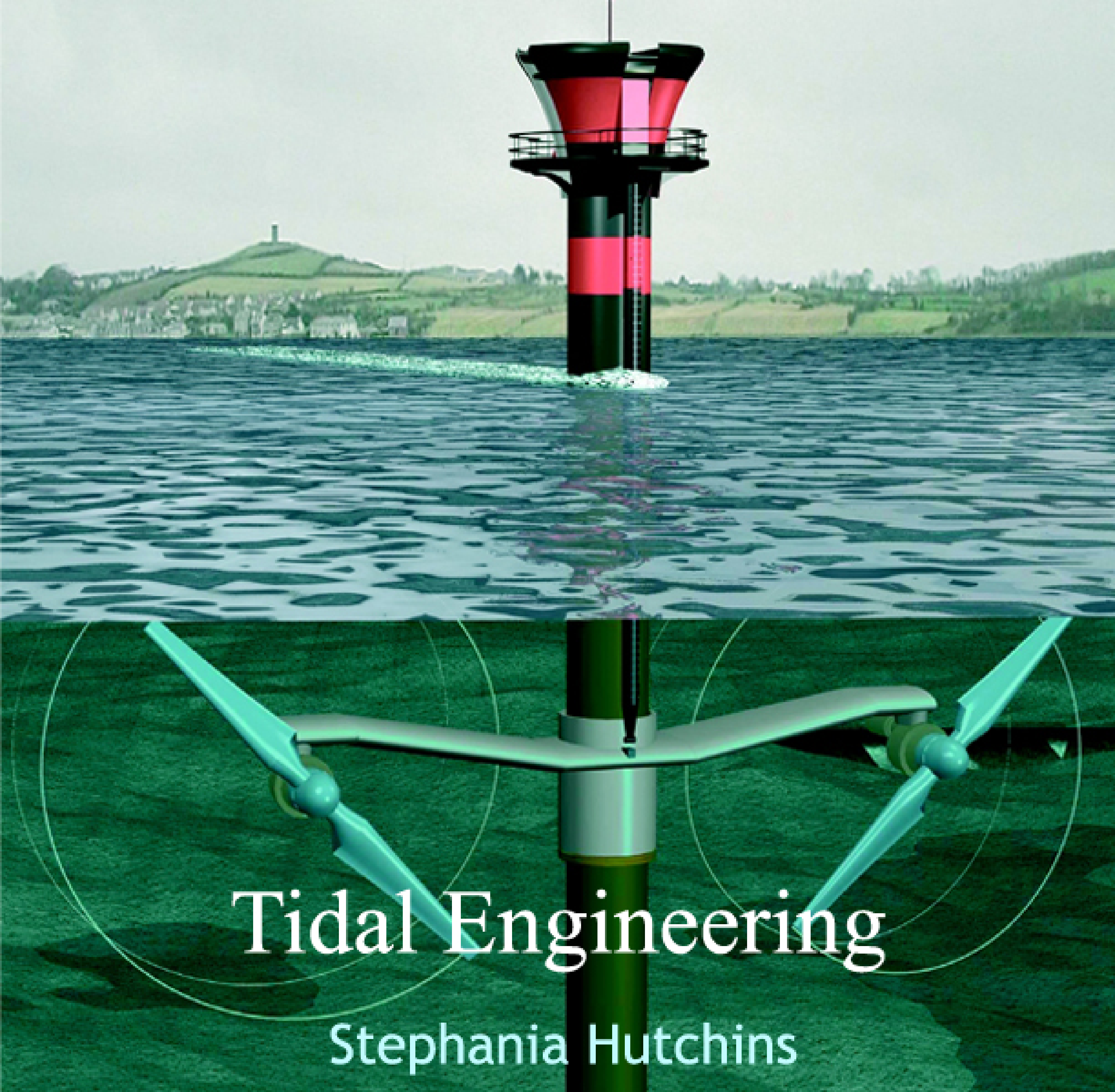

Tidal Engineering

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Chapter 1

Internal Tide

Internal tides are generated as the surface tides move stratified water up and down sloping topography, which produces a wave in the ocean interior. So internal tides are internal waves at a tidal frequency. The other major source of internal waves is the wind which produces internal waves near the inertial frequency. When a small water parcel is displaced from its equilibrium position, it will return either downwards due to gravity or upwards due to buoyancy. The water parcel will overshoot its original equilibrium position and this disturbance will set off an internal gravity wave. Munk (1981) notes, "Gravity waves in the ocean's interior are as common as waves at the sea surface-perhaps even more so, for no one has ever reported an interior calm."

Simple explanation

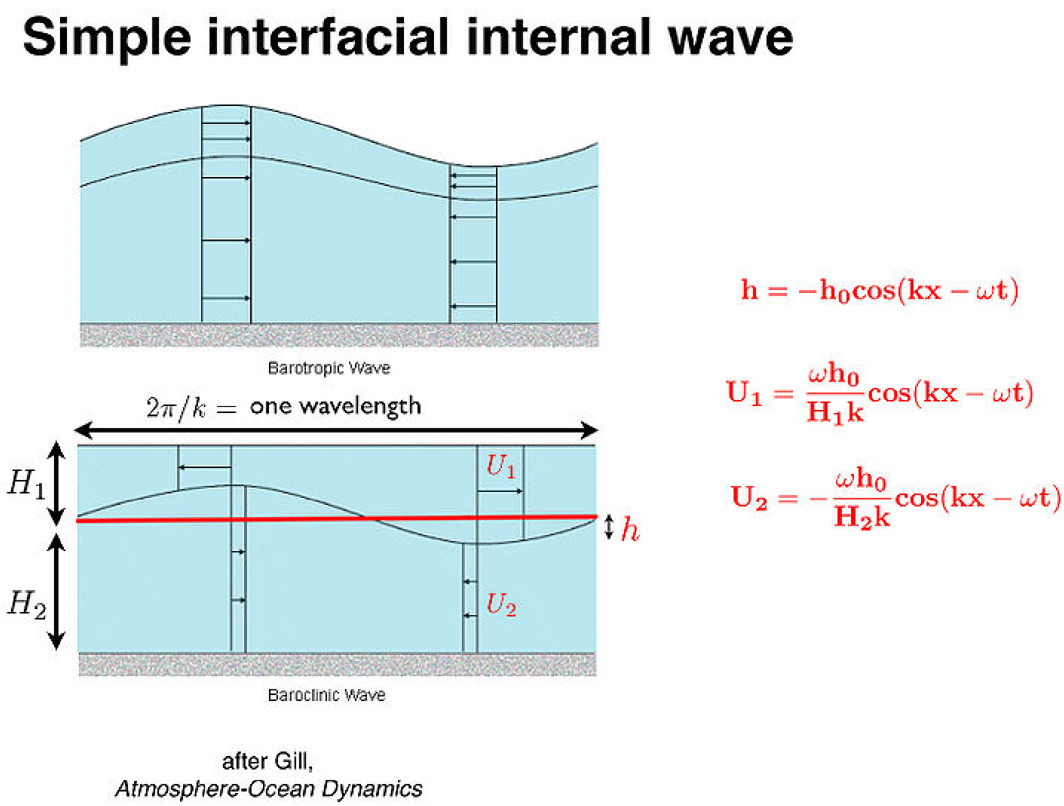


Figure 1: Water parcels in the whole water column move together with the surface tide (top), while shallow and deep waters move in opposite directions in an internal tide (bottom). The surface displacement and interface displacement are the same for a surface wave (top), while for an internal wave the surface displacements are very small, while the interface displacements are large (bottom). This figure is a modified version of one appearing in Gill (1982).

