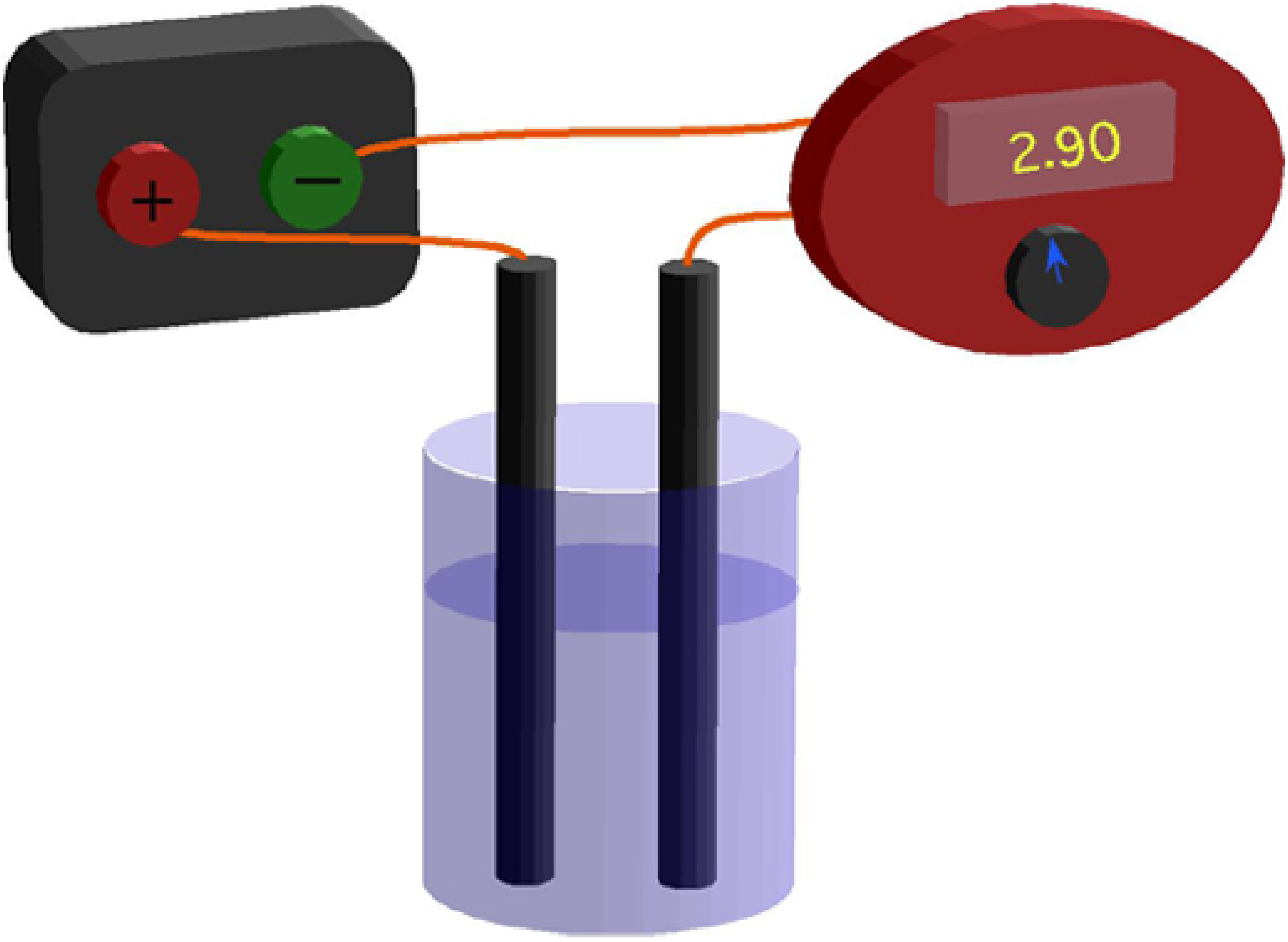
Heat Conduction & Heat Pump Technologies



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Chapter 1

Conduction (Heat)

In heat transfer, **conduction** (or **heat conduction**) is the transfer of thermal energy between regions of matter due to a temperature gradient. Heat spontaneously flows from a region of higher temperature to a region of lower temperature, and reduces temperature differences over time, approaching thermal equilibrium. The previous statement can be argued to apply to heat transfer in general, but to distinguish conduction specifically, it should be stated that the heat flows through the region of matter itself, as opposed to requiring electromagnetic waves as does radiation or to requiring bulk motion of the matter as does convection. Conduction takes place in all forms of matter, viz. solids, liquids, gases and plasmas, but does not require any bulk motion of matter. In solids, it is due to the combination of vibrations of the molecules in a lattice or phonons with the energy transported by free electrons. In gases and liquids, conduction is due to the collisions and diffusion of the molecules during their random motion.

In the engineering sciences, heat transfer includes the processes of thermal radiation, convection, and sometimes mass transfer and often more than one of these processes occurs in a given situation.

Overview

On a microscopic scale, conduction occurs as rapidly moving or vibrating atoms and molecules interact with neighboring particles, transferring some of their kinetic energy. Heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from one atom to another. Conduction is the most significant means of heat transfer within a solid or between solid objects in thermal contact. Conduction is greater in solids because the network of relatively fixed spacial relationships between atoms helps to transfer energy between them by vibration.

As density decreases so does conduction. Therefore, fluids (and especially gases) are less conductive. This is due to the large distance between atoms in a gas: fewer collisions between atoms means less conduction. Conductivity of gases increases with temperature. Conductivity increases with increasing pressure from vacuum up to a critical point that the density of the gas is such that molecules of the gas may be expected to collide with each other before they transfer heat from one surface to another. After this point conductivity increases only slightly with increasing pressure and density.

Thermal contact conductance is the study of heat conduction between solid bodies in contact. A temperature drop is often observed at the interface between the two surfaces in contact. This phenomenon is said to be a result of a thermal contact resistance existing between the contacting surfaces. Interfacial thermal resistance is a measure of an interface's resistance to thermal flow. This thermal resistance differs from contact resistance, as it exists even at atomically perfect interfaces. Understanding the thermal resistance at the interface between two materials is of primary significance in the study of its thermal properties. Interfaces often contribute significantly to the observed properties of the materials.

The inter-molecular transfer of energy could be primarily by elastic impact as in fluids or by free electron diffusion as in metals or phonon vibration as in insulators. In insulators the heat flux is carried almost entirely by phonon vibrations.

Metals (e.g. copper, platinum, gold,etc.) are usually the best conductors of thermal energy. This is due to the way that metals are chemically bonded: metallic bonds (as opposed to covalent or ionic bonds) have free-moving electrons which are able to transfer thermal energy rapidly through the metal. The "electron fluid" of a conductive metallic solid conducts nearly all of the heat flux through the solid. Phonon flux is still present, but carries less than 1% of the energy. Electrons also conduct electric current through conductive solids, and the thermal and electrical conductivities of most metals have about the same ratio. A good electrical conductor, such as copper, usually also conducts heat well. The Peltier-Seebeck effect exhibits the propensity of electrons to conduct heat through an electrically conductive solid. Thermoelectricity is caused by the relationship between electrons, heat fluxes and electrical currents. Heat conduction within a solid is directly analogous to diffusion of particles within a fluid, in the situation where there are no fluid currents.

To quantify the ease with which a particular medium conducts, engineers employ the thermal conductivity, also known as the conductivity constant or conduction coefficient, k. In thermal conductivity k is defined as "the quantity of heat, Q, transmitted in time (t) through a thickness (L), in a direction normal to a surface of area (A), due to a temperature difference (ΔT) [...]." Thermal conductivity is a material *property* that is primarily dependent on the medium's phase, temperature, density, and molecular bonding. Thermal effusivity is a quantity derived from conductivity which is a measure of its ability to exchange thermal energy with its surroundings.

Steady-state conduction

Steady state conduction is the form of conduction that happens when the temperature difference(s) driving the conduction are constant, so that (after an equilibration time), the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. Thus, all partial derivatives of temperature *with respect to space* may either be zero or have values, but all derivatives of temperature at any point *with respect to time* are uniformly zero. In steady state conduction, the amount of heat entering any region of an object is equal to amount of heat coming out (if this were not so, the temperature would be rising or falling, as thermal energy was tapped or trapped in a region).

For example, a bar may be cold at one end and hot at the other, but after a state of steady state conduction is reached, the spacial gradient of temperatures along the bar does not change any further, as time proceeds. Instead, the temperature at any given section of the rod remains constant, and this temperature varies linearly in space, along the direction of heat transfer.

In steady state conduction, all the laws of direct current electrical conduction can be applied to "heat currents". In such cases, it is possible to take "thermal resistances" as the analog to electrical resistances. In such cases, temperature plays the role of voltage, and heat transferred per unit time (heat power) is the analog of electrical current.

Transient conduction

In non-steady-state situations, in which a new temperature change at a boundary, or a new source of heat has been suddenly introduced, a system will change in time to reach a new equilibrium. Such systems eventually establish steady-state thermal gradients, if the introduction and removal of heat is continuous. During the time-period of establishment of such new conditions, the mode of thermal energy flow is termed *transient conduction*.

Transient conduction occurs in any situation where there is a change in temperature from external or internal sources or sinks of heat, causing a new source of heat-input (or output) within an object, or into or out of the object (to the external surroundings). New sources of heat "turning on" within an object can cause the phenomenon (for example, an engine starting in a car). New external conditions also cause this process: the copper bar in the above example would experience transient conduction as soon as one end was subjected to a different temperature from the other. Over time, the field of temperatures inside the bar would reach a new steady-state, in which a constant temperature gradient along the bar will finally be set up, and this gradient would then stay constant, in time. Typically, such a new steady state gradient is approached exponentially with time, after a new temperature-or-heat source or sink, has been introduced. To this extent, the "transient conduction" phase is then over, even though heat flow may be still continue at high power.

For example, in an automobile, the transient thermal conduction phase for the entire machine would be over, and the steady state phase would appear, as soon as the engine had reached steady-state operating temperature. In this state, temperature would vary greatly from cylinders to other parts of the car, but at no point would the temperature be increasing or decreasing.

An example of transient conduction which does not end with steady-state conduction, but rather no conduction, occurs when a hot copper ball is dropped into oil at a low temperature. Here the temperature field within the object begins to change as a function of time, as the heat is removed from the metal, and the interest lies in analyzing this spatial change of temperature within the object over time, until all gradients disappear entirely (the ball has reached the same temperature as the oil). Mathematically, this condition is also approached exponentially; in theory it takes infinite time, but in practice it is over, for all intents and purposes, in a much shorter period. At the end of this process with no heat sink but the internal parts of the ball (which are finite), there is no steady state heat conduction to be reached. Such a state never occurs in this situation, but rather the end of the process is when there is no heat conduction at all.

Analysis of non steady-state conduction systems is more complex than steady state systems, and (except for simple shapes) calls for the application of approximation theories, and/or numerical analysis by computer. One popular graphical method involves the use of Heisler Charts.

Relativistic conduction

The theory of relativistic heat conduction is a model that is compatible with the theory of special relativity. For most of the last century, it was recognized that Fourier equation is in contradiction with the theory of relativity because it admits an infinite speed of propagation of heat signals. For example, according to Fourier equation, a pulse of heat at the origin would be felt at infinity instantaneously. The speed of information propagation is faster than the speed of light in vacuum, which is physically inadmissible within the framework of relativity. Alterations to the Fourier model provided for a relativistic model of heat conduction, avoiding this problem.

Quantum conduction

Second sound is a quantum mechanical phenomenon in which heat transfer occurs by wave-like motion, rather than by the more usual mechanism of diffusion. Heat takes the place of pressure in normal sound waves. This leads to a very high thermal conductivity. It is known as "second sound" because the wave motion of heat is similar to the propagation of sound in air.

Fourier's law

The law of Heat Conduction, also known as Fourier's law, states that the time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which the heat is flowing. We can state this law in two equivalent forms: the integral form, in which we look at the amount of energy flowing into or out of a body as a whole, and the differential form, in which we look at the flow rates or fluxes of energy locally.

Differential form

The differential form of Fourier's Law of thermal conduction shows that the local heat flux density, $\overrightarrow{q}$, is equal to the product of thermal conductivity, *k*, and the negative local temperature gradient, $-\nabla T$. The heat flux density is the amount of energy that flows through a unit area per unit time.

$$\overrightarrow{q} = -k\nabla T$$

where (including the SI units)

$\overrightarrow{q}$ is the local heat flux, W·m^{-2}
k is the material's conductivity, W·m^{-1}·K^{-1},
∇T is the temperature gradient, K·m^{-1}.

The thermal conductivity, *k*, is often treated as a constant, though this is not always true. While the thermal conductivity of a material generally varies with temperature, the variation can be small over a significant range of temperatures for some common materials. In anisotropic materials, the thermal conductivity typically varies with orientation; in this case *k* is represented by a second-order tensor. In nonuniform materials, *k* varies with spatial location.

For many simple applications, Fourier's law is used in its one-dimensional form. In the x-direction,

$$q_x = -k\frac{dT}{dx}$$

Integral form

By integrating the differential form over the material's total surface *S*, we arrive at the integral form of Fourier's law:

$$\frac{\partial Q}{\partial t} = -k \oint_S \overrightarrow{\nabla} T \cdot \overrightarrow{dA}$$

where (including the SI units)

$\frac{\partial Q}{\partial t}$ is the amount of heat transferred per unit time (in W) and
$\overrightarrow{dA}$ is an oriented surface area element (in m^2)

The above differential equation, when integrated for a homogeneous material of 1-D geometry between two endpoints at constant temperature, gives the heat flow rate as:

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}$$

where

A is the cross-sectional surface area,
ΔT is the temperature difference between the ends,
Δx is the distance between the ends.

This law forms the basis for the derivation of the heat equation. Ohm's law is the electrical analogue of Fourier's law.

Conductance

Writing

$$U = \frac{k}{\Delta x},$$

where U is the conductance, in W/(m^2 K).

Fourier's law can also be stated as:

$$\frac{\Delta Q}{\Delta t} = UA\,(-\Delta T).$$

The reciprocal of conductance is resistance, R, given by:

$$R = \frac{1}{U} = \frac{\Delta x}{k} = \frac{A\,(-\Delta T)}{\frac{\Delta Q}{\Delta t}},$$

and it is resistance which is additive when several conducting layers lie between the hot and cool regions, because A and Q are the same for all layers. In a multilayer partition, the total conductance is related to the conductance of its layers by:

$$\frac{1}{U} = \frac{1}{U_1} + \frac{1}{U_2} + \frac{1}{U_3} + \cdots$$

So, when dealing with a multilayer partition, the following formula is usually used:

$$\frac{\Delta Q}{\Delta t} = \frac{A\,(-\Delta T)}{\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3} + \cdots}.$$

When heat is being conducted from one fluid to another through a barrier, it is sometimes important to consider the conductance of the thin film of fluid which remains stationary next to the barrier. This thin film of fluid is difficult to quantify, its characteristics depending upon complex conditions of turbulence and viscosity, but when dealing with thin high-conductance barriers it can sometimes be quite significant.

Intensive-property representation

The previous conductance equations, written in terms of extensive properties, can be reformulated in terms of intensive properties.

Ideally, the formulae for conductance should produce a quantity with dimensions independent of distance, like Ohm's Law for electrical resistance: $R = V/I$, and conductance: $G = I/V$.

From the electrical formula: $R = \rho x/A$, where ρ is resistivity, x = length, and A is cross-sectional area, we have $G = kA/x$, where G is conductance, k is conductivity, x = length, and A = cross-sectional area.

For Heat,

$$U = \frac{kA}{\Delta x},$$

where *U* is the conductance.

Fourier's law can also be stated as:

$$\dot{Q} = U\,\Delta T$$

analogous to Ohm's law: $I = V/R$ or $I = VG.$

The reciprocal of conductance is resistance, R, given by:

$$R = \frac{\Delta T}{\dot{Q}},$$

analogous to Ohm's law: $R = V/I.$

The rules for combining resistances and conductances (in series and in parallel) are the same for both heat flow and electric current.

Cylinders

Conduction through cylinders can be calculated when variables such as the internal radius r_1, the external radius r_2, and the length denoted as ℓ.

The temperature difference between the inner and outer wall can be expressed as $T_2 - T_1$.

The area of the heat flow: $A_r = 2\pi r\ell$

When Fourier's equation is applied:

$$Q = -kA_r\frac{\mathrm{d}T}{\mathrm{d}r} = -2k\pi r\ell\frac{\mathrm{d}T}{\mathrm{d}r}$$

Rearranged:

$$Q\int_{r_1}^{r_2}\frac{1}{r}\mathrm{d}r = -2k\pi\ell\int_{T_1}^{T_2}\mathrm{d}T$$

Therefore the rate of heat transfer is

$$Q = 2k\pi\ell\frac{T_1 - T_2}{\ln(r_2/r_1)}$$

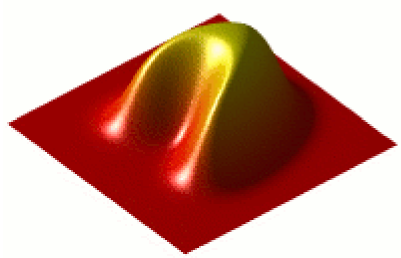
The thermal resistance is

$$R_c = \frac{\Delta T}{Q} = \frac{\ln(r_2/r_1)}{2\pi k\ell}$$

And $Q = 2\pi k\ell r_m\frac{T_1 - T_2}{r_2 - r_1}$, where $r_m = \frac{r_2 - r_1}{\ln(r_2/r_1)}$ and it is important to note that this is the log-mean radius.

Chapter 2

Heat Equation



The heat equation predicts that if a hot body is placed in a box of cold water, the temperature of the body will decrease, and eventually (after infinite time, and subject to no external heat sources) the temperature in the box will equalize.

The **heat equation** is an important partial differential equation which describes the distribution of heat (or variation in temperature) in a given region over time. For a function $u(x,y,z,t)$ of three spatial variables (x,y,z) and the time variable t, the heat equation is

$$\frac{\partial u}{\partial t} - \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) = 0$$

also written

$$\frac{\partial u}{\partial t} - \alpha \nabla^2 u = 0$$

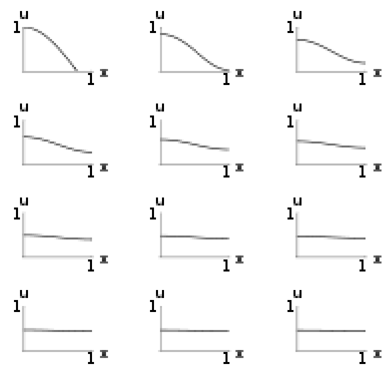
or sometimes

$$\frac{\partial u}{\partial t} - \alpha \Delta u = 0$$

where α is a positive constant and Δ or ∇^2 denotes the Laplace operator. In the physical problem of temperature variation, $u(x,y,z,t)$ is the temperature and α is the thermal diffusivity. For the mathematical treatment it is sufficient to consider the case α = 1.

The heat equation is of fundamental importance in diverse scientific fields. In mathematics, it is the prototypical parabolic partial differential equation. In probability theory, the heat equation is connected with the study of Brownian motion via the Fokker–Planck equation. In financial mathematics it is used to solve the Black–Scholes partial differential equation. The diffusion equation, a more general version of the heat equation, arises in connection with the study of chemical diffusion and other related processes.

General description



Graphical representation of the solution to a 1D heat equation PDE.

Suppose one has a function *u* which describes the temperature at a given location (*x*, *y*, *z*). This function will change over time as heat spreads throughout space. The heat equation is used to determine the change in the function *u* over time. The image to the right is animated and describes the way heat changes in time along a metal bar. One of the interesting properties of the heat equation is the maximum principle which says that the maximum value of *u* is either earlier in time than the region of concern or on the edge of the region of concern. This is essentially saying that temperature comes either from some source or from earlier in time because heat permeates but is not created from nothingness. This is a property of parabolic partial differential equations and is not difficult to prove mathematically.

Another interesting property is that even if *u* has a discontinuity at an initial time $t = t_0$, the temperature becomes smooth as soon as $t > t_0$. For example, if a bar of metal has temperature 0 and another has temperature 100 and they are stuck together end to end, then very quickly the temperature at the point of connection is 50 and the graph of the temperature is smoothly running from 0 to 100.

The heat equation is used in probability and describes random walks. It is also applied in financial mathematics for this reason.

It is also important in Riemannian geometry and thus topology: it was adapted by Richard Hamilton when he defined the Ricci flow that was later used by Grigori Perelman to solve the topological Poincaré conjecture.

The physical problem and the equation

Derivation in one dimension

The heat equation is derived from Fourier's law and conservation of energy (Cannon 1984). By Fourier's law, the flow rate of heat energy through a surface is proportional to the negative temperature gradient across the surface,

$$\mathbf{q} = -k\nabla u$$

where *k* is the thermal conductivity and *u* is the temperature. In one dimension, the gradient is an ordinary spatial derivative, and so Fourier's law is

$$\mathbf{q} = -ku_x$$

where u_x is du/dx. In the absence of work done, a change in internal energy per unit volume in the material, ΔQ, is proportional to the change in temperature, Δu. That is,

$$\Delta Q = c_p \rho \Delta u$$

where c_p is the specific heat capacity and ρ is the mass density of the material. (In this section only, Δ is the ordinary difference operator, not the Laplacian.) Choosing zero energy at absolute zero temperature, this can be rewritten as

$$Q = c_p \rho u.$$

The increase in internal energy in a small spatial region of the material

$$x - \Delta x \le \xi \le x + \Delta x$$

over the time period

$$t - \Delta t \le \tau \le t + \Delta t$$

is given by

$$c_p \rho \int_{x-\Delta x}^{x+\Delta x} [u(\xi, t+\Delta t) - u(\xi, t-\Delta t)]\, d\xi = c_p \rho \int_{t-\Delta t}^{t+\Delta t} \int_{x-\Delta x}^{x+\Delta x} \frac{\partial u}{\partial \tau}\, d\xi d\tau$$

where the fundamental theorem of calculus was used. Additionally, with no work done and absent any heat sources or sinks, the change in internal energy in the interval [*x*-Δ*x*,

$x+\Delta x$] is accounted for entirely by the flux of heat across the boundaries. By Fourier's law, this is

$$k\int_{t-\Delta t}^{t+\Delta t}\left[\frac{\partial u}{\partial x}(x+\Delta x,\tau)-\frac{\partial u}{\partial x}(x-\Delta x,\tau)\right]\,d\tau = k\int_{t-\Delta t}^{t+\Delta t}\int_{x-\Delta x}^{x+\Delta x}\frac{\partial^2 u}{\partial \xi^2}\,d\xi\,d\tau$$

again by the fundamental theorem of calculus. By conservation of energy,

$$\int_{t-\Delta t}^{t+\Delta t}\int_{x-\Delta x}^{x+\Delta x}[c_p\rho u_\tau - k u_{\xi\xi}]\,d\xi\,d\tau = 0.$$

This is true for any rectangle [$t-\Delta t$, $t+\Delta t$] × [$x-\Delta x$, $x+\Delta x$]. Consequently, the integrand must vanish identically:

$$c_p\rho u_t - k u_{xx} = 0.$$

Which can be rewritten as:

$$u_t = \frac{k}{c_p\rho}u_{xx},$$

or:

$$\frac{\partial u}{\partial t} = \frac{k}{c_p\rho}\left(\frac{\partial^2 u}{\partial x^2}\right)$$

which is the heat equation. The coefficient $k/(c_p\rho)$ is called thermal diffusivity and is often denoted α.

Three-dimensional problem

In the special case of heat propagation in an isotropic and homogeneous medium in a 3-dimensional space, this equation is

$$\frac{\partial u}{\partial t} = \alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) = \alpha(u_{xx}+u_{yy}+u_{zz})$$

where:

- $u = u(x, y, z, t)$ is temperature as a function of space and time;

- $\frac{\partial u}{\partial t}$ is the rate of change of temperature at a point over time;

- u_{xx}, u_{yy}, and u_{zz} are the second spatial derivatives (*thermal conductions*) of temperature in the x, y, and z directions, respectively;

- $\alpha = k / c_p\rho$ is the thermal diffusivity, a material-specific quantity depending on the *thermal conductivity*, k, the *mass density*, ρ, and the *specific heat capacity*, c_p.

The heat equation is a consequence of Fourier's law of cooling.

If the medium is not the whole space, in order to solve the heat equation uniquely we also need to specify boundary conditions for u. To determine uniqueness of solutions in the whole space it is necessary to assume an exponential bound on the growth of solutions, this assumption is consistent with observed experiments.

Solutions of the heat equation are characterized by a gradual smoothing of the initial temperature distribution by the flow of heat from warmer to colder areas of an object. Generally, many different states and starting conditions will tend toward the same stable equilibrium. As a consequence, to reverse the solution and conclude something about earlier times or initial conditions from the present heat distribution is very inaccurate except over the shortest of time periods.

The heat equation is the prototypical example of a parabolic partial differential equation.

Using the Laplace operator, the heat equation can be simplified, and generalized to similar equations over spaces of arbitrary number of dimensions, as

$$u_t = \alpha \nabla^2 u = \alpha \Delta u,$$

where the Laplace operator, Δ or ∇^2, the divergence of the gradient, is taken in the spatial variables.

The heat equation governs heat diffusion, as well as other diffusive processes, such as particle diffusion or the propagation of action potential in nerve cells. Although they are not diffusive in nature, some quantum mechanics problems are also governed by a mathematical analog of the heat equation. It also can be used to model some phenomena arising in finance, like the Black-Scholes or the Ornstein-Uhlenbeck processes. The equation, and various non-linear analogues, has also been used in image analysis.

The heat equation is, technically, in violation of special relativity, because its solutions involve instantaneous propagation of a disturbance. The part of the disturbance outside the forward light cone can usually be safely neglected, but if it is necessary to develop a reasonable speed for the transmission of heat, a hyperbolic problem should be considered instead – like a partial differential equation involving a second-order time derivative.

Internal heat generation

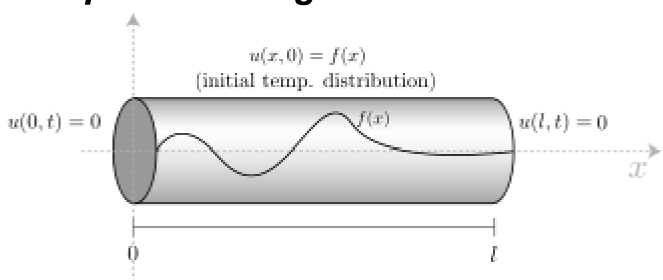
The function u above represents temperature of a body. Alternatively, it is sometimes convenient to change units and represent u as the heat density of a medium. Since heat density is proportional to temperature in a homogeneous medium, the heat equation is still obeyed in the new units.

Suppose that a body obeys the heat equation and, in addition, generates its own heat per unit volume (e.g., in watts/L) at a rate given by a known function q varying in space and time. Then the heat per unit volume u satisfies an equation

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + q.$$

For example, a tungsten light bulb filament generates heat, so it would have a positive nonzero value for q when turned on. While the light is turned off, the value of q for the tungsten filament would be zero.

Solving the heat equation using Fourier series



Idealized physical setting for heat conduction in a rod with homogeneous boundary conditions.

The following solution technique for the heat equation was proposed by Joseph Fourier in his treatise *Théorie analytique de la chaleur*, published in 1822. Let us consider the heat equation for one space variable. This could be used to model heat conduction in a rod. The equation is

$$(1)\ u_t = \alpha u_{xx}$$

where $u = u(x, t)$ is a function of two variables x and t. Here

- x is the space variable, so $x \in [0,L]$, where L is the length of the rod.
- t is the time variable, so $t \geq 0$.

We assume the initial condition

$$(2)\ u(x,0) = f(x) \quad \forall\, x \in [0,L]$$

where the function *f* is given and the boundary conditions

$$(3)\ u(0,t) = 0 = u(L,t) \quad \forall\, t > 0.$$

Let us attempt to find a solution of (1) which is not identically zero satisfying the boundary conditions (3) but with the following property: *u* is a product in which the dependence of *u* on *x*, *t* is separated, that is:

$$(4)\ u(x,t) = X(x)T(t).$$

This solution technique is called separation of variables. Substituting *u* back into equation (1),

$$\frac{T'(t)}{\alpha T(t)} = \frac{X''(x)}{X(x)}.$$

Since the right hand side depends only on *x* and the left hand side only on *t*, both sides are equal to some constant value $-\lambda$. Thus:

$$(5)\ T'(t) = -\lambda\alpha T(t)$$

and

$$(6)\ X''(x) = -\lambda X(x).$$

We will now show that solutions for (6) for values of $\lambda \le 0$ cannot occur:

1. Suppose that $\lambda < 0$. Then there exist real numbers *B*, *C* such that

   $$X(x) = Be^{\sqrt{-\lambda}\,x} + Ce^{-\sqrt{-\lambda}\,x}.$$

   From (3) we get

   $$X(0) = 0 = X(L).$$

   and therefore $B = 0 = C$ which implies *u* is identically 0.

2. Suppose that $\lambda = 0$. Then there exist real numbers *B*, *C* such that

   $$X(x) = Bx + C.$$

   From equation (3) we conclude in the same manner as in 1 that *u* is identically 0.

3. Therefore, it must be the case that $\lambda > 0$. Then there exist real numbers A, B, C such that

$$T(t) = Ae^{-\lambda\alpha t}$$

and

$$X(x) = B\sin(\sqrt{\lambda}\,x) + C\cos(\sqrt{\lambda}\,x).$$

From (3) we get $C = 0$ and that for some positive integer n,

$$\sqrt{\lambda} = n\frac{\pi}{L}.$$

This solves the heat equation in the special case that the dependence of u has the special form (4).

In general, the sum of solutions to (1) which satisfy the boundary conditions (3) also satisfies (1) and (3). We can show that the solution to (1), (2) and (3) is given by

$$u(x,t) = \sum_{n=1}^{+\infty} D_n \left(\sin\frac{n\pi x}{L}\right) e^{-\frac{n^2\pi^2\alpha t}{L^2}}$$

where

$$D_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx.$$

Generalizing the solution technique

The solution technique used above can be greatly extended to many other types of equations. The idea is that the operator u_{xx} with the zero boundary conditions can be represented in terms of its eigenvectors. This leads naturally to one of the basic ideas of the spectral theory of linear self-adjoint operators.

Consider the linear operator $\Delta u = u_{xx}$. The infinite sequence of functions

$$e_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

for $n \geq 1$ are eigenvectors of Δ. Indeed

$$\Delta e_n = -\frac{n^2\pi^2}{L^2}e_n.$$

Moreover, any eigenvector f of Δ with the boundary conditions $f(0)=f(L)=0$ is of the form e_n for some $n \geq 1$. The functions e_n for $n \geq 1$ form an orthonormal sequence with respect to a certain inner product on the space of real-valued functions on $[0, L]$. This means

$$\langle e_n, e_m \rangle = \int_0^L e_n(x) e_m(x) dx = \begin{cases} 0 & n \neq m \\ 1 & m = n \end{cases}.$$

Finally, the sequence $\{e_n\}_{n \in \mathbf{N}}$ spans a dense linear subspace of $L^2(0, L)$. This shows that in effect we have diagonalized the operator Δ.

Heat conduction in non-homogeneous anisotropic media

In general, the study of heat conduction is based on several principles. Heat flow is a form of energy flow, and as such it is meaningful to speak of the time rate of flow of heat into a region of space.

- The time rate of heat flow into a region V is given by a time-dependent quantity $q_t(V)$. We assume q has a density, so that

$$q_t(V) = \int_V Q(x,t)\, dx$$

- Heat flow is a time-dependent vector function $\mathbf{H}(x)$ characterized as follows: the time rate of heat flowing through an infinitesimal surface element with area dS and with unit normal vector $\mathbf{n}$ is

$$\mathbf{H}(x) \cdot \mathbf{n}(x)\, dS$$

Thus the rate of heat flow into V is also given by the surface integral

$$q_t(V) = -\int_{\partial V} \mathbf{H}(x) \cdot \mathbf{n}(x)\, dS$$

where $\mathbf{n}$(x) is the outward pointing normal vector at x.

- The Fourier law states that heat energy flow has the following linear dependence on the temperature gradient

$$\mathbf{H}(x) = -\mathbf{A}(x) \cdot \nabla u(x)$$

where $\mathbf{A}(x)$ is a 3×3 real matrix that is symmetric and positive definite.

By Green's theorem, the previous surface integral for heat flow into V can be transformed into the volume integral

$$q_t(V) = -\int_{\partial V} \mathbf{H}(x) \cdot \mathbf{n}(x)\, dS$$
$$= \int_{\partial V} \mathbf{A}(x) \cdot \nabla u(x) \cdot \mathbf{n}(x)\, dS$$
$$= \int_V \sum_{i,j} \partial_{x_i} \big(a_{ij}(x) \partial_{x_j} u(x,t)\big)\, dx$$

- The time rate of temperature change at x is proportional to the heat flowing into an infinitesimal volume element, where the constant of proportionality is dependent on a constant κ

$$\partial_t u(x,t) = \kappa(x) Q(x,t)$$

Putting these equations together gives the general equation of heat flow:

$$\partial_t u(x,t) = \kappa(x) \sum_{i,j} \partial_{x_i} \big(a_{ij}(x) \partial_{x_j} u(x,t)\big)$$

Remarks.

- The coefficient κ(x) is the inverse of specific heat of the substance at x × density of the substance at x.

- In the case of an isotropic medium, the matrix **A** is a scalar matrix equal to thermal conductivity.

- In the anisotropic case where the coefficient matrix **A** is not scalar (i.e., if it depends on x), then an explicit formula for the solution of the heat equation can seldom be written down. Though, it is usually possible to consider the associated abstract Cauchy problem and show that it is a well-posed problem and/or to show some qualitative properties (like preservation of positive initial data, infinite speed of propagation, convergence toward an equilibrium, smoothing properties). This is usually done by one-parameter semigroups theory: for instance, if A is a symmetric matrix, then the elliptic operator defined by

$$Au(x) := \sum_{i,j} \partial_{x_i} a_{ij}(x) \partial_{x_j} u(x)$$

  is self-adjoint and dissipative, thus by the spectral theorem it generates a one-parameter semigroup.

Fundamental solutions

A fundamental solution, also called a *heat kernel*, is a solution of the heat equation corresponding to the initial condition of an initial point source of heat at a known position. These can be used to find a general solution of the heat equation over certain domains; see, for instance, (Evans 1998) for an introductory treatment.

