*.

While no one is sure who introduced balloon framing in the U.S., the first building using balloon framing was probably a warehouse constructed in 1832 in Chicago by George Washington Snow. The following year, Augustine Taylor (1796–1891) constructed St. Mary's Catholic Church in Chicago using the balloon framing method. Alternately, the

balloon frame has been shown to have been introduced in Missouri as much as fifty years earlier.

The name comes from a French Missouri type of construction, *maison en boulin*. The curious name of this framing technique is conventionally thought to be a derisive one. Historians have fabricated the following story: As Taylor was constructing his first such building, St. Mary's Church, in 1833, skilled carpenters looked on at the comparatively thin framing members, all held together with nails, and declared this method of construction to be no more substantial than a balloon. It would surely blow over in the next wind! Though the criticism proved baseless, the name stuck.

Although lumber was plentiful in 19th century America, skilled labor was not. The advent of cheap machine-made nails, along with water-powered sawmills in the early 19th century made balloon framing highly attractive, because it did not require highly-skilled carpenters, as did the dovetail joints, mortises and tenons required by post-and-beam construction. For the first time, any farmer could build his own buildings without a time-consuming learning curve.

It has been said that balloon framing populated the western United States and the western provinces of Canada. Without it, western boomtowns certainly could not have blossomed overnight. It is also a fair certainty that, by radically reducing construction costs, balloon framing improved the shelter options of poorer North Americans. For example, many 19th century New England working neighborhoods consist of balloon-constructed three-story apartment buildings referred to as triple deckers.

The main difference between platform and balloon framing is at the floor lines. The balloon wall studs extend from the sill of the first story all the way to the top plate or end rafter of the second story. The platform-framed wall, on the other hand, is independent for each floor.

Balloon framing has several disadvantages as a construction method:

1. The creation of a path for fire to readily travel from floor to floor. This is mitigated with the use of firestops at each floor level.
2. The lack of a working platform for work on upper floors. Whereas workers can readily reach the top of the walls being erected with platform framing, balloon construction requires scaffolding to reach the tops of the walls (which are often two or three stories above the working platform).
3. The requirement for long framing members.
4. In certain larger buildings, a noticeable down-slope of floors towards central walls, caused by the differential shrinkage of the wood framing members at the perimeter versus central walls. Larger balloon-framed buildings will have central bearing walls which are actually platform framed and thus will have horizontal sill and top plates at each floor level, plus the intervening floor joists, at these central walls. Wood will shrink much more across its grain than along the grain. Therefore, the cumulative shrinkage in the center of such a building is

considerably more than the shrinkage at the perimeter where there are much fewer horizontal members. Of course, this problem, unlike the first three, takes time to develop and become noticeable.

5. Present day balloon framing buildings have considerably higher heating costs, due to the lack of insulation separating a room from its exterior walls.

Since steel is generally more fire-resistant than wood, and steel framing members can be made to arbitrary lengths, balloon framing is growing in popularity again in light gauge steel stud construction. Balloon framing provides a more direct load path down to the foundation. Additionally, balloon framing allows more flexibility for tradesmen in that it is significantly easier to pull wire, piping and ducting without having to bore through or work around framing members.

Platform framing

In Canada and the United States, the most common method of light-frame construction for houses and small apartment buildings as well as some small commercial buildings is *platform framing*.

The framed structure sits atop a concrete (most common) or treated wood foundation. A sill plate is anchored, usually with 'J' bolts to the foundation wall. Generally these plates must be pressure treated to keep from rotting. The bottom of the sill plate is raised a minimum 6 inches (150 mm) above the finished grade by the foundation. This again is to prevent the sill-plate from rotting as well as providing a termite barrier.

The floors, walls and roof of a framed structure are created by assembling (using nails) consistently sized framing elements of dimensional lumber (2×4, 2×6, etc.) at regular spacings (12 in, 16 in, and 24 in on center. Sometimes the lesser known -19.2" on center-method is used), forming stud-bays (wall) or joist-bays (floor). The floors, walls and roof are typically made torsionally stable with the installation of a plywood or composite wood skin referred to as sheathing. Sheathing has very specific nailing requirements (such as size and spacing); these measures allow a known amount of shear force to be resisted by the element. Spacing the framing members properly allows them to align with the edges of standard sheathing. In the past, tongue and groove planks installed diagonally were used as sheathing. Occasionally, wooden or galvanized steel braces are used instead of sheathing. There are also engineered wood panels made for shear and bracing.

The floor, or the platform of the name, is made up of joists (usually 2x6, 2×8, 2×10 or 2×12, depending on the span) that sit on supporting walls, beams or girders. The floor joists are spaced at (12 in, 16 in, and 24 in on center) and covered with a plywood subfloor. In the past, 1x planks set at 45-degrees to the joists were used for the subfloor.

Where the design calls for a framed floor, the resulting platform is where the framer will construct and stand that floor's walls (interior and exterior load bearing walls and space-dividing, non-load bearing partitions). Additional framed floors and their walls may then

be erected to a general maximum of four in wood framed construction. There will be no framed floor in the case of a single-level structure with a concrete floor known as a *slab on grade*.

Stairs between floors are framed by installing stepped *stringers* and then placing the horizontal *treads* and vertical *risers*.