The surface tide propagates as a wave, in which water parcels in the whole water column oscillate in the same direction at a given phase (i.e, in the trough or at the crest, Fig. 1, top). At the simplest level, an internal wave can be thought of as an interfacial wave (Fig. 1, bottom). If there are two levels in the ocean, such as a warm surface layer and cold deep layer separated by a thermocline,then motions on the interface are possible. The interface movement is large compared to surface movement. The restoring force for internal waves and tides is still gravity but its effect is reduced because the densities of the 2 layers are relatively similar compared to the large density difference at the air-sea interface. Thus larger displacements are possible inside the ocean than at the sea surface.

Tides occur mainly at diurnal and semidiurnal periods. The principal lunar semidiurnal constituent is known as M2 and generally has the largest amplitudes.

Where are internal tides found?

The largest internal tides are generated at steep, midocean topography such as the Hawaiian Ridge, Tahiti, the Macquarie Ridge, and submarine ridges in the Luzon Strait. Continental slopes such as the Australian North West Shelf also generate large internal tides. These internal tide may propagate onshore and dissipate much like surface waves. Or internal tides may propagate away from the topography into the open ocean. For tall, steep, midocean topography, such as the Hawaiian Ridge, it is estimated that about 85% of the energy in the internal tide propagates away into the deep ocean with about 15% of its energy being lost within about 50 km of the generation site. The lost energy contributes to turbulence and mixing near the generation sites. It is not clear where the energy that leaves the generation site is dissipated, but there are 3 possible processes: 1) the internal tides scatter and/or break at distant midocean topography, 2) interactions with other internal waves remove energy from the internal tide, or 3) the internal tides shoal and break on continental shelves.

Where do internal tides go and what happens to them along the way?

Briscoe (1975) succinctly noted that “We cannot yet answer satisfactorily the questions: ‘where does the internal wave energy come from, where does it go, and what happens to it along the way?’” Although technological advances in instrumentation and modeling have produced greater knowledge of internal tide and near-inertial wave generation, Garrett and Kunze (2007) observed 33 years later that “The fate of the radiated [large-scale internal tides] is still uncertain. They may scatter into [smaller scale waves] on further encounter with islands or the rough seafloor , or transfer their energy to smaller-scale internal waves in the ocean interior ” or “break on distant continental slopes ”. It is now known that most of the internal tide energy generated at tall, steep midocean topography radiates away as large-scale internal waves. This radiated internal tide energy is one of the main sources of energy into the deep ocean, roughly half of the wind energy input . Broader interest in internal tides is spurred by their impact on the magnitude and spatial inhomogeneity of mixing, which in turn has first order effect on the meridional overturning circulation .

The internal tidal energy in one tidal period going through an area perpendicular to the direction of propagation is called the energy flux and is measured in Watts/m^2. The energy flux at one point can be summed over depth- this is the depth-integrated energy flux and is measured in Watts/m. The Hawaiian Ridge produces depth-integrated energy fluxes as large as 10 kW/m. The longest wavelength waves are the fastest and thus carry most of the energy flux. Near Hawaii, the typical wavelength of the longest internal tide is about 150 km while the next longest is about 75 km. These waves are called mode 1 and mode 2, respectively. Although Fig. 1 shows there is no sea surface expression of the internal tide, there actually is a displacement of a few centimeters. These sea surface expressions of the internal tide at different wavelengths can be detected with the Topex/Poseidon or Jason-1 satellites (Fig. 2). Near 15 N, 175 W on the Line Islands Ridge, the mode-1 internal tides scatter off the topography, possibly creating turbulence and mixing, and producing smaller wavelength mode 2 internal tides.

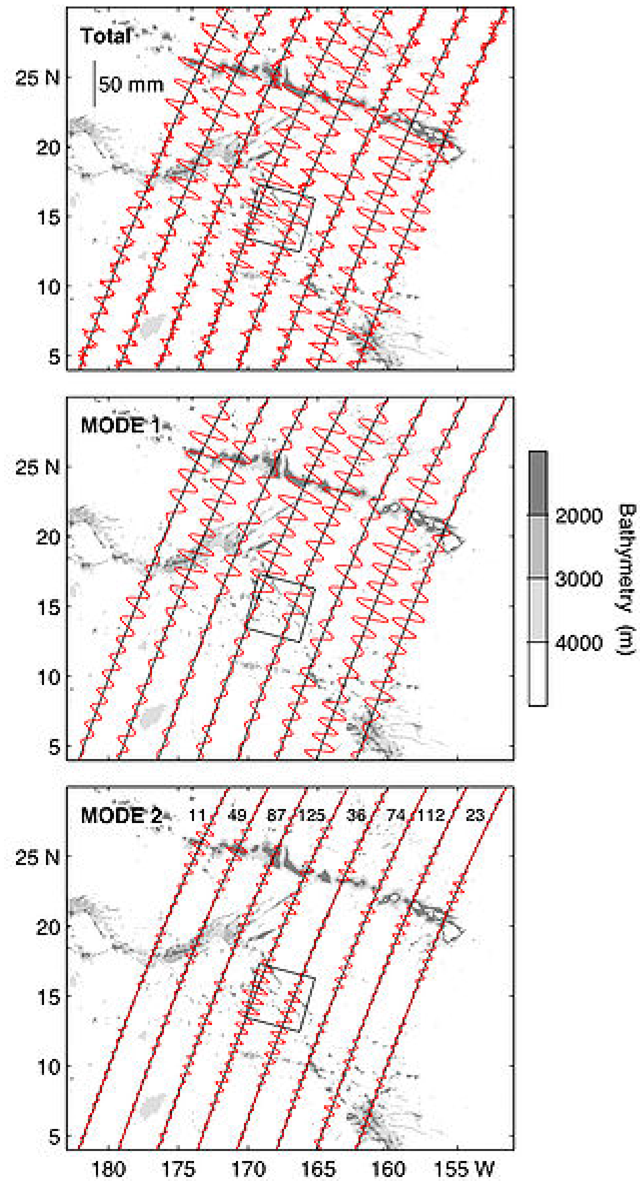


Figure 2: The internal tide sea surface elevation that is in phase with the surface tide (i.e., crests occur in a certain spot at a certain time that are both the same relative to the surface tide) can be detected by satellite (top). (The satellite track is repeated about every 10 days and so M2 tidal signals are shifted to longer periods due to aliasing.) The longest internal tide wavelengths are about 150 km near Hawaii and the next longest waves are about 75 km long. The surface displacements due to the internal tide are plotted as wiggly red lines with amplitudes plotted perpendicular to the satellite groundtracks (black lines). Figure is adapted from Johnston et al. (2003).

The inescapable conclusion is that energy is lost from the surface tide to the internal tide at midocean topography and continental shelves, but the energy in the internal tide is not necessarily lost in the same place. Internal tides may propagate thousands of kilometers or more before breaking and mixing the abyssal ocean.

Their importance to abyssal mixing and the meridional overturning circulation

The importance of internal tides and internal waves in general relates to their breaking, energy dissipation, and mixing of the deep ocean. If there were no mixing in the ocean, the deep ocean would be a cold stagnant pool with a thin warm surface layer. While the meridional overturning circulation (also referred to as the thermohaline circulation) redistributes about 2 PW of heat from the tropics to polar regions, the energy source for this flow is the interior mixing which is comparatively much smaller- about 2 TW. Sandstrom (1908) showed a fluid which is both heated and cooled at its surface cannot develop a deep overturning circulation. Most global models have incorporated uniform mixing throughout the ocean because they do not include or resolve internal tidal flows.