In one variable, the Green's function is a solution of the initial value problem

$$\begin{cases} u_t(x,t) - ku_{xx}(x,t) = 0 & -\infty < x < \infty, \quad 0 < t < \infty \\ u(x,t=0) = \delta(x) \end{cases}$$

where δ is the Dirac delta function. The solution to this problem is the fundamental solution

$$\Phi(x,t) = \frac{1}{\sqrt{4\pi kt}} \exp\left(-\frac{x^2}{4kt}\right).$$

One can obtain the general solution of the one variable heat equation with initial condition $u(x,0) = g(x)$ for $-\infty<x<\infty$ and $0<t<\infty$ by applying a convolution:

$$u(x,t) = \int \Phi(x-y,t)g(y)dy.$$

In several spatial variables, the fundamental solution solves the analogous problem

$$\begin{cases} u_t(\mathbf{x},t) - k\sum_{i=1}^n u_{x_i x_i}(\mathbf{x},t) = 0 \\ u(\mathbf{x},t=0) = \delta(\mathbf{x}) \end{cases}$$

in $-\infty<x_i<\infty$, $i=1,...,n$, and $0<t<\infty$. The n-variable fundamental solution is the product of the fundamental solutions in each variable; i.e.,

$$\Phi(\mathbf{x},t) = \Phi(x_1,t)\Phi(x_2,t)\dots\Phi(x_n,t) = \frac{1}{(4\pi kt)^{n/2}} e^{-\mathbf{x}\cdot\mathbf{x}/4kt}.$$

The general solution of the heat equation on $\mathbf{R}^n$ is then obtained by a convolution, so that to solve the initial value problem with $u(\mathbf{x},t=0)=g(\mathbf{x})$, one has

$$u(\mathbf{x},t) = \int_{\mathbb{R}^n} \Phi(\mathbf{x}-\mathbf{y},t)g(\mathbf{y})d\mathbf{y}.$$

The general problem on a domain Ω in $\mathbf{R}^n$ is

$$\begin{cases} u_t(\mathbf{x},t) - k\sum_{i=1}^n u_{x_i x_i}(\mathbf{x},t) = 0 & \mathbf{x}\in\Omega \quad 0<t<\infty \\ u(\mathbf{x},t=0) = g(\mathbf{x}) & \mathbf{x}\in\Omega \end{cases}$$

with either Dirichlet or Neumann boundary data. A Green's function always exists, but unless the domain Ω can be readily decomposed into one-variable problems, it may not be possible to write it down explicitly. The method of images provides one additional technique for obtaining Green's functions for non-trivial domains.

Some Green's function solutions in 1D

A variety of elementary Green's function solutions in one-dimension are recorded here. In some of these, the spatial domain is the entire real line (-∞,∞). In others, it is the semi-infinite interval (0,∞) with either Neumann or Dirichlet boundary conditions. One further variation is that some of these solve the inhomogeneous equation

$u_t = ku_{xx} + f.$

where f is some given function of x and t.

Homogeneous heat equation

Initial value problem on (-∞,∞)

$$\begin{cases} u_t = ku_{xx} & -\infty < x < \infty,\ 0 < t < \infty \\ u(x,0) = g(x) & IC \end{cases}$$

$$u(x,t) = \frac{1}{\sqrt{4\pi kt}}\int_{-\infty}^{\infty} \exp\left(-\frac{(x-y)^2}{4kt}\right) g(y)\,dy$$

Comment. This solution is the convolution with respect to the variable x of the fundamental solution $\Phi(x,t) := \frac{1}{\sqrt{4kt}}\exp\left(-\frac{x^2}{4kt}\right)$ and the function $g(x)$. Therefore, according to the general properties of the convolution with respect to differentiation, $u = g*\Phi$ is a solution of the same heat equation, for $(\partial_t - k\,\partial_x^2)(\Phi*g) = [(\partial_t - k\,\partial_x^2)\Phi]*g) = 0.$ Moreover, $\Phi(x,t) = \frac{1}{\sqrt{t}}\,\Phi(\frac{x}{\sqrt{t}})$ and $\int_{-\infty}^{+\infty}\Phi(x,t)\,dx = 1,$ so that, by general facts about approximation to the identity, $\Phi(\cdot,t)*g \to g$ as $t\to 0$ in various senses, according to the specific g. For instance, if g is assumed bounded and continuous on $\mathbb{R}$, then $\Phi(\cdot,t)*g$ converges uniformly to g as $t\to 0$, meaning that $u(x,t)$ is continuous on $\mathbb{R}\times[0,\infty)$ with $u(x,0) = g(x)$.

Initial value problem on (0,∞) with homogeneous Dirichlet boundary conditions

$$\begin{cases} u_t = ku_{xx} & 0 \le x < \infty,\ 0 < t < \infty \\ u(x,0) = g(x) & IC \\ u(0,t) = 0 & BC \end{cases}$$

$$u(x,t)=\frac{1}{\sqrt{4\pi kt}}\int_0^\infty\left(\exp\left(-\frac{(x-y)^2}{4kt}\right)-\exp\left(-\frac{(x+y)^2}{4kt}\right)\right)g(y)\,dy$$

Comment. This solution is obtained from the preceding formula as applied to the data $g(x)$ suitably extended to $\mathbb{R}$, so as to be an odd function, that is, letting $g(-x):=-g(x)$ for all x. Correspondingly, the solution of the initial value problem on $(-\infty,+\infty)$ is an odd function with respect to the variable x for all values of t, and in particular it satisfies the homogeneous Dirichlet boundary conditions $u(0,\,t)=0.$

Initial value problem on (0,∞) with homogeneous Neumann boundary conditions

$$\begin{cases}u_t=ku_{xx} & 0\le x<\infty,\ 0<t<\infty\\ u(x,0)=g(x) & IC\\ u_x(0,t)=0 & BC\end{cases}$$

$$u(x,t)=\frac{1}{\sqrt{4\pi kt}}\int_0^\infty\left(\exp\left(-\frac{(x-y)^2}{4kt}\right)+\exp\left(-\frac{(x+y)^2}{4kt}\right)\right)g(y)\,dy$$

Comment. This solution is obtained from the first solution formula as applied to the data $g(x)$ suitably extended to $\mathbb{R}$, so as to be an even function, that is, letting $g(-x):=g(x)$ for all x. Correspondingly, the solution of the initial value problem on $(-\infty,+\infty)$ is an even function with respect to the variable x for all values of $t>0$, and in particular, being smooth, it satisfies the homogeneous Neumann boundary conditions $u_x(0,\,t)=0.$

Problem on (0,∞) with homogeneous initial conditions and non-homogeneous Dirichlet boundary conditions

$$\begin{cases}u_t=ku_{xx} & 0\le x<\infty,\ 0<t<\infty\\ u(x,0)=0 & IC\\ u(0,t)=h(t) & BC\end{cases}$$

$$u(x,t)=\int_0^t\frac{x}{\sqrt{4\pi k(t-s)^3}}\exp\left(-\frac{x^2}{4k(t-s)}\right)h(s)\,ds,\qquad \forall x>0$$

Comment. This solution is the convolution with respect to the variable t of $\psi(x,t):=-2k\,\partial_x\,\Phi(x,t)=\frac{x}{\sqrt{4kt^3}}\exp\left(-\frac{x^2}{4kt}\right)$ and the function $h(t)$. Since $\Phi(x,t)$ is the fundamental solution of $\partial_t-k\,\partial_x^2$, the function $\psi(x,t)$ is also a solution of the same heat equation, and so is $u:=\psi*h$, thanks to general properties of the convolution with respect to differentiation. Moreover, $\psi(x,t)=\frac{1}{x^2}\,\psi(1,\frac{t}{x^2})$ and $\int_0^{+\infty}\psi(x,t)\,dt=1$, so that, by general facts about approximation to the identity, $\psi(x,\cdot)*h\to h$ as $x\to 0$ in various senses, according to the specific h. For instance, if h is assumed continuous on $\mathbb{R}$ with support in $[0,\infty)$, then $\psi(x,\cdot)*h$ converges uniformly on compacta to h as $x\to 0$, meaning that $u(x,t)$ is continuous on $[0,\infty)\times[0,\infty)$ with $u(0,t)=h(t)$.

Inhomogeneous heat equation

Problem on (-∞,∞) homogeneous initial conditions

$$\begin{cases} u_t = k u_{xx} + f(x,t) & -\infty < x < \infty,\ 0 < t < \infty \\ u(x,0) = 0 & IC \end{cases}$$

$$u(x,t) = \int_0^t \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi k(t-s)}} \exp\left(-\frac{(x-y)^2}{4k(t-s)}\right) f(y,s)\,dy\,ds$$

Comment. This solution is the convolution in $\mathbb{R}^2$, that is with respect to both the variables *x* and *t*, of the fundamental solution $\Phi(x,t) := \frac{1}{\sqrt{4kt}} \exp\left(-\frac{x^2}{4kt}\right)$ and the function $f(x,t)$, both meant as defined on the whole $\mathbb{R}^2$ and identically 0 for all $t \le 0$. One verifies that $(\partial_t - k\,\partial_x^2)(\Phi * f) = f$, which is expressed in the language of distributions as $(\partial_t - k\,\partial_x^2)\Phi = \delta$, where the distribution δ is the Dirac's delta function, that is the evaluation at 0.

Problem on (0,∞) with homogeneous Dirichlet boundary conditions and initial conditions

$$\begin{cases} u_t = k u_{xx} + f(x,t) & 0 \le x < \infty,\ 0 < t < \infty \\ u(x,0) = 0 & IC \\ u(0,t) = 0 & BC \end{cases}$$

$$u(x,t) = \int_0^t \int_0^{\infty} \frac{1}{\sqrt{4\pi k(t-s)}} \left(\exp\left(-\frac{(x-y)^2}{4k(t-s)}\right) - \exp\left(-\frac{(x+y)^2}{4k(t-s)}\right)\right) f(y,s)\,dy\,ds$$

Comment. This solution is obtained from the preceding formula as applied to the data $f(x,t)$ suitably extended to $\mathbb{R} \times [0,\infty)$, so as to be an odd function of the variable *x*, that is, letting $f(-x,t) := -f(x,t)$ for all *x* and *t*. Correspondingly, the solution of the inhomogeneous problem on $(-\infty,+\infty)$ is an odd function with respect to the variable *x* for all values of *t*, and in particular it satisfies the homogeneous Dirichlet boundary conditions $u(0,t) = 0$.

Problem on (0,∞) with homogeneous Neumann boundary conditions and initial conditions

$$\begin{cases} u_t = k u_{xx} + f(x,t) & 0 \le x < \infty,\ 0 < t < \infty \\ u(x,0) = 0 & IC \\ u_x(0,t) = 0 & BC \end{cases}$$

$$u(x,t) = \int_0^t \int_0^{\infty} \frac{1}{\sqrt{4\pi k(t-s)}} \left(\exp\left(-\frac{(x-y)^2}{4k(t-s)}\right) + \exp\left(-\frac{(x+y)^2}{4k(t-s)}\right)\right) f(y,s)\,dy\,ds$$

Comment. This solution is obtained from the first formula as applied to the data $f(x,t)$suitably extended to $\mathbb{R}\times[0,\infty)$, so as to be an even function of the variable *x*, that is, letting $f(-x,t):=f(x,t)$for all *x* and *t*. Correspondingly, the solution of the inhomogeneous problem on $(-\infty,+\infty)$is an even function with respect to the variable *x* for all values of *t*, and in particular, being a smooth function, it satisfies the homogeneous Neumann boundary conditions $u_x(0,t)=0$.

Examples

Since the heat equation is linear, solutions of other combinations of boundary conditions, inhomogeneous term, and initial conditions can be found by taking an appropriate linear combination of the above Green's function solutions.

For example, to solve

$$\begin{cases} u_t = ku_{xx} + f & -\infty < x < \infty, \, 0 < t < \infty \\ u(x,0) = g(x) & IC \end{cases}$$

let

$$u = w + v$$

where *u* and *v* solve the problems

$$\begin{cases} v_t = kv_{xx} + f, \, w_t = kw_{xx} & -\infty < x < \infty, \, 0 < t < \infty \\ v(x,0) = 0, \, w(x,0) = g(x) & IC \end{cases}$$

Similarly, to solve

$$\begin{cases} u_t = ku_{xx} + f & 0 \le x < \infty, \, 0 < t < \infty \\ u(x,0) = g(x) & IC \\ u(0,t) = h(t) & BC \end{cases}$$

let

$$u = w + v + r$$

where *w*, *v*, and *r* solve the problems

$$\begin{cases} v_t = kv_{xx} + f,\ w_t = kw_{xx},\ r_t = kr_{xx} & 0 \le x < \infty,\ 0 < t < \infty \\ v(x,0) = 0,\ w(x,0) = g(x),\ r(x,0) = 0 & IC \\ v(0,t) = 0,\ w(0,t) = 0,\ r(0,t) = h(t) & BC \end{cases}$$

Mean-value property for the heat equation

Solutions of the heat equations $(\partial_t - \Delta)u = 0$ satisfy a mean-value property analogous to the mean-value properties of harmonic functions (solutions of $\Delta u = 0$), though a bit more complicated. Precisely, if u solves $(\partial_t - \Delta)u = 0$ and $(x,t) + E_\lambda \subset \mathrm{dom}(u)$ then

$$u(x,t) = \frac{\lambda}{4} \int_{E_\lambda} u(x-y, t-s) \frac{|y|^2}{s^2} ds\, dy,$$

where E_λ is a "heat-ball", that is a super-level set of the fundamental solution of the heat equation:

$$E_\lambda := \{(y,s)\ :\ \Phi(y,s) > \lambda\},$$
$$\Phi(x,t) := (4t\pi)^{-\frac{n}{2}} \exp\Big(-\frac{|x|^2}{4t}\Big).$$

Notice that $\mathrm{diam}(E_\lambda) = o(1)$ as $\lambda \to \infty$, so the above formula holds for any (x,t) in the (open) set $\mathrm{dom}(u)$ for λ large enough. Conversely, any function u satisfying the above mean-value property on an open domain of $\mathbb{R}^n \times \mathbb{R}$ is a solution of the heat equation. This can be shown by an argument similar to the analogous one for harmonic functions.

Stationary Heat Equation

The stationary heat equation is not dependent on time. This happens in all those problems, where the time equilibrium constant is fast enough to approximate the more complex time dependent heat equation to the stationary case. This equation is much simpler and can help to understand better the physic of the materials without focusing on the dynamic of the heat transport process. It is widely used for simple engineering problems assuming there is equilibrium with time.

Stationary condition:

$$\frac{\partial u}{\partial t} = 0$$

Stationary heat equation with heat source (inhomogeneous case), which is also the Poisson's equation:

$$\alpha \nabla^2 u = q$$

Stationary heat equation without heat source (homogeneous case), which is also the Laplace's equation:

$$\nabla^2 u = 0$$

- where *u* is the temperature (°K), α (or *k*) is the thermal conductivity ($\frac{W}{m^\circ K}$), *q* the heat source density ($\frac{W}{m^3}$).

Applications

Particle diffusion

One can model particle diffusion by an equation involving either:

- the volumetric concentration of particles, denoted *c*, in the case of collective diffusion of a large number of particles, or
- the probability density function associated with the position of a single particle, denoted *P*.

In either case, one uses the heat equation

$$c_t = D\Delta c,$$

or

$$P_t = D\Delta P.$$

Both *c* and *P* are functions of position and time. *D* is the diffusion coefficient that controls the speed of the diffusive process, and is typically expressed in meters squared over second. If the diffusion coefficient *D* is not constant, but depends on the concentration *c* (or *P* in the second case), then one gets the nonlinear diffusion equation.

Brownian motion

The random trajectory of a single particle subject to the particle diffusion equation (or heat equation) is a Brownian motion. If a particle is placed at $\vec{R} = \vec{0}$ at time *t* = 0, then the probability density function associated with the position vector of the particle $\vec{R}$ will be the following:

$$P(\vec{R},t) = G(\vec{R},t) = \frac{1}{(2\pi)^{3/2}(\det(D)t)^{1/2}} \exp\Big\{ -\frac{\vec{R}^T D^{-1}\vec{R}}{2t}\Big\}$$

which is a (multivariate) normal distribution evolving in time.

Schrödinger equation for a free particle

With a simple division, the Schrödinger equation for a single particle of mass *m* in the absence of any applied force field can be rewritten in the following way:

$$\psi_t = \frac{i\hbar}{2m}\Delta\psi$$

, where *i* is the unit imaginary number, and $\hbar$ is Planck's constant divided by 2π, and ψ is the wavefunction of the particle.

This equation is formally similar to the particle diffusion equation, which one obtains through the following transformation:

$$c(\vec{R},t) \to \psi(\vec{R},t)$$
$$D \to \frac{i\hbar}{2m}.$$

Applying this transformation to the expressions of the Green functions determined in the case of particle diffusion yields the Green functions of the Schrödinger equation, which in turn can be used to obtain the wavefunction at any time through an integral on the wavefunction at *t=0*:

$$\psi(\vec{R},t) = \int \psi(\vec{R}^0, t=0) G(\vec{R}-\vec{R}^0,t) dR_x^0\, dR_y^0\, dR_z^0$$

, with

$$G(\vec{R},t) = \Big(\frac{m}{2\pi i\hbar t}\Big)^{3/2} e^{-\frac{\vec{R}^2 m}{2i\hbar t}}.$$

Remark: this analogy between quantum mechanics and diffusion is a purely formal one. Physically, the evolution of the wavefunction satisfying Schrödinger's equation might have an origin other than diffusion.

Thermal diffusivity in polymers

A direct practical application of the heat equation, in conjunction with Fourier theory, in spherical coordinates, is the measurement of the thermal diffusivity in polymers (Unsworth and Duarte). The dual theoretical-experimental method demonstrated by these authors is applicable to rubber and various other materials of practical interest.

Further applications

The heat equation arises in the modeling of a number of phenomena and is often used in financial mathematics in the modeling of options. The famous Black–Scholes option pricing model's differential equation can be transformed into the heat equation allowing relatively easy solutions from a familiar body of mathematics. Many of the extensions to the simple option models do not have closed form solutions and thus must be solved numerically to obtain a modeled option price. The heat equation is also widely used in image analysis (Perona & Malik 1990) and in machine-learning as the driving theory behind scale-space or graph Laplacian methods. The heat equation can be efficiently solved numerically using the Crank–Nicolson method of (Crank & Nicolson 1947).

An abstract form of heat equation on manifolds provides a major approach to the Atiyah–Singer index theorem, and has led to much further work on heat equations in Riemannian geometry.

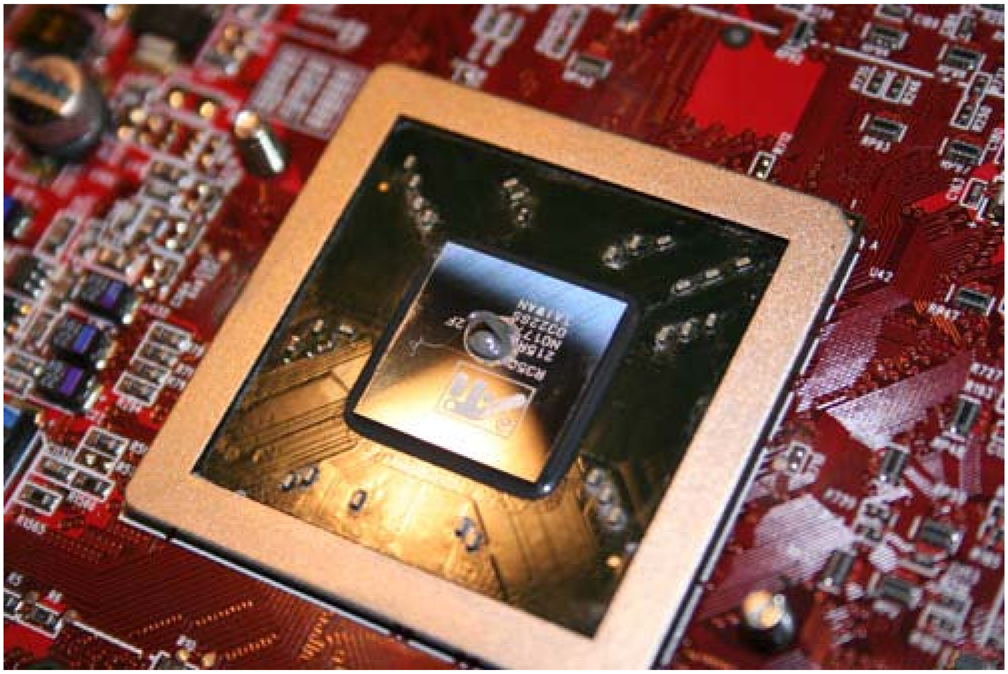
Chapter 3

Thermal Grease

From left to right: Arctic Cooling MX-2 and MX-3, Tuniq TX-3, Cool Laboratory Liquid Metal Pro(Liquid Metal based), Shin-Etsu MicroSi G751, Arctic Silver 5, Powdered Diamond. In background Arctic Silver grease remover

Silicone thermal compound

Metal (silver) thermal compound



Metal thermal grease applied to a chip

Surface imperfections

Thermal grease (also called **thermal gel**, **thermal compound**, **thermal paste**, **heat paste**, **heat sink paste**, **heat transfer compound**, or **heat sink compound**) is a fluid substance, originally with properties akin to grease, which increases the thermal conductivity of a thermal interface by filling microscopic air-gaps present due to the imperfectly flat and smooth surfaces of the components; the compound has far greater thermal conductivity than air (but far less than metal). In electronics, it is often used to aid a component's thermal dissipation via a heat sink.

Thermal conductor types

Thermal greases use one or more different thermally conductive substances:

- Ceramic-based thermal grease has generally good thermal conductivity and is usually composed of a ceramic powder suspended in a liquid or gelatinous silicone compound, which may be described as 'silicone paste' or 'silicone thermal compound'. The most commonly used ceramics and their thermal conductivities (in units of W/(m ·K)) are: beryllium oxide (218), aluminum nitride (170), aluminum oxide (39), zinc oxide (21), and silicon dioxide (1). Thermal grease is usually white in colour since these ceramics are all white in powder form.

- Metal-based thermal grease contain solid metal particles (usually silver or aluminum). It has a better thermal conductivity (and is more expensive) than ceramic-based grease.

- Carbon based. There are products based on with carbon-based conductors, using diamond powder, or short carbon fibers , they have the best thermal conductivity and are generally more expensive than metal-based thermal grease.

- Liquid metal based. Some thermal pastes are made of liquid metal alloys of gallium. These are rare and expensive.

All but the last classification of compound usually use silicone grease as a medium, a heat conductor in itself, though some manufacturers prefer use of fractions of mineral oil.

All these compounds conduct heat far better than air, but far worse than metal. They are intended to fill gaps that would otherwise hold air, not to create a layer between component and heatsink—this will decrease the effectiveness of the heatsink. Ideally perfectly smooth and flat metallic surfaces would not need heatsink compound.

Purpose

Thermal grease is primarily used in the electronics and computer industries to assist a heat sink to draw heat away from a semiconductor component such as an integrated circuit or transistor.

Thermally conductive paste improves the efficiency of a heatsink by filling air gaps that occur when the imperfectly flat and smooth surface of a heat generating component is pressed against the similar surface of a heatsink, air being approximately 8000 times less efficient at conducting heat than, for example, aluminum, a good heatsink material. Surface imperfections and departure from perfect flatness inherently arise from limitations in manufacturing technology and range in size from visible and tactile flaws such as machining marks or casting irregularities to sub-microscopic ones not visible to the naked eye.

Thermal conductivity and "conformability" (i.e., the ability of the material to conform to irregular surfaces) are the important characteristics of thermal grease.

Both high-power handling transistors, such as those in an audio amplifier, and high-speed integrated circuits, such as the central processing unit (CPU) of a personal computer, generate sufficient heat to benefit from the use of thermal grease to improve the effectiveness of a heatsink.

The need for heatsink compound can be minimised or removed by lapping the surfaces of the hot component and the matching heatsink face so that they are virtually perfectly flat and mirror-smooth. Computer overclockers, who increase computer speed by measures which increase heat production, resort to lapping and other extreme cooling methods such as water-cooling.

Properties

The metal oxide and nitride particles suspended in silicone thermal compounds have thermal conductivities of up to 220 W/(m·K). (In comparison, the thermal conductivity of metals used particle additions, copper is 380 W/(m·K), silver 429 and aluminum 237.) The typical thermal conductivities of the silicone compounds are 0.7 to 3 W/(m·K). Silver thermal compounds may have a conductivity of 3 to 8 W/(m·K) or more.

In compounds containing suspended particles, the properties of the fluid may well be the most important. As seen by the thermal conductivity measures above, the conductivity is closer to that of the fluid components rather than the ceramic or metal components. Other properties of fluid components that are important for thermal grease might be:

1. How well it fills the gaps and conforms to both the component's and the heat sink's uneven surfaces.
2. How well it adheres to those surfaces
3. How well it maintains its consistency over the required temperature range
4. How well it resists drying out or flaking over time
5. Whether it degrades with oxidation or breaks down over time

The compound must have a suitable consistency to apply easily and remove all excess to leave only the minimum needed.

Application and removal

Computer processor heatsinks utilize a variety of designs to promote better thermal transfer between components. Some thermal greases have a durability up to at least 8 years. Flat and smooth surfaces may use a small line method to apply material, and exposed heat-pipe surfaces will be best prepared with multiple lines.

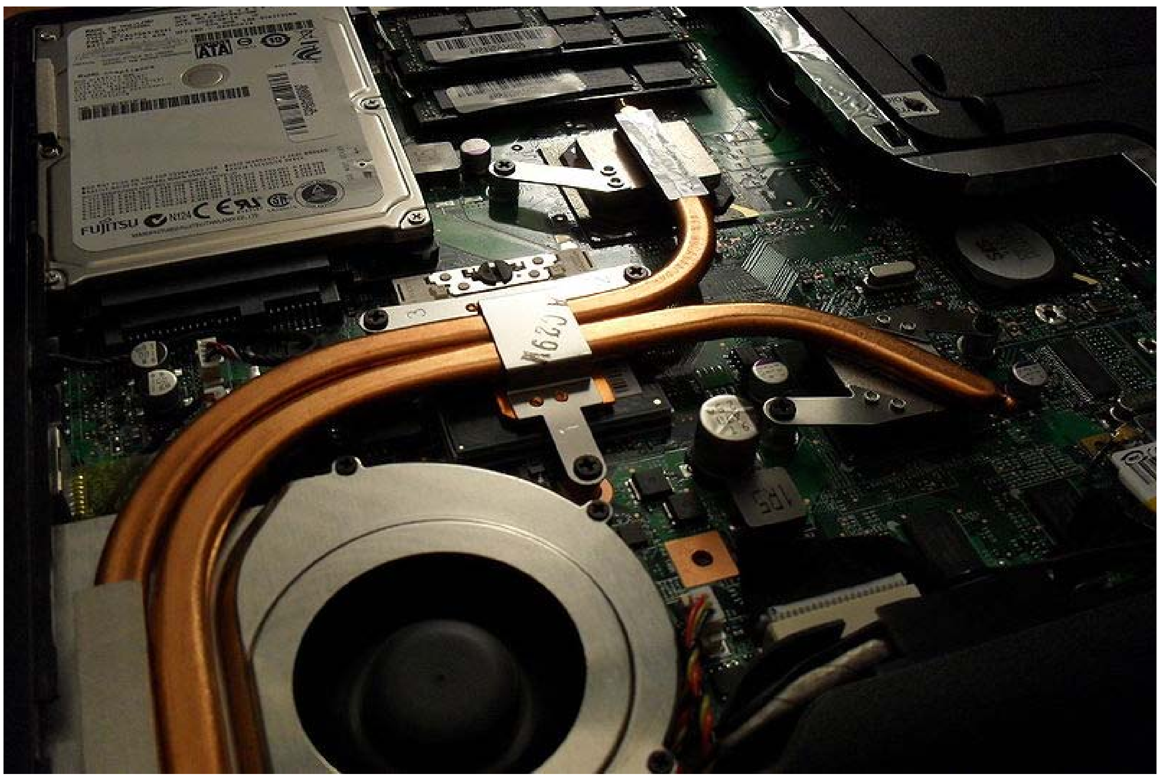
Excess grease separating the metal surfaces more than the minimum necessary to exclude air gaps will only degrade conductivity, increasing the risk of overheating. Silver-based thermal grease can also be either slightly electrically conductive or capacitive; if some flows onto the circuits it can cause malfunctioning and damage.

Over time, some thermal greases may dry out, have reduced heat transferring capabilities, or set like glue and make it difficult to remove the heat sink. If too much force is applied the processor may be damaged. Heating the grease by turning the processor on for a short period often softens the adhesion. It is recommended that thermal grease be re-applied with each removal of the heatsink.

Silicone oil-based thermal grease can be removed from a component or heatsink with an alcohol (such as rubbing alcohol) or acetone. Special-purpose cleaners are made for removing heatsink grease and cleaning the surfaces.

Chapter 4

Heat Pipe



A laptop heat pipe system

A **heat pipe** is a heat transfer mechanism that combines the principles of both thermal conductivity and phase transition to efficiently manage the transfer of heat between two solid interfaces.

At the hot interface within a heat pipe, which is typically at a very low pressure, a liquid in contact with a thermally conductive solid surface turns into a vapor by absorbing heat from that surface. The vapor condenses back into a liquid at the cold interface, releasing the latent heat. The liquid then returns to the hot interface through either capillary action or gravity action where it evaporates once more and repeats the cycle. In addition, the internal pressure of the heat pipe can be set or adjusted to facilitate the phase change depending on the demands of the working conditions of the thermally managed system.

Structure, design and construction

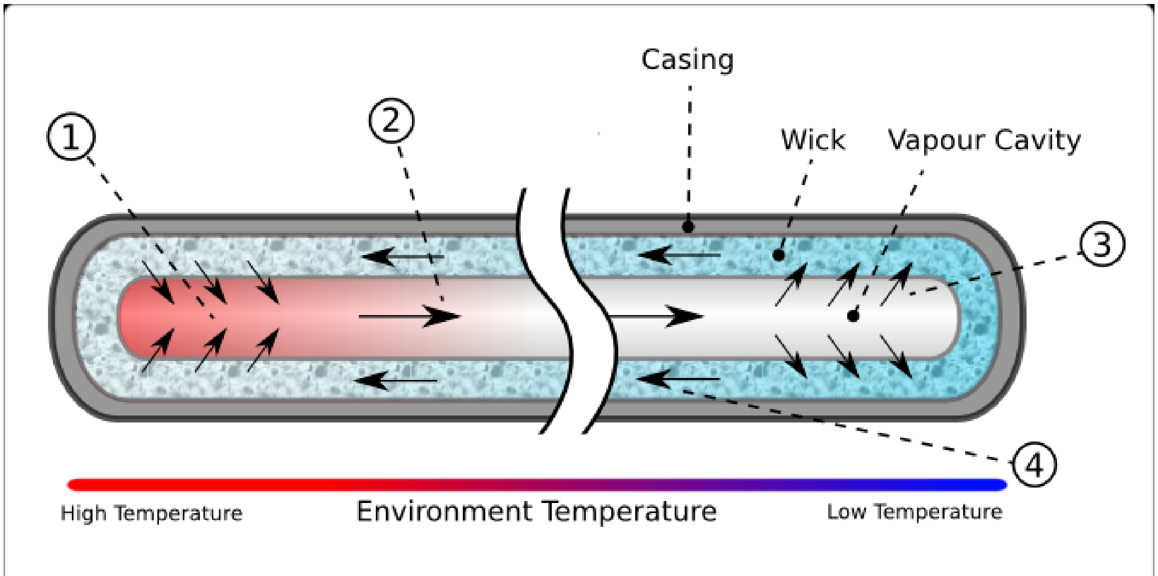
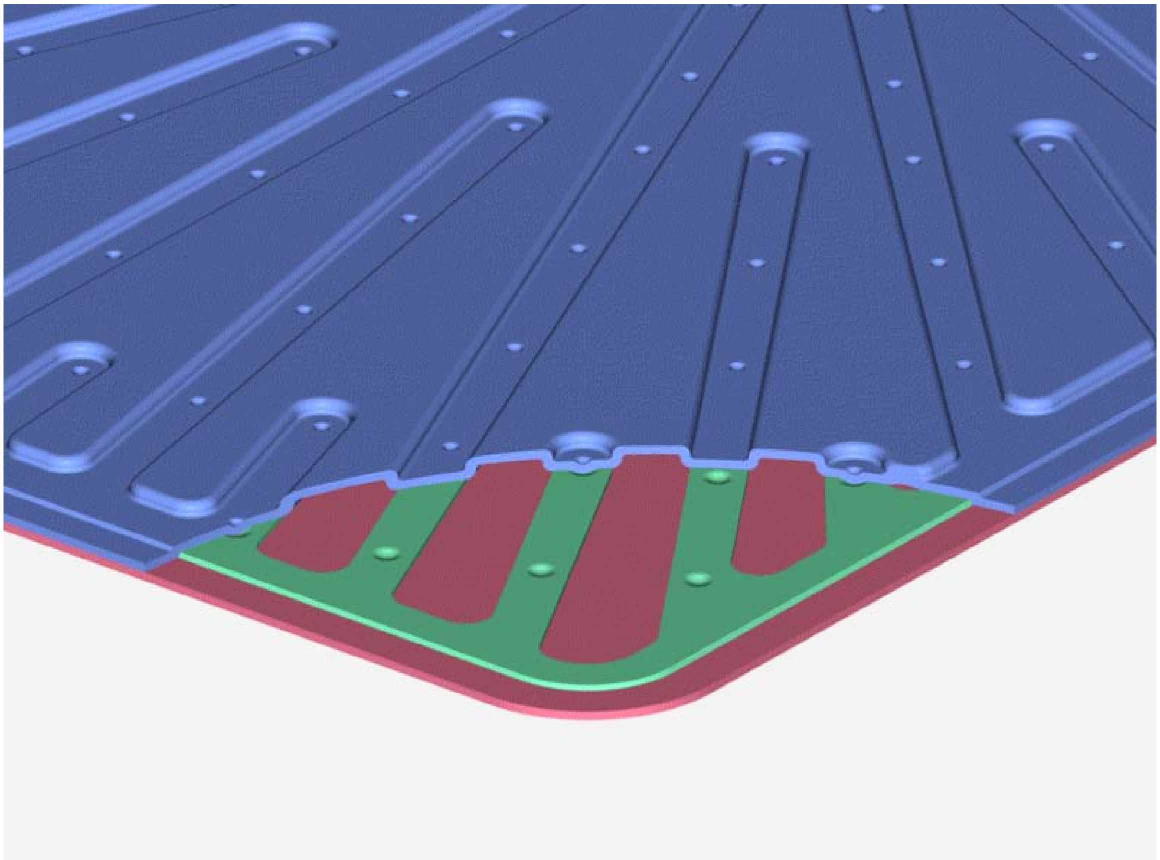
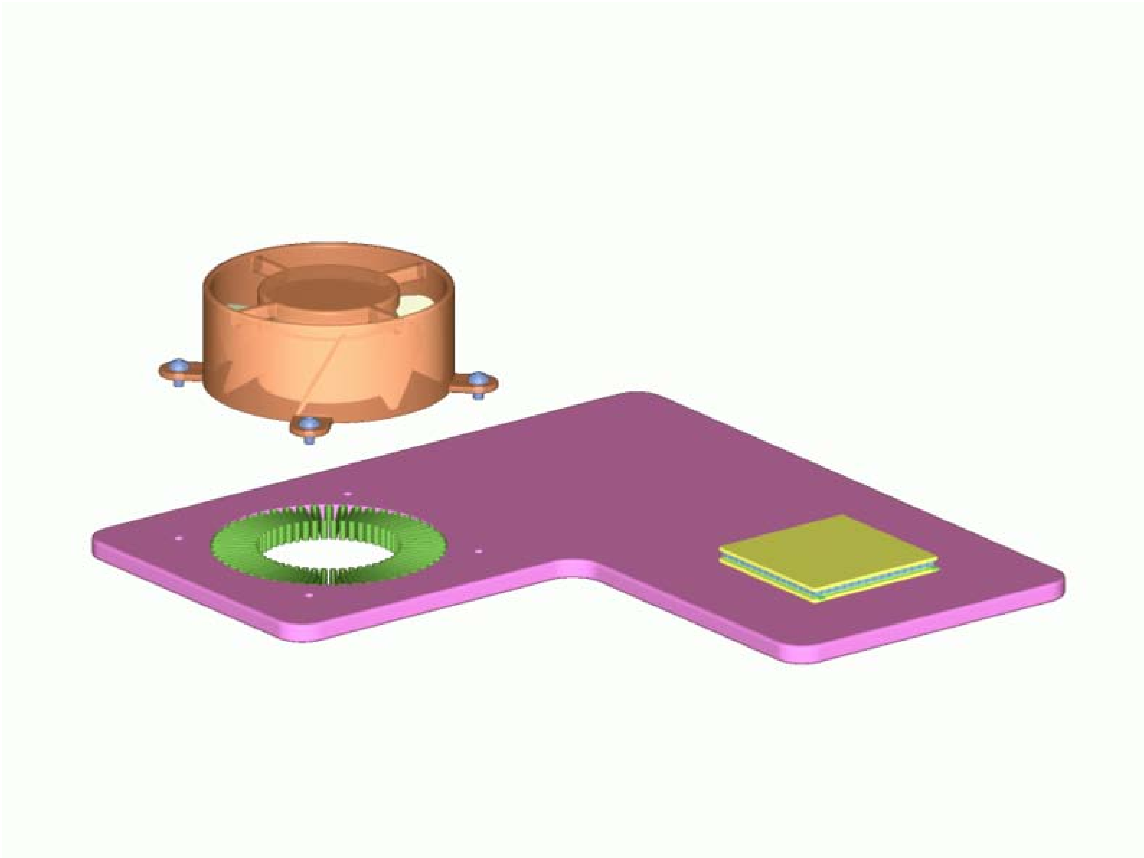


Diagram showing components and mechanism for a heat pipe containing a wick



Cut-away view of a 500 µm thick flat heat pipe, with a thin planar capillary (aqua colored)



Thin flat heat pipe (heat spreader) with remote heat sink and fan

A typical heat pipe consists of a sealed pipe or tube made of a material with high thermal conductivity such as copper or aluminium at both hot and cold ends. A vacuum pump is used to remove all air from the empty heat pipe, and then the pipe is filled with a fraction of a percent by volume of *working fluid* (or coolant) chosen to match the operating temperature. Examples of such fluids include water, ethanol, acetone, sodium, or mercury. Due to the partial vacuum that is near or below the vapor pressure of the fluid, some of the fluid will be in the liquid phase and some will be in the gas phase. The use of a vacuum eliminates the need for the working gas to diffuse through any other gas and so the bulk transfer of the vapor to the cold end of the heat pipe is at the speed of the moving molecules. In this sense, the only practical limit to the rate of heat transfer is the speed with which the gas can be condensed to a liquid at the cold end.

