Mechanical Calculators

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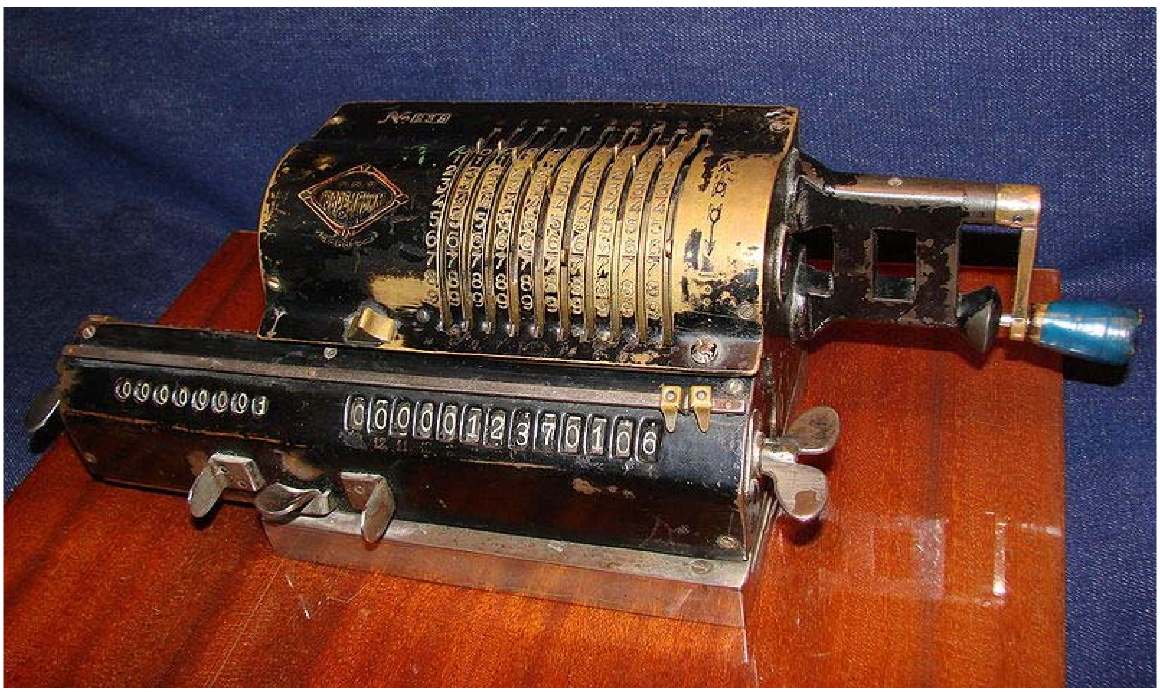
Mechanical Calculator



An old Russian mechanical calculator.

A **mechanical calculator** was a device used to perform the basic operations of arithmetic. The last mechanical calculators were comparable in size to desktop computers and were rendered obsolete by the advent of the electronic calculator.

Description of the illustrated calculator:
This is a pinwheel rotary type, operated by the hand crank at the right. Tips of the setting levers extend through the slots in the brass cover; pinwheels are inside. The result dials are inside the carriage at the front, which can be shifted sidewise. The right array of dials

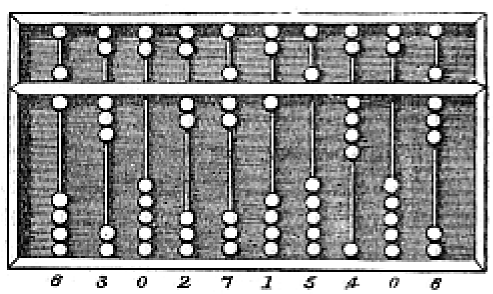
shows sums and differences, as well as the dividend for division. The other set of dials shows the number of cycles at each carriage position. This displayed number serves as a check on multiplication, and shows the quotient from division. "Butterfly" knobs clear the dials to zeros when rotated.

The mechanical calculator was invented in 1642 and the first commercially successful device was manufactured from 1851. Machines with columns of keys were introduced in 1887 while 10 key calculators and electric motors appeared in 1902. The use of electric motors allowed for the design of very powerful machines during the first half of the 20th century. In 1961, A full-keyboard machine like the comptometer, called the Anita, became the first mechanical calculator to receive an all electronic calculator engine, creating the link in between these two industries and marking the beginning of its decline. The Anita was the only full-keyboard electronic calculator of any commercial significance. The last mechanical calculators were built in the middle of the 1970s.

The mechanical calculator was preceded by and competed against clerical aids such as abaci, Napier's bones and slide rules, and various books of mathematical tables. The true precursors to the mechanical calculator were machines made of toothed gears linked by carry mechanisms like odometers, astrolabes, clocks and pedometers. Stylus-operated adders with circular slots for the stylus, and side-by -side wheels, as made by Sterling Plastics (USA), had an ingenious anti-overshoot mechanism to ensure accurate carries.

History

Ancient history



Suanpan (the number represented in the picture is 6,302,715,408)

Devices have been used to aid computation for thousands of years, using one-to-one correspondence with our fingers. The earliest counting device was probably a form of tally stick. Later record keeping aids throughout the Fertile Crescent included clay shapes, which represented counts of items, probably livestock or grains, sealed in containers.

The first clerical aids were abathia, and were often constructed as a wooden frame with beads sliding on wires. Abathias were in use centuries before the adoption of the written Arabic numerals system and are still used by some merchants, fishermen and clerks in

Africa, Asia, and elsewhere. As well, "counting boards", with parallel grooves to hold beads, were used like the abacus.

The counter abacus was devised by Egyptian mathematicians in Egypt in 2000 BC. It was used for arithmetic tasks. The Roman abacus was used in Babylonia as early as 2400 BC. Since then, many other forms of reckoning boards or tables have been invented. In a medieval counting house, a checkered cloth would be placed on a table, and markers moved around on it according to certain rules, as an aid to calculating sums of money (this is the origin of "Exchequer" as a term for a nation's treasury).

Other precursors to the mechanical calculator

A number of analog computers were constructed in ancient and medieval times to perform astronomical calculations. These include the Antikythera mechanism and other astrolabes from ancient Greece (c. 150-100 BC), which are generally regarded as the first mechanical analog computers. Other early versions of mechanical devices used to perform some type of calculations include the planisphere and other mechanical computing devices invented by Abū Rayhān al-Bīrūnī (c. AD 1000); the equatorium and universal latitude-independent astrolabe by Abū Ishāq Ibrāhīm al-Zarqālī (c. AD 1015); the astronomical analog computers of other medieval Muslim astronomers and engineers; and the astronomical clock tower of Su Song (c. AD 1090) during the Song Dynasty. The "castle clock", an astronomical clock invented by Al-Jazari in 1206, is considered to be the earliest programmable analog computer.

The 17th century

Overview

The 17th century was a turning point in the history of mechanical calculators. On one hand, it saw the invention of logarithms, logarithmic tables and the slide rule which, for their ease of use by scientists in multiplying and dividing, ruled over and impeded the use and development of mechanical calculators until the production release of the arithmometer in the mid 19th century, and, on the other hand, it saw the invention of the mechanical calculator by Blaise Pascal quickly followed by Gottfried Leibniz who was the first to describe a pinwheel calculator and who invented the Leibniz wheel. Neither of them was successful in commercializing their machines.

Logarithms and slide rules

17th century calculators, Musée des Arts et Métiers

Scottish mathematician and physicist John Napier noted multiplication and division of numbers could be performed by addition and subtraction, respectively, of logarithms of those numbers. While producing the first logarithmic tables Napier needed to perform many multiplications, and it was at this point that he designed Napier's bones, a device consisting of square rods positioned side by side, marked with digits, used for multiplication and division.

In 1622 William Oughtred invented the slide rule, which was revealed by his student Richard Delamain in 1630. Since real numbers can be represented as distances or intervals on a line, the slide rule allows multiplication and division operations to be carried out significantly faster than was previously possible. The devices were used by generations of engineers and other mathematically inclined professional workers, until the invention of the pocket calculator. The engineers in the Apollo program that sent a man to the moon made many of their calculations on slide rules, which were accurate to three or four significant figures.

Invention of the mechanical calculator

In 1642, Blaise Pascal invented the mechanical calculator while looking for a way to help his father who had been assigned the task of reorganizing the tax revenues of the French province of Haute-Normandie. After three years of effort and 50 prototypes he introduced his machine to the public. He built twenty of these machines (called the Pascaline) in the following ten years. This machine could add and subtract two numbers directly and multiply and divide by repetition.

Gottfried Leibniz invented the first calculator that could perform all four arithmetic operations automatically while adding direct multiplication and division to the Pascaline ; it was called the Stepped Reckoner. He built it around 1672, but careful examination at the end of the 19th century showed a problem with the carry mechanism. It used his Leibniz wheels which, a century and a half later, will be at the heart of the arithmometer, the first calculator to be commercialized. He also was the first to describe a pinwheel calculator in 1685. Leibniz once said "It is unworthy of excellent men to lose hours like slaves in the labour of calculation which could safely be relegated to anyone else if machines were used."

Unknown prototype

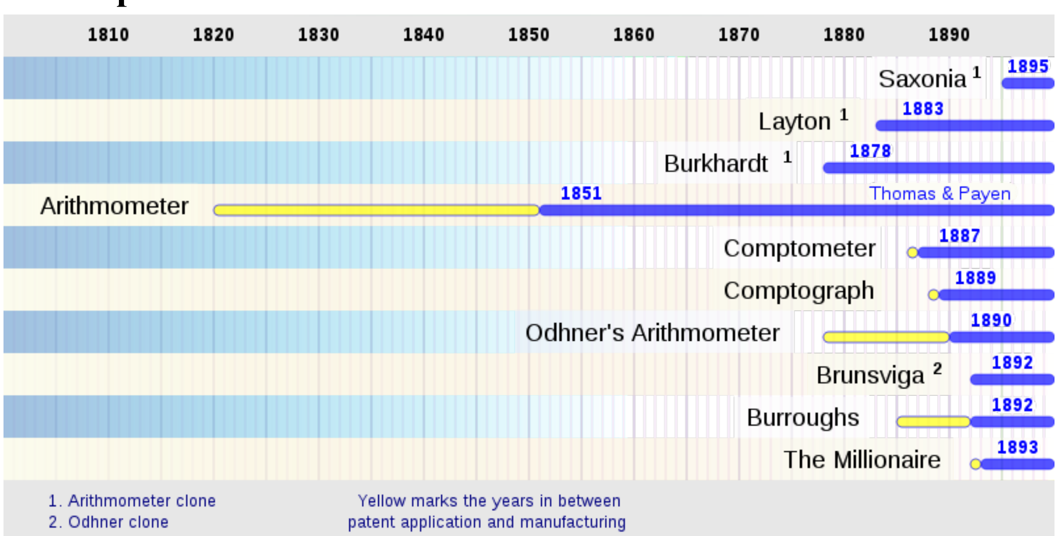
Wilhelm Schickard, a German polymath, designed a calculating clock in 1623; a fire destroyed it during its construction in 1624 and Schickard abandoned his project. Two sketches of it were discovered in 1957; too late to have any impact on the development of mechanical calculators.

The 18th century

The 18th century saw the first fully functional, four operations, mechanical calculators. Both pinwheel calculators and Leibniz wheel calculators were built with a few unsuccessful attempts at their commercialization.

The 19th century

Machines produced



Desktop Mechanical Calculators in production during the 19th century

- in 1820 Thomas de Colmar patented the Arithmometer. It was a true multiplication machine with a one digit multiplier (the millionaire calculator released 70 years later had a similar user interface). He will spent the next 30 years and 300,000 Francs developing his machine. Notable was the calculator's internal mechanical product lookup table.

- in 1851, Thomas de Colmar simplified the arithmometer by removing the multiplier. This made it a simple adding machine, but thanks to its moving carriage used as an indexed accumulator, it still allowed for easy multiplication and division under operator control. The arithmometer was now adapted to the manufacturing capabilities of the time; Thomas can therefore manufacture consistently a sturdy and reliable machine. Manuals are printed and each machine is given a serial number. Its commercialization launches the mechanical calculator industry. Banks, insurance companies, government offices started to use the arithmometer in their day-to-day operations.

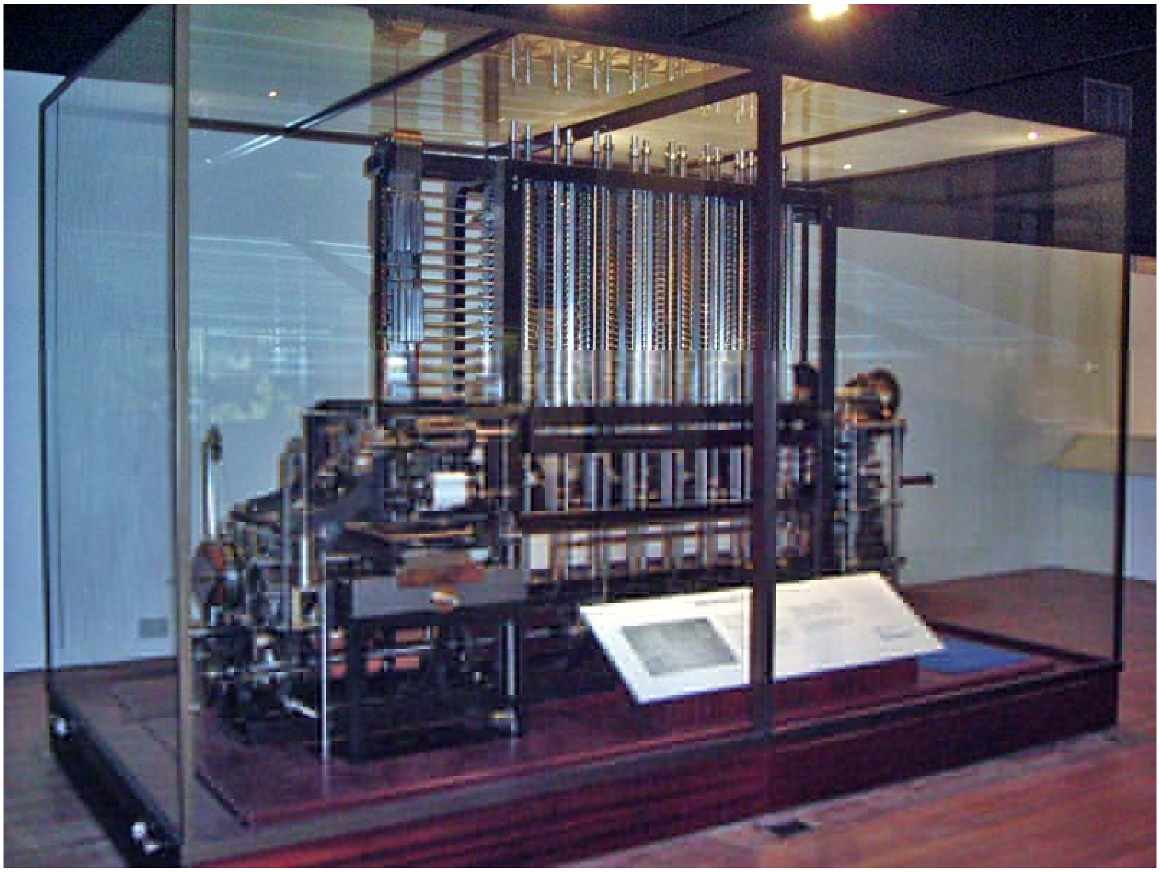
- Dorr E. Felt, in the U.S., patented the Comptometer in 1886. It was the first successful key-driven adding and calculating machine. ["Key-driven" refers to the fact that just pressing the keys causes the result to be calculated, no separate lever or crank has to be operated. Other machines are sometimes called "key-set".] In 1887, he joined with Robert Tarrant to form the Felt & Tarrant Manufacturing Company. The comptometer-type calculator was the first machine to receive an all-electronic calculator engine in 1961 (the ANITA mark VII released by Sumlock comptometer of the UK).

- in 1878 W.T. Odhner patented the Odhner Arithmometer which was a redesigned version of the Arithmometer with a pinwheel engine but with the same user interface. Odhner started manufacturing his machine in his Saint Petersburg workshop in 1890. Many companies, all over the world, manufactured clones of this machine and millions were sold well into the 1970s.

- In 1892 William S. Burroughs began commercial manufacture of his printing adding calculator Burroughs Corporation became one of the leading companies in the accounting machine and computer businesses.

- The "Millionaire" calculator was introduced in 1893. It allowed direct multiplication by any digit - "one turn of the crank for each figure in the multiplier". It contained a mechanical product lookup table, providing units and tens digits by differing lengths of posts. Another direct multiplier was part of the Moon-Hopkins billing machine; that company was acquired by Burroughs in the early 20th century.

Prototypes and limited runs



The London Science Museum's working difference engine, built a century and a half after Charles Babbage's design.

- In 1822 Charles Babbage designed a mechanical calculator, called a difference engine, which was capable of holding and manipulating seven numbers of 31 decimal digits each. Babbage produced two designs for the difference engine and a further design for a more advanced mechanical programmable computer called an analytical engine. None of these designs were completely built by Babbage. In 1991 the London Science Museum followed Babbage's plans to build a working difference engine using the technology and materials available in the 19th century.

- In 1842, Timoleon Maurel invented the Arithmaurel, based on the Arithmometer, which could multiply two numbers by simply entering their values into the machine.

- In 1845 Izrael Abraham Staffel first exhibited a machine that was able to add, subtract, divide, multiply and obtain a square root.

- In 1853 Per Georg Scheutz completed a working difference engine based on Babbage's design. The machine was the size of a piano, and was demonstrated at the Exposition Universelle in Paris in 1855. It was used to create tables of logarithms.

- In 1872, Frank S. Baldwin in the U.S. invented a pinwheel calculator.
- In 1875 Martin Wiberg re-designed the Babbage/Scheutz difference engine and built a version that was the size of a sewing machine.

Operating the calculator at the beginning

Although this is an old machine, nevertheless it represents how one operates any basic rotary calculator. Facits have a pinwheel cylinder that shifts internally, instead of a moving carriage, but the principles still hold.

First, clear the result dials, and then move all setting levers to zero. Position the carriage appropriately. (Use the levers at the front.) The handcrank must be at home position, engaged with its positioning stop.
To add, enter the number into the setting levers. Pull the crank handle to the right, and then toward you, so that it's going away from you when the handle is at the top. One turn will add the number into the accumulator dials, and the counter register to the left will show .
Multiplication: If you continue turning, you'll multiply by the number of turns -- you're adding repetitively. If you need to multiply by several digits, it's simplest to start with the rightmost multiplier digit, then shift the carriage to the right one position for the next digit.
To subtract, pull the crank handle to the right, and push it away from you, so the handle is moving toward you when it's at its highest point. If you subtract more than the number in the accumulator dials, you'll get a complement, which you'll need to convert.
Short-cut multiplication
It's quite unnecessary to crank six or more times for a multiplier digit. Instead, you can shift the carriage one position to the right, add once, then back up the carriage and subtract, until the counter shows the correct digit. For instance, to multiply by 8, shift, add once, shift back, and subtract twice. (10-2 = 8) Thinking ahead, instead, you can subtract twice before adding; the calculator will keep track for you.
Division
Clear the machine and enter the dividend into the setting slides, starting at the left. Move the carriage to the right so the leftmost dividend digit aligns withthe leftmost setting lever. Add once. Clear the counter. If you're lucky, the machine should have a counter-reverse control that will make the counter increment for subtraction and decrement for addition. (Some later, better machines do this automatically for the first turn after entering the dividend and clearing the counter.)
So, now you have the dividend in the accumulator, left-justified.
Change the setting levers to enter the divisor, again to the left.
Be sure the counter is clear, and start subtracting. If the machine has a bell, you can crank mindlessly until the bell rings, then add once. Otherwise, you'll need to watch the

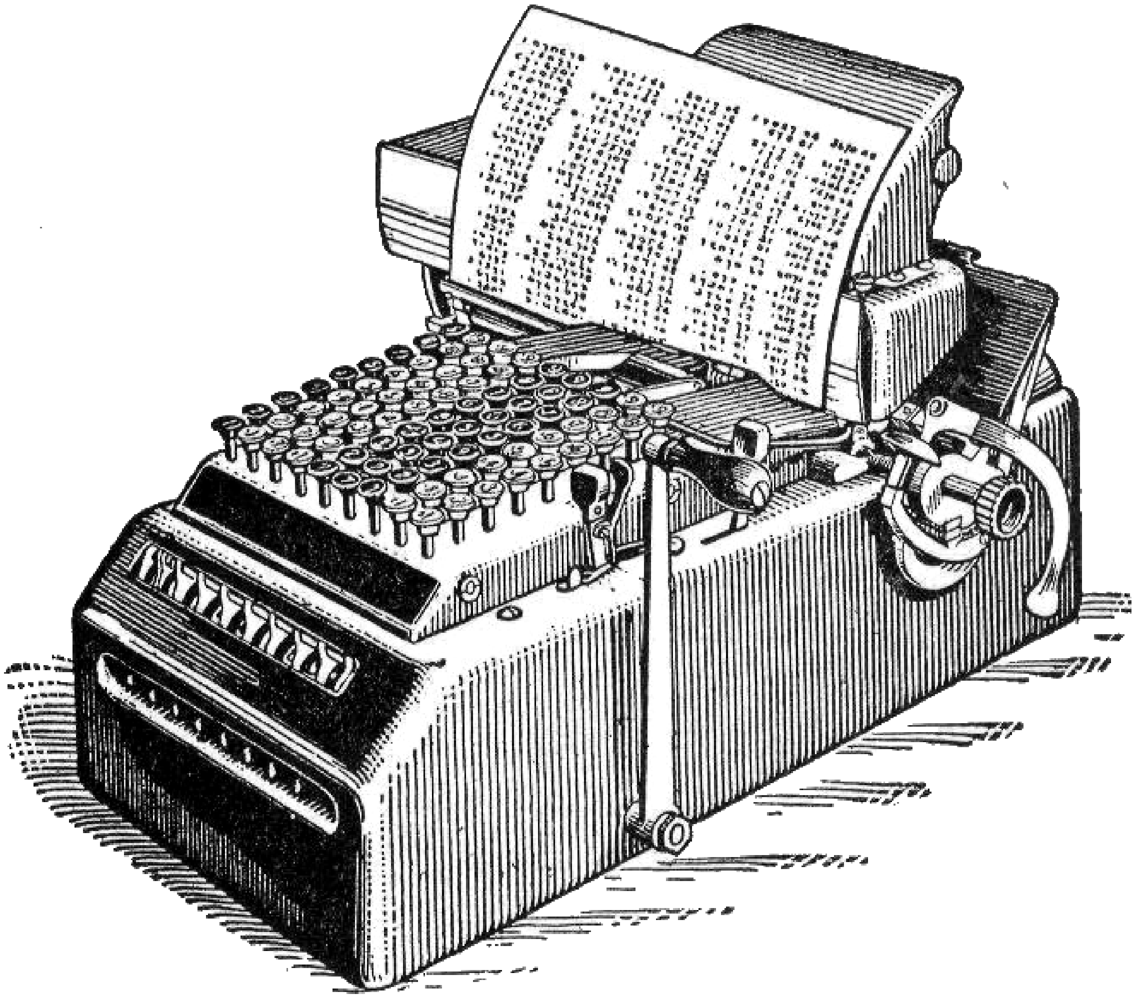
accumulator contents to note (or anticipate) an "overdraft" (subtraction too many times); you have to correct it if it happens, by adding.
Shift the carriage one position to the left, and resume subtracting.
Repeat for each quotient digit, until you either reach the machine's limits or have enough digits. The counter cantains your quotient; the accumulator contains the remainder (if any).
It's of some interest that essentially-automatic division generally appeared before automatic multiplication; as each quotient digit developed, overdraft was allowed to happen, and it triggered a single add cycle followed by a shift.
Square root is possible, by the "fives method", but the description is rather more complicated. This type of machine, in particular, is quite good for this kind of calculation.
Short-cut division
This takes some thought, but can save time. By watching the accumulator, you can anticipate a large quotient digit, and in a fashion similar to short-cut multiplication, you can add and shift, instead of simply subtracting, to save cycles. (The Marchant calculator contains a multi-digit **analog** magnitude comparator that prevents overdrafts! Changing from subtraction to addition and back is messy and slow in that machine.)

1900s to 1970s

Mechanical calculators reach their zenith

Two different classes of mechanisms had become established by this time, reciprocating and rotary. The former type of mechanism was operated typically by a limited-travel hand crank; some internal detailed operations took place on the pull, and others on the release part of a complete cycle. The illustrated 1914 machine is this type; the crank is vertical, on its right side. Later on, some of these mechanisms were operated by electric motors and reduction gearing that operated a crank and connecting rod to convert rotary motion to reciprocating.

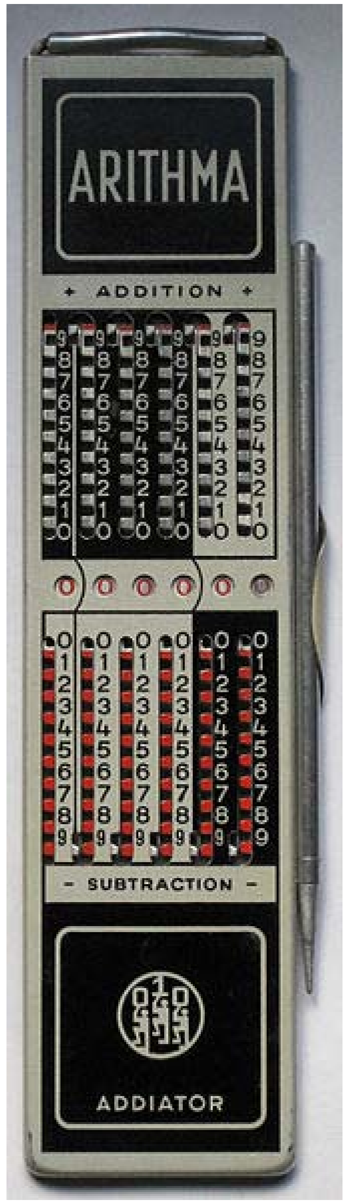
The latter, type, rotary, had at least one main shaft that made one [or more] continuous revolution[s], one addition or subtraction per turn. Numerous designs, notably European calculators, had handcranks, and locks to ensure that the cranks were returned to exact positions once a turn was complete.



Mechanical calculator from 1914

The first half of the 20th century saw the gradual development of the mechanical calculator mechanism.

The Dalton adding-listing machine introduced in 1902 was the first of its type to use only ten keys, and became the first of many different models of "10-key add-listers" manufactured by many companies.

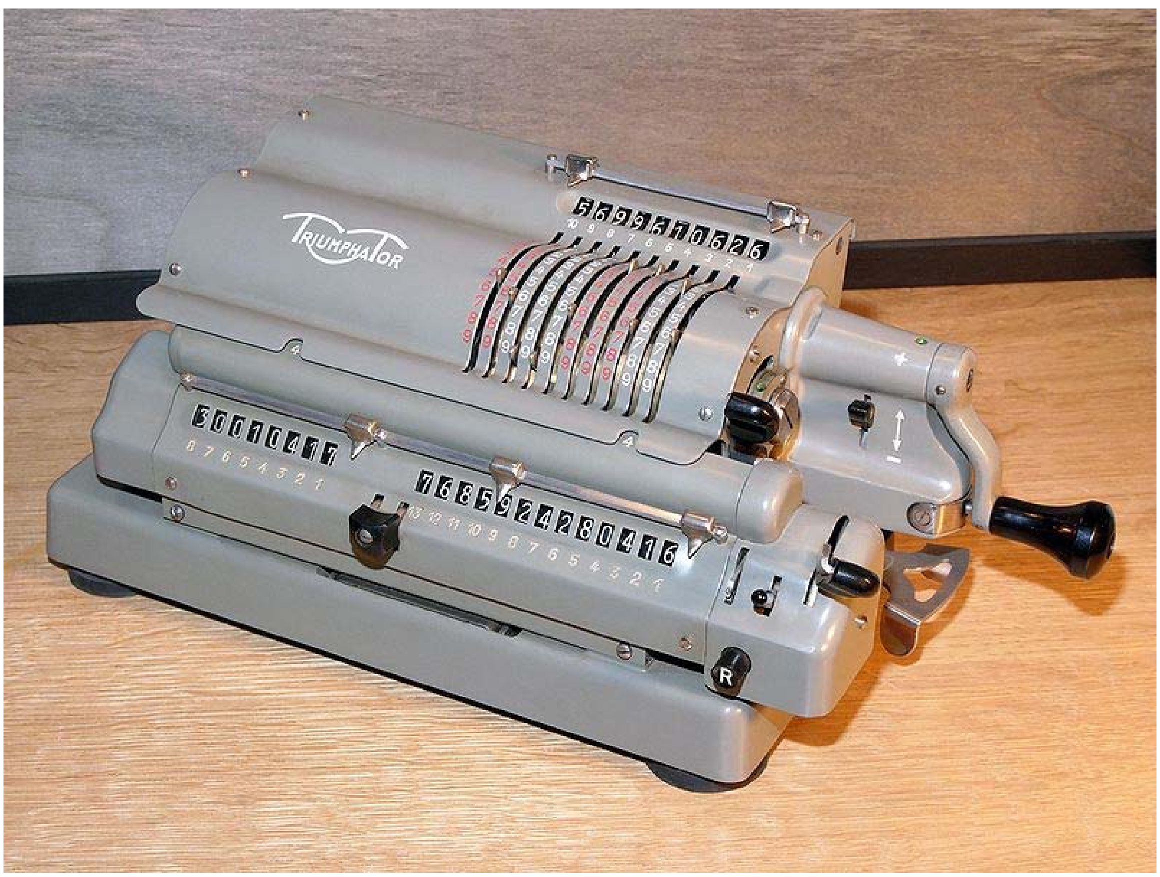


An Addiator could be used for addition and subtraction.

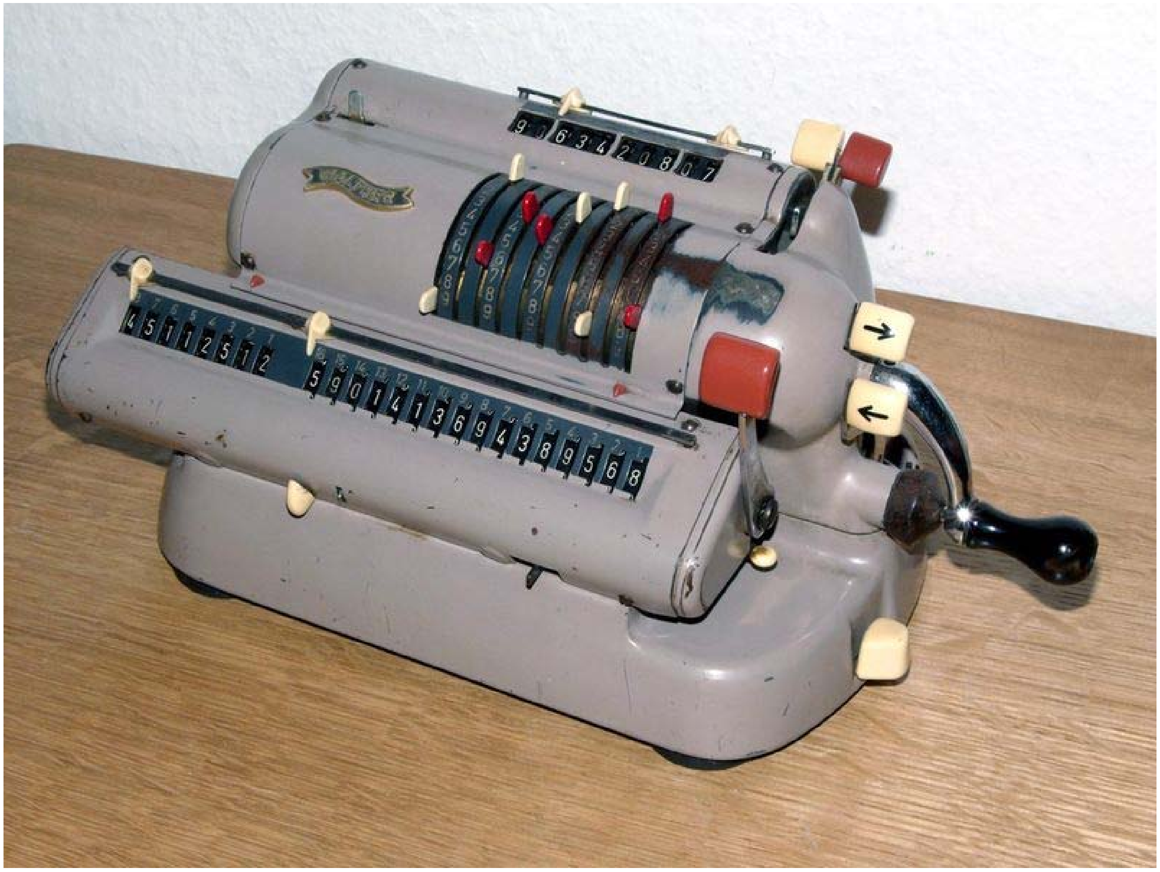
In 1948 the miniature Curta calculator, which was held in one hand for operation, was introduced after being developed by Curt Herzstark in 1938. This was an extreme development of the stepped-gear calculating mechanism. It subtracted by adding complements; between the teeth for addition were teeth for subtraction.