However, models are now beginning to include spatially variable mixing related to internal tides and the rough topography where they are generated and distant topography where they may break. Wunsch and Ferrari (2004) describe the global impact of spatially inhomogeneous mixing near midocean topography: “A number of lines of evidence, none complete, suggest that the oceanic general circulation, far from being a heat engine, is almost wholly governed by the forcing of the wind field and secondarily by deep water tides... The now inescapable conclusion that over most of the ocean significant ‘vertical’ mixing is confined to topographically complex boundary areas implies a potentially radically different interior circulation than is possible with uniform mixing. Whether ocean circulation models... neither explicitly accounting for the energy input into the system nor providing for spatial variability in the mixing, have any physical relevance under changed climate conditions is at issue.” There is a limited understanding of “the sources controlling the internal wave energy in the ocean and the rate at which it is dissipated” and are only now developing some “parameterizations of the mixing generated by the interaction of internal waves, mesoscale eddies, high-frequency barotropic fluctuations, and other motions over sloping topography.”

Internal tides at the beach

Internal tides may also dissipate on continental slopes and shelves or even reach within 100 m of the beach (Fig. 3). Internal tides bring pulses of cold water shoreward and produce large vertical temperature differences. When surface waves break, the cold water is mixed upwards making the water cold for surfers, swimmers, and other beachgoers. Surface waters in the surf zone can change by about 10^{o}C in about an hour.

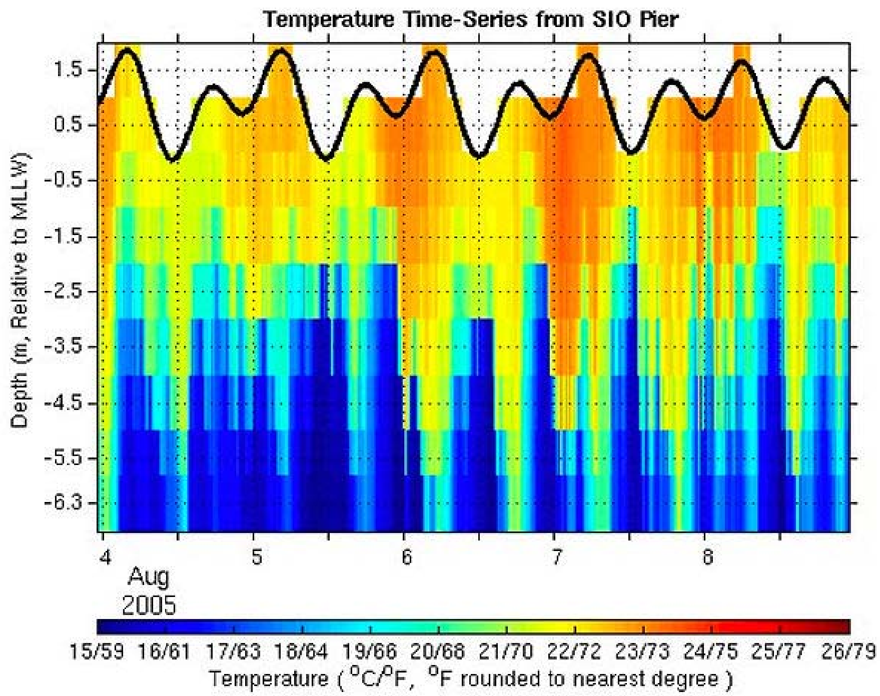


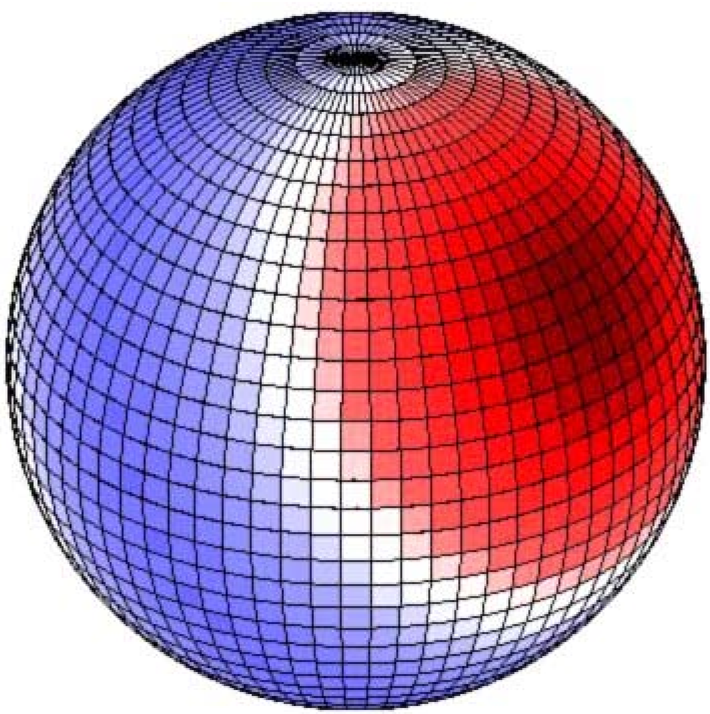
Figure 3: The internal tide produces large vertical differences in temperature at the research pier at the Scripps Institution of Oceanography. The black line shows the surface tide elevation relative to mean lower low water (MLLW). Figure provided by Eric Terrill, Scripps Institution of Oceanography with funding from the U.S. Office of Naval Research

Chapter 2

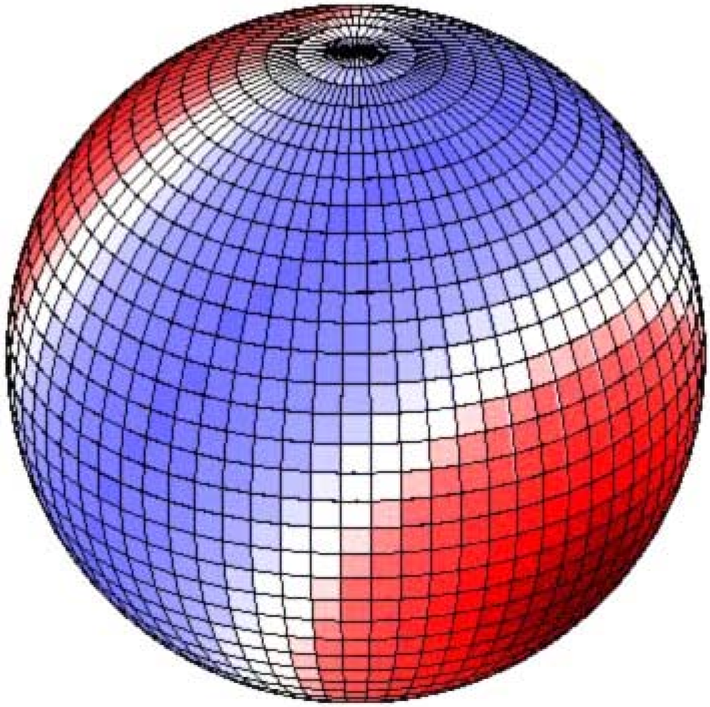
Earth Tide

Earth tide is the sub-meter motion of the Earth of about 12 hours or longer caused by Moon and Sun gravitation, also called *body tide* which is the largest contribution globally. The largest body tide contribution is from the semidiurnal constituents, but there are also significant diurnal constituents. There also semi-annual and fortnightly contributions due to the axial tilt. The use of the word *tide* is by analogy, and although the forcing is quite similar, the responses are quite different.