Inside the pipe's walls, an optional wick structure exerts a capillary pressure on the liquid phase of the working fluid. This is typically a sintered metal powder or a series of grooves parallel to the pipe axis, but it may be any material capable of exerting capillary pressure on the condensed liquid to wick it back to the heated end. The heat pipe may not need a wick structure if gravity or some other source of acceleration is sufficient to overcome surface tension and cause the condensed liquid to flow back to the heated end.

A heat pipe is not a thermosiphon, because there is no siphon. Thermosiphons transfer heat by single-phase convection.

Heat pipes contain no mechanical moving parts and typically require no maintenance, though non-condensing gases (that diffuse through the pipe's walls, result from breakdown of the working fluid, or exist as impurities in the materials) may eventually reduce the pipe's effectiveness at transferring heat. This is significant when the working fluid's vapour pressure is low.

The materials chosen depend on the temperature conditions in which the heat pipe must operate, with coolants ranging from liquid helium for extremely low temperature applications (2–4 K) to mercury (523–923 K) & sodium (873–1473 K) and even indium (2000–3000 K) for extremely high temperatures. The vast majority of heat pipes for low temperature applications use some combination of ammonia (213–373 K), alcohol (methanol (283–403 K) or ethanol (273–403 K)) or water (303–473 K) as working fluid. Since the heat pipe contains a vacuum, the working fluid will boil and hence take up latent heat at well below its boiling point at atmospheric pressure. Water, for instance, will boil at just above 273 K (0 degrees Celsius) and so can start to effectively transfer latent heat at this low temperature.

The advantage of heat pipes over many other heat-dissipation mechanisms is their great efficiency in transferring heat. They are a fundamentally better heat conductor than an equivalent cross-section of solid copper (a heat sink alone, though simpler in design and construction, does not take advantage of the principle of matter phase transition). Some heat pipes have demonstrated a heat flux of more than 230 MW/m^2, nearly four times the heat flux at the surface of the sun.

Active control of heat flux can be effected by adding a variable volume liquid reservoir to the evaporator section. Variable conductance heat pipes employ a large reservoir of inert immiscible gas attached to the condensing section. Varying the gas reservoir pressure changes the volume of gas charged to the condenser which in turn limits the area available for vapor condensation. Thus a wider range of heat fluxes and temperature gradients can be accommodated with a single design.

A modified heat pipe with a reservoir having no capillary connection to the heat pipe wick at the evaporator end can also be used as a thermal diode. This heat pipe will transfer heat in one direction, acting as an insulator in the other.

Vapor Chamber or Flat heat pipes

Thin planar heat pipes (heat spreaders) have the same primary components as tubular heat pipes. These components are a hermetically sealed hollow vessel, a working fluid, and a closed-loop capillary recirculation system.

Compared to a one-dimensional tubular heat pipe, the width of a two-dimensional heat pipe allows an adequate cross section for heat flow even with a very thin device. These thin planar heat pipes are finding their way into “height sensitive” applications, such as notebook computers, and surface mount circuit board cores. It is possible to produce flat heat pipes as thin as 0.5 mm (thinner than a credit card).

Heat transfer

A heat sink (aluminium) with heat pipe (copper)

Heat pipes employ evaporative cooling to transfer thermal energy from one point to another by the evaporation and condensation of a working fluid or coolant. Heat pipes rely on a temperature difference between the ends of the pipe, and cannot lower temperatures at either end beyond the ambient temperature (hence they tend to equalise the temperature within the pipe).

When one end of the heat pipe is heated the working fluid inside the pipe at that end evaporates and increases the vapour pressure inside the cavity of the heat pipe. The latent heat of evaporation absorbed by the vaporisation of the working fluid reduces the temperature at the hot end of the pipe.

The vapour pressure over the hot liquid working fluid at the hot end of the pipe is higher than the equilibrium vapour pressure over condensing working fluid at the cooler end of the pipe, and this pressure difference drives a rapid mass transfer to the condensing end where the excess vapour condenses, releases its latent heat, and warms the cool end of the pipe. Non-condensing gases (caused by contamination for instance) in the vapour impede the gas flow and reduce the effectiveness of the heat pipe, particularly at low temperatures, where vapour pressures are low. The velocity of molecules in a gas is approximately the speed of sound and in the absence of non condensing gases, this is the upper velocity with which they could travel in the heat pipe. In practice, the speed of the vapour through the heat pipe is dependent on the rate of condensation at the cold end.

The condensed working fluid then flows back to the hot end of the pipe. In the case of vertically-oriented heat pipes the fluid may be moved by the force of gravity. In the case of heat pipes containing wicks, the fluid is returned by capillary action.

When making heat pipes, there is no need to create a vacuum in the pipe. One simply boils the working fluid in the heat pipe until the resulting vapour has purged the non condensing gases from the pipe and then seals the end.

An interesting property of heat pipes is the temperature over which they are effective. Initially, it might be suspected that a water charged heat pipe would only work when the hot end reached the boiling point (100 °C) and steam was transferred to the cold end. However, the boiling point of water is dependent on absolute pressure inside the pipe. In an evacuated pipe, water will boil just slightly above its melting point (0 °C). The heat pipe will operate, therefore, when the hot end is just slightly warmer than the melting point of the working fluid. Similarly, a heat pipe with water as a working fluid can work well above the boiling point (100 °C), if the cold end is low enough in temperature to condense the fluid.

The main reason for the effectiveness of heat pipes is the evaporation and condensation of the working fluid. The heat of vaporization greatly exceeds the sensible heat capacity. Using water as an example, the energy needed to evaporate one gram of water is equivalent to the amount of energy needed to raise the temperature of that same gram of water by 540 °C (hypothetically, if the water was under extremely high pressure so it didn't vaporize or freeze over this temperature range). Almost all of that energy is rapidly transferred to the "cold" end when the fluid condenses there, making a very effective heat transfer system with no moving parts.

Origins and research in the United States

The general principle of heat pipes using gravity (commonly classified as two phase thermosiphons) dates back to the steam age. The modern concept for a capillary driven heat pipe was first suggested by R.S. Gaugler of General Motors in 1942 who patented the idea. The benefits of employing capillary action were independently developed and first demonstrated by George Grover at Los Alamos National Laboratory in 1963 and subsequently published in the Journal of Applied Physics in 1964. Grover noted in his notebook:

"Heat transfer via capillary movement of fluids. The "pumping" action of surface tension forces may be sufficient to move liquids from a cold temperature zone to a high temperature zone (with subsequent return in vapor form using as the driving force, the difference in vapor pressure at the two temperatures) to be of interest in transferring heat from the hot to the cold zone. Such a closed system, requiring no external pumps, may be of particular interest in space reactors in moving heat from the reactor core to a radiating system. In the absence of gravity, the forces must only be such as to overcome the capillary and the drag of the returning vapor through its channels."

Between 1964 and 1966, RCA was the first corporation to undertake research and development of heat pipes for commercial applications (though their work was mostly funded by the US government). During the late 1960s NASA played a large role in heat pipe development by funding a significant amount of research on their applications and reliability in space flight following from Grover's suggestion. NASA's attraction to heat pipe cooling systems was understandable given their low weight, high heat flux, and zero power draw. Their primary interest however was based on the fact that the system wouldn't be adversely affected by operating in a zero gravity environment. The first application of heat pipes in the space program was in thermal equilibration of satellite transponders. As satellites orbit, one side is exposed to the direct radiation of the sun while the opposite side is completely dark and exposed to the deep cold of outer space. This causes severe discrepancies in the temperature (and thus reliability and accuracy) of the transponders. The heat pipe cooling system designed for this purpose managed the high heat fluxes and demonstrated flawless operation with and without the influence of gravity. The developed cooling system was the first description and usage of variable conductance heat pipes to actively regulate heat flow or evaporator temperature.

Corporate R&D

Publications in 1967 and 1968 by Feldman, Eastman, & Katzoff first discussed applications of heat pipes to areas outside of government concern and that did not fall under the high temperature classification such as; air conditioning, engine cooling, and electronics cooling. These papers also made the first mentions of flexible, arterial, and flat plate heat pipes. 1969 publications introduced the concepts of the rotational heat pipe with its applications to turbine blade cooling and the first discussions of heat pipe applications to cryogenic processes.

Starting in the 1980s Sony began incorporating heat pipes into the cooling schemes for some of its commercial electronic products in place of both forced convection and passive finned heat sinks. Initially they were used in tuners & amplifiers, soon spreading to other high heat flux electronics applications. During the late 1990s increasingly hot microcomputer CPUs spurred a threefold increase in the number of U.S. heat pipe patent applications. As heat pipes transferred from a specialized industrial heat transfer component to a consumer commodity most development and production moved from the U.S. to Asia. Modern CPU heat pipes are typically made from copper and use water as the working fluid.

Applications

Alaska pipeline support legs cooled by heat pipes to keep permafrost frozen.

Grover and his colleagues were working on cooling systems for nuclear power cells for space craft, where extreme thermal conditions are found. Heat pipes have since been used extensively in spacecraft as a means for managing internal temperature conditions.

Heat pipes are extensively used in many modern computer systems, where increased power requirements and subsequent increases in heat emission have resulted in greater demands on cooling systems. Heat pipes are typically used to move heat away from components such as CPUs and GPUs to heat sinks where thermal energy may be dissipated into the environment.

Solar Thermal

Heat pipes are also being widely used in solar thermal water heating applications in combination with evacuated tube solar collector arrays. In these applications, distilled water is commonly used as the heat transfer fluid inside a sealed length of copper tubing that is located within an evacuated glass tube and oriented towards the sun.

In solar thermal water heating applications, an evacuated tube collector can deliver up to 40% more efficiency compared to more traditional "flat plate" solar water heaters. Evacuated tube collectors eliminate the need for anti-freeze additives to be added as the vacuum helps prevent heat loss. These types of solar thermal water heaters are frost protected down to more than -3 °C and are being used in Antarctica to heat water.

Pipelines over permafrost

Heat pipes are used to dissipate heat on the Trans-Alaska Pipeline System. Without them residual ground heat remaining in the oil, as well as that produced by friction and turbulence in the moving oil would conduct down the pipe's support legs. This would likely melt the permafrost on which the supports are anchored. This would cause the pipeline to sink and possibly sustain damage. To prevent this each vertical support member has been mounted with 4 vertical heat pipes.

Cooking

Heat pipes have been designed to speed the cooking of roasts. The pipe is poked through the roast. One end of the pipe extends into the oven where it draws heat to the middle of the roast.

Ventilation heat recovery

In heating, ventilation and air-conditioning systems, HVAC, heat pipes are positioned within the supply and exhaust air streams of an air handling system, or in the exhaust gases of an industrial process, in order to recover the heat energy.

The device consists of a battery of multi-row finned heat pipe tubes located within both the supply and exhaust air streams. Within the exhaust air side of the heat pipe, the refrigerant evaporates, taking its heat from the extract air. The refrigerant vapour moves towards the cooler end of the tube, within the supply air side of the device, where it condenses and gives up its heat. The condensed refrigerant returns by a combination of gravity and capillary action in the wick. Thus heat is transferred from the exhaust air stream through the tube wall to the refrigerant, and then from the refrigerant through the tube wall to the supply air stream.

Because of the characteristics of the device, better efficiencies are obtained when the unit is positioned upright with the supply air side mounted over the exhaust air side, this allows the liquid refrigerant to flow quickly under gravity back to the evaporator. Generally, gross heat transfer efficiencies of up to 75% are claimed by manufacturers.

Limitations

Heat pipes must be tuned to particular cooling conditions. The choice of pipe material, size and coolant all have an effect on the optimal temperatures in which heat pipes work.

When heated above a certain temperature, all of the working fluid in the heat pipe will vaporize and the condensation process will cease to occur; in such conditions, the heat pipe's thermal conductivity is effectively reduced to the heat conduction properties of its solid metal casing alone. As most heat pipes are constructed of copper (a metal with high heat conductivity), an overheated heatpipe will generally continue to conduct heat at around 1/80 of the original conductivity.

In addition, below a certain temperature, the working fluid will not undergo phase change, and the thermal conductivity will be reduced to that of the solid metal casing. One of the key criteria for the selection of a working fluid is the desired operational temperature range of the application. The lower temperature limit typically occurs a few degrees above the freezing point of the working fluid.

Most manufacturers cannot make a traditional heat pipe smaller than 3mm in diameter due to material limitations (though 1.6mm thin sheets can be fabricated). Experiments have been conducted with micro heat pipes, which use piping with sharp edges, such as triangular or rhombus-like tubing. In these cases, the sharp edges transfer the fluid through capillary action, and no wick is necessary.

Chapter 5

Thermal Conductivity

In physics, **thermal conductivity**, *k*, is the property of a material describing its ability to conduct heat. It appears primarily in Fourier's Law for heat conduction. Thermal conductivity is measured in watts per kelvin-meter ($W \cdot K^{-1} \cdot m^{-1}$, i.e. W/(K·m) or in IP units ($Btu \cdot hr^{-1} \cdot ft^{-1} \cdot F^{-1}$, i.e. Btu/(hr·ft · F). Multiplied by a temperature difference (in kelvins, K) and an area (in square meters, m^2), and divided by a thickness (in meters, m), the thermal conductivity predicts the rate of energy loss (in watts, W) through a piece of material. In the window building industry "thermal conductivity" is expressed as the U-Factor, which measures the rate of heat transfer and tells you how well the window insulates. U-factor values generally range from 0.15 to 1.25 and are measured in Watts per square meter per Kelvin - ($W/m^2 \cdot K$). The lower the U-factor, the better the window insulates.

The reciprocal of thermal conductivity is *thermal resistivity*.

Measurement

There are a number of ways to measure thermal conductivity. Each of these is suitable for a limited range of materials, depending on the thermal properties and the medium temperature. There is a distinction between steady-state and transient techniques.

In general, steady-state techniques are useful when the temperature of the material does not change with time. This makes the signal analysis straightforward (steady state implies constant signals). The disadvantage is that a well-engineered experimental setup is usually needed. The Divided Bar (various types) is the most common device used for consolidated rock samples.

The transient techniques perform a measurement during the process of heating up. Their advantage is quicker measurements. Transient methods are usually carried out by needle probes.

Standards

- IEEE Standard 442-1981, "IEEE guide for soil thermal resistivity measurements", ISBN 0-7381-0794-8..

- IEEE Standard 98-2002, "Standard for the Preparation of Test Procedures for the Thermal Evaluation of Solid Electrical Insulating Materials", ISBN 0-7381-3277-2

- ASTM Standard D5334-08, "Standard Test Method for Determination of Thermal Conductivity of Soil and Soft Rock by Thermal Needle Probe Procedure"

- ASTM Standard D5470-06, "Standard Test Method for Thermal Transmission Properties of Thermally Conductive Electrical Insulation Materials"

- ASTM Standard E1225-04, "Standard Test Method for Thermal Conductivity of Solids by Means of the Guarded-Comparative-Longitudinal Heat Flow Technique"

- ASTM Standard D5930-01, "Standard Test Method for Thermal Conductivity of Plastics by Means of a Transient Line-Source Technique"

- ASTM Standard D2717-95, "Standard Test Method for Thermal Conductivity of Liquids"

- ISO 22007-2:2008 "Plastics -- Determination of thermal conductivity and thermal diffusivity -- Part 2: Transient plane heat source (hot disc) method"

- Note: What is called the k-value of construction materials (e.g. window glass) in the U.S., is called λ-value in Europe. What is called U-value (= the inverse of R-value) in the U.S., used to be called k-value in Europe, but is now also called U-value in Europe.

Definitions

The reciprocal of thermal conductivity is *thermal resistivity*, usually measured in kelvin-meters per watt ($K{\cdot}m{\cdot}W^{-1}$). When dealing with a known amount of material, its *thermal conductance* and the reciprocal property, *thermal resistance*, can be described. Unfortunately, there are differing definitions for these terms.

Conductance

For general scientific use, *thermal conductance* is the quantity of heat that passes in unit time through a plate of *particular area and thickness* when its opposite faces differ in temperature by one kelvin. For a plate of thermal conductivity k, area A and thickness L this is kA/L, measured in $W{\cdot}K^{-1}$ (equivalent to: W/°C). Thermal conductivity and conductance are analogous to electrical conductivity ($A \cdot m^{-1} \cdot V^{-1}$) and electrical conductance ($A \cdot V^{-1}$).

There is also a measure known as heat transfer coefficient: the quantity of heat that passes in unit time through *unit area* of a plate of particular thickness when its opposite faces differ in temperature by one kelvin. The reciprocal is *thermal insulance*. In summary:

- *thermal conductance* = kA/L, measured in $W \cdot K^{-1}$
    - *thermal resistance* = $L/(kA)$, measured in $K \cdot W^{-1}$ (equivalent to: °C/W)
- *heat transfer coefficient* = k/L, measured in $W \cdot K^{-1} \cdot m^{-2}$
    - *thermal insulance* = L/k, measured in $K \cdot m^2 \cdot W^{-1}$.

The heat transfer coefficient is also known as *thermal admittance*

Resistance

When thermal resistances occur in series, they are additive. So when heat flows through two components each with a resistance of 1 °C/W, the total resistance is 2 °C/W.

A common engineering design problem involves the selection of an appropriate sized heat sink for a given heat source. Working in units of thermal resistance greatly simplifies the design calculation. The following formula can be used to estimate the performance:

$$R_{hs} = \frac{\Delta T}{P_{th}} - R_s$$

where:

- R_{hs} is the maximum thermal resistance of the heat sink to ambient, in °C/W
- ΔT is the temperature difference (temperature drop), in °C
- P_{th} is the thermal power (heat flow), in watts
- R_s is the thermal resistance of the heat source, in °C/W

For example, if a component produces 100 W of heat, and has a thermal resistance of 0.5 °C/W, what is the maximum thermal resistance of the heat sink? Suppose the maximum temperature is 125 °C, and the ambient temperature is 25 °C; then the ΔT is 100 °C. The heat sink's thermal resistance to ambient must then be 0.5 °C/W or less.

Transmittance

A third term, *thermal transmittance*, incorporates the thermal conductance of a structure along with heat transfer due to convection and radiation. It is measured in the same units as thermal conductance and is sometimes known as the *composite thermal conductance*. The term *U-value* is another synonym.

Summary

In summary, for a plate of thermal conductivity *k* (the *k value*), area *A* and thickness *t*:

- *thermal conductance* = *k*/*t*, measured in $W·K^{-1}·m^{-2}$;
- *thermal resistance* (*R-value*) = *t*/*k*, measured in $K·m^2·W^{-1}$;
- *thermal transmittance* (*U-value*) = 1/(Σ(*t*/*k*)) + convection + radiation, measured in $W·K^{-1}·m^{-2}$.
- *K-value* refers in Europe to the total insulation value of a building. K-value is obtained by multiplying the form factor of the building (= the total inward surface of the outward walls of the building divided by the total volume of the building) with the average U-value of the outward walls of the building. K value is therefore expressed as $(m^2·m^{-3})·(W·K^{-1}·m^{-2}) = W·K^{-1}·m^{-3}$. A house with a volume of 400 m^3 and a K-value of 0.45 (the new European norm. It is commonly referred to as K45) will therefore theoretically require 180 W to maintain its interior temperature 1 K above exterior temperature. So, to maintain the house at 20 °C when it is freezing outside (0 °C), 3600 W of continuous heating is required.

Examples

In metals, thermal conductivity approximately tracks electrical conductivity according to the Wiedemann-Franz law, as freely moving valence electrons transfer not only electric current but also heat energy. However, the general correlation between electrical and thermal conductance does not hold for other materials, due to the increased importance of phonon carriers for heat in non-metals. As shown in the table below, highly electrically conductive silver is less thermally conductive than diamond, which is an electrical insulator.

Thermal conductivity depends on many properties of a material, notably its structure and temperature. For instance, pure crystalline substances exhibit very different thermal conductivities along different crystal axes, due to differences in phonon coupling along a given crystal axis. Sapphire is a notable example of variable thermal conductivity based on orientation and temperature, with 35 W/(m·K) along the c-axis and 32 W/(m·K) along the a-axis.

Air and other gases are generally good insulators, in the absence of convection. Therefore, many insulating materials function simply by having a large number of gas-filled pockets which prevent large-scale convection. Examples of these include expanded and extruded polystyrene (popularly referred to as "styrofoam") and silica aerogel. Natural, biological insulators such as fur and feathers achieve similar effects by dramatically inhibiting convection of air or water near an animal's skin.

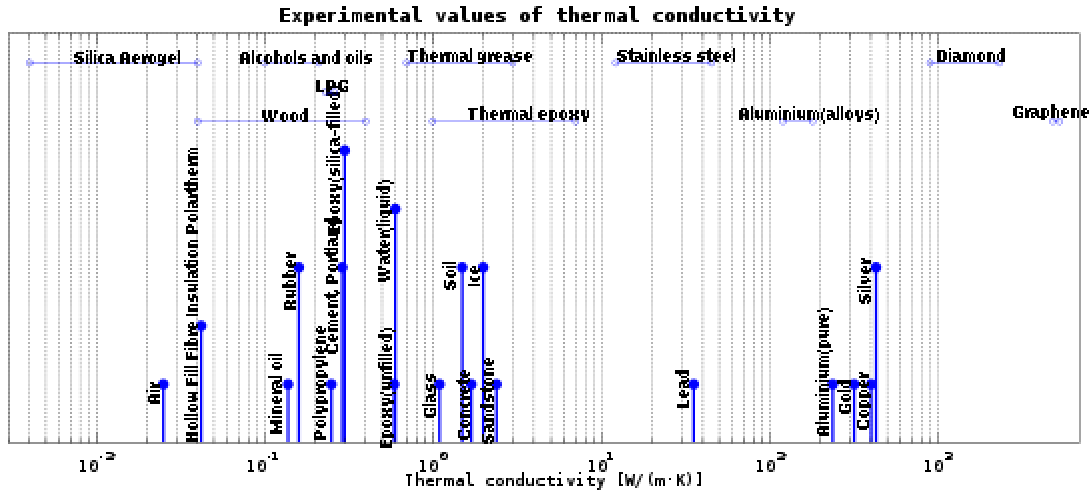
Ceramic is used for its low thermal conductivity on exhaust systems to prevent heat from reaching sensitive components

Light gases, such as hydrogen and helium typically have high thermal conductivity. Dense gases such as xenon and dichlorodifluoromethane have low thermal conductivity. An exception, sulfur hexafluoride, a dense gas, has a relatively high thermal conductivity due to its high heat capacity. Argon, a gas denser than air, is often used in insulated glazing (double paned windows) to improve their insulation characteristics.

Thermal conductivity is important in building insulation and related fields. However, materials used in such trades are rarely subjected to chemical purity standards. Several construction materials' *k* values are listed below. These should be considered approximate due to the uncertainties related to material definitions.

The following table is meant as a small sample of data to illustrate the thermal conductivity of various types of substances.

Experimental values



Experimental values of thermal conductivity.

This is a list of approximate values of thermal conductivity, *k*, for some common materials. Please consult the list of thermal conductivities for more accurate values, references and detailed information.

Material	Thermal conductivity W/(m·K)
Silica Aerogel	0.004 - 0.04
Air	0.025
Wood	0.04 - 0.4
Hollow Fill Fibre Insulation	0.042
Alcohols and oils	0.1 - 0.21
Polypropylene	0.25
Mineral oil	0.138
Rubber	0.16
LPG	0.23 - 0.26
Cement, Portland	0.29
Epoxy (silica-filled)	0.30
Epoxy (unfilled)	0.59
Water (liquid)	0.6
Thermal grease	0.7 - 3
Thermal epoxy	1 - 7
Glass	1.1
Soil	1.5
Concrete, stone	1.7

Ice	2
Sandstone	2.4
Stainless steel	12.11 ~ 45.0
Lead	35.3
Aluminium	237 (pure) 120—180 (alloys)
Gold	318
Copper	401
Silver	429
Diamond	900 - 2320
Graphene	(4840±440) - (5300±480)

Physical origins

Heat flux is exceedingly difficult to control and isolate in a laboratory setting. Thus at the atomic level, there are no simple, correct expressions for thermal conductivity. Atomically, the thermal conductivity of a system is determined by how atoms composing the system interact. There are two different approaches for calculating the thermal conductivity of a system.

- The first approach employs the Green-Kubo relations. Although this employs analytic expressions which in principle can be solved, in order to calculate the thermal conductivity of a dense fluid or solid using this relation requires the use of molecular dynamics computer simulation.

- The second approach is based upon the relaxation time approach. Due to the anharmonicity within the crystal potential, the phonons in the system are known to scatter. There are three main mechanisms for scattering:
    - Boundary scattering, a phonon hitting the boundary of a system;
    - Mass defect scattering, a phonon hitting an impurity within the system and scattering;
    - Phonon-phonon scattering, a phonon breaking into two lower energy phonons or a phonon colliding with another phonon and merging into one higher energy phonon.

Lattice waves

Heat transport in both glassy and crystalline dielectric solids occurs through elastic vibrations of the lattice (phonons). This transport is limited by elastic scattering of acoustic phonons by lattice defects. These predictions were confirmed by the experiments of Chang and Jones on commercial glasses and glass ceramics, where mean free paths were limited by "internal boundary scattering" to length scales of 10^{-2} cm to 10^{-3} cm.

The phonon mean free path has been associated directly with the effective relaxation length for processes without directional correlation. Thus, if V_g is the group velocity of a phonon wave packet, then the relaxation length l is defined as:

$$l = V_g t$$

where *t* is the characteristic relaxation time. Since longitudinal waves have a much greater phase velocity than transverse waves, V_{long} is much greater than V_{trans}, and the relaxation length or mean free path of longitudinal phonons will be much greater. Thus, thermal conductivity will be largely determined by the speed of longitudinal phonons.

Regarding the dependence of wave velocity on wavelength or frequency (dispersion), low-frequency phonons of long wavelength will be limited in relaxation length by elastic Rayleigh scattering. This type of light scattering form small particles is proportional to the fourth power of the frequency. For higher frequencies, the power of the frequency will decrease until at highest frequencies scattering is almost frequency independent. Similar arguments were subsequently generalized to many glass forming substances using Brillouin scattering.

Equations

First, we define heat conduction, *H*:

$$H = \frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{x}$$

where $\frac{\Delta Q}{\Delta t}$ is the rate of heat flow, *k* is the thermal conductivity, *A* is the total cross sectional area of conducting surface, ΔT is temperature difference, and *x* is the thickness of conducting surface separating the 2 temperatures. Dimension of thermal conductivity = $M^1L^1T^{-3}K^{-1}$

Rearranging the equation gives thermal conductivity:

$$k = \frac{\Delta Q}{\Delta t}\frac{1}{A}\frac{x}{\Delta T}$$

(Note: $\Delta T / x$ is the temperature gradient)

I.E. It is defined as the quantity of heat, ΔQ, transmitted during time Δt through a thickness *x*, in a direction normal to a surface of area *A*, per unit area of A, due to a temperature difference ΔT, under steady state conditions and when the heat transfer is dependent only on the temperature gradient.

Alternatively, it can be thought of as a flux of heat (energy per unit area per unit time) divided by a temperature gradient (temperature difference per unit length)

$$k = \frac{\Delta Q}{A\Delta t}\frac{x}{\Delta T}$$

Typical units are SI: W/(m·K) and English units: Btu/(h·ft·°F). To convert between the two, use the relation 1 Btu/(h·ft·°F) = 1.730735 W/(m·K). [Perry's Chemical Engineers' Handbook, 7th Edition, Table 1-4]

In the textile industry, a tog value may be quoted as a measure of thermal resistance in place of a measure in SI units.

Chapter 6

Thermal Contact Conductance

In physics, **thermal contact conductance** is the study of heat conduction between solid bodies in thermal contact. The **thermal contact conductance coefficient**, h_c, is a property indicating the thermal conductivity, or ability to conduct heat, between two bodies in contact. The inverse of this property is termed **thermal contact resistance**.

Definition

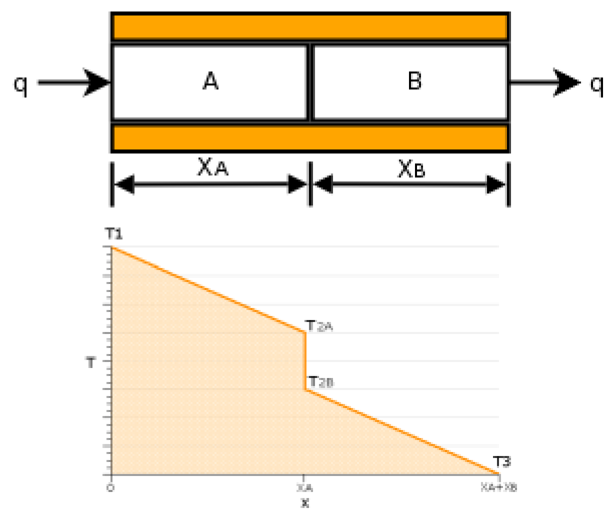


Fig. 1: Heat flow between two solids in contact and the temperature distribution.

When two solid bodies come in contact, such as A and B in Figure 1, heat flows from the hotter body to the colder body. From experience, the temperature profile along the two bodies varies, approximately, as shown in the figure. A temperature drop is observed at the interface between the two surfaces in contact. This phenomenon is said to be a result of a *thermal contact resistance* existing between the contacting surfaces. Thermal contact resistance is defined as the ratio between this temperature drop and the average heat flow across the interface.

According to **Fourier's law**, the heat flow between the bodies is found by the relation:

$$q = -kA\frac{dT}{dx} \quad (1)$$

where *q* is the heat flow, *k* is the thermal conductivity, *A* is the cross sectional area and *dT* / *dx* is the temperature gradient in the direction of flow.