From the early 1900s through the 1960s, mechanical calculators dominated the desktop computing market. Major suppliers in the USA included Friden, Monroe, and SCM/Marchant. (Some comments about European calculators follow below.) These devices were motor-driven, and had movable carriages where results of calculations were displayed by dials. Nearly all keyboards were *full* — each digit that could be entered had its own column of nine keys, 1..9, plus a column-clear key, permitting entry of several digits at once. One could call this parallel entry, by way of contrast with ten-key serial entry that was commonplace in mechanical adding machines, and is now universal in electronic calculators. (Nearly all Friden calculators, as well as some rotary (German) Diehls had a ten-key auxiliary keyboard for entering the multiplier when doing multiplication.) Full keyboards generally had ten columns, although some lower-cost machines had eight. Most machines made by the three companies mentioned did not print their results, although other companies, such as Olivetti, did make printing calculators.

In these machines, addition and subtraction were performed in a single operation, as on a conventional adding machine, but multiplication and division were accomplished by repeated mechanical additions and subtractions. Friden made a calculator that also provided square roots, basically by doing division, but with added mechanism that automatically incremented the number in the keyboard in a systematic fashion. The last of the mechanical calculators were likely to have short-cut muliplication, and some ten-key, serial-entry types had decimal-point keys. However, decimal-point keys required significant internal added complexity, and were offered only in the last designs to be made. The Marchant (Model SKA) also offered square root calculation. Handheld mechanical calculators such as the 1948 Curta continued to be used until they were displaced by electronic calculators in the 1970s.



Triumphator CRN1 (1958)



Walther WSR160 (1960)

Dalton adding machine (1930 ca.)

Typical European four-operations machines use the Odhner mechanism, or variations of it. This kind of machine included the *Original Odhner*, Brunsviga and several following imitators, starting from Triumphator, Thales, Walther, Facit up to Toshiba. Although most of these were operated by handcranks, there were motor-driven versions. Hamann calculators externally resembled pinwheel machines, but the setting lever positioned a cam that disengaged a drive pawl when the dial had moved far enough.

Although Dalton introduced in 1902 first ten-key printing *adding* (two operations, the other being subtraction) machine, these feature were not present in *computing* (four operations) machines for many decades. Facit-T (1932) was the first 10-key computing machine sold in large numbers. Olivetti Divisumma-14 (1948) was the first computing machine with both printer and a 10-key keyboard.

Full-keyboard machines, including motor-driven ones, were also built until the 1960s. Among the major manufacturers were Mercedes-Euklid, Archimedes, and MADAS in Europe; in the USA, Friden, Marchant, and Monroe were the principal makers of rotary calculators with carriages. Reciprocating calculators (most of which were adding machines, many with integral printers) were made by Remington Rand and Burroughs, among others. All of these were key-set. Felt & Tarrant made Comptometers, as well as Victor, which were key-driven.

The basic mechanism of the Friden and Monroe, described above, was a modified Leibniz wheel (better known, perhaps informally, in the USA as a "stepped drum" or "stepped reckoner"). The Friden had an elementary reversing drive between the body of the machine and the accumulator dials, so its main shaft always rotated in the same direction. The Swiss MADAS was similar. The Monroe, however, reversed direction of its main shaft to subtract.

The earliest Marchants were pinwheel machines, but most of them were remarkably-sophisticated rotary types. They ran at 1,300 addition cycles per minute if you held down the [+] bar. Others were limited to 600 cycles per minute, because their accumulator dials started and stopped for every cycle; Marchant dials moved at a steady and proportional speed for continuing cycles. Most Marchants had a row of nine keys on the extreme right, as shown in the photo of the Figurematic. These simply made the machine add for the number of cycles corresponding to the number on the key, and then shifted the carriage one place. Even nine add cycles took only a short time.

In a Marchant, near the beginning of a cycle, the accumulator dials moved downward "into the dip", away from the openings in the cover. They engaged drive gears in the body of the machine, which rotated them at speeds proportional to the digit being fed to them, with added movement (reduced 10:1) from carries created by dials to their right. At the completion of the cycle, the dials would be misaligned like the pointers ina tarditional watthour meter. However, as they came up out of the dip, a constant-lead disc cam realigned them by way of a (limited-travel) spur-gear differential. As well, carries for lower orders were added in by another, planetary differential. (The machine shown has 39 differentials in its (20-digit) accumulator!).

In any mechanical calculator, in effect, a gear, sector, or some similar device moves the accumulator by the number of gear teeth that corresponds to the digit being added or subtracted — three teeth changes the position by a count of three. The great majority of basic calculator mechanisms move the accumulator by starting, then moving at a constant speed, and stopping. In particular, stopping is critical, because to obtain fast operation, the accumulator needs to move quickly. Variants of Geneva drives typically block overshoot (which, of course, would create wrong results).

However, two different basic mechanisms, the Mercedes-Euklid and the Marchant, move the dials at speeds corresponding to the digit being added or subtracted; a moves the accumulator the slowest, and a , the fastest. In the Mercedes-Euklid, a long slotted lever, pivoted at one end, moves nine racks ("straight gears") endwise by distances proportional to their distance from the lever's pivot. Each rack has a drive pin that's moved by the slot. The rack for is closest to the pivot, of course. For each keyboard digit, a sliding selector gear, much like that in the Leibniz wheel, engages the rack that corresponds to the digit entered. Of course, the accumulator changes either on the forward or reverse stroke, but not both. This mechanism is notably simple and relatively easy to manufacture.

The Marchant, however, has, for every one of the its ten columns of keys, a nine-ratio "preselector transmission" with its output spur gear at the top of the machine's body; that gear engages the accumulator gearing. When one tries to work out the numbers of teeth in such a transmission, a straightforward approach leads one to consider a mechanism like that in mechanical gasoline pump registers, used to indicate the total price. However, this mechanism is seriously bulky, and utterly impractical for a calculator; 90-tooth gears are likely to be found is the gas. pump. Practical gears in the computing parts of a calculator can't have 90 teeth. They would be either too big, or too delicate.

Given that nine ratios per column implies significant complexity, a Marchant contains a few hundred individual gears in all, many in its accumulator. Basically, the accumulator dial has to rotate 36 degrees (1/10 of a turn) for a , and 324 degrees (9/10 of a turn) for a , not allowing for incoming carries. At some point in the gearing, one tooth needs to pass for a , and nine teeth for a . There's no way to develop the needed movement from a driveshaft that rotates one revolution per cycle with few gears having practical (relatively small) numbers of teeth.

The Marchant, therefore, has three driveshafts to feed the little transmissions. For one cycle, they rotate 1/2, 1/4, and 1/12 of a revolution. . The 1/2-turn shaft carries (for each column) gears with 12, 14, 16, and 18 teeth, corresponding to digits 6, 7, 8, and 9. The 1/4-turn shaft carries (also, each column) gears with 12, 16, and 20 teeth, for 3, 4, and 5. Digits and are handled by 12 and 24-tooth gears on the 1/12-revolution shaft. Practical design places the 12th-rev. shaft more distant, so the 1/4-turn shaft carries freely-rotating 24 and 12-tooth idler gears. For subtraction, the driveshafts reversed direction.

In the early part of the cycle, one of five pendants moves off-center to engage the appropriate drive gear for the selected digit.

Some machines had as many as 20 columns in their full keyboards. The *monster* in this field was the *Duodecillion* made by Burroughs for exhibit purposes.

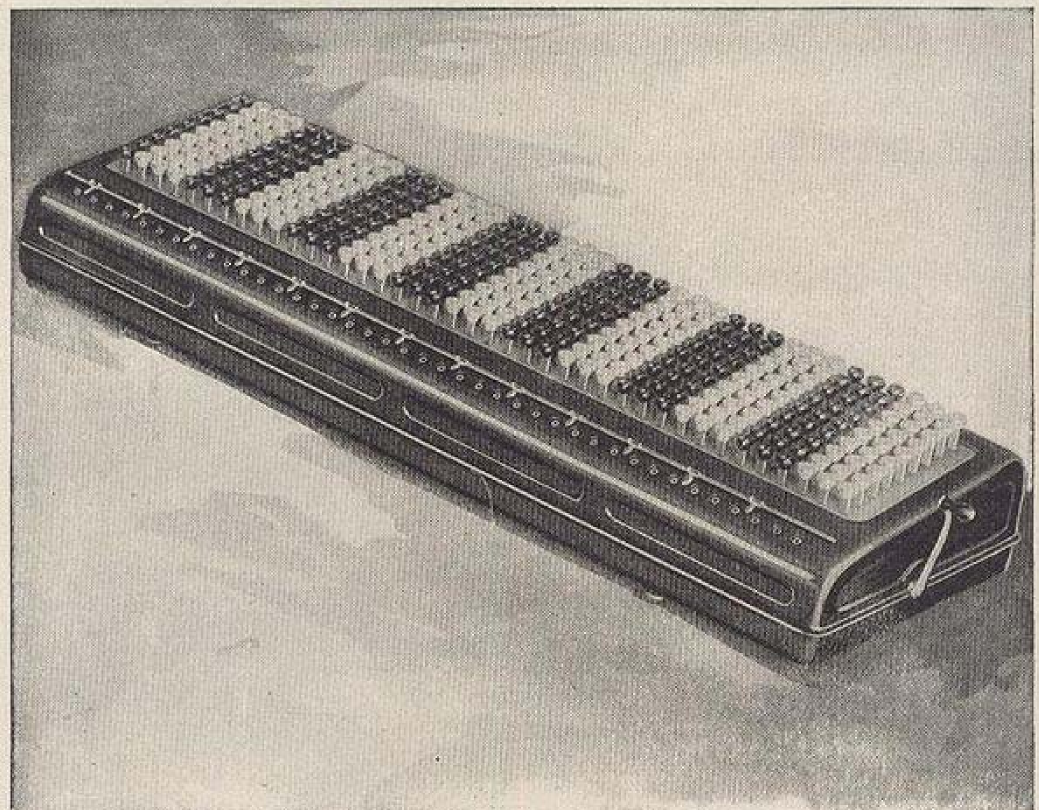
For sterling currency, £/s/d (and even farthings), there were variations of the basic mechanisms, in particular with different numbers of gear teeth and accumulator dial positions. To accomodate shillings and pence, extra columns were added for the tens digit[s], 10 and 20 for shillings, and 10 for pence. Of course, these functioned as radix-20 and radix-12 mechanisms.

A variant of the Marchant, called the Binary-Octal Marchant, was a radix-8 (octal) machine. It was sold to check very early vacuum-tube (valve) binary computers for accuracy. (Back then, the mechanical calculator was much more reliable than a tube/valve computer.)

As well, there was a twin Marchant, comprising two pinwheel Marchants with a common drive crank and reversing gearbox. Twin machines were relatively rare, and apparently were used for surveying calculations (The CORDIC algorithm was invented later, but these machine might be able to execute it.) At least one triple machine (Brunsviga(?)) was made. It's likely that a given accumulator could be engaged with either half of the twin.

The Facit calculator, and one similar to it, are basically pinwheel machines, but the array of pinwheels moves sidewise, instead of the carriage. The pinwheels are biquinary; digits 1 through 4 cause the corresponding number of sliding pins ot extend from the surface; digits 5 through 9 also extend a five-tooth sector as well as the same pins for 6 through 9.

The keys operate cams that operate a swinging lever to first unlock the pin-positioning cam that's partof the pinwheel mechanism; further movement of the lever (by an amount determined by the key's cam) rotates the pin-positioning cam to extend the necessary number of pins.



THE "DUODECILLION"—THE LARGEST CAPACITY ADDING MACHINE IN THE WORLD—HAS FORTY ROWS OF KEYS AND WILL ADD TO WITHIN A UNIT OF TEN DUODECILLIONS

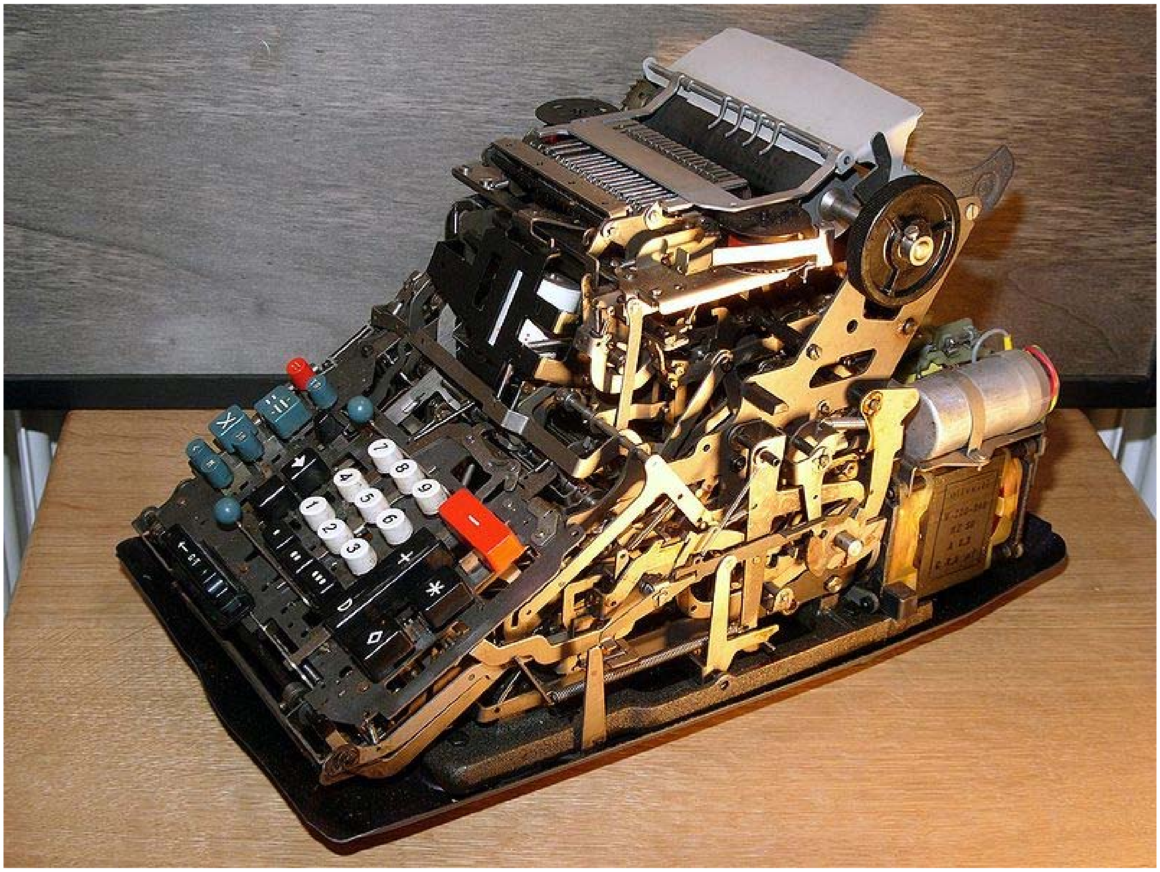
To appreciate this prodigious figure, imagine that a marvelous high-speed flying machine were invented that would go to the sun and back in a day. If you made this 186,000,000-mile trip every day, it would take you just 14,729,700,000,000,000,000,000,000,000 years to travel a duodecillion miles.

Courtesy of the Burroughs Adding Machine Company.

Duodecillion (1915 ca.)

Marchant Figurematic (1950-52)

Facit NTK (1954)



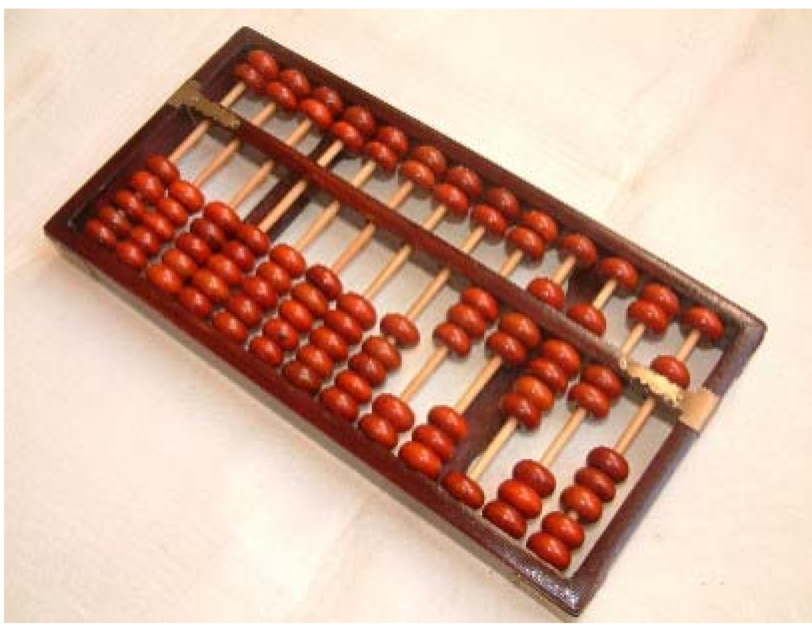
Olivetti Divisumma 24 (1964)

The end of an era

Mechanical calculators continued to be sold, though in rapidly decreasing numbers, into the early 1970s, with many of the manufacturers closing down or being taken over. Comptometer type calculators were often retained for much longer to be used for adding and listing duties, especially in accounting, since a trained and skilled operator could enter all the digits of a number in one movement of the hands on a Comptometer quicker than was possible serially with a 10-key electronic calculator. In fact, it was quicker to enter larger digits in two strokes using only the lower-numbered keys; for instance, a 9 would be entered as 4 followed by 5. Some key-driven calculators had keys for every column, but only 1 through 5; they were correspondingly compact. The spread of the computer rather than the simple electronic calculator put an end to the Comptometer. Also, by the end of the 1970s, the slide rule had become obsolete.

Chapter 2

Abacus



A Chinese abacus

Calculating-Table by Gregor Reisch: Margarita Philosophica, 1508. The woodcut shows *Arithmetica* instructing an algorist and an abacist (inaccurately represented as Boethius and Pythagoras). There was keen competition between the two from the introduction of the *Algebra* into Europe in the 12th century until its triumph in the 16th.

The **abacus**, also called a **counting frame**, is a calculating tool used primarily in parts of Asia for performing arithmetic processes. Today, abaci are often constructed as a bamboo frame with beads sliding on wires, but originally they were beans or stones moved in grooves in sand or on tablets of wood, stone, or metal. The abacus was in use centuries before the adoption of the written modern numeral system and is still widely used by merchants, traders and clerks in Asia, Africa, and elsewhere. The user of an abacus is called an abacist.

Etymology

The use of the word *abacus* dates before 1387 AD, when a Middle English work borrowed the word from Latin to describe a sandboard abacus. The Latin word came from Ἀβακός *abakos*, the Greek genitive form of Ἀβαξ *abax* ("calculating-table"), from Hebrew *ābāq* (אבק), "dust". The preferred plural of *abacus* is a subject of disagreement, with both *abacuses* and *abaci* in use.

Mesopotamian abacus

The period 2700–2300 BC saw the first appearance of the Sumerian abacus, a table of successive columns which delimited the successive orders of magnitude of their sexagesimal number system.

Some scholars point to a character from the Babylonian cuneiform which may have been derived from a representation of the abacus. It is the belief of Carruccio (and other Old Babylonian scholars) that Old Babylonians "may have used the abacus for the operations of addition and subtraction; however, this primitive device proved difficult to use for more complex calculations".

Egyptian abacus

The use of the abacus in Ancient Egypt is mentioned by the Greek historian Herodotus, who writes that the Egyptians manipulated the pebbles from right to left, opposite in direction to the Greek left-to-right method. Archaeologists have found ancient disks of various sizes that are thought to have been used as counters. However, wall depictions of this instrument have not been discovered, casting some doubt over the extent to which this instrument was used.

Persian abacus

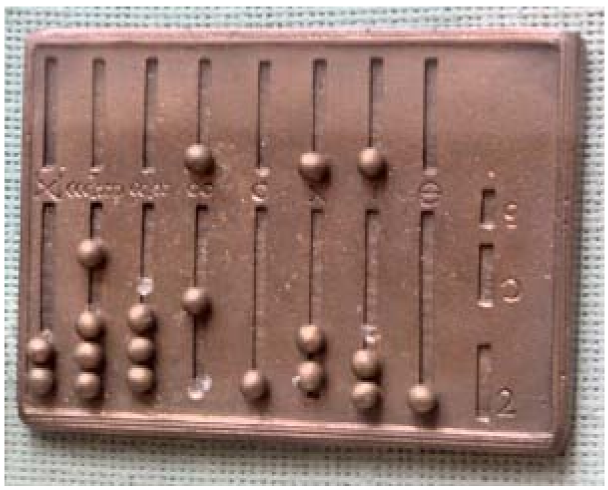
During the Achaemenid Persian Empire, around 600 BC the Persians first began to use the abacus. Under Parthian and Sassanian Iranian empires, scholars concentrated on exchanging knowledge and inventions by the countries around them – India, China, and the Roman Empire, when it is thought to be expanded over the other countries.

Greek abacus

The earliest archaeological evidence for the use of the Greek abacus dates to the 5th century BC. The Greek abacus was a table of wood or marble, pre-set with small counters in wood or metal for mathematical calculations. This Greek abacus saw use in Achaemenid Persia, the Etruscan civilization, Ancient Rome and, until the French Revolution, the Western Christian world.

A tablet found on the Greek island Salamis in 1846 AD dates back to 300 BC, making it the oldest counting board discovered so far. It is a slab of white marble 149 cm (59 in) long, 75 cm (30 in) wide, and 4.5 cm (2 in) thick, on which are 5 groups of markings. In the center of the tablet is a set of 5 parallel lines equally divided by a vertical line, capped with a semicircle at the intersection of the bottom-most horizontal line and the single vertical line. Below these lines is a wide space with a horizontal crack dividing it. Below this crack is another group of eleven parallel lines, again divided into two sections by a line perpendicular to them, but with the semicircle at the top of the intersection; the third, sixth and ninth of these lines are marked with a cross where they intersect with the vertical line.

Roman abacus



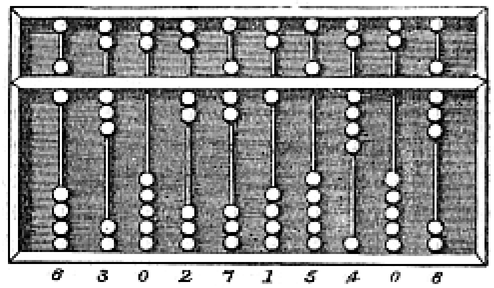
Copy of a Roman Abacus

The normal method of calculation in ancient Rome, as in Greece, was by moving counters on a smooth table. Originally pebbles, calculi, were used. Later, and in medieval Europe, jetons were manufactured. Marked lines indicated units, fives, tens etc. as in the Roman numeral system. This system of 'counter casting' continued into the late Roman empire and in medieval Europe, and persisted in limited use into the tenth century. Due to Pope Sylvester II's reintroduction of the abacus with very useful modifications, it became widely used in Europe once again during the 11th century

Writing in the 1st century BC, Horace refers to the wax abacus, a board covered with a thin layer of black wax on which columns and figures were inscribed using a stylus.

One example of archaeological evidence of the Roman abacus, shown here in reconstruction, dates to the 1st century AD. It has eight long grooves containing up to five beads in each and eight shorter grooves having either one or no beads in each. The groove marked I indicates units, X tens, and so on up to millions. The beads in the shorter grooves denote fives –five units, five tens etc., essentially in a bi-quinary coded decimal system, obviously related to the Roman numerals. The short grooves on the right may have been used for marking Roman ounces.

Chinese abacus



Suanpan (the number represented in the picture is 6,302,715,408)

The earliest known written documentation of the Chinese abacus dates to the 2nd century BC.

The Chinese abacus, known as the *suànpán* (算盤, lit. "Counting tray"), is typically 20 cm (8 in) tall and comes in various widths depending on the operator. It usually has more than seven rods. There are two beads on each rod in the upper deck and five beads each in the bottom for both decimal and hexadecimal computation. The beads are usually rounded and made of a hardwood. The beads are counted by moving them up or down towards the beam. If you move them toward the beam, you count their value. If you move away, you don't count their value. The suanpan can be reset to the starting position instantly by a quick jerk along the horizontal axis to spin all the beads away from the horizontal beam at the center.

Suanpans can be used for functions other than counting. Unlike the simple counting board used in elementary schools, very efficient suanpan techniques have been developed to do multiplication, division, addition, subtraction, square root and cube root operations at high speed. There are currently schools teaching students how to use it.

In the famous long scroll *Along the River During the Qingming Festival* painted by Zhang Zeduan (1085–1145 AD) during the Song Dynasty (960–1297 AD), a suanpan is clearly seen lying beside an account book and doctor's prescriptions on the counter of an apothecary's (Feibao).

The similarity of the Roman abacus to the Chinese one suggests that one could have inspired the other, as there is some evidence of a trade relationship between the Roman Empire and China. However, no direct connection can be demonstrated, and the similarity of the abaci may be coincidental, both ultimately arising from counting with five fingers per hand. Where the Roman model (like most modern Japanese) has 4 plus 1 bead per decimal place, the standard suanpan has 5 plus 2, allowing use with a hexadecimal numeral system. Instead of running on wires as in the Chinese and Japanese models, the beads of Roman model run in grooves, presumably making arithmetic calculations much slower.

Another possible source of the suanpan is Chinese counting rods, which operated with a decimal system but lacked the concept of zero as a place holder. The zero was probably introduced to the Chinese in the Tang Dynasty (618-907 AD) when travel in the Indian Ocean and the Middle East would have provided direct contact with India, allowing them to acquire the concept of zero and the decimal point from Indian merchants and mathematicians.

Indian abacus

First century sources, such as the *Abhidharmakosa* describe the knowledge and use of abacus in India. Around the 5th century, Indian clerks were already finding new ways of recording the contents of the Abacus. Hindu texts used the term *shunya* (zero) to indicate the empty column on the abacus.

Japanese abacus

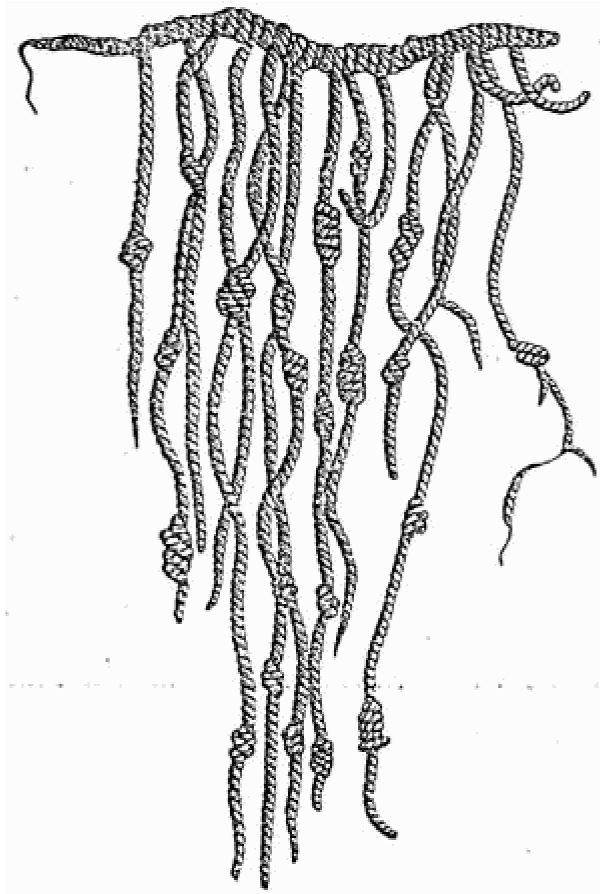
Japanese soroban

In Japanese, the abacus is called *soroban* (算盤, そろばん, lit. "Counting tray"), imported from China around 1600. The 1/4 abacus, which is suited to decimal calculation, appeared circa 1930, and became widespread as the Japanese abandoned hexadecimal weight calculation which was still common in China. The abacus is still manufactured in Japan today even with the proliferation, practicality, and affordability of pocket electronic calculators. The use of the soroban is still taught in Japanese primary schools as part of mathematics, primarily as an aid to faster mental calculation. Using visual imagery of a soroban can arrive at the answer in the same time (or faster) as obtainable with a physical instrument.

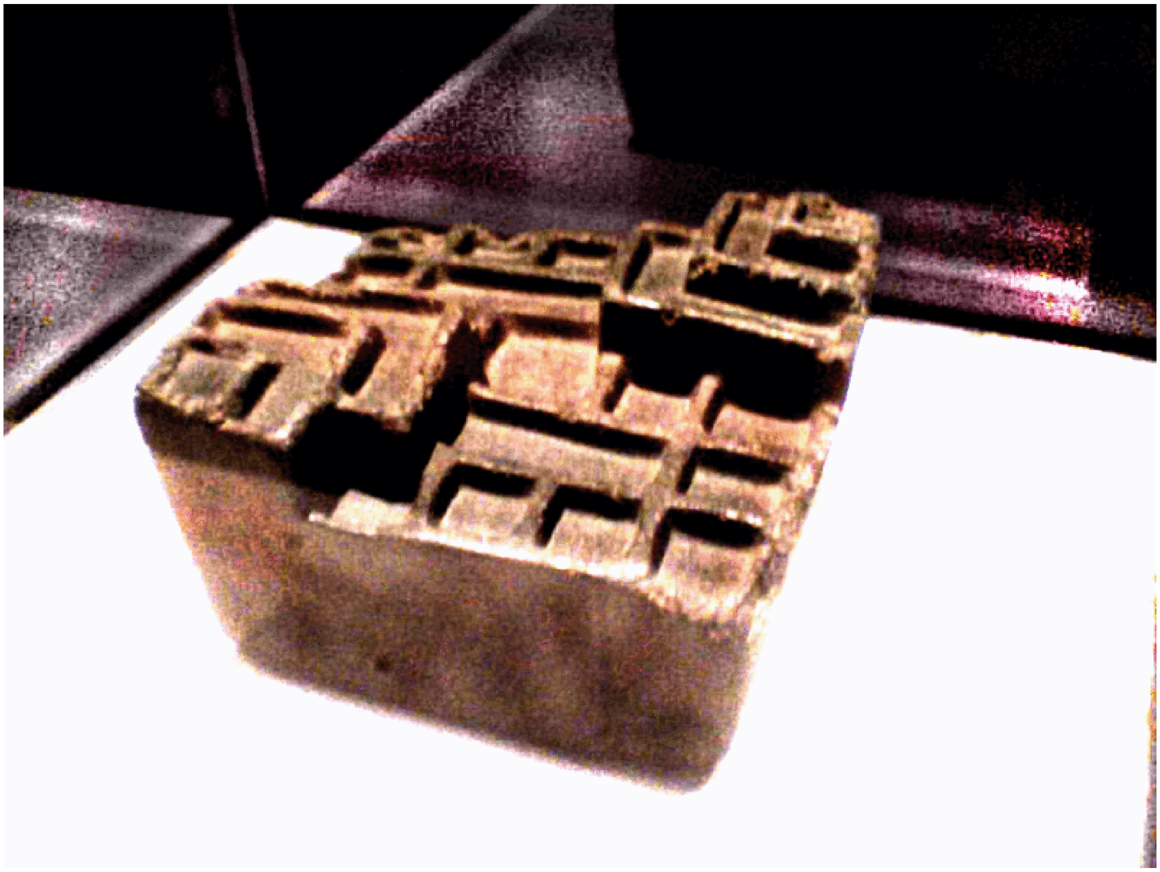
Korean abacus

The Chinese abacus migrated from China to Korea around 1400 AD. Koreans call it *jupan* (주판), *supan* (수판) or *jusan* (주산).

Native American abaci



Representation of an Inca quipu



A yupana as used by the Incas.

Some sources mention the use of an abacus called a *nepohualtzintzin* in ancient Mayan culture. This Mesoamerican abacus used a 5-digit base-20 system. The word Nepohualtzintzin comes from the Nahuatl and it is formed by the roots; Ne - personal -; pohual or pohualli - the account -; and tzintzin - small similar elements. And its complete meaning was taken as: counting with small similar elements by somebody. Its use was taught in the "Kalmekak" to the "temalpouhkeh", who were students dedicated to take the accounts of skies, from childhood. Unfortunately the Nepohualtzintzin and its teaching were among the victims of the conquering destruction, when a diabolic origin was attributed to them after observing the tremendous properties of representation, precision and speed of calculations..

This arithmetic tool was based on the vigesimal system (base 20). For the aztec the count by 20s was completely natural. The amount of 4, 5, 13, 20 and other cyclees meant cycles. The Nepohualtzintzin was divided in two main parts separated by a bar or intermediate cord. In the left part there were four beads, which in the first row have unitary values (1, 2, 3, and 4), and in the right side there are three beads with values of 5, 10, and 15 respectively. In order to know the value of the respective beads of the upper rows, it is enough to multiply by 20 (by each row), the value of the corresponding account in the first row.

Altogether, there were 13 rows with 7 beads in each one, which made up 91 beads in each Nepohualtzintzin. This was a basic number to understand, 7 times 13, a close relation conceived between natural phenomena, the underworld and the cycles of the heavens. One Nepohualtzintzin (91) represented the number of days that a season of the year lasts, two Nepohualtzitzin (182) is the number of days of the corn's cycle, from its sowing to its harvest, three Nepohualtzintzin (273) is the number of days of a baby's gestation, and four Nepohualtzintzin (364) completed a cycle and approximate a year (1 1/4 days short). It is worth mentioning that the Nepohualtzintzin amounted to the rank from 10 to the 18 in floating point, which calculated stellar as well as infinitesimal amounts with absolute precision, meant that no round off was allowed, when translated into modern computer arithmetic.