Tidal forcing



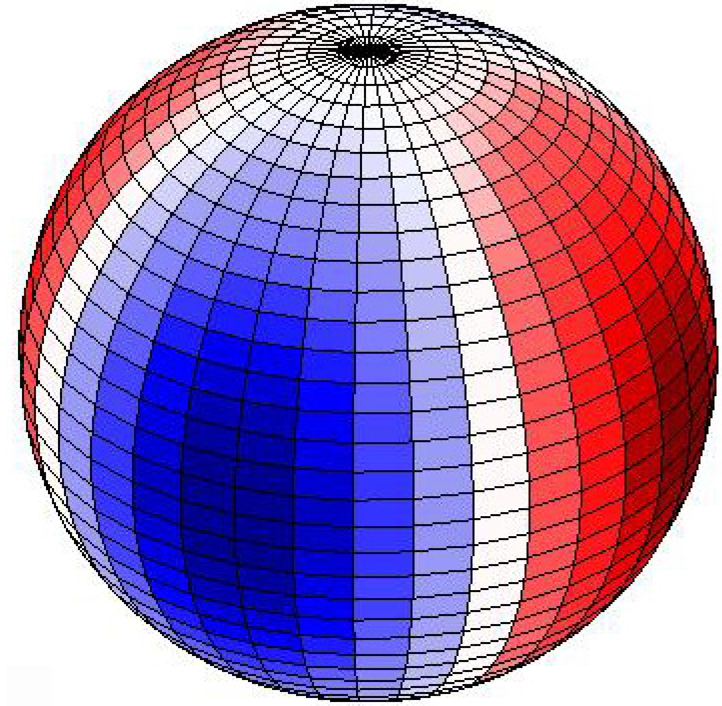
A. Lunar tidal forcing: this depicts the Moon directly over 30° N (or 30° S) viewed from above the Northern Hemisphere.



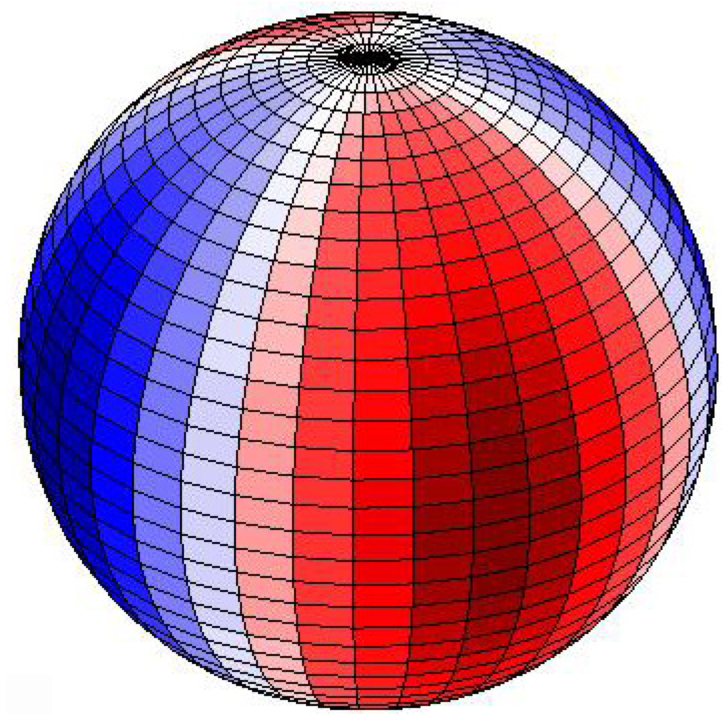
B. This view shows same forcing from 180° from view *A*. Viewed from above the Northern Hemisphere. Red up, blue down.

The larger of the periodic gravitational forcings is from the Moon but that of the Sun is also important. The images here show lunar tidal forcing when the Moon appears directly over 30° N (or 30° S). This pattern remains fixed with the red area directed toward (or directly away from) the Moon. Red indicates upward pull, blue downward. If, for example the Moon is directly over 90° W (or 90° E), the center of the red areas are centered on the western northern hemisphere, on upper right. Red up, blue down. If for example the Moon is directly over 90° W (90° E), the center of the red area is 30° N, 90° W and 30° S, 90° E, and the center of the bluish band follows the great circle equidistant from those points. At 30° latitude a strong peak occurs once per lunar day, giving significant diurnal forcing at that latitude. Along the equator two equally sized peaks (and depressions) are equally sized, giving semi-diurnal forcing there.

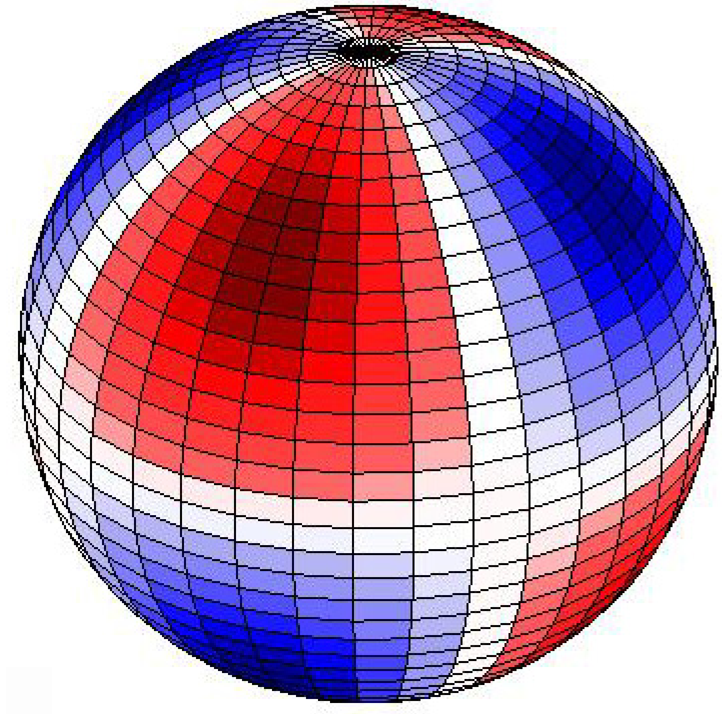
Body tide



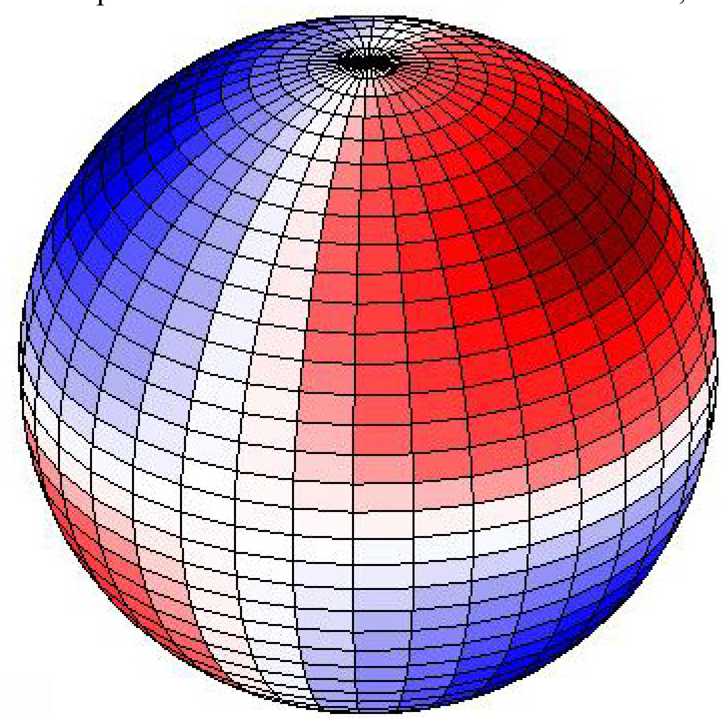
Vertical displacements of sectorial movement. Red up, blue down.



