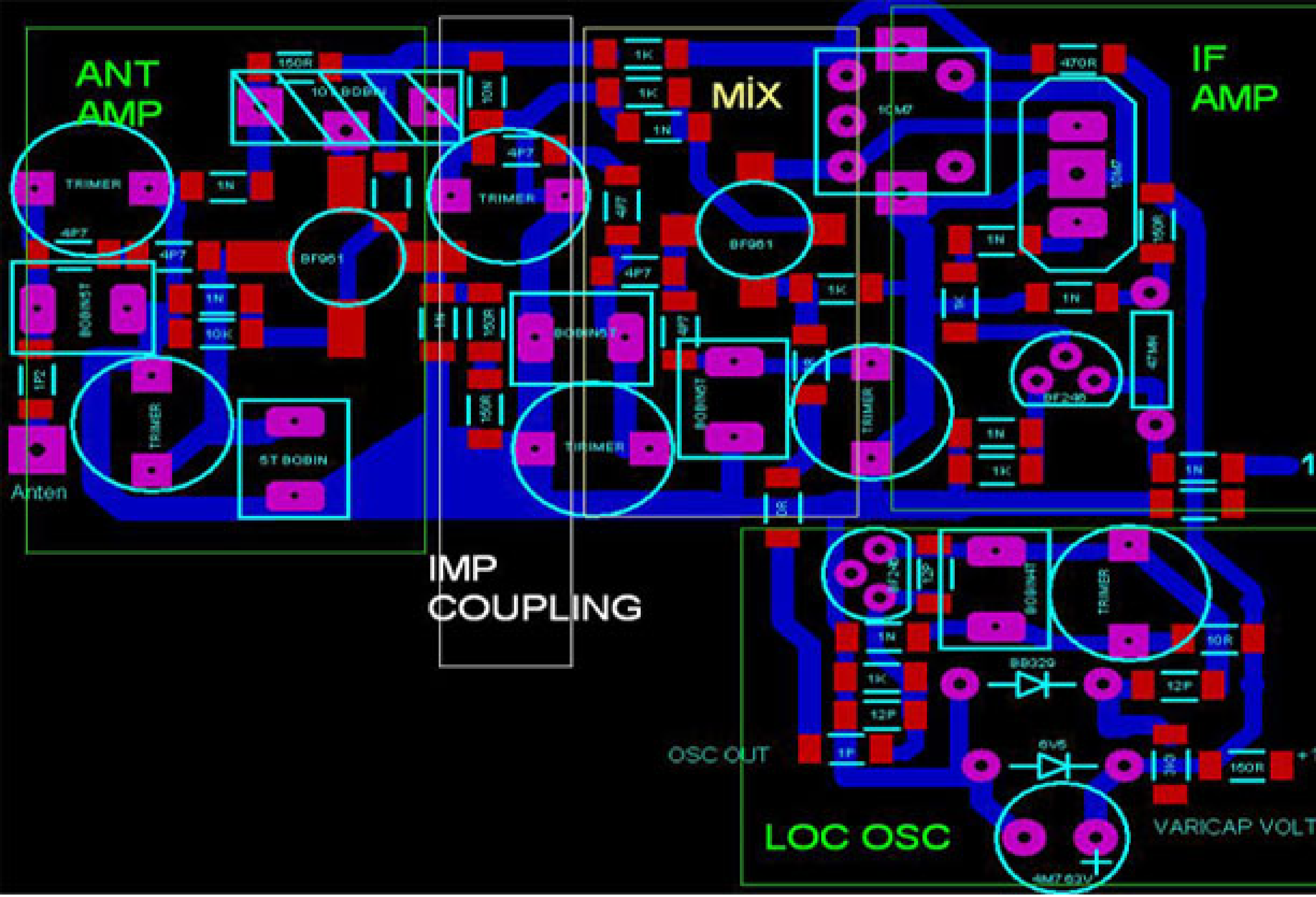


Image Impedance Filters

(Linear Analog Electronic Filters)

Collin Markham

First Edition, 2012

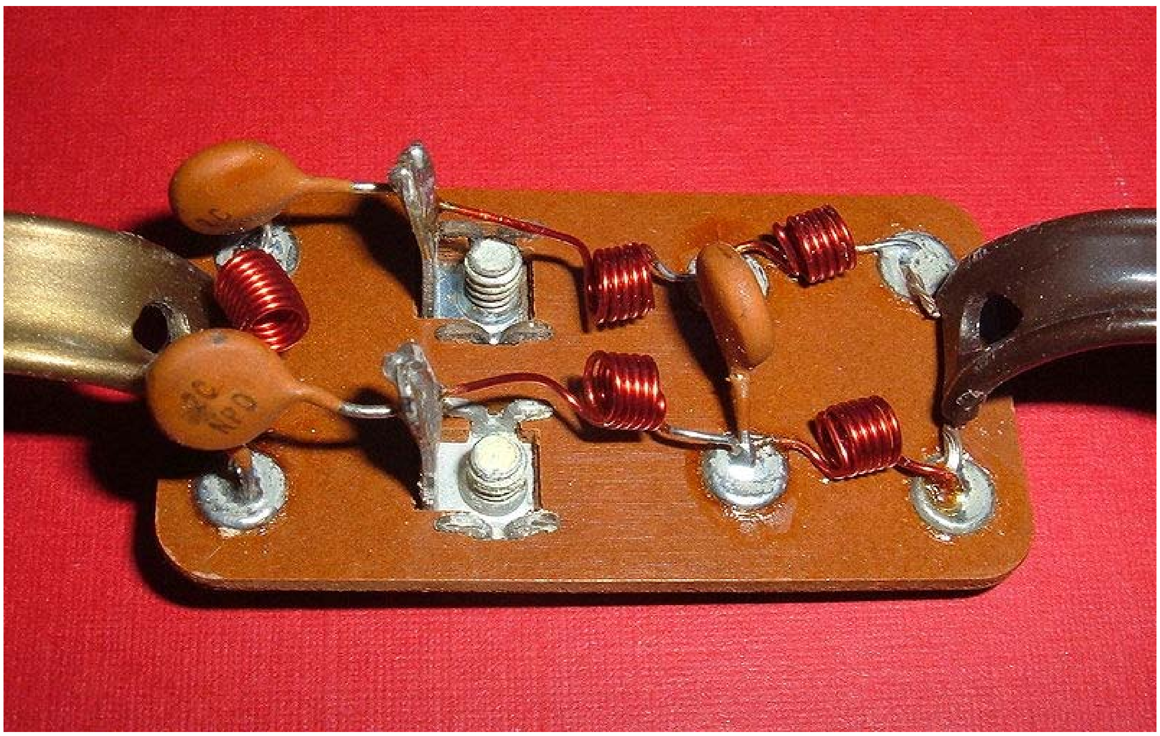
ISBN 978-81-323-2867-4

Published by:
Orange Apple
4735/22 Prakashdeep Bldg,
Ansari Road, Darya Ganj,
Delhi - 110002
Email: info@wtbooks.com

Table of Contents

Chapter- 1

Electronic Filter



Television signal splitter consisting of a high-pass filter (left) and a low-pass filter (right). The antenna is connected to the screw terminals to the left of center.

Electronic filters are electronic circuits which perform signal processing functions, specifically to remove unwanted frequency components from the signal, to enhance wanted ones, or both. Electronic filters can be:

- passive or active
- analog or digital
- high-pass, low-pass, bandpass, band-reject (band reject; notch), or all-pass.
- discrete-time (sampled) or continuous-time
- linear or non-linear

- infinite impulse response (IIR type) or finite impulse response (FIR type)

The most common types of electronic filters are linear filters, regardless of other aspects of their design.

History

The oldest forms of electronic filters are passive analog linear filters, constructed using only resistors and capacitors or resistors and inductors. These are known as RC and RL single-pole filters respectively. More complex multipole LC filters have also existed for many years, and their operation is well understood.

Hybrid filters are also possible, typically involving a combination of analog amplifiers with mechanical resonators or delay lines. Other devices such as CCD delay lines have also been used as discrete-time filters. With the availability of digital signal processing, active digital filters have become common.

Classification by technology

Passive filters

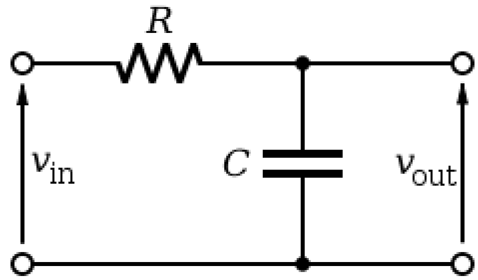
Passive implementations of linear filters are based on combinations of resistors (R), inductors (L) and capacitors (C). These types are collectively known as *passive filters*, because they do not depend upon an external power supply and/or they do not contain active components such as transistors.

Inductors block high-frequency signals and conduct low-frequency signals, while capacitors do the reverse. A filter in which the signal passes through an inductor, or in which a capacitor provides a path to ground, presents less attenuation to low-frequency signals than high-frequency signals and is a *low-pass filter*. If the signal passes through a capacitor, or has a path to ground through an inductor, then the filter presents less attenuation to high-frequency signals than low-frequency signals and is a *high-pass filter*. Resistors on their own have no frequency-selective properties, but are added to inductors and capacitors to determine the *time-constants* of the circuit, and therefore the frequencies to which it responds.

The inductors and capacitors are the reactive elements of the filter. The number of elements determines the order of the filter. In this context, an LC tuned circuit being used in a band-pass or band-stop filter is considered a single element even though it consists of two components.

At high frequencies (above about 100 megahertz), sometimes the inductors consist of single loops or strips of sheet metal, and the capacitors consist of adjacent strips of metal. These inductive or capacitive pieces of metal are called stubs.

Single element types



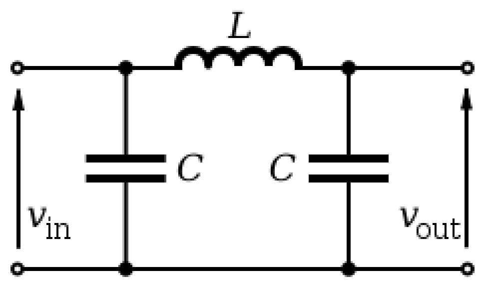
A low-pass electronic filter realised by an RC circuit

The simplest passive filters, RC and RL filters, include only one reactive element, except hybrid LC filter which is characterized by inductance and capacitance integrated in one element.

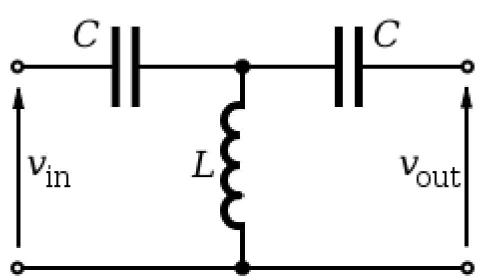
L filter

An L filter consists of two reactive elements, one in series and one in parallel.

T and π filters



Low-pass π filter



High-pass T filter

Three-element filters can have a 'T' or 'π' topology and in either geometries, a low-pass, high-pass, band-pass, or band-stop characteristic is possible. The components can be chosen symmetric or not, depending on the required frequency characteristics. The high-pass T filter in the illustration, has a very low impedance at high frequencies, and a very high impedance at low frequencies. That means that it can be inserted in a transmission line, resulting in the high frequencies being passed and low frequencies being reflected. Likewise, for the illustrated low-pass π filter, the circuit can be connected to a transmission line, transmitting low frequencies and reflecting high frequencies. Using m-derived filter sections with correct termination impedances, the input impedance can be reasonably constant in the pass band.

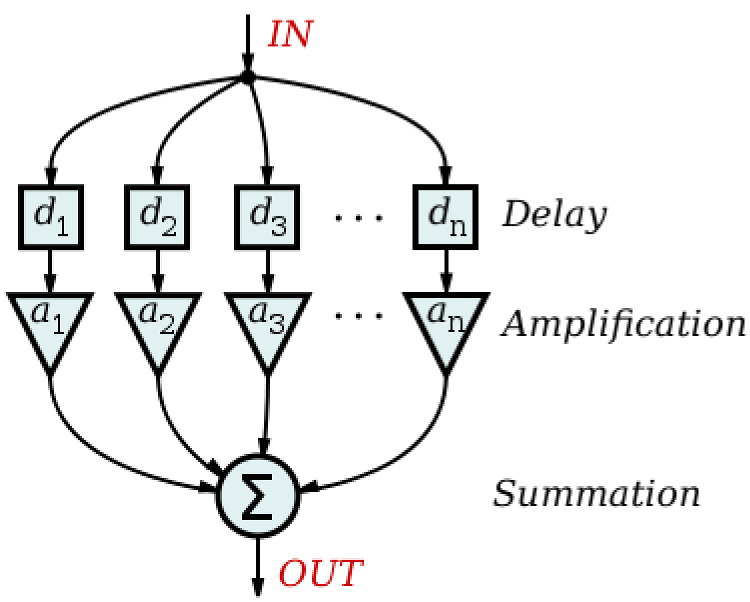
Multiple element types

Multiple element filters are usually constructed as a ladder network. These can be seen as a continuation of the L,T and π designs of filters. More elements are needed when it is desired to improve some parameter of the filter such as stop-band rejection or slope of transition from pass-band to stop-band.

Active filters

Active filters are implemented using a combination of passive and active (amplifying) components, and require an outside power source. Operational amplifiers are frequently used in active filter designs. These can have high Q factor, and can achieve resonance without the use of inductors. However, their upper frequency limit is limited by the bandwidth of the amplifiers used.

Digital filters



A general finite impulse response filter with *n* stages, each with an independent delay, d_i and amplification gain, a_i.

Digital signal processing allows the inexpensive construction of a wide variety of filters. The signal is sampled and an analog-to-digital converter turns the signal into a stream of numbers. A computer program running on a CPU or a specialized DSP (or less often running on a hardware implementation of the algorithm) calculates an output number stream. This output can be converted to a signal by passing it through a digital-to-analog converter. There are problems with noise introduced by the conversions, but these can be controlled and limited for many useful filters. Due to the sampling involved, the input signal must be of limited frequency content or aliasing will occur.

Other filter technologies

Quartz filters and piezoelectrics

In the late 1930s, engineers realized that small mechanical systems made of rigid materials such as quartz would acoustically resonate at radio frequencies, i.e. from audible frequencies (sound) up to several hundred megahertz. Some early resonators were made of steel, but quartz quickly became favored. The biggest advantage of quartz is that it is piezoelectric. This means that quartz resonators can directly convert their own mechanical motion into electrical signals. Quartz also has a very low coefficient of thermal expansion which means that quartz resonators can produce stable frequencies over a wide temperature range. Quartz crystal filters have much higher quality factors than LCR filters. When higher stabilities are required, the crystals and their driving circuits may be mounted in a "crystal oven" to control the temperature. For very narrow band filters, sometimes several crystals are operated in series.

Engineers realized that a large number of crystals could be collapsed into a single component, by mounting comb-shaped evaporations of metal on a quartz crystal. In this scheme, a "tapped delay line" reinforces the desired frequencies as the sound waves flow across the surface of the quartz crystal. The tapped delay line has become a general scheme of making high-*Q* filters in many different ways.

SAW filters

SAW (surface acoustic wave) filters are electromechanical devices commonly used in radio frequency applications. Electrical signals are converted to a mechanical wave in a device constructed of a piezoelectric crystal or ceramic; this wave is delayed as it propagates across the device, before being converted back to an electrical signal by further electrodes. The delayed outputs are recombined to produce a direct analog implementation of a finite impulse response filter. This hybrid filtering technique is also found in an analog sampled filter. SAW filters are limited to frequencies up to 3 GHz.

BAW filters

BAW (Bulk Acoustic Wave) filters are electromechanical devices. BAW filters can implement ladder or lattice filters. BAW filters typically operate at frequencies from around 2 to around 16 GHz, and may be smaller or thinner than equivalent SAW filters. Two main variants of BAW filters are making their way into devices, Thin film bulk acoustic resonator or FBAR and Solid Mounted Bulk Acoustic Resonators.

Garnet filters

Another method of filtering, at microwave frequencies from 800 MHz to about 5 GHz, is to use a synthetic single crystal yttrium iron garnet sphere made of a chemical combination of yttrium and iron (**YIGF**, or **yttrium iron garnet filter**). The garnet sits on a strip of metal driven by a transistor, and a small loop antenna touches the top of the sphere. An electromagnet changes the frequency that the garnet will pass. The advantage of this method is that the garnet can be tuned over a very wide frequency by varying the strength of the magnetic field.

Atomic filters

For even higher frequencies and greater precision, the vibrations of atoms must be used. Atomic clocks use caesium masers as ultra-high *Q* filters to stabilize their primary oscillators. Another method, used at high, fixed frequencies with very weak radio signals, is to use a ruby maser tapped delay line.

The transfer function

The transfer function $H(s)$ of a filter is the ratio of the output signal $Y(s)$ to that of the input signal $X(s)$ as a function of the complex frequency s:

$$H(s) = \frac{Y(s)}{X(s)}$$

with $s = \sigma + j\omega$.

The transfer function of all linear time-invariant filters, when constructed of discrete components, will be the ratio of two polynomials in s, i.e. a rational function of s. The order of the transfer function will be the highest power of s encountered in either the numerator or the denominator.

Classification by topology

Electronic filters can be classified by the technology used to implement them. Filters using passive filter and active filter technology can be further classified by the particular electronic filter topology used to implement them.

Any given filter transfer function may be implemented in any electronic filter topology.

Some common circuit topologies are:

- Cauer topology - Passive
- Sallen Key topology - Active

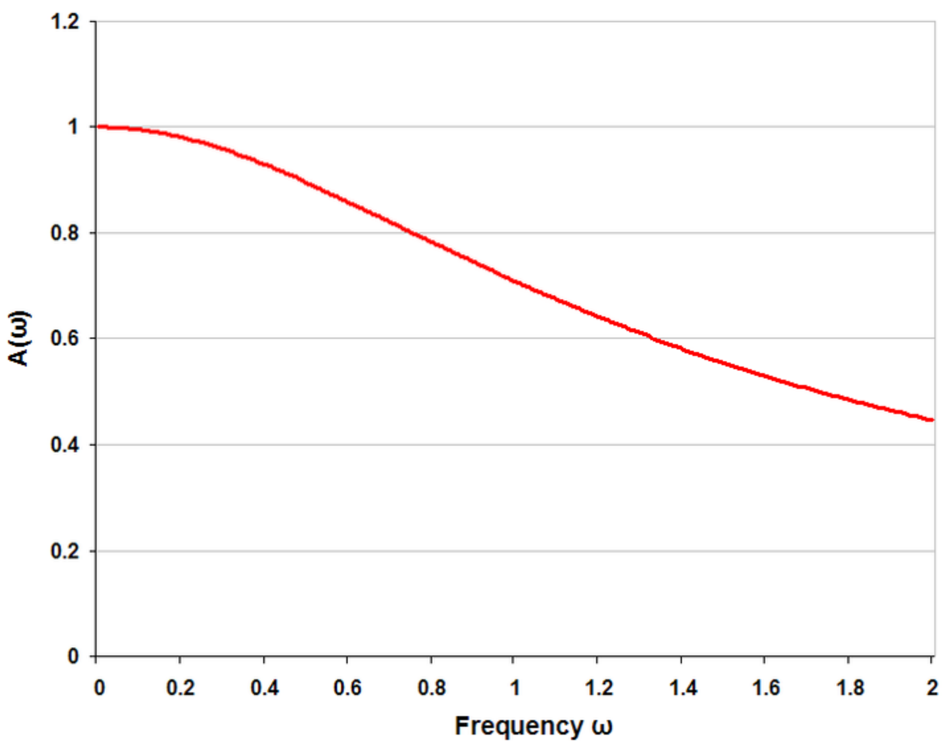
- Multiple Feedback topology - Active
- State Variable Topology - Active
- Biquadratic topology biquad filter - Active

Classification by design methodology

Historically, linear analog filter design has evolved through three major approaches. The oldest designs are simple circuits where the main design criterion was the Q factor of the circuit. This reflected the radio receiver application of filtering as Q was a measure of the frequency selectivity of a tuning circuit. From the 1920s filters began to be designed from the image point of view, mostly being driven by the requirements of telecommunications. After World War II the dominant methodology was network synthesis. The higher mathematics used originally required extensive tables of polynomial coefficient values to be published but modern computer resources have made that unnecessary.

Direct circuit analysis

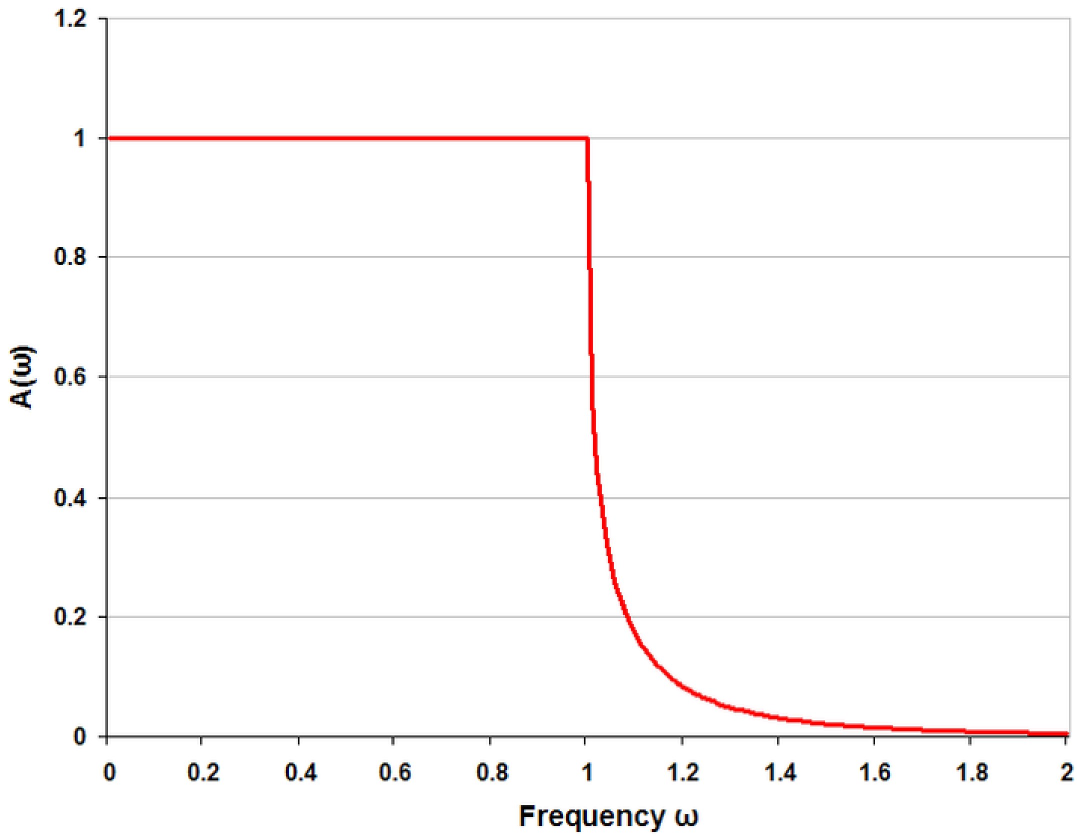
Low order filters can be designed by directly applying basic circuit laws such as Kirchoff's laws to obtain the transfer function. This kind of analysis is usually only carried out for simple filters of 1st or 2nd order.



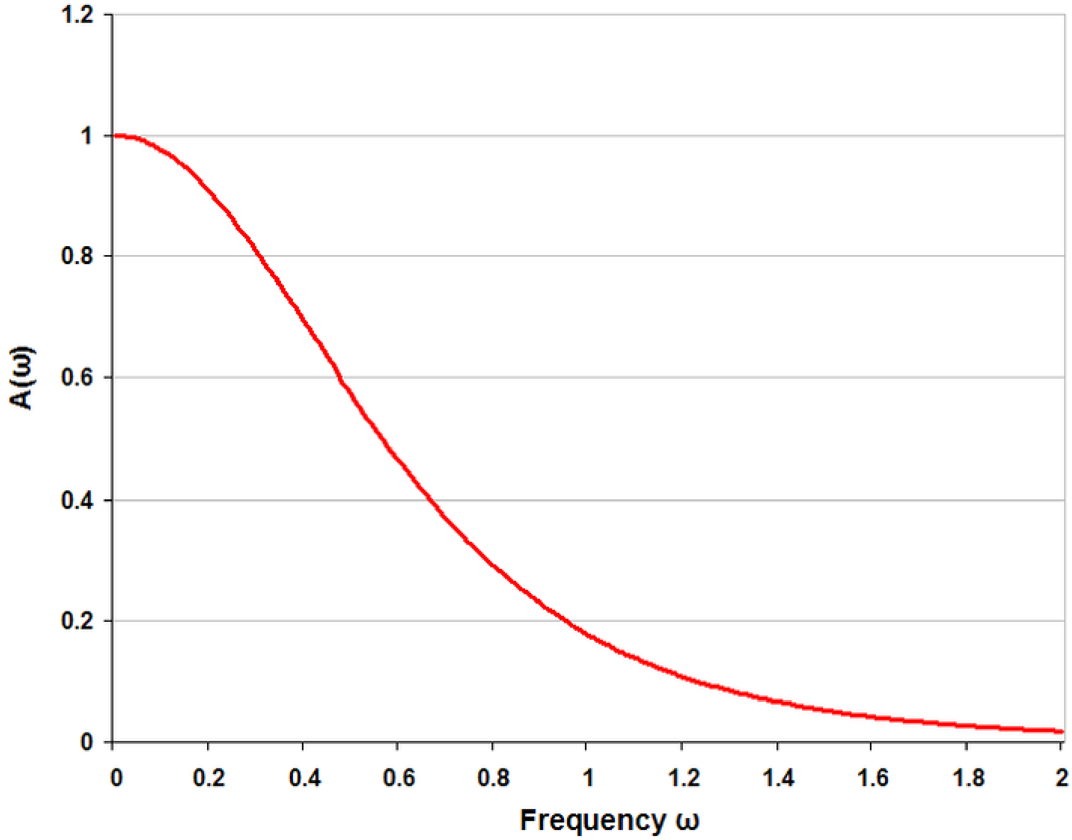
RL filter frequency response

Image impedance analysis

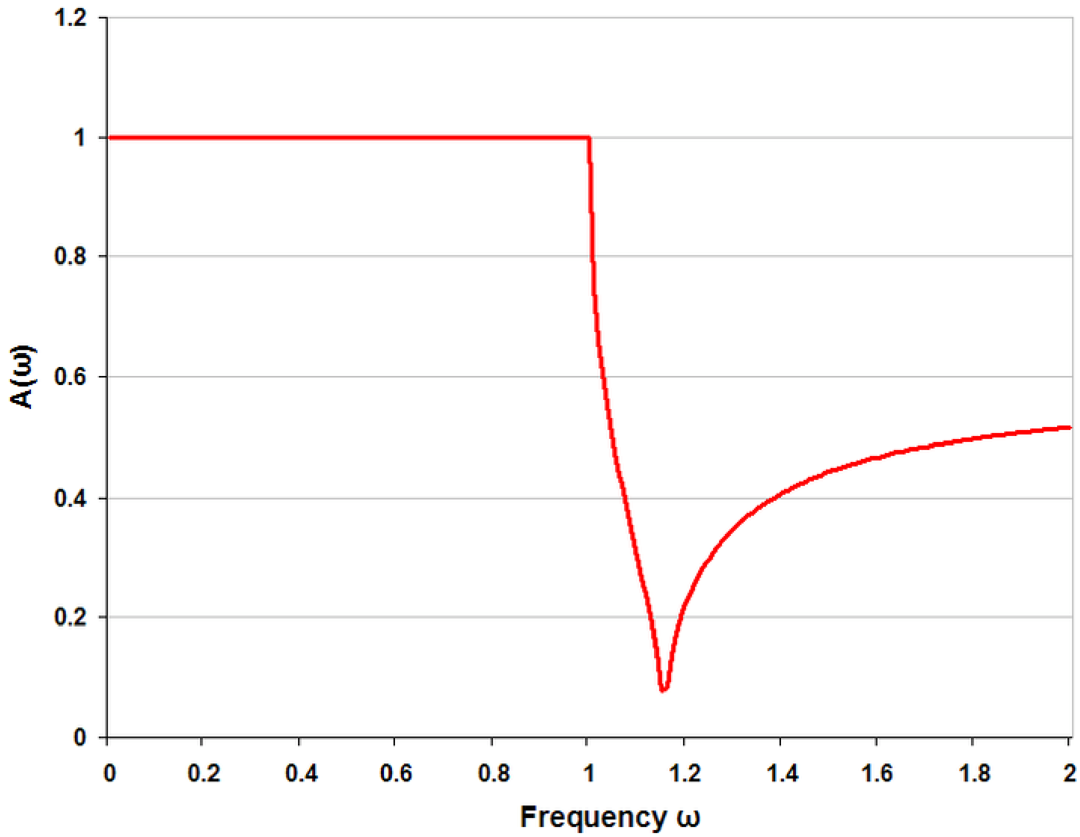
This approach analyses the filter sections from the point of view of the filter being in an infinite chain of identical sections. It has the advantages of simplicity of approach and the ability to easily extend to higher orders. It has the disadvantage that accuracy of predicted responses relies on filter terminations in the image impedance, which is usually not the case.



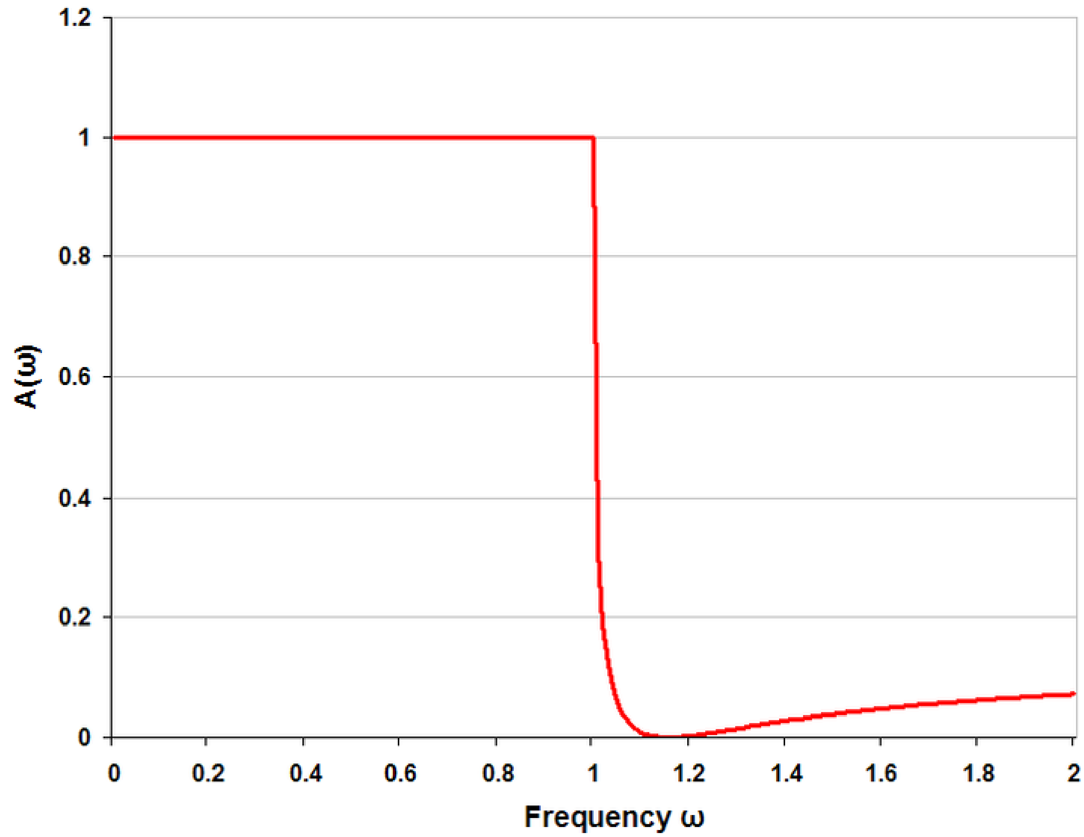
Constant k filter response with 5 elements



Zobel network (constant R) filter, 5 sections



m-derived filter response, m=0.5, 2 elements

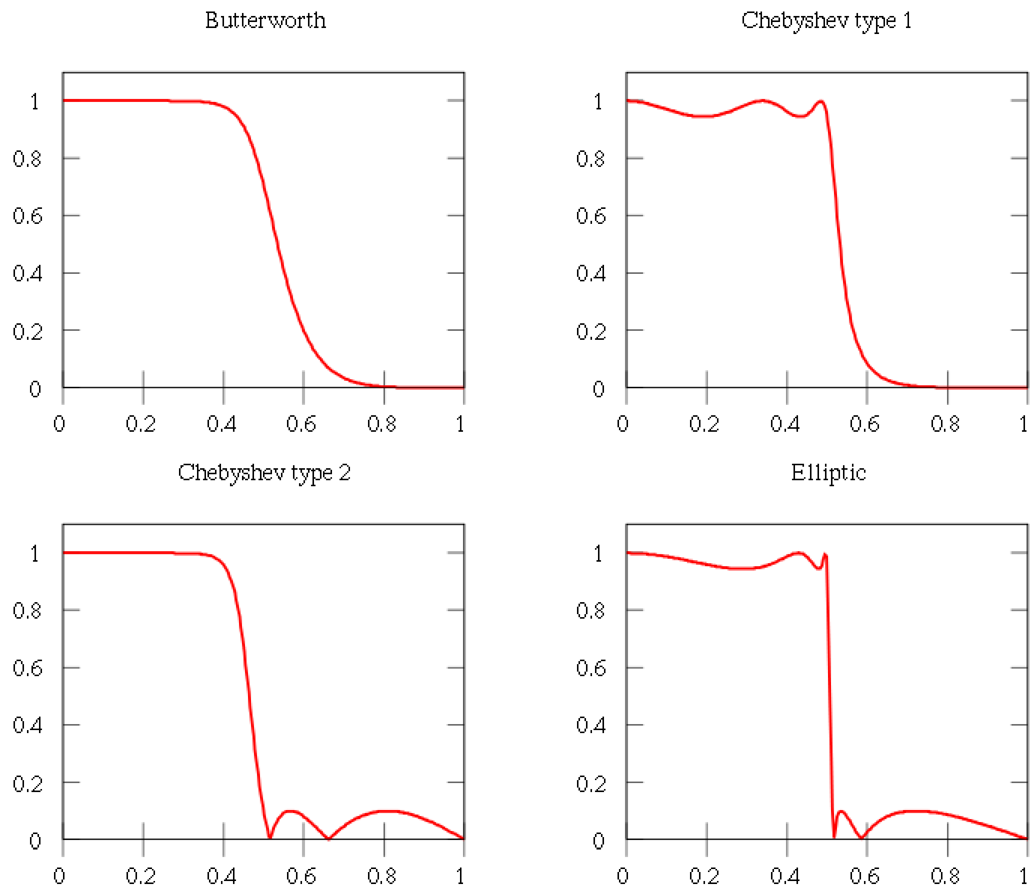


m-derived filter response, m=0.5, 5 elements

Network synthesis

The network synthesis approach starts with a required transfer function and then expresses that as a polynomial equation of the input impedance of the filter. The actual element values of the filter are obtained by continued-fraction or partial-fraction expansions of this polynomial. Unlike the image method, there is no need for impedance matching networks at the terminations as the effects of the terminating resistors are included in the analysis from the start.