The rediscovery of the Nepohualtzintzin was due to the Mexican engineer David Esparza Hidalgo, who in his wanderings throughout Mexico found diverse engravings and paintings of this instrument and reconstructed several of them made in gold, jade, encrustations of shell, etc.. There have also been found very old Nepohualtzintzin attributed to the Olmeca culture, and even some bracelets of Mayan origin, as well as a diversity of forms and materials in other cultures.

George I. Sanchez, "Arithmetic in Maya", Austin-Texas, 1961 found another base 5, base 4 abacus in the Yucatán that also computed calendar data. This was a finger abacus, on one hand 0 1,2, 3, and 4 were used; and on the other hand used 0, 1, 2 and 3 were used. Note the use of zero at the beginning an end of the two cycles. Sanchez worked with Sylvanus Morley a noted Mayanist.

The quipu of the Incas was a system of knotted cords used to record numerical data, like advanced tally sticks – but not used to perform calculations. Calculations were carried out using a yupana which was still in use after the conquest of Peru. The working principle of a yupana is unknown, but in 2001 an explanation of the mathematical basis of these instruments was proposed by Italian mathematician Nicolino De Pasquale. By comparing the form of several yupanas, researchers found that calculations were based using the Fibonacci sequence 1, 1, 2, 3, 5 and powers of 10, 20 and 40 as place values for the different fields in the instrument. Using the Fibonacci sequence would keep the number of grains within any one field at minimum.

Russian abacus

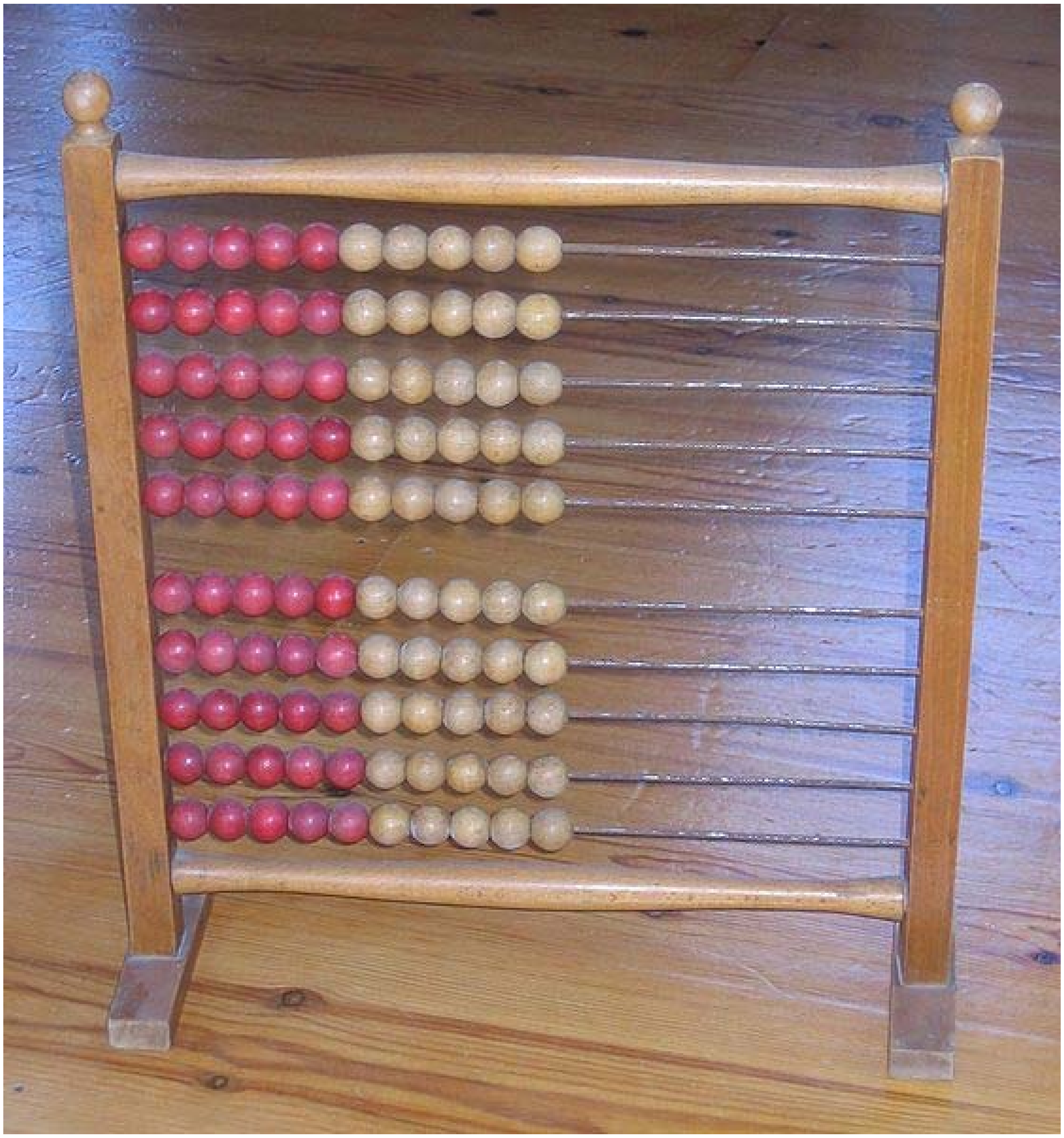
Russian abacus

The Russian abacus, the *schoty* (счёты), usually has a single slanted deck, with ten beads on each wire (except one wire which has four beads, for quarter-ruble fractions. This wire is usually near the user). (Older models have another 4-bead wire for quarter-kopeks, which were minted until 1916.) The Russian abacus is often used vertically, with wires from left to right in the manner of a book. The wires are usually bowed to bulge upward in the center, to keep the beads pinned to either of the two sides. It is cleared when all the beads are moved to the right. During manipulation, beads are moved to the left. For easy viewing, the middle 2 beads on each wire (the 5th and 6th bead) usually are of a different colour from the other eight beads. Likewise, the left bead of the thousands wire (and the million wire, if present) may have a different color.

As a simple, cheap and reliable device, the Russian abacus was in use in all shops and markets throughout the former Soviet Union, and the usage of it was taught in most schools until the 1990s. Even the 1874 invention of mechanical calculator, Odhner arithmometer, had not replaced them in Russia and likewise the mass production of Felix arithmometers since 1924 did not significantly reduce their use in the Soviet Union. Russian abacus began to lose popularity only after the mass production of microcalculators had started in the Soviet Union in 1974. Today it is regarded as an archaism and replaced by the handheld calculator.

The Russian abacus was brought to France around 1820 by the mathematician Jean-Victor Poncelet, who served in Napoleon's army and had been a prisoner of war in Russia. The abacus had fallen out of use in western Europe in the 16th century with the rise of decimal notation and algorismic methods. To Poncelet's French contemporaries, it was something new. Poncelet used it, not for any applied purpose, but as a teaching and demonstration aid.

School abacus



School abacus used in Danish elementary school. Early 19th century.

Around the world, abaci have been used in pre-schools and elementary schools as an aid in teaching the numeral system and arithmetic.

In Western countries, a **bead frame** similar to the Russian abacus but with straight wires and a vertical frame has been common. It is still often seen as a plastic or wooden toy.

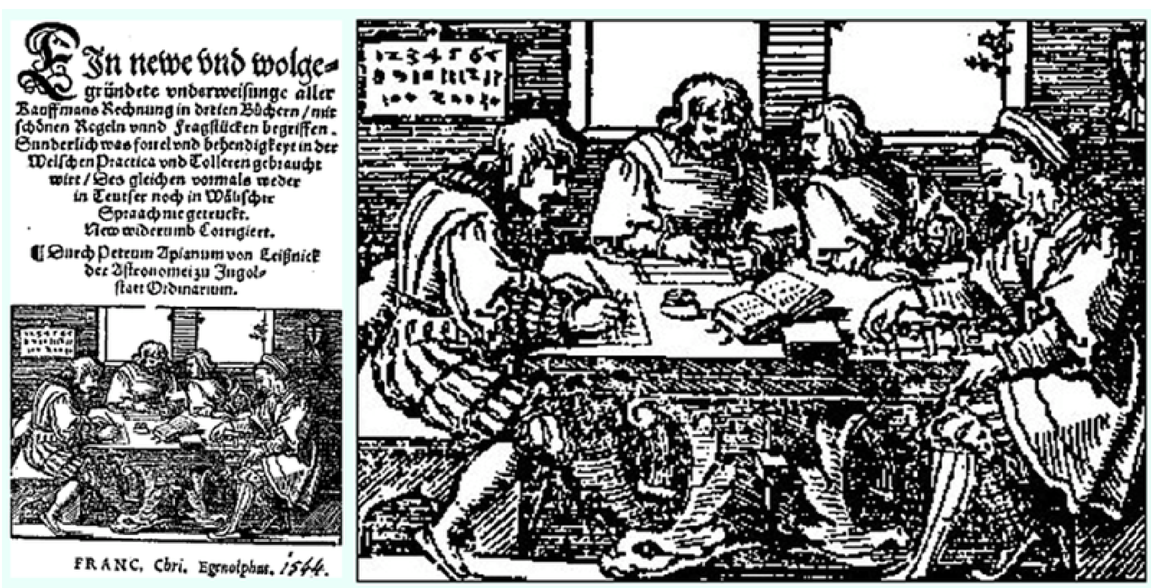
The type of abacus shown here is often used to represent numbers without the use of place value. Each bead and each wire has the same value and used in this way it can represent numbers up to 100.

Abaci in Renaissance pictures

Ain New geordnet Rech
en biechlin auf den linien
mit Rechen pfeningen: den
Jungen angenden zu heis
lichem gebrauch vnd hend
eln leychtlich zu lernen
mit figuren vnd exempeln
Volgt hernach klar-
lichen angezaigt.

Ain New geordnet Rech
en biechlin mit den zyffern
den angenden schülern zů nutz In
haltēt die Siben species Algorith-
mi mit sampt der Regel de Try/ vnd sechs regeln d
prüch/ vñ der regel Fusti mit vil andern gůten fra-
gen den kindern zum anfang nützbarlich durch
Joann Böschensteyn von Esslingen priester
neülych auss gangen vnd geordnet.


VDMIE
1543

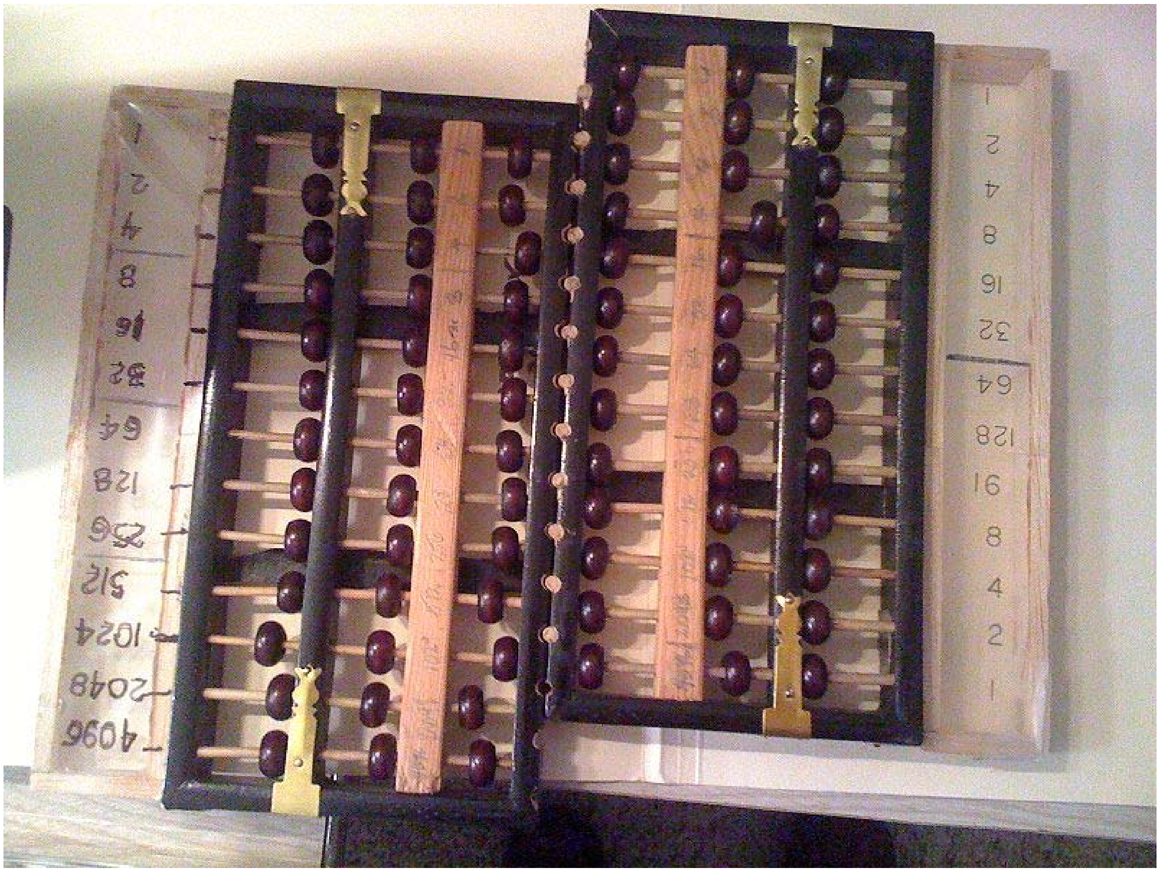

EIn newe vnd wolgegründete vnderweisunge aller
Kauffmans Rechnung in dreien Büchern / mit
schönen Regeln vnnd Fragstücken begriffen.
Sunderlich was fortel vnd behendigkeyt in der
Welschen Practica vnd Tolleren gebraucht
wirt / Des gleichen vormals weder
in Teutscher noch in Wälischer
Spraach nie getruckt.
New widerumb Corrigiert.
Durch Petrum Apianum von Leißnick
der Astronomei zu Ingolstatt Ordinarium.
FRANC. Chri. Egenolphus. 1544.

Uses by the blind

An adapted abacus, invented by Tim Cranmer, called a Cranmer abacus is still commonly used by individuals who are blind. A piece of soft fabric or rubber is placed behind the beads so that they do not move inadvertently. This keeps the beads in place while the users feel or manipulate them. They use an abacus to perform the mathematical functions multiplication, division, addition, subtraction, square root and cubic root.

Although blind students have benefited from talking calculators, the abacus is still very often taught to these students in early grades, both in public schools and state schools for the blind. The abacus teaches mathematical skills that can never be replaced with talking calculators and is an important learning tool for blind students. Blind students also complete mathematical assignments using a braille-writer and Nemeth code (a type of braille code for mathematics) but large multiplication and long division problems can be long and difficult. The abacus gives blind and visually impaired students a tool to compute mathematical problems that equals the speed and mathematical knowledge required by their sighted peers using pencil and paper. Many blind people find this number machine a very useful tool throughout life.

Binary Abacus



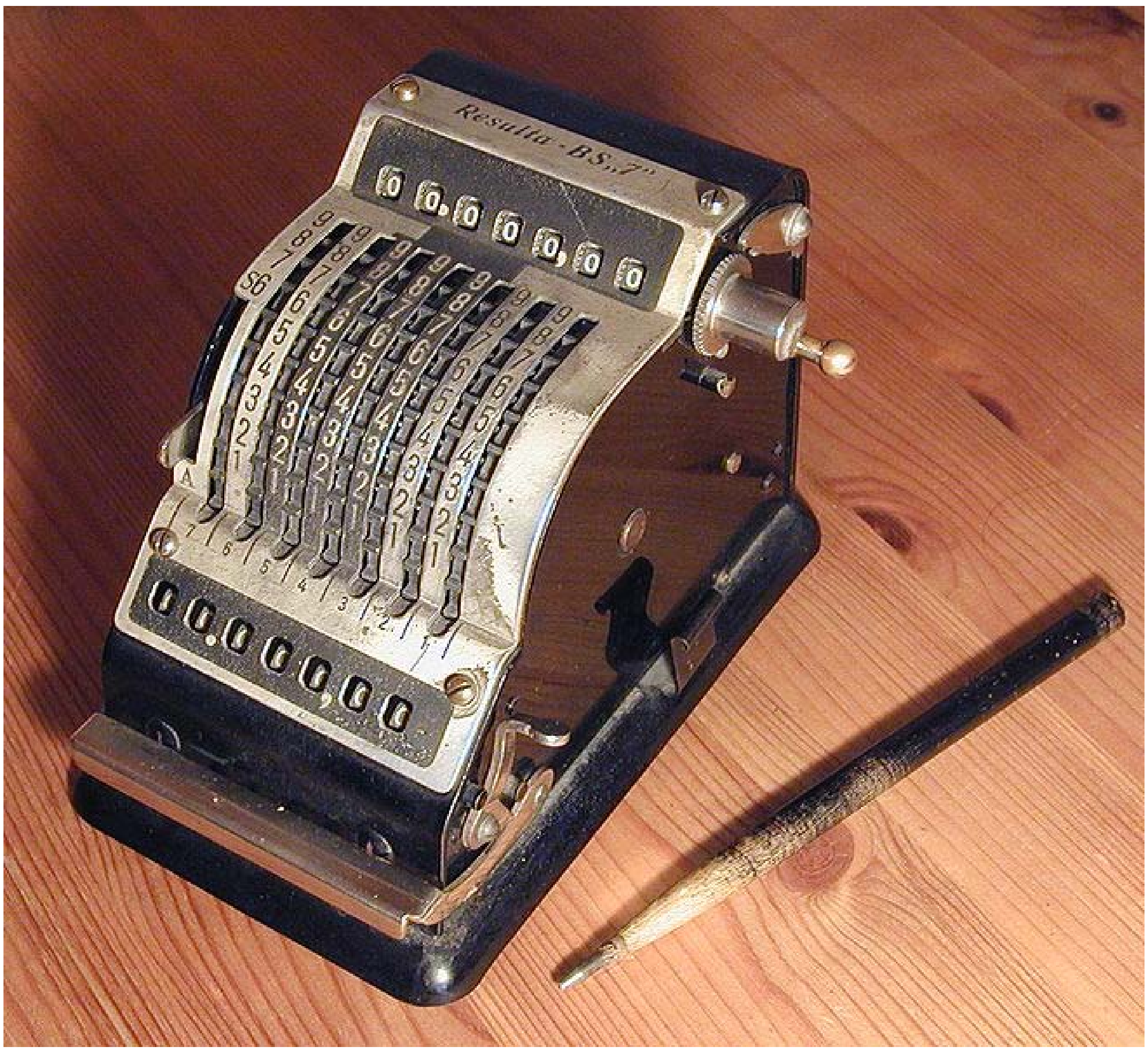
Two binary abaci constructed by Dr. Robert C. Good, Jr., made from two Chinese abaci

The binary abacus is used to explain how computers manipulate numbers. The abacus shows how numbers, letters, and signs can be stored in a binary system on a computer, or via ASCII. The device consists of a series of beads on parallel wires arranged in three separate rows. The beads represent a switch on the computer in either an 'on' or 'off' position.

Chapter 3

Adding Machine and Arithmaurel

Adding machine



An adding machine.

Older adding machine. Its mechanism is similar to a car odometer.

Adding machine for the Australian pound c1910, note the complement numbering

An **adding machine** was a class of mechanical calculator, usually specialized for bookkeeping calculations. In the United States, the earliest adding machines were usually built to read in dollars and cents. Adding machines were ubiquitous office equipment until they were phased out in favor of personal computers, beginning in about 1985. The machines were rarely seen after the year 2000.

Blaise Pascal invented the mechanical calculator in 1642; it was an adding machine that could perform additions and subtractions directly and multiplication and divisions by repetitions. He was followed by Thomas de Colmar who launched the mechanical calculator industry in 1851 when he released his simplified arithmometer (it took him thirty years to turn his complex multiplying machine, patented in 1820, into a simpler and more reliable adding machine). However they didn't gain widespread use until Dorr E. Felt started manufacturing his comptometer (1887) and Burroughs started the commercialization of his adding machine (1892).

Operation

To add a new list of numbers and arrive at a total, the user was first required to "Zero" the machine. Then, to add sets of numbers, the user was required to press numbered keys on a keyboard and then pull the crank. This caused the numbers to be shown on the rotary wheels, and the keys were released in preparation for the next input. To add, for example, the amounts of A£30.72 and A£4.00 (which, in adding machine terms, on a decimal adding machine is 3,072 plus 400 "decimal units"), the following process took place: Press the 3 key in the column fourth from the right (multiples of one thousand), the 7 key in the column 2nd from right (multiples of ten) and the 2 key in the right hand (multiples of 1). Pull the crank. The rotary wheels now showed 3072. Press the 4 key in the 3rd column from the right. Pull the crank. The rotary wheels now show a running 'total' of 3472 which, when interpreted using the decimal currency colour coding of the key columns, equates to A£34.72. Keyboards typically did not have or need "0" (zero) keys. Trailing zeros (those to the right of a number such as the one to the right of the 3 in the example), were there by default because when a machine was "Zero-ed", all numbers visible on the rotary wheels were reset to zero.

Subtraction was impossible, except by adding the complement of a number (for instance, subtract A£2.50 by adding A£9,997.50).

Multiplication was a simple process of keying in the numbers one or more columns to the left and repeating the "addition" process. For example, to multiply A£34.72 by 102, key in 3472, pull crank, repeat once more. Wheels show 6944. Key in 3472(00), pull crank. Wheels now show 354144, or A£3,541.44

A later adding machine, called the comptometer, did not require that a crank be pulled to add. Numbers were input simply by pressing keys. The machine was thus driven by finger power. Multiplication was similar to that on the adding machine, but users would "form" up their fingers over the keys to be pressed and press them down the multiple of times required. Using the above example, four fingers would be used to press down twice on the 3 (fourth column),4 (third column), 7(second column) and 2 (first column) keys. That finger shape would then move left two columns and press once. Usually a small crank near the wheels would be used to zero them.

Some adding machines were electromechanical — an old-style mechanism, but driven by electric power.

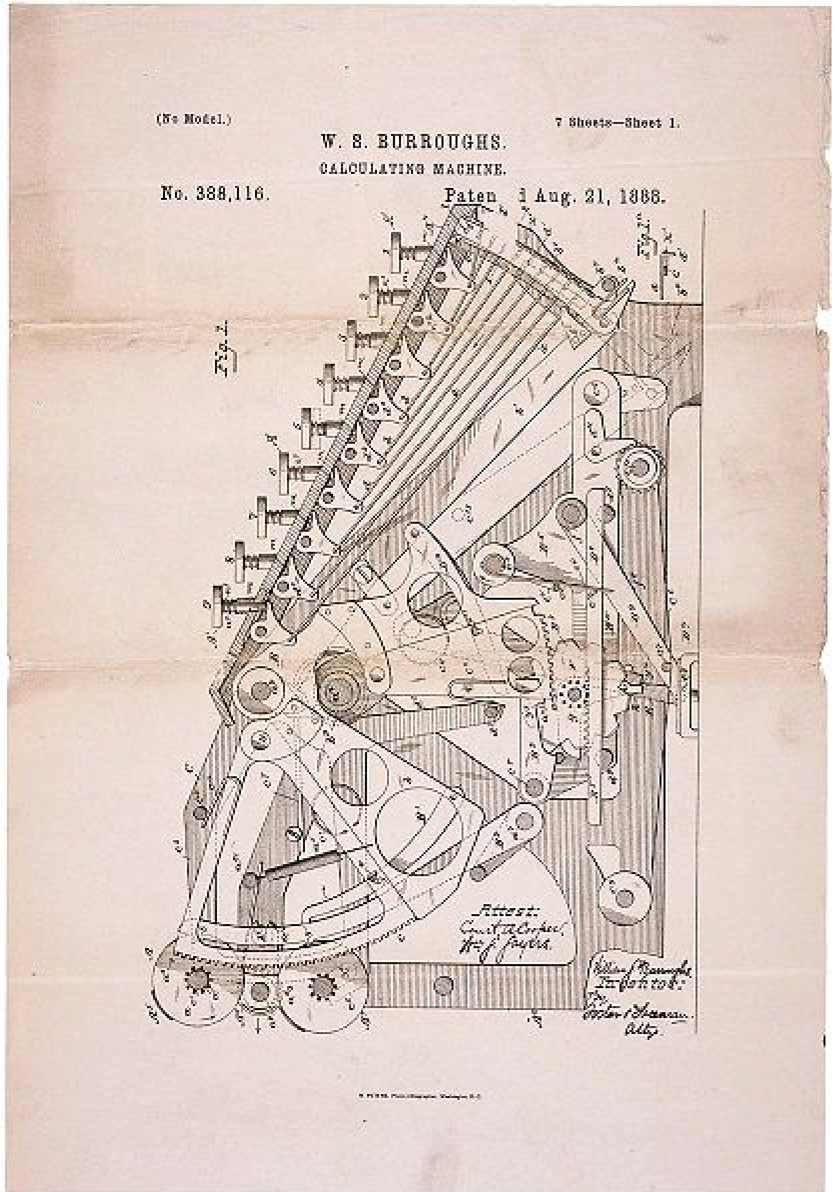
Some "ten-key" machines had input of numbers as on a modern calculator -- 30.72 was input as "3", "0", "7", "2". These machines could subtract as well as add. Some could multiply and divide, although including these operations made the machine more complex. Those that could multiply, used a form of the old adding machine multiplication method. Using the previous example of multiplying A£34.72 by 102, the amount was keyed in, then the 2 key in the "multiplication" key column was pressed. The machine cycled twice, then tabulated the adding mechanism below the keyboard one column to the right. The number keys remained locked down on the keyboard. The user now pressed the multiplication "0" key which caused tabulation of the adding mechanism one more column to the right, but did not cycle the machine. Now the user pressed the multiplication "1" key. The machine cycled once. To see the total the user was required to press a "Total" key and the machine would print the result on a paper tape, release the locked down keys, reset the adding mechanism to zero and tabulate it back to its home position.

Modern adding machines are like simple calculators. They often have a different input system, though.

To figure out this:	Type this on the adding machine:
2+17+5=?	2 + 17 + 5 + T
19-7=?	19 + 7 - T
38-24+10=?	38 + 24 - 10 + T
7×6=?	7 × 6 =
18/3=?	18 ÷ 3 =
(1.99×3)+(.79×8)+(4.29×6)=?	1.99 × 3 = + .79 × 8 = + 4.29 × 6 = + T

Note: Sometimes the adding machine will have a key labeled * instead of T. In this case, substitute * for T in the examples above. Alternatively, the plus key may continuously total instead of either a * or T key. Sometimes, the plus key is even labeled thus: +/=

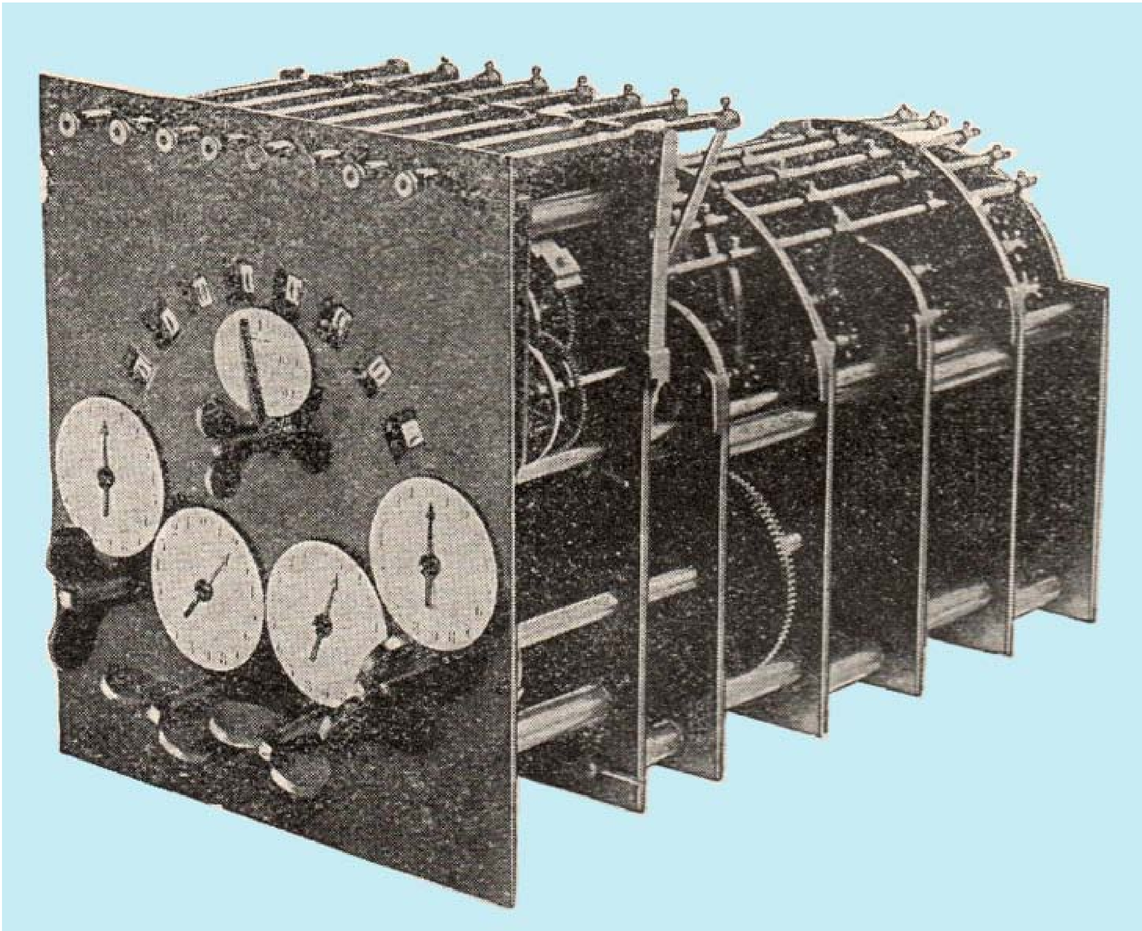
Burroughs's calculating machine



Patent drawing for Burroughs's calculating machine, 1888.

William Seward Burroughs received a patent for his adding machine on August 21, 1888. He was a founder of American Arithmometer Company, which became Burroughs Corporation and evolved to produce electronic billing machines and mainframes, and eventually merged with Sperry to form Unisys. The grandson of the inventor of the adding machine is Beat author William S. Burroughs; a collection of his essays is called *The Adding Machine*.

Arithmaurel



Arithmaurel: the stems at the top are for entering the operands and the dials with their keys are for entering the operators

First patented in France by Timoleon Maurel, in 1842, **the Arithmaurel** was a mechanical calculator that had a very intuitive user interface, especially for multiplying and dividing numbers because the result was displayed as soon as the operands were entered. It received a gold medal at the French national show in Paris in 1849. Unfortunately its complexity and the fragility of its design prevented it from being manufactured.

Its name came from the concatenation of Arithmometer, the machine that inspired its design and of Maurel, the name of its inventor. The heart of the machine uses one Leibniz stepped cylinder driven by a set of differential gears.

History

Timoleon Maurel patented an early version of his machine in 1842 , he then improved its design with the help of Jean Jayet and patented it in 1846. This is the design that won a gold medal at the *Exposition nationale de Paris* in 1849.

Winnerl, a French clockmaker, was asked to manufacture the device in 1850, but only thirty machines were built because the machine was too complex for the manufacturing capabilities of the time. During the first four years, Winnerl was not able to build any of the 8 digit machines (a minimum for any professional usage) that had been ordered while Thomas de Colmar delivered, during the same period, two hundred 10 digit Arithmometers and fifty 16 digit ones.

It is to be noted that none of the machines that were built and none of the machines described in the patents could be used at full capacity because the capacity of the result display register was equal to the capacity of the operand register (for a multiplication, the capacity of the result register should be equal to the capacity of the operand register augmented by the capacity of the operator register).

Description

Following is a description of one of the two machines introduced in the 1846 patent. It has a capacity of five digits for the operator and ten digits for the operand and the result registers.

All the registers are located on the front panel, the reset mechanism is on the side.

- 10 numbered stems, arranged horizontally at the top of the front panel, can be pulled at different lengths to enter the operands with the rightmost stem representing units.
- A 10 digit display register located in the middle is used to display the results.
- 5 dials, each coupled with an input key, are used to enter the operators with the rightmost dial representing units. Turning the units key one division clockwise will add the content of the operand register to the total. Turning the units key one division counterclockwise will subtract the content of the operand register from the current total. Turning the tens key one division clockwise will add 10 times the content of the operand register to the total etc..