From considerations of energy conservation, the heat flow between the two bodies in contact, bodies A and B, is found as:

$$q = \frac{T_1 - T_3}{\Delta x_A/(k_A A) + 1/(h_c A) + \Delta x_B/(k_B A)} \quad (2)$$

One may observe that the heat flow is directly related to the thermal conductivities of the bodies in contact, k_A and k_B, the contact area, *A* and the thermal contact resistance, $1 / h_c$, which, as previously noted, is the inverse of the thermal conductance coefficient, h_c.

Importance

Thermal contact conductance is an important factor in a variety of applications, largely because many physical systems contain a mechanical combination of two materials. Some of the fields where contact conductance is of importance are:

- Electronics
  - Electronic packaging
  - Heat sinks
  - Brackets
- Industry
  - Nuclear reactor cooling
  - Gas turbine cooling
  - Internal combustion engines
  - Heat exchangers
  - Thermal insulation
- Flight
  - Hypersonic flight vehicles
  - Thermal supervision for space vehicles

Factors influencing contact conductance

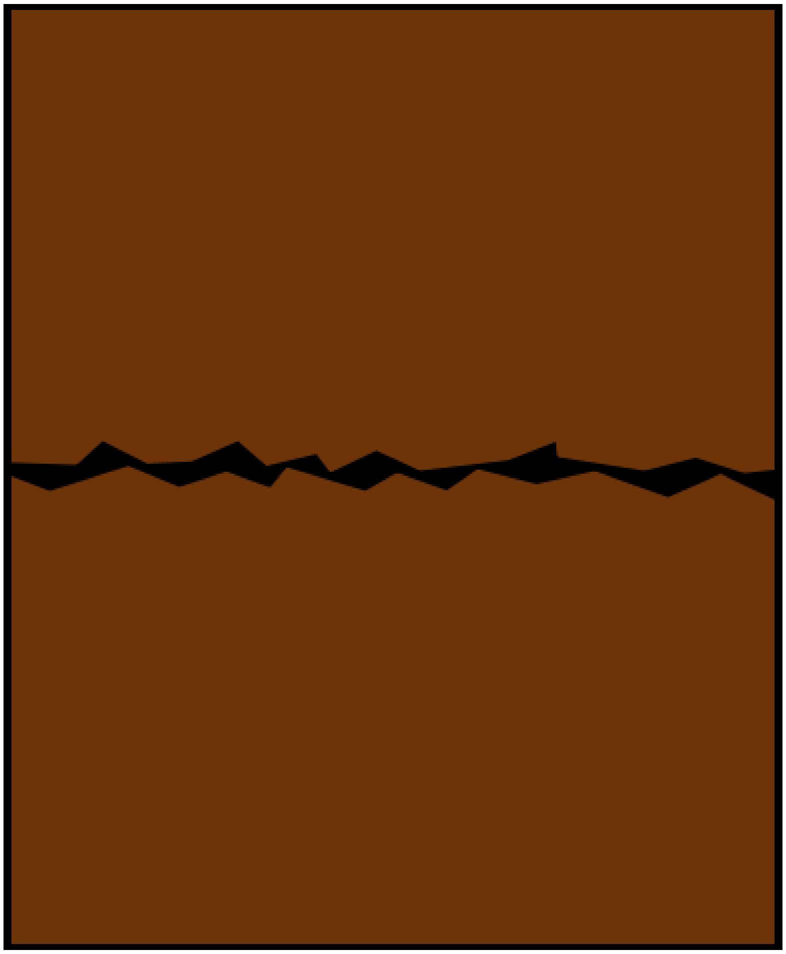


Fig. 2: An enlargement of the interface between two contacting surfaces. The finish quality is exaggerated for the sake of the argument.

Thermal contact conductance is a complicated phenomenon, influenced by many factors. Experience shows that the most important ones are as follows:

Contact pressure

The contact pressure is the factor of most influence on contact conductance. As contact pressure grows, contact conductance grows (And consequentially, contact resistance becomes smaller). This is attributed to the fact that the contact surface between the bodies grows as the contact pressure grows.

Since the contact pressure is the most important factor, most studies, correlations and mathematical models for measurement of contact conductance are done as a function of this factor.

The thermal contact resistance of certain sandwich kinds of materials that are manufactured by rolling under high temperatures may sometimes be ignored because the decrease in thermal conductivity between them is negligible.

Interstitial materials

No truly smooth surfaces really exist, and surface imperfections are visible under a microscope. As a result, when two bodies are pressed together, contact is only performed in a finite number of points, separated by relatively large gaps, as can be shown in Fig. 2. Since the actual contact area is reduced, another resistance for heat flow exists. The gasses/fluids filling these gaps may largely influence the total heat flow across the interface. The thermal conductivity of the interstitial material and its pressure are the two properties governing its influence on contact conductance.

In the absence of interstitial materials, as in a vacuum, the contact resistance will be much larger, since flow through the intimate contact points is dominant.

Surface roughness, waviness and flatness

One can characterise a surface that has undergone certain finishing operations by three properties: roughness, waviness and flatness. Among these, roughness is of most importance, and is usually indicated by an rms value, σ.

Surface deformations

When the two bodies come in contact, surface deformation may occur on both bodies. This deformation may either be plastic or elastic, depending on the material properties and the contact pressure. When a surface undergoes plastic deformation, contact resistance is lowered, since the deformation causes the actual contact area to increase

Surface cleanliness

The presence of dust particles, acids, etc., can also influence the contact conductance.

Measurement of thermal contact conductance

Going back to Formula 2, calculation of the thermal contact conductance may prove difficult, even impossible, due to the difficulty in measuring the contact area, A (A product of surface characteristics, as explained earlier). Because of this, contact conductance/resistance is usually found experimentally, by using a standard apparatus.

The results of such experiments are usually published in Engineering literature, on magazines such as *Journal of Heat Transfer*, *International Journal of Heat and Mass Transfer*, etc. Unfortunately, a centralized database of contact conductance coefficients does not exist, a situation which sometimes causes companies to use outdated, irrelevant data, or not taking contact conductance as a consideration at all.

CoCoE (Contact Conductance Estimator), a project founded to solve this problem and create a centralized database of contact conductance data and a computer program that uses it, was started in 2006.

Thermal boundary conductance

While a finite thermal contact conductance is due to voids at the interface, surface waviness, and surface roughness, etc., a finite conductance exists even at near ideal interfaces as well. This conductance, known as thermal boundary conductance, is due to the differences in electronic and vibrational properties between the contacting materials. This conductance is generally much higher than thermal contact conductance, but becomes important in nanoscale material systems.

Chapter 7

Heat Transfer Coefficient

The **heat transfer coefficient**, in thermodynamics and in mechanical and chemical engineering, is used in calculating the heat transfer, typically by convection or phase change between a fluid and a solid:

$$h = \frac{q}{A \cdot \Delta T}$$

where

q = heat flow in input or lost heat flow , J/s = W
h = heat transfer coefficient, W/(m^2K)
A = heat transfer surface area, m^2
ΔT = difference in temperature between the solid surface and surrounding fluid area, K

From the above equation, the heat transfer coefficient is the proportionality coefficient between the heat flux that is a heat flow per unit area, q/A, and the thermodynamic driving force for the flow of heat (i.e., the temperature difference, ΔT).

The heat transfer coefficient has SI units in watts per meter squared-kelvin: W/(m^2K).

Heat transfer coefficient is the inverse of thermal insulance.

There are numerous methods for calculating the heat transfer coefficient in different heat transfer modes, different fluids, flow regimes, and under different thermohydraulic conditions. Often it can be estimated by dividing the thermal conductivity of the convection fluid by a length scale. The heat transfer coefficient is often calculated from the Nusselt number (a dimensionless number).

Dittus–Boelter correlation (1930): forced convection inside tubes

A common and particularly simple correlation useful for many applications is the Dittus–Boelter heat transfer correlation for fluids in turbulent flow. This correlation is applicable when forced convection is the only mode of heat transfer; i.e., there is no boiling, condensation, significant radiation, etc. The accuracy of this correlation is anticipated to be ±15%.

For a liquid flowing in a straight circular pipe with a Reynolds number between 10 000 and 120 000 (in the turbulent pipe flow range), when the liquid's Prandtl number is between 0.7 and 120, for a location far from the pipe entrance (more than 10 pipe diameters; more than 50 diameters according to many authors) or other flow disturbances, and when the pipe surface is hydraulically smooth, the heat transfer coefficient between the bulk of the fluid and the pipe surface can be expressed as:

$$h = \frac{k_w}{D_H} Nu$$

where

k_w - thermal conductivity of the liquid (i.e.water)
D_H - D_i - Hydraulic diameter
Nu - Nusselt number
$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^n$ (Dittus-Boelter correlation)
Pr - Prandtl number
Re - Reynolds number
n = 0.4 for heating (wall hotter than the bulk fluid) and 0.33 for cooling (wall cooler than the bulk fluid) .

The fluid properties necessary for the application of this equation are evaluated at the bulk temperature thus avoiding iteration

Thom correlation

There exist simple fluid-specific correlations for heat transfer coefficient in boiling. The Thom correlation is for flow boiling of water (subcooled or saturated at pressures up to about 20 MPa) under conditions where the nucleate boiling contribution predominates over forced convection. This correlation is useful for rough estimation of expected temperature difference given the heat flux:

$$\Delta T_{sat} = 22.5 \cdot q^{0.5} \exp(-P/8.7)$$

where:

ΔT_{sat} is the wall temperature elevation above the saturation temperature, K

q is the heat flux, MW/m^2
P is the pressure of water, MPa

Note that this empirical correlation is specific to the units given.

Heat transfer coefficient of pipe wall

The resistance to the flow of heat by the material of pipe wall can be expressed as a "heat transfer coefficient of the pipe wall". However, one needs to select if the heat flux is based on the pipe inner or the outer diameter.

Selecting to base the heat flux on the pipe inner diameter, and assuming that the pipe wall thickness is small in comparison with the pipe inner diameter, then the heat transfer coefficient for the pipe wall can be calculated as if the wall were not curved:

$$h_{wall} = \frac{k}{x}$$

where k is the effective thermal conductivity of the wall material and x is the wall thickness.

If the above assumption does not hold, then the wall heat transfer coefficient can be calculated using the following expression:

$$h_{wall} = \frac{2k}{d_i \ln(d_o/d_i)}$$

where d_i and d_o are the inner and outer diameters of the pipe, respectively.

The thermal conductivity of the tube material usually depends on temperature; the mean thermal conductivity is often used.

Combining heat transfer coefficients

For two or more heat transfer processes acting in parallel, heat transfer coefficients simply add:

$$h = h_1 + h_2 + \ldots$$

For two or more heat transfer processes connected in series, heat transfer coefficients add inversely:

$$\frac{1}{h} = \frac{1}{h_1} + \frac{1}{h_2} + \ldots$$

For example, consider a pipe with a fluid flowing inside. The rate of heat transfer between the bulk of the fluid inside the pipe and the pipe external surface is:

$$q = \left(\frac{1}{\frac{1}{h} + \frac{t}{k}} \right) \cdot A \cdot \Delta T$$

where

q = heat transfer rate (W)
h = heat transfer coefficient ($W/(m^2 \cdot K)$)
t = wall thickness (m)
k = wall thermal conductivity ($W/m \cdot K$)
A = area (m^2)
ΔT = difference in temperature.

Overall heat transfer coefficient

The **overall heat transfer coefficient** U is a measure of the overall ability of a series of conductive and convective barriers to transfer heat. It is commonly applied to the calculation of heat transfer in heat exchangers, but can be applied equally well to other problems.

For the case of a heat exchanger, U can be used to determine the total heat transfer between the two streams in the heat exchanger by the following relationship:

$$q = UA\Delta T_{LM}$$

where

q = heat transfer rate (W)
U = overall heat transfer coefficient ($W/(m^2 \cdot K)$)
A = heat transfer surface area (m^2)
ΔT_{LM} = log mean temperature difference (K)

The overall heat transfer coefficient takes into account the individual heat transfer coefficients of each stream and the resistance of the pipe material. It can be calculated as the reciprocal of the sum of a series of thermal resistances (but more complex relationships exist, for example when heat transfer takes place by different routes in parallel):

$$\frac{1}{UA} = \sum \frac{1}{hA} + \sum R$$

where

R = Resistance(s) to heat flow in pipe wall (K/W)
Other parameters are as above.

The heat transfer coefficient is the heat transferred per unit area per kelvin. Thus *area* is included in the equation as it represents the area over which the transfer of heat takes place. The areas for each flow will be different as they represent the contact area for each fluid side.

The *thermal resistance* due to the pipe wall is calculated by the following relationship:

$$R = \frac{x}{k \cdot A}$$

where

x = the wall thickness (m)
k = the thermal conductivity of the material (W/(m·K))
A = the total area of the heat exchanger (m^2)

This represents the heat transfer by conduction in the pipe.

The *thermal conductivity* is a characteristic of the particular material. Values of thermal conductivities for various materials are listed in the list of thermal conductivities.

As mentioned earlier, the *convection heat transfer coefficient* for each stream depends on the type of fluid, flow properties and temperature properties.

Some typical heat transfer coefficients include:

- Air - h = 10 to 100 W/(m^2K)
- Water - h = 500 to 10,000 W/(m^2K)

Thermal resistance due to fouling deposits

Surface coatings can build on heat transfer surfaces during heat exchanger operation due to fouling. These add extra thermal resistance to the wall and may noticeably decrease the overall heat transfer coefficient and thus performance. (Fouling can also cause other problems.)

The additional thermal resistance due to fouling can be found by comparing the overall heat transfer coefficient determined from laboratory readings with calculations based on theoretical correlations. They can also be evaluated from the development of the overall heat transfer coefficient with time (assuming the heat exchanger operates under otherwise identical conditions). This is commonly applied in practice, e.g.. The following relationship is often used:

$$\frac{1}{U_{exp=}}\frac{1}{U_{pre}}+R_f$$

where

U_{exp} = overall heat transfer coefficient based on experimental data for the heat exchanger in the "fouled" state, $\frac{W}{m^2K}$

U_{pre} = overall heat transfer coefficient based on calculated or measured ("clean heat exchanger") data, $\frac{W}{m^2K}$

R_f = thermal resistance due to fouling, $\frac{m^2K}{W}$

Chapter 8

Lumped Capacitance Model

A **lumped capacitance model**, also called **lumped system analysis**, reduces a thermal system to a number of discrete “lumps” and assumes that the temperature difference inside each lump is negligible. This approximation is useful to simplify otherwise complex differential heat equations. It was developed as a mathematical analog of electrical capacitance.

This is a common approximation in transient conduction, which may be used whenever heat conduction within an object is much faster than heat conduction across the boundary of the object. This is a method of approximation that suitably reduces one aspect of the transient conduction system (that within the object) to an equivalent steady state system (that is, it is assumed that the temperature within the object is completely uniform, although its value may be changing in time).

Method

To determine the number of lumps the Biot number (Bi), a dimensionless parameter of the system, is used. Bi is defined as the ratio of the conductive heat resistance within the object to the convective heat transfer resistance across the object's boundary with a uniform bath of different temperature. When the thermal resistance to heat transferred into the object is larger than the resistance to heat being diffused completely within the object, the Biot number is less than 1. In this case, particularly for Biot numbers which are even smaller, the approximation of *spatially uniform temperature within the object* can begin to be used, since it can be presumed that heat transferred into the object has time to uniformaly distribute itself, due to the lower resistance to doing so, as compared with the resistance to heat entering the object.

If the Biot number is less than 0.1 for a solid object, then the entire material will be nearly the same temperature with the dominant temperature difference will be at the surface. It may be regarded as being "thermally thin". The Biot number must generally be less than 0.1 for usefully accurate approximation and heat transfer analysis. The mathematical solution to the lumped system approximation gives Newton's law of cooling.

A Biot number greater than 0.1 (a "thermally thick" substance) indicates that one cannot make this assumption, and more complicated heat transfer equations for "transient heat conduction" will be required to describe the time-varying and non-spatially-uniform temperature field within the material body.

The single capacitance approach can be expanded to involve many resistive and capacitive elements, with Bi < 0.1 for each lump. As the Biot number is calculated based upon a characteristic length of the system, the system can often be broken into a sufficient number of sections, or lumps, so that the Biot number is acceptably small.

Some characteristic lengths of thermal systems are:

- Plate: thickness
- Fin: thickness/2
- Long cylinder: diameter/4
- Sphere: diameter/6

For arbitrary shapes, it may be useful to consider the characteristic length to be volume / surface area.

Thermal circuits

A very useful concept used in heat transfer applications is the representation of thermal transfer by what is known as thermal circuits. A thermal circuit is the representation of the resistance to heat flow as though it were an electrical resistor. The heat transferred is analogous to the electrical current and the thermal resistance is analogous to the electrical resistor. The values of the thermal resistance for the different modes of heat transfer are calculated as the denominators of the developed equations. The thermal resistances of the different modes of heat transfer are used in analyzing combined modes of heat transfer.

The equations describing the three heat transfer modes and their thermal resistances, as discussed previously, are summarized in the table below:

Equations for different heat transfer modes and their thermal resistances.

Transfer Mode	Rate of Heat Transfer	Thermal Resistance
Conduction	$\dot{Q} = \frac{T_1 - T_2}{\left(\frac{L}{kA}\right)}$	$\frac{L}{kA}$
Convection	$\dot{Q} = \frac{T_{surf} - T_{envr}}{\left(\frac{1}{h_{conv}A_{surf}}\right)}$	$\frac{1}{h_{conv}A_{surf}}$
Radiation	$\dot{Q} = \frac{T_{surf} - T_{surr}}{\left(\frac{1}{h_r A_{surf}}\right)}$	$\frac{1}{h_r A}$, where $h_r = \epsilon\sigma(T_{surf} + T_{surr})(T_{surf}^2 + T_{surr}^2)$

In cases where there is heat transfer through different media (for example, through a composite material), the equivalent resistance is the sum of the resistances of the components that make up the composite. Likely, in cases where there are different heat transfer modes, the total resistance is the sum of the resistances of the different modes. Using the thermal circuit concept, the amount of heat transferred through any medium is the quotient of the temperature change and the total thermal resistance of the medium.

As an example, consider a composite wall of cross-sectional area A. The composite is made of an L_1 long cement plaster with a thermal coefficient k_1 and L_2 long paper faced fiber glass, with thermal coefficient k_2. The left surface of the wall is at T_i and exposed to air with a convective coefficient of h_i. The right surface of the wall is at T_o and exposed to air with convective coefficient h_o.

Using the thermal resistance concept heat flow through the composite is as follows:

$$\dot{Q} = \frac{T_i - T_o}{R_i + R_1 + R_2 + R_o} = \frac{T_i - T_1}{R_i} = \frac{T_i - T_2}{R_i + R_1} = \frac{T_i - T_3}{R_i + R_1 + R_2} = \frac{T_1 - T_2}{R_1} = \frac{T_3 - T_o}{R_0}$$

where

$$R_i = \frac{1}{h_i A},\ R_o = \frac{1}{h_o A},\ R_1 = \frac{L_1}{k_1 A},\ \text{and}\ R_2 = \frac{L_2}{k_2 A}$$

Example: Newton's law of cooling

Many situations in which a object has a large thermal capacity and large conductivity, is immersed in a uniform bath which conducts heat relatively poorly, are described by **Newton's law of cooling.** This law stated in non-mathematical form is that *the rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings.* For the law to be correct, the temperatures at all points inside the body must be approximately the same, including the temperature at its surface. Thus, the temperature difference between the body and surroundings does not depend on which part of the body is chosen, since all parts of the body have effectively the same temperature. In these situations, the material of the body does not act to "insulate" other parts of the body from heat flow, and all of the significant insulation (or "thermal resistance") controlling the rate of heat flow in the situation resides in the area of contact between the body and its surroundings. Across this boundary, the temperature-value jumps in a discontinuous fashion.

In such situations, heat can be transferred from the exterior to the interior of a body, across the insulating boundary, by convection, conduction, or diffusion, so long as the boundary serves as a relatively poor conductor with regard to the object's interior. The presence of a physical insulator is not required, so long as the process which serves to pass heat across the boundary is "slow" in comparison to the conductive transfer of heat inside the body (or inside the region of interest—the "lump" described i the introduction).

Newton's law is mathematically stated by the simple first-order differential equation:

$$\frac{dQ}{dt} = h \cdot A(T_{\text{env}} - T(t)) = -h \cdot A\Delta T(t)$$

Q = Thermal energy in joules
h = Heat transfer coefficient
A = Surface area of the heat being transferred
T = Temperature of the object's surface and interior (since these are the same in this approximation)
T_{env} = Temperature of the environment
$\Delta T(t) = T(t) - T_{\text{env}}$ is the time-dependent thermal gradient between environment and object

Putting heat transfers into this form is sometimes not a very good approximation, depending on ratios of heat conductances in the system. If the differences are not large, an accurate formulation of heat transfers in the system may require analysis of heat flow based on the (transient) heat transfer equation in nonhomogeneous, or poorly conductive mediums.

Solution in terms of object heat capacity

If the entire body is treated as lumped capacitance heat reservoir, with total heat content which is proportional to simple total heat capacity C, and T, the temperature of the body, or $Q = CT$. It is expected that the system will experience exponential decay with time in the temperature of a body.

From the definition of heat capacity C comes the relation $C = dQ / dT$. Differentiating this equation with regard to time gives the identity (valid so long as temperatures in the object are uniform at any given time): $dQ / dt = C(dT / dt)$. This expression may be used to replace dQ / dt in the first equation which begins this section, above. Then, if $T(t)$ is the temperature of such a body at time t, and T_{env} is the temperature of the environment around the body:

$$\frac{dT(t)}{dt} = -r(T(t) - T_{\text{env}}) = -r\Delta T(t)$$

where

$r = hA / C$ is a positive constant characteristic of the system, which must be in units of s^{-1}, and is therefore sometimes expressed in terms of a characteristic time constant t_0 given by: $r = 1 / t_0 = \Delta T / (dT(t) / dt)$. Thus, in thermal systems, $t_0 = C / hA$. (The total heat capacity C of a system may be further represented by its mass-specific heat capacity c_p multiplied by its mass m, so that the time constant t_0 is also given by mc_p / hA).

The solution of this differential equation, by standard methods of integration and substitution of boundary conditions, gives:

$$T(t) = T_{\mathrm{env}} + (T(0) - T_{\mathrm{env}})\ e^{-rt}.$$

If:

$\Delta T(t)$ is defined as : $T(t) - T_{\mathrm{env}}$, where $\Delta T(0)$ is the initial temperature difference at time 0,

then the Newtonian solution is written as:

$$\Delta T(t) = \Delta T(0)\ e^{-rt} = \Delta T(0)\ e^{-t/t_0}.$$

This same solution is almost immediately apparent if the initial differential equation is written in terms of $\Delta T(t)$, as the single function to be solved for. '

$$\frac{dT(t)}{dt} = \frac{d\Delta T(t)}{dt} = -\frac{1}{t_0}\Delta T(t)$$

Applications

This mode of analysis has been applied to forensic sciences to analyze the time of death of humans. Also it can be applied to HVAC (heating, ventilating and air-conditioning, or building climate control), to ensure more nearly instantaneous effects of a change in comfort level setting.

Chapter 9

Relativistic Heat Conduction

The theory of **Relativistic Heat Conduction (RHC)** claims to be the only model for heat conduction (and similar diffusion processes) that is compatible with the theory of special relativity, the second law of thermodynamics, electrodynamics, and quantum mechanics, simultaneously. The main features of RHC are:

1. It admits a finite speed of heat propagation, and allows for relativistic effects when heat flux transients approach that speed.
2. It removes the possibility of paradoxical situations that may violate the second law of thermodynamics.
3. It, implicitly, admits the wave–particle duality of the heat-carrying "phonon".

These outcomes are achieved by (1) upgrading the Fourier equation of heat conduction to the form of a Telegraph equation of electrodynamics, and (2) introducing a new definition of the heat flux vector. Consequently, RHC gives rise to a number of interesting phenomena, such as thermal resonance and thermal shock waves, which are possible during high-frequency pulsed laser heating of thermal insulators. The main appealing feature of the theory is its mathematical *elegance and simplicity*.

Background

Classical model

For most of the last two centuries, heat conduction has been modelled by the well-known Fourier equation:

$$\frac{\partial \theta}{\partial t} = \alpha \nabla^2 \theta,$$

where θ is temperature, t is time, $\alpha = k/(\rho\ c)$ is thermal diffusivity, k is thermal conductivity, ρ is density, and c is specific heat capacity. The Laplace operator, ∇^2, is defined in Cartesian coordinates as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

This Fourier equation can be derived by substituting Fourier's linear approximation of the heat flux vector, $\boldsymbol{q}$, as a function of temperature gradient,

$$\mathbf{q} = -k \nabla\theta,$$

into the first law of thermodynamics

$$\rho c \frac{\partial\theta}{\partial t} + \nabla \cdot \mathbf{q} = 0,$$

where the del operator, ∇, is defined in 3D as

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}.$$

It can be shown that this definition of the heat flux vector also satisfies the second law of thermodynamics,

$$\nabla \cdot \left(\frac{\mathbf{q}}{\theta}\right) + \rho \frac{\partial s}{\partial t} = \sigma,$$

where s is specific entropy and σ is entropy production. Alternatively, the second law can be written as

$$\sigma = \frac{-1}{\theta^2} \mathbf{q} \cdot \nabla\theta,$$

which leads to the condition

$$\sigma = \frac{k}{\theta^2}\left[\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2 + \left(\frac{\partial\theta}{\partial z}\right)^2\right],$$

which is always true, because k is a non-negative material property.

Hyperbolic model

For most of the last century, it was recognized that Fourier equation (and its more general Fick's law of diffusion) is in contradiction with the theory of relativity, for *at least* one reason: it admits infinite speed of propagation of heat signals within the continuum field. For example, consider a pulse of heat at the origin; then according to Fourier equation, it is felt (i.e. temperature changes) at infinity, instantaneously. The speed of information propagation is faster than the speed of light in vacuum, which is physically inadmissible within the framework of relativity.

To overcome this contradiction, workers such as Cattaneo, Vernotte, Chester, and others proposed that Fourier equation should be upgraded from the parabolic to a hyperbolic form,

$$\frac{1}{C^2}\,\frac{\partial^2\theta}{\partial t^2}\;+\;\frac{1}{\alpha}\,\frac{\partial\theta}{\partial t}\;=\;\nabla^2\theta,$$

also known as the Telegrapher's equation. Interestingly, the form of this equation traces its origins to Maxwell's equations of electrodynamics; hence, the wave nature of heat is implied. In this equation, *C* is called the speed of second sound (i.e. the fictitious quantum particles, phonons) and this equation is known as the Hyperbolic Heat Conduction (HHC) equation.

For the HHC equation to remain compatible with the first law of thermodynamics, it is necessary to modify the definition of heat flux vector, ***q***, to

$$\tau_0\,\frac{\partial\mathbf{q}}{\partial t}\;+\;\mathbf{q}\;=\;-\,k\,\nabla\theta,$$

where $\tau_0$is a relaxation time, such that $C\;=\;\alpha/\tau_0$.

The most important implication of the hyperbolic equation is that by switching from a parabolic (dissipative) to a hyperbolic (includes a conservative term) partial differential equation, there is the possibility of phenomena such as thermal resonance and thermal shock waves.

Criticism to the HHC model

- The relaxation time, τ_0, is justified based on microscopic aspects of lattice vibration and electron transport; is an extension of kinetic theory calculations and Boltzmann equation for rarefied gases to the case of solids; and is calculated from statistical Newtonian mechanics.. Further, the speed *C* is only a collection of terms, α and τ_0, and has no physical reality or significance similar to that associated with the speed of light. Hence, the hyperbolic equation is compatible with relativity artificially (in form only), but is still fundamentally classical Newtonian.
- The new definition of heat flux vector is an *ad hoc* mathematical approximation of a far more complicated expression; this raises some doubts about the whole approach.
- The most serious criticism is that the hyperbolic equation can violate the second law of thermodynamics. For example, consider an infinitely long wire conductor,

with a heat source at the origin, and measure temperature at distances significantly remote from origin. If the heat source at origin varies with a frequency much higher than the relaxation time (i.e. faster than the speed of second sound) then the hyperbolic equation admits a temperature field in which heat would appear to be moving from cold to hot, in violation of the second law. This contradiction was demonstrated in more mathematically rigorous form.

The theory of RHC attempts to resolve the controversies surrounding the hyperbolic equation, while maintaining the form of that equation. This is achieved by:

- Deriving the hyperbolic equation starting from space-time duality of a Minkowski space, and simple Lorentz transformations, that are basic to the theory of special relativity. This is done without any reference to microstructure or statistical mechanics.
- Treating the speed of second sound, *C*, as a fundamental property of the temperature field, although still fundamentally inferior to the speed of light.
- Modifying the definition of the heat flux vector so that it is simpler, more elegant, and bring it in compliance with the second law of thermodynamics.

Derivation of the RHC equation

Transformations

In a Euclidean space, distance between any two points, *ds*, is measured by

$$ds^2 = dx^2 + dy^2 + dz^2,$$

where *dx*, *dy*, and *dz* are displacements along three orthogonal axes.

In a Minkowski space, distance between two events, *ds*, is measured by

$$ds^2 = d\tau^2 + dx^2 + dy^2 + dz^2,$$

where, τ, is space-like-time and is related to real time, *t*, by

$$\tau = i\,C\,t,$$

where *C* is speed of light in vacuum and $i = \sqrt{-1}$. Hence,

$$ds^2 = dx^2 + dy^2 + dz^2 - C^2\,dt^2.$$

Consequently, the 3D del, ∇ ,operator is upgraded to the 4D quad, $\Box$, operator (also known as the Four-gradient)

$$\Box = \frac{\partial}{\partial \tau}\mathbf{o} + \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} = \frac{\partial}{\partial \tau}\mathbf{o} + \nabla = \frac{-i}{C}\frac{\partial}{\partial t}\mathbf{o} + \nabla.$$

Likewise, the 3D Laplacian, ∇^2, operator is upgraded to the 4D d'Alembert operator, $\Box^2$,

$$\Box^2 = \frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \tau^2} + \nabla^2 = \frac{-1}{C^2}\frac{\partial^2}{\partial t^2} + \nabla^2.$$

Any physical quantity that is Galilean invariant in Euclidean space can be made Lorentz invariant in a Minkowski space, by upgrading from 3D to 4D operators. Consequently, Fourier's equation can be upgraded to 4D as

$$\frac{\partial \theta}{\partial t} = \alpha\,\Box^2\theta = \frac{-\alpha}{C^2}\frac{\partial^2\theta}{\partial t^2} + \alpha\,\nabla^2\theta,$$

which is called the Relativistic Heat Conduction equation. Likewise, the definition of the heat-flux vector, $\boldsymbol{q}$, is upgraded to the 4D form as

$$\mathbf{q} = -k\,\Box\theta = -k\,\nabla\theta + \frac{i\,k}{C}\frac{\partial\theta}{\partial t}\mathbf{o}.$$

Implications

It can be shown that this definition of $\boldsymbol{q}$ is compatible with the first law of thermodynamics,

$$\rho\,c\frac{\partial\theta}{\partial t} + \Box\cdot\mathbf{q} = 0 = \rho\,c\frac{\partial\theta}{\partial t} + \nabla\cdot\mathbf{q} + \frac{-i}{C}\frac{\partial\mathbf{q}}{\partial t}\cdot\mathbf{o},$$

as well as the second law of thermodynamics,

$$\Box\cdot\left(\frac{\mathbf{q}}{\theta}\right) + \rho\frac{\partial s}{\partial t} = \sigma,$$

in their 4D upgraded form. The imaginary terms in these equations are direct manifestation of the wave nature of heat, and are essential for the heat equation to become compatible with all laws of physics. The real terms in these equations are identical to those in the classical heat model.

The most interesting observation about RHC is that it reduces the second law of thermodynamics to a statement of the form

$$\left(\frac{dt}{dx}\right)^2 + \left(\frac{dt}{dy}\right)^2 + \left(\frac{dt}{dz}\right)^2 \geqslant \frac{1}{C^2},$$

which is the "no action at a distance" principle of special relativity. Essentially, the RHC asserts that relativity and the second law of thermodynamics are two alternative, but equal statements about the nature of time. Both physical principles are mutually derivable from each other and are complementary.

Criticism to RHC

As far as heat conduction is concerned, the RHC equation is identical in form to the hyperbolic equation, and all analytical and experimental results that are relevant to one are equally applicable to the other. The definition of heat flux vector, however, is different; but the RHC definition is merely a 4D upgrade of the original linear Fourier approximation. The mathematics of RHC is much simpler and more elegant. However, RHC raises some significant conceptual challenges:

1. This weak interpretation of relativity, in which the speed of second sound plays a role similar to that of the speed of light, can be viewed as downgrading or degrading to the universality of the theory of relativity. Notice how the symbol c in standard relativity theory is replaced with C without much interpretation.
2. The implied wave nature of heat is controversial. Some workers reject the wave nature of heat on dogmatic grounds. Moreover, RHC implies that a phonon is a full-fledged objective quantum particle whose physical reality is no lesser than that of a photon. Existing experimental evidences are not enough to support for or against such views.
3. Heat quantities become complex numbers, with values including "imaginary temperature", which are hard to interpret experimentally.
4. The equivalence of relativity and the second law is shocking, because it implies that one of them can be a derivative of the other.

In summary, while the RHC is mathematically simple and elegant, and experimentally practical and relevant, it raises a number of conceptual issues that are highly controversial.

Applications

The RHC theory is applicable for any physical problem in which the hyperbolic equation is relevant: when speed of heat propagation is small, e.g. thermal insulators, or when speed of heat-flux variation is very large, e.g. pulsed-laser heating. Applications for those types of problems are abundant, and there is plenty of published work. Most of these results remain relevant to RHC, but because the definition of heat flux vector is different, final closed-form solutions may not be the same. In many cases, RHC provides closed-form solutions that are not possible using the HHC model. A number of useful fundamental solutions for 1D and 2D relativistic moving heat sources are available in closed-form.

Chapter 10

Heat Pump

Outdoor components of a residential air-source heat pump

A **heat pump** is a machine or device that moves heat from one location (the 'source') at a lower temperature to another location (the 'sink' or 'heat sink') at a higher temperature using mechanical work or a high-temperature heat source. A heat pump can be used to provide heating or cooling. Even though the heat pump can heat, it still uses the same basic refrigeration cycle to do this. In other words a heat pump can change which coil is the condenser and which the evaporator. This is normally achieved by a reversing valve. In cooler climates it is common to have heat pumps that are designed only to provide heating.