Here is an image comparing Butterworth, Chebyshev, and elliptic filters. The filters in this illustration are all fifth-order low-pass filters. The particular implementation – analog or digital, passive or active – makes no difference; their output would be the same.



As is clear from the image, elliptic filters are sharper than all the others, but they show ripples on the whole bandwidth.

Chapter- 2

Constant k Filter

Constant k filters, also **k-type filters**, are a type of electronic filter designed using the image method. They are the original and simplest filters produced by this methodology and consist of a ladder network of identical sections of passive components. Historically, they are the first filters that could approach the ideal filter frequency response to within any prescribed limit with the addition of a sufficient number of sections. However, they are rarely considered for a modern design, the principles behind them having been superseded by other methodologies which are more accurate in their prediction of filter response.

History

Constant k filters were invented by George Campbell. He published his work in 1922, but had clearly invented the filters some time before, as his colleague at AT&T Co, Otto Zobel, was already making improvements to the design at this time. Campbell's filters were far superior to the simpler single element circuits that had been used previously. Campbell called his filters electric wave filters, but this term later came to mean any filter that passes waves of some frequencies but not others. Many new forms of wave filter were subsequently invented; an early (and important) variation was the m-derived filter by Zobel who coined the term constant k for the Campbell filter in order to distinguish them.

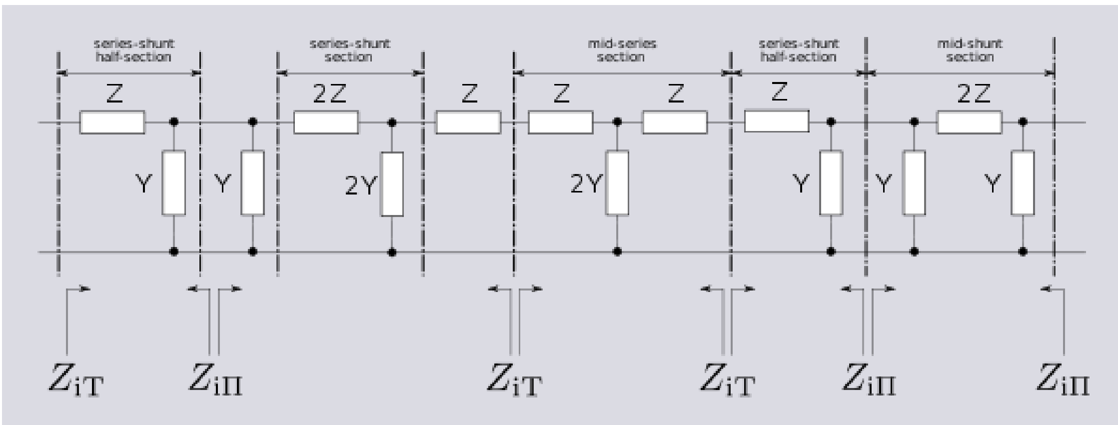
The great advantage Campbell's filters had over the RL circuit and other simple filters of the time was that they could be designed for any desired degree of stop band rejection or steepness of transition between pass band and stop band. It was only necessary to add more filter sections until the desired response was obtained.

The filters were designed by Campbell for the purpose of separating multiplexed telephone channels on transmission lines, but their subsequent use has been much more widespread than that. The design techniques used by Campbell have largely been superseded. However, the ladder topology used by Campbell with the constant k is still in use today with implementations of modern filter designs such as the Tchebyscheff filter.

Campbell gave constant k designs for low-pass, high-pass and band-pass filters. Band-stop and multiple band filters are also possible.

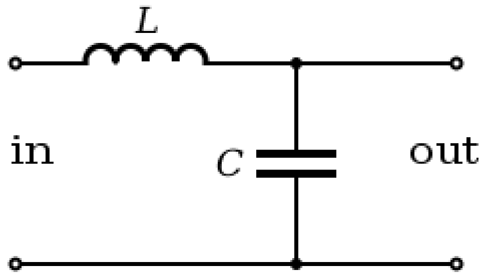
Terminology

Some of the impedance terms and section terms used here are pictured in the diagram below. Image theory defines quantities in terms of an infinite cascade of two-port sections, and in the case of the filters being discussed, an infinite ladder network of L-sections. Here "L" should not be confused with the inductance *L* – in electronic filter topology, "L" refers to the specific filter shape which resembles inverted letter "L".

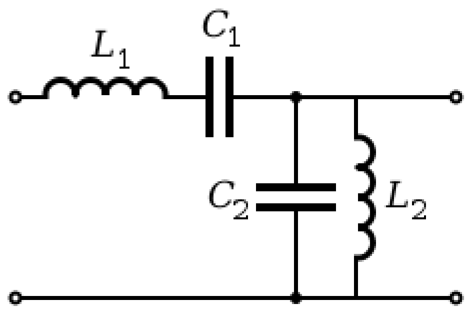


The sections of the hypothetical infinite filter are made of series elements having impedance 2*Z* and shunt elements with admittance 2*Y*. The factor of two is introduced for mathematical convenience, since it is usual to work in terms of half-sections where it disappears. The image impedance of the input and output port of a section will generally not be the same. However, for a *mid-series section* (that is, a section from halfway through a series element to halfway through the next series element) will have the same image impedance on both ports due to symmetry. This image impedance is designated Z_{iT} due to the "T" topology of a mid-series section. Likewise, the image impedance of a *mid-shunt section* is designated $Z_{i\Pi}$ due to the "Π" topology. Half of such a "T" or "Π" section is called a *half-section*, which is also an L-section but with half the element values of the full L-section. The image impedance of the half-section is dissimilar on the input and output ports: on the side presenting the series element it is equal to the mid-series Z_{iT}, but on the side presenting the shunt element it is equal to the mid-shunt $Z_{i\Pi}$. There are thus two variant ways of using a half-section.

Derivation



Constant k low-pass filter half section. Here inductance L is equal Ck^2



Constant k band-pass filter half section.
$L_1 = C_2k^2$ and $L_2 = C_1k^2$

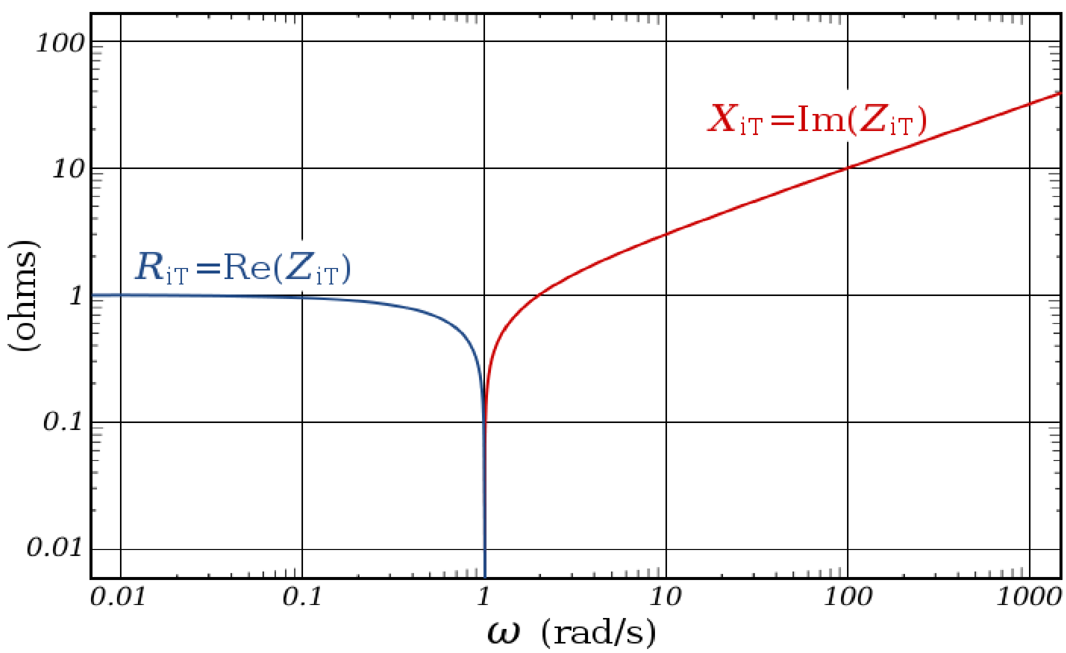


Image impedance Z_{iT} of a constant k prototype low-pass filter is plotted vs. frequency ω. The impedance is purely resistive (real) below ω_c, and purely reactive (imaginary) above ω_c.

The building block of constant k filters is the half-section "L" network, composed of a series impedance *Z*, and a shunt admittance *Y*. The "k" in "constant k" is the value given by,

$$k^2 = \frac{Z}{Y}$$

Thus, *k* will have units of impedance, that is, ohms. It is readily apparent that in order for *k* to be constant, *Y* must be the dual impedance of *Z*. A physical interpretation of k can be given by observing that *k* is the limiting value of Z_i as the size of the section (in terms of values of its components, such as inductances, capacitances, etc.) approaches zero, while keeping *k* at its initial value. Thus, *k* is the characteristic impedance, Z_0, of the transmission line that would be formed by these infinitesimally small sections. It is also the image impedance of the section at resonance, in the case of band-pass filters, or at $\omega = 0$ in the case of low-pass filters. For example, the pictured low-pass half-section has

$$k = \sqrt{\frac{i\omega L}{i\omega C}} = \sqrt{\frac{L}{C}}.$$

Elements *L* and *C* can be made arbitrarily small while retaining the same value of *k*. *Z* and *Y* however, are both approaching zero, and from the formulae (below) for image impedances,

$$\lim_{Z,Y \to 0} Z_{\mathrm{i}} = k.$$

Image impedance

The image impedances of the section are given by

$$Z_{\mathrm{iT}}{}^2 = Z^2 + k^2$$

and

$$\frac{1}{Z_{\mathrm{i\Pi}}{}^2} = Y_{\mathrm{i\Pi}}{}^2 = Y^2 + \frac{1}{k^2}$$

Provided that the filter does not contain any resistive elements, the image impedance in the pass band of the filter is purely real and in the stop band it is purely imaginary. For example, for the pictured low-pass half-section,

$$Z_{\mathrm{iT}}{}^2 = -(\omega L)^2 + \frac{L}{C}$$

The transition occurs at a cut-off frequency given by

$$\omega_c = \frac{1}{\sqrt{LC}}$$

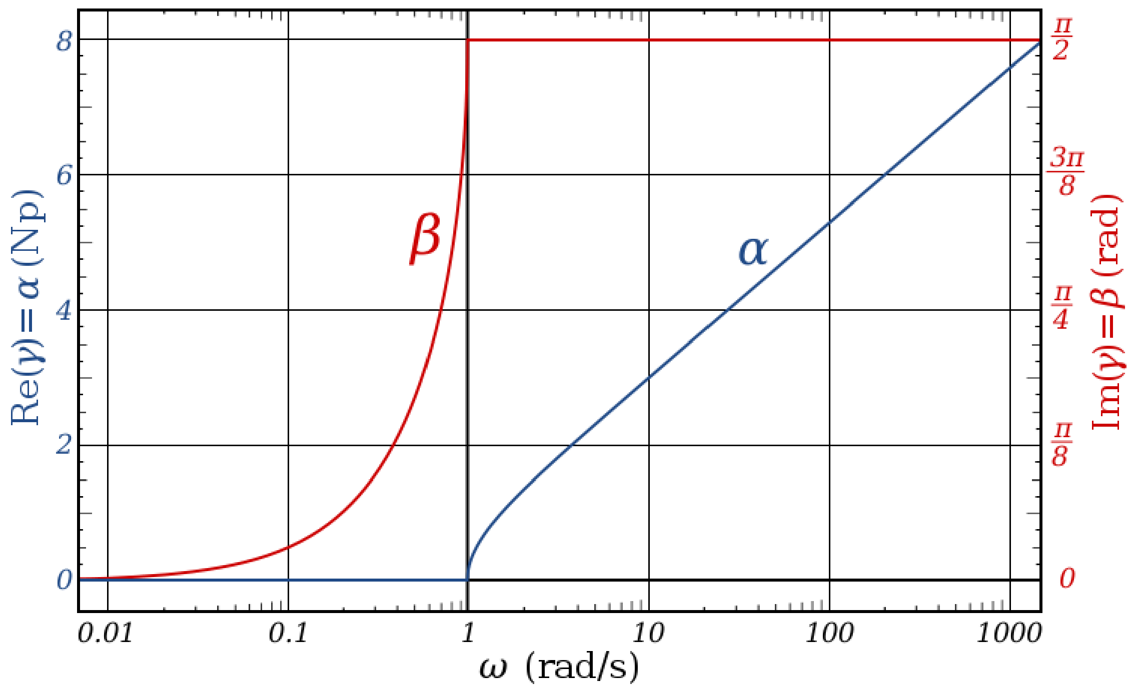
Below this frequency, the image impedance is real,

$$Z_{\mathrm{iT}} = L\sqrt{\omega_c^2 - \omega^2}$$

Above the cut-off frequency the image impedance is imaginary,

$$Z_{\mathrm{iT}} = iL\sqrt{\omega^2 - \omega_c^2}$$

Transmission parameters



The transfer function of a constant k prototype low-pass filter for a single half-section showing attenuation in nepers and phase change in radians.

The transmission parameters for a general constant k half-section are given by

$$\gamma = \sinh^{-1}\frac{Z}{k}$$

and for a chain of *n* half-sections

$$\gamma_n = n\gamma$$

For the low-pass L-shape section, below the cut-off frequency, the transmission parameters are given by

$$\gamma = \alpha + i\beta = 0 + i \sin^{-1} \frac{\omega}{\omega_c}$$

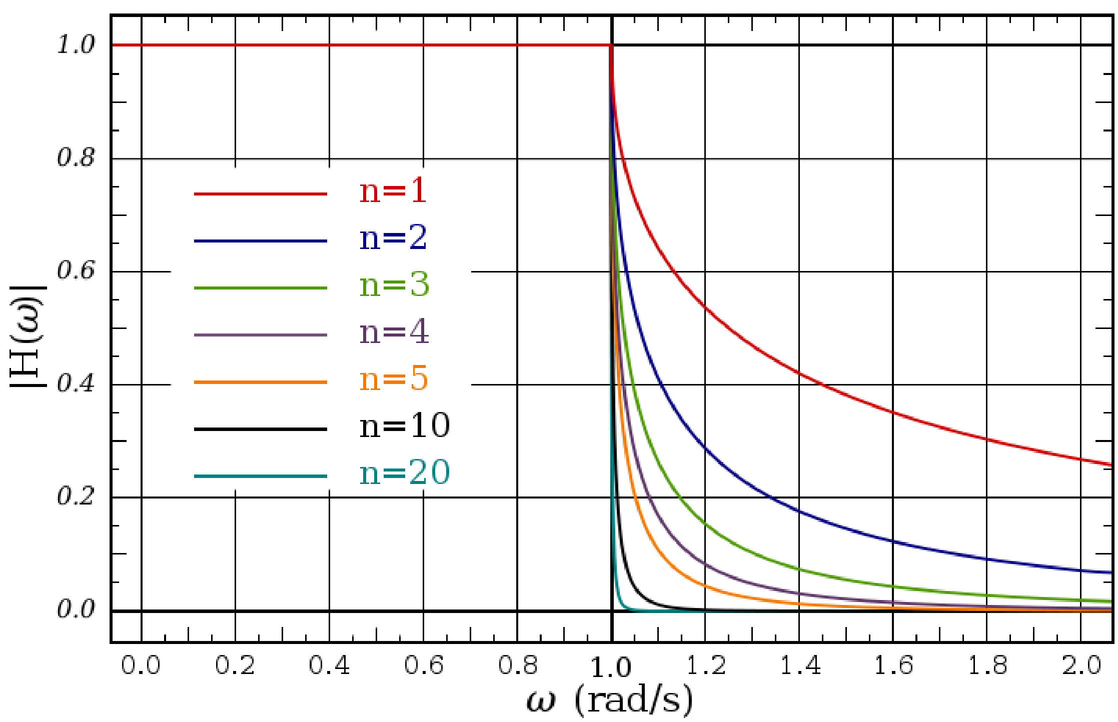
That is, the transmission is lossless in the pass-band with only the phase of the signal changing. Above the cut-off frequency, the transmission parameters are:

$$\gamma = \alpha + i\beta = \cosh^{-1} \frac{\omega}{\omega_c} + i\frac{\pi}{2}$$

Prototype transformations

The presented plots of image impedance, attenuation and phase change correspond to a low-pass prototype filter section. The prototype has a cut-off frequency of $\omega_c = 1$ rad/s and a nominal impedance $k = 1\ \Omega$. This is produced by a filter half-section with inductance $L = 1$ henry and capacitance $C = 1$ farad. This prototype can be impedance scaled and frequency scaled to the desired values. The low-pass prototype can also be transformed into high-pass, band-pass or band-stop types by application of suitable frequency transformations.

Cascading sections



Gain response, H(ω) for a chain of *n* low-pass constant-k filter half-sections.

Several L-shape half-sections may be cascaded to form a composite filter. Like impedance must always face like in these combinations. There are therefore two circuits that can be formed with two identical L-shaped half-sections. Where a port of image impedance Z_{iT} faces another Z_{iT}, the section is called a Π section. Where $Z_{i\Pi}$ faces $Z_{i\Pi}$ the section so formed is a T section. Further additions of half-sections to either of these section forms a ladder network which may start and end with series or shunt elements.

It should be borne in mind that the characteristics of the filter predicted by the image method are only accurate if the section is terminated with its image impedance. This is usually not true of the sections at either end, which are usually terminated with a fixed resistance. The further the section is from the end of the filter, the more accurate the prediction will become, since the effects of the terminating impedances are masked by the intervening sections.

Chapter- 3

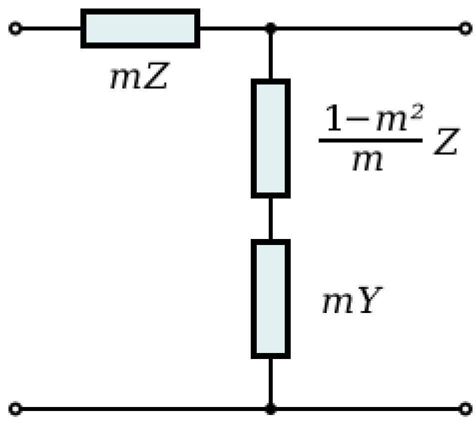
m-derived Filter

m-derived filters or **m-type filters** are a type of electronic filter designed using the image method. They were invented by Otto Zobel in the early 1920s. This filter type was originally intended for use with telephone multiplexing and was an improvement on the existing constant k type filter. The main problem being addressed was the need to achieve a better match of the filter into the terminating impedances. In general, all filters designed by the image method fail to give an exact match, but the m-type filter is a big improvement with suitable choice of the parameter m. The m-type filter section has a further advantage in that there is a rapid transition from the cut-off frequency of the pass band to a pole of attenuation just inside the stop band. Despite these advantages, there is a drawback with m-type filters; at frequencies past the pole of attenuation, the response starts to rise again, and m-types have poor stop band rejection. For this reason, filters designed using m-type sections are often designed as composite filters with a mixture of k-type and m-type sections and different values of m at different points to get the optimum performance from both types.

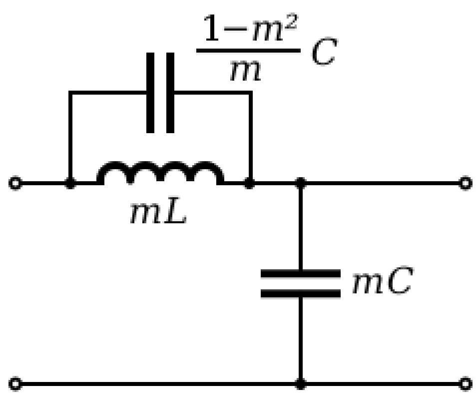
Background

Zobel patented an impedance matching network in 1920 which, in essence, used the topology of what are now called m-type filters, but Zobel did not name them as such or analyse them by the image method. This pre-dated George Campbell's publication of his constant k-type design in 1922 on which the m-type filter is based. Zobel published the image analysis theory of m-type filters in 1923. Once popular, M-type filters and image parameter designed filters in general are now rarely designed, having been superseded by more advanced network synthesis methods.

Derivation



m-derived series general filter half section.



m-derived shunt low-pass filter half section.

$$C = \frac{L}{R_0^2}$$

The building block of m-derived filters, as with all image impedance filters, is the "L" network, called a half-section and composed of a series impedance *Z*, and a shunt admittance *Y*. The m-derived filter is a derivative of the constant k filter. The starting point of the design is the values of *Z* and *Y* derived from the constant k prototype and are given by

$$k^2 = \frac{Z}{Y}$$

where *k* is the nominal impedance of the filter, or R_0. The designer now multiplies *Z* and *Y* by an arbitrary constant *m* ($0 < m < 1$). There are two different kinds of m-derived section; series and shunt. To obtain the m-derived series half section, the designer determines the impedance that must be added to 1/mY to make the image impedance Z_{iT} the same as the image impedance of the original constant k section. From the general formula for image impedance, the additional impedance required can be shown to be

$$\frac{1-m^2}{m}Z.$$

To obtain the m-derived shunt half section, an admittance is added to 1/mZ to make the image impedance $Z_{i\Pi}$ the same as the image impedance of the original half section. The additional admittance required can be shown to be

$$\frac{1-m^2}{m}Y.$$

The general arrangements of these circuits are shown in the diagrams to the right along with a specific example of a low pass section.

A consequence of this design is that the m-derived half section will match a k-type section on one side only. Also, an m-type section of one value of m will not match another m-type section of another value of m except on the sides which offer the Z_i of the k-type.

Operating frequency

For the low-pass half section shown, the cut-off frequency of the m-type is the same as the k-type and is given by

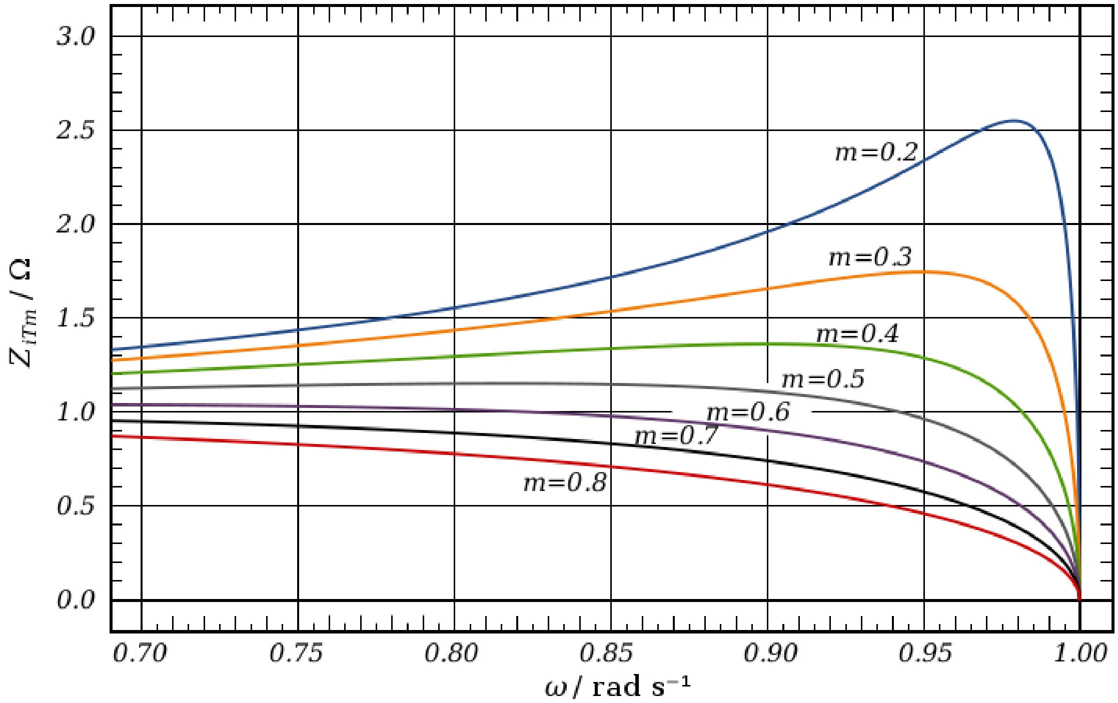
$$\omega_c = \frac{1}{\sqrt{LC}}.$$

The pole of attenuation occurs at;

$$\omega_\infty = \frac{\omega_c}{\sqrt{1-m^2}}.$$

From this it is clear that smaller values of m will produce ω_∞closer to the cut-off frequency $\omega_c$and hence will have a sharper cut-off. Despite this cut-off, it also brings the unwanted stop band response of the m-type closer to the cut-off frequency, making it more difficult for this to be filtered with subsequent sections. The value of m chosen is usually a compromise between these conflicting requirements. There is also a practical limit to how small m can be made due to the inherent resistance of the inductors. This has the effect of causing the pole of attenuation to be less deep (that is, it is no longer a genuinely infinite pole) and the slope of cut-off to be less steep. This effect becomes more marked as $\omega_\infty$is brought closer to ω_c, and there ceases to be any improvement in response with an m of about 0.2 or less.

Image impedance



m-derived prototype shunt low-pass filter Z_{iTm} image impedance for various values of *m*. Values below cut-off frequency only shown for clarity.

The following expressions for image impedances are all referenced to the low-pass prototype section. They are scaled to the nominal impedance $R_0 = 1$, and the frequencies in those expressions are all scaled to the cut-off frequency $\omega_c = 1$.

Series sections

The image impedances of the series section are given by

$$Z_{iT} = \sqrt{1-\omega^2}$$

and is the same as that of the constant k section

$$Z_{i\Pi m} = \frac{1-(\omega/\omega_\infty)^2}{\sqrt{1-\omega^2}}.$$

Shunt sections

The image impedances of the shunt section are given by

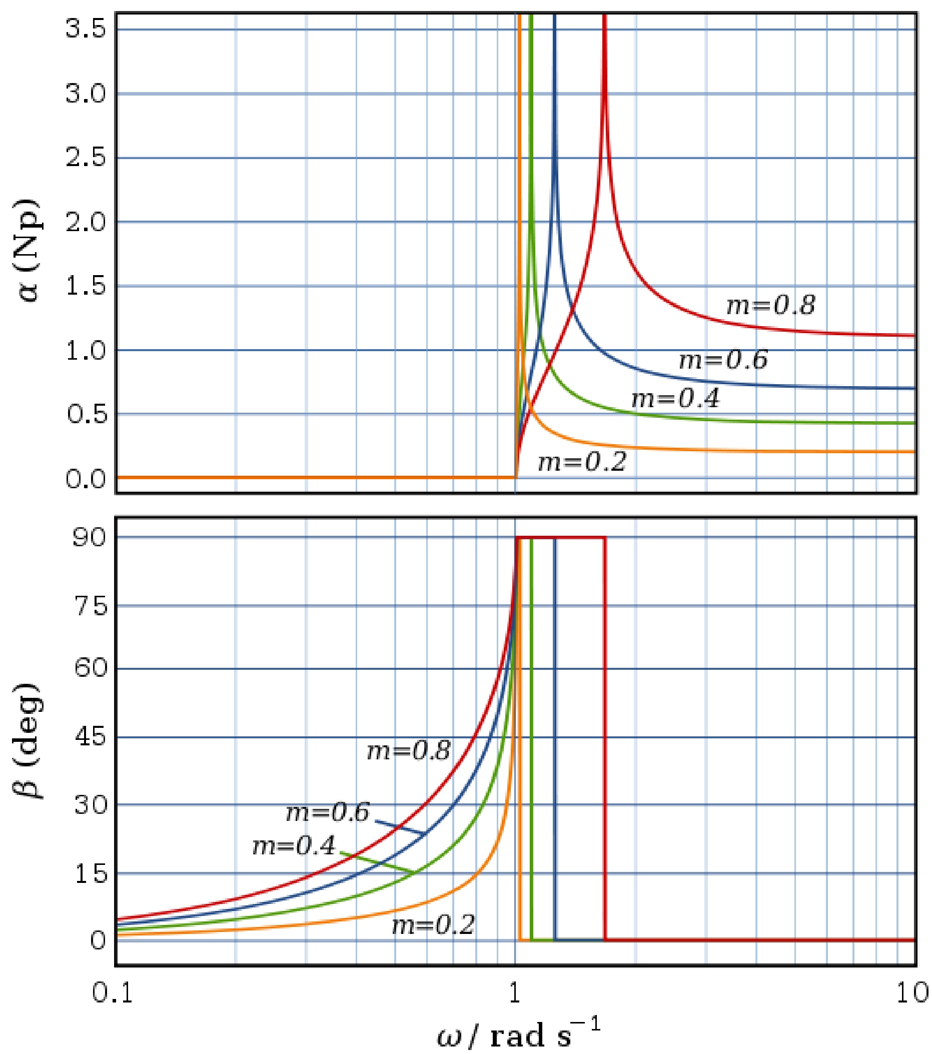
$$Z_{i\Pi} = \frac{1}{\sqrt{1-\omega^2}}$$

and is the same as that of the constant k section

$$Z_{iTm} = \frac{\sqrt{1-\omega^2}}{1-(\omega/\omega_\infty)^2}$$

As with the k-type section, the image impedance of the *m*-type low-pass section is purely real below the cut-off frequency and purely imaginary above it. From the chart it can be seen that in the passband the closest impedance match to a constant pure resistance termination occurs at approximately *m* = 0.6.

Transmission parameters



m-Derived low-pass filter transfer function for a single half-section

For an m-derived section in general the transmission parameters for a half-section are given by

$$\gamma = \sinh^{-1} \frac{mZ}{\sqrt{k^2 + (1 - m^2)Z^2}}$$

and for n half-sections

$$\gamma_n = n\gamma$$

For the particular example of the low-pass L section, the transmission parameters solve differently in three frequency bands.

For $0 < \omega < \omega_c$ the transmission is lossless:

$$\gamma = \alpha + i\beta = 0 + i\frac{1}{2}\cos^{-1}\left(1 - \frac{2m^2}{\left(\frac{\omega_c}{\omega}\right)^2 - \left(\frac{\omega_c}{\omega_\infty}\right)^2}\right)$$

For $\omega_c < \omega < \omega_\infty$ the transmission parameters are

$$\gamma = \alpha + i\beta = \frac{1}{2}\cosh^{-1}\left(\frac{2m^2}{\left(\frac{\omega_c}{\omega}\right)^2 - \left(\frac{\omega_c}{\omega_\infty}\right)^2} - 1\right) + i\frac{\pi}{2}$$

For $\omega_\infty < \omega < \infty$ the transmission parameters are

$$\gamma = \alpha + i\beta = \frac{1}{2}\cosh^{-1}\left(1 - \frac{2m^2}{\left(\frac{\omega_c}{\omega}\right)^2 - \left(\frac{\omega_c}{\omega_\infty}\right)^2}\right) + i0$$

Prototype transformations

The plots shown of image impedance, attenuation and phase change are the plots of a low-pass prototype filter section. The prototype has a cut-off frequency of $\omega_c = 1$ rad/s and a nominal impedance $R_0 = 1\ \Omega$. This is produced by a filter half-section where L = 1 henry and C = 1 farad. This prototype can be impedance scaled and frequency scaled to the desired values. The low-pass prototype can also be transformed into high-pass, band-pass or band-stop types by application of suitable frequency transformations.

Cascading sections

Several L half-sections may be cascaded to form a composite filter. Like impedance must always face like in these combinations. There are therefore two circuits that can be formed with two identical L half-sections. Where Z_{iT} faces Z_{iT}, the section is called a Π section. Where $Z_{i\Pi}$ faces $Z_{i\Pi}$ the section formed is a T section. Further additions of half-sections to either of these forms a ladder network which may start and end with series or shunt elements.

It should be born in mind that the characteristics of the filter predicted by the image method are only accurate if the section is terminated with its image impedance. This is usually not true of the sections at either end which are usually terminated with a fixed resistance. The further the section is from the end of the filter, the more accurate the prediction will become since the effects of the terminating impedances are masked by the intervening sections. It is usual to provide half half-sections at the ends of the filter with m = 0.6 as this value gives the flattest Z_i in the passband and hence the best match in to a resistive termination.