Chapter 4

Arithmometer

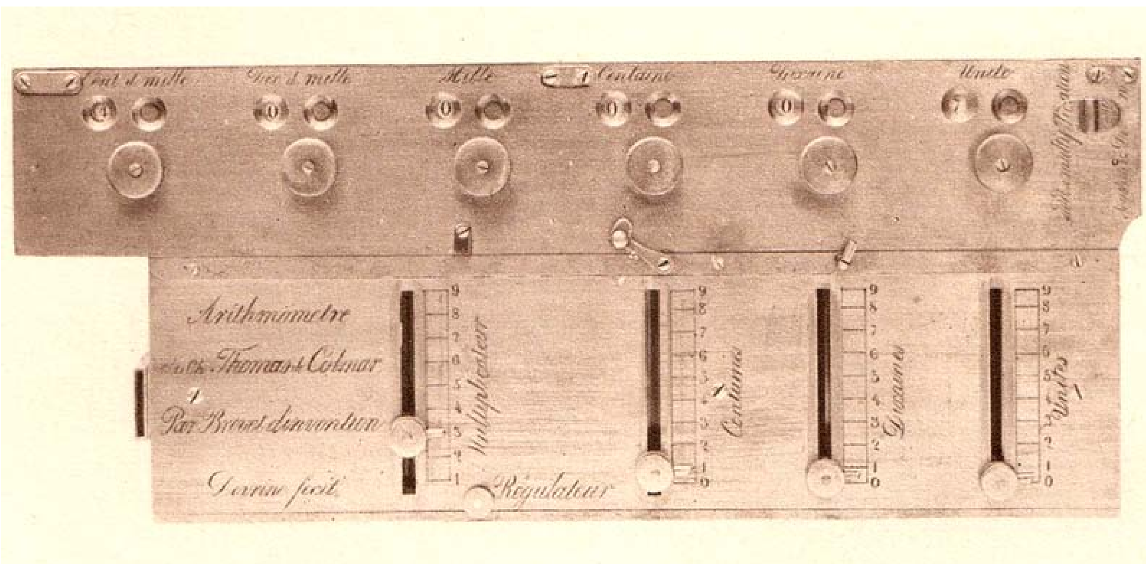
Arithmomètre built by Louis Payen around 1887

An **Arithmometer** or ***Arithmomètre*** was a mechanical calculator that could add and subtract directly and could perform long multiplications and divisions effectively by using a movable accumulator for the result. Patented in France by Thomas de Colmar in

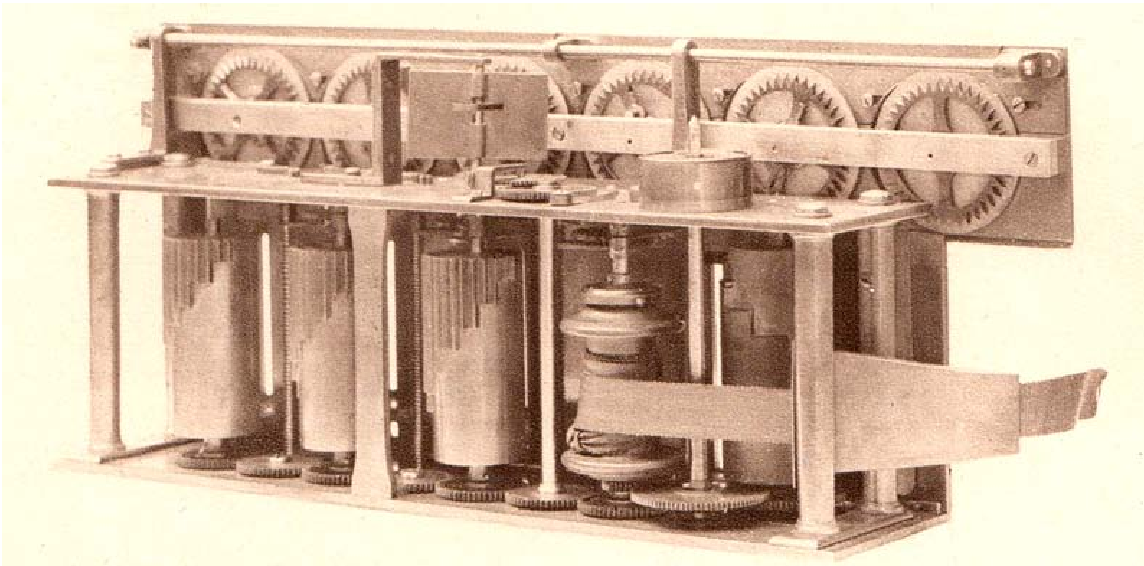
1820 and manufactured from 1851 to 1915, it became the first commercially successful mechanical calculator. Its sturdy design gave it a strong reputation of reliability and accuracy and made it a key player in the move from *human computers* to calculating machines that took place during the second half of the 19th century.

Its production debut of 1851 launched the mechanical calculator industry which ultimately built millions of machines well into the 1970s. For almost forty years, from 1851 to 1887, the Arithmometer was the only type of mechanical calculator in commercial production and it was sold all over the world. During the later part of that period two companies started manufacturing clones of the Arithmometer, they were: Burkhardt from Germany which started in 1878 and Layton from the UK which started in 1883. Eventually about twenty European companies built clones of the arithmometer until the beginning of WWII.

Evolution



The single digit multiplier is set on the left slider while the multiplicand is set on the three sliders on the right



The three Leibniz cylinders can be seen on the left and the pulling ribbon on the right
Drawings of the 1822 machine

Searching for a Solution: 1820-1851

The Arithmometers of this period implemented a direct multiplication, where the multiplicand inscribed on the input sliders could be multiplied by a single digit multiplier by simply pulling on a ribbon (quickly replaced by a crank handle). It was a complicated design and very few machines were built, additionally, no machines were built between 1822 and 1844. It must be noted that this hiatus of 22 years coincides almost exactly with the period of time during which the British government financed the design of Charles Babbage's difference engine which on paper was 10 times more sophisticated than the Arithmometer but that couldn't be built with the technology of the time (The British Parliament gave £1,500 to start the project in 1822 and officially stopped its financing in 1842 after having spent £17,470 which in 2004 money would be about £1,000,000).

In 1844 Thomas reintroduces his machine at the *Exposition des Produits de l'Industrie Française* in the newly created category of *Miscellaneous measuring tools, counters and calculating machines* but only receives an honorable mention.

In 1848 he restarts the development of the machine. By 1850, as part of a marketing effort, Thomas builds a few machines with exquisite boxes that he gives to the crown heads of Europe. He files two patents and two patents of addition in between 1849 and 1851.

This Arithmometer showcases almost one hundred years of improvements and is one of the last machines manufactured (1914).

Creating an Industry: 1851-1887

The multiplier is removed. This makes the Arithmometer a simple adding machine, but thanks to its moving carriage used as an indexed accumulator, it still allows for easy multiplication and division under operator control. It was introduced in the UK at the The Great Exhibition of 1851 . The true industrial production starts in 1851. Each machine is given a serial number and user manuals are printed.

The constant use of some of the machines exposed some minor design flows like a weak carry mechanism (which is given an adequate fix in 1856) and an over rotation of the Leibniz' cylinders when the crank handle is turned too fast which is corrected by the addition of a Maltese cross. A patent covering all these innovations is filed in 1865.

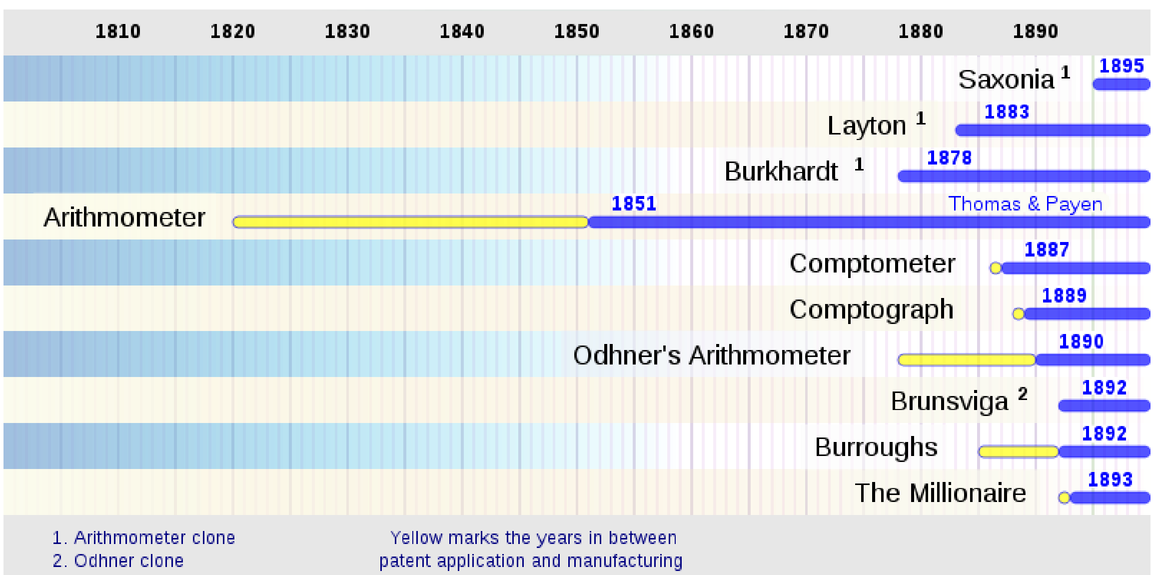
Because of its reliability and accuracy, government offices, banks, observatories and businesses all over the world start using the Arithmometer in their day to day operations. Around 1872, for the first time in calculating machine history, the total number of machines manufactured passes the 1,000 mark. In 1880, twenty years before the competition, a mechanism to move the carriage automatically is patented and installed on some machines, but it is not integrated into the production models.

The Golden Age: 1887-1915

Under the management of Louis Payen, and later his widow, many improvements are introduced like an incline mechanism, a removable top, cursors and result windows that are easier to read and a faster re-zeroing mechanism. Many clone makers appear during that period mostly in Germany and in the United Kingdom.

Unfortunately the fundamental design stays the same and, at the turn of the century, after 50 years at the top, the Arithmometer loses its supremacy over the mechanical calculator industry. While in 1890, the Arithmometer was the most produced mechanical calculator in the world, ten years later, by 1900, four machines, the Comptometer and Burroughs' adding machine in the USA, Odhner's Arithmometer in Russia and Brunsviga in Germany had passed it in volume of machines manufactured.

The manufacturing stopped during WWI and Alphonse Darras, who had bought the business in 1915, was unable to restart it after the war because of the many shortages and a lack of qualified workers.



Desktop Mechanical Calculators in production during the 19th century

Legacy

Because it is the first mass marketed and the first widely copied calculator, its design marks the starting point of the mechanical calculator industry which eventually morphed into the electronic calculator industry and which, through the accidental design of the first microprocessor to be commercialized, the Intel 4004, for one of Busicom's calculator in 1971, lead to the first personal computer, the Altair in 1975.

Its user interface was used throughout during the 120 years that the mechanical calculator industry lasted. First with its clones and then with the Odhner Arithmometer and its clones which was a redesign of the arithmometer with a pinwheel system but with exactly the same user interface.

Over the years, the term Arithmometer or parts of it have been used on many different machines like Odhner's Arithmometer, the *Arith*maurel or the Compt*ometer* and on some portable pocket calculating machines of the 1940s. Burroughs corporation started as the *American Arithmometer Company* in 1886. By the 1920s it had become a generic name for any machine based on its design with about twenty independent companies manufacturing Thomas' clones like Burkhardt, Layton, Saxonia, Gräber, Peerless, Mercedes-Euklid, XxX, Archimedes, etc...

History

Design

Thomas started to work on his machine while serving in the French Army where he had to do a great deal of calculations. He made use of principles from previous mechanical calculators like the stepped reckoner of Leibniz and Pascal's calculator. He patented it on November 18, 1820.

This machine implements a true multiplication where, by just pulling on a ribbon, the multiplicand entered on the input sliders is multiplied by a one digit multiplier number and it uses the 9's complement method for subtracting. Both of these features will be dropped in later designs.

First Machine

The first machine was built by Devrine, a Parisian clockmaker, it took him a year to build it. But, in order to make it work, he had to modify the patented design quite substantially. The *Société d'encouragement pour l'industrie nationale* was given this machine for review and it issued a very positive report on December 26, 1821. The only known prototype of this time is the *1822 machine* on display at the Smithsonian Institution in Washington, D.C.

Production

Some of the logos used over the years

Manufacturing started in 1851 and ended around 1915. There were about 5,000 machines built during this sixty year period; 40% of the production was sold in France and the rest was exported.

The manufacturing was managed by:

- Thomas de Colmar himself until his death in 1870, then by his son Thomas de Bojano until 1881 and by his grand-son Mr. de Rancy until 1887. Misters Devrine (1820), Piolaine (1848), Hoart (1850) and Louis Payen (around 1875) were the engineers responsible for building the machines. All the machines manufactured during this time have the logo *Thomas de Colmar*.
- Louis Payen who bought the business in 1887 until his death in 1902; All these machines have the logo *L. Payen*.
- Veuve L. Payen who took over the business at her husband's death and sold it in 1915 with the logos *L. Payen*, *Veuve L. Payen* and *VLP*. Alphonse Darras built most of these machines.
- Alphonse Darras who bought the business in 1915 and manufactured the last machines. He added a logo made of the letters A and D interlaced and went back to the *L. Payen* logo.

During the early part of manufacturing, Thomas differentiated machines by capacity and therefore gave the same serial number to machines of different capacities. He corrected this in 1865, giving every machine its own unique serial number starting with a serial number of 500. This is why there isn't any machine with a serial number in between 200 and 500.

From 1865 to 1907 the serial numbers were consecutive (from 500 to 4000) then, after patenting a rapid zeroing mechanism in 1907, Veuve L. Payen started a new numbering scheme at 500 (the number of arithmometers she had built with the old scheme) and was at serial number 1700 when she sold the business to Alphonse Darras in 1915. Alphonse Darras went back to the old serial numbers (while approximatively adding the number of machines made by Veuve L. Payen) et restarted at 5500.

Ease of use and Speed

An article published in January 1857 in The Gentleman's Magazine best describes it:

M. Thomas's arithmometer may be used without the least trouble or possibility of error, not only for addition, subtraction, multiplication, and division, but also for much more complex operations, such as the extraction of the square root, involution, the resolution of triangles, etc...

A multiplication of eight figures by eight others is made in eighteen seconds; a division of sixteen figures by eight figures, in twenty four seconds; and in one minute and a quarter one can extract the square root of sixteen figures, and also prove the accuracy of the calculation...
The working of this instrument is, however, most simple. To raise or lower a nut-screw, to turn a winch a few times, and, by means of a button, to slide off a metal plate from left to right, or from right to left, is the whole secret. Instead of simply reproducing the operations of man's intelligence, the arithmometer relieves that intelligence from the necessity of making the operations. Instead of repeating responses dictated to it, this instrument instantaneously dictates the proper answer to the man who asks it a question. It is not matter producing material effects, but matter which thinks, reflects, reasons, calculates, and executes all the most difficult and complicated arithmetical operations with a rapidity and infallibility which defies all the calculators in the world.
The arithmometer is, moreover, a simple instrument, of very little volume and easily portable. It is already used in many great financial establishments, where considerable economy is realized by its employment.
It will soon be considered as indispensable, and be as generally used as a clock, which was formerly only to be seen in palaces, and is now in every cottage.

Models

20 digit Arithmometer built around 1875

The various models had capacities of 10, 12, 16 and 20 digits which gave results ranging from 10 billion (minus 1) to 100 quintillion (minus 1). Only two machines were built outside this range:

The first prototype (the 1822 machine) had a capacity of 6 digits even though the machine described in the 1820 patent is an 8 digits machine.

The piano Arithmometer with a capacity of 30 digits, allowing for numbers up to 1 nonillion (minus 1), which was built for the 1855 *Exposition universelle de Paris* and which is now part of the IBM collection of mechanical calculator. Jules Verne must have been quite impressed by this machine because in his novel Paris in the Twentieth Century, after mentioning Pascal and Thomas de Colmar, he talks of mechanical calculators that will be some huge pianos with keyboards of keys that will deliver answers instantaneously to anyone that can play them!

All the machines, regardless of capacity, were about 7 inches (18 cm) wide and from 4 up to 6 inches (10 to 15 cm) tall (the tallest ones had an incline mechanism). A 20 digit machine was 2 ft 4 in (70 cm) long while the length a 10 digit machine was around 1 ft 6 in (45 cm).

Prices

A 12 digit Arithmometer sold for 300 francs in 1853 which was 30 times the price of a table of logarithms book and 1,500 times the cost of a first class stamp (20 French cents), but, unlike a table of logarithms book, it was simple enough to be used for hours by an operator without any special qualifications.

An advertisement taken from a magazine published in 1855 shows that a 10 digit machine sold for 250 francs and a 16 digit machine sold for 500 francs.

Development Costs

In 1856, Thomas de Colmar estimated that he had spent 300,000 francs of his own money during the thirty years that he perfected his invention.

Physical design

The Arithmometer is a brass instrument housed in a wooden box often made of oak or mahogany and for the oldest ones ebony (solid or veneer). The instrument itself is divided into two parts.

Front panel of a Thomas Arithmometer with its movable result carriage extended

Input - Control - Execution

The bottom part is composed of a set of sliders that are used to input the value of the operands. On the left of it is a control lever which allows to select the current operation, namely *Addition/Multiplication* or *Subtraction/Division*. A crank located on the right of the sliders is used to execute the operation selected by the control lever.

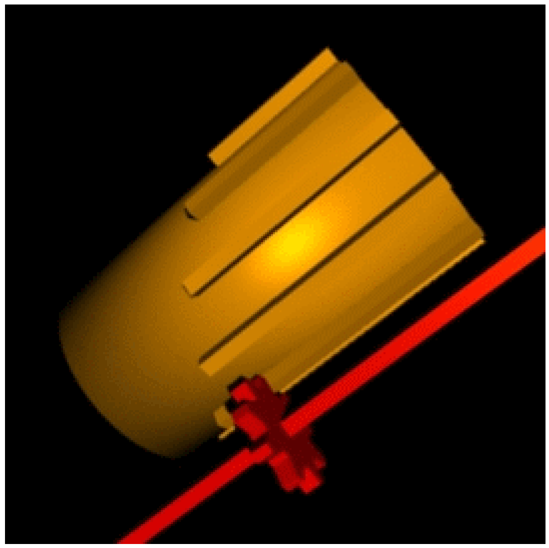
Output - Accumulator

The top part is a movable carriage composed of two display registers and two reset buttons. The top display register holds the result of the previous operation and acts as accumulator for the current operation. Each command adds or subtracts the number inscribed on the sliders to the part of the accumulator directly above it. The lower display register counts the number of operations performed at each index therefore it displays the multiplier at the end of a multiplication and the quotient at the end of a division.

Each number in the accumulator can be individually set with a knob situated right below it. This feature is optional for the operation counter register.

The accumulator and the result counter are in between two buttons used to reset their content at once. The left button resets the accumulator, the right button resets the operation counter. These buttons are also used as handles when lifting and sliding the carriage.

Arithmometer's Leibniz wheel



In the position shown, the counting wheel meshes with 3 of the 9 Leibniz wheel's teeth

The animation on the side shows a nine teeth Leibniz wheel coupled to a red counting wheel. The counting wheel is positioned to mesh with three teeth at each rotation and therefore would add or subtract 3 from the counter at each rotation.

The computing engine of an Arithmometer has a set of linked Leibniz wheels coupled to a crank handle. Each turn of the crank handle rotates all the Leibniz wheels by one full turn. The input sliders move counting wheels up and down the Leibniz wheels which are themselves linked by a carry mechanism.

In the arithmometer the Leibniz wheels always turn the same way. The difference in between addition and subtraction is achieved by a reverser operated by the execution lever and located in the movable display carriage.

Operations

Sliding the Top Carriage

First lift the carriage using the reset buttons located at its extremities, then slide it. The carriage can only be moved to the right initially. Release it when it is above the index you want (ones, tens, hundreds, ...).

Resetting the Displays

First lift the carriage using the reset buttons located at its extremities, then turn them to reset the display registers. The left button resets the accumulator, the right button resets the operation counter.

Addition

Set the control lever to *Addition/Multiplication* and reset the display registers. Each turn of the execution lever adds the number from the sliders to the accumulator. So input the first number and turn the lever once (it adds it to zero) then enter the second number and turn the lever once more.

Multiplication

Set the control lever to *Addition/Multiplication* and reset the display registers. To multiply 921 by 328, first input 921 on the input sliders and then turn the execution lever 8 times. The accumulator shows 7,368 and the operation counter shows 8. Now, shift the carriage to the right once and turn the lever 2 times, the accumulator shows 25,788 and the operation counter shows 28. Shift the carriage one last time to the right and turn the lever 3 times, the product 302,088 appears on the accumulator and the operation counter displays the multiplier 328.

Subtraction

Set the control lever to *Soustraction/Division*. Lift the carriage then reset the display registers and input the minuend, right justified, into the accumulator using the corresponding knobs. Lower the carriage to its default position and then set the subtrahend onto the input sliders and turn the execution lever once.

Integer Division

Set the control lever to *Soustraction/Division* and set the divisor onto the input sliders. While keeping the carriage lifted, reset the display registers, set the dividend, right justified, using the corresponding knobs and shift the carriage so that the highest number in the dividend corresponds to the highest number in the divisor. Lower the carriage then turn the execution lever as many times as required until the number situated above the divisor is less than the divisor, then shift the carriage once to the left and repeat this operation until the carriage is back to its default position and the number in the accumulator is less than the divisor, then the quotient will be in the operations counter and the rest will be in the accumulator.

Decimal Division

In order to increase the decimal division accuracy add as many zeros as required to the right of the dividend but still input it right justified and then proceed as with an integer division. It's important to know where the decimal point is, when you read the quotient (some markers, first ivory and then metal, were usually sold with the machine and used for this purpose).

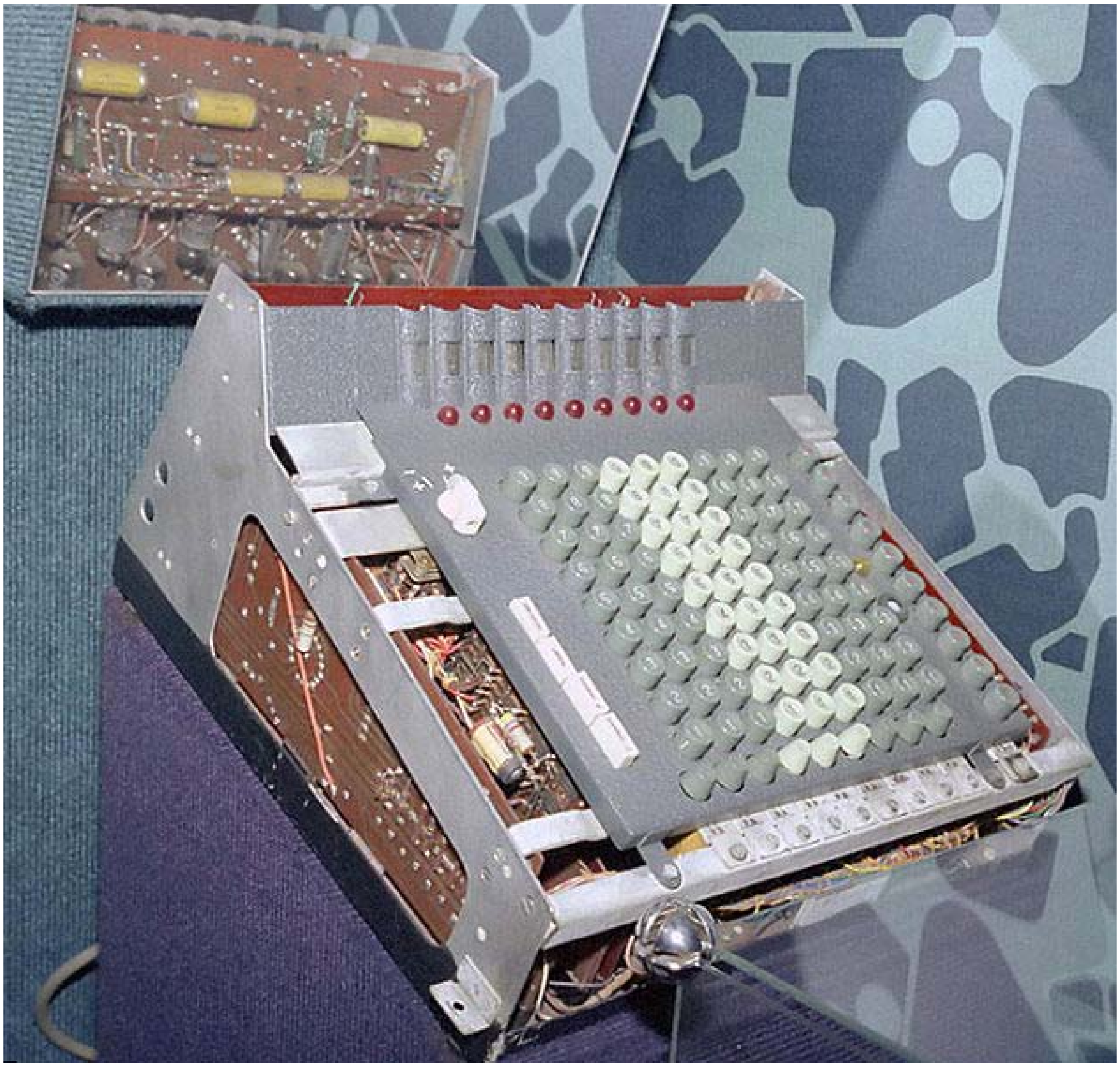
Variants

In 1885, Joseph Edmondson of Halifax, UK, patented his 'Circular Calculator' - essentially a 20-digit arithmometer with a circular carriage (the slides being arranged radially around it) instead of the straight sliding carriage. One benefit of this was that the carriage always remained within the footprint (to use a modern term) of the machine instead of overhanging the case at one side when the higher decimal places were in use. Another was that one could make a calculation of up to ten places, using half the circumference of the carriage, and then turn the carriage through 180°; the result of the calculation was locked in place by means of brass prongs mounted on the framework, and one could leave it there while making an entirely new calculation using the fresh set of display windows now brought into alignment with the sliders. Thus the machine could be said to have a rudimentary memory.

Chapter 5

Comptometer

Model ST (1930s)



Prototype of the first all-electronic desktop calculator marketed by Sumlock Comptometer Ltd of the UK

Patented in the USA by Dorr E. Felt in 1887, the **comptometer** was the first commercially successful key-driven mechanical calculator. A key-driven calculator is extremely fast because each key adds or subtracts its value to the accumulator as soon as it is pressed and a skilled operator can enter all of the digits of a number simultaneously, using as many fingers as required, making them sometimes faster to use than electronic calculators. Consequently, in specialized applications, comptometers remained in use in limited numbers into the early 1990s, but with the exception of museum pieces, they have all now been superseded by electronic calculators and computers.

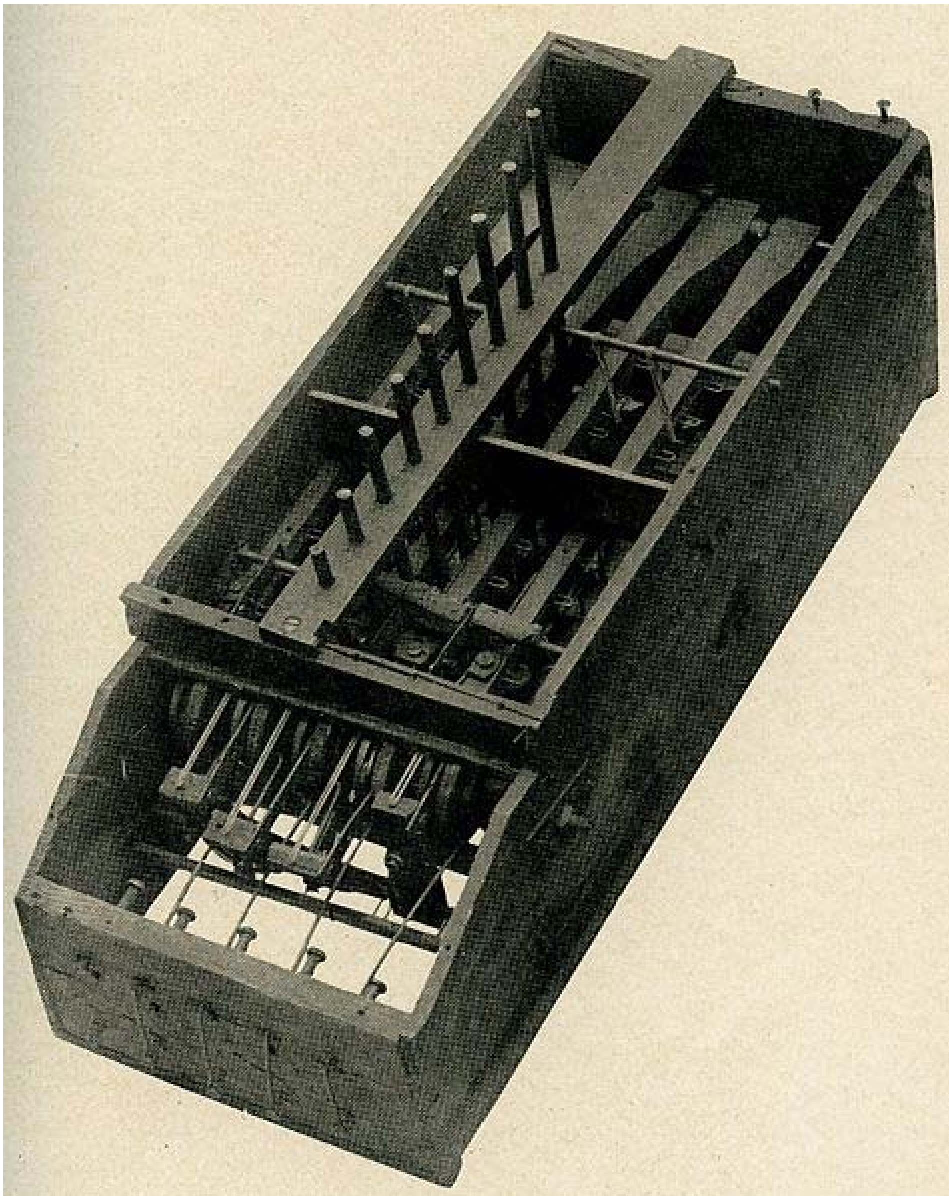
Manufactured without interruption from 1887 to the mid 1970s it was constantly improved; first it was made faster and more reliable, then a line of electro-mechanical models was added in the 1930s, but especially it was the first mechanical calculator to receive an all-electronic calculator engine in 1961, with the ANITA Mark VII model released by Sumlock Comptometer, therefore creating the link in between the mechanical and the electronic calculator industries.

Although the comptometer was primarily an adding machine, it could also do subtractions, multiplication and division. Its keyboard consisted of eight or more columns of nine keys each. Special comptometers with varying key arrays were produced for a variety of special purposes, including calculating currency exchanges, times and Imperial weights. The name comptometer was formerly in wide use as a generic name for this class of calculating machine.

History

An early machine (1887)

Origins



The macaroni box (1885)

The comptometer is the direct descendant of the key-driven machine of Thomas Hill patented in the USA in 1857 and of the Pascaline invented by Blaise Pascal in France in 1642. By just replacing the input wheels of the Pascaline by the columns of keys of Hill's machine, the comptometer was invented. Addition is performed exactly the same way, and both the Pascaline and the Comptometer make use of the 9's complement method for subtraction, but in the case of the comptometer it is the operator who must choose the right keys for the subtrahend (each key has its 9's complement written in miniature letter next to it).

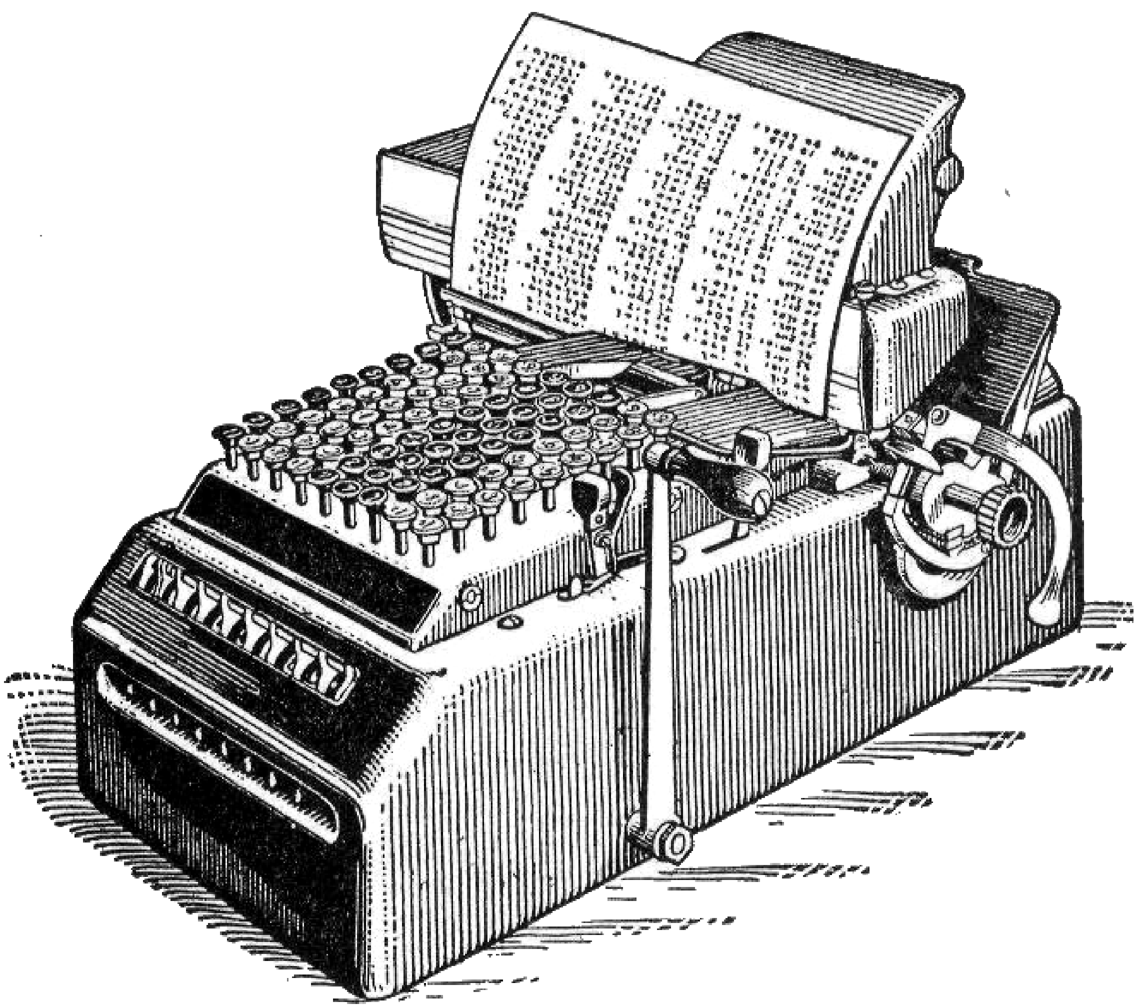
The first comptometers

Dorr Felt started his first prototype during the US Thanksgiving holidays of 1884. Because of his limited amount of money, he used a macaroni box for the outside box, and skewers, staples and rubber bands for the mechanism inside. It was finished soon after New Year's Day, 1885. This prototype, called the macaroni box, is in the Smithsonian Institution in Washington, D.C., USA.