Common examples are food refrigerators and freezers, air conditioners, and reversible-cycle heat pumps for providing building space heating. In heating, ventilation, and air conditioning (HVAC) applications, a heat pump normally refers to a vapor-compression refrigeration device that includes a reversing valve and optimized heat exchangers so that the direction of heat flow may be reversed. Most commonly, heat pumps draw heat from the air or from the ground.

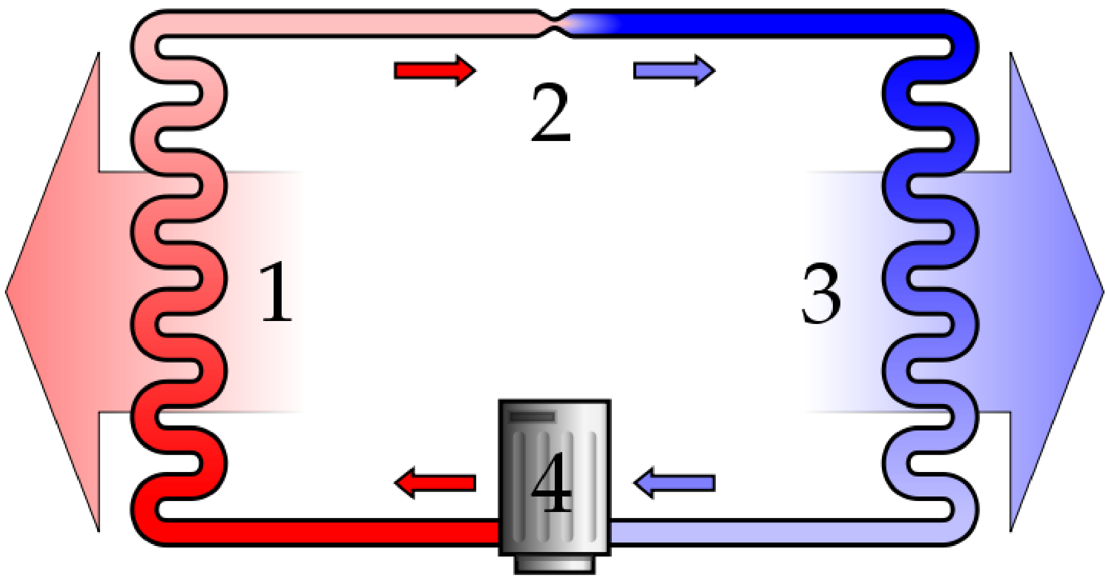
Overview

Heat pumps have the ability to move heat energy from one environment to another, and in either direction. This allows the heat pump to both bring heat into an occupied space, and take it out. In the cooling mode a heat pump works the same as an ordinary air conditioner (A/C). A heat pump uses an intermediate fluid called a refrigerant which absorbs heat as it vaporizes and releases the heat when it condenses. It uses an evaporator to absorb heat from inside an occupied space and rejects this heat to the outside through the condenser. The refrigerant flows outside of the space to be conditioned, where the condenser and compressor are located, while the evaporator is inside. The key component that makes a heat pump different from an A/C is the reversing valve. The reversing valve allows for the flow direction of the refrigerant to be changed. This allows the heat to be pumped in either direction.

- In **heating mode** the outdoor coil becomes the evaporator, while the indoor becomes the condenser which absorbs the heat from the refrigerant and dissipates to the air flowing through it. The air outside even at 0 °C has heat energy in it. With the refrigerant flowing in the opposite direction the evaporator (outdoor coil) is absorbing the heat from the air and moving it inside. Once it picks up heat it is compressed and then sent to the condenser (indoor coil). The indoor coil then rejects the heat into the air handler, which moves the heated air through out the house.
- In **cooling mode** the outdoor coil is now the condenser. This makes the indoor coil now the evaporator. The indoor coil is now the evaporator in the sense that it is going to be used to absorb the heat from inside the enclosed space. The evaporator absorbs the heat from the inside, and takes it to the condenser where it is rejected into the outside air.

Operating principles

Since the heat pump or refrigerator uses a certain amount of work to move the refrigerant, the amount of energy deposited on the hot side is greater than taken from the cold side. One common type of heat pump works by exploiting the physical properties of an evaporating and condensing fluid known as a refrigerant.



A simple stylized diagram of a heat pump's vapor-compression refrigeration cycle: 1) condenser, 2) expansion valve, 3) evaporator, 4) compressor.

The working fluid, in its gaseous state, is pressurized and circulated through the system by a compressor. On the discharge side of the compressor, the now hot and highly pressurized vapor is cooled in a heat exchanger, called a condenser, until it condenses into a high pressure, moderate temperature liquid. The condensed refrigerant then passes through a pressure-lowering device also called a metering device like an expansion valve, capillary tube, or possibly a work-extracting device such as a turbine. The low pressure, liquid refrigerant leaving the expansion device enters another heat exchanger, the evaporator, in which the fluid absorbs heat and boils. The refrigerant then returns to the compressor and the cycle is repeated.

In such a system it is essential that the refrigerant reach a sufficiently high temperature when compressed, since the second law of thermodynamics prevents heat from flowing from a cold fluid to a hot heat sink. Practically, this means the refrigerant must reach a temperature greater than the ambient around the high-temperature heat exchanger. Similarly, the fluid must reach a sufficiently low temperature when allowed to expand, or heat cannot flow from the cold region into the fluid, i.e. the fluid must be colder than the ambient around the cold-temperature heat exchanger. In particular, the pressure difference must be great enough for the fluid to condense at the hot side and still evaporate in the lower pressure region at the cold side. The greater the temperature difference, the greater the required pressure difference, and consequently the more energy needed to compress the fluid. Thus as with all heat pumps, the Coefficient of Performance (amount of heat moved per unit of input work required) decreases with increasing temperature difference.

Insulation is used to reduce the work and energy required to achieve and maintain a lower temperature in the cooled space.

Due to the variations required in temperatures and pressures, many different refrigerants are available. Refrigerators, air conditioners, and some heating systems are common applications that use this technology.

Heat sources

Many heat pumps also use an auxiliary heat source for heating mode. This means that, even though the heat pump is the primary source of heat, another form is available as a back-up. Electricity, oil, or gas are the most common sources. This is put in place so that if the heat pump fails or can't provide enough heat, the auxiliary heat will kick on to make up the difference.

Geothermal heat pumps use the ground as a heat source and sink and water as the heat transport medium. They work in the same manner as an air to air heat pump, but instead of indoor and outdoor coils they use water pumped through earth materials as a heat transfer medium. These are very eco-friendly and are a cheaper alternative in the long run due to lower operating cost. Operating costs can be further reduced by storing summer heat in the ground for use during winter, and (for larger buildings requiring lots of air conditioning) by storing winter cold underground for use during summer.

Solar Assisted Heat Pumps use thermal waste energy from water source heating and cooling systems as "fuel" for a Thermal HVAC system. This is a new technology which uses the energy from the water in holding tanks and a refrigerant to water heat exchange system. The tanks serve as thermal flywheels and thermal buffers, as needed. In this configuration, the water in the middle tank serves as the “fuel” for the system. This fuel is pumped into the cold heat exchanger where the heat in the water is extracted and transferred to warm up the cold refrigerant. The cold water is then pumped into the cold tank. On the opposite side, the hot water is heated by way of the hot heat exchanger and the heated water is put back into the hot tank to either be rejected or used further in other heat exchange processes. In most cases water returns from the zone where work is being done to the neutral tank .

Applications

In HVAC applications, a heat pump is typically a vapor-compression refrigeration device that includes a reversing valve and optimized heat exchangers so that the direction of heat flow may be reversed. The reversing valve switches the direction of refrigerant through the cycle and therefore the heat pump may deliver either heating or cooling to a building. In the cooler climates the default setting of the reversing valve is heating. The default setting in warmer climates is cooling. Because the two heat exchangers, the condenser and evaporator, must swap functions, they are optimized to perform adequately in both modes. As such, the efficiency of a reversible heat pump is typically slightly less than two separately optimized machines.

In plumbing applications, a heat pump is sometimes used to heat or preheat water for swimming pools or domestic water heaters.

In somewhat rare applications, both the heat extraction and addition capabilities of a single heat pump can be useful, and typically results in very effective use of the input energy. For example, when an air cooling need can be matched to a water heating load, a single heat pump can serve two useful purposes. That is, a heat pump domestic water heater located in the living area of a home could cool the home, reducing or eliminating the need for additional air conditioning. This installation would be best-suited to a climate that is warm or hot most of the year. Unfortunately, these situations are rare because the demand profiles for heating and cooling are often significantly different.

Refrigerants

Until the 1990s, the refrigerants were often chlorofluorocarbons such as R-12 (dichlorodifluoromethane), one in a class of several refrigerants using the brand name Freon, a trademark of DuPont. Its manufacture was discontinued in 1995 because of the damage that CFCs were alleged to cause to the ozone layer if released into the atmosphere. One widely adopted replacement refrigerant is the hydrofluorocarbon (HFC) known as R-134a (1,1,1,2-tetrafluoroethane). R-134a is not as efficient as the R-12 it replaced (in automotive applications) and therefore, more energy is required to operate systems utilizing R-134a than those using R-12. Other substances such as liquid R-717 ammonia are widely used in large-scale systems, or occasionally the less corrosive but more flammable propane or butane, can also be used.

Since 2001, carbon dioxide, R-744, has increasingly been used, utilizing the transcritical cycle. In residential and commercial applications, the hydrochlorofluorocarbon (HCFC) R-22 is still widely used, however, HFC R-410A does not deplete the ozone layer and is being used more frequently. Hydrogen, helium, nitrogen, or plain air is used in the Stirling cycle, providing the maximum number of options in environmentally friendly gases.

More recent refrigerators are now exploiting the R600A which is isobutane, and does not deplete the ozone and is friendly to the environment.

Dimethyl ether (DME) is also gaining popularity as a refrigerant.

Efficiency

When comparing the performance of heat pumps, it is best to avoid the word "efficiency" which has a very specific thermodynamic definition. The term coefficient of performance (COP) is used to describe the ratio of useful heat movement to work input. Most vapor-compression heat pumps utilize electrically powered motors for their work input. However, in most vehicle applications, shaft work, via their internal combustion engines, provide the needed work.

When used for heating a building on a mild day of say 10 °C, a typical air-source heat pump has a COP of 3 to 4, whereas a typical electric resistance heater has a COP of 1.0. That is, one joule of electrical energy will cause a resistance heater to produce one joule of useful heat, while under ideal conditions, one joule of electrical energy can cause a heat pump to move much more than one joule of heat from a cooler place to a warmer place.

Note that the heat pump is more efficient on average in hotter climates than cooler ones, so when the weather is much warmer (in a desert city or southern city) the unit will perform better than average COP. Conversely in cold weather the COP approaches 1. Thus when there is a wide temperature differential between the hot & cold reservoir's the COP is lower (worse).

When there is a high temperature differential on a cold day, e.g., when an air-source heat pump is used to heat a house on a very cold winter day of say 0 °C, it takes more work to move the same amount of heat indoors than on a mild day. Ultimately, due to Carnot efficiency limits, the heat pump's performance will approach 1.0 as the outdoor-to-indoor temperature difference increases for colder climates (temperature gets colder). This typically occurs around −18 °C (0 °F) outdoor temperature for air source heat pumps. Also, as the heat pump takes heat out of the air, some moisture in the outdoor air may condense and possibly freeze on the outdoor heat exchanger. The system must periodically melt this ice. In other words, when it is extremely cold outside, it is simpler, and wears the machine less, to heat using an electric-resistance heater than to strain an air-source heat pump.

Geothermal heat pumps, on the other hand, are dependent upon the temperature underground, which is "mild" (typically 10 °C at a depth of more than 1.5m for the UK) all year round. Their COP is therefore normally in the range of 4.0 to 5.0.

The design of the evaporator and condenser heat exchangers is also very important to the overall efficiency of the heat pump. The heat exchange surface areas and the corresponding temperature differential (between the refrigerant and the air stream) directly affect the operating pressures and hence the work the compressor has to do in order to provide the same heating or cooling effect. Generally the larger the heat exchanger the lower the temperature differential and the more efficient the system. Since heat exchangers are expensive, and the heat pump industry generally competes on price rather than efficiency, the drive towards more efficient heat pumps and air conditioners is often led by legislative measures on minimum efficiency standards.

In cooling mode a heat pump's operating performance is described as its energy efficiency ratio (EER) or seasonal energy efficiency ratio (SEER), and both measures have units of BTU/(h·W) (1 BTU/(h·W) = 0.293 W/W). A larger EER number indicates better performance. The manufacturer's literature should provide both a COP to describe performance in heating mode and an EER or SEER to describe performance in cooling mode. Actual performance varies, however, and depends on many factors such as installation, temperature differences, site elevation, and maintenance.

Heat pumps are more *effective* for heating than for cooling if the temperature difference is held equal. This is because the compressor's input energy is largely converted to useful heat when in heating mode, and is discharged along with the moved heat via the condenser. But for cooling, the condenser is normally outdoors, and the compressor's dissipated work is rejected rather than put to a useful purpose.

For the same reason, opening a food refrigerator or freezer heats up the room rather than cooling it because its refrigeration cycle rejects heat to the indoor air. This heat includes the compressor's dissipated work as well as the heat removed from the inside of the appliance.

The COP for a heat pump in a heating or cooling application, with steady-state operation, is:

$$COP_{\text{heating}} = \frac{\Delta Q_{\text{hot}}}{\Delta A} \leq \frac{T_{\text{hot}}}{T_{\text{hot}} - T_{\text{cool}}}$$
$$COP_{\text{cooling}} = \frac{\Delta Q_{\text{cool}}}{\Delta A} \leq \frac{T_{\text{cool}}}{T_{\text{hot}} - T_{\text{cool}}}$$

where

- ΔQ_{cool} is the amount of heat extracted from a cold reservoir at temperature T_{cool},
- ΔQ_{hot} is the amount of heat delivered to a hot reservoir at temperature T_{hot},
- ΔA is the compressor's dissipated work.
- All temperatures are absolute temperatures usually measured in kelvins (K).

COP and lift

The COP increases as the temperature difference, or "lift", decreases between heat source and destination. The COP can be maximised at design time by choosing a heating system requiring only a low final water temperature (e.g. underfloor heating), and by choosing a heat source with a high average temperature (e.g. the ground). Domestic hot water (DHW) and radiators require high water temperatures, affecting the choice of heat pump technology.

COP variation with Output Temperature

Pump type and source	Typical use case	35 °C (e.g. heated screed floor)	45 °C (e.g. heated screed floor)	55 °C (e.g. heated timber floor)	65 °C (e.g. radiator or DHW)	75 °C (e.g. radiator & DHW)	85 °C (e.g. radiator & DHW)
High efficiency air source heat pump (ASHP). Air at −20 °C		2.2	2.0	-	-	-	-

Two-stage ASHP air at −20 °C	Low source temp.	*2.4*	2.2	1.9	-	-	-
High efficiency ASHP air at 0 °C	Low output temp.	*3.8*	2.8	2.2	2.0	-	-
Prototype transcritical CO_2 (R744) heat pump with tripartite gas cooler, source at 0 °C	High output temp.	3.3	-	-	*4.2*	-	3.0
Ground source heat pump (GSHP). Water at 0 °C		5.0	3.7	2.9	2.4	-	-
GSHP ground at 10 °C	Low output temp.	*7.2*	5.0	3.7	2.9	2.4	-
Theoretical Carnot cycle limit, source −20 °C		5.6	4.9	4.4	4.0	3.7	3.4
Theoretical Carnot cycle limit, source 0 °C		8.8	7.1	6.0	5.2	4.6	4.2
Theoretical Lorentz cycle limit (CO_2 pump), return fluid 25 °C, source 0 °C		10.1	8.8	7.9	7.1	6.5	6.1
Theoretical Carnot cycle limit, source 10 °C		12.3	9.1	7.3	6.1	5.4	4.8

Types

The two main types of heat pumps are compression heat pumps and absorption heat pumps. Compression heat pumps always operate on mechanical energy (through electricity), while absorption heat pumps may also run on heat as an energy source (through electricity or burnable fuels). An absorption heat pump may be fueled by natural gas or LP gas, for example. While the Gas Utilization Efficiency in such a device, which is the ratio of the energy supplied to the energy consumed, may average only 1.5, that is better than a natural gas or LP gas furnace, which can only approach 1. Although an absorption heat pump may not be as efficient as an electric compression heat pump, an absorption heat pump fueled by natural gas may be advantageous in locations where electricity is relatively expensive and natural gas is relatively inexpensive. A natural gas-fired absorption heat pump might also avoid the cost of an electrical service upgrade which is sometimes necessary for an electric heat pump installation. In the case of air-to-air heat pumps, an absorption heat pump might also have an advantage in colder regions, due to a lower minimum operating temperature.ROBUR heat pumps comparison

A number of sources have been used for the heat source for heating private and communal buildings.

- air source heat pump (extracts heat from outside air)
    - air–air heat pump (transfers heat to inside air)
    - air–water heat pump (transfers heat to a tank of water)
- exhaust air heat pump (extracts heat from the exhaust air of a building, requires mechanical ventilation)
    - exhaust air - water heat pump (transfers heat to a tank of water)
- geothermal heat pump (extracts heat from the ground or similar sources)
    - geothermal–air heat pump (transfers heat to inside air)
        - ground–air heat pump (ground as a source of heat)
        - rock–air heat pump (rock as a source of heat)
        - water–air heat pump (body of water as a source of heat)
    - geothermal–water heat pump (transfers heat to a tank of water)
        - ground–water heat pump (ground as a source of heat)
        - rock–water heat pump (rock as a source of heat)
        - water–water heat pump (body of water as a source of heat)

Heat sources

Most commonly, heat pumps draw heat from the air (outside or inside air) or from the ground (groundwater or soil). The heat drawn from the ground is in most cases stored solar heat, and it should not be confused with geothermal heat, though the latter will contribute in some small measure to all heat in the ground. Other heat sources include water; nearby streams and other natural water bodies have been used, and sometimes domestic waste water which is often warmer than the ambient temperature.

Air-source heat pumps

Air source heat pumps are relatively easy (and inexpensive) to install and have therefore historically been the most widely used heat pump type. However, they suffer limitations due to their use of the outside air as a heat source or sink. The higher temperature differential during periods of extreme cold or heat leads to declining efficiency, as explained above. In mild weather, COP may be around 4.0, while at temperatures below around −8 °C (17 °F) an air-source heat pump can achieve a COP of 2.5 or better, which is considerably more than the energy efficiency that may be achieved by a 1980's heating systems, and very similar to state of the art oil or gas heaters. The average COP over seasonal variation is typically 2.5-2.8, with exceptional models able to exceed 6.0 in very mild climate, but not in freezing climates. (2.8 kW).

Air source heat pumps for cold climates

At least two manufacturers are selling heat pumps that maintain better heating output at lower outside temperatures than conventional air source heat pumps. These low temperature optimized models make air source heat pumps more practical for cold climates because they don't freeze to a stop that quickly. Some models however, defrost their outdoor unit electrically at regular intervals, which increases electricity consumption dramatically during the coldest weeks. In areas where only one fossil fuel is currently available (e.g. heating oil; no natural gas pipes available) these heat pumps could be used as an alternative, supplemental heat source to reduce a building's direct dependence on fossil fuel. Depending on fuel and electricity prices, using the heat pump for heating may be less expensive than fossil fuel. A backup, fossil-fuel heat source may still be required for the coldest days.

The heating output of low temperature optimized heat pumps (and hence their energy efficiency) still declines dramatically as the temperature drops, but the threshold at which the decline starts is lower than conventional pumps, as shown in the following table (temperatures are approximate and may vary by manufacturer and model):

Air Source Heat Pump Type	Full heat output at or above this temperature	Heat output down to 60% of maximum at
Conventional	47 °F (8.3 °C)	17 °F (-8.3 °C)
Low Temp Optimized	14 °F (-10 °C)	-13 °F (-25 °C)

Ground source heat pumps

Ground source heat pumps, which are also referred to as Geothermal heat pumps, typically have higher efficiencies than air-source heat pumps. This is because they draw heat from the ground or groundwater which is at a relatively constant temperature all year round below a depth of about thirty feet (9 m). This means that the temperature differential is lower, leading to higher efficiency. Ground-source heat pumps typically have COPs of 3.5-4.0 at the beginning of the heating season, with lower COPs as heat is drawn from the ground. The trade off for this improved performance is that a ground-source heat pump is more expensive to install due to the need for the digging of wells or trenches in which to place the pipes that carry the heat exchange fluid. When compared versus each other, groundwater heat pumps are generally more efficient than heat pumps using heat from the soil. Their efficiency can be further improved, by pumping summer heat into the ground. One way is to use ground water to cool the floors on hot days. Another way is to make large solar collectors, for instance by putting plastic pipes just under the roof tiles or in the tarmac of the parking lot. The most cost effective way is to put a large air to water heat exchanger on the rooftop.

Heat distribution

Heat pumps are only highly efficient when they distribute produced heat at a low temperature, ideally around or below 32 °C (90 °F). Normal steel plate radiators are no good: they would need to have four to six times their current size. Underfloor heating is the ideal solution. When wooden floors or carpets would spoil their efficiency, wall heaters (plastic pipes covered with a thick layer of chalk) and piped ceilings can be used. Both systems have the disadvantage that they are slow starters, and that they would require extensive renovation in existing buildings. The alternative is a warm air system in which water runs through a ventilator driven water to air heater. Such a thing can either complement floor heating during warm up, or it can be a quick and economical way to implement a heat pump system into existing buildings. Oversizing them reduces their noise. To efficiently distribute warm water or air from a heat pump, water pipes or air shafts should have significantly larger diameters then in conventional systems, and underfloor heaters should have much more pipes per square meter.

Solid state heat pumps

In 1881, the German physicist Emil Warburg put a block of iron into a strong magnetic field and found that it increased very slightly in temperature. Some commercial ventures to implement this technology are underway, claiming to cut energy consumption by 40% compared to current domestic refrigerators. The process works as follows: Powdered gadolinium is moved into a magnetic field, heating the material by 2 to 5 °C (4 to 9 °F). The heat is removed by a circulating fluid. The material is then moved out of the magnetic field, reducing its temperature below its starting temperature.

Solid state heat pumps using the Thermoelectric Effect have improved over time to the point where they are useful for certain refrigeration tasks. Commercially available technologies have efficiencies that are currently well below that of mechanical heat pumps, however this area of technology is currently the subject of active research in materials science.

Near-solid-state heat pumps using Thermoacoustics are commonly used in cryogenic laboratories.

History

Milestones:

- 1748: William Cullen demonstrates artificial refrigeration.
- 1834: Jacob Perkins builds a practical refrigerator with diethyl ether.
- 1852: Lord Kelvin describes the theory underlying heat pump.
- 1855–1857: Peter Ritter von Rittinger develops and builds the first heat pump.

Chapter 11

Heat Pump and Refrigeration Cycle

Thermodynamic **heat pump cycles** or **refrigeration cycles** are the conceptual and mathematical models for heat pumps and refrigerators. A heat pump is a machine or device that moves heat from one location (the 'source') at a lower temperature to another location (the 'sink' or 'heat sink') at a higher temperature using mechanical work or a high-temperature heat source. Thus a heat pump may be thought of a "heater" if the objective is to warm the heat sink (as when warming the inside of a home on a cold day), or a "refrigerator" if the objective is to cool the heat source (as in the normal operation of a freezer). In either case, the operating principles are identical. Heat is moved from a colder place to a warmer place.

Thermodynamic cycles

According to the second law of thermodynamics heat cannot spontaneously flow from a colder location to a hotter area; work is required to achieve this. An air conditioner does work to cool a living space, moving heat from the cooler interior (the heat source) to the warmer outdoors (the heat sink). Similarly, a refrigerator moves heat from inside the cold icebox (the heat source) to the warmer room-temperature air of the kitchen (the heat sink). The operating principle of the refrigeration cycle was described mathematically by Sadi Carnot in 1824 as a heat engine. A heat pump can be thought of as a heat engine which is operating in reverse.

Heat pump and refrigeration cycles can be classified as *vapor compression*, *vapor absorption*, *gas cycle*, or *Stirling cycle* types.

Vapor-compression cycle

The vapor-compression cycle is used in most household refrigerators as well as in many large commercial and industrial refrigeration systems. Figure 1 provides a schematic diagram of the components of a typical vapour-compression refrigeration system.

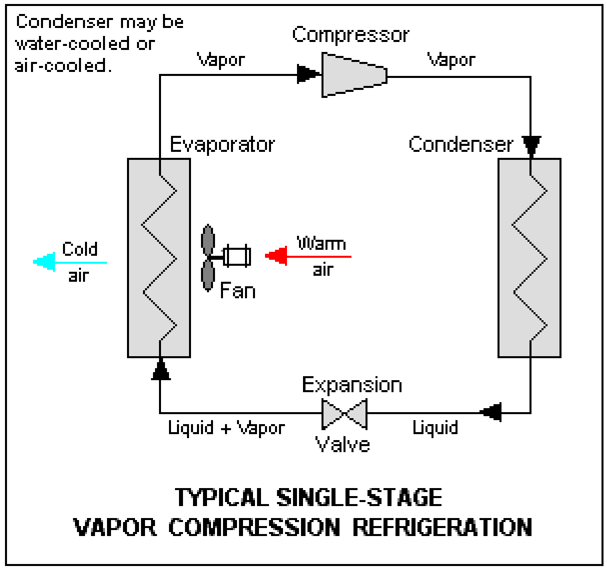


Figure 1: Vapor compression refrigeration

The thermodynamics of the cycle can be analyzed on a diagram as shown in Figure 2. In this cycle, a circulating refrigerant such as Freon enters the compressor as a vapor. The vapor is compressed at constant entropy and exits the compressor superheated. The superheated vapor travels through the condenser which first cools and removes the superheat and then condenses the vapor into a liquid by removing additional heat at constant pressure and temperature. The liquid refrigerant goes through the expansion valve (also called a throttle valve) where its pressure abruptly decreases, causing flash evaporation and auto-refrigeration of, typically, less than half of the liquid.

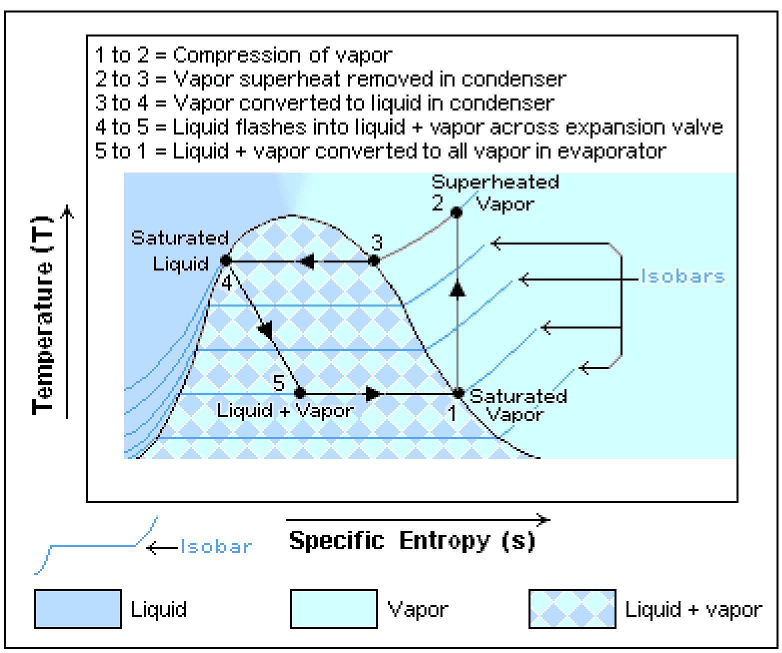


Figure 2:Temperature–Entropy diagram

That results in a mixture of liquid and vapor at a lower temperature and pressure. The cold liquid-vapor mixture then travels through the evaporator coil or tubes and is completely vaporized by cooling the warm air (from the space being refrigerated) being blown by a fan across the evaporator coil or tubes. The resulting refrigerant vapor returns to the compressor inlet to complete the thermodynamic cycle.

The above discussion is based on the ideal vapor-compression refrigeration cycle, and does not take into account real-world effects like frictional pressure drop in the system, slight thermodynamic irreversibility during the compression of the refrigerant vapor, or non-ideal gas behavior (if any).

Vapor absorption cycle

In the early years of the twentieth century, the vapor absorption cycle using water-ammonia systems was popular and widely used but, after the development of the vapor compression cycle, it lost much of its importance because of its low coefficient of performance (about one fifth of that of the vapor compression cycle). Nowadays, the vapor absorption cycle is used only where waste heat is available or where heat is derived from solar collectors.

The absorption cycle is similar to the compression cycle, except for the method of raising the pressure of the refrigerant vapor. In the absorption system, the compressor is replaced by an absorber which dissolves the refrigerant in a suitable liquid, a liquid pump which raises the pressure and a generator which, on heat addition, drives off the refrigerant vapor from the high-pressure liquid. Some work is required by the liquid pump but, for a given quantity of refrigerant, it is much smaller than needed by the compressor in the vapor compression cycle. In an absorption refrigerator, a suitable combination of refrigerant and absorbent is used. The most common combinations are ammonia (refrigerant) and water (absorbent), and water (refrigerant) and lithium bromide (absorbent).

Gas cycle

When the working fluid is a gas that is compressed and expanded but does not change phase, the refrigeration cycle is called a *gas cycle*. Air is most often this working fluid. As there is no condensation and evaporation intended in a gas cycle, components corresponding to the condenser and evaporator in a vapor compression cycle are the hot and cold gas-to-gas heat exchangers in gas cycles.

The gas cycle is less efficient than the vapor compression cycle because the gas cycle works on the reverse Brayton cycle instead of the reverse Rankine cycle. As such the working fluid does not receive and reject heat at constant temperature. In the gas cycle, the refrigeration effect is equal to the product of the specific heat of the gas and the rise in temperature of the gas in the low temperature side. Therefore, for the same cooling load, a gas refrigeration cycle will require a large mass flow rate and would be bulky.

Because of their lower efficiency and larger bulk, *air cycle* coolers are not often applied in terrestrial refrigeration. The air cycle machine is very common, however, on gas turbine-powered jet airliners since compressed air is readily available from the engines' compressor sections. These jet aircraft's cooling and ventilation units also serve the purpose of pressurizing the aircraft cabin.

Stirling cycle

The Stirling cycle heat engine can be driven in reverse, using a mechanical energy input to drive heat transfer in a reversed direction (i.e. a heat pump, or refrigerator). There are several design configurations for such devices that can be built. Several such setups require rotary or sliding seals, which can introduce difficult tradeoffs between frictional losses and refrigerant leakage.

The Free Piston Stirling Cooler (FPSC) is an elegant, completely-sealed heat transfer system that has only two moving parts (a piston and a displacer), and uses helium as the working fluid. The piston is typically driven by an oscillating magnetic field that is the source of the power needed to drive the refrigeration cycle. The magnetic drive allows the piston to be driven without requiring any seals, gaskets, O-rings, or other compromises to the hermetically sealed system. Claimed advantages for the system include environmental friendliness, cooling capacity, light weight, compact size, precise controllability, and high efficiency.

The FPSC was invented in 1964 by William Beale, a professor of Mechanical Engineering at Ohio University in Athens, Ohio. He founded and continues to be associated with Sunpower Inc., which specializes primarily in researching and developing FPSC systems for a wide variety of military, aerospace, industrial, and commercial applications. Sunpower also makes cryocoolers and special pulse tube coolers capable of reaching below 40°K (around –390°F, or –230°C). A FPSC cooler made by Sunpower was used by NASA to cool instrumentation in satellites.

Since 2002. another leading supplier of FPSC technology has been the Twinbird Company in Japan, which also markets a broad line of household appliances. Both Sunpower and Twinbird appear to work in collaboration with Global Cooling NV, which is located in the Netherlands, but has a research center in Athens, Ohio.

For several years starting around 2004, the Coleman Company sold a version of the Twinbird "SC-C925 Portable Freezer Cooler 25L" under its own brand name, but it has since discontinued offering the product, in spite of favorable customer reviews on Amazon. The portable cooler can be operated more than a day, maintaining sub-freezing temperatures while powered only by an automotive battery. This cooler is still being manufactured and distributed worldwide, with Global Cooling now coordinating distribution to North America and Europe. Other variants offered by Twinbird include a portable deep freezer (to –80°C), collapsible coolers, and a special model for transporting blood and vaccine.

In addition to the technical information available on the websites referenced above, a step-by-step photographic teardown of the Coleman (Twinbird) FPSC cooler is viewable online.

Chapter 12

Absorption Refrigerator

An **absorption refrigerator** is a refrigerator that uses a heat source (e.g., solar, kerosene-fueled flame) to provide the energy needed to drive the cooling system. Absorption refrigerators are a popular alternative to regular compressor refrigerators where electricity is unreliable, costly, or unavailable, where noise from the compressor is problematic, or where surplus heat is available (e.g., from turbine exhausts or industrial processes, or from solar plants).

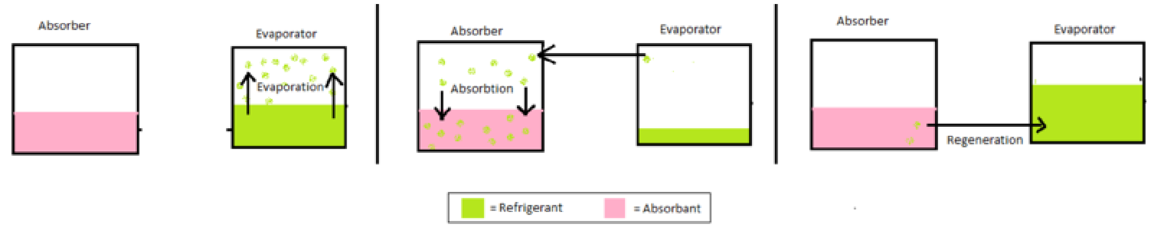
For example, absorption refrigerators powered by heat from the combustion of liquefied petroleum gas are often used for food storage in recreational vehicles. Absorptive refrigeration can also be used to air-condition buildings using the waste heat from a gas turbine or water heater. This use is very efficient, since the gas turbine produces electricity, hot water and air-conditioning (called Trigeneration).

Both absorption and compressor refrigerators use a refrigerant with a very low boiling point (less than 0 °F/−18 °C). In both types, when this refrigerant evaporates (boils), it takes some heat away with it, providing the cooling effect. The main difference between the two types is the way the refrigerant is changed from a gas back into a liquid so that the cycle can repeat. An absorption refrigerator changes the gas back into a liquid using a different method that needs only heat, and has no moving parts. The other difference between the two types is the refrigerant used. Compressor refrigerators typically use an HCFC or HFC, while absorption refrigerators typically use ammonia.