Chapter- 4

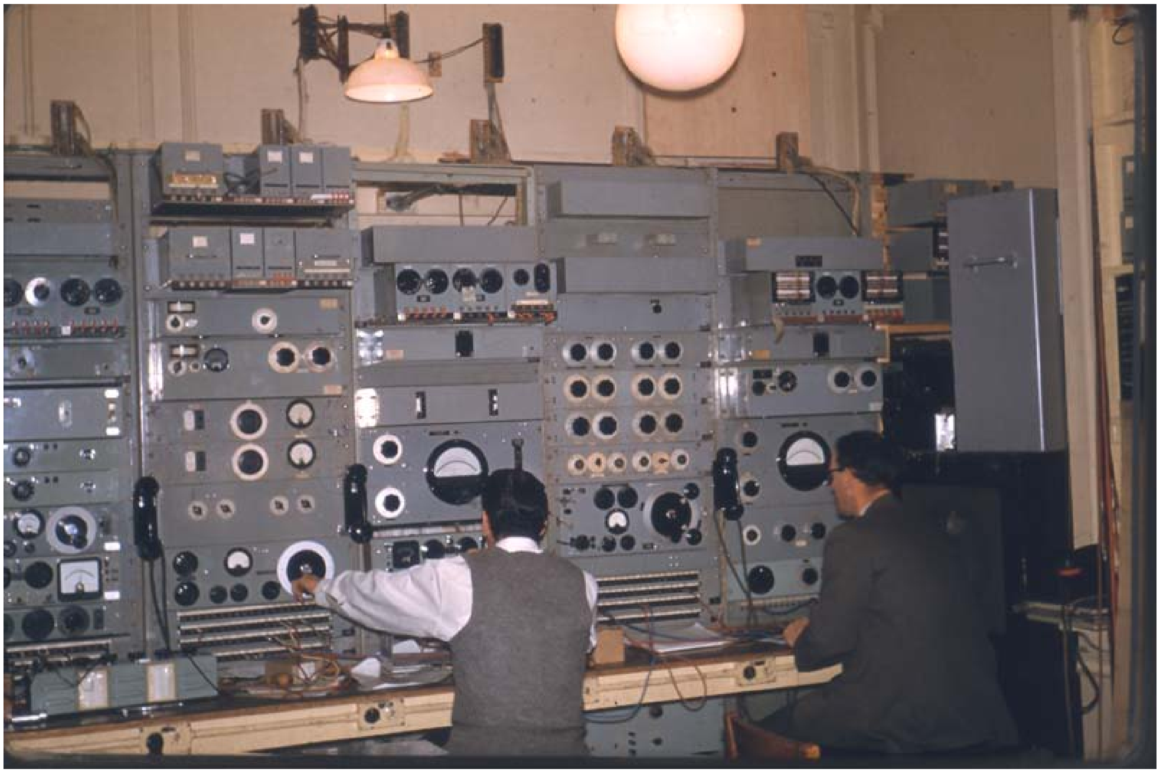
Zobel Network

Zobel networks are a type of filter section based on the image impedance design principle. They are named after Otto Zobel of Bell Labs who published a much referenced paper on image filters in 1923. The distinguishing feature of Zobel networks is that the input impedance is fixed in the design independently of the transfer function. This characteristic is achieved at the expense of a much higher component count compared to other types of filter sections. The impedance would normally be specified to be constant and purely resistive. For this reason they are also known as constant resistance networks. However, any impedance achievable with discrete components is possible.

Zobel networks were formerly widely used in telecommunications to flatten and widen the frequency response of copper land lines, producing a higher quality line from one originally intended for ordinary telephone use. However, as analogue technology has given way to digital they are now little used.

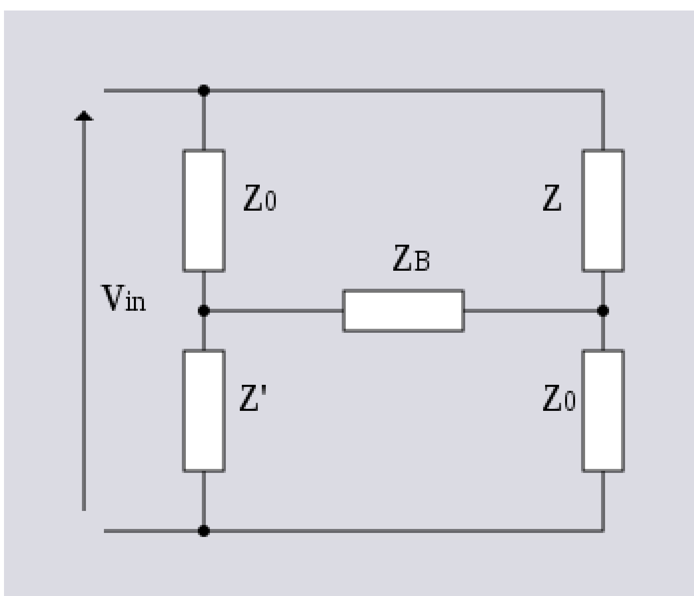
When used to cancel out the reactive portion of loudspeaker impedance, the design is sometimes called a Boucherot cell. In this case, only half the network is implemented as fixed components, the other half being the real and imaginary components of the loudspeaker impedance. This network is more akin to the power factor correction circuits used in electrical power distribution, hence the association with Boucherot's name.

A common circuit form of Zobel networks is in the form of a bridged T. This term is often used to mean a Zobel network, sometimes incorrectly when the circuit implementation is, in fact, something other than a bridged T.



BBC engineers equalising audio landlines circa 1959. The boxes with two large black dials towards the top of the equipment racks are adjustable Zobel equalisers. They are used both for temporary outside broadcast lines and for checking the engineer's calculations prior to building permanent units

Derivation



The basis of a Zobel network is a balanced bridge circuit as shown in the circuit to the right. The condition for balance is that;

$$\frac{Z}{Z_0} = \frac{Z_0}{Z'}.$$

If this is expressed in terms of a normalised $Z_0 = 1$ as is conventionally done in filter tables, then the balance condition is simply;

$$Z = \frac{1}{Z'}.$$

In other words, Z' is simply the inverse, or dual impedance of Z.

The bridging impedance Z_B is across the balance points and hence has no potential across it. Consequently, it will draw no current and its value makes no difference to the function of the circuit. However, its value is often chosen to be Z_0 for reasons which will become clear in the discussion of bridged T circuits.

Input impedance

The input impedance is given by

$$\frac{1}{Z_{\text{in}}} = \frac{1}{Z_0 + Z'} + \frac{1}{Z + Z_0}.$$

Substituting the balance condition,

$$Z' = \frac{Z_0^2}{Z},$$

yields

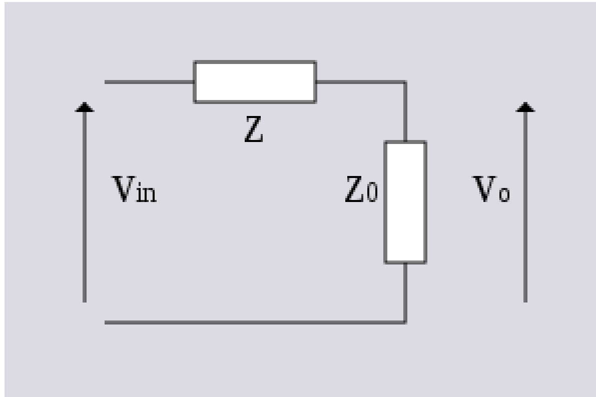
$$Z_{\text{in}} = Z_0$$

The input impedance can be designed to be purely resistive by setting

$$Z_0 = R_0.$$

The input impedance will then be real and independent of ω in band and out of band no matter what complexity of filter section is chosen.

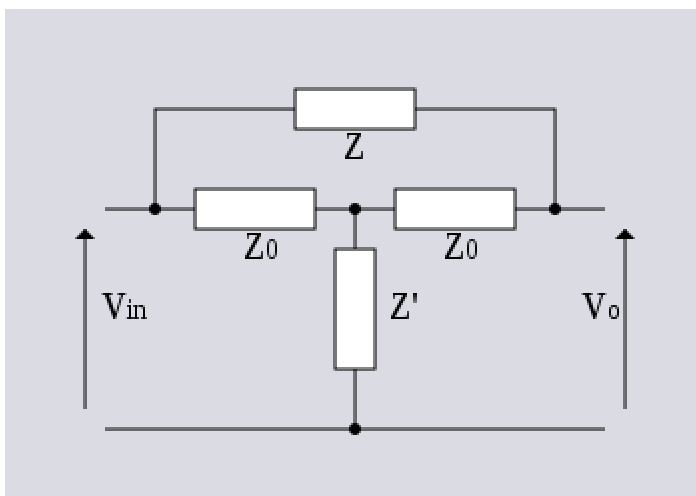
Transfer function



If the Z_0 in the bottom right of the bridge is taken to be the output load then a transfer function of V_{in}/V_o can be calculated for the section. Only the rhs branch needs to be considered in this calculation. The reason for this can be seen by considering that there is no current flow through R_B. None of the current flowing through the lhs branch is going to flow into the load. The lhs branch therefore, cannot possibly affect the output. It certainly affects the input impedance (and hence the input terminal voltage) but not the transfer function. The transfer function can now easily be seen to be;

$$A(\omega) = \frac{Z_0}{Z + Z_0}.$$

Bridged T implementation



The load impedance is actually the impedance of the following stage or of a transmission line and can sensibly be omitted from the circuit diagram. If we also set;

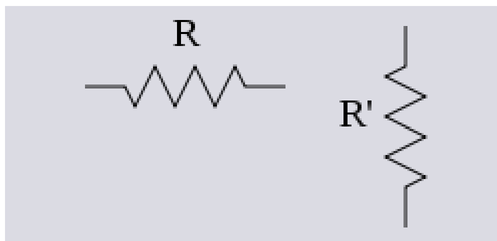
$$Z_B = Z_0$$

then the circuit to the right results. This is referred to as a bridged T circuit because the impedance Z is seen to "bridge" across the T section. The purpose of setting $Z_B = Z_0$ is to make the filter section symmetrical. This has the advantage that it will then present the same impedance, Z_0, at both the input and the output port.

Types of section

A Zobel filter section can be implemented for low-pass, high-pass, band-pass or band-stop. It is also possible to implement a flat frequency response attenuator. This last is of some importance for the practical filter sections described later.

Attenuator



Z and *Z'* for a Zobel attenuator

For an attenuator section, Z is simply

$$Z = R$$

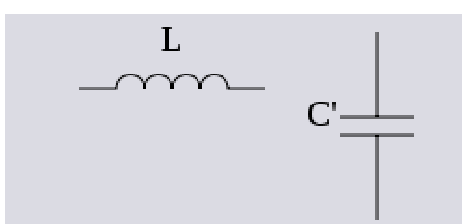
and,

$$Z' = R' = \frac{R_0^2}{R}$$

The attenuation of the section is given by;

$$L = 20 \log \left(\frac{R}{R_0} + 1 \right) \text{ dB.}$$

Low pass



Z and *Z'* for a Zobel low-pass filter section

For a low-pass filter section, *Z* is an inductor and *Z'* is a capacitor;

$$Z = i\omega L,$$

and

$$Z' = \frac{1}{i\omega C'}$$

where

$$C' = \frac{L}{R_0^2}.$$

The transfer function of the section is given by

$$A(\omega) = \frac{R_0}{i\omega L + R_0}.$$

The 3dB point occurs when $\omega L = R_0$ *so the 3dB cut-off frequency is given by*

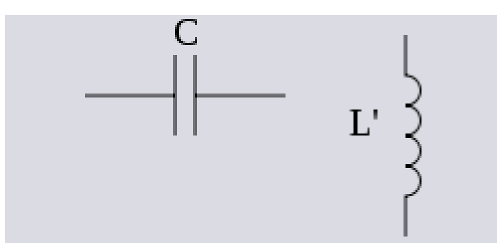
$$\omega_c = \frac{R_0}{L},$$

where ω is in the stop band well above ω_c,

$$A(\omega) \approx \frac{R_0}{i\omega L},$$

it can be seen from this that $A(\omega)$ is falling away in the stop band at the classic 6dB/8ve (or 20dB/decade).

High pass



Z and *Z'* for a Zobel high-pass filter section

For a high-pass filter section, *Z* is a capacitor and *Z'* is an inductor:

$$Z = \frac{1}{i\omega C},$$

and

$$Z' = i\omega L'$$

where

$$L' = CR_0^2.$$

The transfer function of the section is given by

$$A(\omega) = \frac{i\omega CR_0}{1 + i\omega CR_0}.$$

The 3dB point occurs when $\omega C = 1/R_0$ so the 3dB cut-off frequency is given by

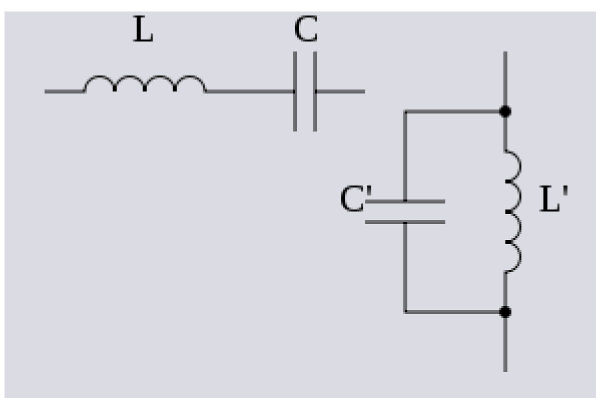
$$\omega_c = \frac{1}{CR_0}.$$

In the stop band,

$$A(\omega) \approx i\omega CR_0$$

falling at 6dB/8ve with decreasing frequency.

Band pass



Z and *Z'* for a Zobel band-pass filter section

For a band-pass filter section, *Z* is a series resonant circuit and *Z'* is a shunt resonant circuit;

$$Z = i\omega L + \frac{1}{i\omega C},$$

and

$$Y' = \frac{1}{Z'} = i\omega C' + \frac{1}{i\omega L'}.$$

The transfer function of the section is given by

$$A(\omega) = \frac{i\omega C R_0}{1 + i\omega C R_0 - \omega^2 LC}.$$

The 3dB point occurs when $|1 - \omega^2 LC| = \omega C R_0$ so the 3dB cut-off frequencies are given by

$$\omega_c = \frac{\pm R_0 C + \sqrt{R_0^2 C^2 + 4LC}}{2LC},$$

from which the centre frequency, ω_m, and bandwidth, $\Delta\omega$, can be determined:

$$\Delta\omega = \frac{R_0}{L}$$

$$\omega_m = \sqrt{\frac{R_0^2}{4L^2} + \frac{1}{LC}}.$$

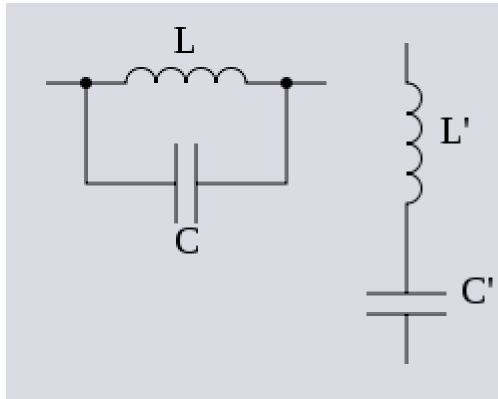
Note that this is different from the resonant frequency

$$\omega_0 = \sqrt{\frac{1}{LC}};$$

the relationship between them being given by

$$\omega_m^2 = \left(\frac{\Delta\omega}{2}\right)^2 + \omega_0^2.$$

Band stop



Z and Z' for a Zobel band-stop filter section

For a band-stop filter section, Z is a shunt resonant circuit and Z' is a series resonant circuit:

$$Y = \frac{1}{Z} = i\omega C + \frac{1}{i\omega L},$$

and

$$Z' = i\omega L' + \frac{1}{i\omega C'}.$$

The transfer function and bandwidth can be found by analogy with the band-pass section.

$$\Delta\omega = \frac{1}{CR_0}.$$

And,

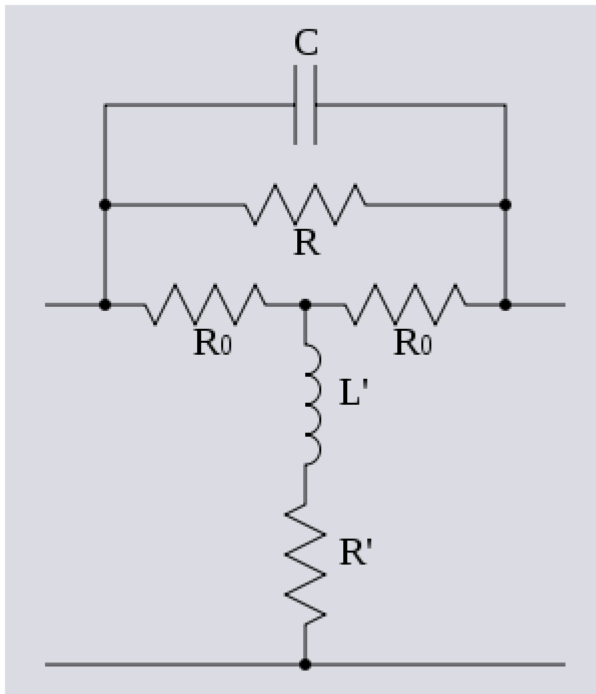
$$\omega_m = \sqrt{\left(\frac{1}{2R_0C}\right)^2 + \frac{1}{LC}}.$$

Practical sections

Zobel networks are rarely used for traditional frequency filtering. Other filter types are significantly more efficient for this purpose. Where Zobels come into their own is in frequency equalisation applications, particularly on transmission lines. The difficulty with transmission lines is that the impedance of the line varies in a complex way across the band and is tedious to measure. For most filter types, this variation in impedance will cause a significant difference in response to the theoretical, and is mathematically difficult to compensate for, even assuming that the impedance is known precisely. If

Zobel networks are used however, it is only necessary to measure the line response into a fixed resistive load and then design an equaliser to compensate it. It is entirely unnecessary to know anything at all about the line impedance as the Zobel network will present exactly the same impedance to line as the measuring instruments. Its response will therefore be precisely as theoretically predicted. This is a tremendous advantage where high quality lines with flat frequency responses are desired.

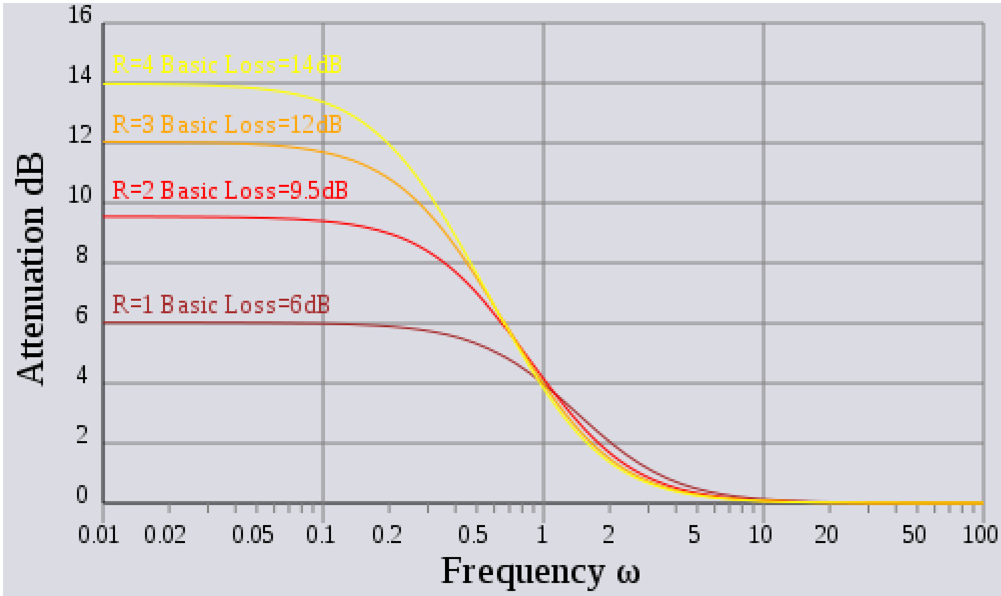
Basic loss



A practical high pass section incorporating basic loss used to correct high end roll-off

For audio lines, it is invariably necessary to combine L/C filter components with resistive attenuator components in the same filter section. The reason for this is that the usual design strategy is to require the section to attenuate all frequencies down to the level of the frequency in the passband with the lowest level. Without the resistor components, the filter, at least in theory, would increase attenuation without limit. The attenuation in the stop band of the filter (that is, the limiting maximum attenuation) is referred to as the "basic loss" of the section. In other words, the flat part of the band is attenuated by the basic loss down to the level of the falling part of the band which it is desired to equalise. The following discussion of practical sections relates in particular to audio transmission lines.

6dB/octave roll-off



High-pass Zobel network response for various basic losses. Normalised to $R_0 = 1$ and $\omega_c = 1$

The most significant effect that needs to be compensated for is that at some cut-off frequency the line response starts to roll-off like a simple low-pass filter. The effective bandwidth of the line can be increased with a section that is a high-pass filter matching this roll-off, combined with an attenuator. In the flat part of the pass-band only the attenuator part of the filter section is significant. This is set at an attenuation equal to the level of the highest frequency of interest. All frequencies up to this point will then be equalised flat to an attenuated level. Above this point, the output of the filter will again start to roll-off.

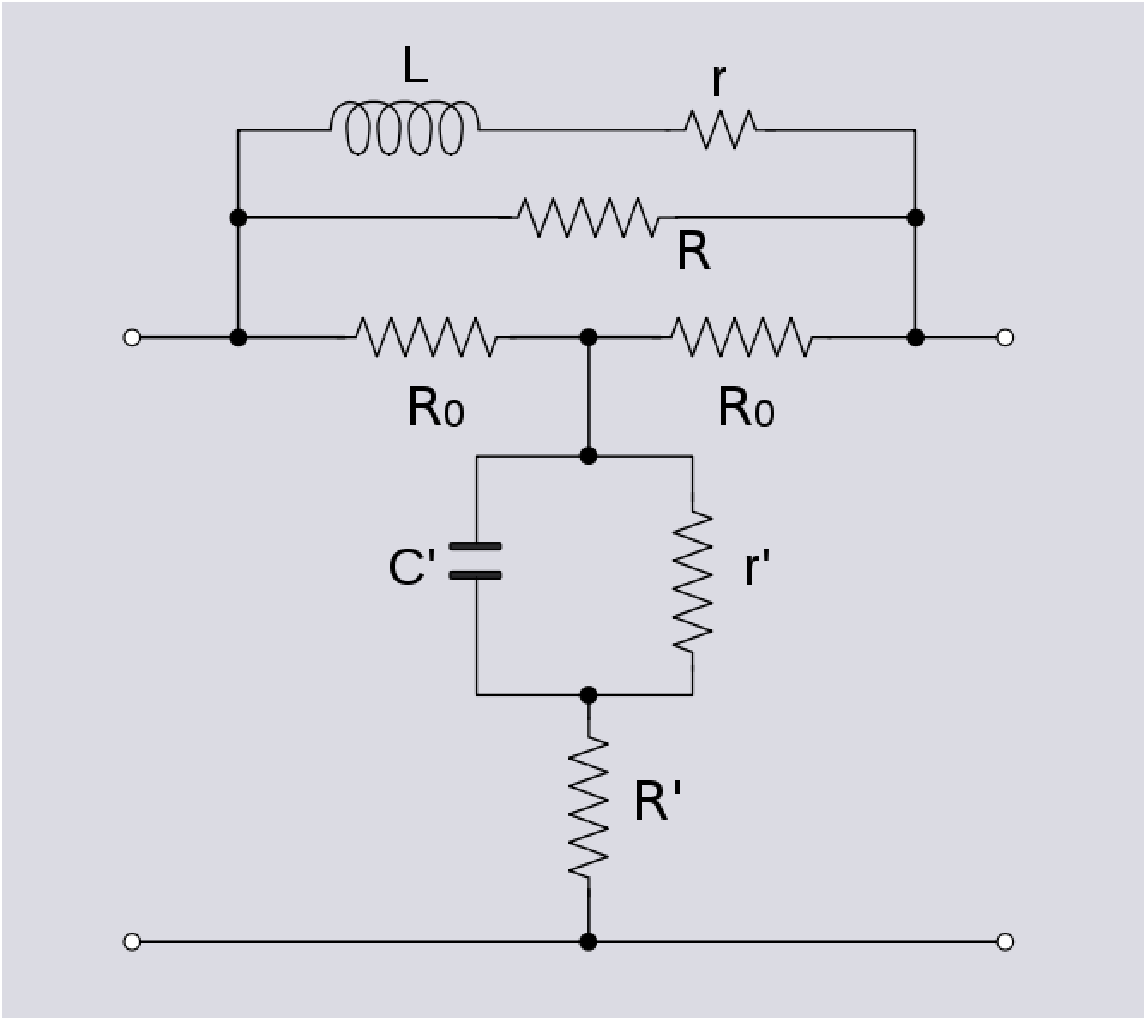
Mismatched lines

Quite commonly in telecomms networks, a circuit is made up of two sections of line which do not have the same characteristic impedance. For instance 150Ω and 300Ω. One effect of this is that the roll-off can start at 6dB/octave at an initial cut-off frequency f_{c1}, but then at f_{c2} can become suddenly steeper. This situation then requires (at least) two high-pass sections to compensate each operating at a different f_c.

Bumps and dips

Bumps and dips in the passband can be compensated for with band-stop and band-pass sections respectively. Again, an attenuator element is also required, but usually rather smaller than that required for the roll-off. These anomalies in the pass-band can be caused by mismatched line segments as described above. Dips can also be caused by ground temperature variations.

Transformer roll-off



Low frequency equaliser section with compensation for inductor resistance. The resistance **r** represents the stray resistance of the non-ideal inductor. The resistance **r'** is a real resistor calculated to compensate for **r**.

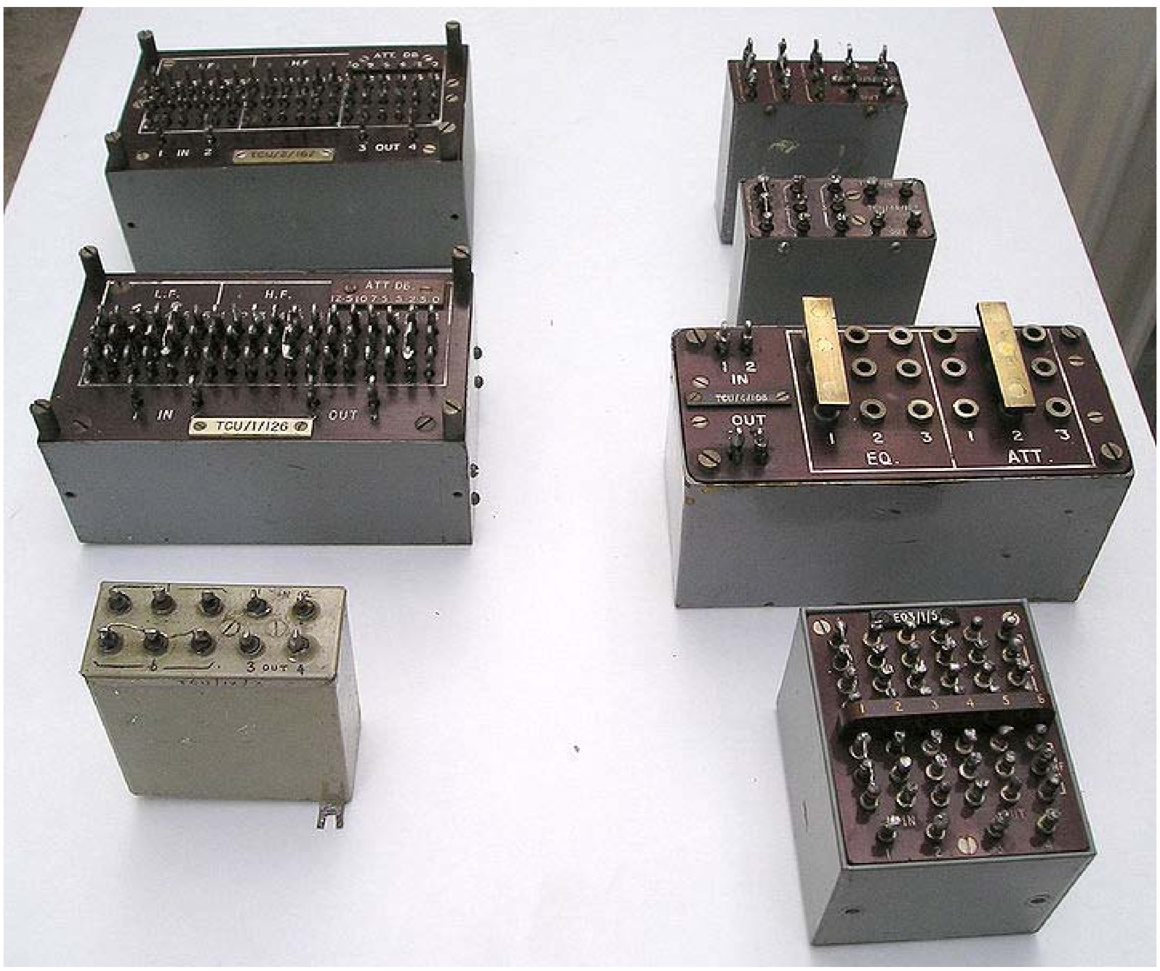
Occasionally, a low-pass section is included to compensate for excessive line transformer roll-off at the low frequency end. However, this effect is usually very small compared to the other effects noted above.

Low frequency sections will usually have inductors of high values. Such inductors have many turns and consequently tend to have significant resistance. In order to keep the section constant resistance at the input, the dual branch of the bridge T must contain a dual of the stray resistance, that is, a resistor in parallel with the capacitor. Even with the compensation, the stray resistance still has the effect of inserting attenuation at low frequencies. This in turn has the effect of slightly reducing the amount of LF lift the section would otherwise have produced. The basic loss of the section can be increased by the same amount as the stray resistance is inserting and this will return the LF lift achieved to that designed for.

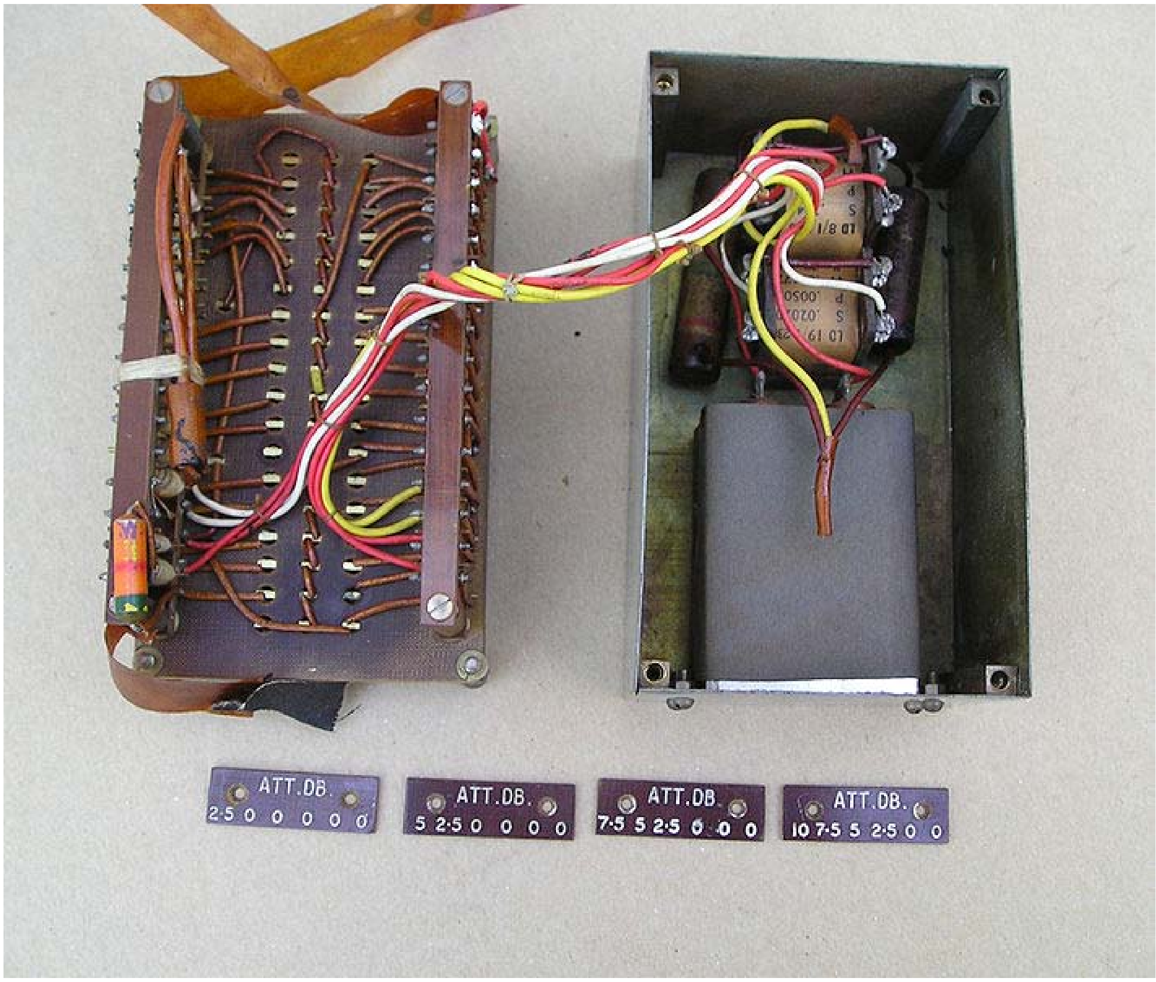
Compensation of inductor resistance is not such an issue at high frequencies were the inductors will tend to be smaller. In any case, for a high-pass section the inductor is in series with the basic loss resistor and the stray resistance can merely be subtracted from that resistor. On the other hand, the compensation technique may be required for resonant sections, especially a high Q resonator being used to lift a very narrow band. For these sections the value of inductors can also be large.

Temperature compensation

An adjustable attenuation high-pass section can be used to compensate for changes in ground temperature. Ground temperature is very slow varying in comparison to surface temperature. Adjustments are usually only required 2-4 times per year for audio applications.