Shortly after, Robert Tarrant, the owner of a Chicago workshop, gave Mr. Felt a salary of $6 a week, a bench to work on and what will add up to $5,000 to build his first practical machine which he finished in the autumn of 1886.

By September 1887, eight production machines had been built.

Comptometer and comptograph



Comptograph (1914)

The original comptometer design was patented by Dorr E. Felt, a citizen of the United States. The first two patents were granted on July 19, 1887 and on October 11, 1887.

Two years later, on June 11, 1889, he was granted a patent for the Comptograph. A Comptograph is a comptometer with a printing mechanism making it more like a key-set calculating machine (even though the keys are registered as they are typed and not when a handle is pulled) therefore slower and more complicated to operate. It was the first printing-adding machine design to use individualized type impression which made its printed output very legible.

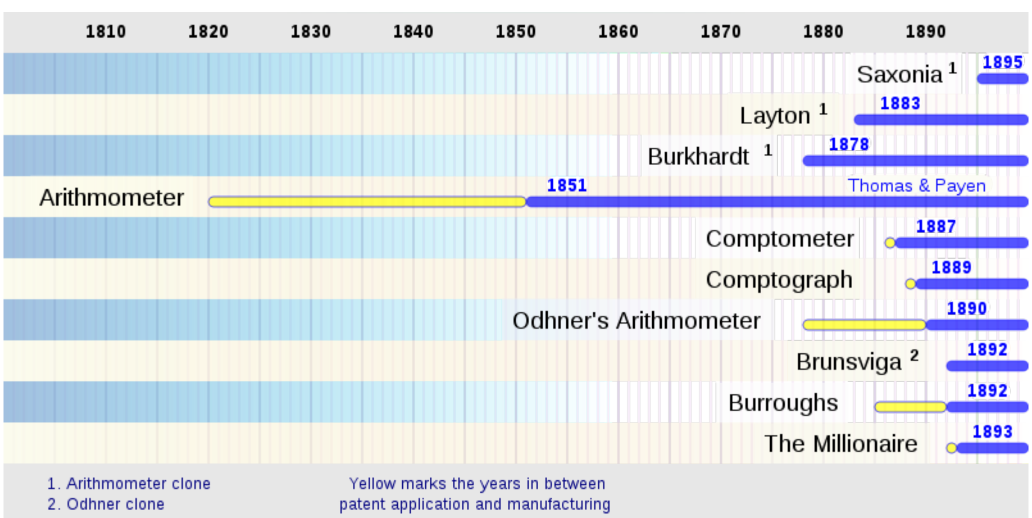
The first comptograph was sold to the Merchants & Manufacturers National Bank of Pittsburgh, PA. in December 1889. It was the first sale of a recording-adding machine ever. This machine is now on display at the Smithsonian Institution in Washington, D.C..

The Felt & Tarrant Manufacturing Company built both comptometers and comptographs throughout the 1890s. In 1902, the Comptograph Company was setup to manufacture comptographs exclusively, unfortunately it was shut down at the beginning of World War I.

Forty years later, in the mid 1950s, the Comptometer Corporation reused the name Comptograph for a line of 10 key printing machines.

Competition

The comptometer was the first machine in production to challenge the supremacy of the arithmometer and its clones; but not immediately, it took almost three years to sell the first hundred machines.



Desktop Mechanical Calculators in production during the 19th century

Companies and trademarks

Comptometer Pins

Felt & Tarrant Manufacturing Company

Felt and Tarrant signed a partnership contract on November 28, 1887. The partnership was incorporated as Felt & Tarrant Manufacturing Company on January 25, 1889.

Comptograph Company

In 1902, Felt and Tarrant parted ways in creating a second company and decided to split the shares of both companies so that one would own the controlling interest of a company with 51% of the shares and the other would still share the profits with the remaining 49% of the shares. Mr. Felt became the majority owner of the Felt & Tarrant Manufacturing Company and Mr. Tarrant became the majority owner of the Comptograph Company. The Felt & Tarrant Manufacturing Company reabsorbed the Comptograph company at the beginning of World War I

Comptometer Corporation

The Felt & Tarrant Mfg Co became public in 1947 and changed its name to the Comptometer Corporation in 1957.

Bell Punch Company - Sumlock Comptometer Ltd

In 1960, the Bell Punch Company bought the British rights to the Comptometer design and trademark, and continued its development. In 1961, Sumlock, a division of the Bell Punch Company, was renamed The Sumlock Comptometer Ltd, and began marketing the first all-electronic desktop calculator, the ANITA Mark VII. The entire calculator division of the Bell Punch Company was bought by Rockwell International in 1973. Unfortunately they exited the calculator business in 1976 and shut down all operations.

Victor Comptometer Corporation

In 1961, the Comptometer Coporation merged with the Victor Adding Machine Company, and the two became the Victor Comptometer Corporation. After barely surviving the microprocessor revolution, it is still doing business today as Victor Technology LLC.

Evolution

Keyboard of a Woody (1895)

Wooden boxes

The first machines were built using wooden boxes, which made them lighter but more fragile. They were called the Woodies and about 6,500 machines were built in between 1887 and 1903.

Two major improvements happened during the first years of production: the first was the insertion of carry inhibit push buttons for quicker operations of subtraction and the second was the color grouping of the columns of keys (three by three except for the first two columns that represent hundredths). Mr. Felt continually improved his machine with seven patents filed during the first ten years. These included the patents that he took for the Comptograph.

Each row of keys is differentiated from the one above and the one below by a different tactile feel: the even rows have round and raised keys and the odd rows have flat and oblong keys. The keys of the first machines, with their metal rims, are similar to the typewriters keys of the same period. Plastic keys were also introduced very early on but their rows do not have that tactile difference.

The woodies had a three-part zeroing mechanism: a lever, a lever stop, and a knob. In order to reset the machine, first push the lever toward the stop, start rotating the knob and release the lever as soon as the result numbers start to move. Continue rotating the knob until all the result numbers are reset.

Model A (1905)

Model A

All the models, from this one on, have a metal casing, but the model A has a polished glass panel for the output display in the front. This model is produced from 1904 to 1906.

Described in patents 762,520 and 762,521, it introduces a new carry mechanism that reduces the power needed to operate the keys to one fourth of its predecessor's. It also introduces the duplex feature that allows for keys to be pressed simultaneously. And finally it introduces a simplified clearing mechanism that only requires one lever, and one back and forth motion of it, to reset the machine. "The machine gun of the office" as it was called in some WWI advertisements, was starting to develop the form and mechanism that it would keep for the next forty years.

Model B,C,D

These models are built from 1907 to 1915. They all have the shoebox metal casing that will be used until the end of World War II.

Model E

Model E with a white Controlled-Key (1914)

Very few machines of this type were built, but all were made between 1913 and 1915. This was a transition machine that introduced the Controlled-Key which was part of an error detection mechanism that blocked most of the keyboard if a key was not pressed enough to add its total to the result. When this mechanism was activated, all the columns of keys were blocked except for the one where the error occurred. Therefore the operator could find the column in question, reenter the number and unlock the keyboard by pressing the Controlled-Key in order to resume the operations.

Each key was supported by a metal plate that wrapped around it on three sides. This created a support that was used to block the key when the error detection mechanism was activated. The blocking mechanism was moved inside the box in the following models.

Model F

The production of this model started in 1915 and lasted five years. The error detection mechanism is moved inside the box and the Controlled-Key is red. This machine is the last machine without the Comptometer logo inscribed on its front and back panels.

Model J (1930s)

Model H, J, ST

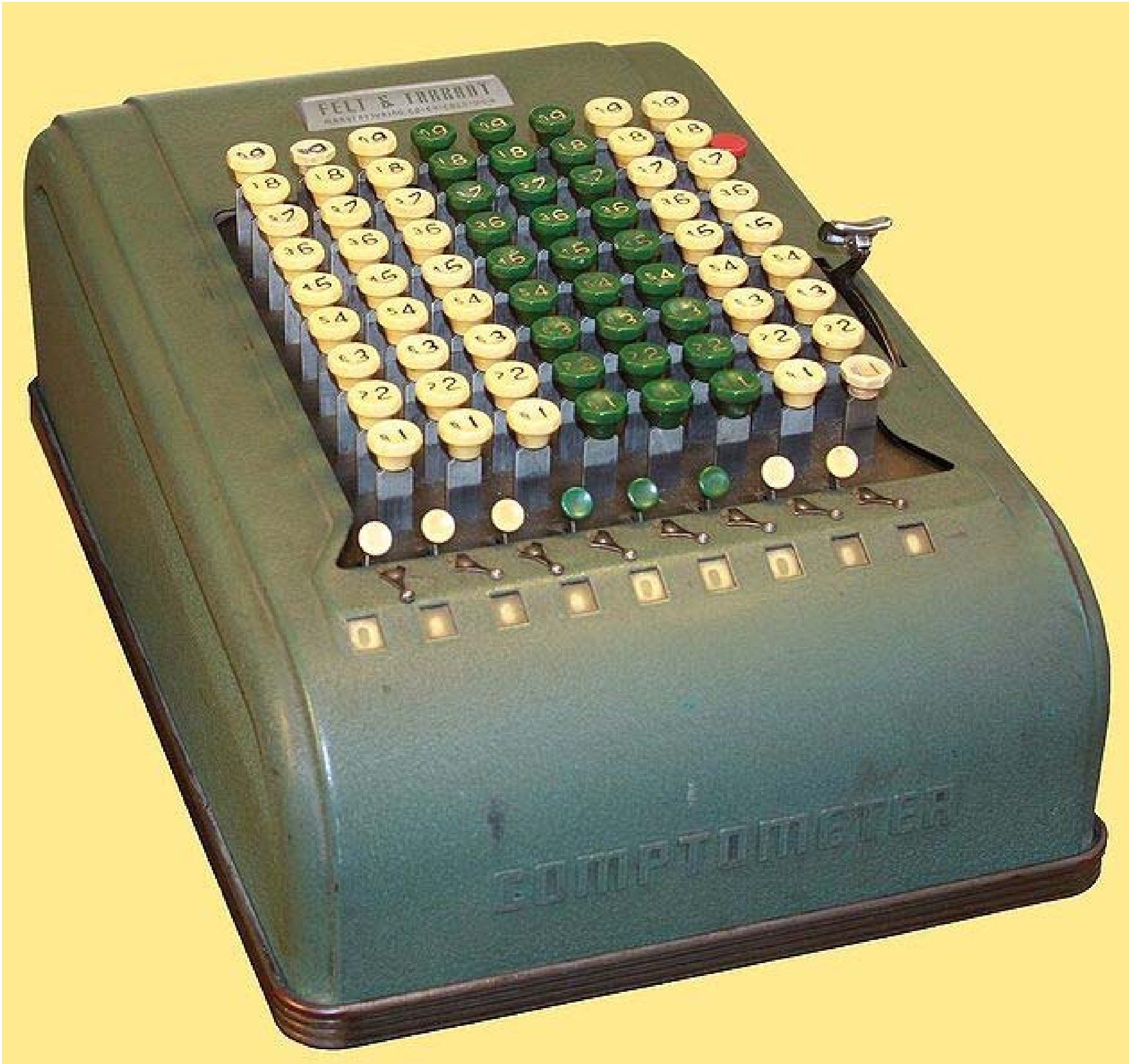
Manufactured from 1920 until the beginning of World War II, these models incorporate all of the improvements of the previous machines. The Comptometer logo is inscribed on their front and back panels and they have the red Controlled-Key that was introduced with the previous two models.

The ST (SuperTotalizer) has two display output registers and two levers that allow for the creation of intermediate results. Each key pressed by the operator is added to the top display register creating an intermediate result which is added to the bottom display register when the operator activates the top lever (it also resets it). The bottom display register accumulates the intermediate results until it is cleared by using the bottom lever.

These three models are the last of the shoebox design that was introduced in 1907 with the B model.

Model K

This electric model was introduced in the middle of the 1930s and had a very moderate success since the mechanical models were very easy to operate and less expensive to maintain.



Model WM produced during WW II

Model M, WM

Introduced just before World War II, it had a new box design which was more symmetrical and rounder in shape. The WM was manufactured during World War II with a less wasteful use of material for the mechanism.

The WM had many aluminum parts which were more prone to failure so the use of steel was resumed as soon as steel again became available. While the machine was lighter, is was also weaker.

The ANITA mark VIII sold from 1962 in Britain

Post World War II models

The models become lighter with the use of aluminum and more refined inner mechanical designs. Both mechanical and electric models are offered. Their outside is similar to the model M machines. The model 3D11 succeeds to the model M and the electric model 992 succeeds to the model K. The major external appearance change of the 3D11 & 992 , the keys were two tones of green with a fuller depth almost twice the old original style. making identification very easy. Also the lock release key (red) was moved from the upper right corner of the keyboard to the center and just below the 'one' keys so it could be thumb operated. On model K electrics the motor was turned on with a switch and ran continuously during use. The 992 had a micro switch which started the motor at the start of any key depression and shut off after key action competed. This reduced the motor wear to almost nil.

The most important new designs of this period came from the Sumlock comptometer Ltd company that introduced the first all-electronic desktop calculators almost two years before the competition. They were the ANITA Mark VII, first introduced in 1961 and marketed in Continental Europe and the ANITA Mark VIII, a slightly improved design, introduced at the same time, marketed in Britain and the rest of the world. They both started shipping in 1962.

The last ANITA machine with a Comptometer keyboard was the *ANITA mk 10* introduced in 1965, still using cold-cathode switching tubes and that will be replaced in 1968 by the *ANITA mk 11*, a 10 key machine.

Incidentally, Sharp's first all transistor desktop calculator, the *CS-10A COMPET*, introduced in the summer of 1964, also had a Comptometer type keyboard.

Chapter 6

Curta Calculator

Curta mechanical calculator shown in the operational position (left hand). The crank is turned with the right hand.

The **Curta** is a small, hand-cranked mechanical calculator introduced in 1948. It has an extremely compact design: a small cylinder that fits in the palm of the hand. It can be used to perform addition, subtraction, multiplication, division, and —with more difficulty— square roots and other operations. The Curta's design is a descendant of Gottfried Leibniz's Stepped Reckoner and Thomas's Arithmometer, accumulating values on cogs, which are added or complemented by a stepped drum mechanism.

History of the Invention

The Curta was conceived by Curt Herzstark (1902–1988) in the 1930s in Vienna. By 1938, he had filed a key patent, covering his complemented stepped drum, Deutsches Reichspatent (German Empire Patent) No. 747073. This single drum replaced the multiple drums, typically around 10 or so, of contemporary calculators, and it enabled not only addition, but subtraction through nines complement math, essentially subtracting by adding. The nines' complement math breakthrough eliminated the significant mechanical complexity created when "borrowing" during subtraction. This drum would prove to be the key to the small, hand-held mechanical calculator the Curta would become.

His work on the pocket calculator stopped in 1938 when the Nazis forced him and his company to concentrate on manufacturing measuring instruments and distance gauges for the German army.

Herzstark, the son of a Catholic mother but Jewish father, was taken into custody in 1943, eventually finding himself at the Buchenwald concentration camp. Ironically, it was in the concentration camp that he was encouraged to continue his earlier research: "While I was imprisoned inside [Buchenwald] I had, after a few days, told the [people] in the work production scheduling department of my ideas. The head of the department, Mr. Munich said, 'See, Herzstark, I understand you've been working on a new thing, a small calculating machine. Do you know, I can give you a tip. We will allow you to make and draw everything. If it is really worth something, then we will give it to the Führer as a present after we win the war. Then, surely, you will be made an Aryan.' For me, that was the first time I thought to myself, my God, if you do this, you can extend your life. And then and there I started to draw the CURTA, the way I had imagined it."

Herzstark worked furiously to move his invention from his knowing how to build the device "in principle" to concise working drawings for a manufacturable device.

The department head's celebration plan didn't work out, but Herzstark's construction plans did: Between April 11, 1945, when Buchenwald was liberated by the Americans, and the following November, Herzstark was, after making only a few "detailed improvements" to the design, able to locate a factory in Sommertal, near Weimar, where machinists were skilled enough to work at the necessary level of precision, and walk away with three working models of the calculator.

The Russians had arrived in July, and Hertstark feared being sent to Russia, so, later that same month, he fled to Austria. He began to look for financial backers, at the same time filing continuing patents as well as several additional patents to protect his work. The Prince of Liechtenstein eventually showed interest in the manufacture of the device, and soon a newly-formed company, Contina AG Mauren, (aka, Contina Ltd Mauren) began production in Liechtenstein.

It wasn't long before the money men, apparently having gotten from him all they thought they needed, contrived to force him out by reducing the value of all existing stock to zero, including his one-third interest in the company. These were the same money men who, earlier, had elected not to transfer ownership of the Herzstark's patents to the company, so that, should anyone sue, they'd be suing Herzstark, not the company, thereby protecting themselves at Herzstark's expense. This ploy now backfired: Without the patent rights, they could manufacture nothing. Herzstark was able to negotiate a new agreement, and money continued to flow to him.

Curtas were widely considered the best portable calculators available until they were displaced by electronic calculators in the 1970s. Herzstark continued to make money from his invention until that time, although, like many inventors before him, he was not among those who profited the most from his invention. The Curta, however, lives on, being a highly-popular collectible, with thousands of machines working just as smoothly as they did 40, 50, and 60 years ago when they were manufactured.

Description and use

Numbers are entered using slides (one slide per digit) on the side of the device. The *revolution counter* and *result counter* appear on the top. A single turn of the crank adds the input number to the result counter, at any position, and increments the revolution counter accordingly. Pulling the crank upwards slightly before turning it performs a subtraction instead of an addition. Multiplication, division, and other functions require a series of crank and carriage-shifting operations.

The Curta was affectionately known as the "pepper grinder" or "peppermill" due to its shape and means of operation. It was also termed the "math grenade", due to its superficial resemblance to a certain type of hand grenade.

Curta Type I and Type II

The Type I Curta has 8 digits for data entry (known as "setting sliders"), a 6-digit revolution counter, and an 11-digit result counter. According to the advertising literature, it weighs only 8 ounces (about 230 grams). Serial number 70154, produced in 1969, weighs 245 grams.

The larger Type II Curta, introduced in 1954, has 11 digits for data entry (known as "setting sliders"), an 8-digit revolution counter, and a 15-digit result counter. It weighs 13.15 ounces or 373 grams, based on weighing serial number 550973, produced in early 1966.

An estimated 140,000 Curta calculators were made (80,000 Type I and 60,000 Type II). According to Curt Herzstark, the last Curta was produced in 1972.

A partially disassembled Curta calculator, showing the digit slides and the stepped drum behind them.

Curta Type I calculator showing view from top.

Curta Type I calculator showing view from bottom.

Use in car rallies

The Curta was popular among contestants in sports car rallies during the 1960s, 1970s and into the 1980s. Even after the introduction of the electronic calculator for other purposes, they were used in time-speed-distance (TSD) rallies to aid in computation of times to checkpoints, distances off-course, etc., since the early electronic calculators did not fare well with the bounces and jolts of rally racing.

Contestants who used such calculators were often called "Curta-crankers" by those who were limited to paper and pencil, or who used computers linked to the car's wheels.

Curta calculators contributed to the saying when describing the process of calculating, "Cranking out the answer."

Use by pilots

The Curta was also favored by both commercial and general-aviation pilots, before the advent of electronic calculators, because of both its precision and the user's ability to confirm the accuracy of his or her manipulations via the revolution counter. Because calculations such as weight and balance are a matter of life and death, precise results free of pilot error are essential.

The real cost of a Curta

While only 3% of Curtas were returned to the factory because of repairs under warranty, a small, but significant stream returned the Curta in pieces. Many purchasers attempted to disassemble the Curta. Reassembling the machine was more difficult, as assembly required intimate knowledge of the orientation and installation order for each part and sub-assembly. Many identical looking parts, each with slightly different dimensions, required test fitting and selection as well as special tools to adjust tolerances. One chagrined owner, on showing up to the dealer to retrieve his $600 Curta, now reassembled for an additional $300, was told, "don't feel bad. Curtas really cost $900. Everyone takes them apart."

The Curta in print

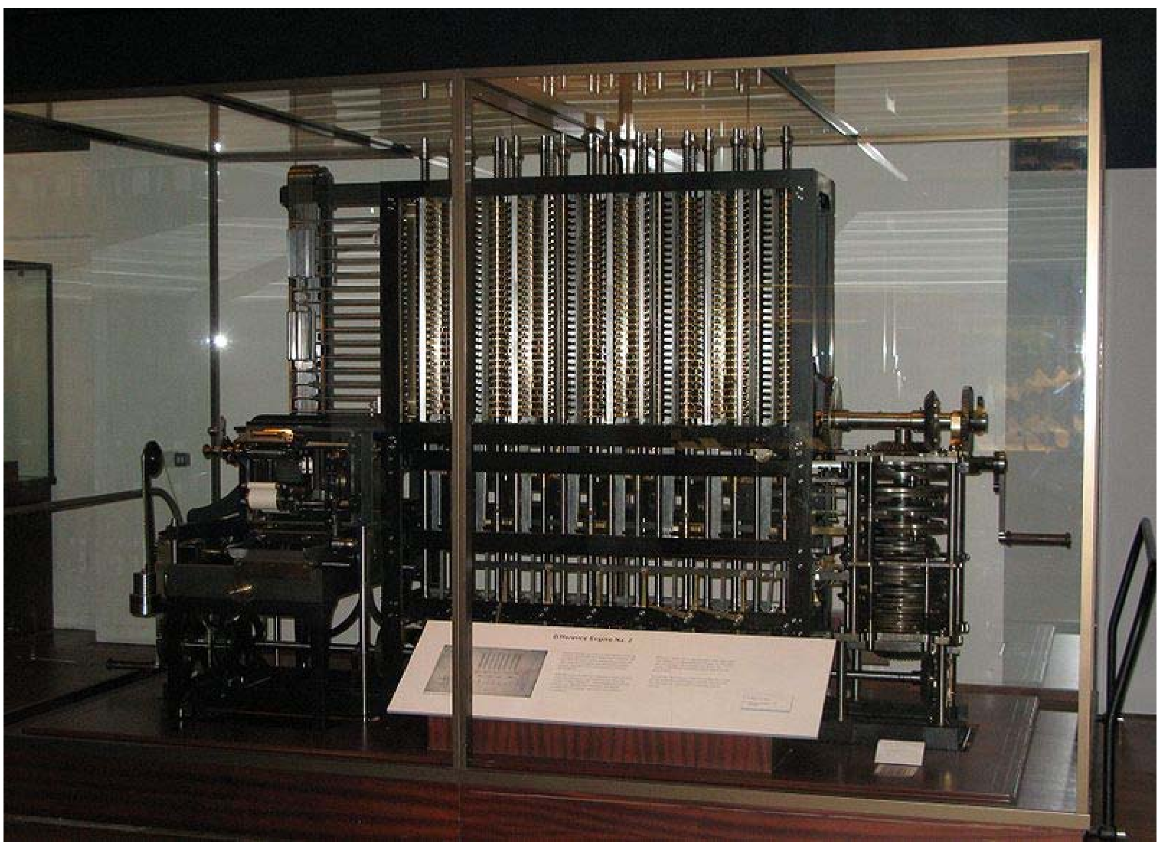
The Curta is featured in the novel *Pattern Recognition* by William Gibson, where one of the minor characters has an interest in them.

The Curta is also the subject of an article by Cliff Stoll in the January 2004 edition of *Scientific American*.

Lastly, the Curta is on the Popular Science magazine list of top 100 pieces of retro technology.

Chapter 7

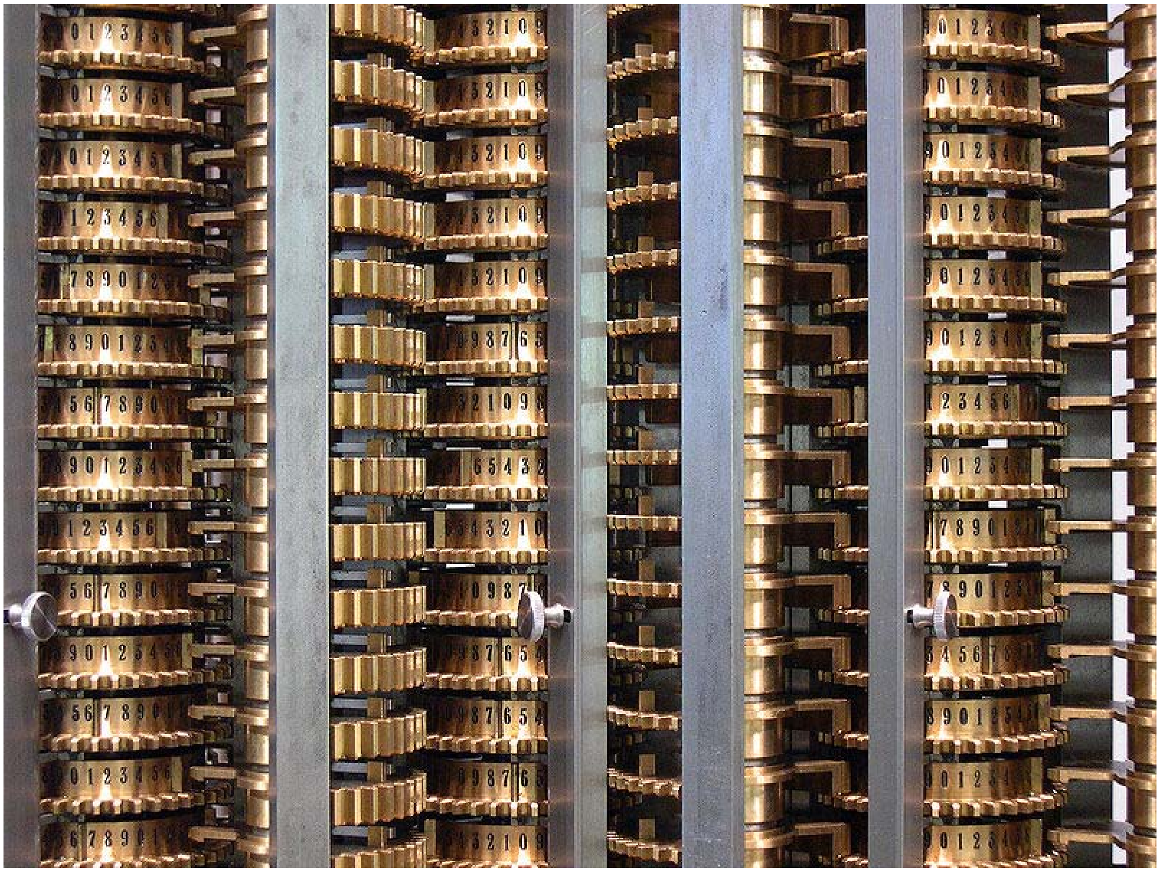
Difference Engine



The London Science Museum's difference engine, built from Babbage's design. The design has the same precision on all columns, but when calculating converging polynomials, the precision on the higher-order columns could be lower. The Engine is not a replica (there never was one built during Babbage's lifetime); therefore this is the first one - the original.

A **Difference Engine** is an automatic, mechanical calculator designed to tabulate polynomial functions. Both logarithmic and trigonometric functions can be approximated by polynomials, so a difference engine can compute many useful sets of numbers.

History



Closeup of the London Science Museum's difference engine showing some of the number wheels and the sector gears between columns. The sector gears on the left show the double-high teeth very clearly. The sector gears on the middle-right are facing the back side of the engine, but the single-high teeth are clearly visible. Notice how the wheels are mirrored, with counting up from left-to-right, or counting down from left-to-right. Also notice the metal tab between "6" and "7". That tab trips the carry lever in the back when "9" passes to "0" in the front during the add steps (Step 1 and Step 3).

Per Georg Scheutz's third Difference engine

J. H. Müller, an engineer in the Hessian army conceived the idea in a book published in 1786, but failed to find funding to progress this further.

In 1822, Charles Babbage proposed the use of such a machine in a paper to the Royal Astronomical Society on 14 June entitled "Note on the application of machinery to the computation of astronomical and mathematical tables". This machine used the decimal number system and was powered by cranking a handle. The British government initially financed the project, but withdrew funding when it became apparent that the machine would cost much more than originally anticipated. Babbage went on to design his much more general analytical engine but later produced an improved difference engine design (his "Difference Engine No. 2") between 1847 and 1849. Inspired by Babbage's difference engine plans, Per Georg Scheutz built several Difference Engines from 1855 onwards; one was sold to the British government in 1859. Martin Wiberg improved Scheutz's construction but used his device only for producing and publishing printed logarithmic tables.

Based on Babbage's original plans, the London Science Museum constructed a working Difference Engine No. 2 from 1989 to 1991, under Doron Swade, the then Curator of Computing. This was to celebrate the 200th anniversary of Babbage's birth. In 2000, the printer which Babbage originally designed for the difference engine was also completed. The conversion of the original design drawings into drawings suitable for engineering manufacturers' use revealed some minor errors in Babbage's design, which had to be corrected. Once completed, both the engine and its printer worked flawlessly, and still do. The difference engine and printer were constructed to tolerances achievable with 19th century technology, resolving a long-standing debate whether Babbage's design would actually have worked. (One of the reasons formerly advanced for the non-completion of Babbage's engines had been that engineering methods were insufficiently developed in the Victorian era.)

Although the "printer" is here referred to as such, its primary purpose is to produce stereotype plates for use in printing presses; Babbage's intention being that the Engine's results be conveyed *directly* to mass printing, rather than through a fallible human typesetter. The printer's paper output is mainly a means of checking the Engine's performance.

In addition to funding the construction of the output mechanism for the Science Museum's Difference Engine No. 2, Nathan Myhrvold commissioned the construction of a second complete Difference Engine No. 2, which has been on exhibit at the Computer History Museum in Mountain View, California starting on 10 May 2008 and continuing through the end of 2011.

Operation

The difference engine consists of a number of columns, numbered from **1** to ***N***. The machine is able to store one decimal number in each column. The machine can only add the value of a column $n + 1$ to column n to produce the new value of n. Column N can only store a constant, column 1 displays (and possibly prints) the value of the calculation on the current iteration.

The engine is programmed by setting initial values to the columns. Column 1 is set to the value of the polynomial at the start of computation. Column 2 is set to a value derived from the first and higher derivatives of the polynomial at the same value of X. Each of the columns from 3 to N is set to a value derived from the $(n - 1)$ first and higher derivatives of the polynomial.

Timing

In the Babbage design, one iteration i.e. one full set of addition and carry operations happens once for four rotations of the crank. Odd and even columns alternatively perform an addition in one cycle. The sequence of operations for column n is thus:

1. Count up, receiving the value from column $n + 1$ (Addition step)
2. Perform carry propagation on the counted up value
3. Count down to zero, adding to column $n - 1$
4. Reset the counted down value to its original value

Steps 1,2,3,4 occur for every odd column while steps 3,4,1,2 occur for every even column.