The standard for the absorption refrigerator is given by the ANSI/AHRI standard 560-2000.

Principles

Absorptive refrigeration uses a source of heat to provide the energy needed to drive the cooling process.



Absorption

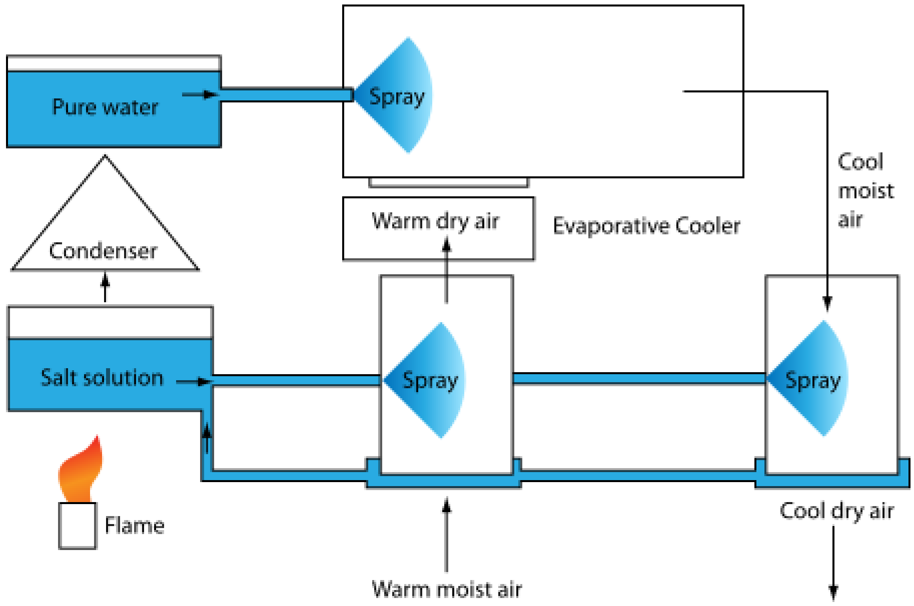
The absorption cooling cycle can be described in three phases:

1. Evaporation: A liquid refrigerant evaporates in a low partial pressure environment, thus extracting heat from its surroundings – the refrigerator.
2. Absorption: The gaseous refrigerant is absorbed – dissolved into another liquid - reducing its partial pressure in the evaporator and allowing more liquid to evaporate.
3. Regeneration: The refrigerant-laden liquid is heated, causing the refrigerant to evaporate out. It is then condensed through a heat exchanger to replenish the supply of liquid refrigerant in the evaporator.

Simple salt and water system

A simple absorption refrigeration system common in large commercial plants uses a solution of lithium bromide salt and water. Water under low pressure is evaporated from the coils that are being chilled. The water is absorbed by a lithium bromide/water solution. The water is driven off the lithium bromide solution using heat.

Water spray absorption refrigeration



Water Spray Absorption Refrigeration

Another variant, depicted to the right, uses air, water, and a salt water solution. The intake of warm, moist air is passed through a sprayed solution of salt water. The spray lowers the humidity but does not significantly change the temperature. The less humid, warm air is then passed through an evaporative cooler, consisting of a spray of fresh water, which cools and re-humidifies the air. Humidity is removed from the cooled air with another spray of salt solution, providing the outlet of cool, dry air.

The salt solution is regenerated by heating it under low pressure, causing water to evaporate. The water evaporated from the salt solution is re-condensed, and rerouted back to the evaporative cooler.

Single pressure absorption refrigeration

Labeled photo of a domestic absorption refrigerator.

A single-pressure absorption refrigerator uses three substances: ammonia, hydrogen gas, and water. At standard atmospheric conditions, ammonia is a gas with a boiling point of -33°C, but a single-pressure absorption refrigerator is pressurised to the point where the ammonia is a liquid. The cycle is closed, with all hydrogen, water and ammonia collected and endlessly reused.

The cooling cycle starts with liquefied ammonia entering the evaporator at room temperature. The ammonia is mixed in the evaporator with hydrogen. The partial pressure of the hydrogen is used to regulate the total pressure, which in turn regulates the vapour pressure and thus the boiling point of the ammonia. The ammonia boils in the evaporator, providing the cooling required.

The next three steps exist to separate the gaseous ammonia and the hydrogen. First, in the absorber, the mixture of gasses enters the bottom of an uphill series of tubes, into which water is added at the top. The ammonia dissolves in the water, producing a mixture of ammonia solution and hydrogen. The hydrogen is collected at the top of the absorber, with the ammonia solution collected at the bottom.

The second step is to separate the ammonia and water. In the generator, heat is applied to the solution, to distill the ammonia from the water. Some water remains with the ammonia, in the form of vapour and bubbles. This is dried in the final separation step, called the separator, by passing it through an uphill series of twisted pipes with minor obstacles to pop the bubbles, allowing the collected water to drain back to the generator.

Finally the pure ammonia gas enters the condenser. In this heat exchanger, the hot ammonia gas is cooled to room temperature and hence condenses to a liquid, allowing the cycle to restart.

History

Absorption cooling was invented by the French scientist Ferdinand Carré in 1858. The original design used water and sulfuric acid.

In 1922 Baltzar von Platen and Carl Munters, while they were still students at the Royal Institute of Technology in Stockholm, Sweden, enhanced the principle with a 3 fluids configuration. This "Platen-Munters" design can operate without a pump.

Commercial production began in 1923 by the newly formed company AB Arctic, which was bought by Electrolux in 1925. In the 60s the absorption refrigeration saw a renaissance due to the substantial demand for refrigerators for caravans. AB Electrolux established a subsidiary in the U.S, named Dometic Sales Corporation. The company marketed refrigerators for caravans under the **Dometic** brand. In 2001 Electrolux sold most of its Leisure Products line to the venture-capital company EQT. The Dometic Group was created.

In 1926 Albert Einstein and his former student Leó Szilárd proposed an alternative design known as Einstein refrigerator.

In 2007, Adam Grosser presented his research of a new, very small, "intermittent absorption" refrigeration system for use in third world countries at the TED Conference. The refrigerator is a small unit placed over a campfire, that can later be used to cool 3 gallons of water to just above freezing for 24 hours in a 30 degree Celsius environment.

Chapter 13

Coefficient of Performance

The **coefficient of performance** or COP (sometimes CP), of a heat pump is the ratio of the change in heat at the "output" (the heat reservoir of interest) to the supplied work.

Equation

The equation is:

$$COP = \frac{Q_H}{W}$$

where

- Q_H is the heat supplied to the hot reservoir
- W is the work consumed by the heat pump.

The COP for heating and cooling are thus different, because the heat reservoir of interest is different. When one is interested in how well a machine cools, the COP is the ratio of the heat removed from the cold reservoir to input work. However, for heating, the COP is the ratio of the heat removed from the cold reservoir plus the heat added to the hot reservoir by the input work to input work:

$$COP_{heating} = \frac{|Q_C| + W}{W}$$
$$COP_{cooling} = \frac{|Q_C|}{W}$$

where

- Q_C is the heat supplied to the cold reservoir.

Derivation

According to the first law of thermodynamics, in a reversible system we can show that $Q_{hot} = Q_{cold} + W$ and $W = Q_{hot} - Q_{cold}$, where Q_{hot} is the heat given off by the hot heat reservoir and Q_{cold} is the heat taken in by the cold heat reservoir.
Therefore, by substituting for W,

$$COP_{heating} = \frac{Q_{hot}}{Q_{hot} - Q_{cold}}$$

For a heat pump operating at maximum theoretical efficiency (i.e. Carnot efficiency), it can be shown that $\frac{Q_{hot}}{T_{hot}} = \frac{Q_{cold}}{T_{cold}}$ and $Q_{cold} = \frac{Q_{hot}T_{cold}}{T_{hot}}$, where T_{hot} and T_{cold} are the absolute temperatures of the hot and cold heat reservoirs respectively.

At maximum theoretical efficiency,

$$COP_{heating} = \frac{T_{hot}}{T_{hot} - T_{cold}}$$

Which is equal to the inverse of the ideal Carnot cycle efficiency because a heat pump is a heat engine operating in reverse. Similarly,

$$COP_{cooling} = \frac{Q_{cold}}{Q_{hot} - Q_{cold}} = \frac{T_{cold}}{T_{hot} - T_{cold}}$$

It can also be shown that $COP_{cooling} = COP_{heating} - 1$. Note that these equations must use the absolute temperature (the Kelvin or Rankine scale.)

$COP_{heating}$ applies to heat pumps and $COP_{cooling}$ applies to air conditioners or refrigerators. Values for actual systems will always be less than these theoretical maximums. In Europe, ground source heat pump units are standard tested at T_{hot} is 35 Celsius (95 Fahrenheit) and T_{cold} is 0 Celsius (32 Fahrenheit). According to the above formula, the maximum achievable COP would be 7.8. Test results of the best systems are around 4.5. When measuring installed units over a whole season and one also counts the energy needed to pump water through the piping systems, then seasonal COP's are around 3.5 or less. This indicates room for improvement.

Improving COP

As the formula shows, to improve the COP of a heat pump system, one needs to reduce the temperature gap T_{hot} minus T_{cold} at which the system works. For a heating system this would mean two things. One is to reduce output temperature to around 30 Celsius (86 Fahrenheit) which requires piped floor- or wall- or ceiling heating, or oversized water to air heaters. The other is to increase input temperature (by using an oversized ground source). For an air cooler, COP could be improved by using ground water as an input instead of air, and by reducing temperature drop on output side through increasing air flow. For both systems, also increasing the size of pipes and air canals would help to reduce noise and the energy consumption of pumps (and ventilators).

Also the heat pump itself can be improved a lot. The two most simple ways to improve heat pump units, is to double the size of the internal heat exchangers relative to the power of the compressor, and to reduce the system's internal temperature gap over the compressor. This last measure however, makes such heat pumps unsuitable to produce output above roughly 40 Celsius (104 Fahrenheit) which means that a separate machine is needed for producing hot tap water.

One reason that heat pump manufacturers often show little interest into making such improvements, is that they often also earn a lot of money in producing boilers, turbines and transformers for electricity plants. More efficient heat pumps would mean that less growth can be achieved in the industrial boilers and turbines sector.

For the same reason, there is little progress in engineering small heat pumps, driven on oil or piped gas. Such systems can also produce electricity, and, especially when combined with seasonally storing excess heat underground, they would be even more energy efficient then electricity driven heat pumps, because they cut away the heat wastage of central electricity production, by using exhaust and cool water heat for house warming. However, they would also vastly reduce the need for centrally produced electricity.

Example

A geothermal heat pump operating at $COP_{heating}$ 3.5 provides 3.5 units of heat for each unit of energy consumed (i.e. 1 kWh consumed would provide 3.5 kWh of output heat). The output heat comes from both the heat source and 1 kWh of input energy, so the heat-source is cooled by 2.5 kWh, not 3.5 kWh.

A heat pump of $COP_{heating}$ 3.5, such as in the example above, could be less expensive to use than even the most efficient gas furnace.

A heat pump cooler operating at $COP_{cooling}$ 2.0 removes 2 units of heat for each unit of energy consumed (e.g. an air conditioner consuming 1 kWh would remove 2 kWh of heat from a building's air).

Given the same energy source and operating conditions, a higher COP heat pump will consume less purchased energy than one with a lower COP. The overall environmental impact of a heating or air conditioning installation depends on the source of energy used as well as the COP of the equipment. The operating cost to the consumer depenends on the cost of energy as well as the COP or efficiency of the unit. Some areas provide two or more sources of energy, for example, natural gas and electricity. A high COP of a heat pump may not entirely overcome a relatively high cost for electicity compared with the same heating value from natural gas.

For example, the 2009 US average price per therm (100,000 BTU) of electricity was \$3.38 while the average price per therm of natural gas was \$1.16. Using these prices, a heat pump with a COP of 3.5 in moderate climate would cost \$0.97 to provide one therm of heat, while a high efficiency gas furnace with 95% efficiency would cost \$1.22 to provide one therm of heat. With these average prices, the heat pump costs 20% less to provide the same amount of heat. At 0 fahrenheit (-18 Celsius) COP is much lower. Then, the same system costs as much to operate as an efficient gas heater. The yearly savings will depend on the actual cost of electricity and natural gas, which can both vary widely.

However, a COP may help make a determination of system choice based on carbon contribution. Although a heat pump may cost more to operate than a conventional natural gas or electric heater, depending on the source of electricity generation in one's area, it may contribute less net carbon dioxide to the environment than burning natural gas or heating fuel. If locally no green electricity is available, then carbon wise the best option would be to drive a heat pump on piped gas or oil, to store excess heat in the ground source for use in winter, while using the same machine also for producing electricity with a built-in Sterling engine.

Conditions of use

While the COP is partly a measure of the efficiency of a heat pump, it is also a measure of the conditions under which it is operating: the COP of a given heat pump will rise as the input temperature increases or the output temperature decreases because it is linked to a warm temperature distribution system like underfloor heating.

Chapter 14

EcoCute and Fluidyne Engine

EcoCute

Domestic EcoCute outdoor unit (front) and hot water storage tank (back)

The **EcoCute** is an energy efficient electric heat pump, water heating and supply system that uses heat extracted from the air to heat water for domestic, industrial and commercial use. Instead of the more conventional ammonia or haloalkane gases, EcoCute uses carbon dioxide as a refrigerant. The technology offers a means of energy conservation and reduces the emission of greenhouse gas.

Etymology

The name of the EcoCute comes from the Japanese phrase *Shizen Reibai Hīto Ponpu Kyūtō-ki* (自然冷媒ヒートポンプ給湯機?), which literally means "natural refrigerant heat pump water heater". *Eco* is a contraction of either Ecology or Economical and *Cute* also means *kyūtō* (給湯?); literally “supply hot water.”

History

Modern chemical refrigeration techniques developed after the proposal of the Carnot cycle in 1824. Jacob Perkins invented an ice-making machine that used ether in 1843, and *Edmond Carré* built a refrigerator that used water and sulfuric acid in 1850. In Japan, Fusanosuke Kuhara, founder of Hitachi, Ltd., made an air conditioner for his own home use using compressed CO_2 as a refrigerant.

In 1930 Thomas Midgley, Jr. discovered dichlorodifluoromethane, a chlorinated fluorocarbon (CFC) known as Freon. CFCs rapidly replaced traditional refrigerant substances, including CO_2 (which proved hard to compress for domestic use), for use in heat pumps and refrigerators. But from the 1980s CFCs began to lose favor as refrigerant when their damaging effects on the ozone layer were discovered. An alternative type of refrigerant, Hydro fluoro compounds (HFC), also lost favour when they were identified as greenhouse gases. The Vienna Convention for the Protection of the Ozone Layer, the Montreal Protocol and the Kyoto Protocol call for the complete abandonment of such refrigerants by 2030.

In 1989, amid international concern about the effects of chlorofluorocarbons (CFC and HFC) on the ozone layer, scientist Gustav Lorentzen and SINTEF patented a method for using CO_2 as a refrigerant in heating and cooling. Further research into CO_2 refrigeration was then conducted at Shecco (Sustainable Heating and Cooling with CO_2) in Brussels, Belgium, leading to increasing use of CO_2 refrigerant technology in Europe.

In 1993 the Japanese company Denso, in collaboration with Gustav Lorentzen, developed an automobile air conditioner using CO_2 as a refrigerant. They demonstrated the invention at the June 1998 International Institute of Refrigeration/Gustav Lorentzen Conference. After the conference Denso were approached by CRIEPI (Central Research Institute of Electric Power Industry) and TEPCO (The Tokyo Electric Power Company) to develop a prototype air conditioner using natural refrigerant materials instead of Freon. Together they produced 30 prototype EcoCute units for a year-long experimental installation at locations throughout Japan, from the cold climate of Hokkaidō to hotter

Okinawa. After this successful feasibility study, Denso obtained a patent to compress CO_2 refrigerant for use in a heat pump from SINTEF in September 2000.

The first commercial domestic EcoCute was marketed in Japan by CORONA Corp. in May 2001, and several manufacturers sold 1.5 million units there by October, 2008.

FEPC reported 2 million units of EcoCute had been delivered by end of October 2009, and its equivalents CO_2 absorption is 9400 km² of forest.

Features and demand

In Japan in 1998, water heating (Kyuto (給湯 *kyūtō*?)) accounted for 33.8% of typical domestic energy consumption, with air conditioner and kerosene heater heating accounting for another 26.9% and cooling by air conditioner another 2.3%. Most of the remaining 37% was spent on electrical home appliances, a field where 21st century innovations in energy conservation began to make considerable energy savings. This left hot water supply as the most difficult area for energy conservation, leaving a gap in the market for the EcoCute. By January 2005, 26 Japanese companies were producing more than 450 models of EcoCute machines, and sales of domestic units increased 130-150% each year between 2001 and 2005.

Denso first introduced the EcoCute outside Japan at the COP9 Milan, Italy on December 9, 2003. From 2007, Denso began concentrating on marketing the EcoCute in the EU. In Japan, the Japanese government incorporated the EcoCute into its CO_2 reduction program under the Kyoto Protocol, mandating the installation of 5.2 million units in commercial and domestic properties by 2010. The cost of EcoCute is approximately 500 thousand Japanese yen as of February - March 2009.

EcoCute machine basics

An EcoCute machine or system consists of a heat pump and hot water storage unit. The components are serially concatenated with sealing refrigerant CO_2 gas in circulation.

1. At the first stage, a heat exchanger collects heat from the air outside to use as energy for the refrigerant. Air flow is usually obtained using a centrifugal fan; in cold areas with ambient temperatures around -20 to -25 ℃ an auxiliary fan heater is attached.
2. A gas compressor is used to heat the gas CO_2 refrigerant to around 100℃ under pressure of 10MPa via adiabatic compression. The carbon dioxide becomes a supercritical fluid. Several types of compressor can be used, including dual layer cylindrical compressors, scroll compressors, and dual stage rotary compressors
3. At the second stage a heat exchanger transfers energy from the hot refrigerant into water to produce hot water. Water temperatures around 5℃ and up are suitable at this stage.

4. Finally, ejector or expansion valves reduce pressure on the refrigerant, letting it cool via adiabatic expansion and revert to CO_2 gas.

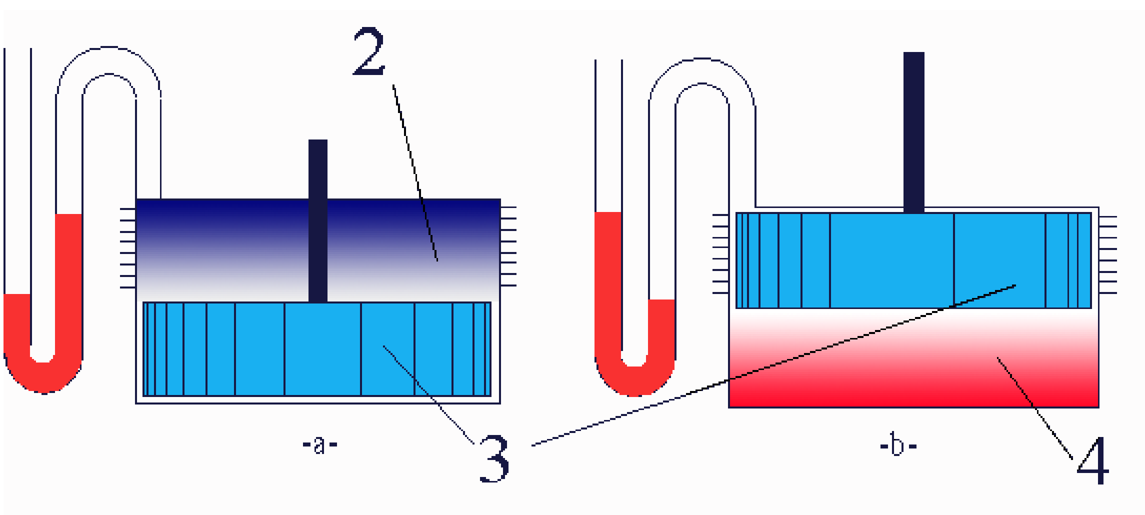
The EcoCute derives two units of energy from ambient air temperature for every unit of electrical power it requires. Each of these **one unit plus two cost free units** produces more than three units of hot water energy, resulting in reduced CO_2 emissions compared to water heating via electricity or town gas. To produce 90℃ hot water, an EcoCute consumes 66% less energy than an electric water heater, and costs 80% less than heating water via town gas in Japan. Also, by reducing use of fossil fuels, the EcoCute results in more than 50% reductions in CO_2 emissions

The EcoCute's COP is 3.8 in industrial use, while electric power water heating is 1.0, and gas boiler is 0.88 including pilot light loss.

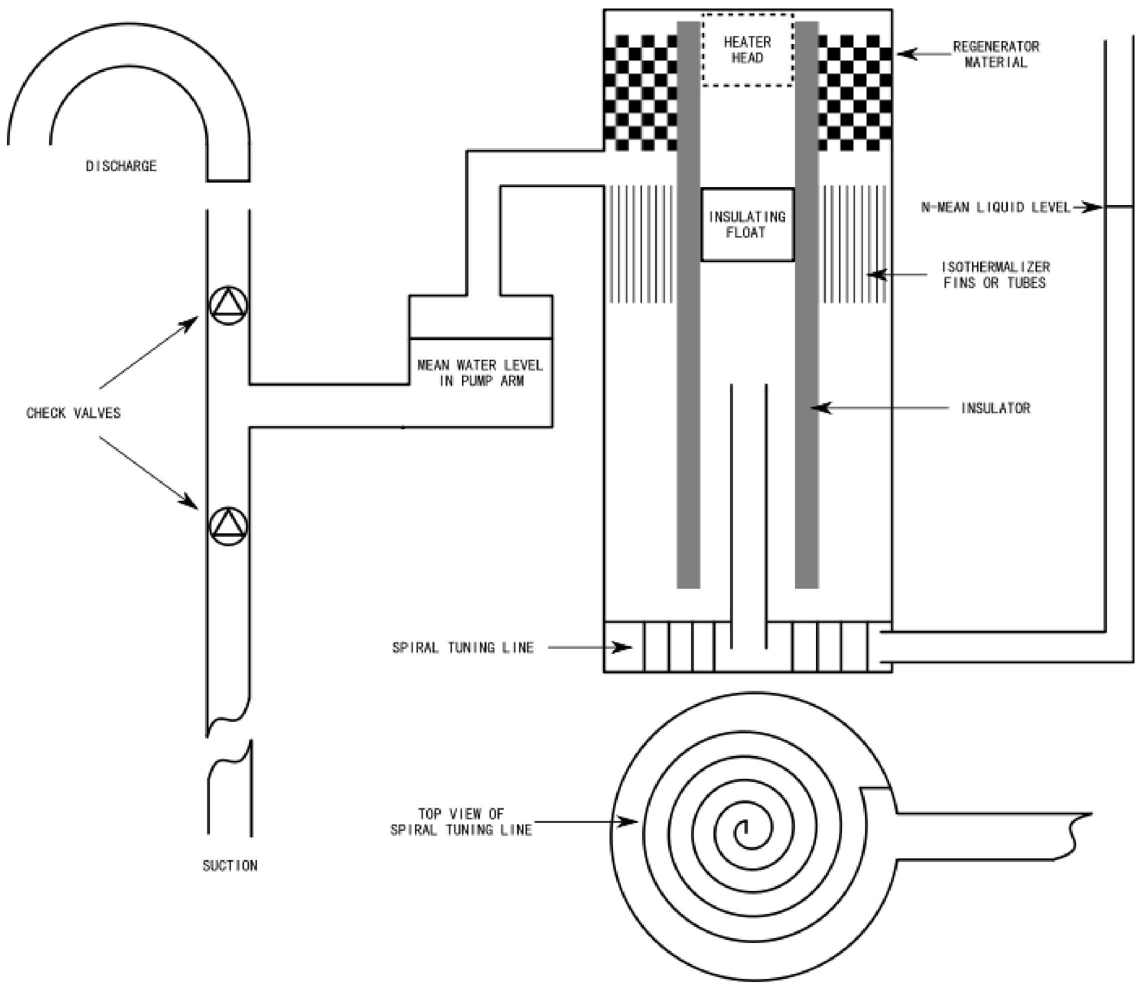
Others

EcoCute (エコキュート *ekokyūto*?) is a registered trademark (No. 4575216 - Japan) of Kansai Electric Power Company, but the term is also used generically to refer to water heaters designed for energy conservation or greenhouse gas emission reduction.

Fluidyne Engine



This is a Fluidyne variant with a solid displacer piston (3). In figure -a-, as the displacer moves from the cold compression space (2), to the hot expansion space (4) in figure -b-, the temperature of the gaseous working fluid is increased. This increases the pressure of the gaseous working fluid, and as it expands, work is done on the (red) liquid piston as it is pushed through the tube.



A Concentric-cylinder Fluidyne Pumping engine

A **Fluidyne engine** is an alpha or gamma type Stirling engine with one or more liquid pistons. It contains a working gas (often air), and either two liquid pistons or one liquid piston and a displacer.

Engine operation

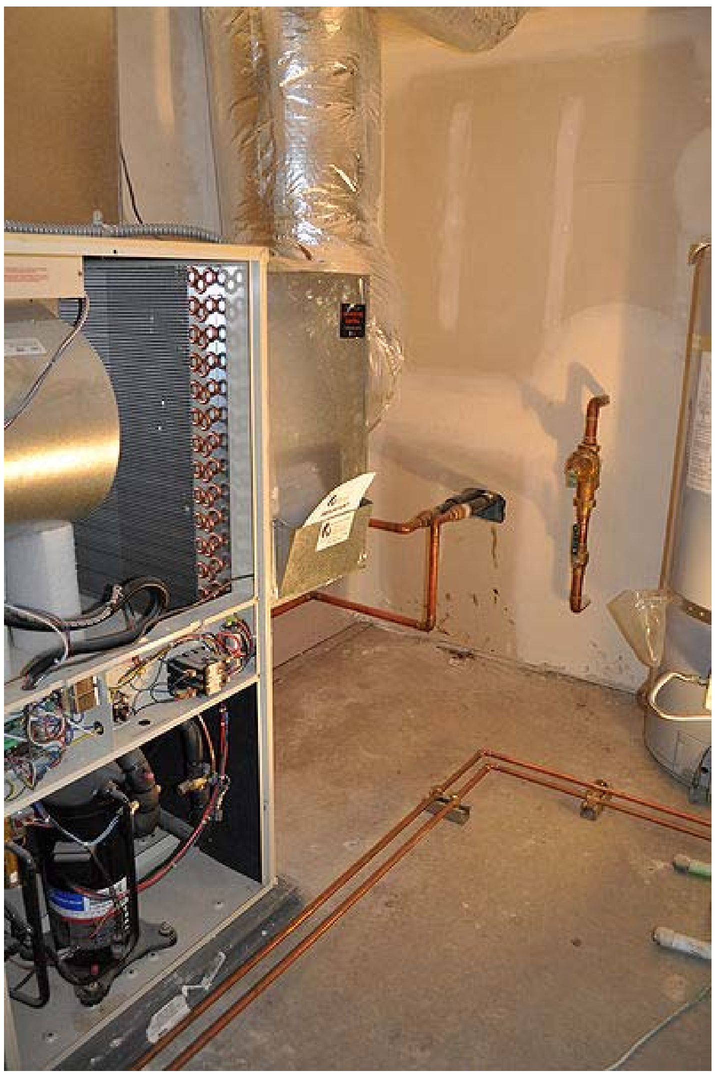
Working gas in the engine is heated, and this causes it to expand and push on the water column. This expansion cools the air which contracts, at the same time being pushed back by the weight of the displaced water column. Cycle then repeats.

Engine as a pump

In the classic configuration, the work produced via the water pistons is integrated with a water pump. The simple pump is external to the engine, and consists of two check valves, one on the intake and one on the outlet. In the engine, the loop of oscillating liquid can be thought of as acting as a displacer piston. The liquid in the single tube extending to the pump acts as the power piston. Traditionally the pump is open to the atmosphere, and the hydraulic head is small, so that the absolute engine pressure is close to atmospheric pressure.

Chapter 15

Geothermal Heat Pump



Ground source heating and cooling

Ground source heating and cooling

A **geothermal heat pump**, **ground source heat pump** (GSHP), or **ground heat pump** is a central heating and/or cooling system that pumps heat to or from the ground. It uses the earth as a heat source (in the winter) or a heat sink (in the summer). This design takes advantage of the moderate temperatures in the ground to boost efficiency and reduce the operational costs of heating and cooling systems, and may be combined with solar heating to form a geosolar system with even greater efficiency. Geothermal heat pumps are also known by a variety of other names, including **geoexchange, earth-coupled, earth energy** or **water-source heat pumps**. The engineering and scientific communities prefer the terms "geoexchange" or "ground source heat pumps" to avoid confusion with traditional geothermal power, which uses a high temperature heat source to generate electricity. Ground source heat pumps harvest a combination of geothermal energy (from the Earth's core) and solar energy (heat absorbed at the Earth's surface) when heating, but work against these heat sources when used for air conditioning.

Depending on latitude, the upper 3 metres (9.8 ft) of Earth's surface maintains a nearly constant temperature between 10 and 16 °C (50 and 60 °F). Like a refrigerator or air conditioner, these systems use a heat pump to force the transfer of heat from there. Heat pumps can transfer heat from a cool space to a warm space, against the natural direction of flow, or they can enhance the natural flow of heat from a warm area to a cool one. The core of the heat pump is a loop of refrigerant pumped through a vapor-compression refrigeration cycle that moves heat. Heat pumps are always more efficient at heating than pure electric heaters, even when extracting heat from cold winter air. But unlike an air-source heat pump, which transfers heat to or from the outside air, a ground source heat pump exchanges heat with the ground. This is much more energy-efficient because underground temperatures are more stable than air temperatures through the year. Seasonal variations drop off with depth and disappear below seven meters due to thermal inertia. Like a cave, the shallow ground temperature is warmer than the air above during the winter and cooler than the air in the summer. A ground source heat pump extracts ground heat in the winter (for heating) and transfers heat back into the ground in the summer (for cooling). Some systems are designed to operate in one mode only, heating or cooling, depending on climate.

The geothermal pump systems reach fairly high Coefficient of performance (CoP), 3-6, on the coldest of winter nights, compared to 1.75-2.5 for air-source heat pumps on cool days. Ground source heat pumps (GSHPs) are among the most energy efficient technologies for providing HVAC and water heating. Actual CoP of a geothermal system which includes the power required to circulate the fluid through the underground tubes can be lower than 2.5. The setup costs are higher than for conventional systems, but the difference is usually returned in energy savings in 3 to 10 years. System life is estimated at 25 years for inside components and 50+ years for the ground loop. As of 2004, there are over a million units installed worldwide providing 12 GW of thermal capacity, with an annual growth rate of 10%.

Differing terms and definitions

There is a great deal of controversy and confusion with regard to exactly what geothermal heat pumps do. There are several concepts commonly attached to the idea of geothermal:

- Using geologically hot rocks, which have little relationship to the surface climate and derive their heat from deep in the earth, to run a heat engine which produces electricity. Such a system can be operated only until the rock around the bore cools, then it gradually loses its generating ability. All of these systems are in tectonically or volcanically active areas. Most people are pretty clear that this should be called "geothermal power".

- Using geologically hot rocks to heat some type of liquid or gas which is pumped up to be used to heat a building is often called "geothermal heating".

- Using a heat exchanger with a finite amount of external material to incorporate additional thermal mass to a building. This makes the building change temperature slowly, and allows the inhabitants to go through a time period with less overall temperature variation. The most common ones appear to be "geothermal heat pump" by laymen and "ground-source heat pump" by experts, but even these are broad, barely understood terms about which there is no consensus.

Builders may try to smooth out the indoor climate over surface temperature variations resulting from the day-night cycle, variations due to short-term weather patterns, or variations due to entire seasons. The amount of thermal mass incorporated is on a spectrum, so one cannot say their system addresses any of these cycles specifically – a system sized for day-night cycling will still help somewhat in a week-long blizzard. Such a system requires power to pump the coolant, but can be operated indefinitely.

To further complicate things, even though most home-sized systems termed "geothermal" operate primarily on the former principle, the thermal mass in such systems is rarely perfectly finite and closed. Groundwater flows through the area, and heat leaks out and warms/cools the surrounding area. True geothermal heat may play a small or large role in such systems.

When trying to explain this subject, experts may go through a series of explanations and divisions.

First, people separate out terms for geothermal electricity generation:

- geothermal power

Then, they split out geothermal heating, which is commonly used in tectonically or volcanically active regions:

- geothermal heating

Then, they explain the traditional concept of a heat pump which uses only inside and outside air:

- heat pump

After that, they try to identify simple systems in which the coolant is air which is pumped directly out of and back into the building, going through a simple hole in the ground:

- earth tube or earth air heat exchanger
- ground-coupled heat exchanger

After that, they remove systems which depend on large quantities of water or wet ground, primarily for cooling:

- lake water cooling
- deep water source cooling

At this point they may explain the concept of a seasonal thermal store or a thermal mass climate control strategy:

- trombe wall
- seasonal thermal store
- thermal mass

Then, they may try to figure out the size of the system. Is it targeted at a home? A building? Is it a full-scale district heating system?

Then they go into the specifics of the system. First, is the coolant water, and if so is it "open loop" – exposed to groundwater – or "closed loop" – not exposed.

Are other energy sources helping? Is solar absorbed from the house or from a dedicated thermal collector?

- annualized geothermal solar or annualized geo solar

- geosolar or solar combisystem

After this they concentrate on the specific form factor of the system. Is it a grid of pipes buried 3 feet (0.91 m) underneath the owner's garden? Does it consist of a hundred-foot borehole? A thousand-foot borehole? Dozens of 8-foot (2.4 m) boreholes?

- downhole heat exchanger or borehole heat exchanger

Finally they may try to decide what the locals call the system, as identical systems are often called different things in different countries, and in some countries generic terms may be trademarked in others:

- Geoexchange is a trademarked product in the US, but is a standards coalition in Canada.
- Earth tubes, Air-earth heat exchangers and "heat exchanger" in general, appear to be primarily used in the UK.