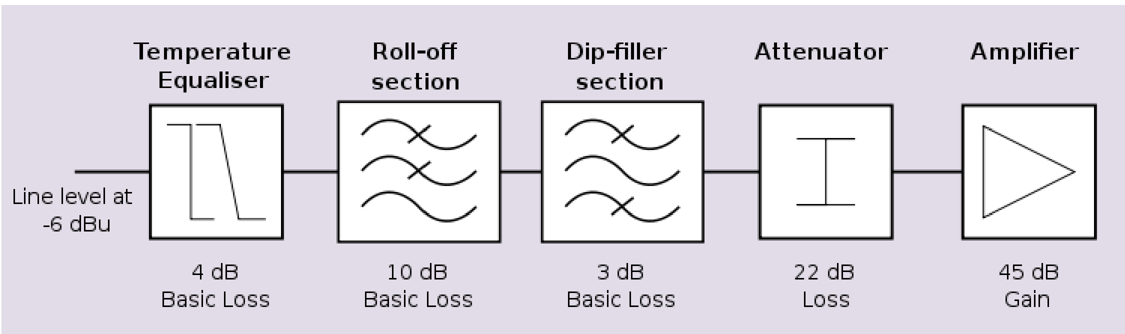


A collection of various designs of temperature compensation equaliser. Some can be adjusted with plugable links, others require soldering. Adjustment is not very frequent.



The internal components of a temperature equaliser. The inductor and capacitor on the right set the frequency at which the equaliser starts to operate, the banks of selectable resistors on the left set the basic loss and hence amount of equalisation.

Typical filter chain



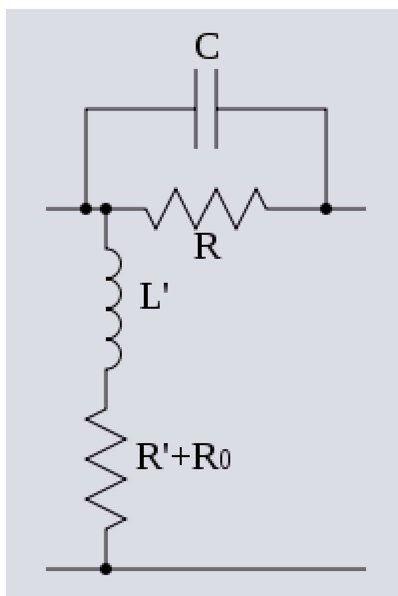
An example of a typical chain of Zobel networks being used for line equalisation

A typical complete filter will consist of a number of Zobel sections for roll-off, frequency dips and temperature followed by a flat attenuator section to bring the level down to a standard attenuation. This is followed by a fixed gain amplifier to bring the signal back up to a usable level, typically 0dBu. The gain of the amplifier is usually no more than 45dB maximum. Any more and the amplification of line noise will tend to cancel out the quality benefits of improved bandwidth. This limit on amplification essentially limits how much the bandwidth can be increased by these techniques. It should also be noted that no one part of the incoming signal band will be amplified by the full 45dB. The 45dB is made up of the line loss in the flat part of its spectrum plus the basic loss of each section. In general, each section will be minimum loss at a different frequency band, hence the amplification in that band will be limited to the basic loss of just that one filter section, assuming insignificant overlap. A typical choice for R_0 is 600 Ω. A good quality transformer (usually essential, but not shown on the diagram), known as a repeating coil, is at the beginning of the chain where the line terminates.

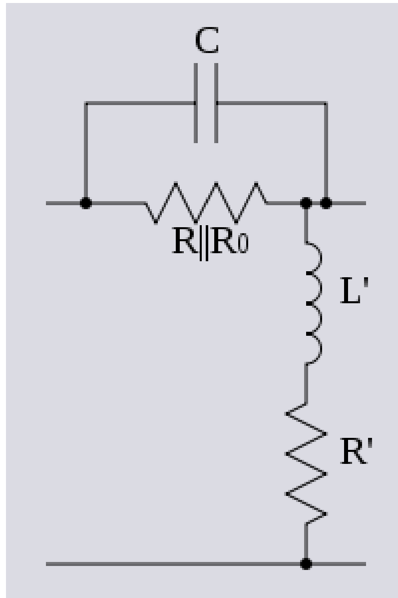
Other section implementations

Besides the Bridged T, there are a number of other possible section forms that can be used.

L-sections



Open circuit derived Zobel L-section for a high-pass section with basic loss

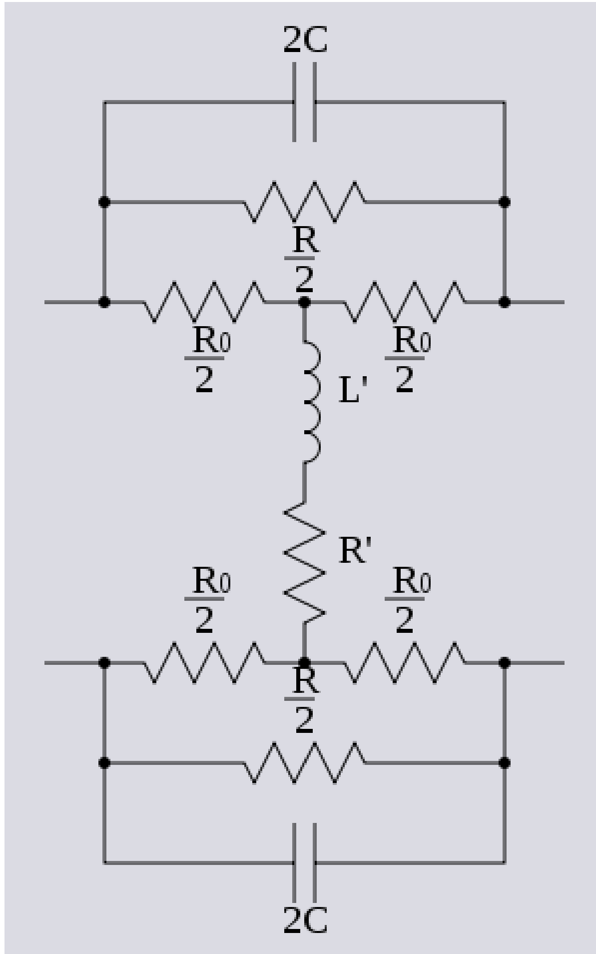


Short circuit derived Zobel L-section for a high-pass section with basic loss

As mentioned above, Z_B can be set to any desired impedance without affecting the input impedance. In particular, setting it as either an open circuit or a short circuit results in a simplified section circuit, called L–sections. These are shown above for the case of a high pass section with basic loss.

The input port still presents an impedance of R_0 (provided that the output is terminated in R_0) but the output port no longer presents a constant impedance. Both the open-circuit and the short-circuit L–sections are capable of being reversed so that R_0 is then presented at the output and the variable impedance is presented at the input.

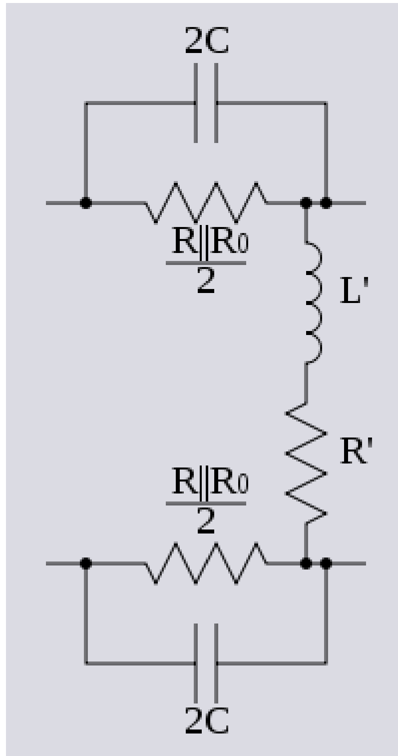
To retain the benefit of Zobel networks constant impedance, the variable impedance port must not face the line impedance. Nor should it face the variable impedance port of another half section. Facing the amplifier is acceptable since the input impedance of the amplifier is normally arranged to be R_0 within acceptable tolerances. In other words, variable impedance must not face variable impedance.



A balanced bridged T high-pass full section with basic loss

Balanced bridged T

The Zobel networks described here can be used to equalise land lines composed of twisted pair or star quad cables. The balanced circuit nature of these lines delivers a good common mode rejection ratio (CMRR). To maintain the CMRR, circuits connected to the line should maintain the balance. For this reason, balanced versions of Zobel networks are sometimes required. This is achieved by halving the impedance of the series components and then putting identical components in the "earthy" leg of the circuit.



A balanced Zobel high-pass short-circuit derived C-section with basic loss

Balanced C-sections

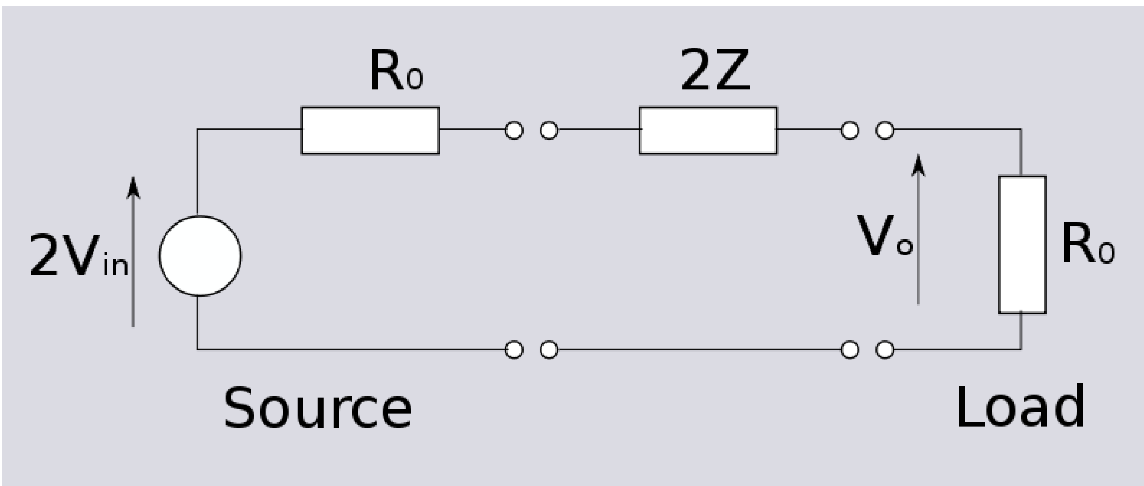
A C–section is a balanced version of an L–section. The balance is achieved in the same way as a balanced full bridged T section by placing half of the series impedance in, what was, the common conductor. C–sections, like the L–section from which they are derived, can come in both open-circuit and short circuit varieties. The same restrictions apply to C–sections regarding impedance terminations as to L–sections.

X-section

It is possible to transform a bridged–T section into a Lattice, or X–section. The X–section is a kind of bridge circuit, but usually drawn as a lattice, hence the name. Its topology makes it intrinsically balanced but it is never used to implement the constant resistance filters of the kind described here because of the increased component count. The component count increase arises out of the transformation process rather than the balance. There is however, one common application for this topology, the lattice phase equaliser, which is also constant resistance and also invented by Zobel. This circuit differs from those described here in that the bridge circuit is not generally in the balanced condition.

Half sections

In respect of constant resistance filters, the term half section has a somewhat different meaning to other kinds of image filter. Generally, a half section is formed by cutting through the mid-point of the series impedance and shunt admittance of a full section of a ladder network. It is literally half a section. Here, however, there is a somewhat different definition. A half section is either the series impedance (series half-section) or shunt admittance (shunt half-section) that, when connected between source and load impedances of R_0, will result in the same transfer function as some arbitrary constant resistance circuit. The purpose of using half sections is that the same functionality is achieved with a drastically reduced component count.



A general Zobel series half section showing the equality of transfer function to an equivalent constant resistance section

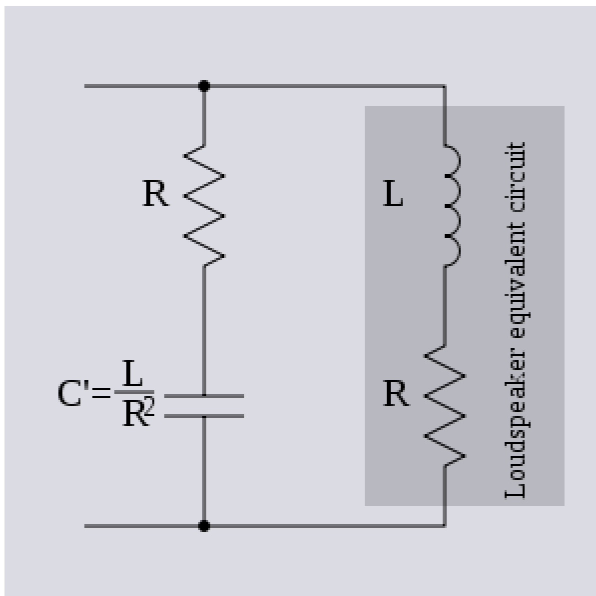
If a constant resistance circuit has an input V_{in}, then a generator with an impedance R_0 must have an open-circuit voltage of $E=2V_{in}$ in order to produce V_{in} at the input of the constant resistance circuit. If now the constant resistance circuit is replaced by an impedance of 2Z, as in the diagram above, it can be seen by simple symmetry that the voltage V_{in} will appear half way along the impedance 2Z. The output of this circuit can now be calculated as,

$$\frac{V_O}{V_{in}} = \frac{R_0}{Z + R_0}$$

which is precisely the same as a bridged T section with series element Z. The series half-section is thus a series impedance of 2Z. By corresponding reasoning, the shunt half-section is a shunt impedance of ½Z' (or twice the admittance).

It must be emphasised that these half sections are far from being constant resistance. They have the same transfer function as a constant resistance network, but only when correctly terminated. An equaliser will not give good results if a half-section is positioned facing the line since the line will have a variable (and probably unknown) impedance. Likewise, two half-sections cannot be connected directly to each other as these both will have variable impedances. However, if a sufficiently large attenuator is placed between the two variable impedances, this will have the effect of masking the effect. A high value attenuator will have an input impedance $\approx R_0$ no matter what the terminating impedance on the other side. In the example practical chain shown above there is a 22dB attenuator required in the chain. This does not need to be at the end of the chain, it can be placed anywhere desired and used to mask two mismatched impedances. It can also be split into two or more parts and used for masking more than one mismatch.

Zobel networks and loudspeaker drivers



Zobel network correcting loudspeaker impedance

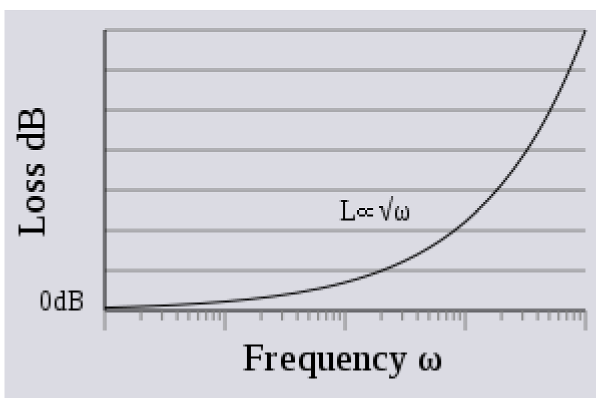
Zobel networks can be used to make the impedance a loudspeaker presents to its amplifier output appear as a steady resistance. This is beneficial to the amplifier performance. The impedance of a loudspeaker is partly resistive. The resistance representing the energy transferred from the amplifier to the sound output plus some heating losses in the loudspeaker. However, the speaker also possesses inductance due to the windings of its coil. The impedance of the loudspeaker is thus typically modelled as a series resistor and inductor. A parallel circuit of a series resistor and capacitor of the correct values will form a Zobel bridge. It is obligatory to choose $R_B = \infty$ because the centre point between the inductor and resistor is inaccessible (and, in fact, fictitious - the resistor and inductor are distributed quantities as in a transmission line). The loudspeaker may be modelled more accurately by a more complex equivalent circuit. The compensating Zobel network will also become more complex to the same degree.

Note that the circuit will work just as well if the capacitor and resistor are interchanged. In this case the circuit is no longer a Zobel balanced bridge but clearly the impedance has not changed. The same circuit could have been arrived at by designing from Boucherot's minimising reactive power point of view. From this design approach there is no difference in the order of the capacitor and the resistor and Boucherot cell might be considered a more accurate description.

Video equalisers

Zobel networks can be used for the equalisation of video lines as well as audio lines. There is, however, a noticeably different approach taken with the two types of signal. The difference in the cable characteristics can be summarised a s follows;

- Video commonly uses co-axial cable which requires an unbalanced topology for the filters whereas audio commonly uses twisted pair which requires a balanced topology.
- Video requires a wider bandwidth and tighter differential phase specification which in turn results in a tighter dimensional specification for the cable.
- The tighter specifications for video cable tends to produce a substantially constant characteristic impedance over a wide band (usually nominally 75 Ω). On the other hand, audio cable may be nominally 600 Ω (300 Ω and 150 Ω are also standard values), but it will only actually measure this value at 800 Hz. At a lower frequencies it will be much higher and at higher frequencies will be lower and more reactive.

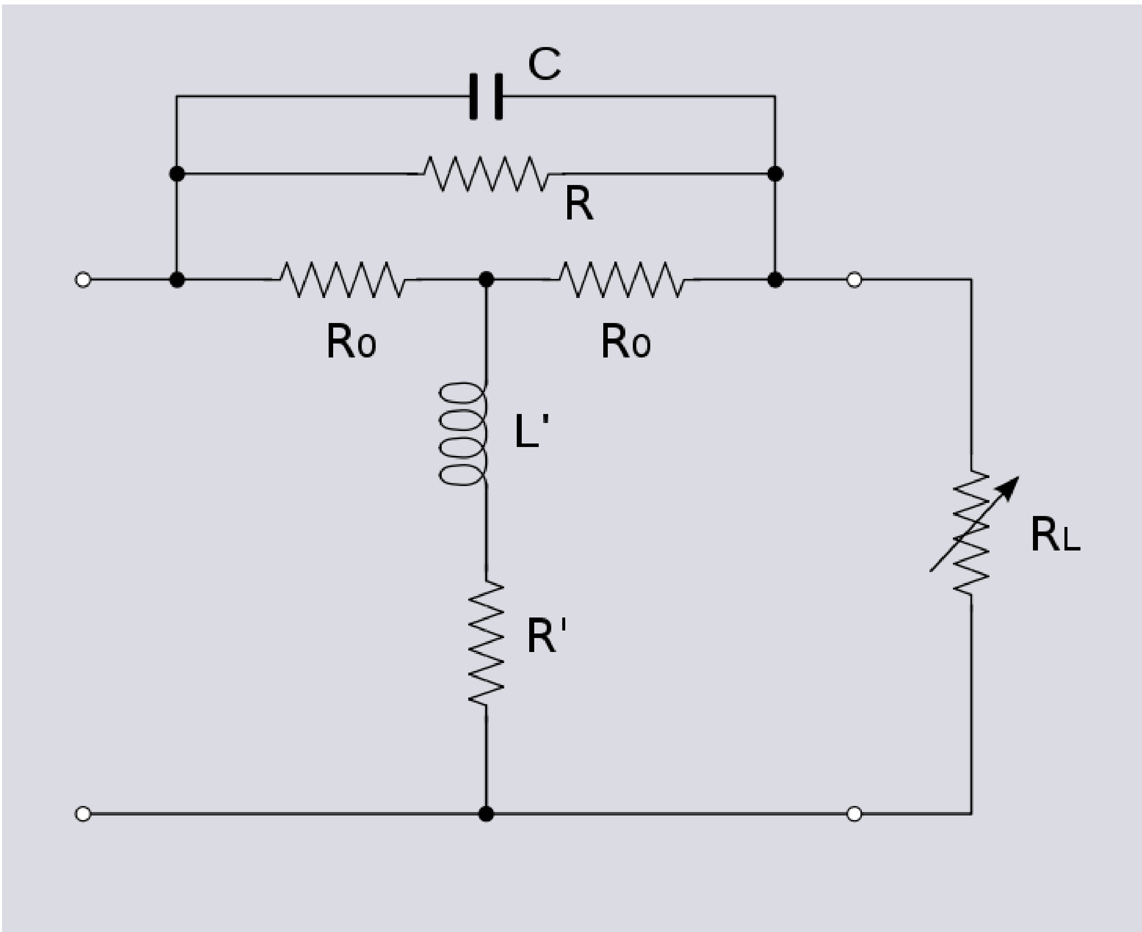


Response plot of a video line showing the typical $\sqrt{f}$ response

- These characteristics result in a smoother, more well behaved response for video lines with none of the nasty discontinuities typically found with audio lines. These discontinuities in the frequency response are often caused by the habit of the telecom companies of forming a connection by joining two shorter lines of differing characteristic impedance. Video lines on the other hand tend to roll off smoothly with frequency in a predictable way.

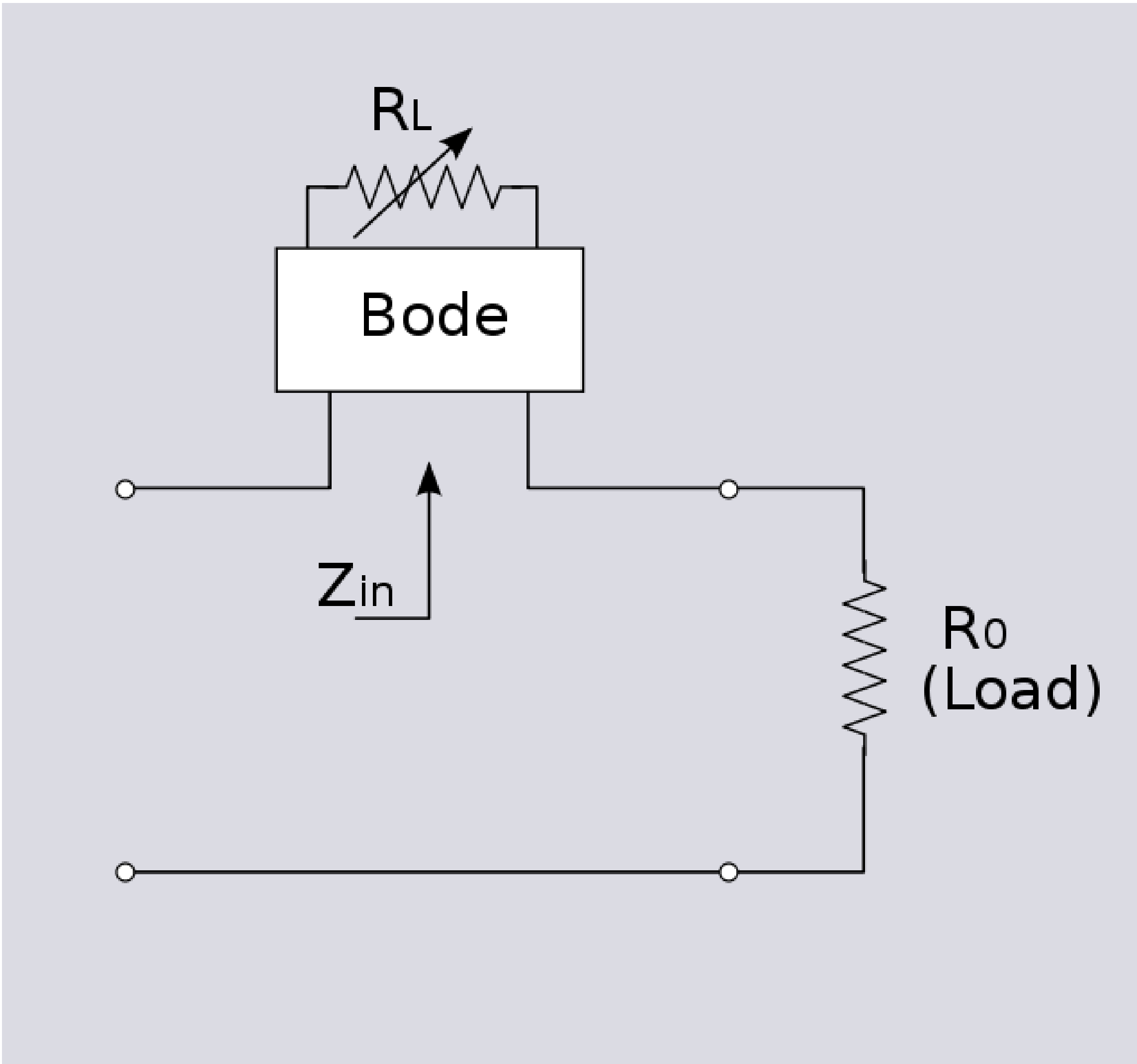
This more predictable response of video allows a different design approach. The video equaliser is built as a single bridged T section but with a rather more complex network for Z. For short lines, or for a trimming equaliser, a Bode filter topology might be used. For longer lines a network with Cauer filter topology might be used. Another driver for this approach is the fact that a video signal occupies a large number of octaves, around 20 or so. If equalised with simple basic sections, a large number of filter sections would be required. Simple sections are designed, typically, to equalise a range of one or two octaves.

Bode equaliser

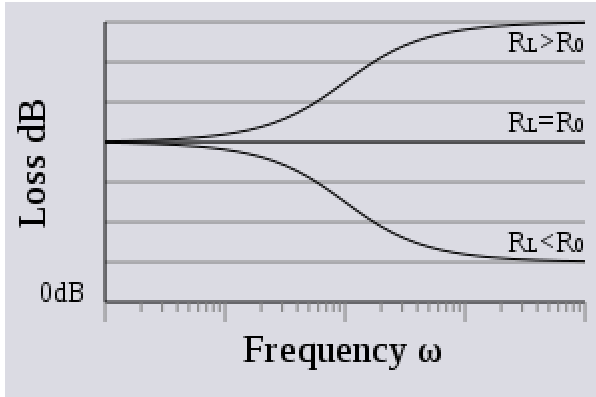


Bode network for high frequency equalisation

A Bode network, as with a Zobel network, is a symmetrical bridge T network which meets the constant k condition. It does not however meet the constant resistance condition, that is, the bridge is not in balance. Any impedance network, Z, can be used in a Bode network, just as with a Zobel network, but the high pass section shown for correcting high-end frequencies is the most common. A Bode network terminated in a variable resistor can be used to produce a variable impedance at the input terminals of the network. A useful property of this network is that the input impedance can be made to vary from a capacitive impedance through a purely resistive impedance to an inductive impedance all by adjusting the single load potentiometer, R_L. The bridging resistor, R_0, is chosen to equal the nominal impedance so that in the special case when R_L is set to R_0 the network behaves as a Zobel network and Z_{in} is also equal to R_0.



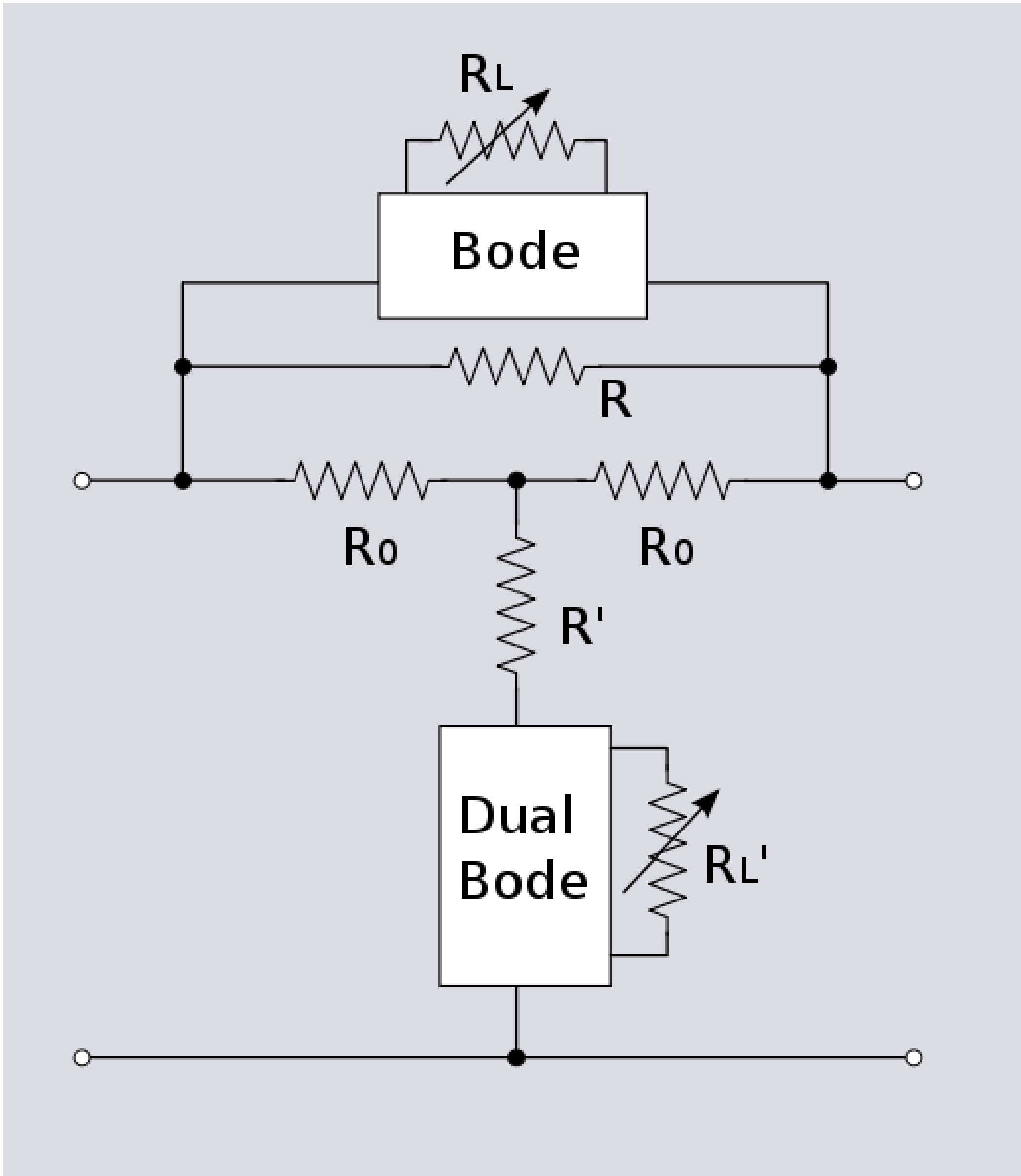
Bode network used in an equaliser circuit



Bode trimming equaliser response plot

The Bode network is used in an equaliser by connecting the whole network such that the input impedance of the Bode network, Z_{in}, is in series with the load. Since the impedance of the Bode network can be either capacitive or inductive depending on the position of the adjustment potentiometer, the response may be a boost or a cut to the band of frequencies it is acting on. The transfer function of this arrangement is:

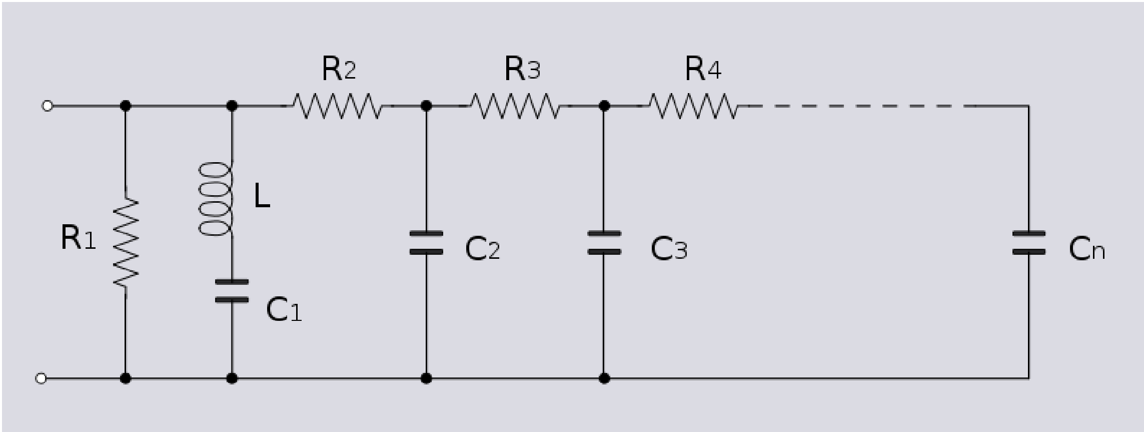
$$A(\omega) = \frac{R_0}{Z_{in} + R_0}$$



Bode equaliser implemented as a Zobel constant resistance equaliser

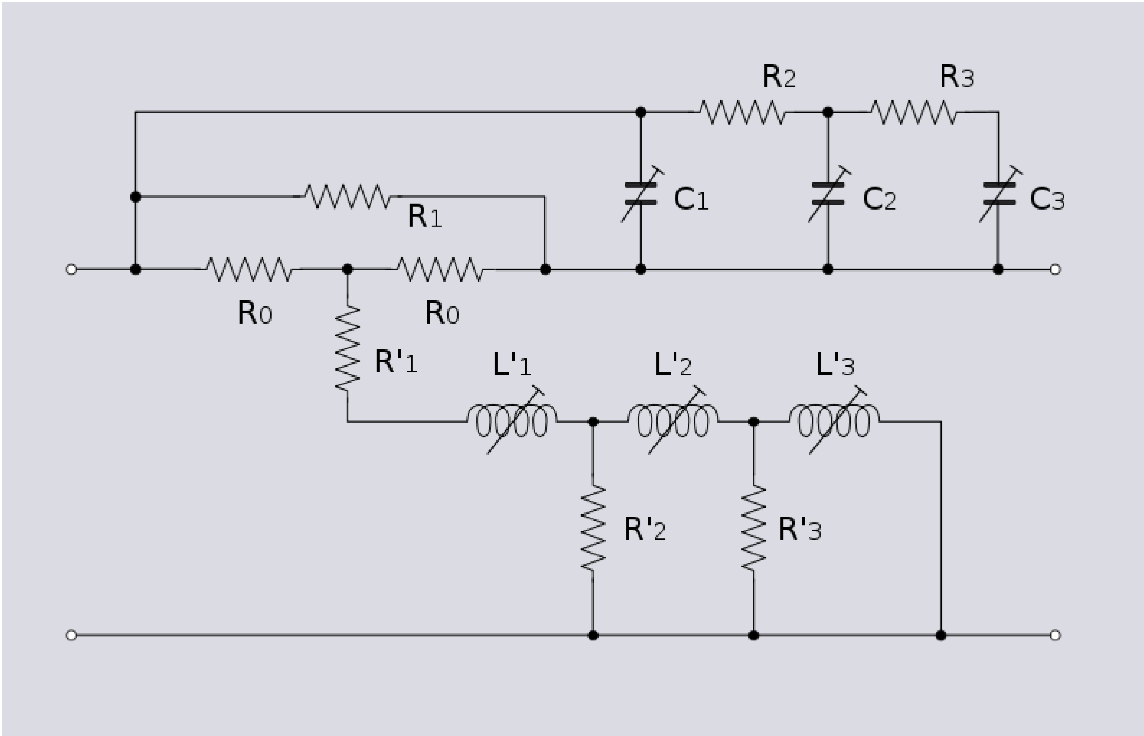
The Bode equaliser can be converted into a constant resistance filter by using the entire Bode network as the Z branch of a Zobel network, resulting in a rather complex network of bridge T networks embedded in a larger bridge T. It can be seen that this results in the same transfer function by noting that the transfer function of the Bode equaliser is identical to the transfer function of the general form of Zobel equaliser. Note that the dual of a constant resistance bridge T network is the identical network. The dual of a Bode network is therefore the same network except for the load resistance R_L, which must be the inverse, R_L', in the dual circuit. To adjust the equaliser R_L and R_L' must be ganged, or otherwise kept in step such that as R_L increases R_L' will decrease and vice versa.