Steps

Each iteration creates a new result, and is accomplished in four steps corresponding to four complete turns of the handle shown at the far right in the picture below. The four steps are:

- Step 1. All even numbered columns (2,4,6,8) are added to all odd numbered columns (1,3,5,7) simultaneously. An interior sweep arm turns each even column to cause whatever number is on each wheel to count down to zero. As a wheel turns to zero, it transfers its value to a sector gear located between the odd/even columns. These values are transferred to the odd column causing them to count up. Any odd column value that passes from "9" to "0" activates a carry lever.
- Step 2. Carry propagation is accomplished by a set of spiral arms in the back that poll the carry levers in a helical manner so that a carry at any level can increment the wheel above by one. That can create a carry, which is why the arms move in a spiral. At the same time, the sector gears are returned to their original position, which causes them to increment the even column wheels back to their original values. The sector gears are double-high on one side so they can be lifted to disengage from the odd column wheels while they still remain in contact with the even column wheels.
- Step 3. This is like Step 1, except it is odd columns (3,5,7) added to even columns (2,4,6), and column one has its values transferred by a sector gear to the print mechanism on the left end of the engine. Any even column value that passes from

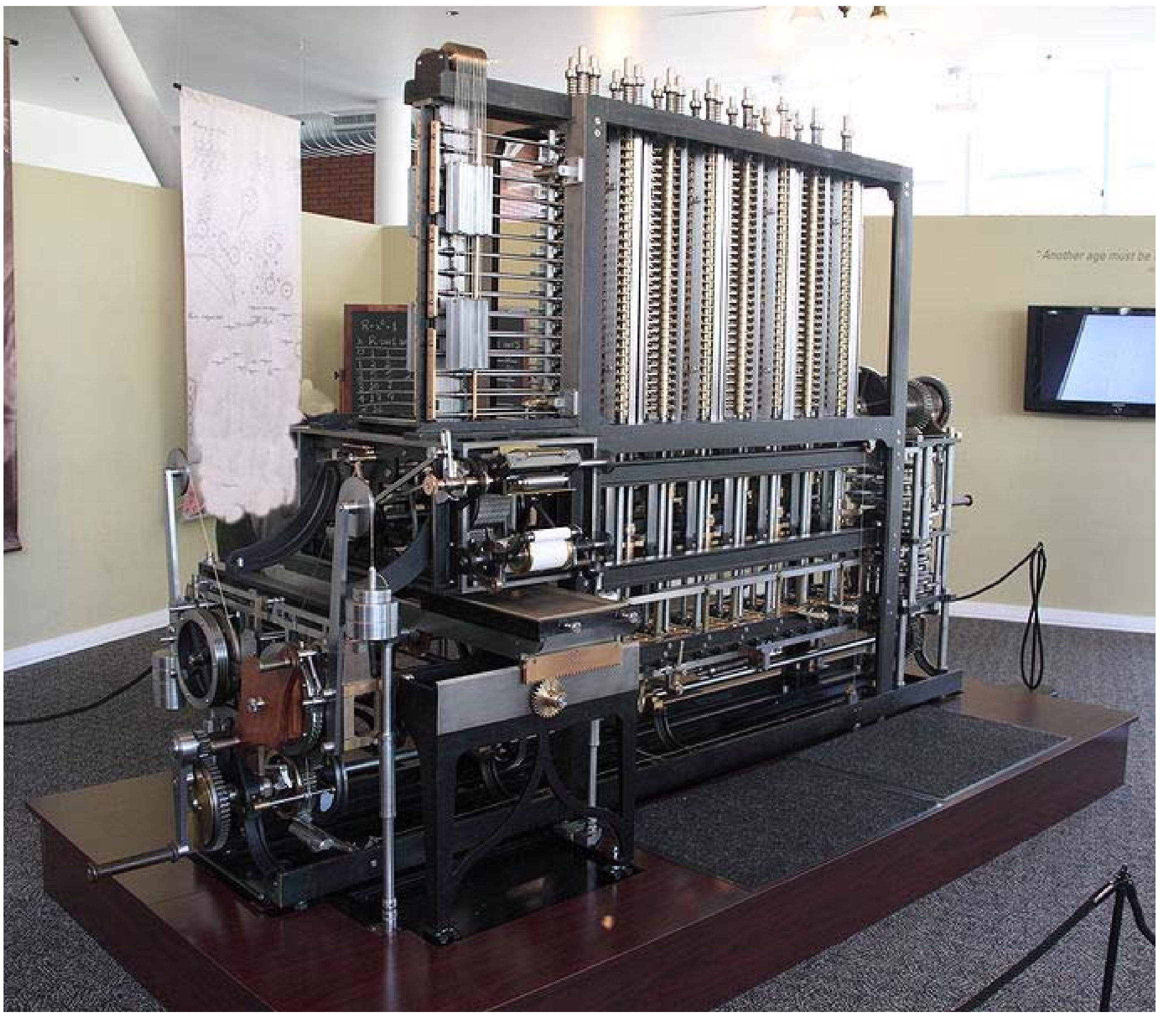
"9" to "0" activates a carry lever. The column 1 value, the result for the polynomial, is sent to the attached printer mechanism.

- Step 4. This is like Step 2, but for doing carries on even columns, and returning odd columns to their original values.

Subtraction

The engine represents negative numbers as ten's complements. Subtraction amounts to addition of a negative number. This works in exactly the same manner that modern computers perform subtraction, known as two's complement.

Method of differences



Fully operational difference engine at the Computer History Museum in Mountain View, California

The principle of a difference engine is Newton's method of divided differences. If the initial value of a polynomial (and of its finite differences) is calculated by some means for some value of ***X***, the difference engine can calculate any number of nearby values, using the method generally known as the **method of finite differences**. It may be illustrated with a small example. Consider the quadratic polynomial

$p(x) = 2x^2 - 3x + 2$

and suppose we want to tabulate the values $p(0)$, $p(1)$, $p(2)$, $p(3)$, $p(4)$ etc. The table below is constructed as follows: the second column contains the values of the polynomial, the third column contains the differences of the two left neighbors in the second column, and the fourth column contains the differences of the two neighbors in the third column:

x	**$p(x) = 2x^2 - 3x + 2$**	**diff1(x) = (p(x+1) - p(x))**	**diff2(x) = (diff1(x+1) - diff1(x))**
0	2	-1	4
1	1	3	4
2	4	7	4
3	11	11	
4	22		

Notice how the values in the fourth column are constant. This is no mere coincidence. In fact, if you start with any polynomial of degree n, the column number $n + 1$ will always be constant.

We constructed this table from the left to the right, but now we can continue it from the right to the left down a diagonal in order to compute more values of our polynomial.

To calculate $p(5)$ we use the values from the lowest diagonal. We start with the fourth column constant value of 4 and copy it down the column. Then we continue the third column by adding 4 to 11 to get 15. Next we continue the second column by taking its previous value, 22 and adding the 15 from the third column. Thus $p(5)$ is 22+15 = 37. In order to compute $p(6)$, we iterate the same algorithm on the $p(5)$ values: take 4 from the fourth column, add that to the third column's value 15 to get 19, then add that to the second column's value 37 to get 56, which is $p(6)$.

This process may be continued ad infinitum. The values of the polynomial are produced without ever having to multiply. A difference engine only needs to be able to add. From one loop to the next, it needs to store 2 numbers in our case (the last elements in the first and second columns); if we wanted to tabulate polynomials of degree n, we'd need enough storage to hold n numbers.

Babbage's difference engine No. 2, finally built in 1991, could hold 8 numbers of 31 decimal digits each and could thus tabulate 7th degree polynomials to that precision. The best machines from Scheutz were able to store 4 numbers with 15 digits each.

Initial values

The initial values of columns can be calculated by first manually calculating N consecutive values of the function and by backtracking, i.e. calculating the required differences.

Col 1_0 gets the value of the function at the start of computation *f*(0). Col 2_0 is the difference between *f*(1) and *f*(0)...

If the function to be calculated is a polynomial function, expressed as

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

the initial values can be calculated directly from the constant coefficients a_0, a_1,a_2, ..., a_n without calculating any data points. The initial values are thus:

- Col $1_0 = a_0$
- Col $2_0 = a_1 + a_2 + a_3 + a_4 + ... + a_n$
- Col $3_0 = 2a_2 + 6a_3 + 14a_4 + 30a_5 + ...$
- Col $4_0 = 6a_3 + 36a_4 + 150a_5 + ...$
- Col $5_0 = 24a_4 + 240a_5 + ...$
- Col $6_0 = 120a_5 + ...$
- ...

Use of derivatives

Many commonly used functions are analytic functions, which can be expressed as power series, for example as a Taylor series. The initial values can be calculated to any degree of accuracy; if done correctly the engine will give exact results for first N steps. After that, the engine will only give an approximation of the function.

The Taylor series expresses the function as a sum of its derivatives. For many functions the higher derivatives are trivial to obtain; the sine function at 0 has derivates of 0 or + / − 1 for all derivates. Setting 0 as the start of computation we get the simplified Maclaurin series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n$$

The same method of calculating the initial values from the coefficients can be used as for polynomial functions. The polynomial constant coefficients will now have the value

$$a_n \equiv \frac{f^{(n)}(0)}{n!}$$

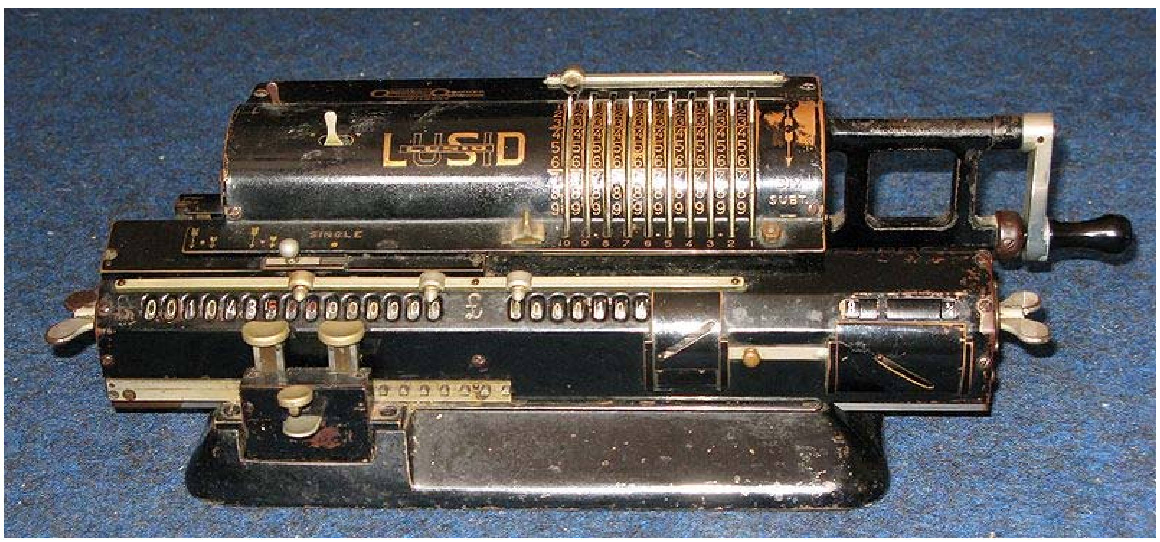
Curve fitting

The problem with the methods described above is that errors will accumulate and the series will tend to diverge from the true function. A solution which guarantees a constant maximum error is to use curve fitting. A minimum of N values are calculated evenly spaced along the range of the desired calculations. Using a curve fitting technique like Gaussian reduction a N-1th degree polynomial interpolation of the function is found. With the optimized polynomial, the initial values can be calculated as above.

Chapter 8

Odhner Arithmometer and Pinwheel Calculator

Odhner Arithmometer

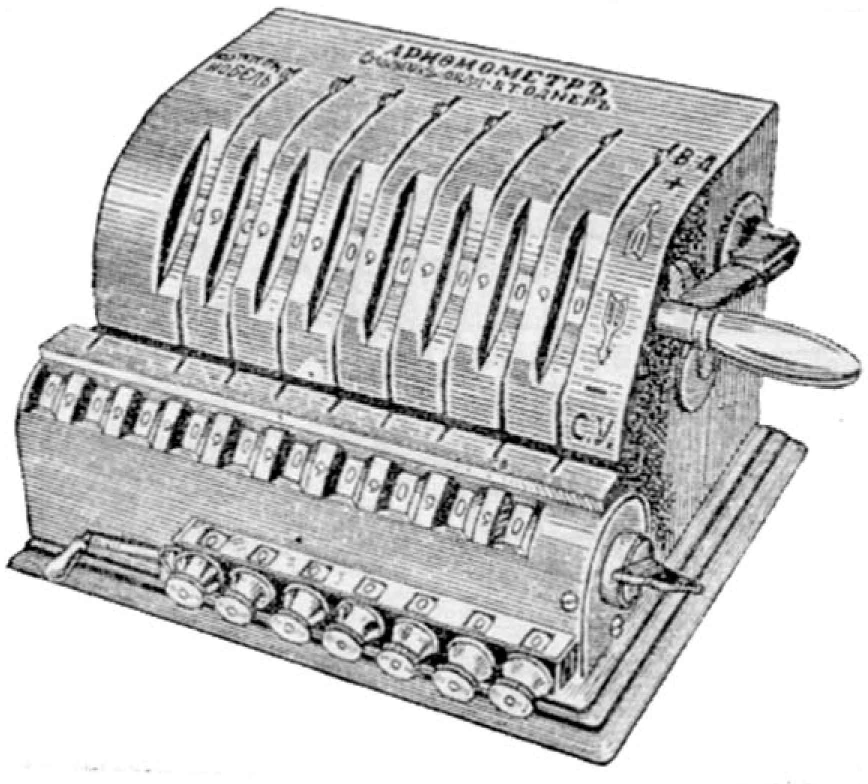


Original-Odhner LUSID

The **Odhner Arithmometer** was a very successful pinwheel calculator invented in Russia in 1873 by W. T. Odhner, a Swedish immigrant. Its industrial production officially started in 1890 in Odhner's Saint Petersburg workshop. Even though the machine was very popular, the production only lasted thirty years until the factory was nationalised and closed down during the Russian revolution of 1917.

From 1892 to the middle of the 20th century, independent companies were set up all over the world to manufacture Odhner's clones and, by the 1960s, with millions sold, it became one of the most successful type of mechanical calculator ever designed.

History



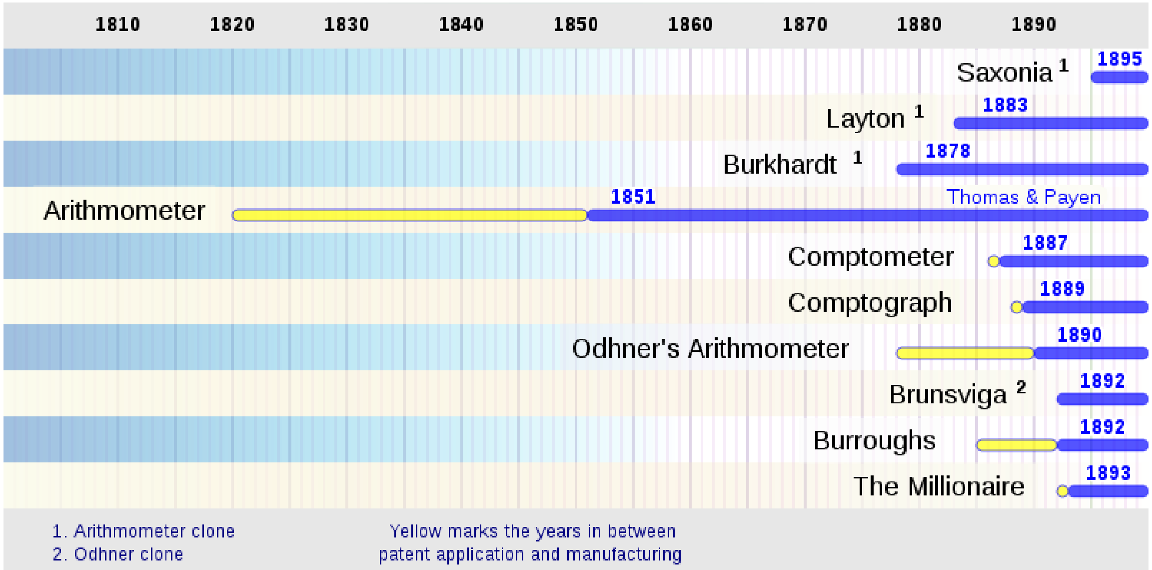
Original design

Odhner thought of his machine in 1871 while repairing a Thomas' Arithmometer (which was the only mechanical calculator in production at the time) and decided to replace its heavy, bulky Leibniz cylinder by a lighter, smaller pinwheel disk. This is why the two machines share the same user interface but look completely different.

Odhner developed the first version of his mechanical calculator in 1873. In 1876, he agreed to build 14 machines for Ludvig Nobel, his employer at the time, which he delivered in 1877. He patented his original machine in several countries in 1878–1879 and an improved version of it in 1890. The serial production began with this improved machine in 1890.

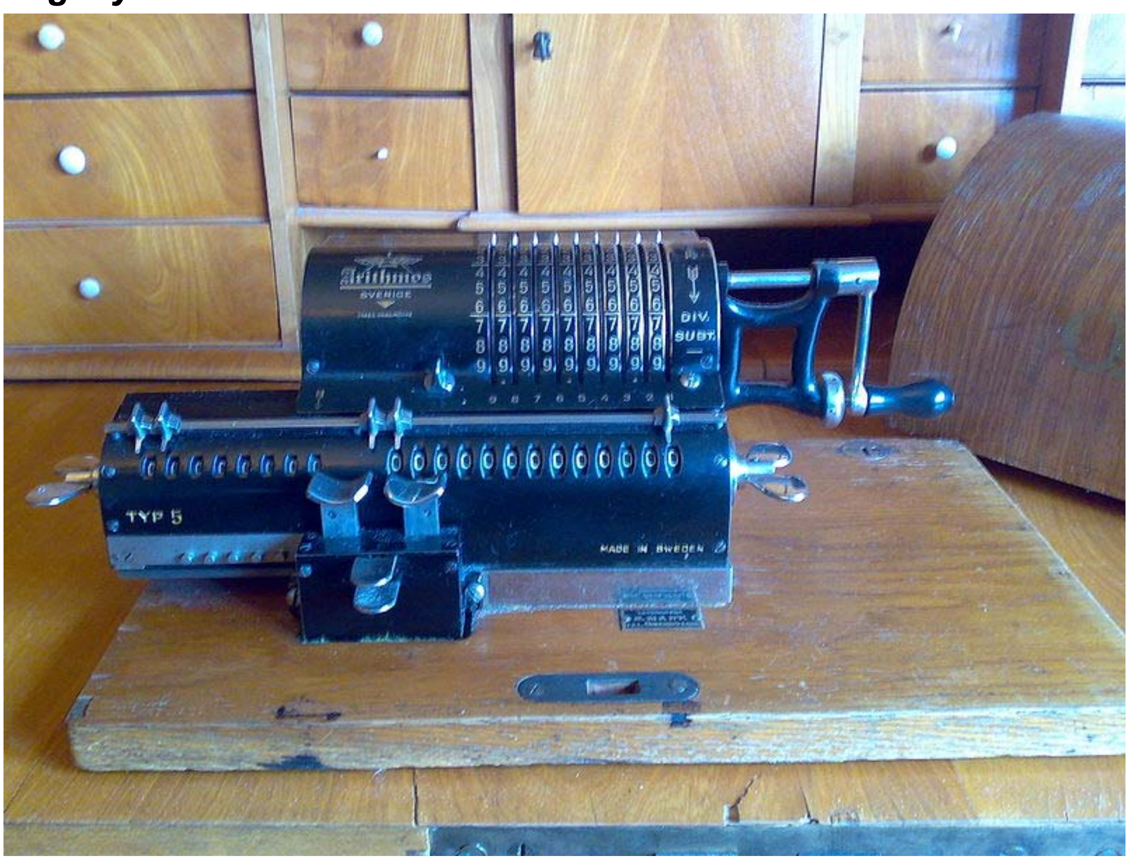
In 1891 Odhner opened a branch of his factory in Germany, unfortunately, he had to sell it in 1892 to *Grimme, Natalis & Co.* because of the difficulty of having two manufacturing facilities so far apart. *Grimme, Natalis & Co.* started production in Braunschweig and sold their machines under the *Brunsviga* brand name (Brunsviga is the latin name of the town of Braunschweig); they became very successful on their own.

After Odhner's death, in 1905, his sons Alexander and Georg and son-in-law Karl Siewert continued the production and about 23,000 calculators were made until the factory was nationalized during the Russian revolution and was forced to close down in 1918. This makes the Brunsviga arithmometer, with its 1892 start, the longest-lasting Odhner type calculator in production.



Desktop Mechanical Calculators in production during the 19th century

Legacy



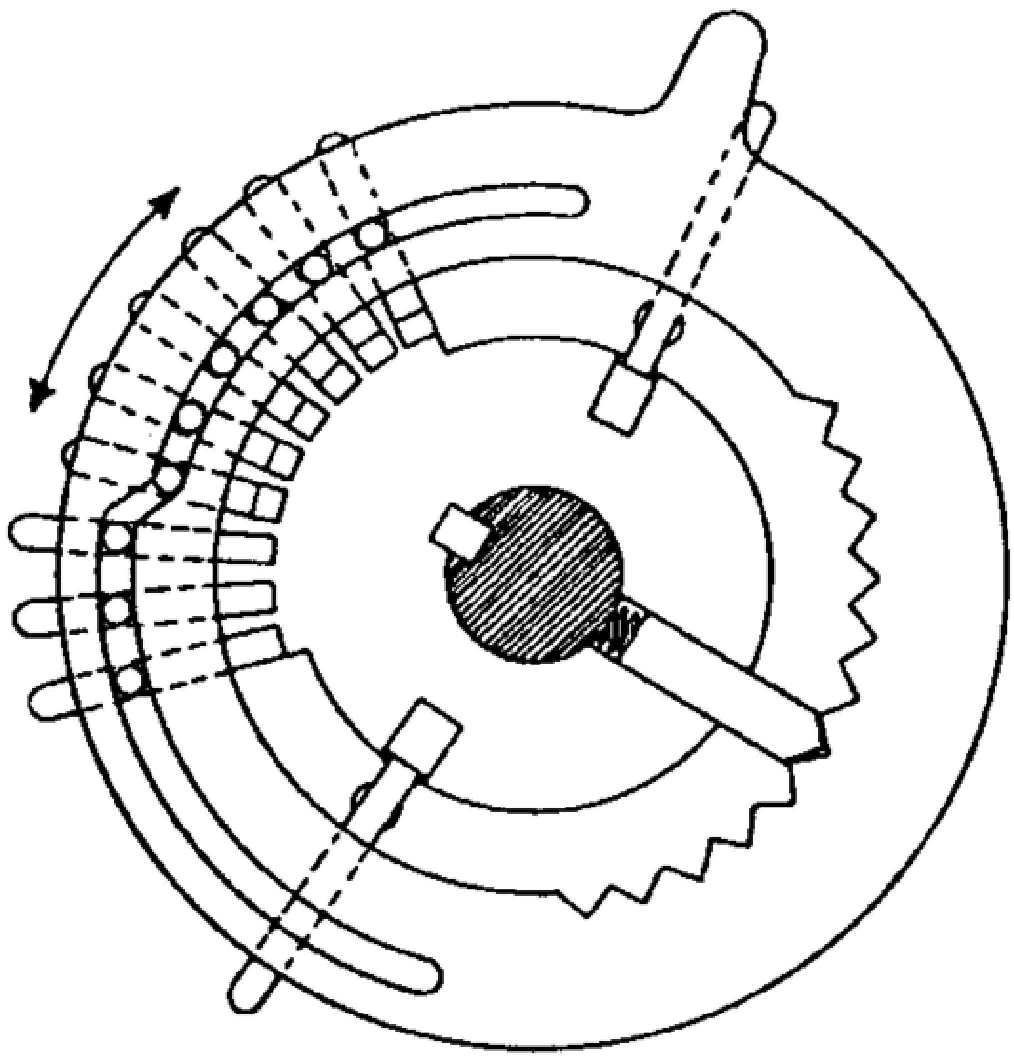
Original-Odhner Arithmos, Type 5, Made in Sweden

Towards the end of 1917, the Odhner family went back to Sweden and restarted the manufacturing of their calculator under the *Original Odhner* name. In 1924, the Russian government moved the old production facility to Moscow and commercialized their calculator under the *Felix Arithmometer* name which went on well into the 1970s.

In 1950, with millions of clones manufactured, the Odhner arithmometer was one of the most popular type of mechanical calculator ever made. The number of machines produced increased constantly until the apparition of the electronic calculators in the early 1970s. For instance, the production of one of them, the *Felix* arithmometer of Russia, peaked in 1969 with 300,000 machines made.

Odhner's arithmometer was copied, manufactured and sold by many other companies all over the world. In Germany there was *Thales*, *Triumphator*, *Walther* and *Brunsviga*. In England there was *Britannic* and *Muldivo*. In Sweden *Multo* and *Original Odhner*. In Russia *Felix* and in Japan *Tiger* and *Busicom* which, incidentally, was made famous because Intel created the first microprocessor, the Intel 4004, while desiging one of their electronic calculators in 1970.

Pinwheel calculator

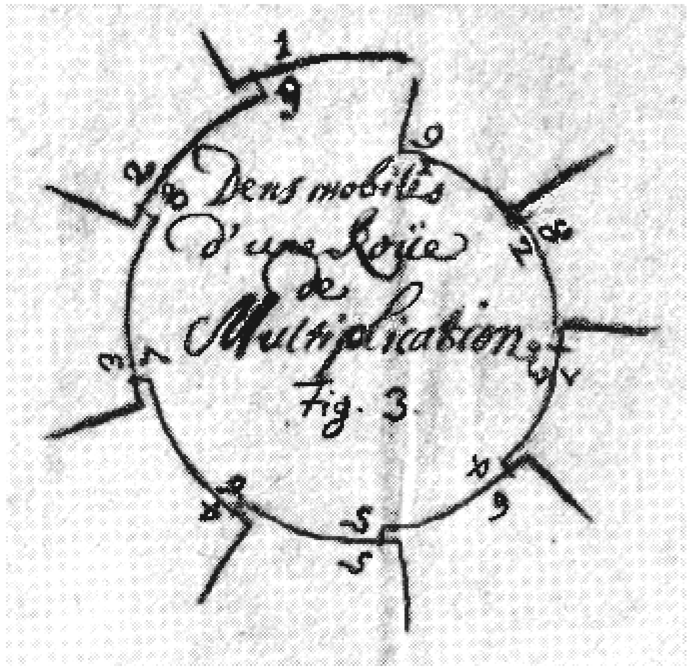


The teeth are moved by a second wheel located inside the first and moved by a side lever.

A **Pinwheel calculator** was a class of mechanical calculator popular in the 19th and 20th century using, for its calculating engine, a set of wheels that had an adjustable number of teeth. These wheels, also called pinwheels, could be set by using a side lever which could expose anywhere from 0 to 9 teeth, and therefore when coupled to a counter they could, at each rotation, add a number from 0 to 9 to the result. By linking these wheels with carry mechanisms a new kind of calculator engine was invented. Turn the wheels one way and one performs an addition, the other way a subtraction. As part of a redesign of the arithmometer, they reduced the cost and the size of a mechanical calculators on which one could easily do the four basic operations (add, subtract, multiply and divide) by an order of magnitude.

Pinwheel calculators became extremely popular with the success of the Odhner Arithmometer.

History



This drawing from Leibniz reads in French: Adjustable teeth of a multiplication wheel

- - In *"Machina arithmetica in qua non additio tantum et subtracto sed et mutiplicatio nullo, diviso vero paene nullo animi labore peragantur"*, written in 1685, Leibniz described an arithmetic machine he had invented that was made by linking two separate machines, one to perform additions/subtractions and one for multiplications/divisions. Pascal's calculator was to be used for additions and subtractions (he called it the calculating box of Pascal) and a machine using wheels with movable teeth was to be used for multiplications and divisions. There is no evidence that Leibniz constructed this pinwheel machine, but his Leibniz wheel, which, when coupled to a sliding counting wheel, can mesh with a variable number of teeth, seems to have been his way of implementing a variable number of teeth design.

- - Giovanni Poleni was the first to build a calculator that used a pinwheel design. Made of wood, his *calculating clock* was built in 1709; he destroyed it after hearing that Antonius Braun had received 10,000 Guldens for dedicating a pinwheel machine of his own design to the emperor Charles VI of Vienna. Poleni described his machine in his *Miscellanea* in 1709, but it was also described by Jacob Leupold in his *Theatrum Machinarum Generale*, ("The General Theory of Machines") which was published in 1727.

- - Antonius Braun was native of Swabia; his machine, which he presented to the emperor in 1727, was cylindrical in shape and was made of steel, silver and brass; it was finely decorated and looked like a renaissance table clock. It could perform all four operations. His dedication to the emperor engraved on the top of the machine also reads "..to make easy to ignorant people, addition, subtraction, multiplication and even division".

- - Lord Stanhope of the United Kingdom designed a pinwheel machine in 1775. It was set in a rectangular box with a handle on the side. He also designed a machine using Leibniz wheels in 1777.

- - Dr. Didier Roth, a French inventor, patented and built a machine based on that design in 1842.

- - Izrael Staffel, a polish clockmaker introduced his pinwheel machine in 1845 at an industrial exposition in Warsaw, Poland and won a gold medal in 1851 at The Great Exhibition in London.

- - Frank S. Baldwin invented a pinwheel calculator in the USA in 1872.

- - In St. Petersburg, Russia, Wilgott Theophil Odhner invented his arithmometer in 1874 and in 1890 it became the first pinwheel calculator to be mass-manufactured. Its industrial production started in Odhner's workshop: *W.T. Odhner, Maschinenfabrik & Metallgiesserei* and then moved to the Odhner-Gill factory (фабрика Однера-Гиля) in 1891. Odhner type calculators were more popular in Europe (particularly in Germany) than in the United States.

- - *Grimme, Natalis & Co.* bought the rights to Odhner's patents in 1892 and soon after started production in Braunschweig, Germany. They sold their machines under the Brunsviga brand name (Brunsviga is the latin name of the town of Braunschweig); they became very successful on their own and were the first of a long line of Odhner clone makers.

- - In 1924, Felix Dzerzhinsky, the head of the Russian Cheka, initiated the manufacturing of arithmometers. Later they were named *arithmometer Feliks* and served in the Soviet Union well into the 1970s, popularly known under the name "Iron Feliks".

Operation

"The operation of machines of this type was accomplished by means of pulling levers or knobs to set up the desired number. Addition, subtraction, multiplication, and division were accomplished by means of revolving drums. For addition they revolved in one direction, and for subtraction the direction was reversed. For multiplication the revolutions were repeated in the same direction as for addition, and for division they were repeated in the same direction as for subtraction. Two sets of dials provided a means of reading totals. In one the accumulation of totals appeared; in the other, there appeared the figure which was added, subtracted, multiplied, or divided." (The Office Appliance Manual, p. 88)

Chapter 9

Antikythera Mechanism

The Antikythera mechanism (main fragment).

The **Antikythera mechanism** is an ancient mechanical computer designed to calculate astronomical positions. It was recovered in 1900–01 from the Antikythera wreck. Its significance and complexity were not understood until decades later. Its time of construction is now estimated between 150 and 100 BCE. The degree of mechanical sophistication is comparable to a 19th century Swiss clock. Technological artifacts of similar complexity and workmanship did not reappear until the 14th century, when mechanical astronomical clocks were built in Europe.

Jacques-Yves Cousteau visited the wreck for the last time in 1978, but found no additional remains of the Antikythera mechanism. Professor Michael Edmunds of Cardiff University who led the most recent study of the mechanism said: "This device is just extraordinary, the only thing of its kind. The design is beautiful, the astronomy is exactly right. The way the mechanics are designed just makes your jaw drop. Whoever has done this has done it extremely carefully ... in terms of historic and scarcity value, I have to regard this mechanism as being more valuable than the Mona Lisa."

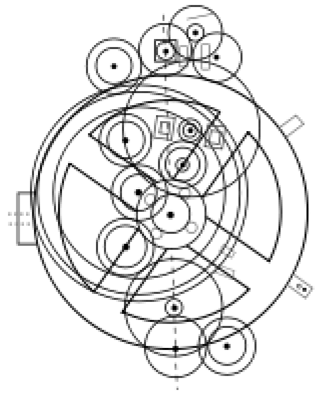
The device is displayed at the National Archaeological Museum of Athens, accompanied by a reconstruction made and donated to the museum by Derek de Solla Price. Other reconstructions are on display at the American Computer Museum in Bozeman, Montana, the Computer History Museum in Mountain View, California, the Children's Museum of Manhattan in New York, and in Kassel, Germany.

Origins

The mechanism is the oldest known complex scientific calculator. It contains many gears, and is sometimes called the first known analog computer, although its flawless manufacturing suggests that it may have had a number of undiscovered predecessors during the Hellenistic Period. It appears to be constructed upon theories of astronomy and mathematics developed by Greek astronomers and it is estimated that it was made around 150-100 BCE.

Consensus among scholars is that the mechanism itself was made in the Greek-speaking world. All the instructions of the mechanism are written in Koine Greek. One hypothesis is that the device was constructed at an academy founded by the ancient Stoic philosopher Posidonius on the Greek island of Rhodes, which at the time was known as a center of astronomy and mechanical engineering, and that perhaps the astronomer Hipparchus was the engineer who designed it since it contains a lunar mechanism which uses Hipparchus's theory for the motion of the Moon. However, the most recent findings of The Antikythera Mechanism Research Project, as published in the July 30, 2008, edition of *Nature* alternatively suggest that the concept for the mechanism originated in the colonies of Corinth, which might imply a connection with Archimedes.

The circumstances under which it came to be on the cargo ship are unknown. Investigators have suggested that the ship could have been carrying it to Rome, together with other treasure looted from the island to support a triumphal parade being staged by Julius Caesar.



Schematic of the artifact's mechanism.

Function

The device is remarkable for the level of miniaturization and for the complexity of its parts, which is comparable to that of 19th century clocks. It has over 30 gears, although Michael Wright (see below) has suggested as many as 72 gears, with teeth formed through equilateral triangles. When a date was entered via a crank (now lost), the mechanism calculated the position of the Sun, Moon, or other astronomical information such as the location of other planets. Since the purpose was to position astronomical bodies with respect to the celestial sphere, with reference to the observer's position on the surface of the Earth, the device was based on the geocentric model.

The mechanism has three main dials, one on the front, and two on the back. The front dial has two concentric scales. The outer ring is marked off with the days of the 365-day Egyptian calendar, or the Sothic year, based on the Sothic cycle. Inside this, there is a second dial marked with the Greek signs of the Zodiac and divided into degrees. The calendar dial can be moved to compensate for the effect of the extra quarter day in the solar year (there are 365.2422 days per year) by turning the scale backwards one day every four years. Note that the Julian calendar, the first calendar of the region to contain leap years, was not introduced until about 46 BCE, up to a century after the device was said to have been built.