History

The heat pump was described by Lord Kelvin in 1853 and developed by Peter Ritter von Rittinger in 1855. After experimenting with a freezer, Robert C. Webber built the first direct exchange ground-source heat pump in the late 1940s. The first successful commercial project was installed in the Commonwealth Building (Portland, Oregon) in 1946, and has been designated a National Historic Mechanical Engineering Landmark by ASME. The technology became popular in Sweden in the 1970s, and has been growing slowly in worldwide acceptance since then. Open loop systems dominated the market until the development of polybutylene pipe in 1979 made closed loop systems economically viable. As of 2004, there are over a million units installed worldwide providing 12 GW of thermal capacity. Each year, about 80,000 units are installed in the USA (geothermal energy is used in all 50 U.S. states today, with great potential for near-term market growth and savings) and 27,000 in Sweden.

Ground heat exchanger

Loop field for a 12-ton system (unusually large for most residential applications)

Heat pumps provide wintertime heating by extracting heat from a source and transferring it to the building. In theory, heat can be extracted from any source, no matter how cold, but a warmer source allows higher efficiency. A ground source heat pump uses the shallow ground as a source of heat, thus taking advantage of its seasonally moderate temperatures.

In the summer, the process can be reversed so the heat pump extracts heat from the building and transfers it to the ground. Transferring heat to a cooler space takes less energy, so the cooling efficiency of the heat pump gain benefits from the lower ground temperatures.

Shallow horizontal heat exchangers experience seasonal temperature cycles due to solar gains and transmission losses to ambient air at ground level. These temperature cycles lag behind the seasons because of thermal inertia, so the heat exchanger can harvest heat deposited by the sun several months earlier. Deep vertical systems rely heavily on migration of heat from surrounding geology, unless they are recharged annually by exhaust heat from air conditioning.

Ground source heat pumps must have a heat exchanger in contact with the ground or groundwater to extract or dissipate heat. This component accounts for a third to a half of the total system cost. Several major design options are available for these, which are classified by fluid and layout. Direct exchange systems circulate refrigerant underground, closed loop systems use a mixture of anti-freeze and water, and open loop systems use natural groundwater.

Direct exchange

The Direct exchange geothermal heat pump is the oldest type of geothermal heat pump technology. It is also the simplest and easiest to understand. The ground-coupling is achieved through a single loop circulating refrigerant in direct thermal contact with the ground (as opposed to a combination of a refrigerant loop and a water loop). The refrigerant leaves the heat pump appliance cabinet, circulates through a loop of copper tube buried underground, and exchanges heat with the ground before returning to the pump. The name "direct exchange" refers to heat transfer between the refrigerant and the ground without the use of an intermediate fluid. There is no direct interaction between the fluid and the earth; only heat transfer through the pipe wall. Direct exchange heat pumps are not to be confused with "water-source heat pumps" or "water loop heat pumps" since there is no water in the ground loop. ASHRAE defines the term *ground-coupled heat pump* to encompass closed loop and direct exchange systems, while excluding open loops.

Direct exchange systems are significantly more efficient and have potentially lower installation costs than closed loop water systems. Copper's high thermal conductivity contributes to the higher efficiency of the system, but heat flow is predominantly limited by the thermal conductivity of the ground, not the pipe. The main reasons for the higher efficiency are the elimination of the water pump (which uses electricity), the elimination of the water heat exchanger (which is a source of heat losses), and most importantly, the latent heat phase change of the refrigerant in the ground itself.

While they require much more refrigerant and their tubing is more expensive per foot, a direct exchange loop is shorter than a closed water loop for a given capacity. A direct exchange system requires only 15 to 30% of the length of tubing and half the diameter of drilled holes, and the drilling or excavation costs are therefore lower. Refrigerant loops are less tolerant of leaks than water loops because gas can leak out through smaller imperfections. This dictates the use of brazed copper tubing, even though the pressures are similar to water loops. The copper loop must be protected from corrosion in acidic soil through the use of a sacrificial anode or cathodic protection.

Closed loop

Most installed systems have two loops on the ground side: the primary refrigerant loop is contained in the appliance cabinet where it exchanges heat with a secondary water loop that is buried underground. The secondary loop is typically made of High-density polyethylene pipe and contains a mixture of water and anti-freeze (propylene glycol, denatured alcohol or methanol). After leaving the internal heat exchanger, the water flows through the secondary loop outside the building to exchange heat with the ground before returning. The secondary loop is placed below the frost line where the temperature is more stable, or preferably submerged in a body of water if available. Systems in wet ground or in water are generally more efficient than drier ground loops since it is less work to move heat in and out of water than solids in sand or soil. If the ground is naturally dry, soaker hoses may be buried with the ground loop to keep it wet.

An installed liquid pump pack

Closed loop systems need a heat exchanger between the refrigerant loop and the water loop, and pumps in both loops. Some manufacturers have a separate ground loop fluid pump pack, while some integrate the pumping and valving within the heat pump. Expansion tanks and pressure relief valves may be installed on the heated fluid side. Closed loop systems have lower efficiency than direct exchange systems, so they require longer and larger pipe to be placed in the ground, increasing excavation costs.

Closed loop tubing can be installed horizontally as a loop field in trenches or vertically as a series of long U-shapes in wells. The size of the loop field depends on the soil type and moisture content, the average ground temperature and the heat loss and or gain characteristics of the building being conditioned. A rough approximation of the initial soil temperature is the average daily temperature for the region.

Vertical

A vertical closed loop field is composed of pipes that run vertically in the ground. A hole is bored in the ground, typically 75 to 500 feet (23–150 m) deep. Pipe pairs in the hole are joined with a U-shaped cross connector at the bottom of the hole. The borehole is commonly filled with a bentonite grout surrounding the pipe to provide a thermal connection to the surrounding soil or rock to improve the heat transfer. Thermally enhanced grouts are available to improve this heat transfer. Grout also protects the ground water from contamination, and prevents artesian wells from flooding the property. Vertical loop fields are typically used when there is a limited area of land available. Bore holes are spaced at least 5–6 m apart and the depth depends on ground and building characteristics. For illustration, a detached house needing 10 kW (3 ton) of heating capacity might need three boreholes 80 to 110 m (260 to 360 ft) deep. (A ton of heat is 12,000 British thermal units per hour (BTU/h) or 3.5 kilowatts.) During the cooling season, the local temperature rise in the bore field is influenced most by the moisture travel in the soil. Reliable heat transfer models have been developed through sample bore holes as well as other tests.

Horizontal

A 3-ton slinky loop prior to being covered with soil. The three slinky loops are running out horizontally with three straight lines returning the end of the slinky coil to the heat pump

A horizontal closed loop field is composed of pipes that run horizontally in the ground. A long horizontal trench, deeper than the frost line, is dug and U-shaped or slinky coils are placed horizontally inside the same trench. Excavation for horizontal loop fields is about half the cost of vertical drilling, so this is the most common layout used wherever there is adequate land available. For illustration, a detached house needing 10 kW (3 ton) of heating capacity might need 3 loops 120 to 180 m (390 to 590 ft) long of NPS 3/4 (DN 20) or NPS 1.25 (DN 32) polyethylene tubing at a depth of 1 to 2 m (3.3 to 6.6 ft).

As an alternative to trenching, the horizontal loop field may be laid by mini horizontal directional drilling. (mini-HDD) This technique can lay piping under yards, driveways or other structures without disturbing them, with a cost between those of trenching and vertical drilling.

A slinky (also called coiled) closed loop field is a type of horizontal closed loop where the pipes overlay each other (not a recommended method). The easiest way of picturing a slinky field is to imagine holding a slinky on the top and bottom with your hands and then move your hands in opposite directions. A slinky loop field is used if there is not adequate room for a true horizontal system, but it still allows for an easy installation. Rather than using straight pipe, slinky coils, use overlapped loops of piping laid out horizontally along the bottom of a wide trench. Depending on soil, climate and your heat pump's run fraction, slinky coil trenches can be anywhere from one third to two thirds shorter than traditional horizontal loop trenches. Slinky coil ground loops are essentially a more economic and space efficient version of a horizontal ground loop.

Pond

12-ton pond loop system being sunk to the bottom of a pond

A closed pond loop is not common because it depends on proximity to a body of water, where an open loop system is usually preferable. A pond loop may be advantageous where poor water quality precludes an open loop, or where the system heat load is small. A pond loop consists of coils of pipe similar to a slinky loop attached to a frame and located at the bottom of an appropriately sized pond or water source.

Open loop

In an open loop system (also called a groundwater heat pump), the secondary loop pumps natural water from a well or body of water into a heat exchanger inside the heat pump. ASHRAE calls open loop systems *groundwater heat pumps* or *surface water heat pumps*, depending on the source. Heat is either extracted or added by the primary refrigerant loop, and the water is returned to a separate injection well, irrigation trench, tile field or body of water. The supply and return lines must be placed far enough apart to ensure thermal recharge of the source. Since the water chemistry is not controlled, the appliance may need to be protected from corrosion by using different metals in the heat exchanger and pump. Limescale may foul the system over time and require periodic acid cleaning. Also, as fouling decreases the flow of natural water, it becomes difficult for the heat pump to exchange building heat with the groundwater. If the water contains high levels of salt, minerals, iron bacteria or hydrogen sulfide, a closed loop system is usually preferable.

Deep lake water cooling uses a similar process with an open loop for air conditioning and cooling. Open loop systems using ground water are usually more efficient than closed systems because they are better coupled with ground temperatures. Closed loop systems, in comparison, have to transfer heat across extra layers of pipe wall and dirt.

A growing number of jurisdictions have outlawed open-loop systems that drain to the surface because these may drain aquifers or contaminate wells. This forces the use of more environmentally sound injection wells.

Standing column well

A standing column well system is a specialized type of open loop system. Water is drawn from the bottom of a deep rock well, passed through a heat pump, and returned to the top of the well, where traveling downwards it exchanges heat with the surrounding bedrock. The choice of a standing column well system is often dictated where there is near-surface bedrock and limited surface area is available. A standing column is typically not suitable in locations where the geology is mostly clay, silt, or sand. If bedrock is deeper than 200 feet (61 m) from the surface, the cost of casing to seal off the overburden may become prohibitive.

A multiple standing column well system can support a large structure in an urban or rural application. The standing column well method is also popular in residential and small commercial applications. There are many successful applications of varying sizes and well quantities in the many boroughs of New York City, and is also the most common application in the New England states. This type of ground source system has some heat storage benefits, where heat is rejected from the building and the temperature of the well is raised, within reason, during the Summer cooling months which can then be harvested for heating in the Winter months, thereby increasing the efficiency of the heat pump system. As with closed loop systems, sizing of the standing column system is critical in reference to the heat loss and gain of the existing building. As the heat exchange is actually with the bedrock, using water as the transfer medium, a large amount of production capacity (water flow from the well) is not required for a standing column system to work. However, if there is adequate water production, then the thermal capacity of the well system can be enhanced by discharging a small percentage of system flow during the peak Summer and Winter months.

Since this is essentially a water pumping system, standing column well design requires critical considerations to obtain peak operating efficiency. Should a standing column well design be misapplied, leaving out critical shut-off valves for example, the result could be an extreme loss in efficiency and thereby cause operational cost to be higher than anticipated.

Building distribution

Liquid-to-air heat pump

The heat pump is the central unit that becomes the heating and cooling plant for the building. Some models may cover space heating, space cooling, (space heating via conditioned air, hydronic systems and / or radiant heating systems), domestic or pool water preheat (via the desuperheater function, demand hot water, and driveway ice melting all within one appliance with a variety of options with respect to controls, staging and zone control. The heat may be carried to its end use by circulating water or forced air. Almost all types of heat pumps are produced for commercial and residential applications.

Liquid-to-air heat pumps (also called *water-to-air*) output forced air, and are most commonly used to replace legacy forced air furnaces and central air conditioning systems. There are variations that allow for split systems, high-velocity systems, and ductless systems. Heat pumps cannot achieve as high of a fluid temperature as a conventional furnace, so they require a higher volume flow rate of air to compensate. When retrofitting a residence, the existing duct work may have to be enlarged to reduce the noise from the higher air flow.

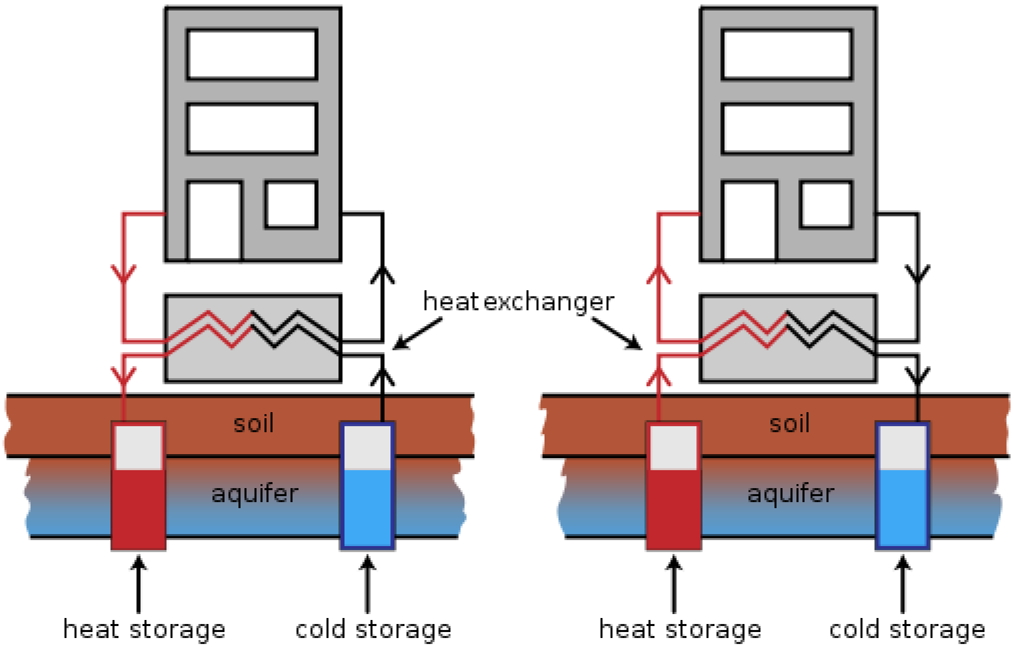
Liquid-to-water heat pump

Liquid-to-water heat pumps (also called *water-to-water*) are hydronic systems that use water to carry heating or cooling through the building. Systems such as radiant underfloor heating, baseboard radiators, conventional cast iron radiators would use a liquid-to-water heat pump. These heat pumps are preferred for pool heating or domestic hot water pre-heat. Heat pumps can only heat water to about 50 °C (122 °F) efficiently, whereas a boiler normally reaches 65–95 °C (149–203 °F). Legacy radiators designed for these higher temperatures may have to be doubled in numbers when retrofitting a home. A hot water tank will still be needed to raise water temperatures above the heat pump's maximum, but pre-heating will save 25-50% of hot water costs.

Ground source heat pumps are especially well matched to underfloor heating and baseboard radiator systems which only require warm temperatures (40 °C) to work well. Thus they are ideal for open plan offices. Using large surfaces such as floors, as opposed to radiators, distributes the heat more uniformly and allows for a lower water temperature. Wood or carpet floor coverings dampen this effect because the thermal transfer efficiency of these materials is lower than that of masonry floors (tile, concrete). Underfloor piping, ceiling or wall radiators can also be used for cooling in dry climates, although the temperature of the circulating water must be above the dew point to ensure that atmospheric humidity does not condense on the radiator.

Combination heat pumps are available that can produce forced air and circulating water simultaneously and individually. These systems are largely being used for houses that have a combination of air and liquid conditioning needs, for example central air conditioning and pool heating.

Seasonal thermal storage



A heat pump in combination with heat and cold storage

The efficiency of ground source heat pumps can be improved by using seasonal thermal storage. If heat loss from the ground source is sufficiently low, the heat pumped out of the building in the summer can be retrieved in the winter. Heat storage efficiency increases with scale, so this advantage is most significant in commercial or district heating systems. Geosolar combisystems further augment this efficiency by collecting extra solar energy during the summer (more than is needed for air conditioning) and concentrating it in the store.

Such a system has been used to heat and cool a greenhouse using an aquifer for thermal storage. In summer, the greenhouse is cooled with cold ground water. This heats the water in the aquifer which can become a warm source for heating in winter. The combination of cold and heat storage with heat pumps can be combined with water/humidity regulation. These principles are used to provide renewable heat and renewable cooling to all kinds of buildings.

Also the efficiency of existing small heat pump installations can sometimes be improved a lot by adding large, cheap, water filled solar collectors. These may be integrated into a to be overhauled parking lot, or in walls or roof constructions simply by putting lots of one inch PE pipes into the outer layer. A very simple option is to add a large mechanically ventilated out door water-air heat exchanger (like the one that is in front of your car engine, but larger). In the summer they allow to pump lots of heat, almost free of running cost, into the ground. This only works well when ground water mobility is not too high, and it works better when more houses install this system next to each other. (In the winter such outdoor components have to be drained of water.)

Thermal efficiency

The net thermal efficiency of a heat pump should take into account the efficiency of electricity generation and transmission, typically about 40%. Since a heat pump moves 3 to 5 times more heat energy than the electric energy it consumes, the total energy output is much greater than the input. This results in net thermal efficiencies greater than 100% for most electricity sources. Traditional combustion furnaces and electric heaters can never exceed 100% efficiency, but heat pumps provide extra energy by extracting it from the ground.

Geothermal heat pumps can reduce energy consumption— and corresponding air pollution emissions—up to 44% compared to air source heat pumps and up to 72% compared to electric resistance heating with standard air-conditioning equipment.

The dependence of net thermal efficiency on the electricity infrastructure tends to be an unnecessary complication for consumers and is not applicable to hydroelectric power, so performance of heat pumps is usually expressed as the ratio of heating output or heat removal to electricity input. Cooling performance is typically expressed in units of BTU/hr/watt as the Energy Efficiency Ratio, (EER) while heating performance is typically reduced to dimensionless units as the Coefficient of Performance. (COP) The conversion factor is 3.41 BTU/hr/watt. Performance is influenced by all components of the installed system, including the soil conditions, the ground-coupled heat exchanger, the heat pump appliance, and the building distribution, but is largely determined by the "lift" between the input temperature and the output temperature.

For the sake of comparing heat pump appliances to each other, independently from other system components, a few standard test conditions have been established by the American Refrigerant Institute (ARI) and more recently by the International Organization for Standardization. Standard ARI 330 ratings were intended for closed loop ground-source heat pumps, and assumes secondary loop water temperatures of 77 °F (25 °C) for air conditioning and 32 °F (0 °C) for heating. These temperatures are typical of installations in the northern USA. Standard ARI 325 ratings were intended for open loop ground-source heat pumps, and include two sets of ratings for groundwater temperatures of 50 °F (10 °C) and 70 °F (21 °C). ARI 325 budgets more electricity for water pumping than ARI 330. Neither of these standards attempt to account for seasonal variations. Standard ARI 870 ratings are intended for direct exchange ground-source heat pumps. ASHRAE transitioned to ISO 13256-1 in 2001, which replaces ARI 320, 325 and 330. The new ISO standard produces slightly higher ratings because it no longer budgets any electricity for water pumps.

Efficient compressors, variable speed compressors and larger heat exchangers all contribute to heat pump efficiency. Residential ground source heat pumps on the market today have standard COPs ranging from 2.4 to 5.0 and EERs ranging from 10.6 to 30. To qualify for an Energy Star label, heat pumps must meet certain minimum COP and EER ratings which depend on the ground heat exchanger type. For closed loop systems, the

ISO 13256-1 heating COP must be 3.3 or greater and the cooling EER must be 14.1 or greater.

Actual installation conditions may produce better or worse efficiency than the standard test conditions. COP improves with a lower temperature difference between the input and output of the heat pump, so the stability of ground temperatures is important. If the loop field or water pump is undersized, the addition or removal of heat may push the ground temperature beyond standard test conditions, and performance will be degraded. Similarly, an undersized blower may allow the plenum coil to overheat and degrade performance.

Soil without artificial heat addition or subtraction and at depths of several meters or more remains at a relatively constant temperature year round. This temperature equates roughly to the average annual air-temperature of the chosen location, usually 7–12 °C (45–54 °F) at a depth of six meters in the northern USA. Because this temperature remains more constant than the air temperature throughout the seasons, geothermal heat pumps perform with far greater efficiency during extreme air temperatures than air conditioners and air-source heat pumps.

Standards ARI 210 and 240 define Seasonal Energy Efficiency Ratio (SEER) and Heating Seasonal Performance Factors (HSPF) to account for the impact of seasonal variations on air source heat pumps. These numbers are normally not applicable and should not be compared to ground source heat pump ratings. However, Natural Resources Canada has adapted this approach to calculate typical seasonally adjusted HSPFs for ground-source heat pumps in Canada. The NRC HSPFs ranged from 8.7 to 12.8 BTU/hr/watt (2.6 to 3.8 in nondimensional factors, or 255% to 375% seasonal average electricity utilization efficiency) for the most populated regions of Canada. When combined with the thermal efficiency of electricity, this corresponds to net average thermal efficiencies of 100% to 150%.

Environmental impact

The U.S. Environmental Protection Agency (EPA) has called ground source heat pumps the most energy-efficient, environmentally clean, and cost-effective space conditioning systems available. Heat pumps offer significant emission reductions potential, particularly where they are used for both heating and cooling and where the electricity is produced from renewable resources.

Ground-source heat pumps have unsurpassed thermal efficiencies and produce zero emissions locally, but their electricity supply includes components with high greenhouse gas emissions, unless the owner has opted for a 100% renewable energy supply. Their environmental impact therefore depends on the characteristics of the electricity supply.

Annual greenhouse gas savings from using a ground source heat pump instead of a high-efficiency furnace in a detached residence (assuming no specific supply of renewable energy)

Country	Electricity CO_2 Emissions Intensity	GHG savings relative to		
		natural gas	**heating oil**	**electric heating**
Canada	223 ton/GWh	2.7 ton/yr	5.3 ton/yr	3.4 ton/yr
Russia	351 ton/GWh	1.8 ton/yr	4.4 ton/yr	5.4 ton/yr
USA	676 ton/GWh	-0.5 ton/yr	2.2 ton/yr	10.3 ton/yr
China	839 ton/GWh	-1.6 ton/yr	1.0 ton/yr	12.8 ton/yr

The GHG emissions savings from a heat pump over a conventional furnace can be calculated based on the following formula:

$$GHG\ Savings = HL\left(\frac{FI}{AFUE \times 1000\frac{kg}{ton}} - \frac{EI}{COP \times 3600\frac{sec}{hr}}\right)$$

- HL = seasonal heat load ≈ 80 GJ/yr for a modern detached house in the northern USA
- FI = emissions intensity of fuel = 50 kg(CO_2)/GJ for natural gas, 73 for heating oil, 0 for 100% renewable energy such as wind, hydro, photovoltaic or solar thermal
- AFUE = furnace efficiency ≈ 95% for a modern condensing furnace
- COP = heat pump coefficient of performance ≈ 3.2 seasonally adjusted for northern USA heat pump
- EI = emissions intensity of electricity ≈ 200-800 ton(CO_2)/GWh, depending on region

Ground-source heat pumps always produce less greenhouse gases than air conditioners, oil furnaces, and electric heating, but natural gas furnaces may be competitive depending on the greenhouse gas intensity of the local electricity supply. In countries like Canada and Russia with low emitting electricity infrastructure, a residential heat pump may save 5 tons of carbon dioxide per year relative to an oil furnace, or about as much as taking an average passenger car off the road. But in countries like China or USA that are highly reliant on coal for electricity production, a heat pump may result in 1 or 2 tons more carbon dioxide emissions than a natural gas furnace.

The fluids used in closed loops may be designed to be biodegradable and non-toxic, but the refrigerant used in the heat pump cabinet and in direct exchange loops was, until recently, chlorodifluoromethane, which is an ozone depleting substance. Although harmless while contained, leaks and improper end-of-life disposal contribute to enlarging the ozone hole. This refrigerant is being phased out in favor of ozone-friendly R410A for new construction. The EcoCute water heater is an air-source heat pump that uses Carbon Dioxide as its working fluid instead of Chlorofluorocarbons.

Open loop systems that draw water from a well and drain to the surface may contribute to aquifer depletion, water shortages, groundwater contamination, and subsidence of the soil. A geothermal heating project in Staufen im Breisgau, Germany, is suspected to have caused considerable damage to buildings in the city center. The ground has subsided by up to eight millimeters under the city hall while other areas have been uplifted by a few millimeters.

Ground-source heat pump technology, like building orientation, is a natural building technique (bioclimatic building).

Economics

Ground source heat pumps are characterized by high capital costs and low operational costs compared to other HVAC systems. Their overall economic benefit depends primarily on the relative costs of electricity and fuels, which are highly variable over time and across the world. Based on recent prices, ground-source heat pumps currently have lower operational costs than any other conventional heating source almost everywhere in the world. Natural gas is the only fuel with competitive operational costs, and only in a handful of countries where it is exceptionally cheap, or where electricity is exceptionally expensive. In general, a homeowner may save anywhere from 20% to 60% annually on utilities by switching from an ordinary system to a ground-source system. However, many family size installations are reported to use much more electricity then their owners had expected from advertisements. This is often partly due to bad design or installation: Heat exchange capacity with groundwater is often too small, heating pipes in house floors are often too thin and too few, or heated floors are covered with wooden panels or carpets.

Capital costs and system lifespan have received much less study, and the return on investment is highly variable. One study found the total installed cost for a system with 10 kW (3 ton) thermal capacity for a detached rural residence in the USA averaged $8000–$9000 in 1995 US dollars. More recent studies found an average cost of $14,000 in 2008 US dollars for the same size system. The US Department of Energy estimates a price of $7500 on its website, last updated in 2008. Prices over $20,000 are quoted in Canada, with one source placing them in the range of $30,000-$34,000 Canadian dollars. The rapid escalation in system price has been accompanied by rapid improvements in efficiency and reliability. Capital costs are known to benefit from economies of scale, particularly for open loop systems, so they are more cost-effective for larger commercial buildings and harsher climates. The initial cost can be two to five times that of a conventional heating system in most residential applications, new construction or existing. In retrofits, the cost of installation is affected by the size of living area, the home's age, insulation characteristics, the geology of the area, and location of the home/property. Proper duct system design and mechanical air exchange should be considered in the initial system cost.

Payback period for installing a ground source heat pump in a detached residence

Country	Payback period for replacing		
	natural gas	**heating oil**	**electric heating**
Canada	13 years	3 years	6 years
USA	12 years	5 years	4 years
Germany	net loss	8 years	2 years

Notes:

- Highly variable with energy prices.
- Government subsidies not included.
- Climate differences not evaluated.

Capital costs may be offset by substantial subsidies from many governments, for example totaling over $7000 in Ontario for residential systems installed in the 2009 fiscal year. Some electric companies offer special rates to customers who install a ground-source heat pump for heating/cooling their building. This is due to the fact that electrical plants have the largest loads during summer months and much of their capacity sits idle during winter months. This allows the electric company to use more of their facility during the winter months and sell more electricity. It also allows them to reduce peak usage during the summer (due to the increased efficiency of heat pumps), thereby avoiding costly construction of new power plants. For the same reasons, other utility companies have started to pay for the installation of ground-source heat pumps at customer residences. They lease the systems to their customers for a monthly fee, at a net overall savings to the customer.

The lifespan of the system is longer than conventional heating and cooling systems. Good data on system lifespan is not yet available because the technology is too recent, but many early systems are still operational today after 25–30 years with routine maintenance. Most loop fields have warranties for 25 to 50 years and are expected to last at least 50 to 200 years. Ground-source heat pumps use electricity for heating the house. The higher investment above conventional oil, propane or electric systems may be returned in energy savings in 2–10 years for residential systems in the USA. If compared to natural gas systems, the payback period can be much longer or non-existent. The payback period for larger commercial systems in the USA is 1–5 years, even when compared to natural gas.

Ground source heat pumps are recognized as one of the most efficient heating and cooling systems on the market. They are often the second-most cost effective solution in extreme climates, (after co-generation), despite reductions in thermal efficiency due to ground temperature. (The ground source is warmer in climates that need strong air conditioning, and cooler in climates that need strong heating.)

Commercial systems maintenance costs in the USA have historically been between $0.11 to $0.22 per m^2 per year in 1996 dollars, much less than the average $0.54 per m^2 per year for conventional HVAC systems.

Governments that promote renewable energy will likely offer incentives for the consumer (residential), or industrial markets. For example, in the United States, incentives are offered both on the state and federal levels of government.

Installation

Because of the technical knowledge and equipment needed to properly design and size the system (and install the piping if heat fusion is required), a GSHP system installation requires a professional's services. The International Ground Source Heat Pump Association (IGSHPA), Geothermal Exchange Organization (GEO) and the Canadian GeoExchange Coalition maintain listings of qualified installers in the USA and Canada.

Chapter 16

Air Source Heat Pumps

An **air source heat pump** uses outside air as a heat source or heat sink. A compressor, condenser and refrigerant system is used to absorb heat at one place and release it at another.

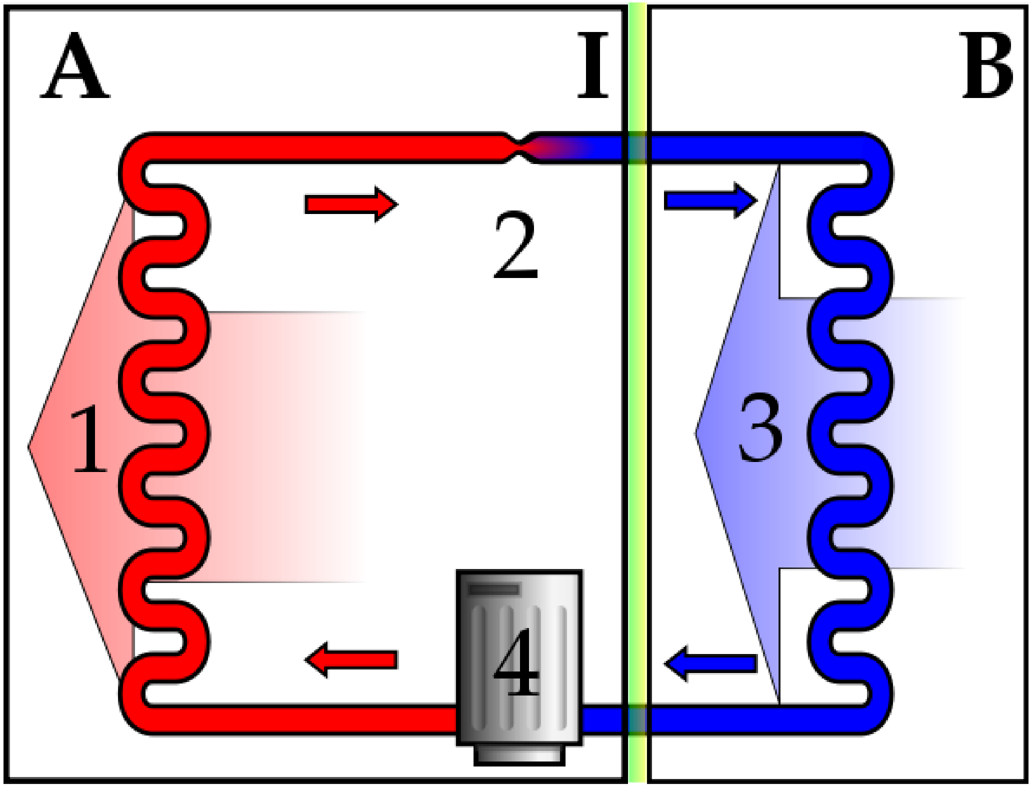
General

Outside air, necessarily existing at some temperature above absolute zero, is a heat container. An air-source heat pump moves ("pumps") some of this heat to provide hot water or household heating. This can be done in either direction, to cool or heat the interior of a building.

The main components of an air-source heat pump are:

- a heat exchanger, over which outside air is blown, to extract the heat from the air
- a compressor, which acts like a refrigerator but in reverse and raises the temperature from the outside air
- a way to transfer the heat into a hot water tank or heating system, such as radiators or under-floor heating tubes

How air source heat pumps work



A: indoor compartment, B: outdoor compartment, I: insulation, 1: condenser, 2: expansion valve, 3: evaporator, 4: compressor

Heating and cooling is accomplished by moving a refrigerant through the heat pump's various indoor and outdoor coils and components. A compressor, condenser, expansion valve and evaporator are used to change states of the refrigerant from a liquid to hot gas and from a gas to a cold liquid. The refrigerant is used to heat or cool coils in a building or room and fans pull the room air over the coils. An external outdoor heat exchanger is used to heat or cool the refrigerant. This use of outside air has led to the term "Air Source" Heat Pump. The overall operation uses the concepts described in classic vapor compression refrigeration.

When the liquid refrigerant at a low temperature passes through the outdoor evaporator coils, the temperature of the outside air causes the liquid to boil. This change of state from liquid to a vapor requires a considerable amount of energy or "latent heat" which is provided by outside air passing over the coils.

This vapor is then drawn into the compressor where the temperature of the vapor is boosted to well over 100 degrees Celsius. At this point we have used heat from the outside air to change the liquid refrigerant to a gas and added an amount of compression "work" to raise the temperature of the vapor. The vapor now enters the condenser heat exchanger coils where it begins to transfer heat to the air being drawn across the coils. As the vapor cools, it condenses back to a liquid and in so doing releases and transfers considerable latent heat to the air passing over the condenser unit coils. We have used the heat energy of outside air to change the phase of the refrigerant and then released this heat for heating, a typical heat pump operation.

At this stage we now have a very cold liquid refrigerant compressed to a high pressure. The refrigerant is next passed through an expansion valve which turns it back to a low pressure cold liquid ready to re-enter the evaporator to begin a new cycle.

The heat pump can also operate in a cooling mode where the cold refrigerant is moved through the indoor coils to cool the room air.

Efficiency

The 'Efficiency' of air source heat pumps is measured by the Coefficient of performance (COP). In simple terms, a COP of 3 means the heat pump produces 3 units of heat energy for every 1 unit of electricity it consumes. In mild weather, the COP of an air source heat pump can be up to 4. However, on a very cold winter day, it takes more work to move the same amount of heat indoors than on a mild day. The heat pump's performance is limited by the Carnot cycle and will approach 1.0 as the outdoor-to-indoor temperature difference increases at around −18 °C (0 °F) outdoor temperature for air source heat pumps. Within most normal temperature ranges of say -3°C to 10°C heat pump performance and thus the COP for many machines is fairly stable at 3-3.5. However, heat pump construction methods that enable use of carbon dioxide refrigerant extend the figure downward to -30 °C (-22 °F). A Geothermal heat pump will have less change in COP as the ground temperature from which they extract heat is more constant than outdoor air temperature.