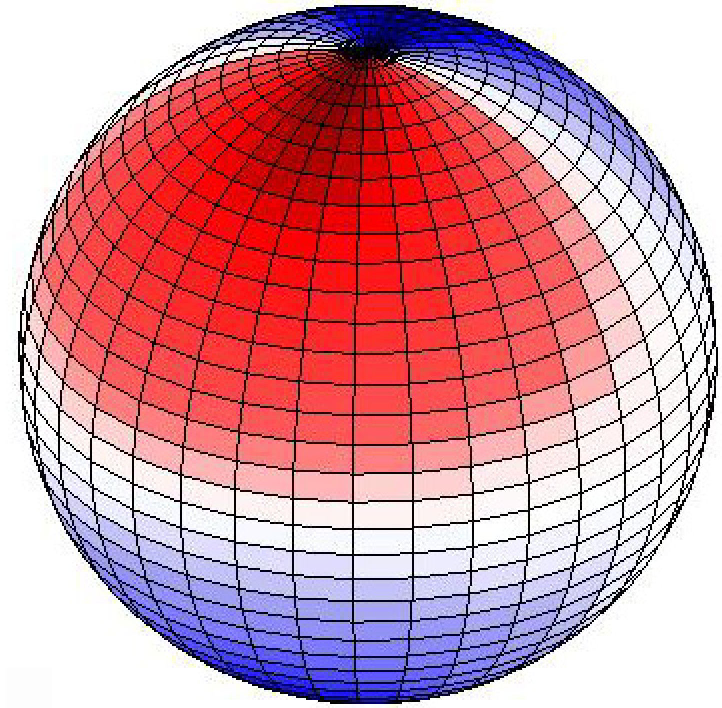
East-west displacements of sectorial movement. Red east, blue west.



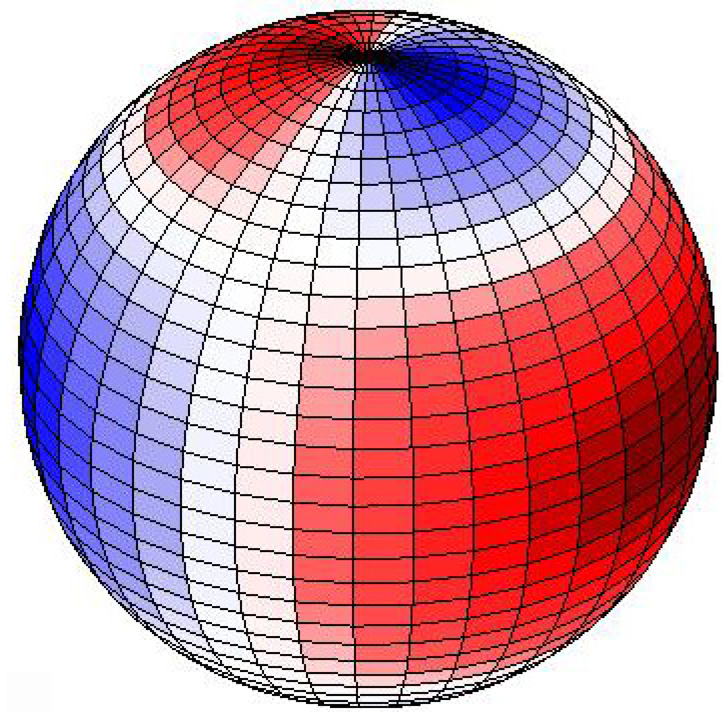
North-south displacements of sectorial movement. Red north, blue south.



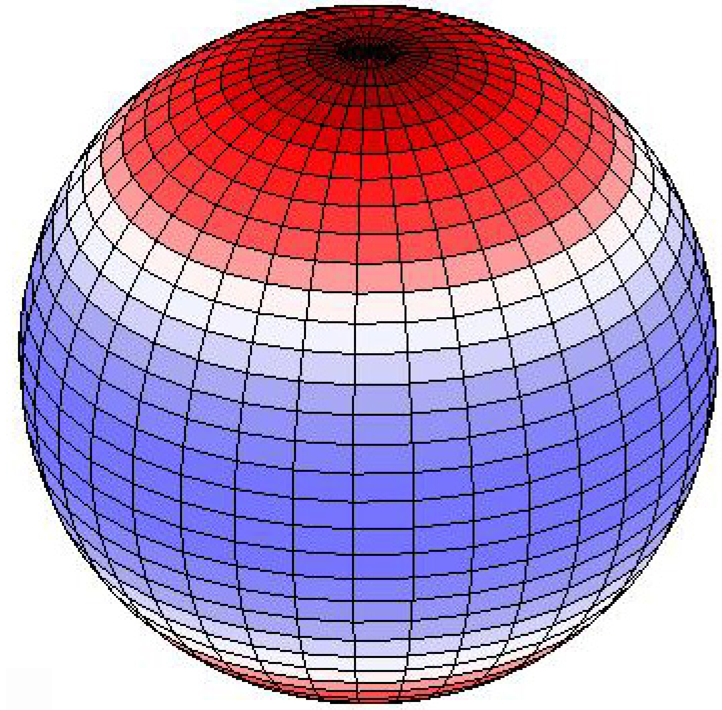
Vertical displacements of tessearal movement. Red up, blue down.



East-West displacements of tessearal movement. Red east, blue west.



North-South displacements of tessearal movement. Red north, blue south.



Vertical displacements of zonal movement. Red up, blue down.

The Earth tide encompasses the entire body of the Earth and is unhindered by the thin crust and land masses of the surface, on scales that make the rigidity of rock irrelevant. Ocean tides are a consequence of the resonance of the same driving forces with water movement periods in ocean basins accumulated over many days, so that their amplitude and timing are quite different and vary over short distances of just a few hundred kilometres. The oscillation periods of the earth as a whole are not near the astronomical periods, so its flexing is due to the forces of the moment.

The tide components with a period near twelve hours have a lunar amplitude (earth bulge/depression distances) that are a little more than twice the height of the solar amplitudes, as tabulated below. At new and full moon, the Sun and the Moon are aligned, and the lunar and the solar tidal maxima and minima (bulges and depressions) add together for the greatest tidal range at particular latitudes. At first- and third-quarter phases of the moon, lunar and solar tides are in opposition, and the tidal range is at a minimum. The semi-diurnal tides go through one full cycle (a high and low tide) about once every 12 hours and one full cycle of maximum height (a spring and neap tide) about once every 14 days.

The classical theory of Earth tides first became established in 1905. , primarily to explain nutations, but are also used in Earth rotation predictions. The semi-diurnal tide (one maximum every 12 or so hours) is primarily Lunar (only S_2 is purely solar) and gives rise to *sectorial* deformations which rise and fall at the same time along the same longitude. Sectorial variations of vertical and east-west displacements are maximum at the equator and vanish at the poles. There are two cycles along each latitude, the bulges opposite one another, and the depressions similarly opposed. The diurnal tide is Lunisolar, and gives rise to *tesseral* deformations. The vertical and east-west movement is maximum at 45° latitude and is zero on the equator and at the poles. Tesseral variation have one cycle per latitude, one bulge and one depression; the bulge are opposed (antipodal), that is to say the western part of the northern hemisphere and the eastern part of the southern hemisphere, for example, and similarly the depressions are opposed,the western part of the northern hemisphere and the western part of the southern hemisphere, in this case. Finally, fortnightly and semi-annual tides have 'zonal'deformations (constant along a circle of latitude), as the Moon or Sun gravitation is directed alternately away from the Northern and Southern hemispheres due to tilt. There is zero vertical displacement at 35°16' latitude.

Since these displacements affect the vertical direction east-west and north-south variations are often tabulated in milliarc seconds for astronomical use. The vertical displacement is frequent tabulated in µgal, since the gradient of gravity is location dependent so that the distance conversion is only approximately 3 µgal per cm

Other Earth tide contributors

In coastal areas because the ocean tide is quite out of step with the earth tide, at high ocean tide there is an excess (or at low tide a deficit) of water about what would be the gravitational equilibrium level and the adjacent ground falls (or rises) in response to the resulting differences in weight. Displacements caused by ocean tidal loading can exceed the displacements due to the earth body tide. Sensitive instruments far inland often have to make similar corrections. Atmospheric loading and storm events may also be measurable, though the masses in movement are less weighty.