Cauer equaliser



Cauer topology network to be used as the Z impedance of a Zobel network equaliser

To equalise long video lines, a network with Cauer topology is used as the Z impedance of a Zobel constant resistance network. Just as the input impedance of a Bode network is used as the Z impedance of a Zobel network to form a Zobel Bode equaliser, so the input impedance of a Cauer network is used to make a Zobel Cauer equaliser. The equaliser is required to correct an attenuation increasing with frequency and for this a Cauer ladder network consisting of series resistors and shunt capacitors is required. Optionally, there may be an inductor included in series with the first capacitor which increases the equalisation at the high end due to the steeper slope produced as resonance is approached. This may be required on longer lines. The shunt resistor R_1 provides the basic loss of the Zobel network in the usual way.



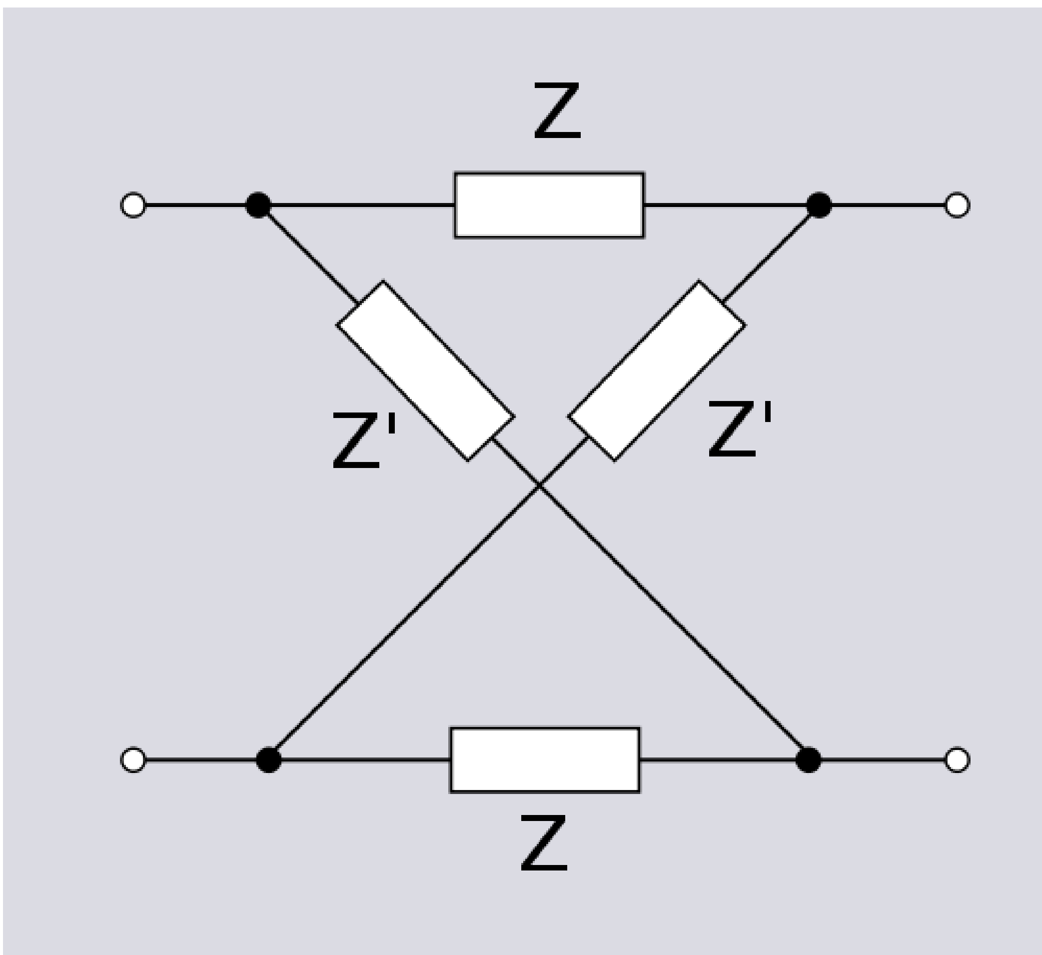
Cauer Zobel bridge T implemented video equaliser. The impedance Z of this example consists of a three section ladder and is suitable for the equalisation of short lines (between nearby buildings for instance)

The dual of a RC Cauer network is a LR Cauer network which is required for the Z' impedance as shown in the example. Adjustment is a bit problematic with this equaliser. In order to maintain the constant resistance, the pairs of components C_1/L_1', C_2/L_2' etc., must remain dual impedances as the component is adjusted, so both parts of the pair must be adjusted together. With the Zobel Bode equaliser, this is a simple matter of ganging two pots together - a component configuration available off-the-shelf. Ganging together a variable capacitor and inductor is not, however, a very practical solution. These equalisers tend to be "hand built", one solution being to select the capacitors on test and fit fixed values according to the measurements and then adjust the inductors until the required match is achieved. The furthest element of the ladder from the driving point is equalising the lowest frequency of interest. This is adjusted first as it will also have an effect on higher frequencies and from there progressively higher frequencies are adjusted working along the ladder towards the driving point.

Chapter- 5

Lattice Phase Equalizer and General mn-type Image Filter

Lattice phase equaliser



Lattice filter topology

A **lattice phase equaliser** or **lattice filter** is an example of an all-pass filter. That is, the attenuation of the filter is constant at all frequencies but the relative phase between input and output varies with frequency. The lattice filter topology has the particular property of being a constant-resistance network and for this reason is often used in combination with other constant resistance filters such as bridge-T equalisers. The topology of a lattice filter, also called an **X-section** is identical to bridge topology. The lattice phase equaliser was invented by Otto Zobel. using a filter topology proposed by George Campbell.

The characteristic impedance of this structure is given by;

$$Z_o^2 = ZZ'$$

and the transfer function is given by;

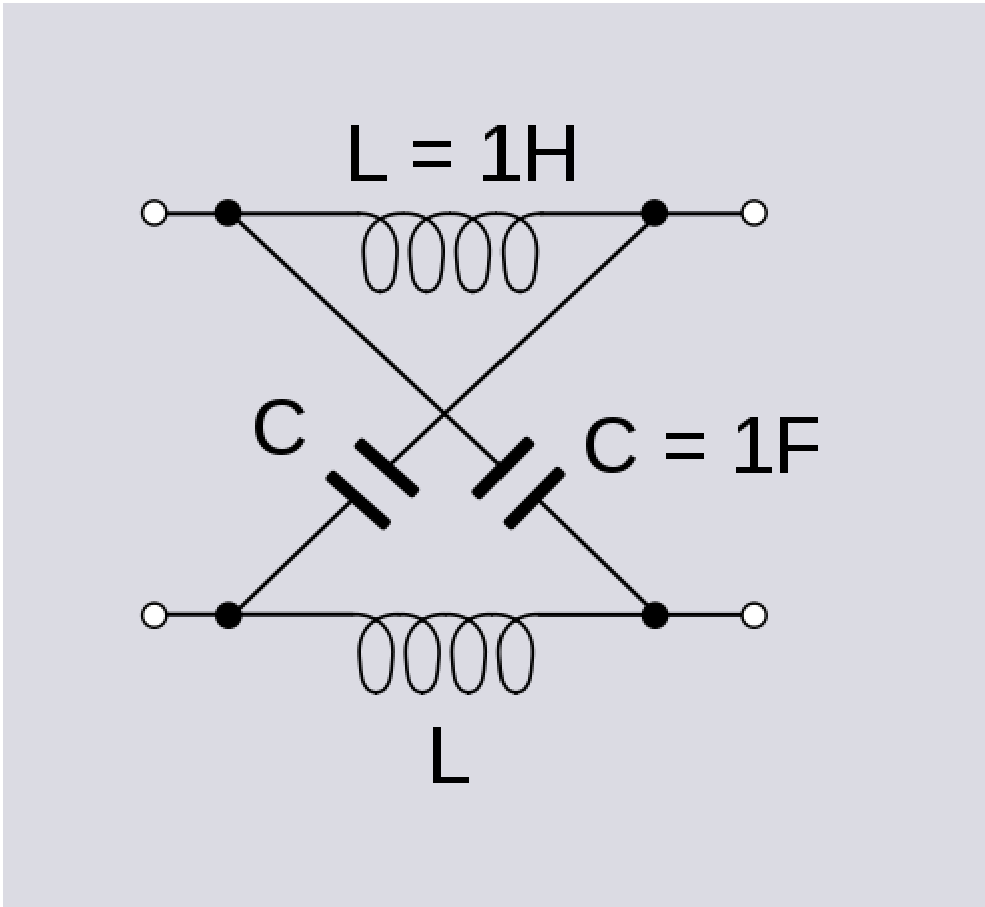
$$H(\omega) = \frac{Z_o - Z}{Z_o + Z}$$

Applications

The lattice filter has an important application on lines used by broadcasters for stereo audio feeds. Phase distortion on a monophonic line does not have a serious effect on the quality of the sound unless it is very large. The same is true of the absolute phase distortion on each leg (left and right channels) of a stereo pair of lines. However, the differential phase between legs has a very dramatic effect on the stereo image. This is because the formation of the stereo image in the brain relies on the phase difference information from the two ears. A phase difference translates to a delay, which in turn can be interpreted as a direction the sound came from. Consequently, landlines used by broadcasters for stereo transmissions are equalised to very tight differential phase specifications.

Another property of the lattice filter is that it is an intrinsically balanced topology. This is useful when used with landlines which invariably use a balanced format. Many other types of filter section are intrinsically unbalanced and have to be transformed into a balanced implementation in these applications which increases the component count. This is not required in the case of lattice filters.

Design

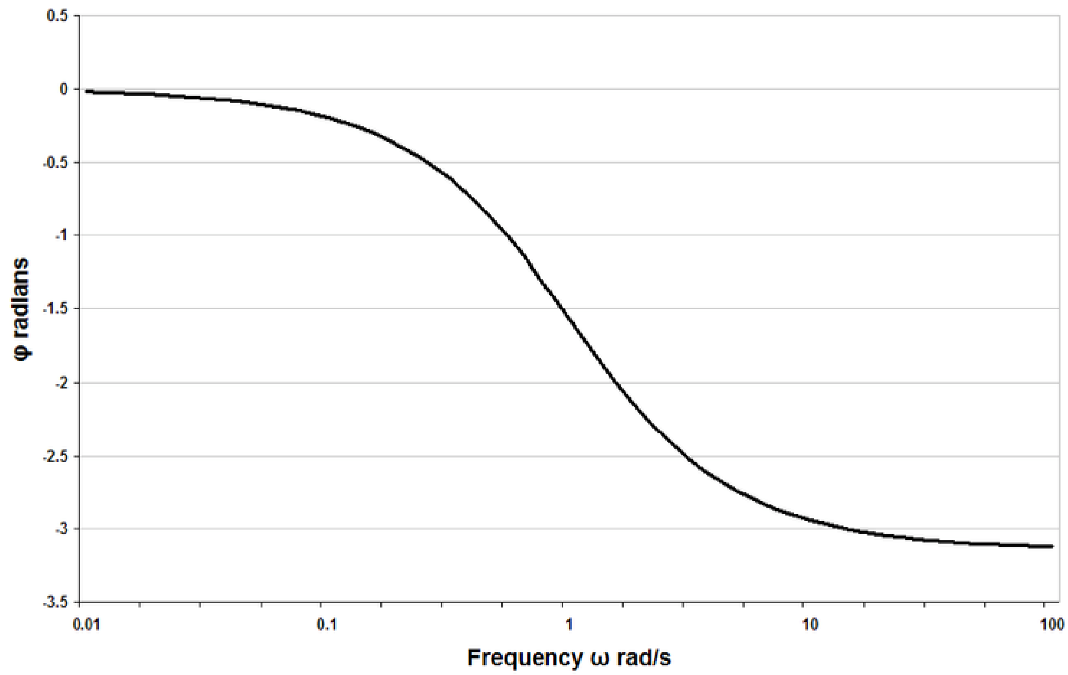


A prototype lattice filter which passes low frequencies without phase shifting

The essential requirement for a lattice filter is that for it to be constant resistance, the lattice element of the filter must be the dual of the series element with respect to the characteristic impedance. That is,

$$\frac{Z}{R_0} = \frac{R_0}{Z'}$$

Such a network, when terminated in R_0, will have an input resistance of R_0 at all frequencies. If the impedance Z is purely reactive such that Z = iX then the phase shift, φ, inserted by the filter is given by,

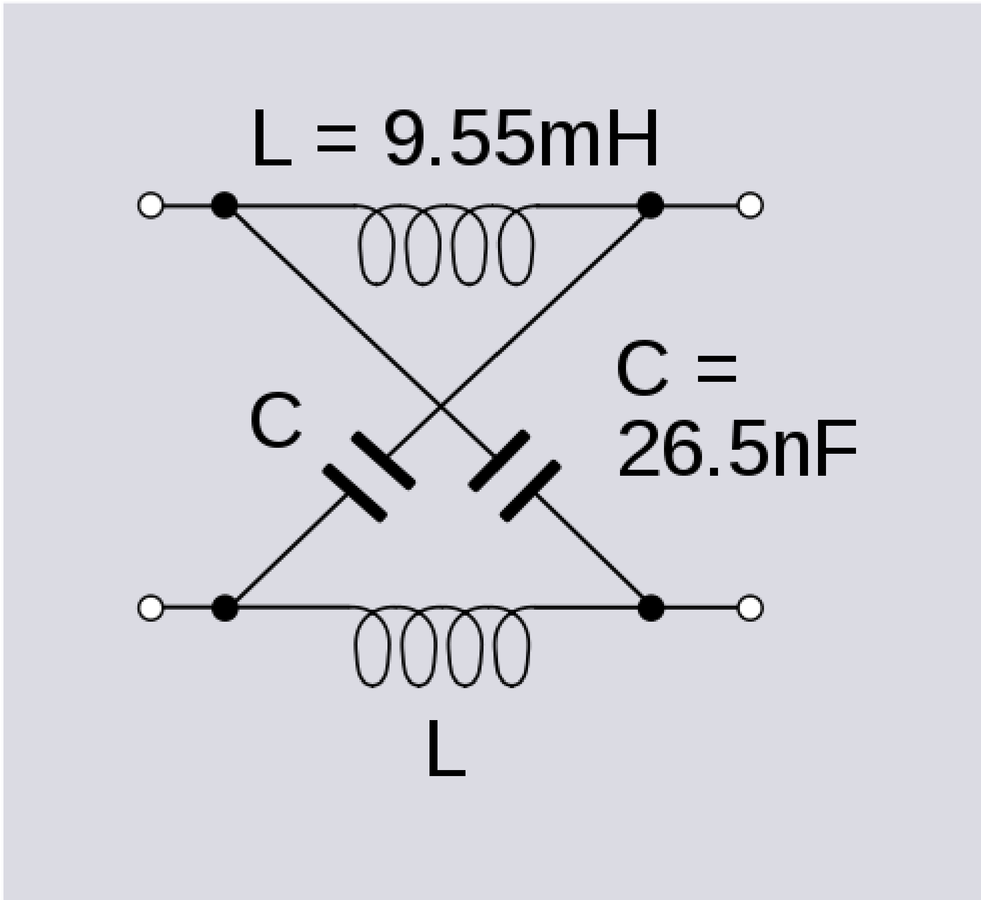


Prototype lattice filter response ranging from 0 radians at low frequencies to -π radians at high frequencies

$$\tan\frac{\varphi}{2} = -\frac{X}{R_0}$$

The prototype lattice filter shown here passes low frequencies without modification but phase shifts high frequencies. That is, it is phase correction for the high end of the band. At low frequencies the phase shift is 0° but as the frequency increases the phase shift approaches 180°. It can be seen qualitatively that this is so by replacing the inductors with open circuits and the capacitors with short circuits, which is what they become at high frequency. At high frequency the lattice filter is a cross-over network and will produce 180° phase shift. A 180° phase shift is the same as an inversion in the frequency domain, but is a delay in the time domain. At an angular frequency of ω = 1 rad/s the phase shift is exactly 90° and this is the mid-point of the filter's transfer function.

Low-in-phase section



Lattice filter transformed from the prototype to operate at 10 kHz mid-point and 600Ω terminations

The prototype section can be scaled and transformed to the desired frequency, impedance and bandform by applying the usual prototype filter transforms. A filter which is in-phase at low frequencies (that is, one that is correcting phase at high frequencies) can be obtained from the prototype with simple scaling factors.

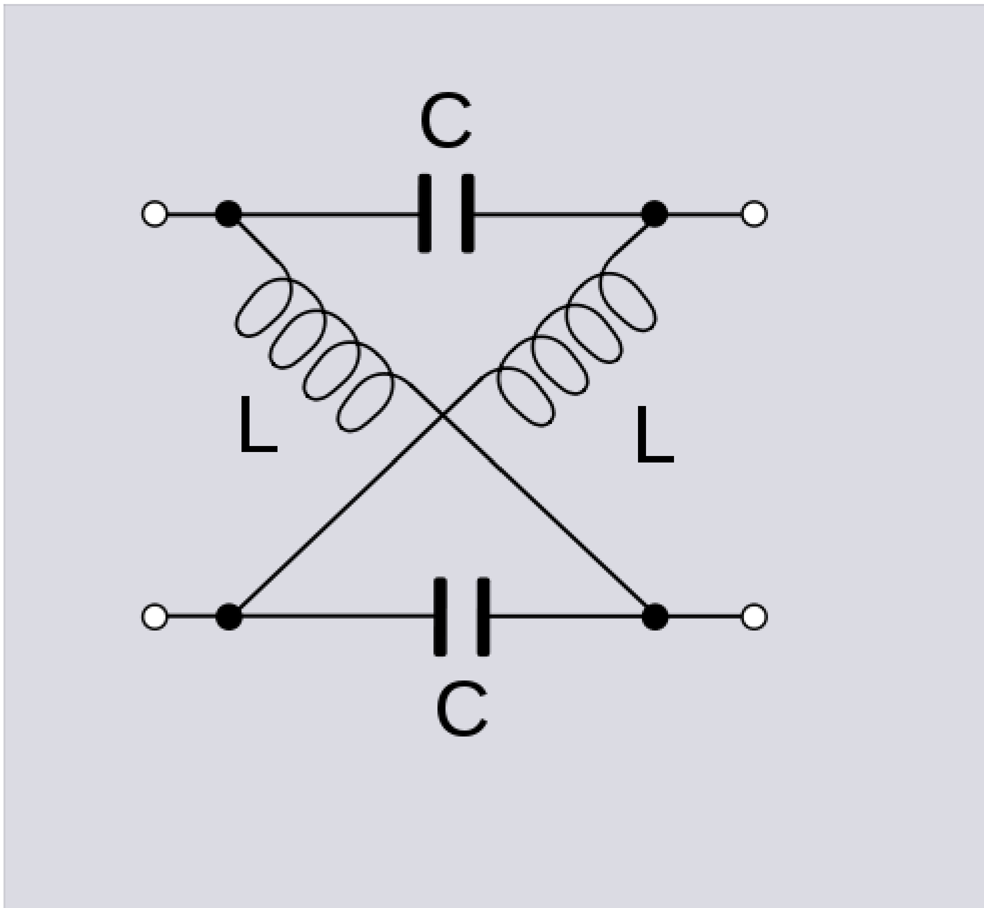
The phase response of a scaled filter is given by,

$$\tan\frac{\varphi}{2} = -\frac{\omega}{\omega_m}$$

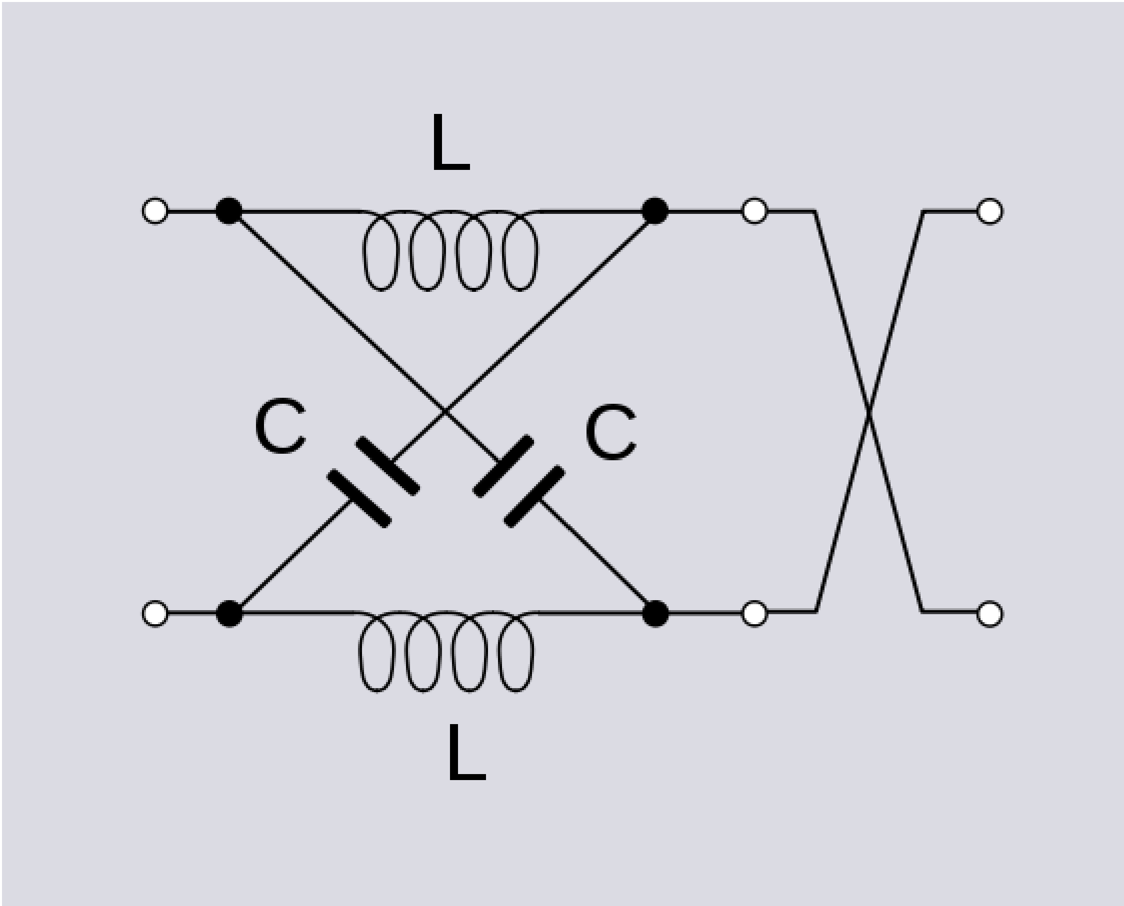
where ω_m is the mid-point frequency and is given by,

$$\omega_m = \frac{1}{\sqrt{LC}}$$

High-in-phase section



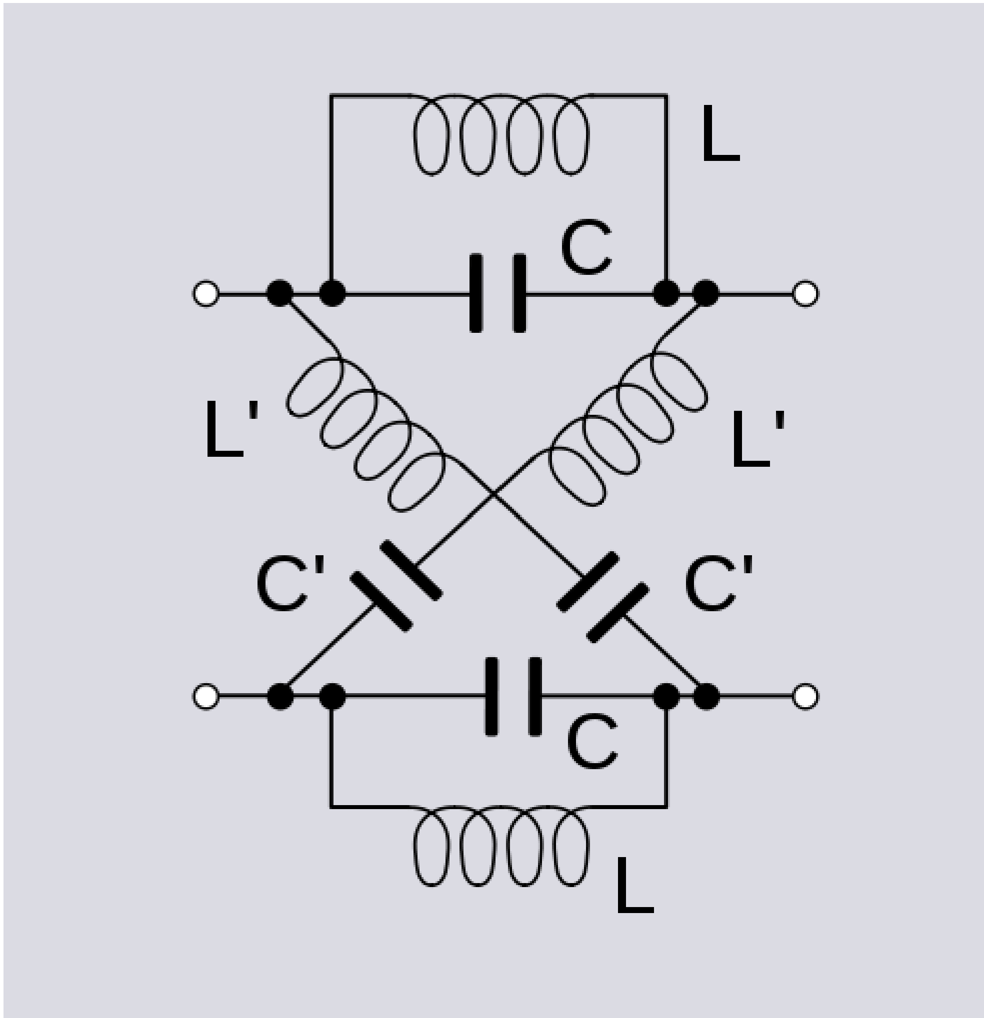
Lattice filter for low end phase correction



Demonstration that a low-in-phase section in cascade with a crossover is equivalent to a high-in-phase section

A filter that is in-phase at high frequencies (that is, a filter to correct low-end phase) can be obtained by applying the high-pass transformation to the prototype filter. However, it can be seen that due to the lattice topology this is also equivalent to a crossover on the output of the corresponding low-in-phase section. This second method may not only make calcualtion easier but it is also a useful property where lines are being equalised on a temporary basis, for instance for outside broadcasts. It is desirable to keep the number of different types of adjustable sections to a minimum for temporary work and being able to use the same section for both high end and low end correction is a distinct advantage.

Band equalise section

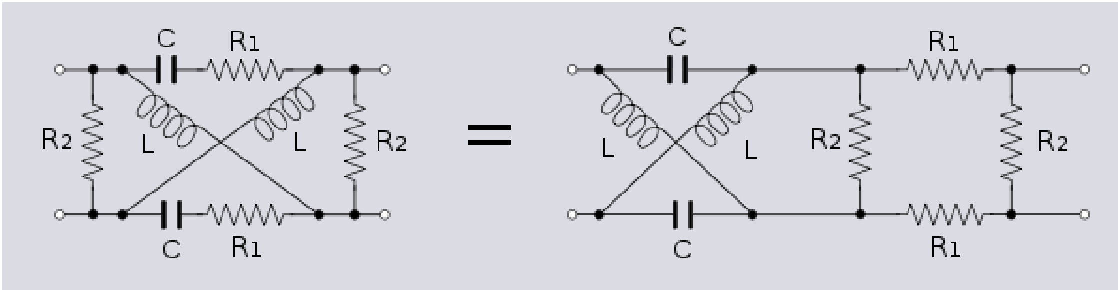


Lattice filter for phase correction of a limited band

A filter that corrects a limited band of frequencies (that is, a filter that is in phase everywhere except in the band being corrected) can be obtained by applying the band-stop transformation to the prototype filter. This results in resonant elements appearing in the filter's network.

An alternative, and possibly more accurate, view of this filter's response is to describe it as a phase change that varies from 0° to 360° with increasing frequency. At 360° phase shift, of course, the input and output are now back in phase with each other.

Resistance compensation

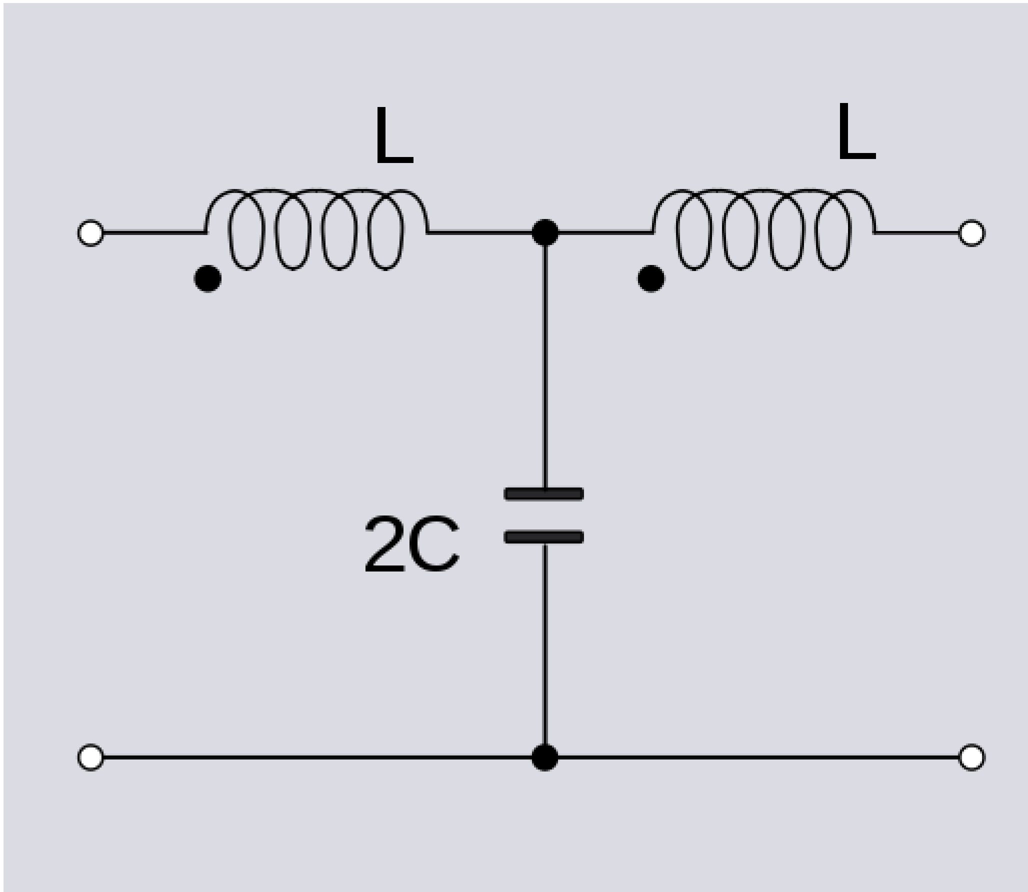


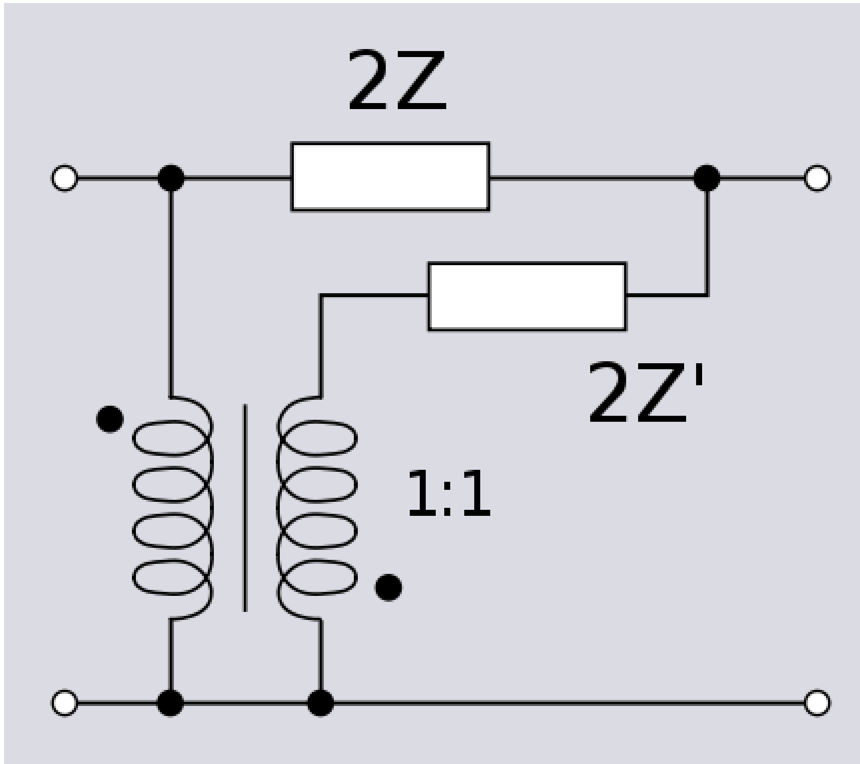
A lattice filter with compensation for the resistance of its inductors and its equivalent circuit

With ideal components there is no need to use resistors in the design of lattice filters. However, practical considerations of properties of real components leads to resistors being incorporated. Sections designed to equalise low audio frequencies will have larger inductors with a high number of turns. This results in significant resistance being in the inductive branches of the filter, which in turn causes attenuation at low frequencies.