Seasonally adjusted heating and cooling efficiencies are given by the heating seasonal performance factor (HSPF) and seasonal energy efficiency ratio (SEER) respectively.

The efficiency of a heat pump can be significantly affected by its original design. Many air source heat pumps began life as air conditioning units, designed for summer temperatures. In designing a heat pump as a heat pump from inception great COPs and life cycles can be attained. The principal changes are in the scale and type of compressor and evaporator to allow COP of greater than 2 even down to -20°C.

Advantages and disadvantages

Advantages

- Typically draws approximately 1/3 to 1/4 of the electricity of a standard resistance heater for the same amount of heating, reducing utility bills. This typical efficiency compares to 70-95% for a fossil fuel-powered boiler.

- Few moving parts, reducing maintenance requirements. However, it should be ensured that the outdoor heat exchanger and fan is kept free from leaves and debris. Moreover, it must be borne in mind that a heat pump will have significantly more moving parts than an equivalent electric resistance heater or fuel burning heater.
- As an electric system, no flammable or potentially asphyxiating fuel is used at the point of heating, reducing the potential danger to users, and removing the need to obtain gas or fuel supplies (except for electricity).
- May be used to heat air, or water.
- The same system may be used for air conditioning in summer, as well as a heating system in winter.
- Lower running costs, the compressor being the thing that uses most power - when in comparison with traditional electrical resistance heaters..
- When correctly specified an ASHP can offer a full central heating solution and domestic hot water up to 80°C. This can in theory be down to well below -10°C if the unit is large enough.

Disadvantages

The following disadvantages are associated with all air source heat pump designs:

- Air source heat pumps require electricity for operation. Electricity generation accounts for a significant amount of emissions pollutants and greenhouse gases.
- External space needs to be found for the outside condenser unit which can be somewhat noisy(comparable to an air conditioner unit) and possibly unsightly.
- The cost of installation is high (though less than a Ground Source heat pump because a ground source heat pump requires installation of a ground loop).
- The outdoor section on some units may "frost up" when outdoor temperatures are between 0°C and 5°C (between 32°F and 41°F respectively) and there is sufficient moisture in the air which causes restriction of air flow across the outdoor coil. These units employ a time/delay or demand defrost cycle where the system a) switches to "A/C" mode for up to 10 minutes or more to move heat from the home to the outdoor section to melt the ice and b) turns on the supplemental heater (resistance electric, gas, etc.) in the indoor section to temper the cold air being distributed. The defrost cycle reduces the efficiency of the heat pump significantly, although the newer (demand) systems are more intelligent and need to defrost less. As temperatures drop below freezing the tendency for frosting of the outdoor section decreases due to reduced humidity in the air. An air source heat pump switching out of defrost mode in normal operation emits a characteristic "whoosh" sound from the outdoor section..
- Air source heat pumps lose their efficiency as the external temperatures fall below 5 degrees Celsius (about 41 degrees Fahrenheit). In colder climates, the system needs to be installed with an auxiliary source of heat to supplement the heat pump in extremely cold temperatures or when it is simply too cold for the heat pump to work at all. When this happens, the heat pump will simply cease to function, and the system will operate solely on "Emergency Heat." The Auxiliary

Heat/Emergency Heat can also be used if the heat pump is malfunctioning and/or being repaired. In Northern climates, split-system heat pumps matched with gas or oil furnaces will work just fine in extremely cold temperatures. However, all-electric heat pump systems in colder Northern climates be considerably more expensive to operate when the system is operating solely on the electric heat. All-electric heat pump systems have an electric furnace or electric resistance heat, or strip heat, which typically consists of rows of electric coils that heat up. A fan blows over the heated coils and circulates warm air throughout the home. This serves as an adequate heating source, but as temperatures go down, the utility bill will go up. As mentioned above, the gas, oil, or electric heating system will also kick in when the heat pump is defrosting. In milder climates, and regions where heat is rarely needed, an all-electric system is usually all that is needed. Some homes in regions where heat is rarely needed (like South Florida) do not even have heat pumps, and instead have cool-only central air-conditioners with electric strip/resistance heat.

- Retrofit is difficult when used with conventional heating systems using radiators, hot water baseboard heaters, or radiant panels. The lower Heat Pump output temperatures would mean radiators would have to be increased in size or a low temperature underfloor heating system be installed instead.

The following disadvantages are associated with units charged with HFC refrigerants:

- Usually marketed as low energy or a sustainable technology, the HFCs have the potential to contribute to global warming. The effect the refrigerant could have is measured in global warming potential (GWP) and ozone depletion potential (ODP). However, recent government mandates have seen the phase-out of R-22 refrigerant and its replacement with more environmentally sound R410a refrigerant.
- The COP is reduced when heat pumps are used to reach over 60°C for heating domestic water or in conventional central heating systems using radiators to distribute heat (instead of an underfloor heating array).

Conclusions

Air source heat pumps can provide fairly low cost space heating. A high efficiency heat pump can provide four times the heat compared to an electric heater..

Air source heat pumps are sometimes used to provide hot water from a pressurized system up to temperatures of 55°C. To minimize the risk from Legionellosis it is advised that hot water is heated to above 60°C.

The overall lifetime costs for using air source heat pumps should be considered carefully as gas (where available) may be cheaper than electricity (although it has higher carbon emissions).

Air source heat pumps should last for over 20 years with low maintenance requirements. There are numerous heat pumps from the 1970s and 1980s that are still in service as of 2011, even in Northern states where winters seasons are extremely cold.

Chapter 17

Geothermal Heating

Geothermal heating is the direct use of geothermal power for heating applications. Humans have taken advantage of geothermal heat this way since the Paleolithic era. Approximately seventy countries made direct use of a total of 270 PJ of geothermal heating in 2004. As of 2007, 28 GW of geothermal heating capacity is installed around the world, satisfying 0.07% of global primary energy consumption. Thermal efficiency is high since no energy conversion is needed, but capacity factors tend to be low (around 20%) since the heat is mostly needed in the winter.

Geothermal energy originates from the heat retained within the Earth since the original formation of the planet, from radioactive decay of minerals, and from solar energy absorbed at the surface. Most high temperature geothermal heat is harvested in regions close to tectonic plate boundaries where volcanic activity rises close to the surface of the Earth. In these areas, ground and groundwater can be found with temperatures higher than the target temperature of the application. However, even cold ground contains heat, below 10' or 3 Meters, the ground is consistently 12.8°C (55°F), and it may be extracted with a geothermal heat pump. Due to recent advances in heat pump performance, this is now a rapidly growing market in the US.

Briefly and Simply Explained

Geothermal heating relies on an energy exchange between the air within the building being heated and the ground. Below ten feet the earth's temperature is fairly constant, generally between ~10ºC (~50ºF). During the summer when the ambient temperature of the building exceeds that of the ground heat pumps are used to pump heat from the building in to the transfer medium (typically water with small amounts of ethanol or glycol) and is subsequently pumped through narrow pipes into the ground so that the heat can be dissipated in the earth. When the ambient temperature falls below the ground temperature the process works in reverse. Heat pumps extract heat from the ground and use it to heat the building.

Applications

Top countries using the most geothermal heating in 2005

Country	Production PJ/yr	Capacity GW	Capacity Factor	Dominant applications
China	45.38	3.69	39%	bathing
Sweden	43.2	4.2	33%	heat pumps
USA	31.24	7.82	13%	heat pumps
Turkey	24.84	1.5	53%	district heating
Iceland	24.5	1.84	42%	district heating
Japan	10.3	0.82	40%	bathing (onsens)
Hungary	7.94	0.69	36%	spas/greenhouses
Italy	7.55	0.61	39%	spas/space heating
New Zealand	7.09	0.31	73%	industrial uses
63 others	71	6.8		
Total	**273**	**28**	**31%**	**space heating**

There are a wide variety of applications for cheap geothermal heat. In 2004 more than half of direct geothermal heat was used for space heating, and a third was used for spas. The remainder was used for a variety of industrial processes, desalination, domestic hot water, and agricultural applications. The cities of Reykjavík and Akureyri pipe hot water from geothermal plants under roads and pavements to melt snow. Geothermal desalination has been demonstrated.

Geothermal systems tend to benefit from economies of scale, so space heating power is often distributed to multiple buildings, sometimes whole communities. This technique, long practiced throughout the world in locations such as Reykjavik, Iceland, Boise, Idaho, and Klamath Falls, Oregon is known as district heating.

Extraction

Some parts of the world, including substantial portions of the western USA, are underlain by relatively shallow geothermal resources. Similar conditions exist in Iceland, parts of Japan, and other geothermal hot spots around the world. In these areas, water or steam may be captured from natural hot springs and piped directly into radiators or heat exchangers. Alternatively, the heat may come from waste heat supplied by co-generation from a geothermal electrical plant or from deep wells into hot aquifers. Direct geothermal heating is far more efficient than geothermal electricity generation and has less demanding temperature requirements, so it is viable over a large geographical range. If the shallow ground is hot but dry, air or water may be circulated through earth tubes or downhole heat exchangers which act as heat exchangers with the ground.

In areas where the shallow ground is too cold to provide comfort directly, it is still warmer than the winter air. The thermal inertia of the shallow ground retains solar energy accumulated in the summertime, and seasonal variations in ground temperature disappear completely below 10m of depth. That heat can be extracted with a geothermal heat pump more efficiently than it can be generated by conventional furnaces. Geothermal heat pumps are economically viable essentially anywhere in the world. One geothermal district heating system at Drake Landing enhances storage of solar energy in the ground to such an extent that no heat pumps are needed.

Geothermal heat pumps

Even in regions without large high temperature geothermal resources, a geothermal heat pump can still provide space heating and air conditioning. Like a refrigerator or air conditioner, these systems use a heat pump to force the transfer of heat from the ground to the application. In theory, heat can be extracted from any source, no matter how cold, but a warmer source allows higher efficiency. A ground-source heat pump uses the shallow ground or ground water (typically starting at 10–12 °C, 50–54 °F) as a source of heat, thus taking advantage of its seasonally moderate temperatures. In contrast, an air-source heat pump draws heat from the colder outside air and thus requires more energy.

Closed loop geothermal heat pumps circulate a carrier fluid (usually a water/antifreeze mix) through pipes buried in the ground. As the fluid circulates underground it absorbs heat from the ground and, on its return, the now warmer fluid passes through the heat pump which uses electricity to extract the heat from the fluid. The re-chilled fluid is sent back through the ground thus continuing the cycle. The heat extracted and that generated by the heat pump appliance as a byproduct is used to heat the house. The addition of the ground heating loop in the energy equation means that more heat is generated than if electricity alone had been used directly for heating. Switching the direction of heat flow, the same system can be used to circulate the cooled water through the house for cooling in the summer months. The heat is exhausted to the same relatively cool soil (or groundwater) rather than delivering it to the hot outside air as an air conditioner does. As a result, the heat is pumped across a smaller temperature difference and this leads to higher efficiency and lower energy use.

This technology makes geothermal heating economically viable in any geographical location. In 2004, an estimated million geothermal heat pumps with a total capacity of 15 GW extracted 88 PJ of geothermal energy for space heating. Global geothermal heat pump capacity is growing by 10% annually.

History

The oldest known pool fed by a hot spring, built in the Qin dynasty in the 3rd century BC.

Hot springs have been used for bathing at least since Paleolithic times. The oldest known spa is a stone pool on China's Lisan mountain built in the Qin dynasty in the 3rd century BC, at the same site where the Huaqing Chi palace was later built. In the first century AD, Romans conquered Aquae Sulis and used the hot springs there to feed public baths and underfloor heating. The admission fees for these baths probably represents the first commercial use of geothermal power. The world's oldest geothermal district heating system in Chaudes-Aigues, France, has been operating since the 14th century. The earliest industrial exploitation began in 1827 with the use of geyser steam to extract boric acid from volcanic mud in Larderello, Italy.

In 1892, America's first district heating system in Boise, Idaho was powered directly by geothermal energy, and was soon copied in Klamath Falls, Oregon in 1900. A deep geothermal well was used to heat greenhouses in Boise in 1926, and geysers were used to heat greenhouses in Iceland and Tuscany at about the same time. Charlie Lieb developed the first downhole heat exchanger in 1930 to heat his house. Steam and hot water from the geysers began to be used to heat homes in Iceland in 1943.

By this time, Lord Kelvin had already invented the heat pump in 1852, and Heinrich Zoelly had patented the idea of using it to draw heat from the ground in 1912. But it was not until the late 1940s that the geothermal heat pump was successfully implemented. The earliest one was probably Robert C. Webber's home-made 2.2 kW direct-exchange system, but sources disagree as to the exact timeline of his invention. J. Donald Kroeker designed the first commercial geothermal heat pump to heat the Commonwealth Building (Portland, Oregon) and demonstrated it in 1946. Professor Carl Nielsen of Ohio State University built the first residential open loop version in his home in 1948. The technology became popular in Sweden as a result of the 1973 oil crisis, and has been growing slowly in worldwide acceptance since then. The 1979 development of polybutylene pipe greatly augmented the heat pump’s economic viability. As of 2004, there are over a million geothermal heat pumps installed worldwide providing 12 GW of thermal capacity. Each year, about 80,000 units are installed in the USA and 27,000 in Sweden.

Economics

Geothermal drill machine.

Geothermal energy is a type of renewable energy that encourages conservation of natural resources. According to the U.S. Environmental Protection Agency, geo-exchange systems save homeowners 30-70 percent in heating costs, and 20-50 percent in cooling costs, compared to conventional systems. Geo-exchange systems also save money because they require much less maintenance. In addition to being highly reliable they are built to last for decades.

Some utilities, such as Kansas City Power and Light, offer special, lower winter rates for geothermal customers, offering even more savings.

Subsidence

In geothermal heating projects the underground is penetrated by trenches or drillholes. Large projects may cause problems if the geology of the area is poorly understood as with all underground work. In connection with a geothermal heating project for the historical city hall of Staufen im Breisgau, Germany, subsidence of the ground up to eight millimeters has occurred while other areas have been uplifted by a few millimeters. A relation to the geothermal wells is suspected. The subsidence has caused considerable damage to buildings in the city center.

Chapter 18

Vapor-Compression Refrigeration

Vapor-compression refrigeration is one of the many refrigeration cycles available for use. It has been and is the most widely used method for air-conditioning of large public buildings, offices, private residences, hotels, hospitals, theaters, restaurants and automobiles. It is also used in domestic and commercial refrigerators, large-scale warehouses for chilled or frozen storage of foods and meats, refrigerated trucks and railroad cars, and a host of other commercial and industrial services. Oil refineries, petrochemical and chemical processing plants, and natural gas processing plants are among the many types of industrial plants that often utilize large vapor-compression refrigeration systems.

Refrigeration may be defined as lowering the temperature of an enclosed space by removing heat from that space and transferring it elsewhere. A device that performs this function may also be called a *heat pump*.

Description of the vapor-compression refrigeration system

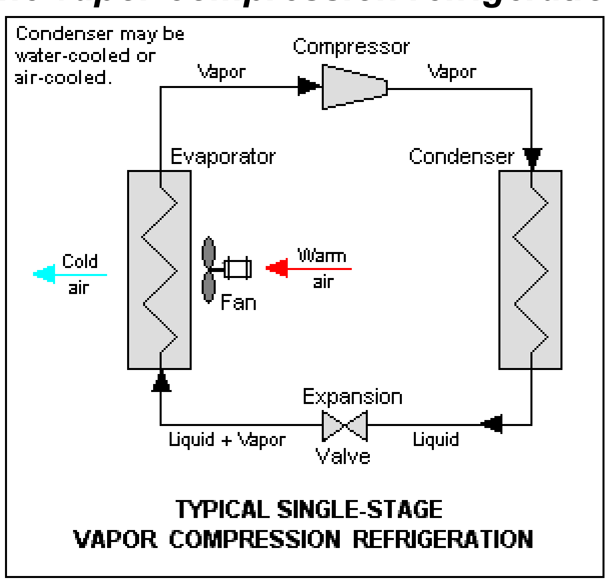


Figure 1: Vapor compression refrigeration

The vapor-compression uses a circulating liquid refrigerant as the medium which absorbs and removes heat from the space to be cooled and subsequently rejects that heat elsewhere. Figure 1 depicts a typical, single-stage vapor-compression system. All such systems have four components: a compressor, a condenser, a Thermal expansion valve (also called a throttle valve), and an evaporator. Circulating refrigerant enters the compressor in the thermodynamic state known as a saturated vapor and is compressed to a higher pressure, resulting in a higher temperature as well. The hot, compressed vapor is then in the thermodynamic state known as a superheated vapor and it is at a temperature and pressure at which it can be condensed with typically available cooling water or cooling air. That hot vapor is routed through a condenser where it is cooled and condensed into a liquid by flowing through a coil or tubes with cool water or cool air flowing across the coil or tubes. This is where the circulating refrigerant rejects heat from the system and the rejected heat is carried away by either the water or the air (whichever may be the case).

The condensed liquid refrigerant, in the thermodynamic state known as a saturated liquid, is next routed through an expansion valve where it undergoes an abrupt reduction in pressure. That pressure reduction results in the adiabatic flash evaporation of a part of the liquid refrigerant. The auto-refrigeration effect of the adiabatic flash evaporation lowers the temperature of the liquid and vapor refrigerant mixture to where it is colder than the temperature of the enclosed space to be refrigerated.

The cold mixture is then routed through the coil or tubes in the evaporator. A fan circulates the warm air in the enclosed space across the coil or tubes carrying the cold refrigerant liquid and vapor mixture. That warm air evaporates the liquid part of the cold refrigerant mixture. At the same time, the circulating air is cooled and thus lowers the temperature of the enclosed space to the desired temperature. The evaporator is where the circulating refrigerant absorbs and removes heat which is subsequently rejected in the condenser and transferred elsewhere by the water or air used in the condenser.

To complete the refrigeration cycle, the refrigerant vapor from the evaporator is again a saturated vapor and is routed back into the compressor.

Refrigerants

"Freon" is a trade name for a family of haloalkane refrigerants manufactured by DuPont and other companies. These refrigerants were commonly used due to their superior stability and safety properties: they were not flammable nor obviously toxic as were the fluids they replaced, such as sulfur dioxide. Unfortunately, these chlorine-bearing refrigerants reach the upper atmosphere when they escape. In the stratosphere, CFCs break up due to UV-radiation, releasing their chlorine atoms. These chlorine atoms act as catalysts in the breakdown of ozone, thus causing severe damage to the ozone layer that shields the Earth's surface from the Sun's strong UV radiation. The chlorine will remain active as a catalyst until and unless it binds with another particle, forming a stable molecule. CFC refrigerants in common but receding usage include R-11 and R-12. Newer refrigerants that have reduced ozone depletion effect include HCFCs (R-22, used in most homes today) and HFCs (R-134a, used in most cars) have replaced most CFC use. HCFCs in turn are being phased out under the Montreal Protocol and replaced by hydrofluorocarbons (HFCs), such as R-410A, which lack chlorine. However, CFCs, HCFCs, and HFCs all have large global warming potential.

Newer refrigerants are currently the subject of research, such as supercritical carbon dioxide, known as R-744. These have similar efficiencies compared to existing CFC and HFC based compounds, and have many orders of magnitude lower **global warming potential**.

Thermodynamic analysis of the system

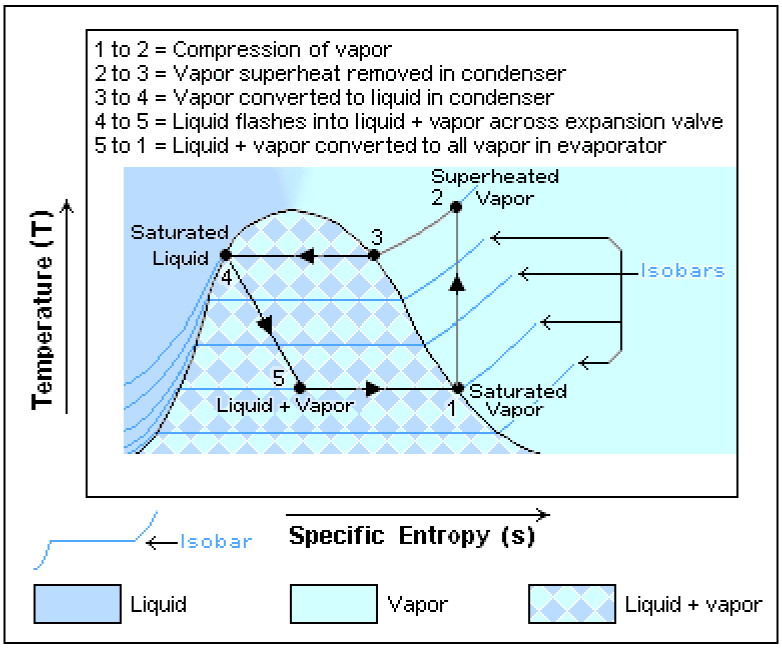


Figure 2: Temperature–Entropy diagram

Types of gas compressors

The most common compressors used in chillers are reciprocating, rotary screw, centrifugal, and scroll compressors. Each application prefers one or another due to size, noise, efficiency and pressure issues. Compressors are often described as being either open, hermetic, or semi-hermetic, to describe how the compressor and/or motor is situated in relation to the refrigerant being compressed. Variations of motor/compressor types can lead to the following configuations:

- Hermetic motor, hermetic compressor
- Hermetic motor, semi-hermetic compressor
- Open motor (belt driven or close coupled), hermetic compressor
- Open motor (belt driven or close coupled), semi-hermetic compressor

Typically in hermetic, and most semi-hermetic compressors (sometimes known as accessible hermetic compressors), the compressor and motor driving the compressor are integrated, and operate within the refrigerant system. The motor is hermetic and is designed to operate, and be cooled by, the refrigerant being compressed. The obvious disadvantage of hermetic motor compressors is that the motor drive cannot be maintained in situ, and the entire compressor must be removed if a motor fails. A further disadvantage is that burnt out windings can contaminate whole refrigeration systems requiring the system to be entirely pumped down and the refrigerant replaced.

An open compressor has a motor drive which is outside of the refrigeration system, and provides drive to the compressor by means of an input shaft with suitable gland seals. Open compressor motors are typically air cooled and can be fairly easily exchanged or repaired without degassing of the refrigeration system. The disadvantage of this type of compressor is a failure of the shaft seals, leading to loss of refrigerant.

Open motor compressors are generally easier to cool (using ambient air) and therefore tend to be simpler in design and more reliable, especially in high pressure applications where compressed gas temperatures can be very high. However the use of liquid injection for additional cooling can generally overcome this issue in most hermetic motor compressors.

Reciprocating compressors

Reciprocating compressors are piston-style, positive displacement compressors.

Rotary screw compressors

Figure 3: Screw Compressors

Rotary screw compressors are also positive displacement compressors. Two meshing screw-rotors rotate in opposite directions, trapping refrigerant vapor, and reducing the volume of the refrigerant along the rotors to the discharge point.

Centrifugal compressors

Centrifugal compressors are dynamic compressors. These compressors raise the pressure of the refrigerant by imparting velocity or dynamic energy, using a rotating impeller, and converting it to pressure energy.

Scroll compressors

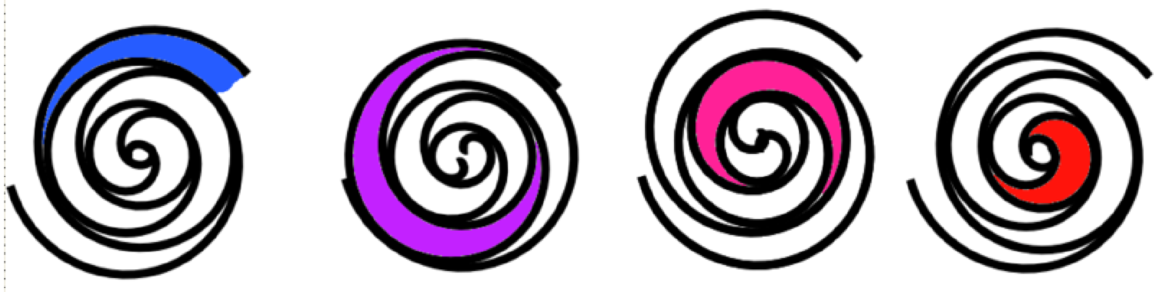


Figure 4: Operating principle of a Scroll Compressor

Scroll compressors are also positive displacement compressors. The refrigerant is compressed when one spiral orbits around a second stationary spiral, creating smaller and smaller pockets and higher pressures. By the time the refrigerant is discharged, it is fully pressurized.

Compressor Lubrication

In order to lubricate the moving parts of the compressor, an oil is added to the refrigerant during installation or commissioning. The type of oil may be mineral or synthetic to suit the compressor type, and also chosen so as not to react with the refrigerant type and other components in the system. In small refrigeration systems the oil is allowed to circulate throughout the whole circuit, but care must be taken to design the pipework and components such that oil can drain back under gravity to the compressor. In larger more distributed systems, especially in retail refrigeration, then oil is normally captured at an oil separator immediately after the compressor, and is in turn re-delivered, by an oil level management system, back to the compressor(s). Oil separators are not 100% efficient so system pipework must still be designed so that oil can drain back by gravity to the oil separator or compressor.

Some newer compressor technologies use magnetic bearings and require no lubrication, for example the Danfoss Turbocor range of centrifugal compressors. Avoiding the need for oil lubrication and the design requirements and ancillaries associated with it, simplifies the design of the refrigerant system and reduces maintenance requirements.

Control

In simple commercial refrigeration systems the compressor is normally controlled by a simple pressure switch, with the expansion performed by a capillary tube or simple thermostatic expansion valve. In more complex systems, including multiple compressor installations, the use of electronic controls is typical, with adjustable set points to control the pressure at which compressors cut in and cut out, and temperature control by the use of electronic expansion valves.

In addition to the operational controls, separate high pressure and low pressure switches are normally utilised to provide secondary protection to the compressors and other components of the system from operating outside of safe parameters.

In more advanced electronic control systems the use of floating head pressure, and proactive suction pressure, control routines allow the compressor operation to be adjusted to accurately meet differing cooling demands whilst reducing energy consumption.

Other features and facts of interest

The schematic diagram of a single-stage refrigeration system shown in Figure 1 does not include other equipment items that would be provided in a large commercial or industrial vapor compression refrigeration system, such as:

- A horizontal or vertical pressure vessel, equipped internally with a demister, between the evaporator and the compressor inlet to capture and remove any residual, entrained liquid in the refrigerant vapor because liquid may damage the compressor. Such vapor-liquid separators are most often referred to as "suction line accumulators". (In other industrial processes, they are called "compressor suction drums" or "knockout drums".)

- Large commercial or industrial refrigeration systems may have multiple expansion valves and multiple evaporators in order to refrigerate multiple enclosed spaces or rooms. In such systems, the condensed liquid refrigerant may be routed into a pressure vessel, called a receiver, from which liquid refrigerant is withdrawn and routed through multiple pipelines to the multiple expansion valves and evaporators.

- Filter Dryers, installed before the compressors to catch any moisture or contaminents in the system and thus protect the compressors from internal damage

- Some refrigeration units may have multiple stages which requires the use of multiple compressors in various arrangements.

The cooling capacity of refrigeration systems is often defined in units called "tons of refrigeration". The most common definition of that unit is: 1 ton of refrigeration is the rate of heat removal required to freeze a short ton (i.e., 2000 pounds) of water at 32 °F in 24 hours. Based on the heat of fusion for water being 144 Btu per pound, 1 ton of refrigeration = 12,000 Btu/h = 12,660 kJ/h = 3.517 kW. Most residential air conditioning units range in capacity from about 1 to 5 tons of refrigeration.

A much less common definition is: 1 tonne of refrigeration is the rate of heat removal required to freeze a metric ton (i.e., 1000 kg) of water at 0 °C in 24 hours. Based on the heat of fusion being 334.9 kJ/kg, 1 tonne of refrigeration = 13,954 kJ/h = 3.876 kW. As can be seen, 1 tonne of refrigeration is 10 percent larger than 1 ton of refrigeration.

Applications

Refrigeration application	Short descriptions	Typical refrigerants used
Domestic refrigeration	Appliances used for keeping food in dwelling units	R-600a, R-134a
Commercial refrigeration	Holding and displaying frozen and fresh food in retail outlets	R-134a, R-404A, R-507
Food processing and cold storage	Equipment to preserve, process and store food from its source to the wholesale distribution point	R-134a, R-407C, R-410A, R-507
Industrial refrigeration	Large equipment, typically 25 kW to 30 MW, used for chemical processing, cold storage, food processing, building and district heating and cooling	R-134a, R-404A, R-407C, R-507, R-717
Transport refrigeration	Equipment to preserve and store goods, primarily foodstuffs, during transport by road, rail, air and sea	R-134a, R-407C, R-410A
Electronic cooling	Low-temperature cooling of CMOS circuitry and other components in large computers and servers	R-134a, R-404A, R-507
Medical refrigeration		R-134a, R-404A, R-507
Cryogenic refrigeration		Ethylene, Helium

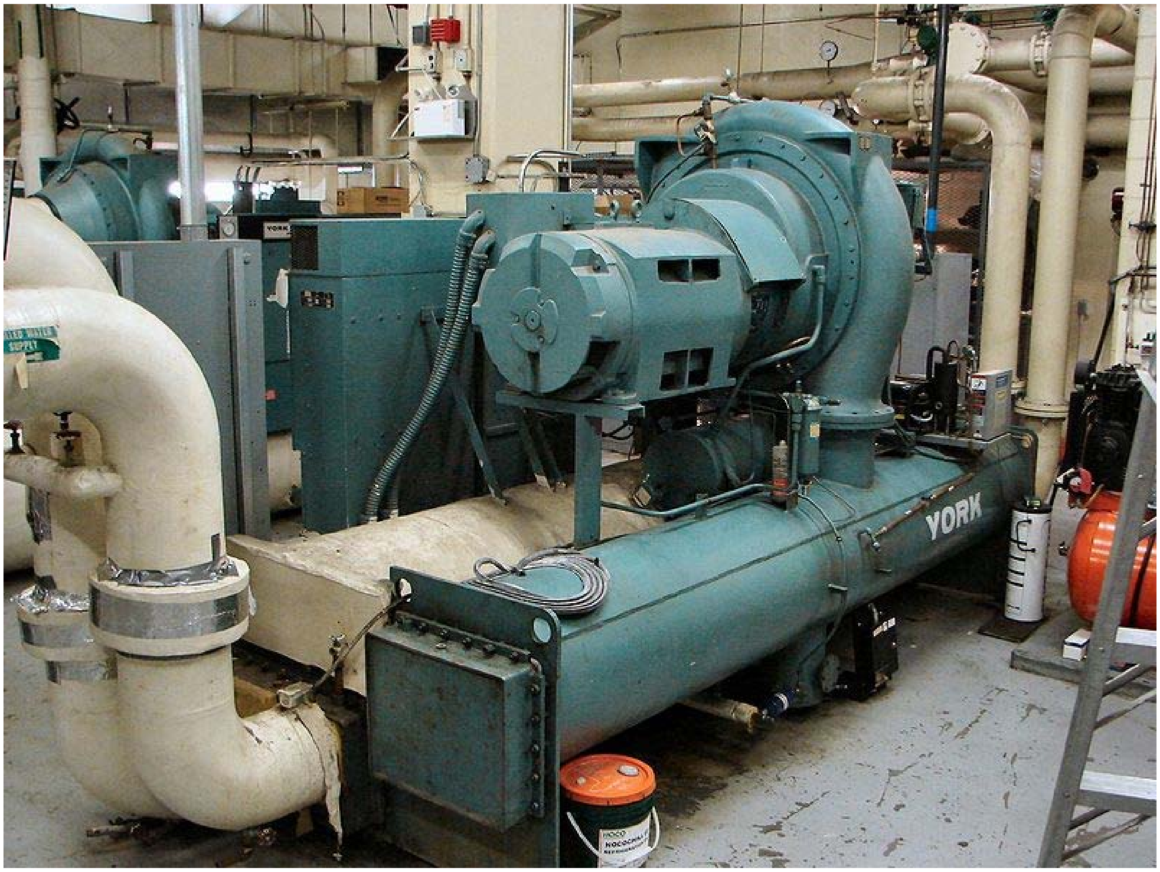


Figure 5: Commercial water cooled liquid chiller installation for building air conditioning

Economic analysis

Advantages

- Very mature technology.
- Relatively inexpensive.
- Can be driven directly using mechanical energy (water, car/truck motor) or with electrical energy.
- Efficient up to 60% of Carnot's theoretical limit (as evaluated in ASHRAE testing conditions: evaporation temperature of -23.3 °C, condensing temperature of 54.4°C, and ambient temperature of 32°C) based on some of the best compressors produced by Danfoss, Matsushita, Copeland, Embraco, Bristol and Tecumseh compressor manufacturers. However, many refrigeration systems use compressors having lower efficiencies of between 40-55%, since the 60% efficient ones cost almost twice as much as the lower efficiency ones.

Disadvantages

Many systems still use HCFC refrigerants, which contribute to depletion of the Earth's ozone layer. In countries adhering to the Montreal Protocol, HCFCs are due to be phased out and are largely being replaced by ozone-friendly HFCs. However, systems using HFC refrigerants tend to be slightly less efficient than systems using HCFCs. HFCs also have an extremely large global warming potential (GWP) because they remain in the atmosphere for many years and trap heat more effectively than carbon dioxide.

With disruption of the status quo already a certainty, alternative non-haloalkane refrigerants are gaining popularity. In particular, once-abandoned refrigerants such as hydrocarbons (HCs, such as butane) and CO_2 are coming back into broader use. For example, Coca-Cola's vending machines at the 2006 FIFA World Cup in Germany used refrigeration utilizing CO_2

History

Jacob Perkins received a patent for the first refrigeration machine in 1836 using the vapor-compression cycle, based on an 1834 machine he built. Perkins had improved upon the design of Oliver Evans, who conceived of the idea in 1805 but never built a refrigerator. John Gorrie received US patent 8,080 in 1851 for work he began in 1845 on ice machines.

Alexander Twining received a patent in 1853 for an ice maker, US Patent 10221. James Harrison of Australia received a patent in 1855 for an ice maker. Both inventors used ether as the working fluid in their vapor-compression refrigeration cycle, but Harrison was the first to develop a practical refrigeration system which could be used in the brewing and meat-packing industries in Geelong, Victoria.