The front dial probably carried at least three hands, one showing the date, and two others showing the positions of the Sun and the Moon. The Moon indicator is adjusted to show the first anomaly of the Moon's orbit. It is reasonable to suppose the Sun indicator had a similar adjustment, but any gearing for this mechanism (if it existed) has been lost. The front dial also includes a second mechanism with a spherical model of the Moon that displays the lunar phase.

There is reference in the inscriptions for the planets Mars and Venus, and it would have certainly been within the capabilities of the maker of this mechanism to include gearing to show their positions. There is some speculation that the mechanism may have had indicators for all the five planets known to the Greeks. None of the gearing for such planetary mechanisms survives, except for one gear otherwise unaccounted for.

Finally, the front dial includes a parapegma, a precursor to the modern day almanac, which was used to mark the rising and setting of specific stars. Each star is thought to be identified by Greek characters which cross reference details inscribed on the mechanism.

The upper back dial is in the form of a spiral, with 47 divisions per turn, displaying the 235 months of the 19 year Metonic cycle. This cycle is important in fixing calendars.

The lower back dial is also in the form of a spiral, with 223 divisions showing the Saros cycle; it also has a smaller subsidiary dial which displays the 54 year "Triple Saros" or "Exeligmos" cycle. (The Saros cycle, discovered by the Chaldeans, is a period of approximately 18 years 11 days 8 hours — the length of time between occurrences of a particular eclipse.)

The Antikythera Mechanism Research Project, with experts from Britain, Greece and the United States, detected in July 2008 the word "Olympia" on a bronze dial thought to display the 76 year Callippic cycle, as well as the names of other games in ancient Greece, and probably used to track dates of the ancient Olympic Games. According to BBC news:

"The four sectors of the dial are inscribed with a year number and two Panhellenic Games: the 'crown' games of Isthmia, Olympia, Nemea, and Pythia; and two lesser games: Naa (held at Dodona) and a second game which has not yet been deciphered."

Speculation about its purpose

Derek J. de Solla Price suggested that it might have been on public display, possibly in a museum or public hall in Rhodes. The island was known for its displays of mechanical engineering, particularly automata, which apparently were a speciality of the Rhodians. Pindar, one of the nine lyric poets of ancient Greece, said this of Rhodes in his seventh Olympic Ode:

> "The animated figures stand
> Adorning every public street
> And seem to breathe in stone, or
> Move their marble feet."

Arguments against it being on public display include:

1. The device is rather small, indicating that the designer was aiming for compactness and, as a result, the size of the front and back dials is unsuitable for public display. A simple comparison with size of the Tower of the Winds in Athens could give us a hint to suggest that the aim of the Antikythera mechanism

manufacturer was the mobility of this device rather than its public display in a fixed place (such as a temple, museum or public hall).

2. The mechanism had door plates attached to it that contain at least 2,000 characters, forming what members of the Antikythera mechanism research project often refer to as an instruction manual for the mechanism. The neat attachment of this manual to the mechanism itself implies ease of transport and personal use.
3. The existence of this "instruction manual" implies that the device was constructed by an expert scientist and mechanic in order to be used by a non-expert traveler (the text gives a lot of information associated with well known geographical locations of the Mediterranean area).

The device is unlikely to have been intended for navigation use because:

1. Some data, such as eclipse predictions, are unnecessary for navigation.
2. The salt-laden dampness of marine environments would corrode the gears in a short period of time, rendering it useless.

On 30 July 2008, scientists reported new findings in the journal *Nature* showing that the mechanism tracked the Metonic calendar, predicted solar eclipses, and calculated the timing of the Ancient Olympic Games. Inscriptions on the instrument closely match the names of the months on calendars from Illyria and Epirus in northwestern Greece and with the island of Corfu.

Similar devices in ancient literature

Cicero's *De re publica*, a 1st century BCE philosophical dialogue, mentions two machines that some modern authors consider as some kind of planetarium or orrery, predicting the movements of the Sun, the Moon, and the five planets known at that time. They were both built by Archimedes and brought to Rome by the Roman general Marcus Claudius Marcellus after the death of Archimedes at the siege of Syracuse in 212 BCE. Marcellus had a high respect for Archimedes and one of these machines was the only item he kept from the siege (the second was offered to the temple of Virtus). The device was kept as a family heirloom, and Cicero has Philus (one of the participants in a conversation that Cicero imagined had taken place in a villa belonging to Scipio Aemilianus in the year 129 BCE) saying that Caius Sulpicius Gallus (consul with Marcellus' nephew in 166 BCE, and credited by Pliny the Elder as the first Roman to have written a book explaining solar and lunar eclipses) gave a 'learned explanation' of it and demonstrated it working.

I had often heard this celestial globe or sphere mentioned on account of the great fame of Archimedes. Its appearance, however, did not seem to me particularly striking. There is another, more elegant in form, and more generally known, moulded by the same Archimedes, and deposited by the same Marcellus, in the Temple of Virtue at Rome. But as soon as Gallus had begun to explain, by his sublime science, the composition of this machine, I felt that the Sicilian geometrician must have possessed a genius superior to any thing we usually conceive to belong to our nature. Gallus assured us, that the solid

and compact globe, was a very ancient invention, and that the first model of it had been presented by Thales of Miletus. That afterwards Eudoxus of Cnidus, a disciple of Plato, had traced on its surface the stars that appear in the sky, and that many years subsequent, borrowing from Eudoxus this beautiful design and representation, Aratus had illustrated them in his verses, not by any science of astronomy, but the ornament of poetic description. He added, that the figure of the sphere, which displayed the motions of the Sun and Moon, and the five planets, or wandering stars, could not be represented by the primitive solid globe. And that in this, the invention of Archimedes was admirable, because he had calculated how a single revolution should maintain unequal and diversified progressions in dissimilar motions.
When Gallus moved this globe it showed the relationship of the Moon with the Sun, and there were exactly the same number of turns on the bronze device as the number of days in the real globe of the sky. Thus it showed the same eclipse of the Sun as in the globe [of the sky], as well as showing the Moon entering the area of the Earth's shadow when the Sun is in line...
[i.e. It showed both solar and lunar eclipses.]

So at least one of Archimedes' machines, probably (considering Gallus' interests and the fact that that portion of the *De Republica* seems to be concerned with astronomical *prodigia* and in particular eclipses) quite similar to the Antikythera mechanism, was still operated around 150 BCE.

Pappus of Alexandria stated that Archimedes had written a now lost manuscript on the construction of these devices entitled *On Sphere-Making*. The surviving texts from the Library of Alexandria describe many of his creations, some even containing simple blueprints. One such device is his odometer, the exact model later used by the Romans to place their mile markers (described by Vitruvius, Heron of Alexandria and in the time of Emperor Commodus). The blueprints in the text appeared functional, but attempts to build them as pictured had failed. When the gears pictured, which had square teeth, were replaced with gears of the type in the Antikythera mechanism, which were angled, the device was perfectly functional. Whether this is an example of a device created by Archimedes and described by texts lost in the burning of the Library of Alexandria, or if it is a device based on his discoveries, or if it has anything to do with him at all, is debatable.

If Cicero's account is correct, then this technology existed as early as the 3rd century BCE. Archimedes' device is also mentioned by later Roman era writers such as Lactantius (*Divinarum Institutionum Libri VII*), Claudian (*In sphaeram Archimedes*), and Proclus (*Commentary on the first book of Euclid's Elements of Geometry*) in the 4th and 5th centuries.

Cicero also said that another such device was built 'recently' by his friend Posidonius, "... each one of the revolutions of which brings about the same movement in the Sun and Moon and five wandering stars [planets] as is brought about each day and night in the heavens..."

It is unlikely that any one of these machines was the Antikythera mechanism found in the shipwreck because both the devices fabricated by Archimedes and mentioned by Cicero were located in Rome at least 30 years later than the estimated date of the shipwreck and the third one was almost certainly in the hands of Posidonius by that date. So we know of at least four such devices. The modern scientists who have reconstructed the Antikythera mechanism also agree that it was too sophisticated to have been a unique device.

It is probable that the Antikythera mechanism was not unique, as shown by Cicero's references to such mechanisms. This adds support to the idea that there was an ancient Greek tradition of complex mechanical technology that was later, at least in part, transmitted to the Byzantine and Islamic worlds, where mechanical devices which were complex, albeit simpler than the Antikythera mechanism, were built during the Middle Ages. Fragments of a geared calendar attached to a sundial, from the 5th or 6th century Byzantine Empire, have been found; the calendar may have been used to assist in telling time. In the Islamic world, Banū Mūsā's *Kitab al-Hiyal*, or *Book of Ingenious Devices*, was commissioned by the Caliph of Baghdad in the early 9th century. This text described over a hundred mechanical devices, some of which may date back to ancient Greek texts preserved in monasteries. A geared calendar similar to the Byzantine device was described by the scientist al-Biruni around 1000 CE, and a surviving 13th century astrolabe also contains a similar clockwork device. It is possible that this medieval technology may have been transmitted to Europe and contributed to the development of mechanical clocks there.

Investigations and reconstructions

Reconstruction of the Antikythera mechanism in the National Archaeological Museum, Athens (made by Robert J. Deroski, based on Derek J. de Solla Price model).

The Antikythera mechanism is one of the world's oldest known geared devices. It has puzzled and intrigued historians of science and technology since its discovery. A number of individuals and groups have been instrumental in advancing the knowledge and understanding of the mechanism including: Derek J. de Solla Price (with Charalampos Karakalos); Allan George Bromley (with Frank Percival, Michael Wright and Bernard Gardner); Michael Wright and The Antikythera Mechanism Research Project.

Derek J. de Solla Price

Following decades of work cleaning the device, in 1951 British science historian Derek J. de Solla Price undertook systematic investigation of the mechanism.

Price published several papers on "Clockwork before the Clock". and "On the Origin of Clockwork", before the first major publication in June 1959 on the mechanism: "An Ancient Greek Computer". This was the lead article in *Scientific American* and appears to have been initially published at the prompting of Arthur C. Clarke, according to the book *Arthur C. Clarke's Mysterious World*. In "An Ancient Greek Computer" Price advanced the theory that the Antikythera mechanism was a device for calculating the motions of stars and planets, which would make the device the first known analog computer. Until that time, the Antikythera mechanism's function was largely unknown, though it had been correctly identified as an astronomical device, perhaps being an astrolabe.

In 1971, Price, by then the first Avalon Professor of the History of Science at Yale University, teamed up with Charalampos Karakalos, professor of nuclear physics at the Greek National Centre of Scientific Research "DEMOKRITOS". Karakalos took both gamma- and X-ray radiographs of the mechanism, which revealed critical information about the device's interior configuration.

In 1974, Price wrote "Gears from the Greeks: the Antikythera mechanism — a calendar computer from ca. 80 B.C.", where he presented a model of how the mechanism could have functioned.

Price's model, as presented in his "Gears from the Greeks", was the first theoretical attempt at reconstructing the device. According to that model, the front dial shows the annual progress of the Sun and Moon through the zodiac against the Egyptian calendar. The upper rear dial displays a four-year period and has associated dials showing the Metonic cycle of 235 synodic months, which approximately equals 19 solar years. The lower rear dial plots the cycle of a single synodic month, with a secondary dial showing the lunar year of 12 synodic months.

One of the remarkable proposals made by Price was that the mechanism employed differential gears, which enabled the mechanism to add or subtract angular velocities. The differential was used to compute the synodic lunar cycle by subtracting the effects of the Sun's movement from those of the sidereal lunar movement.

Allan George Bromley

A variant on Price's reconstruction was built by Australian computer scientist Allan George Bromley of the University of Sydney and Sydney clockmaker Frank Percival. Bromley went on to make new, more accurate X-ray images in collaboration with Michael Wright. Some of these were studied by Bromley's student, Bernard Gardner, in 1993.

Michael Wright

Michael Wright, formerly Curator of Mechanical Engineering at The London Science Museum and now of Imperial College, London, made a completely new study of the original fragments together with Allan George Bromley. They used a technique called linear X-ray tomography which was suggested by retired consultant radiologist, Alan Partridge. For this, Wright designed and made an apparatus for linear tomography, allowing the generation of sectional 2D radiographic images. Early results of this survey were presented in 1997, which showed that Price's reconstruction was fundamentally flawed.

Further study of the new imagery allowed Wright to advance a number of proposals. Firstly he developed the idea, suggested by Price in "Gears from the Greeks", that the mechanism could have served as a planetarium. Wright's planetarium not only modelled the motion of the Sun and Moon, but also the Inferior Planets (Mercury and Venus), and the Superior Planets (Mars, Jupiter and Saturn).

Wright proposed that the Sun and Moon could have moved in accordance with the theories of Hipparchus and the five known planets moved according to the simple epicyclic theory suggested by the theorem of Apollonios. In order to prove that this was possible using the level of technology apparent in the mechanism, Wright produced a working model of such a planetarium.

Wright also increased upon Price's gear count of 27 to 31 including 1 in Fragment C that was eventually identified as part of a Moon phase display. He suggested that this is a mechanism that shows the phase of the Moon by means of a rotating semi-silvered ball, realized by the differential rotation of the sidereal cycle of the Moon and the Sun's yearly cycle. This precedes previously known mechanisms of this sort by a millennium and a half.

More accurate tooth counts were also obtained, allowing a new gearing scheme to be advanced. This more accurate information allowed Wright to confirm Price's perceptive suggestion that the upper back dial displays the Metonic cycle with 235 lunar months divisions over a five-turn scale. In addition to this Wright proposed the remarkable idea that the main back dials are in the form of spirals, with the upper back dial out as a five-turn spiral containing 47 divisions in each turn. It therefore presented a visual display of the 235 months of the Metonic cycle (19 years ≈ 235 Synodic Months). Wright also observed that fragmentary inscriptions suggested that the pointer on the subsidiary dial showed a count of four cycles of the 19-year period, equal to the 76-year Callippic cycle.

Based on more tentative observations, Wright also came to the conclusion that the lower back dial counted Draconic Months and could perhaps have been used for eclipse prediction.

All these findings have been incorporated into Wright's working model, demonstrating that a single mechanism with all these functions could be built, and would work.

Despite the improved imagery provided by the linear tomography Wright could not reconcile all the known gears into a single coherent mechanism, and this led him to advance the theory that the mechanism had been altered, with some astronomical functions removed and others added.

Finally, as an outcome of his considerable research, Wright also conclusively demonstrated that Price's suggestion of the existence of a differential gearing arrangement was incorrect.

Michael Wright's research on the mechanism is continuing in parallel with the efforts of the Antikythera Mechanism Research Project (AMRP). Recently Wright slightly modified his model of the mechanism to incorporate the latest findings of the AMRP regarding the function of the pin and slot engaged gears that brilliantly simulate the anomaly in the Moon's angular velocity. On 6 March 2007 he presented his model in the National Hellenic Research Foundation in Athens.

The Antikythera Mechanism Research Project

The Antikythera mechanism is now being studied by the Antikythera Mechanism Research Project, a joint program between Cardiff University (M. Edmunds, T. Freeth), the National and Kapodistrian University of Athens (X. Moussas, Y. Bitsakis), the Aristotle University of Thessaloniki (J.H. Seiradakis), the National Archaeological Museum of Athens, X-Tek Systems UK and Hewlett-Packard USA, funded by the Leverhulme Trust and supported by the Cultural Foundation of the National Bank of Greece.

The mechanism's fragility precluded its removal from the museum, so the Hewlett-Packard research team and X-Tek Systems had to bring their devices to Greece. HP built a 3-D surface imaging device, known as the "PTM Dome", that surrounds the object under examination. X-Tek Systems developed a 12 ton 450 kV microfocus computerised tomographer especially for the Antikythera Mechanism.

It was announced in Athens on 21 October 2005 that new pieces of the Antikythera mechanism had been found. There are now 82 fragments. Most of the new pieces had been stabilized but were awaiting conservation.

On 30 May 2006, it was announced that the imaging system had enabled much more of the Greek inscription to be viewed and translated, from about 1,000 characters that were visible previously, to over 2,160 characters, representing about 95% of the extant text. The team's findings shed new light concerning the function and purpose of the Antikythera mechanism. Research is ongoing. The first results were announced at an international conference in Athens, November 30 and December 1, 2006.

New discoveries

On 30 November 2006, the science journal *Nature* published a new reconstruction of the mechanism by the Antikythera Mechanism Research Project, based on the high resolution X-ray tomography described above. This work doubled the amount of readable text, corrected prior transcriptions, and provided a new translation. The inscriptions led to a dating of the mechanism to around 100 BCE. It is evident that they contain a manual with an astronomical, mechanical and geographical section. The name HISPANIA (ΙΣΠΑΝΙΑ, Spain in Greek) in these texts is the oldest reference to the Iberian Peninsula under this form, as opposed to Iberia.

The new discoveries confirm that the mechanism is an astronomical analog calculator or orrery used to predict the positions of celestial bodies. This work proposes that the mechanism possessed 37 gears, of which 30 survive, and was used for prediction of the position of the Sun and the Moon. Based on the inscriptions, which mention the stationary points of the planets, the authors speculate that planetary motions may also have been indicated.

On the front face were graduations for the solar scale and the zodiac together with pointers that indicated the position of the Sun, the Moon, the lunar phase, and possibly the planetary motions.

On the back, two spiral scales (made of half-circles with two centers) with sliding pointers indicated the state of two further important astronomical cycles: the Saros cycle, the period of approximately 18 years separating the return of the Sun, Moon and Earth to the same relative positions and the more accurate exeligmos cycle of 54 years and one day. It also contains another spiral scale for the Metonic cycle (19 years, equal to 235 lunar months) and the Callippic cycle with a period of 1016 lunar orbits in approximately 76 years.

The Moon mechanism, using an ingenious train of gears, two of them linked with a slightly offset axis and pin in a slot, shows the position and phase of the Moon during the month. The velocity of the Moon varies according to the theory of Hipparchus, and to a good approximation follows Kepler's second law for the angular velocity, being faster near the perigee and slower at the apogee.

On 31 July 2008, a paper providing further details about the mechanism was published in *Nature* (*Nature* Vol 454, Issue 7204, July 31, 2008). In this paper, among other revelations, it is demonstrated that the mechanism also contained a dial divided into four parts, and demonstrated a four-year cycle through four segments of one year each, which is thought to be a means of describing which of the games (such as the ancient Olympics) that took place in two and four-year cycles were to take place in any given year.

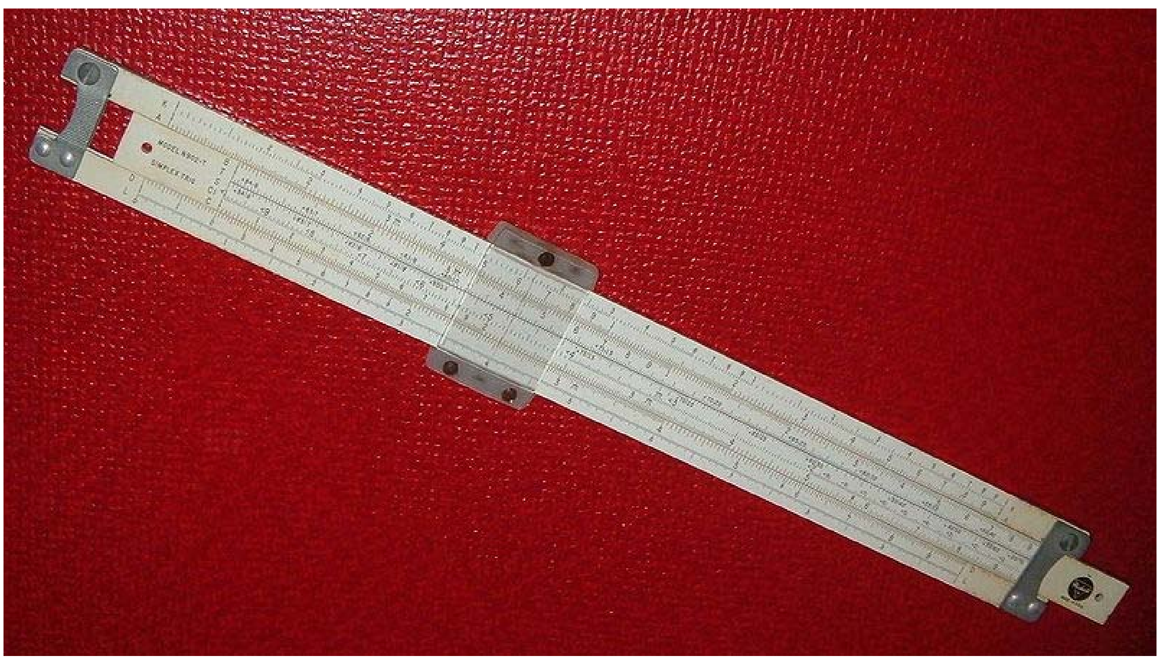
The names of the months have been read; they are the months attested for the colonies of Corinth (and therefore also traditionally assumed for Corinth, Kerkyra, Epidamnos, and Syracuse, which have left less direct evidence). The investigators suggest that the device may well be of Syracusan design and may descend from the work of Archimedes; alternatively it may have been ordered by and customized for any of these markets and was being shipped.

Nature published another study on 24 November 2010. The study interprets the mechanism to be based in computation methods used in Babylonian astronomy, not ancient Greek astronomy, implying that the Babylonian astronomy inspired the Greek counterpart – including the mechanical constructs.

Andrew Carol, an Apple software engineer, created a replica of the mechanism out of 1,500 LEGO pieces , and has correctly predicted the Solar eclipse of April 8, 2024 as a demonstration of its accuracy.

Chapter 10

Slide Rule

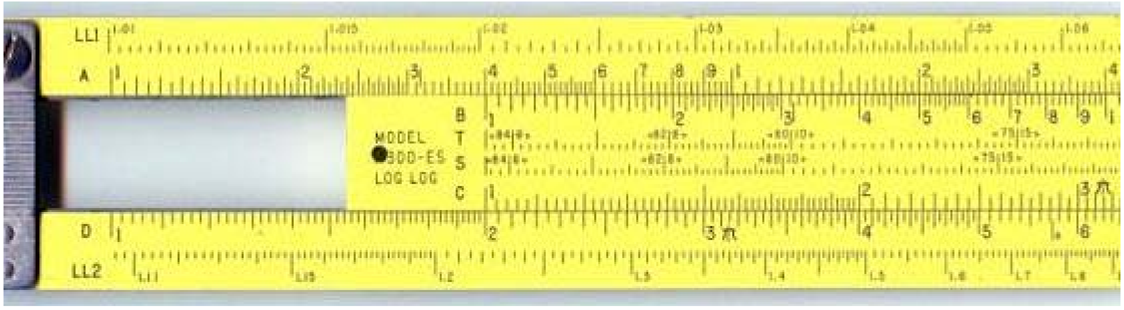


A typical ten-inch student slide rule (Pickett N902-T simplex trig).

The **slide rule**, also known colloquially as a *slipstick*, is a mechanical analog computer. The slide rule is used primarily for multiplication and division, and also for functions such as roots, logarithms and trigonometry, but is not normally used for addition or subtraction.

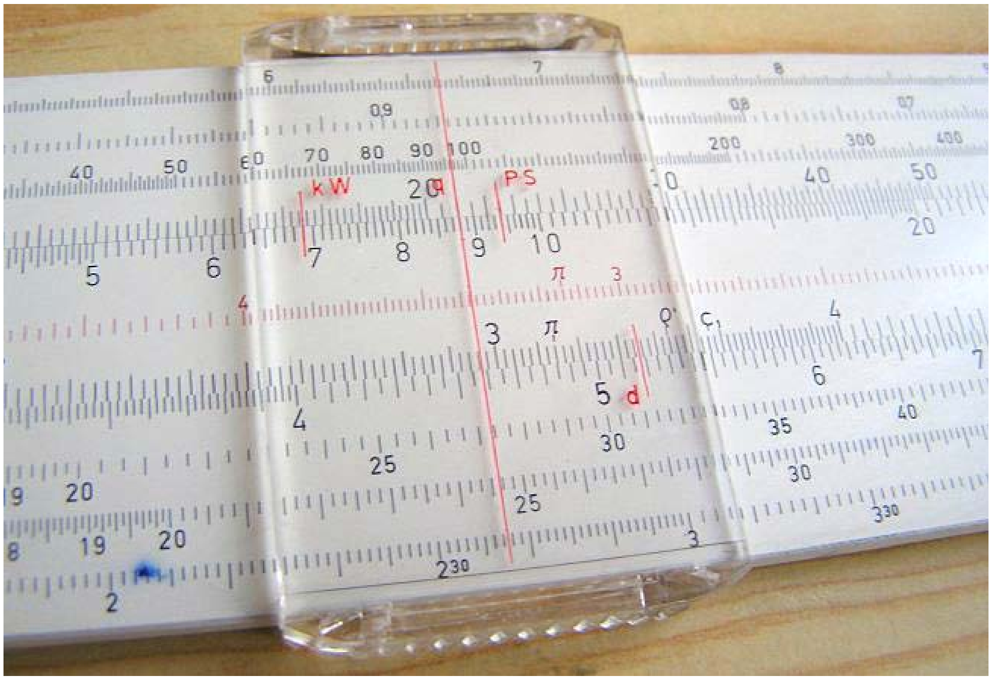
Slide rules come in a diverse range of styles and generally appear in a linear or circular form with a standardized set of markings (scales) essential to performing mathematical computations. Slide rules manufactured for specialized fields such as aviation or finance typically feature additional scales that aid in calculations common to that field.

William Oughtred and others developed the slide rule in the 17th century based on the emerging work on logarithms by John Napier. Before the advent of the pocket calculator, it was the most commonly used calculation tool in science and engineering. The use of slide rules continued to grow through the 1950s and 1960s even as digital computing devices were being gradually introduced; but around 1974 the electronic scientific calculator made it largely obsolete and most suppliers left the business.



A slide rule positioned so as to multiply by 2. Each number on the D (bottom) scale is double the number above it on the C (middle) scale.

Basic concepts



Cursor on a slide rule.

In its most basic form, the slide rule uses two logarithmic scales to allow rapid multiplication and division of numbers. These common operations can be time-consuming and error-prone when done on paper. More elaborate slide rules allow other calculations, such as square roots, exponentials, logarithms, and trigonometric functions.

Scales may be grouped in decades, which are numbers ranging from 1 to 10 (i.e. 10^n to 10^{n+1}). Thus single decade scales C and D range from 1 to 10 across the entire width of the slide rule while double decade scales A and B range from 1 to 100 over the width of the slide rule.

In general, mathematical calculations are performed by aligning a mark on the sliding central strip with a mark on one of the fixed strips, and then observing the relative positions of other marks on the strips. Numbers aligned with the marks give the approximate value of the product, quotient, or other calculated result.

The user determines the location of the decimal point in the result, based on mental estimation. Scientific notation is used to track the decimal point in more formal calculations. Addition and subtraction steps in a calculation are generally done mentally or on paper, not on the slide rule.

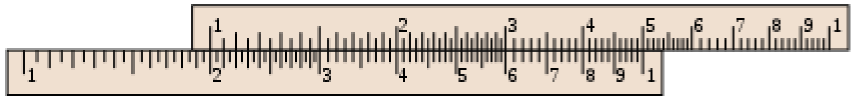
Most slide rules consist of three linear strips of the same length, aligned in parallel and interlocked so that the central strip can be moved lengthwise relative to the other two. The outer two strips are fixed so that their relative positions do not change.

Some slide rules ("duplex" models) have scales on both sides of the rule and slide strip, others on one side of the outer strips and both sides of the slide strip (which can usually be pulled out, flipped over and reinserted for convenience), still others on one side only ("simplex" rules). A sliding cursor with a vertical alignment line is used to find corresponding points on scales that are not adjacent to each other or, in duplex models, are on the other side of the rule. The cursor can also record an intermediate result on any of the scales.

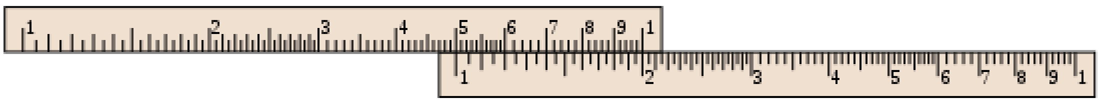
Operation

Multiplication

A logarithm transforms the operations of multiplication and division to addition and subtraction according to the rules $\log(xy) = \log(x) + \log(y)$ and $\log(x / y) = \log(x) - \log(y)$. Moving the top scale to the right by a distance of $\log(x)$, by matching the beginning of the top scale with the label x on the bottom, aligns each number y, at position $\log(y)$ on the top scale, with the number at position $\log(x) + \log(y)$ on the bottom scale. Because $\log(x) + \log(y) = \log(xy)$, this position on the bottom scale gives xy, the product of x and y. For example, to calculate 3×2, the 1 on the top scale is moved to the 2 on the bottom scale. The answer, 6, is read off the bottom scale where 3 is on the top scale. In general, the 1 on the top is moved to a factor on the bottom, and the answer is read off the bottom where the other factor is on the top.



Operations may go "off the scale;" for example, the diagram above shows that the slide rule has not positioned the 7 on the upper scale above any number on the lower scale, so it does not give any answer for 2×7. In such cases, the user may slide the upper scale to the left until its right index aligns with the 2, effectively multiplying by 0.2 instead of by 2, as in the illustration below:



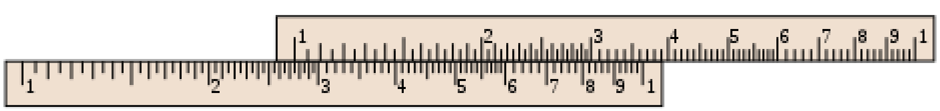
Here the user of the slide rule must remember to adjust the decimal point appropriately to correct the final answer. We wanted to find 2×7, but instead we calculated 0.2×7=1.4. So the true answer is not 1.4 but 14. Resetting the slide is not the only way to handle multiplications that would result in off-scale results, such as 2×7; some other methods are:

1. Use the double-decade scales A and B.
2. Use the folded scales. In this example, set the left 1 of C opposite the 2 of D. Move the cursor to 7 on CF, and read the result from DF.
3. Use the CI inverted scale. Position the 7 on the CI scale above the 2 on the D scale, and then read the result off of the D scale, below the 1 on the CI scale. Since 1 occurs in two places on the CI scale, one of them will always be on-scale.
4. Use both the CI inverted scale and the C scale. Line up the 2 of CI with the 1 of D, and read the result from D, below the 7 on the C scale.

Method 1 is easy to understand, but entails a loss of precision. Method 3 has the advantage that it only involves two scales.

Division

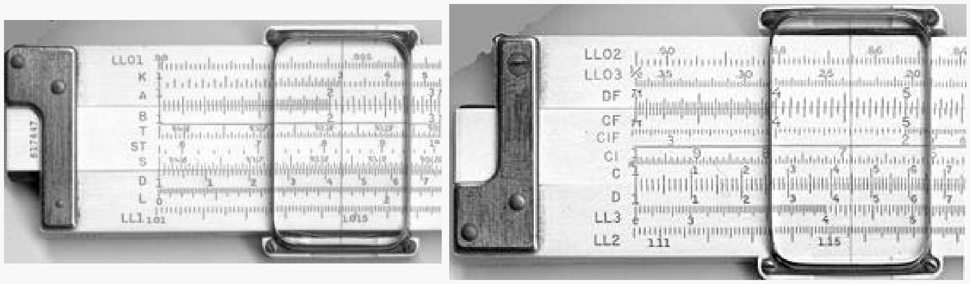
The illustration below demonstrates the computation of 5.5/2. The 2 on the top scale is placed over the 5.5 on the bottom scale. The 1 on the top scale lies above the quotient, 2.75. There is more than one method for doing division, but the method presented here has the advantage that the final result cannot be off-scale, because one has a choice of using the 1 at either end.



Other operations

In addition to the logarithmic scales, some slide rules have other mathematical functions encoded on other auxiliary scales. The most popular were trigonometric, usually sine and tangent, common logarithm ($\log_{10}$) (for taking the log of a value on a multiplier scale), natural logarithm (ln) and exponential (e^x) scales. Some rules include a Pythagorean scale, to figure sides of triangles, and a scale to figure circles. Others feature scales for calculating hyperbolic functions. On linear rules, the scales and their labeling are highly standardized, with variation usually occurring only in terms of which scales are included and in what order:

A, B	two-decade logarithmic scales, used for finding square roots and squares of numbers
C, D	single-decade logarithmic scales
K	three-decade logarithmic scale, used for finding cube roots and cubes of numbers
CF, DF	"folded" versions of the C and D scales that start from π rather than from unity; these are convenient in two cases. First when the user guesses a product will be close to 10 but is not sure whether it will be slightly less or slightly more than 10, the folded scales avoid the possibility of going off the scale. Second, by making the start π rather than the square root of 10, multiplying or dividing by π (as is common in science and engineering formulas) is simplified.
CI, DI, DIF	"inverted" scales, running from right to left, used to simplify $1/x$ steps
S	used for finding sines and cosines on the D scale
T, T1, T2	used for finding tangents and cotangents on the D and DI scales
ST, SRT	used for sines and tangents of small angles and degree–radian conversion
L	a linear scale, used along with the C and D scales for finding base-10 logarithms and powers of 10
LLn	a set of log-log scales, used for finding logarithms and exponentials of numbers
Ln	a linear scale, used along with the C and D scales for finding natural (base e) logarithms and e^x



The scales on the front and back of a Keuffel and Esser (K&E) 4081-3 slide rule.