In the example diagram, the resistors placed in series with the capacitors, R_1, are made equal to the unwanted stray resistance present in the inductors. This ensures that the attenuation at high frequency is the same as the attenuation at low frequency and brings the filter back to a flat response. The purpose of the shunt resistors, R_2, is to bring the image impedance of the filter back to the original design R_0. The resulting filter is the equivalent of a box attenuator formed from the R_1's and R_2's connected in cascade with an ideal lattice filter as shown in the diagram.

Unbalanced topology



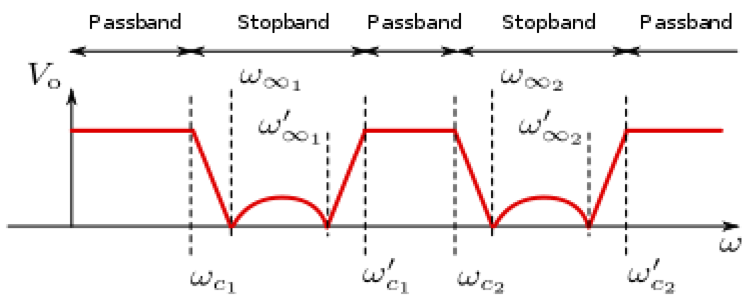


The lattice phase equaliser cannot be directly transformed into T-section topology without introducing active components. However, a T-section is possible if ideal transformers are introduced. Transformer action can be conveniently achieved in the low-in-phase T-section by winding both inductors on a common core. The response of this section is identical to the original lattice, however, the input is no longer constant resistance. This circuit was first used by George Washington Pierce who needed a delay line as part of the improved sonar he developed between the world wars. Pierce used a cascade of these sections to provide the required delay. The circuit can be considered a low-pass m-derived filter with $m>1$ which puts the transmission zero on the $j\omega$ axis of the complex frequency plane. Other unbalanced transformations utilising ideal transformers are possible, one such is shown on the right.

General mn-type image filter

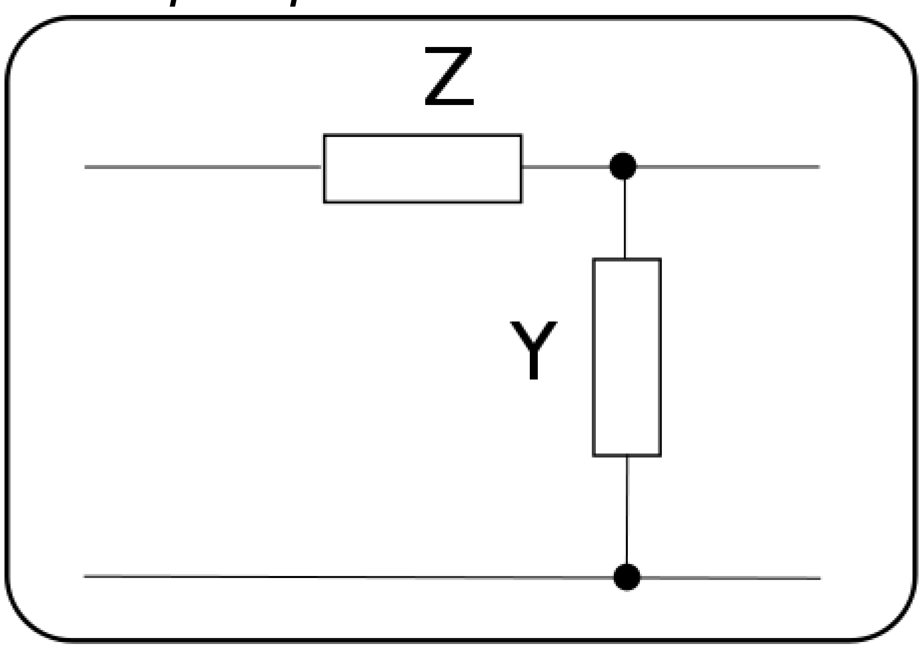
These filters are electrical wave filters designed using the image method. They are an invention of Otto Zobel at AT&T Corp.. They are a generalisation of the m-type filter in that a transform is applied that modifies the transfer function while keeping the image impedance unchanged. For filters that have only one stopband there is no distinction with the m-type filter. However, for a filter that has multiple stopbands, there is the possibility that the form of the transfer function in each stopband can be different. For instance, it may be required to filter one band with the sharpest possible cut-off, but in another to minimise phase distortion while still achieving some attenuation. If the form is identical at each transition from passband to stopband the filter will be the same as an m-type filter (k-type filter in the limiting case of *m*=1). If they are different, then the general case described here pertains.

The k-type filter acts as a prototype for producing the general m_n designs. For any given desired bandform there are two classes of m_n transformation that can be applied, namely, the mid-series and mid-shunt derived sections; this terminology being more fully explained in the m-derived filter article. Another feature of m-type filters that also applies in the general case is that a half section will have the original k-type image impedance on one side only. The other port will present a new image impedance. The two transformations have equivalent transfer functions but different image impedances and circuit topology.



Bandform diagram showing frequency response of a general image filter. The ω_c are the critical frequencies (the frequency where cut-off begins) and the ω_∞ are the poles of attenuation in the stop bands.

Mid-series multiple stopband



If Z and Y are the series impedance and shunt admittance of a constant k half section and;

$$Z = Z_1 + Z_2 + Z_3 + \ldots + Z_N$$

where Z_1, Z_2 etc are a cascade of antiresonators,

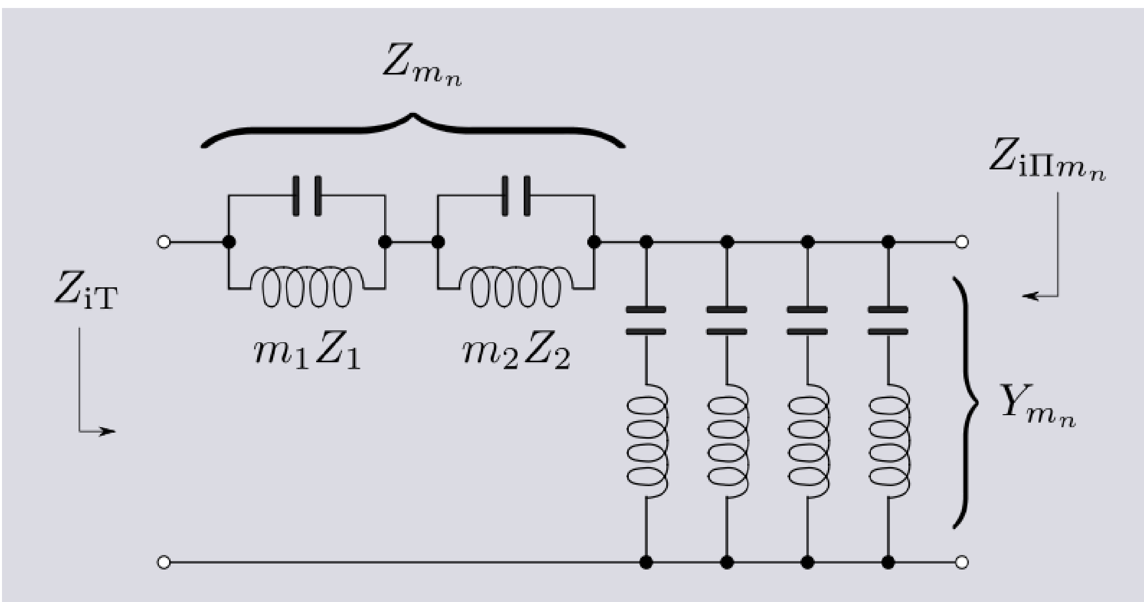
the transformed series impedance for a mid-series derived filter becomes;

$$Z_{m_n} = m_1 Z_1 + m_2 Z_2 + m_3 Z_3 + \ldots + m_N Z_N$$

Where the m_n are arbitrary positive coefficients. For an invariant image impedance Z_{iT} and invariant bandform (that is, invariant cut-off frequencies ω_c) the transformed shunt admittance, expressed in terms of Z_{mn}, is given by;

$$Y_{m_n} = \frac{Z_{m_n}}{k^2 + Z^2 - Z_{m_n}^2}$$

where $k = \sqrt{\frac{Z}{Y}}$ and is a constant by definition. When the m_n are all equal this reduces to the expression for an m-type filter and where they are all equal to one it reduces further to the k-type filter.



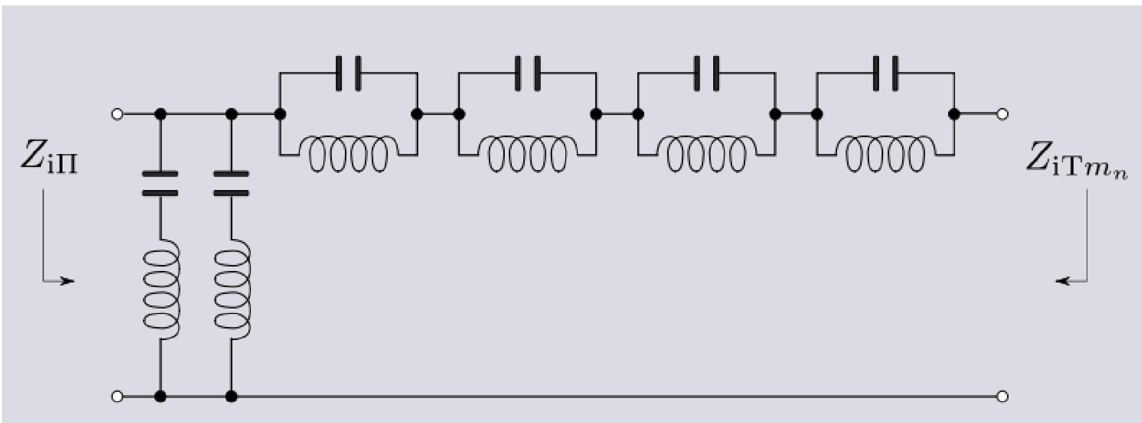
2-bandstop mid-series derived m_1, m_2 filter half-section

A result of this relationship is that the N antiresonators in Z_{mn} will transform into $2N$ resonators in Y_{mn}. The coefficients m_n can be adjusted by the designer to set the frequency of one of the two poles of attenuation, ω_∞, in each stopband. The second pole of attenuation is dependant and cannot be set separately.

Special cases

In the case of a filter with a stopband extending to zero frequency, one of the antiresonators in *Z* will reduce to a single inductor. In this case the resonators in Y_{mn} are reduced by one to 2*N*-1. Similarly, for a filter with a stopband extending to infinity, one antiresonator will reduce to a single capacitor and the resonators will again be reduced by one. In a filter where both conditions pertain, the number of resonators will be 2*N*-2. For these end stopbands, there is only one pole of attenuation in each, as would be expected from the reduced number of resonators. These forms are the maximum allowable complexity while maintaining invariance of bandform and one image impedance.

Mid-shunt multiple stopband



2-bandstop mid-shunt derived m1, m2 filter half-section. Frequency response is equivalent to the corresponding mid-series derived filter

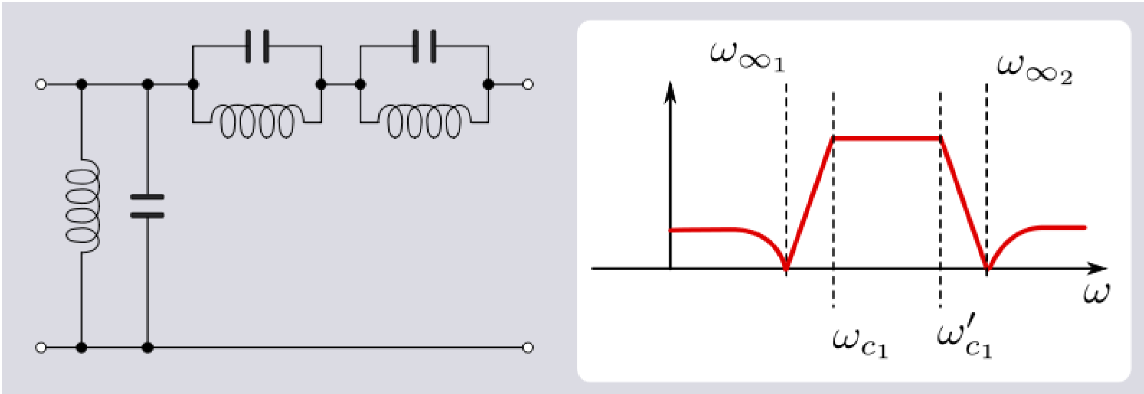
By dual analogy, the shunt derived filter starts from;

$$Y_{m_n} = m_1 Y_1 + m_2 Y_2 + m_3 Y_3 + \ldots + m_N Y_N$$

For an invariant image admittance $Y_{i\Pi}$ and invariant bandform the transformed series impedance is given by;

$$Z_{m_n} = \frac{Y_{m_n}}{k^2 + Y^2 - Y_{m_n}^2}$$

Simple bandpass section



General image bandpass filter, mid-shunt derived

The bandpass filter can be characterised as a 2-bandstop filter with $\omega_c = 0$ for the lower critical frequency of the lower band and $\omega_c = \infty$ for the upper critical frequency of the upper band. The two resonators reduce to an inductor and a capacitor respectively. The number of antiresonators reduces to two.

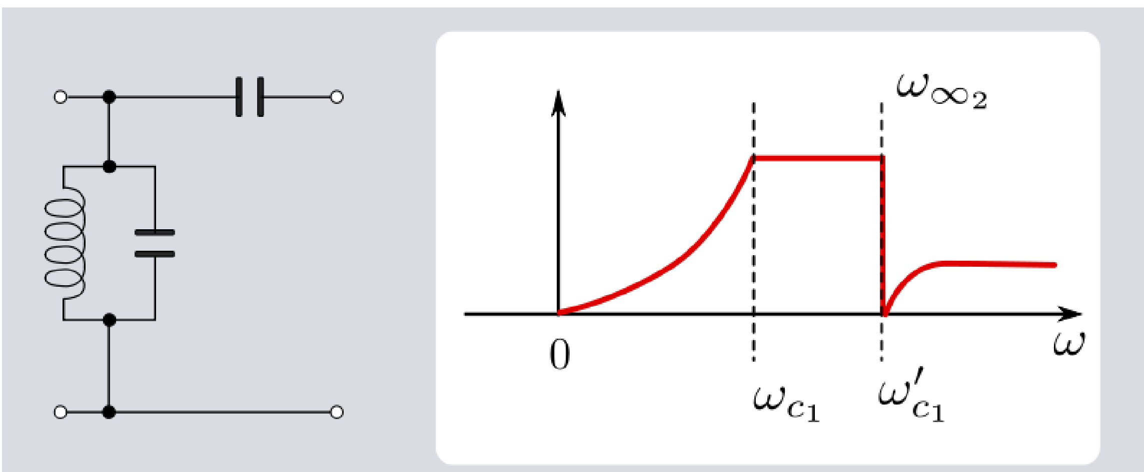


Image bandpass filter with $\omega_{\infty 1}$ set to zero and $\omega_{\infty 2}$ set to correspond to ω'_{c1}.

If, however, $\omega_{\infty 1}$ is set to zero (that is, there is no pole of attenuation in the lower stopband) and $\omega_{\infty 2}$ is set to correspond to the upper critical frequency ω'_{c1}, then a particularly simple form of the bandpass filter is obtained consisting of just antiresonators coupled by capacitors. This was a popular topology for multi-section band-pass filters due its low component count, particularly of inductors. Many other such reduced forms are possible by setting one of the poles of attenuation to correspond to one of the critical frequencies for various classes of basic filter.

Chapter- 6

Composite Image Filter

A **composite image filter** is an electronic filter consisting of multiple image filter sections of two or more different types.

The image method of filter design determines the properties of filter sections by calculating the properties they have in an infinite chain of such sections. In this, the analysis parallels transmission line theory on which it is based. Filters designed by this method are called *image parameter filters*, or just *image filters*. An important parameter of image filters is their image impedance, the impedance of an infinite chain of identical sections.

The basic sections are arranged into a ladder network of several sections, the number of sections required is mostly determined by the amount of stopband rejection required. In its simplest form, the filter can consist entirely of identical sections. However, it is more usual to use a composite filter of two or three different types of section to improve different parameters best addressed by a particular type. The most frequent parameters considered are stopband rejection, steepness of the filter skirt (transition band) and impedance matching to the filter terminations.

Image filters are linear filters and are invariably also passive in implementation.

History

The image method of designing filters originated at AT&T, who were interested in developing filtering that could be used with the multiplexing of many telephone channels on to a single cable. The researchers involved in this work and their contributions are briefly listed below;

- John Carson provided the mathematical underpinning to the theory. He invented single sideband modulation for the purpose of multiplexing telephone channels. It was the need to recover these signals that gave rise to the need for advanced filtering techniques. He also pioneered the use of operational calculus (what has now become Laplace transforms in its more formal mathematical guise) to analyse these signals.

- George Campbell worked on filtering from 1910 onwards and invented the constant k filter. This can be seen as a continuation of his work on loading coils on transmission lines, a concept invented by Oliver Heaviside. Heaviside, incidentally, also invented the operational calculus used by Carson.
- Otto Zobel provided a theoretical basis (and the name) for Campbell's filters. In 1920 he invented the m-derived filter. Zobel also published composite designs incorporating both constant k and m-derived sections.
- R S Hoyt also contributed.

The image method

The image analysis starts with a calculation of the input and output impedances (the image impedances) and the transfer function of a section in an infinite chain of identical sections. This can be shown to be equivalent to the performance of a section terminated in its image impedances. The image method, therefore, relies on each filter section being terminated with the correct image impedance. This is easy enough to do with the internal sections of a multiple section filter, because it is only necessary to ensure that the sections facing the one in question have identical image impedances. However, the end sections are a problem. They will usually be terminated with fixed resistances that the filter cannot match perfectly except at one specific frequency. This mismatch leads to multiple reflections at the filter terminations and at the junctions between sections. These reflections result in the filter response deviating quite sharply from the theoretical, especially near the cut-off frequency.

The requirement for better matching to the end impedances is one of the main motivations for using composite filters. A section designed to give good matching is used at the ends but something else (for instance stopband rejection or passband to stopband transition) is designed for the body of the filter.

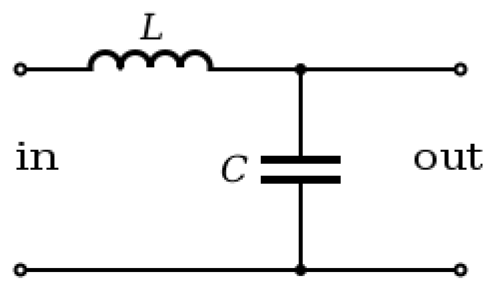
Filter section types

Each filter section type has particular advantages and disadvantages and each has the capability to improve particular filter parameters. The sections described below are the prototype filters for low-pass sections. These prototypes may be scaled and transformed to the desired frequency bandform (low-pass, high-pass, band-pass or band-stop).

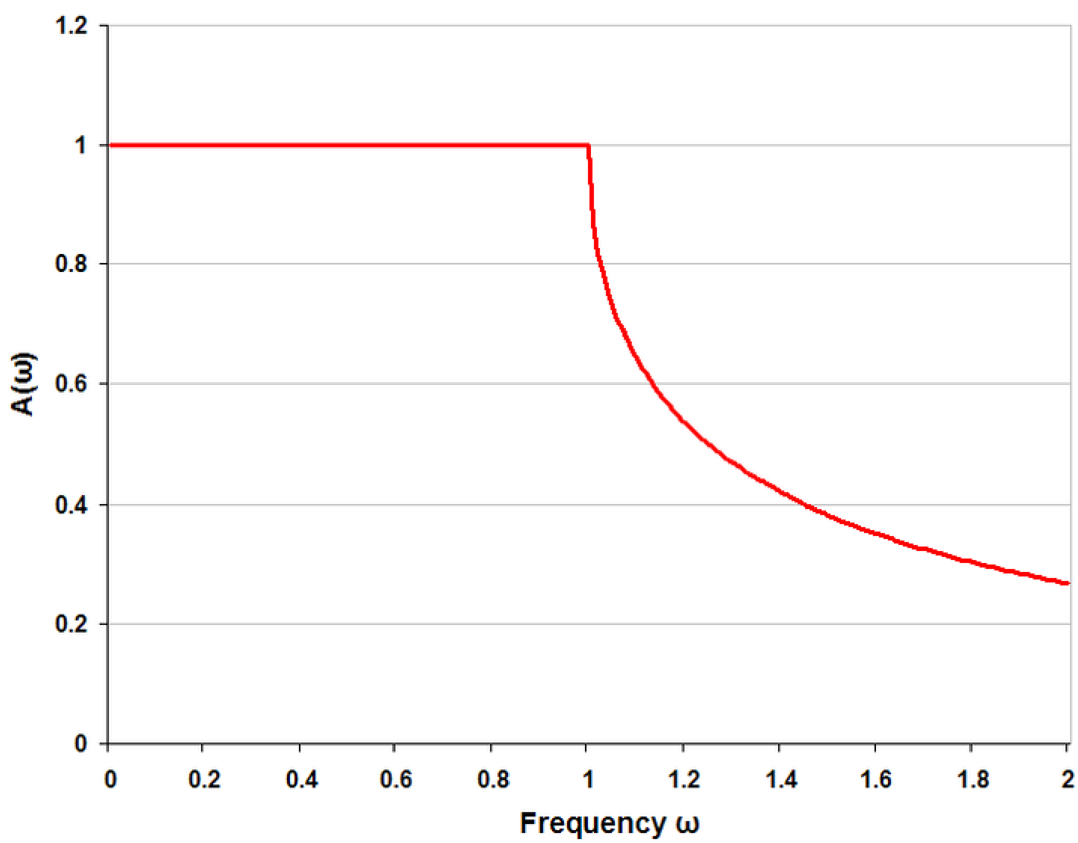
The smallest unit of an image filter is an L half-section. Because the L section is not symmetrical, it has different image impedances (Z_{i}) on each side. These are denoted $Z_{\mathrm{iT}}$and $Z_{\mathrm{i\Pi}}$. The T and the Π in the suffix refer to the shape of the filter section that would be formed if two half sections were to be connected back-to-back. T and Π are the smallest symmetrical sections that can be constructed, as shown in diagrams in the topology chart (below). Where the section in question has an image impedance different from the general case a further suffix is added identifying the section type, for instance $Z_{\mathrm{iT}m}$.

Constant k section

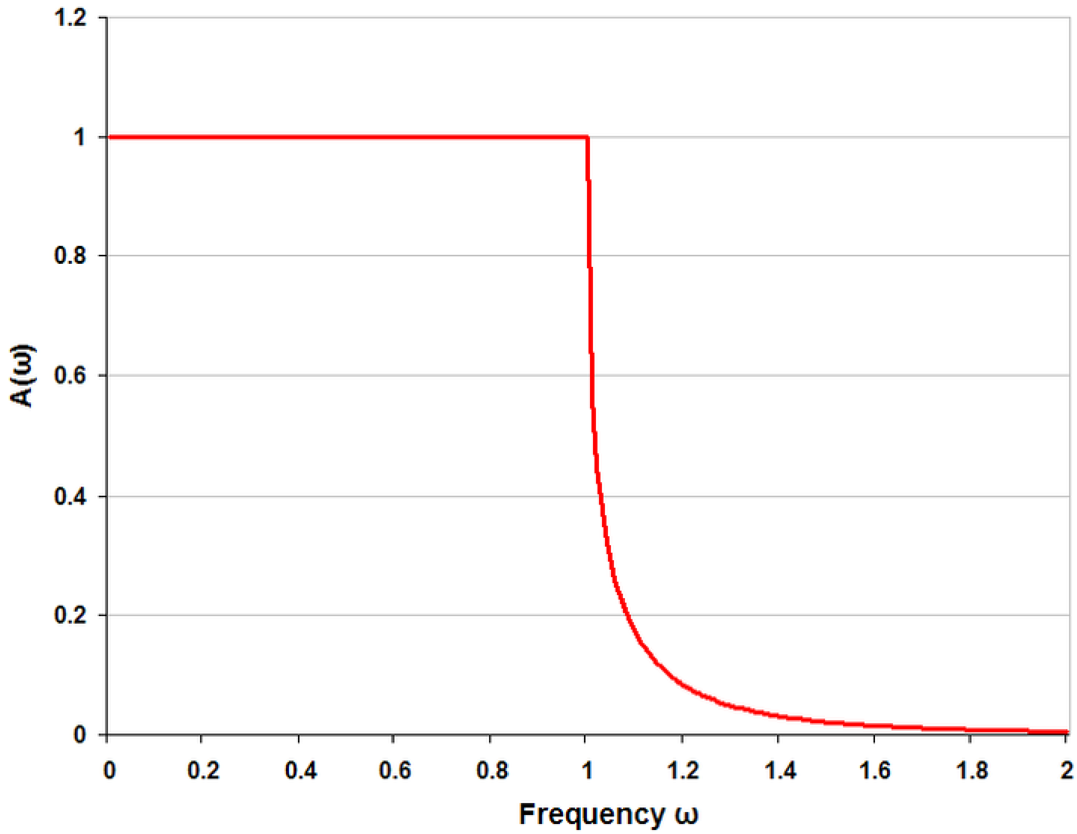
The **constant k** or **k-type** filter section is the basic image filter section. It is also the simplest circuit topology. The k-type has moderately fast transition from the passband to the stopband and moderately good stopband rejection.



k-type low-pass filter half section



k-type low-pass response, single half-section



k-type low-pass response with four (half) sections

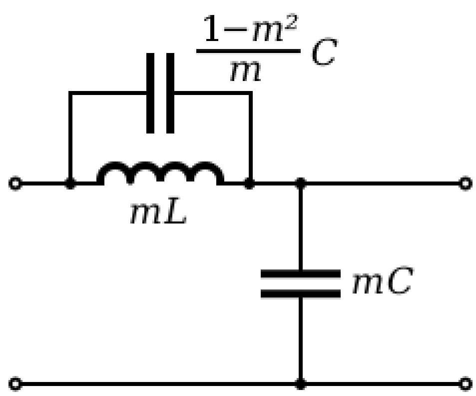
m-derived section

The **m-derived** or **m-type** filter section is a development of the k-type section. The most prominent feature of the m-type is a pole of attenuation just past the cut-off frequency inside the stopband. The parameter *m* ($0<m<1$) adjusts the position of this pole of attenuation. Smaller values of *m* put the pole closer to the cut-off frequency. Larger values of *m* put it further away. In the limit, as *m* approaches unity, the pole approaches *ω* of infinity and the section approaches a k-type section.

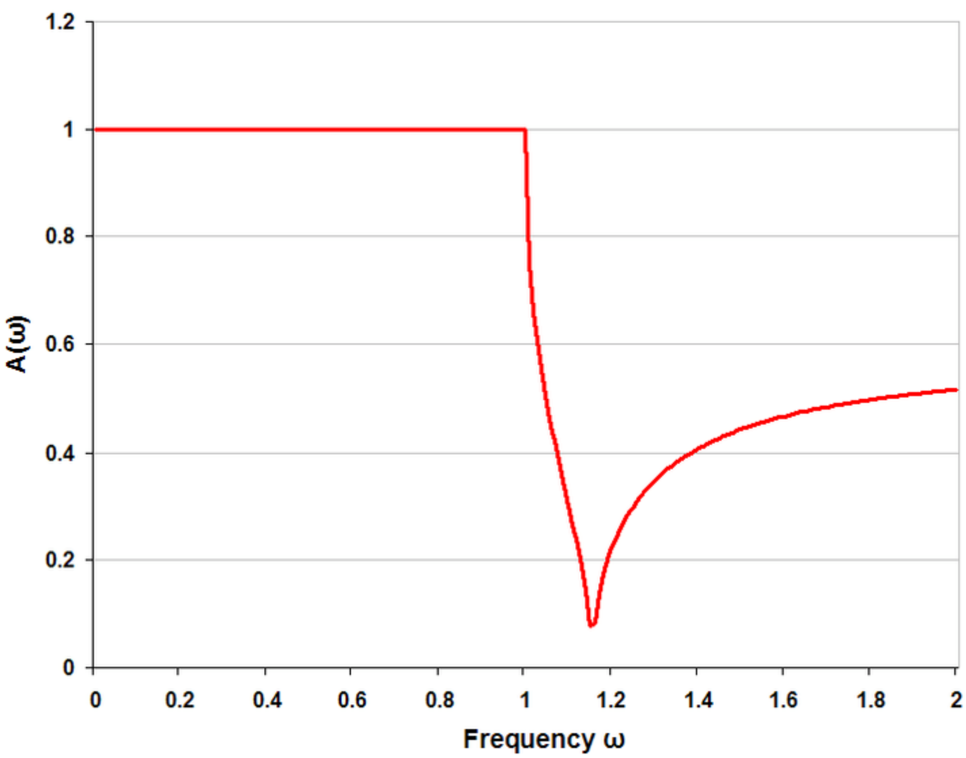
The m-type has a particularly fast cut-off, going from fully pass at the cut-off frequency to fully stop at the pole frequency. The cut-off can be made faster by moving the pole nearer to the cut-off frequency. This filter has the fastest cut-off of any filter design; note that the fast transition is achieved with just a single section, there is no need for multiple sections. The drawback with m-type sections is that they have poor stopband rejection past the pole of attenuation.

There is a particularly useful property of m-type filters with $m=0.6$. These have maximally flat image impedance Z_{im} in the passband. They are therefore good for matching in to the filter terminations, in the passband at least, the stopband is another story.

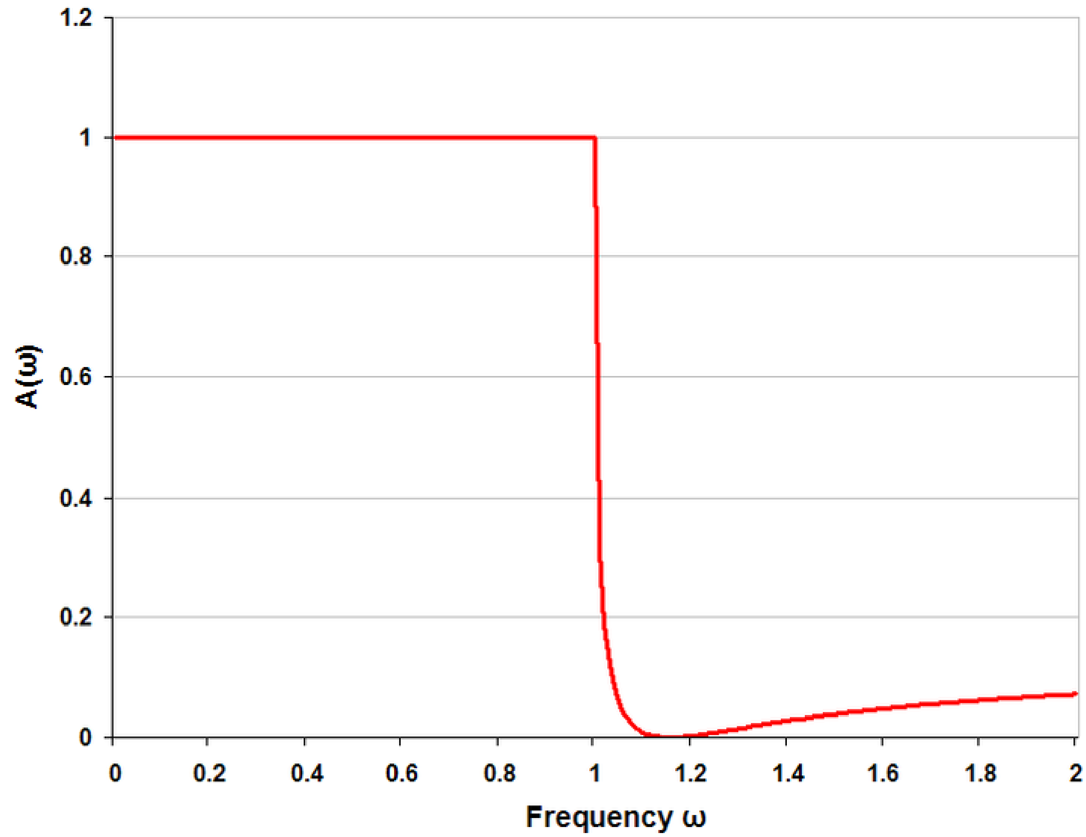
There are two variants of the m-type section, *series* and *shunt*. They have identical transfer functions but their image impedances are different. The shunt half-section has an image impedance which matches $Z_{i\Pi}$ on one side but has a different impedance, Z_{iTm} on the other. The series half-section matches Z_{iT} on one side and has $Z_{i\Pi m}$ on the other.