Tidal constituents

Principal body tide constituents. The amplitudes may vary from those listed within several per cent.

Semi-diurnal			
Tidal constituent	**Period**	**Vertical amplitude (mm)**	**Horizontal amplitude(mm)**
M_2	12.421 hr	384.83	53.84
S_2 (Solar Semi-diurnal)	12.000 hr	179.05	25.05
N_2	12.658 hr	73.69	10.31
K_2	11.967 hr	48.72	6.82
Diurnal			
Tidal constituent	**Period**	**Vertical amplitude**	**Horizontal**

		(mm)	amplitude(mm)
K_1	23.934 hr	191.78	32.01
O_1	25.819 hr	158.11	22.05
P_1	24.066 hr	70.88	10.36
φ_1	23.804 hr	3.44	0.43
ψ_1	23.869 hr	2.72	0.21
S_1 (Solar diurnal)	24.000 hr	1.65	0.25
Long term			
Tidal constituent	**Period**	**Vertical amplitude (mm)**	**Horizontal amplitude(mm)**
M_f	13.661 days	40.36	5.59
M_m (Moon Monthly)	27.555 days	21.33	2.96
S_{sa} (Solar semi-Annual)	0.50000 yr	18.79	2.60
lunar node	18.613 yr	16.92	2.34
S_a (Solar Annual)	1.0000 yr	2.97	0.41

Earth tide effects

Volcanologists use the regular, predictable Earth tide movements to calibrate and test sensitive volcano deformation monitoring instruments. The tides may also trigger volcanic events. Seismologists have determined that microseismic events are correlated to tidal variations in Central Asia (north of the Himalayas). The semidiurnal amplitude of terrestrial tides can reach about 55 cm at the equator which is important in GPS calibration and VLBI measurements. Also to make precise astronomical angular measurements requires knowledge of the Earth's rate of rotation and nutation, both of which are influenced by earth tides. It is a matter of conjecture at present but there is some correlation between terrestrial tides and earthquake activity.

Terrestrial tides also need to be taken in account in the case of some particle physics experiments. For instance, at the CERN or SLAC, the very large particle accelerators were designed while taking terrestrial tides into account for proper operation. Among the effects that need to be taken into account are circumference deformation for circular accelerators and particle beam energy.

Since tidal forces generate currents of conducting fluids within the interior of the Earth, they affect in turn the Earth's magnetic field itself.

Chapter 3

Tidal Force

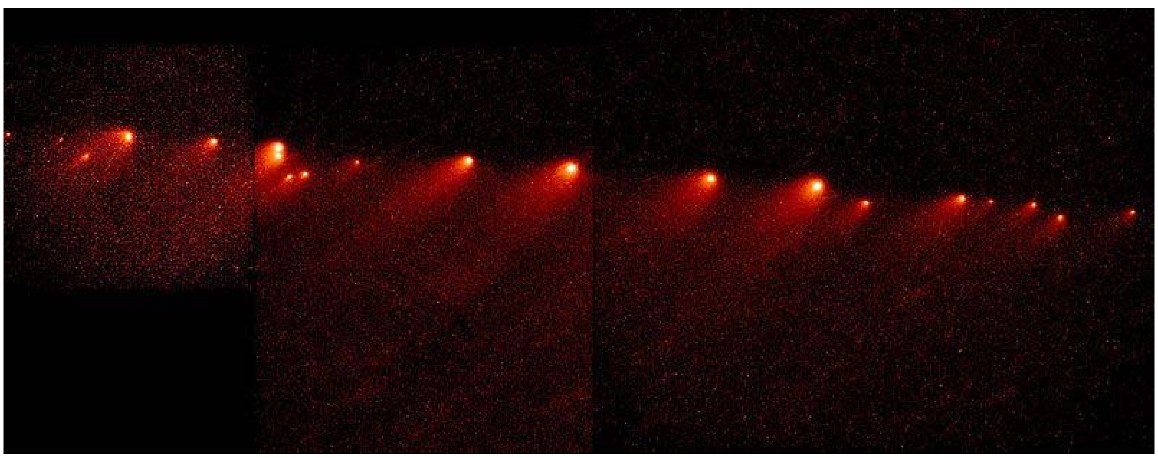


Figure 1: Comet Shoemaker-Levy 9 in 1994 after breaking up under the influence of Jupiter's tidal forces during a previous pass in 1992.

The **tidal force** is a secondary effect of the force of gravity and is responsible for the tides. It arises because the gravitational force per unit mass exerted on one body by a second body is not constant across its diameter, the side nearest to the second being more attracted by it than the side farther away.

In a more general usage in celestial mechanics, the expression 'tidal force' can refer to a situation in which a body or material (for example, tidal water, or the Moon) is mainly under the gravitational influence of a second body (for example, the Earth), but is also perturbed by the gravitational effects of a third body (for example, by the Moon in the case of tidal water, or by the Sun in the case of the Moon). The perturbing force is sometimes in such cases called a tidal force (for example, the perturbing force on the

Moon): it is the difference between the force exerted by the third body on the second and the force exerted by the third body on the first.

Explanation

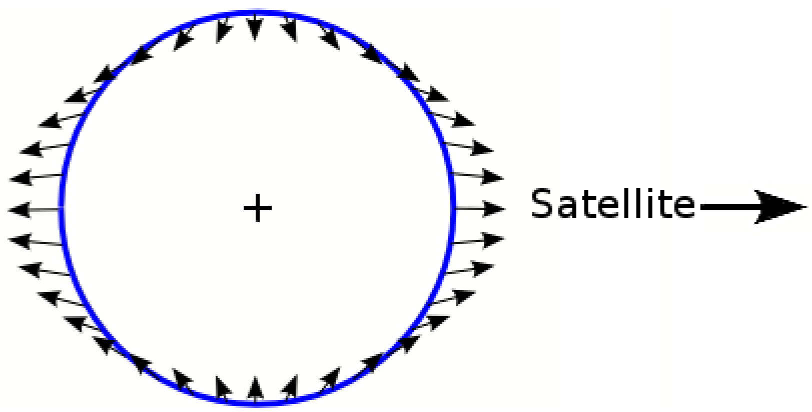


Figure 2: The Moon's gravity differential field at the surface of the Earth is known (along with another and weaker differential effect due to the Sun) as the Tide Generating Force. This is the primary mechanism driving tidal action, explaining two tidal equipotential bulges, and accounting for two high tides per day. In this figure, the Moon is either on the right side or on the left side of the Earth (at center). The **outward** direction of the arrows on the right and left indicates that where the Moon is overhead (or at the nadir) its perturbing force opposes and weakens the Earth's net attraction; and the **inward** direction of the arrows at top and bottom indicates that where the Moon is 90 degrees away from overhead, its perturbing effect reinforces and strengthens the Earth's net attraction.

When a body (body 1) is acted on by the gravity of another body (body 2), the field can vary significantly on body 1 between the side of the body facing body 2 and the side facing away from body 2. Figure 2 shows the differential force of gravity on a spherical body (body 1) exerted by another body (body 2). These so called *tidal forces* cause strains on both bodies and may distort them or even, in extreme cases, break one or the other apart. The Roche limit is the distance from a planet at which tidal effects would cause an object to disintegrate because the differential force of gravity from the planet overcomes the attraction of the parts of the object for one another. These strains would not occur if the gravitational field were uniform, because a uniform field only causes the entire body to accelerate together in the same direction and at the same rate.