A framed roof is an assembly of rafters and wall-ties supported by the top story's walls. Prefabricated and site-built trussed rafters are also used along with the more common stick framing method. *Trusses* are engineered to redistribute tension away from wall-tie members and the ceiling members. The roof members are covered with sheathing or strapping to form the roof deck for the finish roofing material.

Floor joists can be engineered lumber (trussed, I-joist, etc.), conserving resources with increased rigidity and value. They allow access for runs of plumbing, HVAC, etc. and some forms are pre-manufactured.

Double framing is a style of framing used to reduce heat loss and air infiltration. Two walls are built around the perimeter of the building with a small gap in between. The inner wall carries the structural load of the building and is constructed as described above. The exterior wall is not load bearing and can be constructed using lighter materials. Insulation is installed in the entire space between the outside edge of the exterior wall and the inside edge of the interior wall. The size of the gap depends upon how much insulation is desired. The vapour barrier is installed on the outside of the inner wall, rather than between the studs and drywall of a standard framed structure. This increases its effectiveness as it is not perforated by electrical and plumbing connections.

Materials

Light-frame materials are most often wood or rectangular steel tubes or C-channels. Wood pieces are typically connected with nails or screws; steel pieces are connected by screws. Preferred species for linear structural members are softwoods such as spruce, pine and fir. Light frame material dimensions range from 38 mm by 89 mm (1.5 in by 3.5 in; i.e., a two-by-four) to 5 cm by 30 cm (two-by-twelve inches) at the cross-section, and lengths ranging from 2.5 m (8.2 ft) for walls to 7 m (23 ft) or more for joists and rafters. Recently, architects have begun experimenting with pre-cut modular aluminum framing to reduce on-site construction costs.

Wall panels built of studs are interrupted by sections that provide rough openings for doors and windows. Openings are typically spanned by a header or lintel that bears the weight of structure above the opening. Headers are usually built to rest on trimmers, also called jacks. Areas around windows are defined by a sill beneath the window, and cripples, which are shorter studs that span the area from the bottom plate to the sill and sometimes from the top of the window to a header, or from a header to a top plate. Diagonal bracings made of wood or steel provide shear (horizontal strength) as do panels of sheeting nailed to studs, sills and headers.

Light-gauge metal stud framing

Wall sections usually include a bottom plate which is secured to the structure of a floor, and one, or more often two top plates that tie walls together and provide a bearing for structures above the wall. Wood or steel floor frames usually include a rim joist around the perimeter of a system of floor joists, and often include bridging material near the center of a span to prevent lateral buckling of the spanning members. In two-story construction, openings are left in the floor system for a stairwell, in which stair risers and treads are most often attached to squared faces cut into sloping stair stringers.

Interior wall coverings in light-frame construction typically include wallboard, lath and plaster or decorative wood paneling.

Exterior finishes for walls and ceilings often include plywood or composite sheathing, brick or stone veneers, and various stucco finishes. Cavities between studs, usually placed 40–60 cm (16–24 in) apart, are usually filled with insulation materials, such as fiberglass batting, or cellulose filling sometimes made of recycled newsprint treated with boron additives for fire prevention and vermin control.

In natural building, straw bales, cob and adobe may be used for both exterior and interior walls. The part of a structural building that goes diagonally across a wall is called a T-bar. It stops the walls from collapsing in gusty winds.

Roofs

Roofs are usually built to provide a sloping surface intended to shed rain or snow, with slopes ranging from 1 cm of rise per 15 cm (less than an inch per linear foot) of rafter length, to steep slopes of more than 2 cm per cm (two feet per foot) of rafter length. A light-frame structure built mostly inside sloping walls comprising a roof is called an A-frame.

Roofs are most often covered with shingles made of asphalt, fiberglass and small gravel coating, but a wide range of materials are used. Molten tar is often used to waterproof flatter roofs, but newer materials include rubber and synthetic materials. Steel panels are popular roof coverings in some areas, preferred for their durability. Slate or tile roofs offer more historic coverings for light-frame roofs.

Light-frame methods allow easy construction of unique roof designs. Hip roofs, which slope toward walls on all sides and are joined at hip rafters that span from corners to a ridge. Valleys are formed when two sloping roof sections drain toward each other. Dormers are small areas in which vertical walls interrupt a roof line, and which are topped off by slopes at usually right angles to a main roof section. Gables are formed when a length-wise section of sloping roof ends to form a triangular wall section. Clerestories are formed by an interruption along the slope of a roof where a short vertical wall connects it to another roof section. Flat roofs, which usually include at least a nominal slope to shed water, are often surrounded by parapet walls with openings (called scuppers) to allow water to drain out. Sloping crickets are built into roofs to direct water away from areas of poor drainage, such as behind a chimney at the bottom of a sloping section.

Structure

Light-frame buildings are often erected on monolithic concrete slab foundations that serve both as a floor and as a support for the structure. Other light-frame buildings are built over a crawlspace or a basement, with wood or steel joists used to span between foundation walls, usually constructed of poured concrete or concrete blocks.

Engineered components are commonly used to form floor, ceiling and roof structures in place of solid wood. I-joists (closed-web trusses) are often made from laminated woods, most often chipped poplar wood, in panels as thin as 1 cm (0.4 in), glued between horizontally laminated members of less than 4 cm by 4 cm (*two-by-twos*), to span distances of as much as 9 m (30 ft). Open web trussed joists and rafters are often formed of 4 cm by 9 cm (*two-by-four* [sic]) wood members to provide support for floors, roofing systems and ceiling finishes.