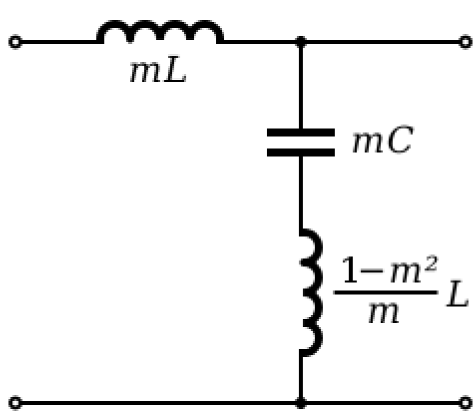
m-type low-pass filter shunt half section



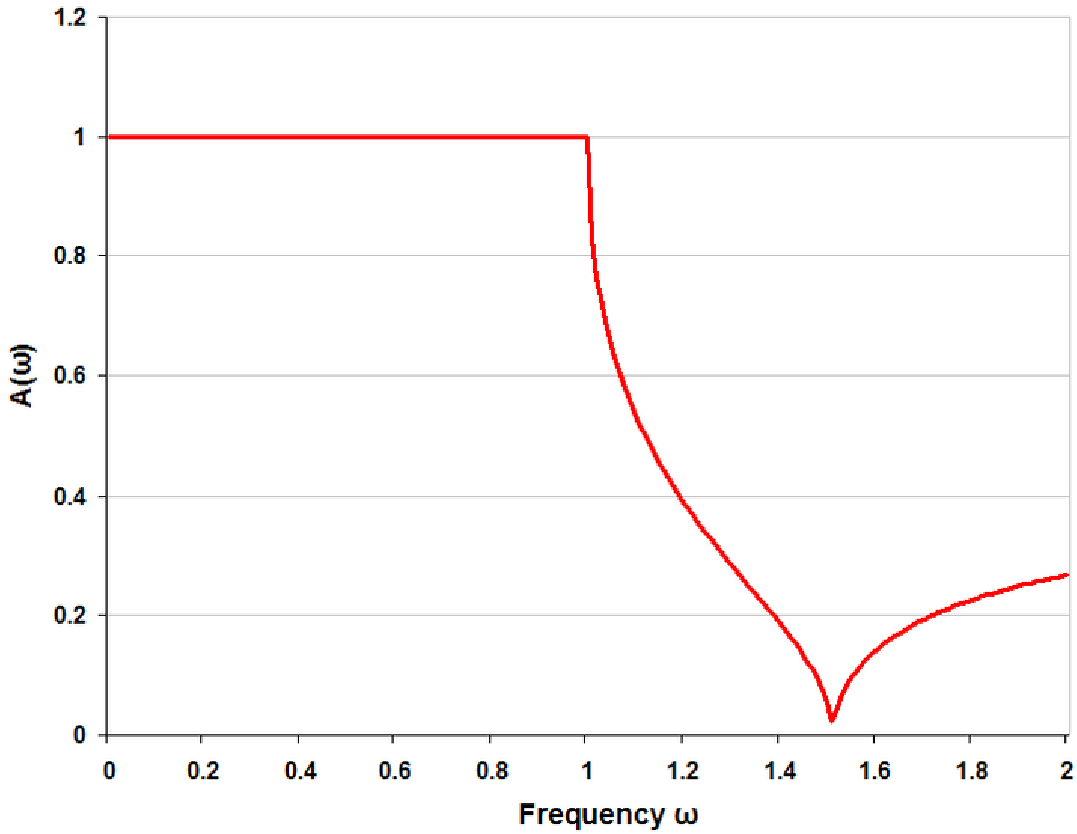
m-type low-pass response single half-section *m*=0.5



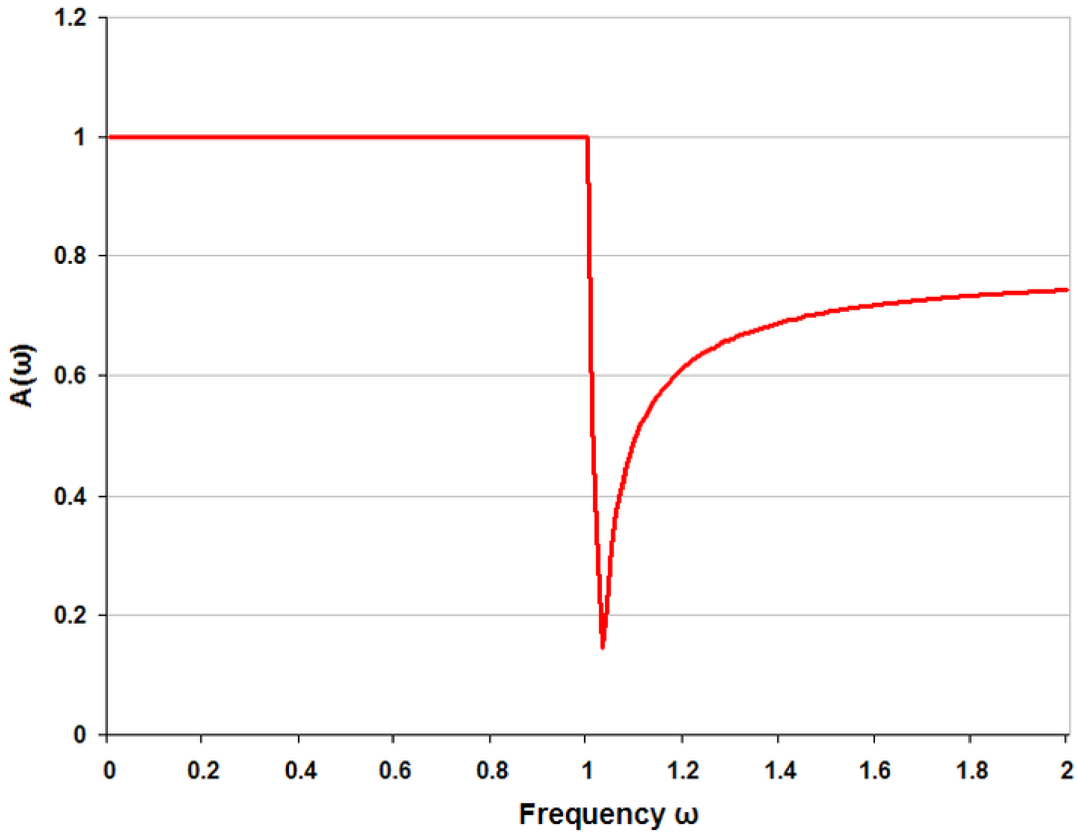
m-type low-pass response with four (half) sections *m*=0.5



m-type low-pass filter series half section



m-type low-pass response single half-section m=0.75



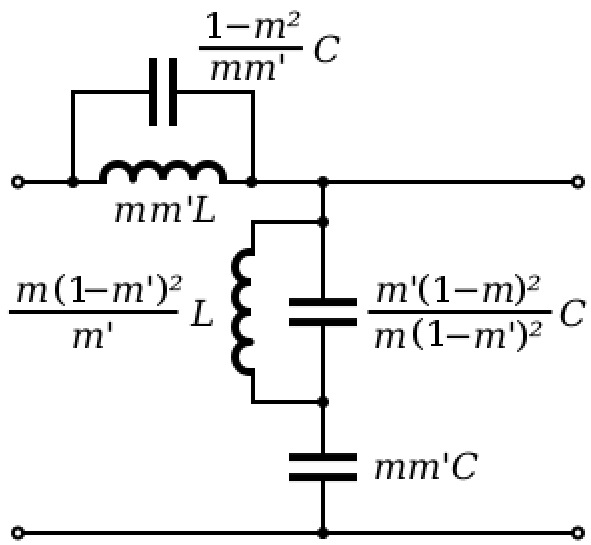
m-type low-pass response single half-section *m*=0.25

mm'-type section

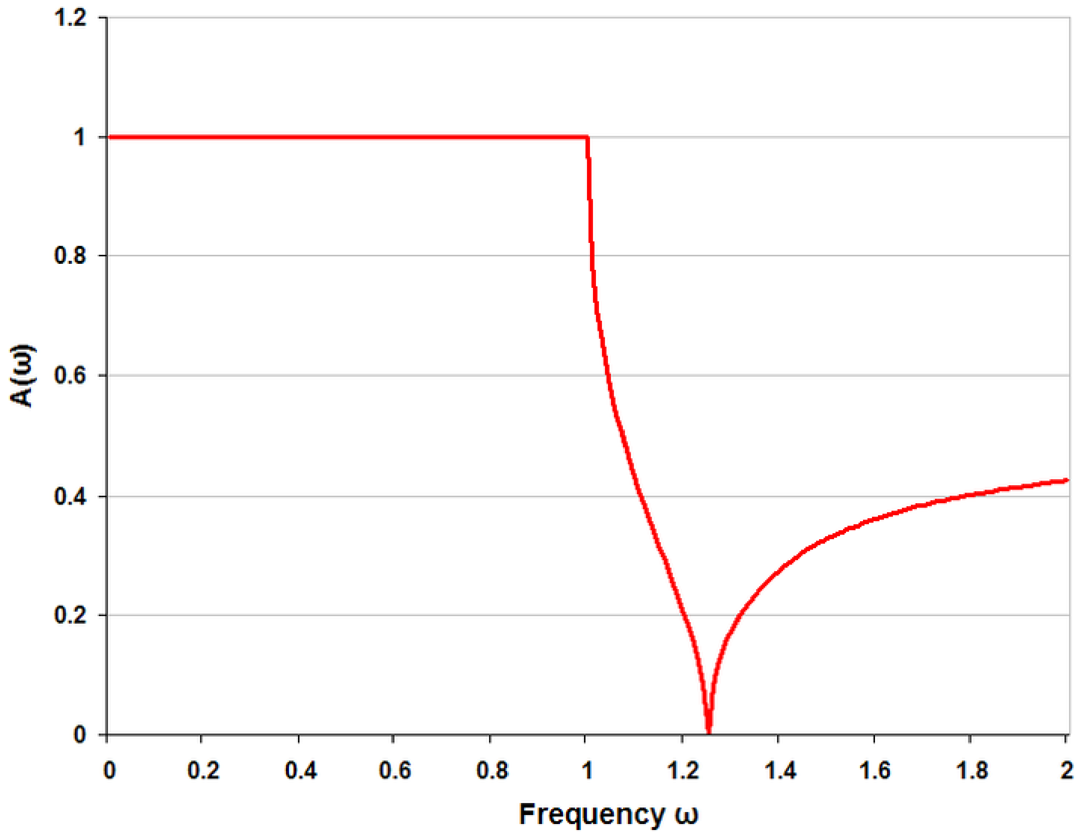
The **mm'-type** section has two independent parameters (*m* and *m*') that the designer can adjust. It is arrived at by double application of the *m*-derivation process. Its chief advantage is that it rather better at matching in to resistive end terminations than the k-type or m-type. The image impedance of a half-section is $Z_{\mathrm{i}m}$on one side and a different impedance, $Z_{\mathrm{i}mm'}$on the other. Like the m-type, this section can be constructed as a series or shunt section and the image impedances will come in T and Π variants. Either a series construction is applied to a shunt m-type or a shunt construction is applied to a series m-type. The advantages of the *mm*'-type filter are achieved at the expense of greater circuit complexity so it would normally only be used where it is needed for impedance matching purposes and not in the body of the filter.

The transfer function of an mm'-type is the same as an m-type with *m* set to the product *mm*'. To choose values of *m* and *m*' for best impedance match requires the designer to choose two frequencies at which the match is to be exact, at other frequencies there will be some deviation. There is thus some leeway in the choice but Zobel suggests the values *m*=0.7230 and *m*'=0.4134 which give a deviation of the impedance of less than 2% over the useful part of the band. Since *mm*'=0.3, this section will also have a much faster cut-off than an m-type of *m*=0.6 which is an alternative for impedance matching.

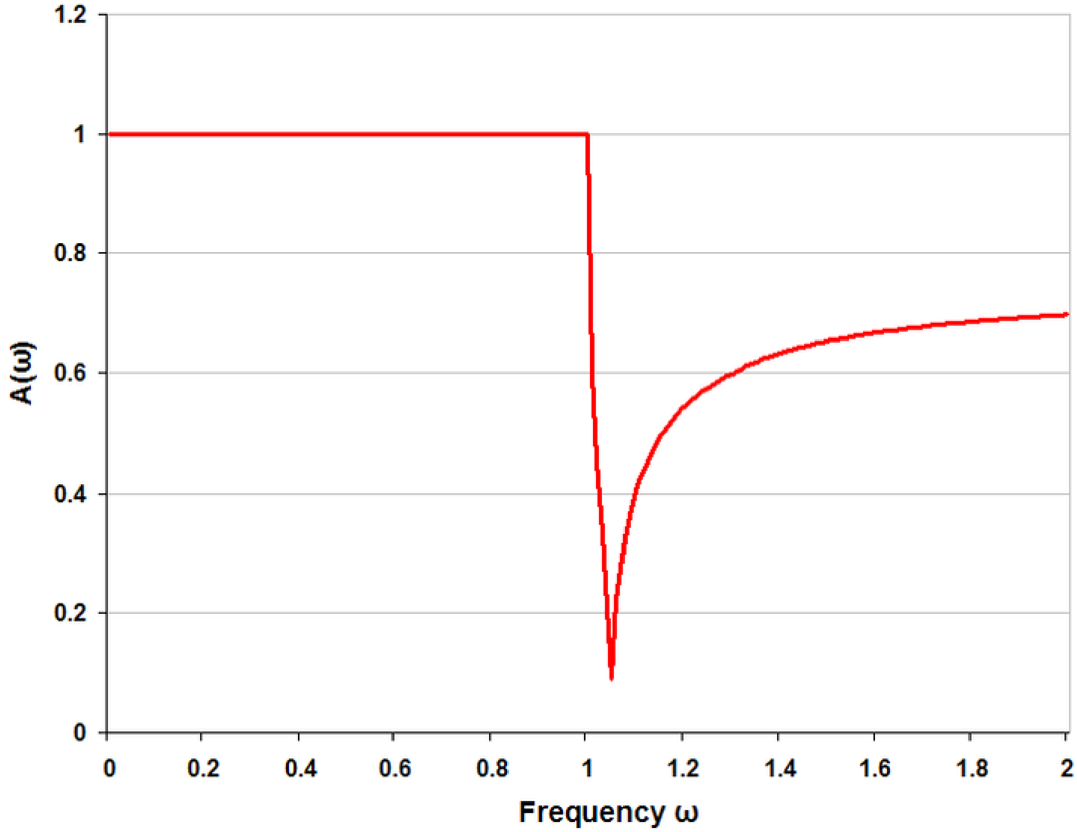
It is possible to continue the m-derivation process repeatedly and produce mm'm''-types and so on. However, the improvements obtained diminish at each iteration and are not usually worth the increase in complexity.



mm'-type low-pass filter series half section

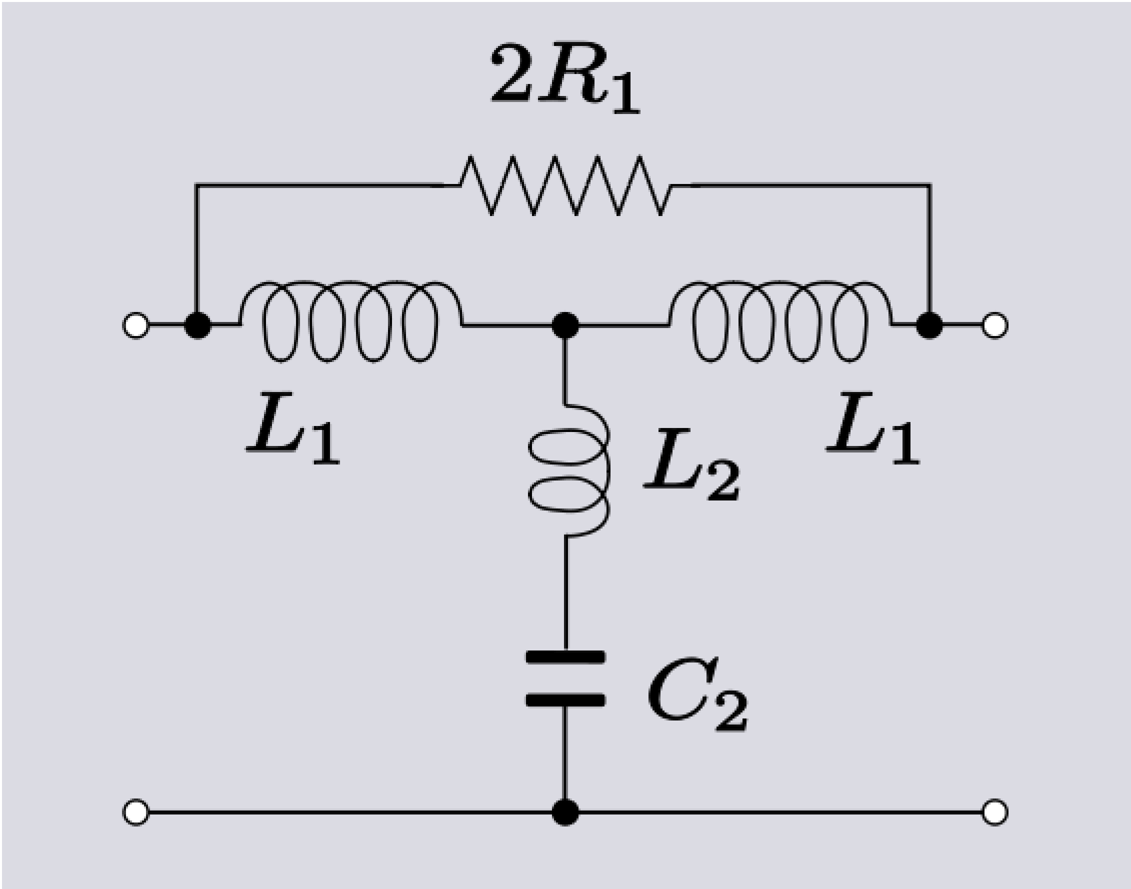


m-type low-pass response single half-section *m*=0.6



mm'-type low-pass response single half-section *mm*'=0.3

Bode's filter

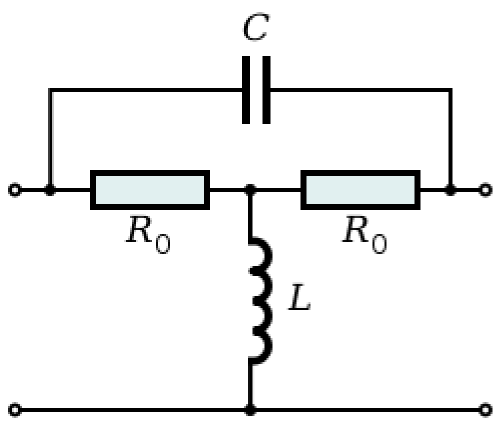


One incarnation of Bode's filter as a low-pass filter.

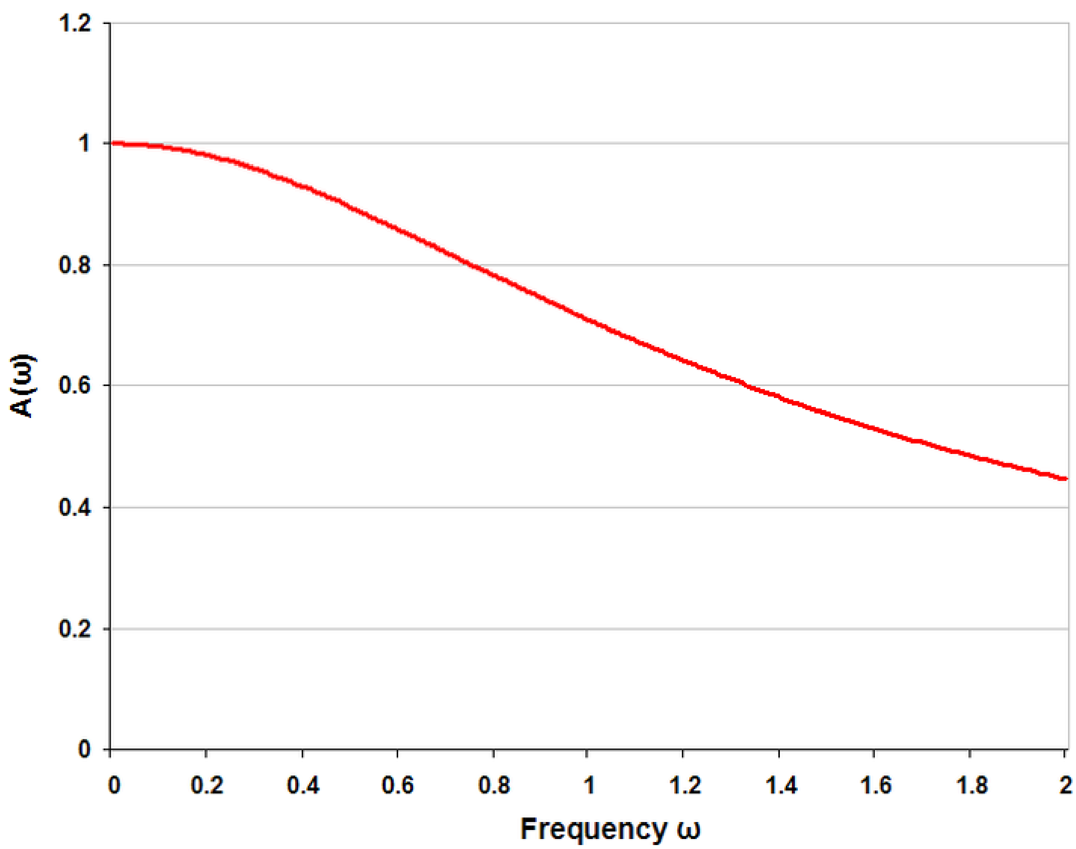
Another variation on the m-type filter was described by Hendrik Bode. This filter uses as a prototype a mid-series m-derived filter and transforms this into a bridged-T topology with the addition of a bridging resistor. This section has the advantage of being able to place the pole of attenuation much closer to the cut-off frequency than the Zobel filter, which starts to fail to work properly with very small values of *m* because of inductor resistance.

Zobel network

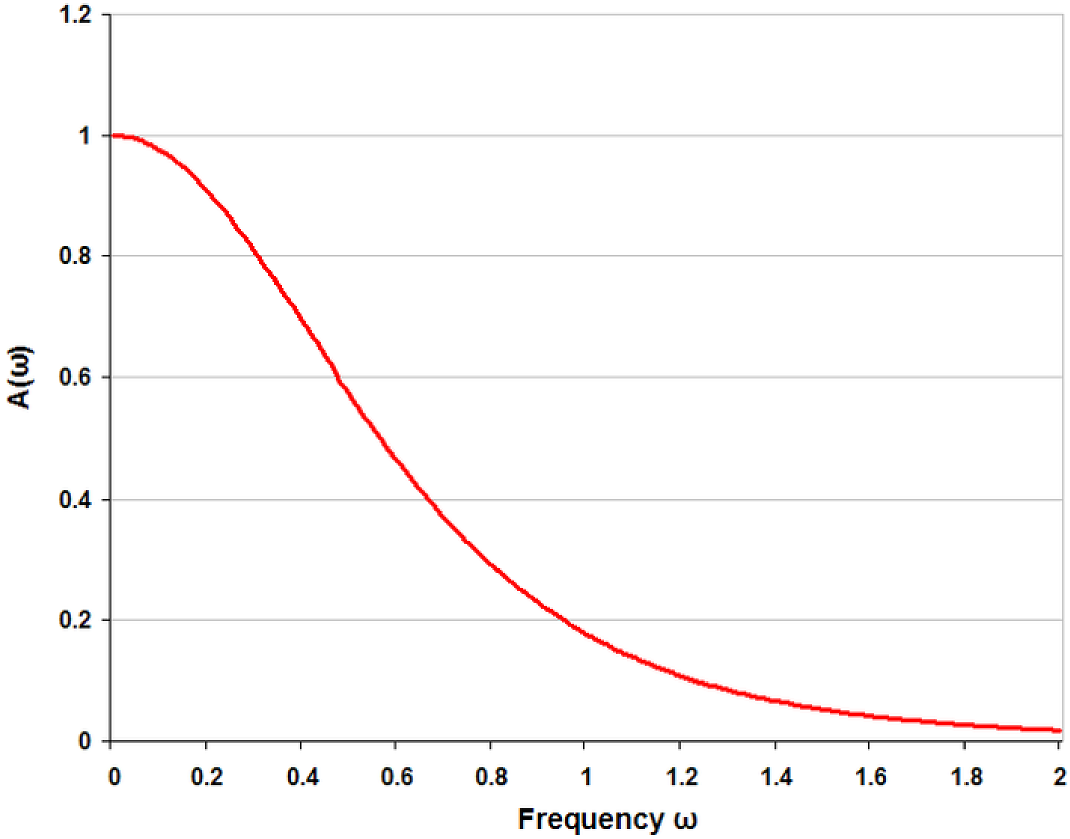
The distinguishing feature of **Zobel network** filters is that they have a constant resistance image impedance and for this reason are also known as **constant resistance networks**. Clearly, the Zobel network filter does not have a problem matching to its terminations and this is its main advantage. However, other filter types have steeper transfer functions and sharper cut-offs. In filtering applications, the main role of Zobel networks is as equalisation filters. Zobel networks are in a different group from other image filters. The constant resistance means that when used in combination with other image filter sections the same problem of matching arises as with end terminations. Zobel networks also suffer the disadvantage of using far more components than other equivalent image sections.



Zobel network bridge T high-pass filter section



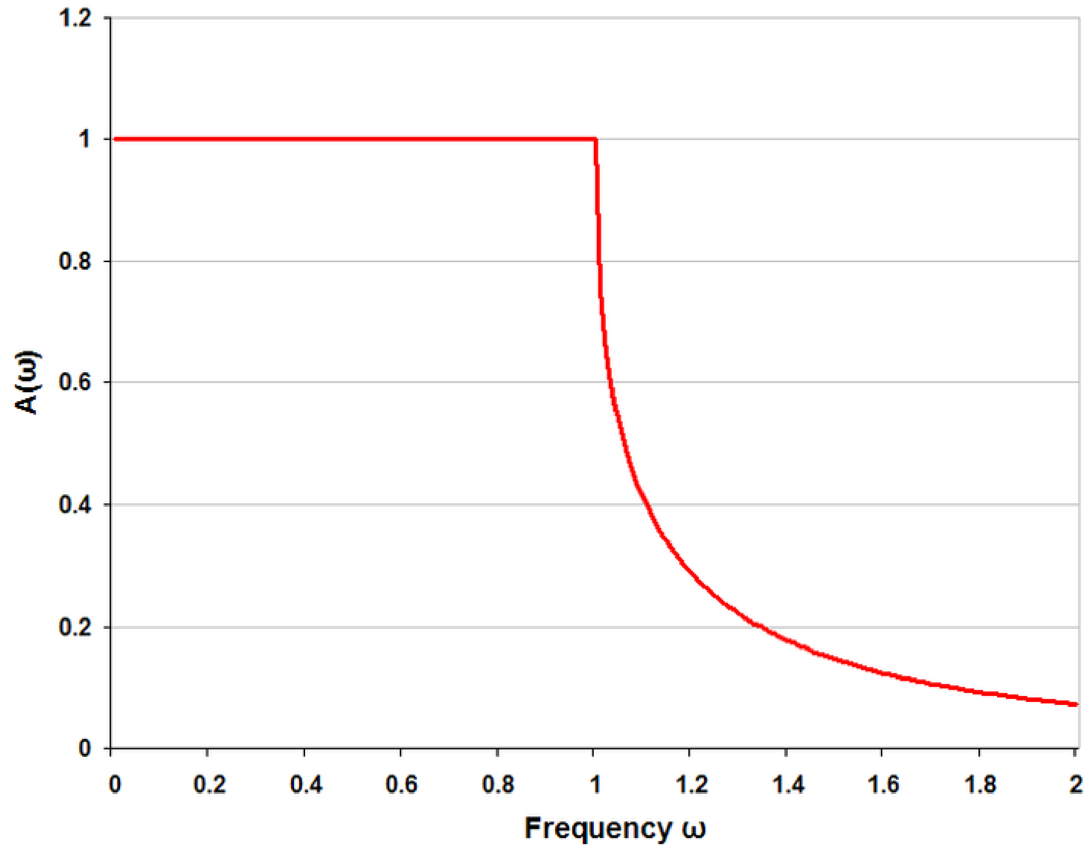
Zobel network low-pass response single section



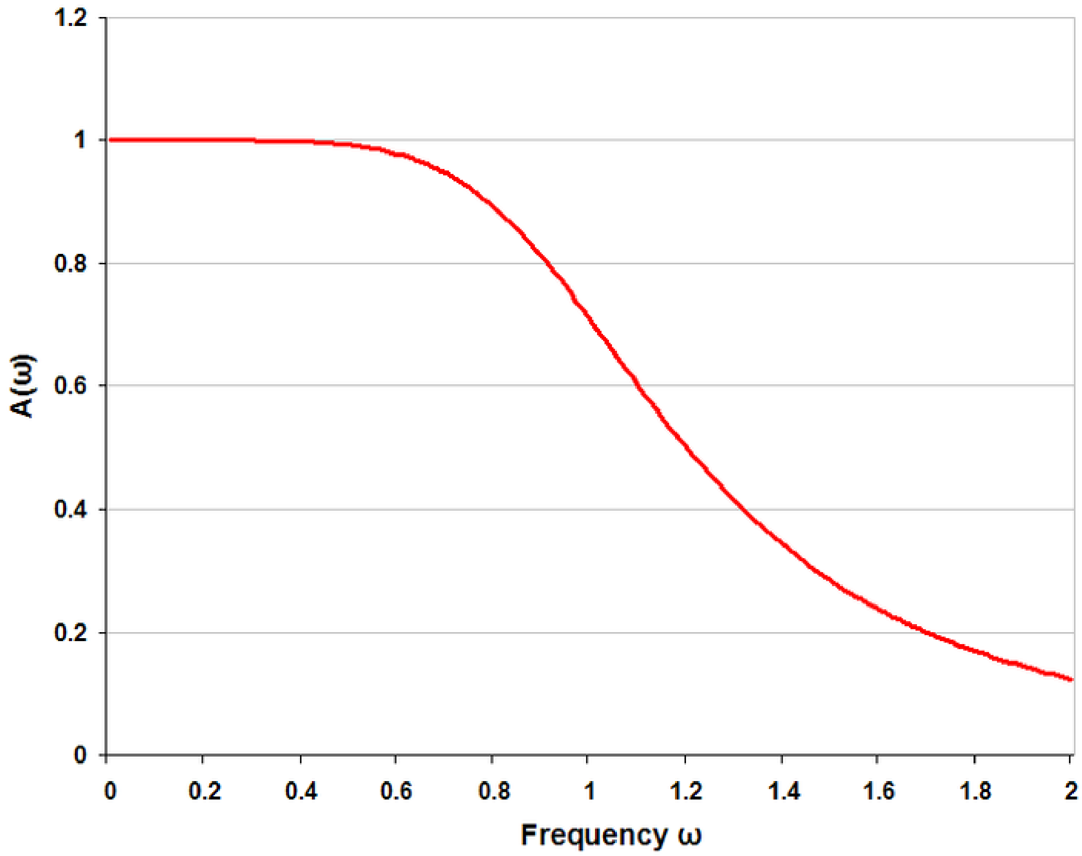
Zobel network low-pass response five sections

Effect of end terminations

A consequence of the image method of filter design is that the effect of the end terminations has to be calculated separately if its effects on response are to be taken into account. The most severe deviation of the response from that predicted occurs in the passband close to cut-off. The reason for this is twofold. Further into the passband the impedance match progressively improves, thus limiting the error. On the other hand, waves in the stopband are reflected from the end termination due to mismatch but are attenuated twice by the filter stopband rejection as they pass through it. So while stopband impedance mismatch may be severe, it has only limited effect on the filter response.



Theoretical k-type low-pass T-filter (two half-sections) response when correctly terminated in image impedance



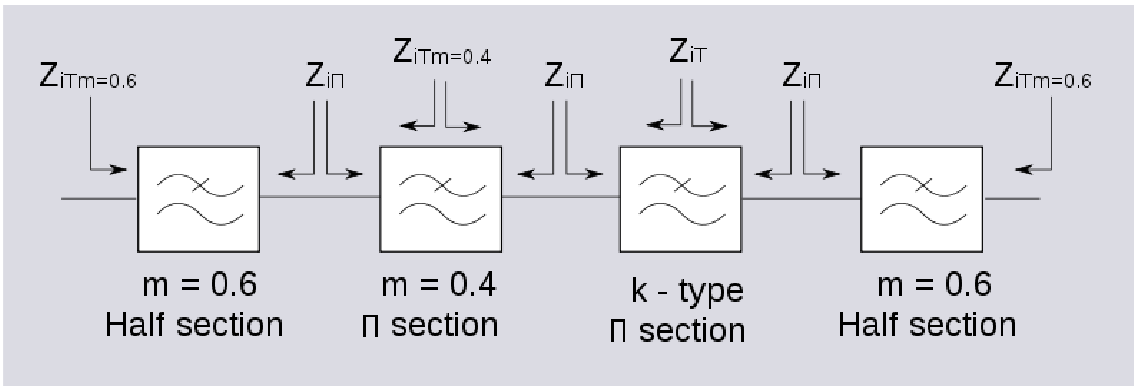
Practical k-type low-pass T-filter (two half-sections) response when terminated with fixed resistors

Cascading sections

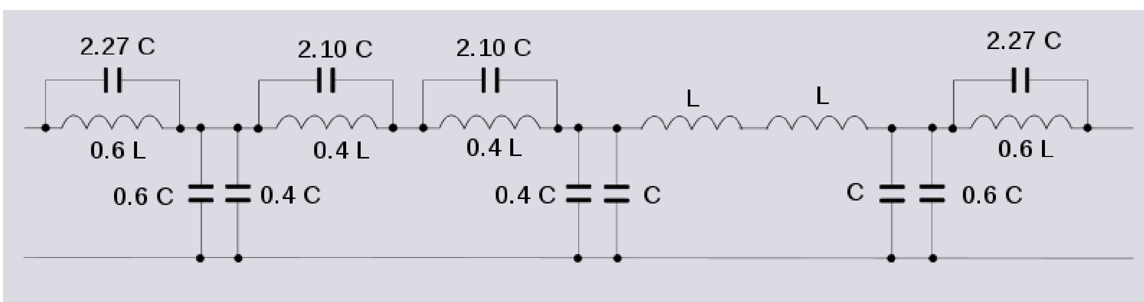
Several L half-sections may be cascaded to form a composite filter. The most important rule when constructing a composite image filter is that the image impedances must always face an identical impedance; like must always face like. T sections must always face T sections, Π sections must always face Π sections, k-type must always face k-type (or the side of an m-type which has the k-type impedance) and m-type must always face m-type. Furthermore, m-type impedances of different values of m cannot face each other. Nor can sections of any type which have different values of cut-off frequency.