Effects of tidal forces

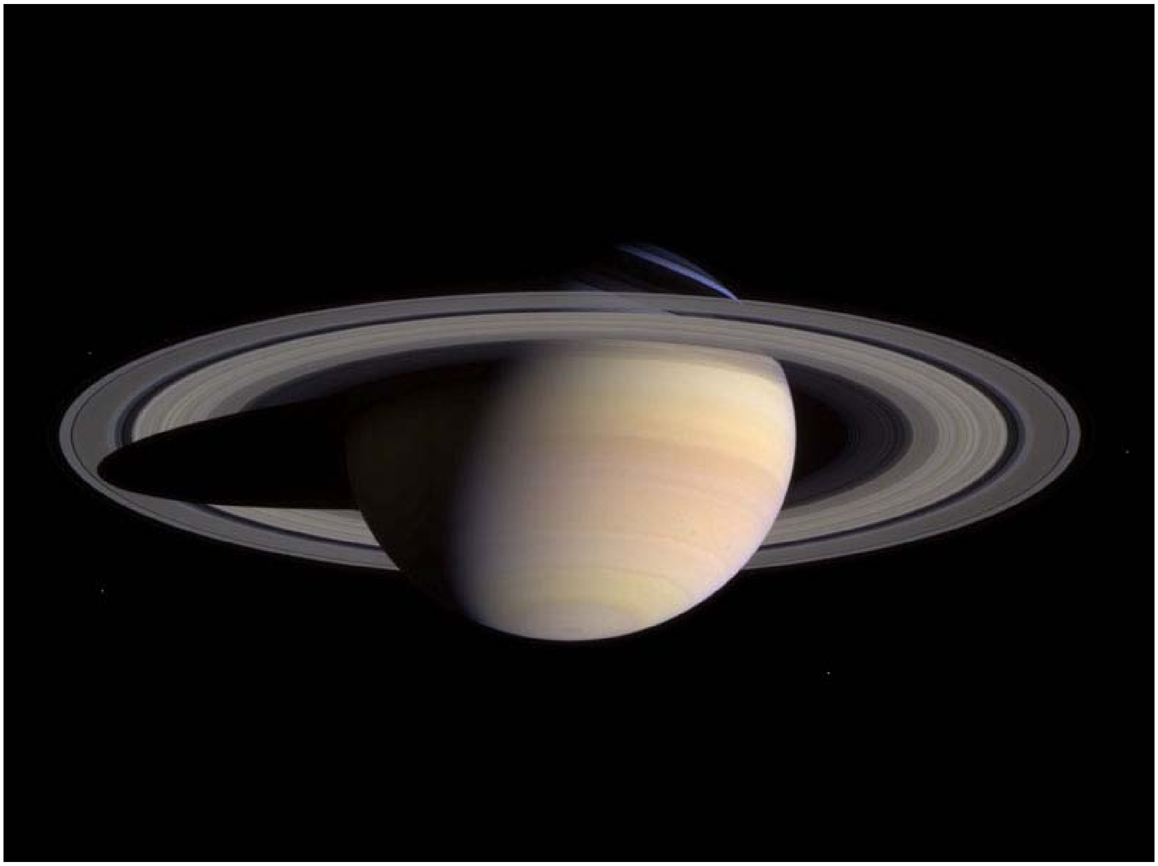


Figure 3: Saturn's rings are inside the orbits of its moons. Tidal forces prevented the material in the rings from coalescing gravitationally to form moons.

In the case of an infinitesimally small elastic sphere, the effect of a tidal force is to distort the shape of the body without any change in volume. The sphere becomes an ellipsoid with two bulges, pointing towards and away from the other body. Larger objects distort into an ovoid, and are slightly compressed, this is approximately what happens to the Earth's oceans under the action of the Moon. The Earth and Moon rotate about their common center of mass or barycenter, and their gravitational attraction provides the centripetal force necessary to maintain this motion. To an observer on the Earth, very close to this barycenter, the situation is one of the Earth as body 1 acted upon by the gravity of the Moon as body 2. All parts of the Earth are subject to the Moon's gravitational forces, causing the water in the oceans to redistribute, forming bulges on the sides near the Moon and far from the Moon.

When a body rotates while subject to tidal forces, internal friction results in the gradual dissipation of its rotational kinetic energy as heat. If the body is close enough to its primary, this can result in a rotation which is tidally locked to the orbital motion, as in the case of the Earth's moon. Tidal heating produces dramatic volcanic effects on Jupiter's moon Io. Stresses caused by tidal forces also cause a regular monthly pattern of moonquakes on Earth's Moon.

Tidal forces contribute to ocean currents, which moderate global temperatures by transporting heat energy toward the poles. It has been suggested that in addition to other factors, harmonic beat variations in tidal forcing may contribute to climate changes.

Tidal effects become particularly pronounced near small bodies of high mass, such as neutron stars or black holes, where they are responsible for the "spaghettification" of infalling matter. Tidal forces create the oceanic tide of Earth's oceans, where the attracting bodies are the Moon and, to a lesser extent, the Sun.

Tidal forces are also responsible for tidal locking and tidal acceleration.

Mathematical treatment

For a given (externally-generated) gravitational field, the **tidal acceleration** at a point with respect to a body is obtained by vectorially subtracting the gravitational acceleration at the center of the body (due to the given externally-generated field) from the gravitational acceleration (due to the same field) at the given point. Correspondingly, the term ***tidal force*** is used to describe the forces due to tidal acceleration. Note that for these purposes the only gravitational field considered is the external one; the gravitational field of the body (as shown in the graphic) is not relevant. (In other words the comparison is with the conditions at the given point as they would be if there were no externally-generated field acting unequally at the given point and at the center of the reference body. The externally-generated field is usually that produced by a perturbing third body, often the Sun or the Moon in the frequent example-cases of points on or above the Earth's surface in a geocentric reference frame.).

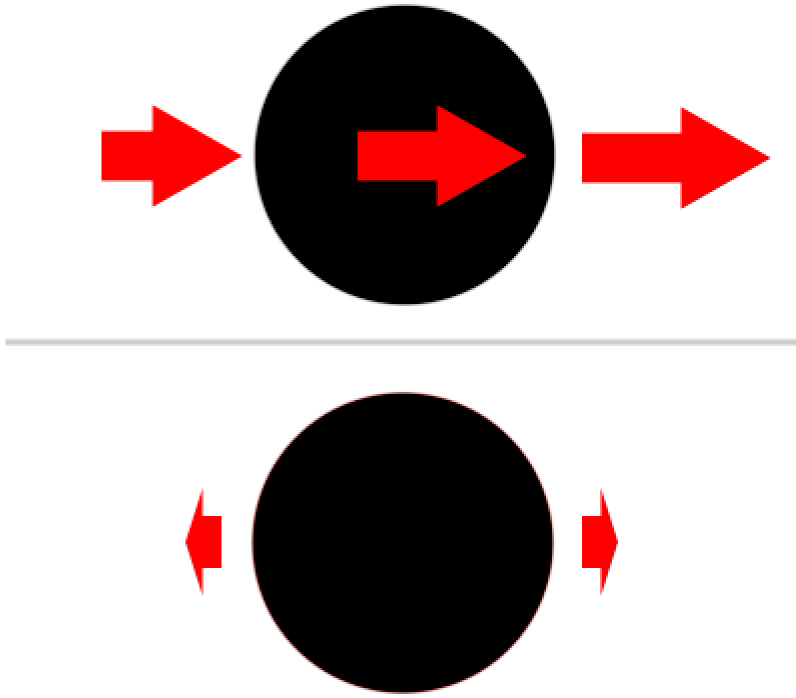


Figure 4: Graphic of tidal forces; the gravity field is generated by a body to the right. The top picture shows the gravitational forces; the bottom shows their residual once the field of the sphere is subtracted; this is the tidal force.

Tidal acceleration does not require rotation or orbiting bodies; for example, the body may be freefalling in a straight line under the influence of a gravitational field while still being influenced by (changing) tidal acceleration.