The Binary Slide Rule manufactured by Gilson in 1931 performed an addition and subtraction function limited to fractions.

Roots and powers

There are single-decade (C and D), double-decade (A and B), and triple-decade (K) scales. To compute x^2, for example, locate x on the D scale and read its square on the A scale. Inverting this process allows square roots to be found, and similarly for the powers 3, 1/3, 2/3, and 3/2. Care must be taken when the base, x, is found in more than one place on its scale. For instance, there are two nines on the A scale; to find the square root of nine, use the first one; the second one gives the square root of 90.

For x^y problems, use the LL scales. When several LL scales are present, use the one with x on it. First, align the leftmost 1 on the C scale with x on the LL scale. Then, find y on the C scale and go down to the LL scale with x on it. That scale will indicate the answer. If y is "off the scale," locate $x^{y/2}$ and square it using the A and B scales as described above.

Trigonometry

The S, T, and ST scales are used for trig functions and multiples of trig functions, for angles in degrees.

For angles from around 5.7 up to 90 degrees, sines are found by comparing the S scale with C. The S scale has a second set of angles (sometimes in a different color), which run in the opposite direction, and are used for cosines. Tangents are found by comparing the T scale with C for angles less than 45 degrees. for angles greater than 45 degrees the CI scale is used. Common forms such as $k\sin x$ can be read directly from x on the S scale to the result on the D scale, when the C-scale index is set at k. For angles below 5.7 degrees, sines, tangents, and radians are approximately equal, and are found on the ST or SRT (sines, radians, and tangents) scale, or simply divided by 57.3 degrees/radian. Inverse trigonometric functions are found by reversing the process.

Many slide rules have S, T, and ST scales marked with degrees and minutes. So-called *decitrig* models use decimal fractions of degrees instead.

Logarithms and exponentials

Base-10 logarithms and exponentials are found using the L scale, which is linear. Some slide rules have a Ln scale, which is for base e.

The Ln scale was invented by an 11th grade student, Stephen B. Cohen, in 1958. The original intent was to allow the user to select an exponent x (in the range 0 to 2.3) on the Ln scale and read e^x on the C (or D) scale and e^{-x} on the CI (or DI) scale. Pickett, Inc. was given exclusive rights to the scale. Later, the inventor created a set of "marks" on the Ln scale to extend the range beyond the 2.3 limit, but Pickett never incorporated these marks on any of its slide rules.

Addition and subtraction

Slide rules are not typically used for addition and subtraction, but it is nevertheless possible to do so using two different techniques.

The first method to perform addition and subtraction on the C and D (or any comparable scales) requires converting the problem into one of division. For addition, the quotient of the two variables plus one times the divisor equals their sum:

$$x + y = \left(\frac{x}{y} + 1\right) y$$

For subtraction, the quotient of the two variables minus one times the divisor equals their difference:

$$x - y = \left(\frac{x}{y} - 1\right) y$$

This method is similar to the addition/subtraction technique used for high-speed electronic circuits with the logarithmic number system in specialized computer applications like the Gravity Pipe (GRAPE) supercomputer and hidden Markov models.

The second method utilizes a sliding linear L scale available on some models. Addition and subtraction are performed by sliding the cursor left (for subtraction) or right (for addition) then returning the slide to 0 to read the result.

Physical design

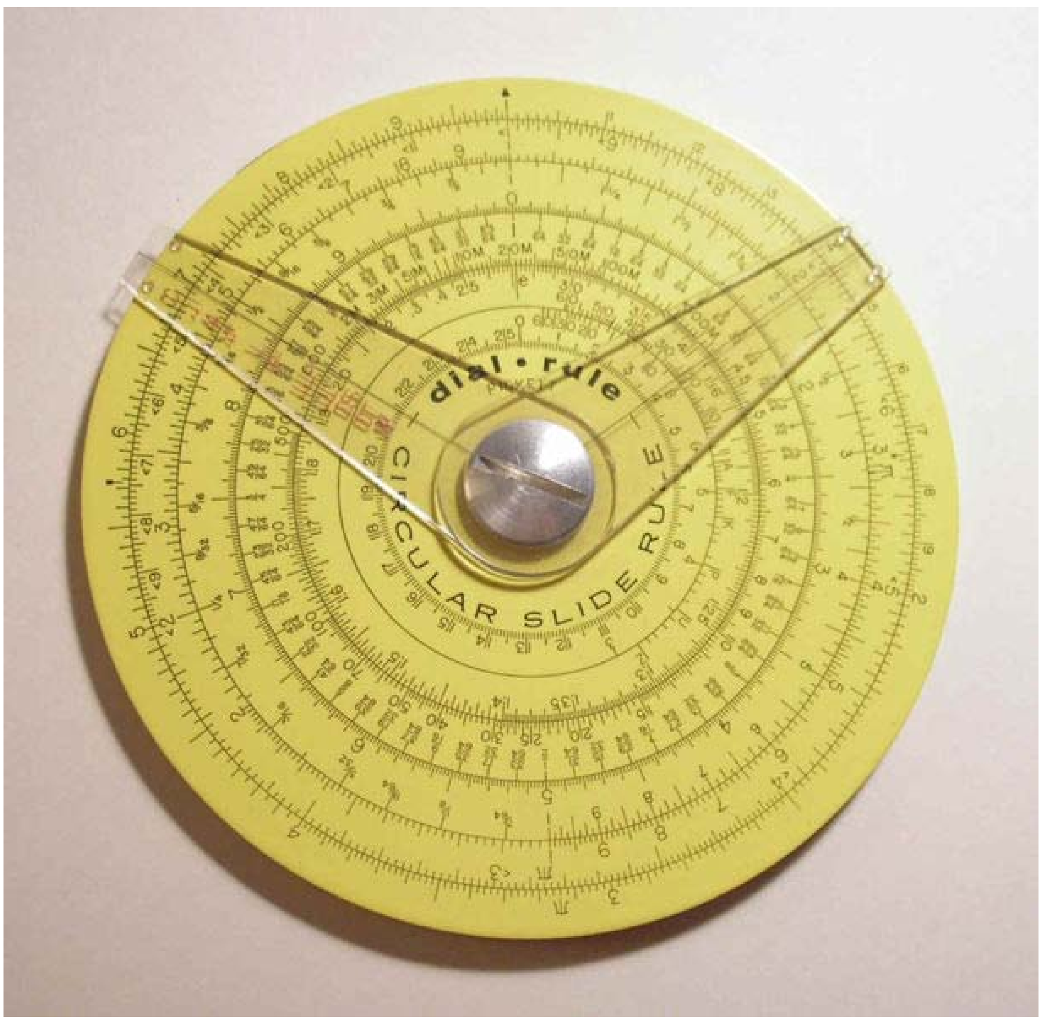
Standard linear rules

The width of the slide rule is quoted in terms of the nominal width of the scales. Scales on the most common "10-inch" models are actually 25 cm, as they were made to metric standards, though some rules offer slightly extended scales to simplify manipulation when a result overflowed. Pocket rules are typically 5 inches. Models a couple of metres wide were sold to be hung in classrooms for teaching purposes.

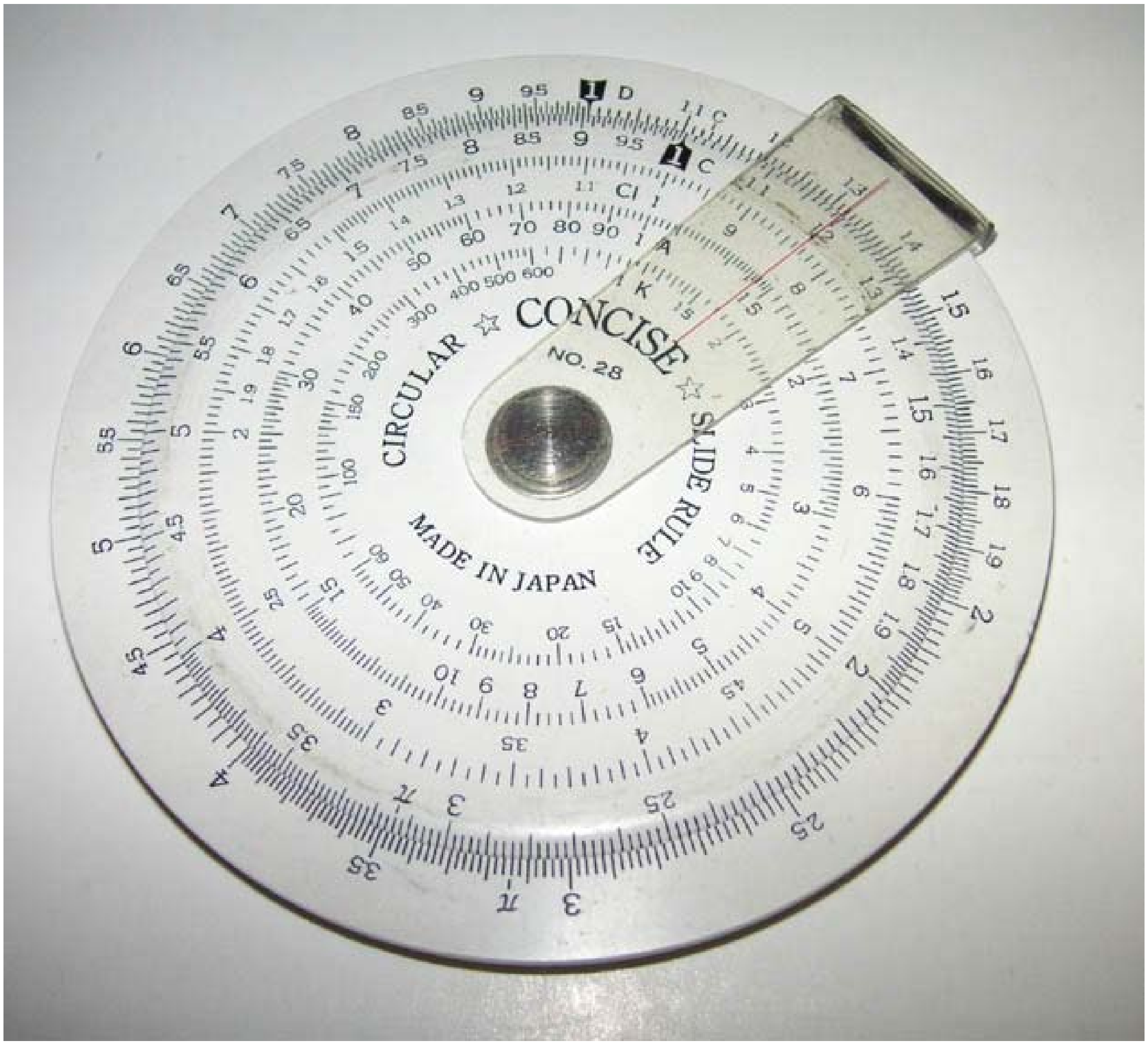
Typically the divisions mark a scale to a precision of two significant figures, and the user estimates the third figure. Some high-end slide rules have magnifier cursors that make the markings easier to see. Such cursors can effectively double the accuracy of readings, permitting a 10-inch slide rule to serve as well as a 20-inch.

Various other conveniences have been developed. Trigonometric scales are sometimes dual-labeled, in black and red, with complementary angles, the so-called "Darmstadt" style. Duplex slide rules often duplicate some of the scales on the back. Scales are often "split" to get higher accuracy.

Circular slide rules



Pickett circular slide rule with two cursors. (4.25in./10.9cm width) Reverse has additional scale and one cursor.



A simple circular slide rule, made by Concise Co., Ltd., Tokyo, Japan, with only inverse, square, and cubic scales. On the reverse is a handy list of 38 metric/imperial conversion factors.

Breitling Navitimer wristwatch with circular slide rule.

A Russian circular slide rule built like a pocket watch that works as single cursor slide rule since the two needles are ganged together.

Circular slide rules come in two basic types, one with two cursors (left), and another with a free dish and one cursor (right). The dual cursor versions perform multiplication and division by holding a fast angle between the cursors as they are rotated around the dial. The onefold cursor version operates more like the standard slide rule through the appropriate alignment of the scales.

The basic advantage of a circular slide rule is that the widest dimension of the tool was reduced by a factor of about 3 (i.e. by π). For example, a 10 cm circular would have a maximum precision equal to a 30 cm ordinary slide rule. Circular slide rules also eliminate "off-scale" calculations, because the scales were designed to "wrap around"; they never have to be reoriented when results are near 1.0—the rule is always on scale. However, for non-cyclical non-spiral scales such as S, T, and LL's, the scale width is narrowed to make room for end margins.

Circular slide rules are mechanically more rugged and smoother-moving, but their scale alignment precision is sensitive to the centering of a central pivot; a minute 0.1 mm off-centre of the pivot can result in a 0.2mm worst case alignment error. The pivot, however, does prevent scratching of the face and cursors. The highest accuracy scales are placed on the outer rings. Rather than "split" scales, high-end circular rules use spiral scales for more complex operations like log-of-log scales. One eight-inch premium circular rule had a 50-inch spiral log-log scale.

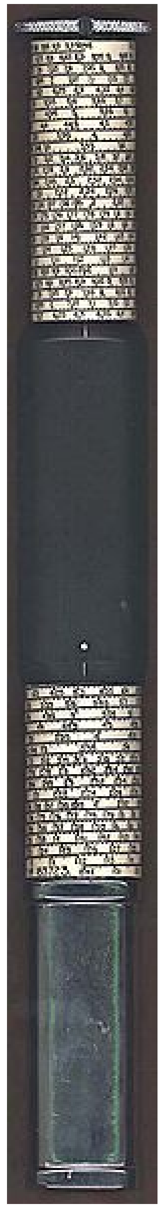
The main disadvantages of circular slide rules are the difficulty in locating figures along a dish, and limited number of scales. Another drawback of circular slide rules is that less-important scales are closer to the center, and have lower precisions. Most students learned slide rule use on the linear slide rules, and did not find reason to switch.

One slide rule remaining in daily use around the world is the E6B. This is a circular slide rule first created in the 1930s for aircraft pilots to help with dead reckoning. With the aid of scales printed on the frame it also helps with such miscellaneous tasks as converting time, distance, speed, and temperature values, compass errors, and calculating fuel use. The so-called "prayer wheel" is still available in flight shops, and remains widely used. While GPS has reduced the use of dead reckoning for aerial navigation, and handheld calculators have taken over many of its functions, the E6B remains widely used as a primary or backup device and the majority of flight schools demand that their students have some degree of proficiency in its use.

Proportion wheels are simple circular slide rules used in graphic design to broaden or slimmen images and photographs. Lining up the desired values on the emmer and inner wheels (which correspond to the original and desired sizes) will display the proportion as a percentage in a small window. They are not as common since the advent of computerized layout, but are still made and used.

In 1952, Swiss watch company Breitling introduced a pilot's wristwatch with an integrated circular slide rule specialized for flight calculations: the Breitling Navitimer. The Navitimer circular rule, referred to by Breitling as a "navigation computer", featured airspeed, rate/time of climb/descent, flight time, distance, and fuel consumption functions, as well as kilometer—nautical mile and gallon—liter fuel amount conversion functions.

Cylindrical slide rules



Otis King

There are two main types of cylindrical slide rules: those with helical scales such as the Fuller, the Otis King and the Bygrave slide rule, and those with bars, such as the Thacher and some Loga models. In either case, the advantage is a much longer scale, and hence potentially higher accuracy, than a straight or circular rule.

Materials

Traditionally slide rules were made out of hard wood such as mahogany or boxwood with cursors of glass and metal. At least one high precision instrument was made of steel.

In 1895, a Japanese firm, Hemmi, started to make slide rules from bamboo, which had the advantages of being dimensionally stable, strong and naturally self-lubricating. These bamboo slide rules were introduced in Sweden in September, 1933, and probably only a little earlier in Germany. Scales were made of celluloid or plastic. Later slide rules were made of plastic, or aluminium painted with plastic. Later cursors were acrylics or polycarbonates sliding on Teflon bearings.

All premium slide rules had numbers and scales engraved, and then filled with paint or other resin. Painted or imprinted slide rules were viewed as inferior because the markings could wear off. Nevertheless, Pickett, probably America's most successful slide rule company, made all printed scales. Premium slide rules included clever catches so the rule would not fall apart by accident, and bumpers to protect the scales and cursor from rubbing on tabletops. The recommended cleaning method for engraved markings is to scrub lightly with steel-wool. For painted slide rules, and the faint of heart, use diluted commercial window-cleaning fluid and a soft cloth.

History

William Oughtred (1575–1660), inventor of the circular slide rule.

The slide rule was invented around 1620–1630, shortly after John Napier's publication of the concept of the logarithm. Edmund Gunter of Oxford developed a calculating device with a single logarithmic scale, which, with additional measuring tools, could be used to multiply and divide. The first description of this scale was published in Paris in 1624 by Edmund Wingate (c.1593–1656), an English mathematician, in a book entitled *L'usage de la reigle de proportion en l'arithmetique & geometrie*. The book contains a double scale on one side of which is a logarithmic scale and on the other a tabular scale. In 1630, William Oughtred of Cambridge invented a circular slide rule, and in 1632 he combined two Gunter rules, held together with the hands, to make a device that is recognizably the modern slide rule. Like his contemporary at Cambridge, Isaac Newton, Oughtred taught his ideas privately to his students, but delayed in publishing them, and like Newton, he became involved in a vitriolic controversy over priority, with his one-time student Richard Delamain and the prior claims of Wingate. Oughtred's ideas were only made public in publications of his student William Forster in 1632 and 1653.

In 1677, Henry Coggeshall created a two-foot folding rule for timber measure, called the Coggeshall slide rule. His design and uses for the tool gave the slide rule purpose outside of mathematical inquiry.

In 1722, Warner introduced the two- and three-decade scales, and in 1755 Everard included an inverted scale; a slide rule containing all of these scales is usually known as a "polyphase" rule.

In 1815, Peter Mark Roget invented the log log slide rule, which included a scale displaying the logarithm of the logarithm. This allowed the user to directly perform calculations involving roots and exponents. This was especially useful for fractional powers.

In 1821, Nathaniel Bowditch, in the *American Practical Navigator*, described the use of a "sliding rule" which contained scales trigonometric functions on the fixed part and a line of log-sines and log-tans on the slider. This device was used to solve navigation problems.

Modern form

The more modern form was created in 1859 by French artillery lieutenant Amédée Mannheim, "who was fortunate in having his rule made by a firm of national reputation and in having it adopted by the French Artillery." It was around that time, as engineering became a recognized professional activity, that slide rules came into wide use in Europe. They did not become common in the United States until 1881, when Edwin Thacher introduced a cylindrical rule there. The duplex rule was invented by William Cox in 1891, and was produced by Keuffel and Esser Co. of New York.

Astronomical work also required fine computations, and in the 19th century Germany a steel slide rule about 2 meters long was used at one observatory. It had a microscope attached, giving it accuracy to six decimal places.

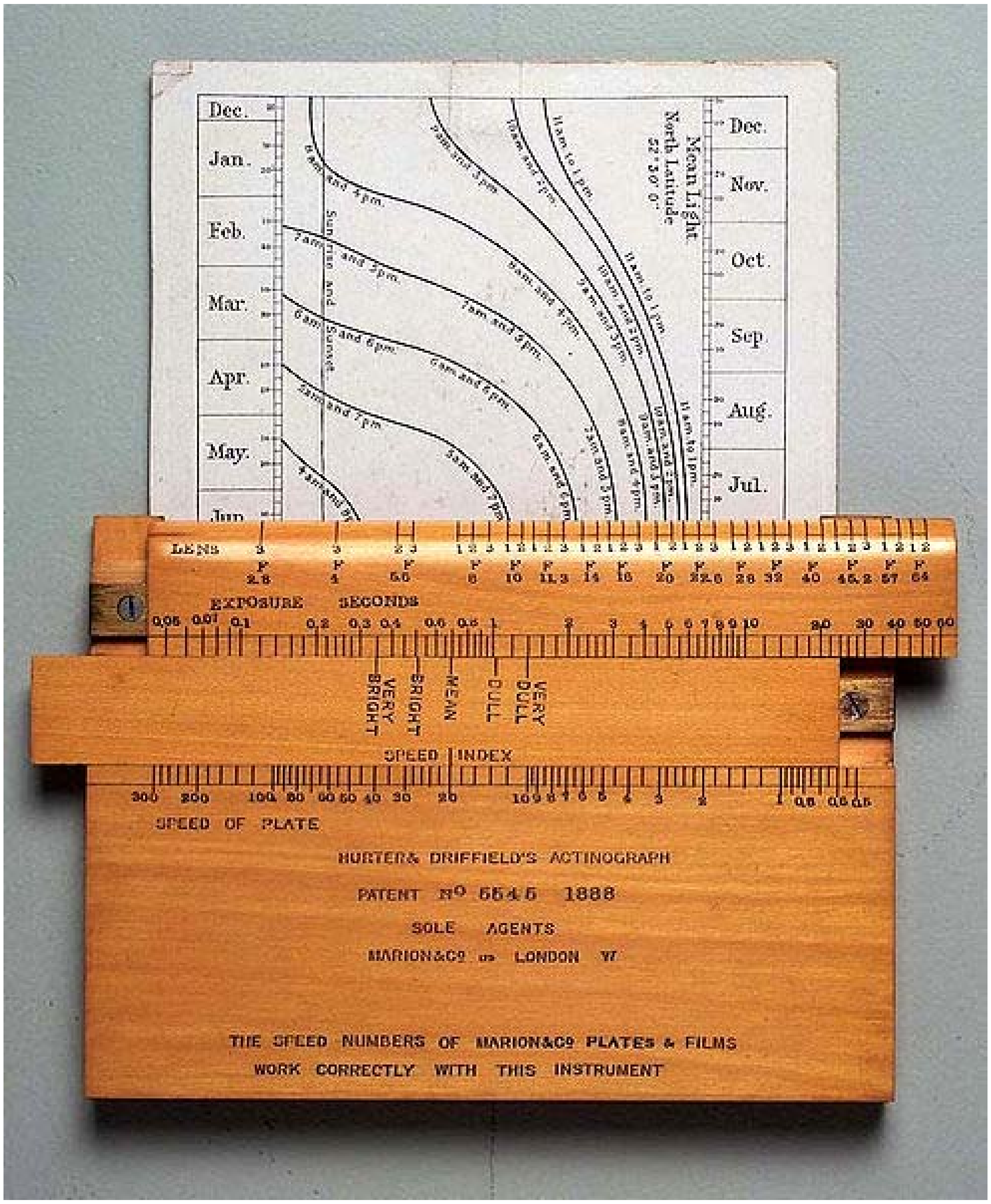
Engineer using a slide rule. Note mechanical calculator in background.

Throughout the 1950s and 1960s the slide rule was the symbol of the engineer's profession (in the same way that the stethoscope symbolizes the medical profession). German rocket scientist Wernher von Braun brought two 1930s vintage *Nestler* slide rules with him when he moved to the U.S. after World War II to work on the American space program. Throughout his life he never used any other pocket calculating devices; slide rules served him perfectly well for making quick estimates of rocket design parameters and other figures. Aluminium Pickett-brand slide rules were carried on five Apollo space missions, including to the moon, according to advertising on Pickett's N600 slide rule boxes.

Some engineering students and engineers carried ten-inch slide rules in belt holsters, and even into the mid 1970s this was a common sight on campuses. Students also might keep a ten- or twenty-inch rule for precision work at home or the office while carrying a five-inch pocket slide rule around with them.

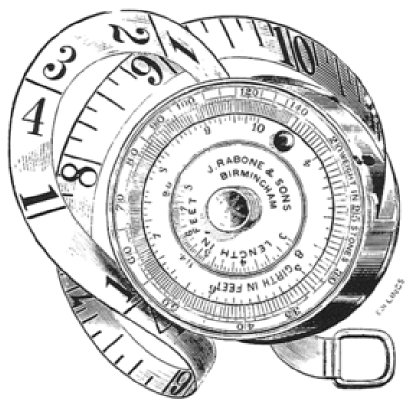
In 2004, education researchers David B. Sher and Dean C. Nataro conceived a new type of slide rule based on *prosthaphaeresis*, an algorithm for rapidly computing products that predates logarithms. There has been little practical interest in constructing one beyond the initial prototype, however.

Specialized calculators



Hurter and Driffield's actinograph

Slide rules have often been specialized to varying degrees for their field of use, such as excise, proof calculation, engineering, navigation, etc., but some slide rules are extremely specialized for very narrow applications. For example, the John Rabone & Sons 1892 catalog lists a "Measuring Tape and Cattle Gauge", a device to estimate the weight of a cow from its measurements.



Measuring Tape and Cattle Gauge.

Spring Measuring Tape, 12 feet long, Marked English feet, inches, and hands; and Gauge for ascertaining weight of Cattle by measure, including full directions for use.

No. 340	Brass Case, Spring Stop, Linen Tape	... ...	7/- each.
No. 341	,, ,, Steel Tape	... ...	9/6 ,,

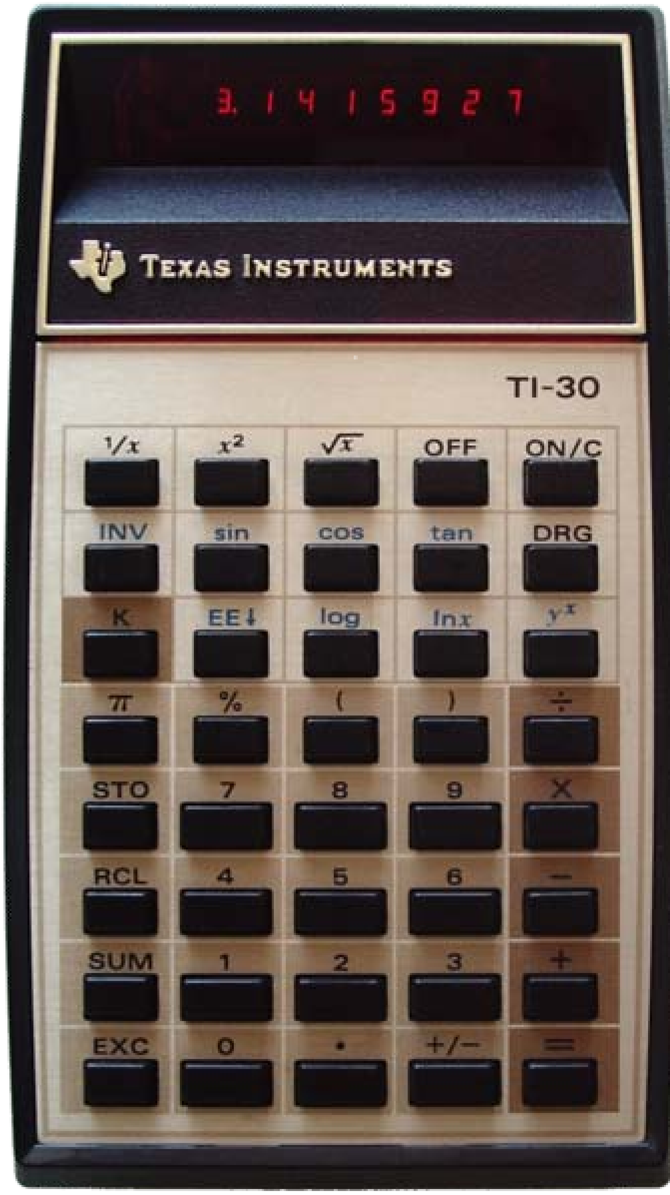
John Rabone & Sons 1892 Cattle Gauge

There were many specialized slide rules for photographic applications; for example, the actinograph of Hurter and Driffield was a two-slide boxwood, brass, and cardboard device for estimating exposure from time of day, time of year, and latitude.

Specialized slide rules were invented for various forms of engineering, business and banking. These often had common calculations directly expressed as special scales, for example loan calculations, optimal purchase quantities, or particular engineering equations. For example, the Fisher Controls company distributed a customized slide rule adapted to solving the equations used for selecting the proper size of industrial flow control valves.

In World War II, bombardiers and navigators who required quick calculations often used specialized slide rules. One office of the U.S. Navy actually designed a generic slide rule "chassis" with an aluminium body and plastic cursor into which celluloid cards (printed on both sides) could be placed for special calculations. The process was invented to calculate range, fuel use and altitude for aircraft, and then adapted to many other purposes.

Decline



TI-30

The importance of the slide rule began to diminish as electronic computers, a new but very scarce resource in the 1950s, became widely available to technical workers during the 1960s. The introduction of Fortran in 1957 made computers practical for solving modest size mathematical problems. IBM introduced a series of more affordable computers, the IBM 650 (1954), IBM 1620 (1959), IBM 1130 (1965) addressed to the science and engineering market. The BASIC programming language (1964) made it easy for students to use computers. The DEC PDP-8 minicomputer was introduced in 1965.

Computers also changed the nature of calculation. With slide rules, there was a great emphasis on working the algebra to get expressions into the most computable form. Users of slide rules would simply approximate or drop small terms to simplify the calculation. Fortran allowed complicated formulas to be typed in from textbooks without the effort of reformulation. Numerical integration was often easier than trying to find closed form solutions for difficult problems. The young engineer asking for computer time to solve a problem that could have been done by a few swipes on the slide rule became a humorous cliché. Many computer centers had a framed slide rule hung on a wall with the note "In case of emergency, break glass."

Another step toward the replacement of slide rules with electronics was the development of electronic calculators for scientific and engineering use. The first included the Wang Laboratories LOCI-2, introduced in 1965, which used logarithms for multiplication and division and the Hewlett-Packard HP-9100, introduced in 1968. The HP-9100 had trigonometric functions (sin, cos, tan) in addition to exponentials and logarithms. It used the CORDIC (coordinate rotation digital computer) algorithm, which allows for calculation of trigonometric functions using only shift and add operations. This method facilitated the development of ever smaller scientific calculators.

The era of the slide rule ended with the launch of pocket-sized scientific calculators, of which the 1972 Hewlett-Packard HP-35 was the first. Such calculators became known as "slide rule" calculators, since they could perform most, or all, of the functions of a slide rule. Introduced at US$395, even this was considered expensive for most students. But by 1975, basic four-function electronic calculators could be purchased for less than $50. By 1976 the TI-30 offered a scientific calculator for less than $25. After this time, the market for slide rules dwindled quickly as small scientific calculators became affordable.

Advantages

- The spatial, manual operation of slide rules cultivates in the user an intuition for numerical relationships and scale that people who have used only digital calculators often lack.Since users must explicitly note the order of magnitude at each step in order to interpret the results, they are less likely to make wildly wrong errors; users are forced to use common sense and an understanding of the subject as they calculate. Since order of magnitude gets the greatest prominence when using a slide rule, and precision is limited only to the few digits that are normally useful, users are less likely to make errors of false precision.
- When performing a sequence of multiplications or divisions by the same number, the answer can often be determined by merely glancing at the slide rule without any manipulation. This can be especially useful when calculating percentages, e.g., for test scores, or when comparing prices, e.g., in dollars per kilogram. Multiple speed-time-distance calculations can be performed hands-free at a glance with a slide rule.
- Other useful constants such as pounds to kilograms can be easily marked on the rule and used directly in calculations.
- A slide rule does not depend on electricity or batteries.
- They are extraordinarily useful for concealing the answers to exams within the moveable section.

- The principle of operation of a slide-rule can be demonstrated with a pair of hand-made paper scales.

For many of these reasons slide rules are still commonly used in aviation, particularly for smaller planes. They are only being replaced by integrated, special purpose and expensive flight computers, and not general purpose calculators.

Disadvantages

- The typical precision of a slide rule is about three significant digits. A typical pocket calculator displays results to seven or more digits.
- A slide rule requires the user to mentally calculate the order of magnitude of the results. For example, 1.5 × 30 (which equals 45) will show the same result as 1,500,000 × 0.03 (which equals 45,000). This forces the user to keep track of magnitude in short-term memory (which is error-prone), keep notes (which is cumbersome) or reason about it in every step (which distracts from the other calculation requirements).
- Errors may arise from mechanical imprecision in slide rules that are warped by heat or use or that were poorly constructed.

Finding and collecting slide rules

Faber Castell slide rule from flea market

There are still people who prefer a slide rule over an electronic calculator as a practical computing device. Many others keep their old slide rules out of a sense of nostalgia, or collect slide rules as a hobby.

A popular collectible model is the Keuffel & Esser *Deci-Lon*, a premium scientific and engineering slide rule available both in a ten-inch "regular" (*Deci-Lon 10*) and a five-inch "pocket" (*Deci-Lon 5*) variant. Another prized American model is the eight-inch Scientific Instruments circular rule. Of European rules, Faber-Castell's high-end models are the most popular among collectors.

Although there is a large supply of slide rules circulating on the market, specimens in good condition tend to be surprisingly expensive. Many rules found for sale on online auction sites are damaged or have missing parts, and the seller may not know enough to supply the relevant information. Replacement parts are scarce, and therefore expensive, and are generally only available for separate purchase on individual collectors' web sites. The Keuffel and Esser rules from the period up to about 1950 are particularly problematic, because the end-pieces on the cursors, made of celluloid, tend to break down chemically over time.

There are still a handful of sources for brand new slide rules. The Concise Company of Tokyo, which began as a manufacturer of circular slide rules in July 1954, continues to make and sell them today. And in September 2009, on-line retailer ThinkGeek introduced its own brand of straight slide rules, which they describe as "faithful replica[s]" that are "individually hand tooled" due to a stated lack of any existing manufacturers. The E6B circular slide rule used by pilots has been in continuous production and remains available in a variety of models. Proportion wheels are still used in graphic design.