Sections at the beginning and end of the filter are often chosen for their impedance match in to the terminations rather than the shape of their frequency response. For this purpose, m-type sections of $m = 0.6$ are the most common choice. An alternative is mm'-type sections of m=0.7230 and m'=0.4134 although this type of section is rarely used. While it has several advantages noted below, it has the disadvantages of being more complex and also, if constant k sections are required in the body of the filter, it is then necessary to include m-type sections to interface the mm'-type to the k-types.

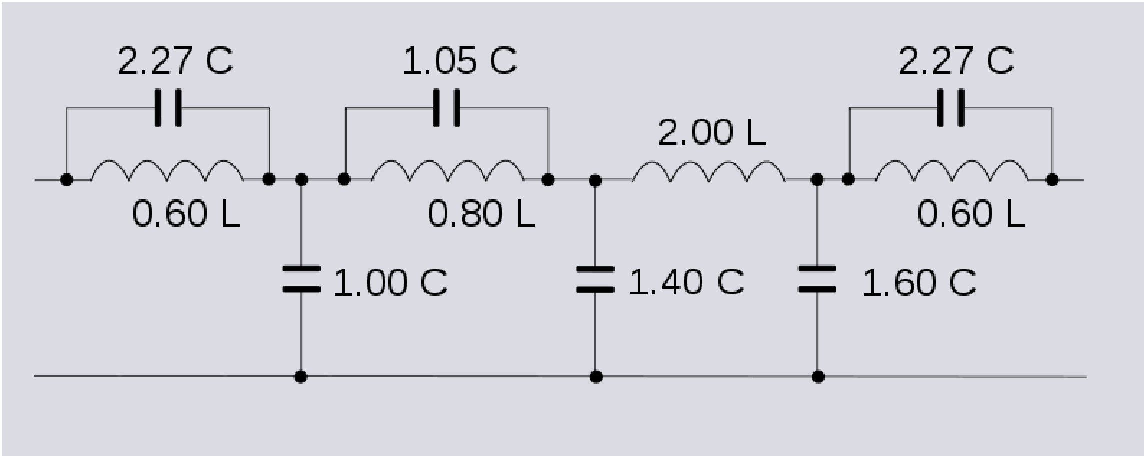
The inner sections of the filter are most commonly chosen to be constant k since these produce the greatest stopband attenuation. However, one or two m-type sections might also be included to improve the rate of fall from pass to stopband. A low value of m is chosen for m-types used for this purpose. The lower the value of *m*, the faster the transition, while at the same time, the stopband attenuation becomes less, increasing the need to use extra k-type sections as well. An advantage of using mm'-types for impedance matching is that these type of end sections will have a fast transition anyway (much more so than *m*=0.6 m-type) because *mm*'=0.3 for impedance matching. So the need for sections in the body of the filter to do this may be dispensed with.



Typical example of a composite image filter in block diagram form. The image impedances and how they match are shown.



The above filter realised as a ladder low-pass filter. Component values are given in terms of L and C, the component values of a constant k half-section.



The same filter minimised by combining components in series or parallel where appropriate.

Another reason for using m-types in the body of the filter is to place an additional pole of attenuation in the stopband. The frequency of the pole directly depends on the value of *m*. The smaller the value of *m*, the closer the pole is to the cut-off frequency. Conversely, a large value of *m* places the pole further away from cut-off until in the limit when *m*=1 the pole is at infinity and the response is the same as the k-type section. If a value of *m* is chosen for this pole which is different from the pole of the end sections it will have the effect of broadening the band of good stopband rejection near to the cut-off frequency. In this way the m-type sections serve to give good stopband rejection near to cut-off and the k-type sections give good stopband rejection far from cut-off. Alternatively, m-type sections can be used in the body of the filter with different values of m if the value found in the end sections is unsuitable. Here again, the mm'-type would have some advantages if used for impedance matching. The mm'-type used for impedance matching places the pole at *m*=0.3. However, the other half of the impedance matching section needs to be an m-type of *m*=0.723. This automatically gives a good spread of stopband rejection and as with the steepness of transition issue, use of mm'-type sections may remove the need for additional m-type sections in the body.

Constant resistance sections may also be required, if the filter is being used on a transmission line, to improve the flatness of the passband response. This is necessary because the transmission line response is not usually anywhere near perfectly flat. These sections would normally be placed closest to the line since they present a predictable impedance to the line and also tend to mask the indeterminate impedance of the line from the rest of the filter. There is no issue with matching constant resistance sections to each other even when the sections are operating on totally different frequency bands. All sections can be made to have precisely the same image impedance of a fixed resistance.

Chapter- 7

mm'-type Filter and Bridged T Delay Equaliser

mm'-type filter

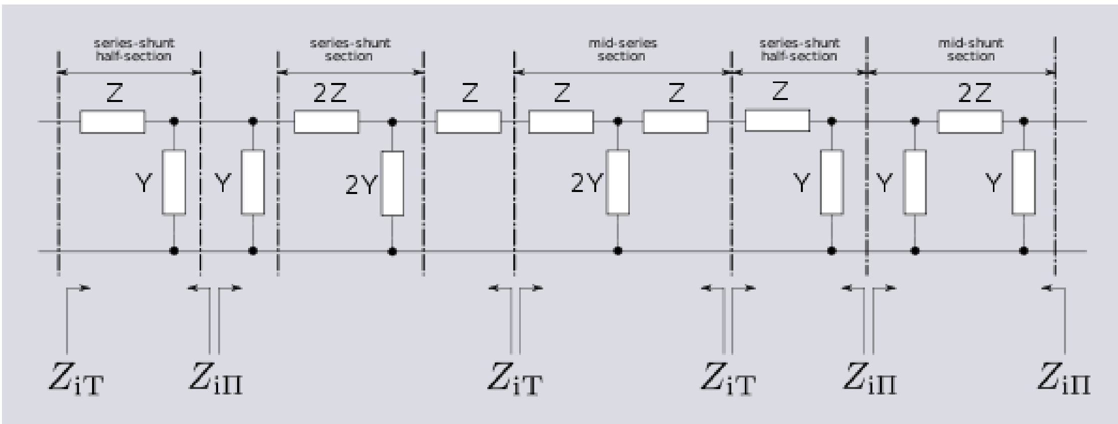
mm'-type filters, also called **double-m-derived filters**, are a type of electronic filter designed using the image method. They were patented by Otto Zobel in 1932. Like the m-type filter from which it is derived, the mm'-type filter type was intended to provide an improved impedance match into the filter termination impedances and originally arose in connection with telephone frequency division multiplexing. The filter has a similar transfer function to the m-type, having the same advantage of rapid cut-off, but the input impedance remains much more nearly constant if suitable parameters are chosen. In fact, the cut-off performance is better for the mm'-type if like-for-like impedance matching are compared rather than like-for-like transfer function. It also has the same drawback of a rising response in the stopband as the m-type. However, its chief disadvantage is its much increased complexity which is the chief reason its use never became widespread. It was only ever intended to be used as the end sections of a composite filters, the rest of the filter being made up of other sections such as k-type and m-type sections.

An incidental advantage of the mm'-type is that it has two independent parameters (m and m') that the designer can adjust. This allows for two different design criteria to be independently optimised.

Background

The mm'-type filter was an extension of Zobel's previous m-type filter, which itself grew out of George Campbell's k-type design. Zobel's m-type is arrived at by applying the m-derivation process to the k-type filter. Completely analogously, the mm'-type is arrived at by applying the m-derived process to the m-type filter. The value of *m* used in the second transformation is designated *m'* to distinguish it from *m*, hence the naming of the filter mm'-type. However, this filter is not a member of the class of filters, general m_n-type image filters, which are a generalisation of m-type filters. Rather, it is a double application of the m-derived process and for those filters the arbitrary parameters are usually designated m_1, m_2, m_3 etc., rather than m, m', m'' as here.

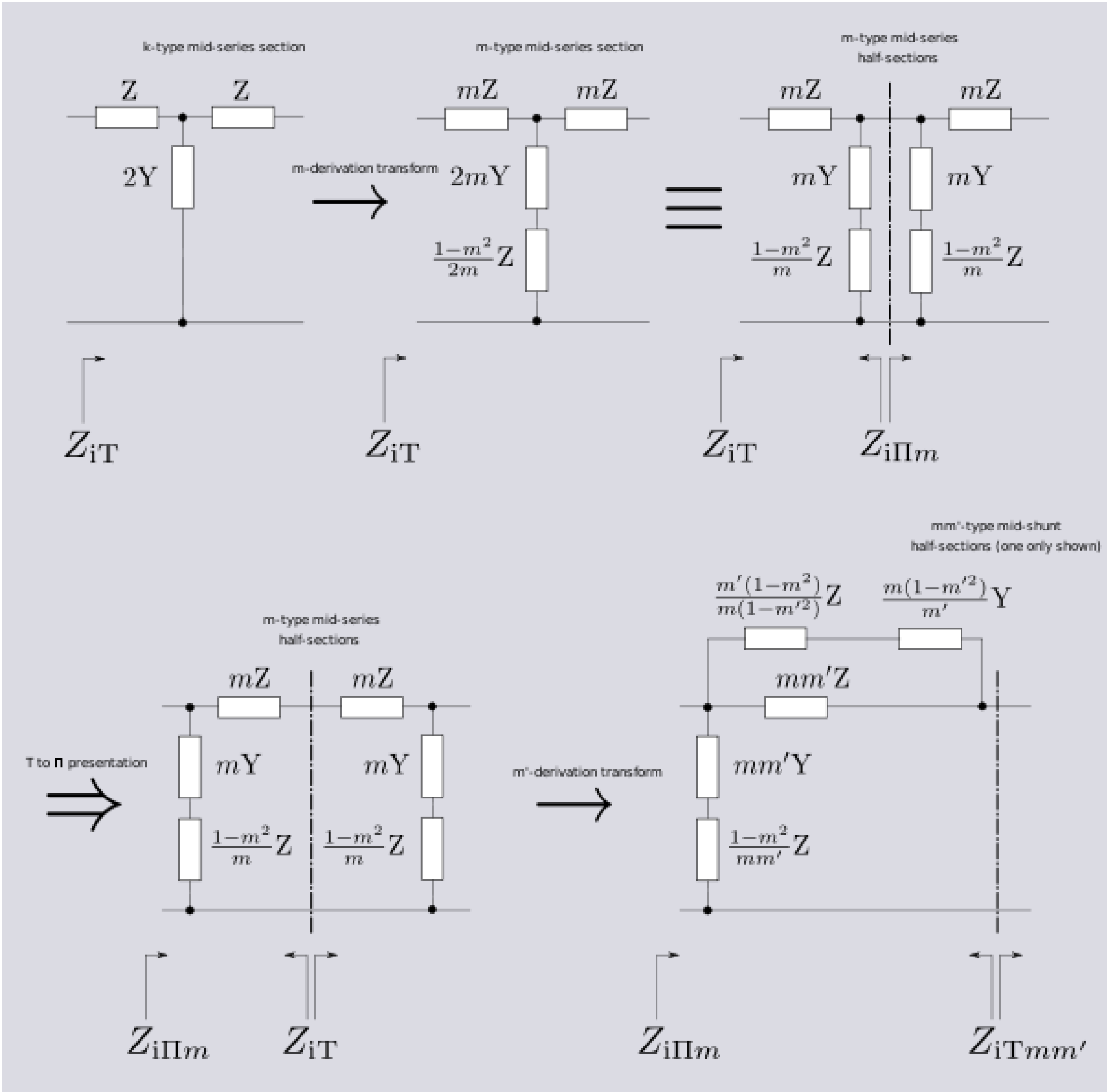
The importance of the filter lies in its impedance properties. Some of the impedance terms and section terms used in image design theory are pictured in the diagram below. As always, image theory defines quantities in terms of an infinite cascade of two-port sections, and in the case of the filters being discussed, an infinite ladder network of L-sections.



The sections of the hypothetical infinite filter are made up of series impedance elements of $2Z$ and shunt admittance elements of $2Y$. The factor of two is introduced since it is normal to work in terms of half-sections where it disappears. The image impedance of the input and output port of a section, Z_{i1} and Z_{i2}, will generally not be the same. However, for a mid-series section (that is, a section from halfway through a series element to halfway through the next series element) will have the same image impedance on both ports due to symmetry. This image impedance is designated Z_{iT} due to the "T" topology of a mid-series section. Likewise, the image impedance of a mid-shunt section is designated $Z_{i\Pi}$ due to the "Π" topology. Half of such a "T" or "Π" section is (unsurprisingly) called a half-section. The image impedances of the half section are dissimilar on the input and output ports but are equal to the mid-series Z_{iT} on the side presenting the series element and the mid-shunt $Z_{i\Pi}$ on the side presenting the shunt element.

A mid-series *derived* section (that is, a series m-type filter) has precisely the same image impedance, Z_{iT}, as a k-type mid-series "T" filter. However, the image impedance of a half-section of such a filter (on the shunt side) is not the same and is designated $Z_{i\Pi m}$. Similarly, the series element side of a shunt m-derived filter half-section is designated Z_{iTm}.

Derivation

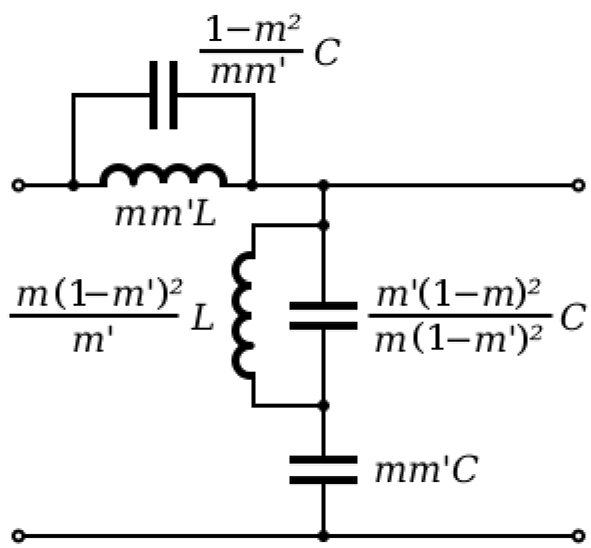


Derivation of a mm'-type mid-shunt filter section. Starting from a prototype constant k filter T-filter, the circuit is transformed to a m-type mid-series derived filter, re-implemented as Π-section, transformed again to a mid-shunt derived mm'-type.

The m-derived process starts with a k-type filter mid-section and transforms it into a m-derived filter with a different transfer function but retaining the same image impedance and passband. Two different results are obtained depending on whether the process began with a T-section or a Π-section. From a T-section, the series *Z* and shunt *Y* are multiplied by an arbitrary parameter *m* ($0 < m < 1$). An additional impedance is then inserted in series with *Y* whose value is that which restores the original image impedance. The half-sections resulting from splitting the T-section, however, will show a different image impedance at the split, $Z_{i\Pi m}$. Two such half-sections cascaded with the Z_{iT} impedances facing, will form a Π-section with image impedance $Z_{i\Pi m}$. The m-derived process can now be applied to this new section, but with a new parameter *m*'. The series *Z* and shunt *Y* are multiplied by *m*' and an additional admittance is inserted in parallel with the series elements to bring the image impedance back to its original value of $Z_{i\Pi m}$. Again, the half sections will have a different image impedance at the split ports and this is designated $Z_{iTmm'}$.

The dual realisation of this filter is obtained in a completely analogous way by first transforming a mid-shunt k-type Π-section, forming the resulting mid-series m-type T-section and then transforming using *m*', resulting in a new Π flavour of $Z_{imm'}$, $Z_{i\Pi mm'}$, which is the dual of $Z_{iTmm'}$.

The m-derivation transformation can, in principle, be applied ad-infinitum and produce mm'm''-types etc. There is, however, no practical benefit in doing this. The improvements obtained diminish at each iteration and are not worth the increase in complexity. Note that applying the m-derived transformation twice to a T-section (or Π-section) will merely result in an m-type filter with a different value of *m*. The transformation must be applied alternately to T-sections and Π-sections for a completely new form of filter to be obtained.



mm'-type low-pass filter series half section. $C = L/R_0^2$

Operating frequency

For a low-pass prototype, the cut-off frequency is given by the same expression as for the m-type and the k-type;

$$\omega_c = \frac{1}{\sqrt{LC}}.$$

The pole of attenuation occurs at;

$$\omega_\infty = \frac{\omega_c}{\sqrt{1-(mm')^2}}.$$

Image impedance

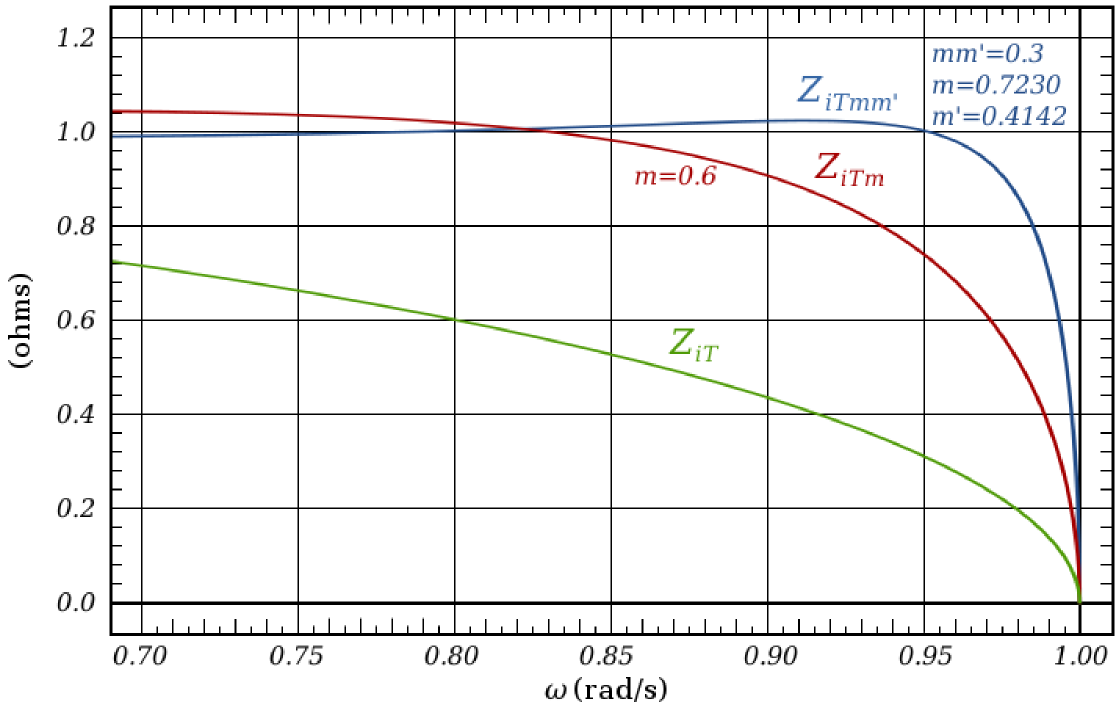


Image impedance plot of an optimised mm'-type shunt section ($Z_{\mathrm{iTmm'}}$, in blue) compared with an m-derived prototype shunt image impedance (Z_{iTm}, in red) for $m = 0.6$. A constant-k filter's image impedance is also shown (Z_{iT}, in green).

The following expressions for image impedances are all referenced to the low-pass prototype section. They are scaled to the nominal impedance $R_0 = 1$ and the frequencies in those expressions are all scaled to the cut-off frequency $\omega_c = 1$.

"T" port image impedance

The image impedance looking into a "T" topology port of a shunt derived section is given by,

$$Z_{iTmm'} = \frac{\sqrt{1-\omega^2}\,(1-(1-m^2)\omega^2)}{1-(\omega/\omega_\infty)^2}$$

For comparison;

$$Z_{iT} = \sqrt{1-\omega^2} \text{ and } Z_{iTm} = \frac{\sqrt{1-\omega^2}}{1-(\omega/\omega_\infty)^2}$$

"Π" port image impedance

The image impedance looking into a "Π" topology port of a series derived section is given by,

$$Z_{i\Pi mm'} = \frac{1-(\omega/\omega_\infty)^2}{\sqrt{1-\omega^2}\,(1-(1-m^2)\omega^2)}$$

For comparison;

$$Z_{i\Pi} = \frac{1}{\sqrt{1-\omega^2}} \text{ and } Z_{i\Pi m} = \frac{1-(\omega/\omega_\infty)^2}{\sqrt{1-\omega^2}}.$$

Optimisation

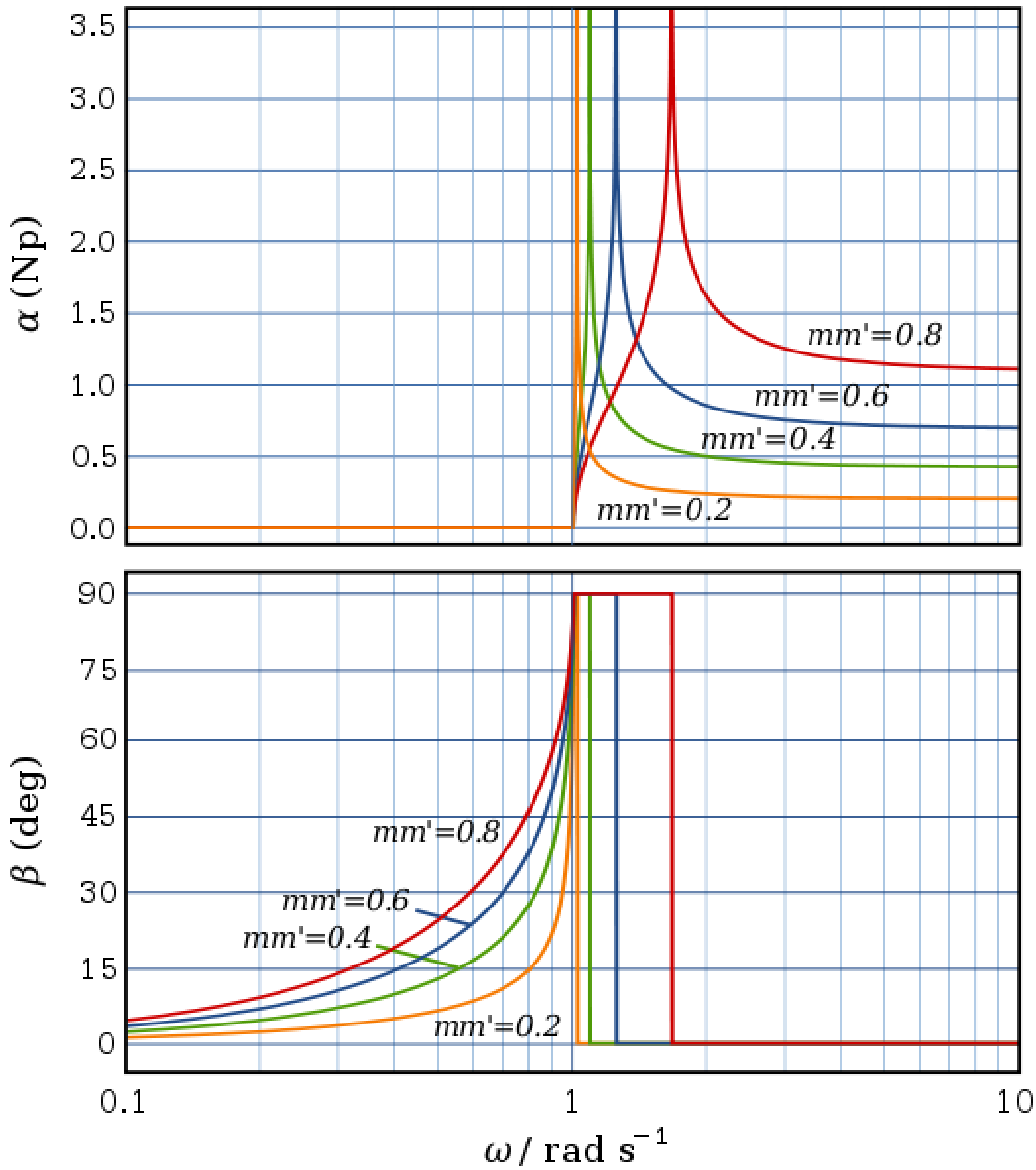
Note that ω_∞ can be adjusted independently of *m* by adjusting *m*'. It is therefore possible to adjust the impedance characteristic and frequency response characteristic independently. However, for optimum impedance match it is necessary to adjust both parameters solely for maximally flat image resistance in the passband. The term resistance is used because the image impedance is purely real in the passband between cut-off frequencies and purely imaginary outside the passband. It is not possible to obtain an exact impedance match across the entire band. With two degrees of freedom it is only possible to match the impedance exactly at two spot frequencies. It has been determined empirically that a good match is made with the values;

$$m = 0.7230$$
$$m' = 0.4134$$

This amounts to setting the match to be exact at frequencies 0.8062 and 0.9487 rad/s for the prototype filter and the impedance departs less than 2% from nominal from 0 to 0.96 rad/s, that is, nearly all of the passband.

The transfer function of an mm'-type is the same as an m-type with *m* set to the product *mm*' and in this case *mm*'=0.3. Where an m-type section is used for impedance matching the optimum value of *m* is *m*=0.6. Steepness of cut-off increases with decreasing *m* so an mm'-type section has this as an incidental advantage over an m-type in this application.

Transmission parameters



mm'-type low-pass filter transfer function for a single half-section with various values of *mm'*

$$\gamma = \alpha + i\beta$$

The operating frequencies, transmission parameters and transfer function are identical to those for the m-type and details can be found in that article if the parameter *m* is replaced by the product *mm*'. Only the image impedance is different in the mm'-type filter in terms of black box behaviour.

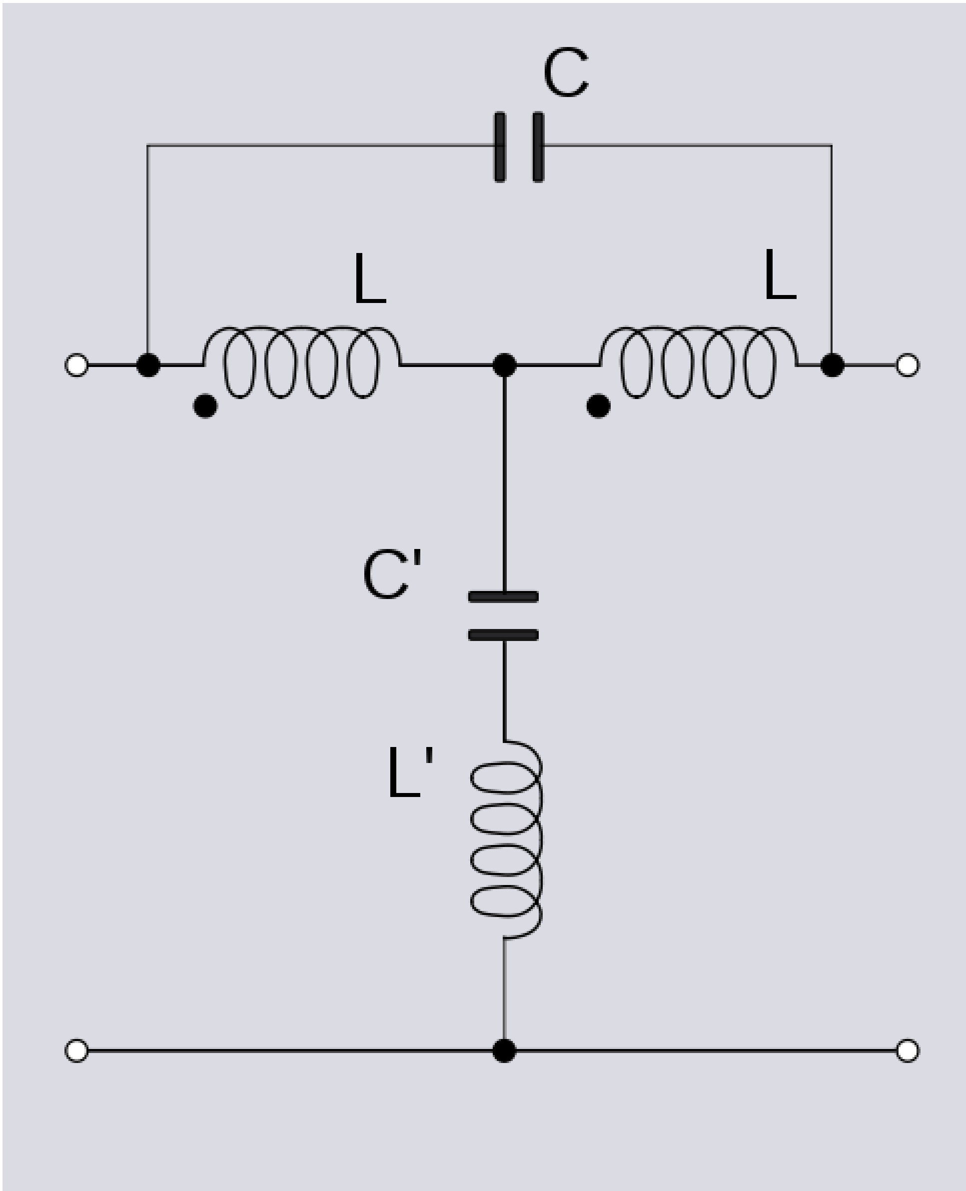
Prototype transformations

The plots shown of image impedance and attenuation are the plots of a low-pass prototype filter section. The prototype has a cut-off frequency of ω_c=1 rad/s and a nominal impedance R_0=1Ω. This is produced by a filter half-section where L=1 henry and C=1 farad. This prototype can be impedance scaled and frequency scaled to the desired values. The low-pass prototype can also be transformed into high-pass, band-pass or band-stop types by application of suitable frequency transformations.

Cascading sections

As with all image filters, it is required to match each section into a section of identical image impedance if the theoretical filter response is to be achieved. This is a particular difficulty for the end sections of the filter which are often working into resistive terminations which cannot be matched exactly by an image impedance. Hence the use of the mm'-type as end sections for the filter because of its nearly flat impedance with frequency characteristic in the passband. However, it is not desirable to use it throughout the entire filter. The workhorse of image filters are the k-type sections and these will usually be required somewhere in the filter for good rejection in the stopband well away from cut-off and also because they are the simplest topology and lowest component count. Unfortunately, neither side of a mm'-type can match into a k-type. The solution is to form a composite section from a mm'-type half-section and a m-type section which will match each other on one side if *m* has the same value for both half-sections. This can, for instance, produce a composite T-section with $Z_{iTmm'}$ facing the termination and Z_{iT} facing the rest of the filter. The T-section will be matched internally with Z_{iTm}. This has the additional incidental advantage of producing two poles of attenuation in the stopband at different frequencies. This is a consequence of *m* and *mm*' necessarily being different values.

Bridged T delay equaliser



The **bridged-T delay equaliser** is an electrical all-pass filter circuit utilising bridged-T topology whose purpose is to insert an, ideally, constant delay at all frequencies in the signal path. It is a class of image filter.

Applications

The network is used when it is required that two or more signals are matched to each other on some form of timing criterion. Delay is added to all other signals so that the total delay is matched to the signal which already has the longest delay. In television broadcasting, for instance, it is desirable that the timing of the television waveform synchronisation pulses from different sources are aligned as they reach studio control rooms or network switching centres. This ensures that cuts between sources do not result in disruption at the receivers. Another application occurs when stereophonic sound is connected by landline, for instance from an outside broadcast to the studio centre. It is important that delay is equalised between the two stereo channels as a difference will destroy the stereo image. When the landlines are long and the two channels arrive by substantially different routes it can require many filter sections to fully equalise the delay.

Operation

The operation is best explained in terms of the phase shift the network introduces. At low frequencies L is low impedance and C' is high impedance and consequently the signal passes through the network with no shift in phase. As the frequency increases, the phase shift gradually increases, until at some frequency, ω_0, the shunt branch of the circuit, L'C', goes in to resonance and causes the centre-tap of L to be short-circuited to ground. Transformer action between the two halves of L, which had been steadily becoming more significant as the frequency increased, now becomes dominant. The winding of the coil is such that the secondary winding produces an inverted voltage to the primary. That is, at resonance the phase shift is now 180°. As the frequency continues to increase, the phase delay also continues to increase and the input and output start to come back into phase as a whole cycle delay is approached. At high frequencies L and L' approach open-circuit and C approaches short-circuit and the phase delay tends to level out at 360°.

The relationship between phase shift (φ) and time delay (T_D) with angular frequency (ω) is given by the simple relation,

$$\phi = \omega T_D$$

It is required that T_D is constant at all frequencies over the band of operation. φ must therefore be kept linearly proportional to ω. With suitable choice of parameters, the network phase shift can be made linear up to about 180° phase shift.

Design

The four component values of the network provide four degrees of freedom in the design. It is required from image theory that the L/C branch and the L'/C' branch are the dual of each other (ignoring the transformer action) which provides two parameters for calculating component values. A third parameter is set by choosing a resonant frequency, this is set to (at least) the maximum frequency the network is required to operate at.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L'C'}}$$

There is one remaining degree of freedom that the designer can use to maximimally linearise the phase/frequency response. This parameter is usually stated as the L/C ratio. As stated above, it is not practical to linearise the phase response above 180°, ie half a cycle, so once a maximum frequency of operation, f_m is chosen, this sets the maximum delay that can be designed in to the circuit and is given by,

$$T_{D(max)} = \frac{1}{2f_m}$$

For broadcast sound purposes, 15 kHz is often chosen as the maximum usable frequency on landlines. A delay equaliser designed to this specification can therefore insert a delay of 33μs. In reality, the differential delay that might be required to equalise may be many hundreds of microseconds. A chain of many sections in tandem will be required. For television purposes, a maximum frequency of 6 MHz might be chosen, which corresponds to a delay of 83ns. Again, many sections may be required to fully equalise. In general, much greater attention is paid to the routing and exact length of television cables because many more equaliser sections are required to remove the same delay difference as compared to audio.