By Newton's law of universal gravitation and laws of motion, a body of mass *m* a distance *R* from the center of a sphere of mass *M* feels a force $\vec{F}_g$ equivalent to an acceleration $\vec{a}_g$, where:

$$\vec{F}_g = -\hat{r}\ G\ \frac{Mm}{R^2} \ldots \text{, and} \ldots \ \vec{a}_g = -\hat{r}\ G\ \frac{M}{R^2} \ldots,$$

where $\hat{r}$ is a unit vector pointing from the body *M* to the body *m* (here, acceleration from *m* towards *M* has negative sign).

Consider now the acceleration due to the sphere of mass *M* experienced by a particle in the vicinity of the body of mass *m*. With *R* as the distance from the center of *M* to the center of *m*, let Δr be the (relatively small) distance of this other particle from the center of the body of mass *m*. For simplicity, distances are first considered only in the direction pointing towards or away from the sphere of mass M. If the body of mass m is itself a sphere of radius Δr, then the new particle considered may be located on its surface, at a distance $(R \pm \Delta r)$ from the centre of the sphere of mass M, and Δr may be taken as positive where the particle's distance from M is greater than R. Leaving aside whatever gravitational acceleration may be experienced by the particle towards m on account of m's own mass, we have the acceleration on the particle due to gravitational force towards M as:

$$\vec{a}_g = -\hat{r}\ G\ \frac{M}{(R \pm \Delta r)^2}$$

Pulling out the R^2 term from the denominator gives:

$$\vec{a}_g = -\hat{r}\ G\ \frac{M}{R^2}\ \frac{1}{(1 \pm \Delta r/R)^2}$$

The Maclaurin series of $1/(1 + x)^2$ is $1 - 2x + 3x^2 - \ldots$, which gives a series expansion of:

$$\vec{a}_g = -\hat{r}\ G\ \frac{M}{R^2} \pm \hat{r}\ G\ \frac{2M}{R^2}\ \frac{\Delta r}{R} \mp \cdots$$

The first term is the gravitational acceleration due to M at the center of the reference body m, i.e. at the point where Δr is zero. This term does not affect the observed acceleration of particles on the surface of m because with respect to M, m (and everything on its surface) is in free fall. Effectively, this first term cancels. The remaining (residual) terms represent the difference mentioned above and are tidal force (acceleration) terms. Where Δr, is small compared to R, the first of the tidal acceleration terms is usually much more significant than the others, giving for the tidal acceleration $\vec{a}_t$(axial) for the distances Δr considered, along the axis joining the centers of m and M:

$$\vec{a}_{t(\text{axial})} \approx\ \pm\ \hat{r}\ 2\Delta r\ G\ \frac{M}{R^3}$$

When calculated in this way for the case where Δr is a distance along the axis joining the centers of m and M, $\vec{a}_t$ is directed outwards, relative to the center of m where Δr is zero. Tidal accelerations can also be calculated away from the axis connecting the bodies m and M, requiring a vector calculation. In the plane perpendicular to that axis, the tidal acceleration is directed inwards (towards the center where Δr is zero), and its magnitude is $|\vec{a}_t$(axial)$|$ / 2 in linear approximation as in Figure 2.

The tidal accelerations at the surface of planets in the Solar System are generally very small. For example, the lunar tidal acceleration at the Earth's surface along the Moon-Earth axis is about 1.1×10^{-7} g, while the solar tidal acceleration at the Earth's surface along the Sun-Earth axis is about 0.52×10^{-7} g, where g is the gravitational acceleration at the Earth's surface. Modern estimates put the size of the tide-raising force (acceleration) due to the Sun at about 45% of that due to the Moon. The solar tidal acceleration at the Earth's surface was first given by Newton in the 'Principia'

Chapter 4

Tidal Locking

Tidal locking (or **captured rotation**) occurs when the gravitational gradient makes one side of an astronomical body always face another; for example, one side of the Earth's Moon always faces the Earth. A tidally locked body takes just as long to rotate around its own axis as it does to revolve around its partner. This synchronous rotation causes one hemisphere constantly to face the partner body. Usually, at any given time only the satellite is tidally locked around the larger body, but if the difference in mass between the two bodies and their physical separation is small, *each* may be tidally locked to the other, as is the case between Pluto and Charon. This effect is employed to stabilize some artificial satellites.

Mechanism

The change in rotation rate necessary to tidally lock a body B to a larger body A is caused by the torque applied by A's gravity on bulges it has induced on B by tidal forces.

Tidal bulges

A's gravity produces a tidal force on B which distorts its gravitational equilibrium shape slightly so that it becomes elongated along the axis oriented toward A, and conversely, is slightly reduced in dimension in directions perpendicular to this axis. These distortions are known as tidal bulges. When B is not yet tidally locked, the bulges travel over its surface, with one of the two "high" tidal bulges traveling close to the point where body A is overhead. For large astronomical bodies which are near-spherical due to self-gravitation, the tidal distortion produces a slightly prolate spheroid - i.e., an axially symmetric ellipsoid that is elongated along its major axis. Smaller bodies also experience distortion, but this distortion is less regular.

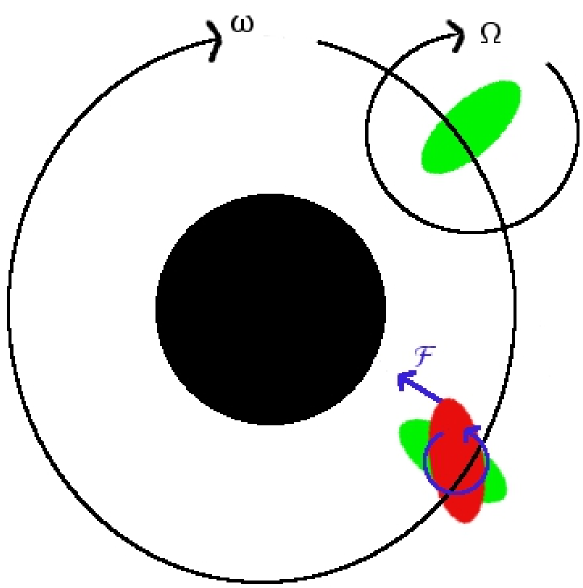
Bulge dragging

The material of B exerts resistance to this periodic reshaping caused by the tidal force. In effect, some time is required to reshape B to the gravitational equilibrium shape, by which time the forming bulges have already been carried some distance away from the A-B axis by B's rotation. Seen from a vantage point in space, the points of maximum bulge extension are displaced from the axis oriented towards A. If B's rotation period is shorter than its orbital period, the bulges are carried forward of the axis oriented towards A in the direction of rotation, whereas if B's rotation period is longer the bulges lag behind instead.

Resulting torque

Since the bulges are now displaced from the A-B axis, A's gravitational pull on the mass in them exerts a torque on B. The torque on the A-facing bulge acts to bring B's rotation in line with its orbital period, while the "back" bulge which faces away from A acts in the opposite sense. However, the bulge on the A-facing side is closer to A than the back bulge by a distance of approximately B's diameter, and so experiences a slightly stronger gravitational force and torque. The net resulting torque from both bulges, then, is always in the direction which acts to synchronize B's rotation with its orbital period, leading eventually to tidal locking.

Orbital changes



If rotational frequency is larger than orbital frequency, a small torque counteracting the rotation arises, eventually locking the frequencies (situation depicted in green)

The angular momentum of the whole A-B system is conserved in this process, so that when B slows down and loses rotational angular momentum, its *orbital* angular momentum is boosted by a similar amount (there are also some smaller effects on A's rotation). This results in a raising of B's orbit about A in tandem with its rotational slowdown. For the other case where B starts off rotating too slowly, tidal locking both speeds up its rotation, and *lowers* its orbit.

Locking of the larger body

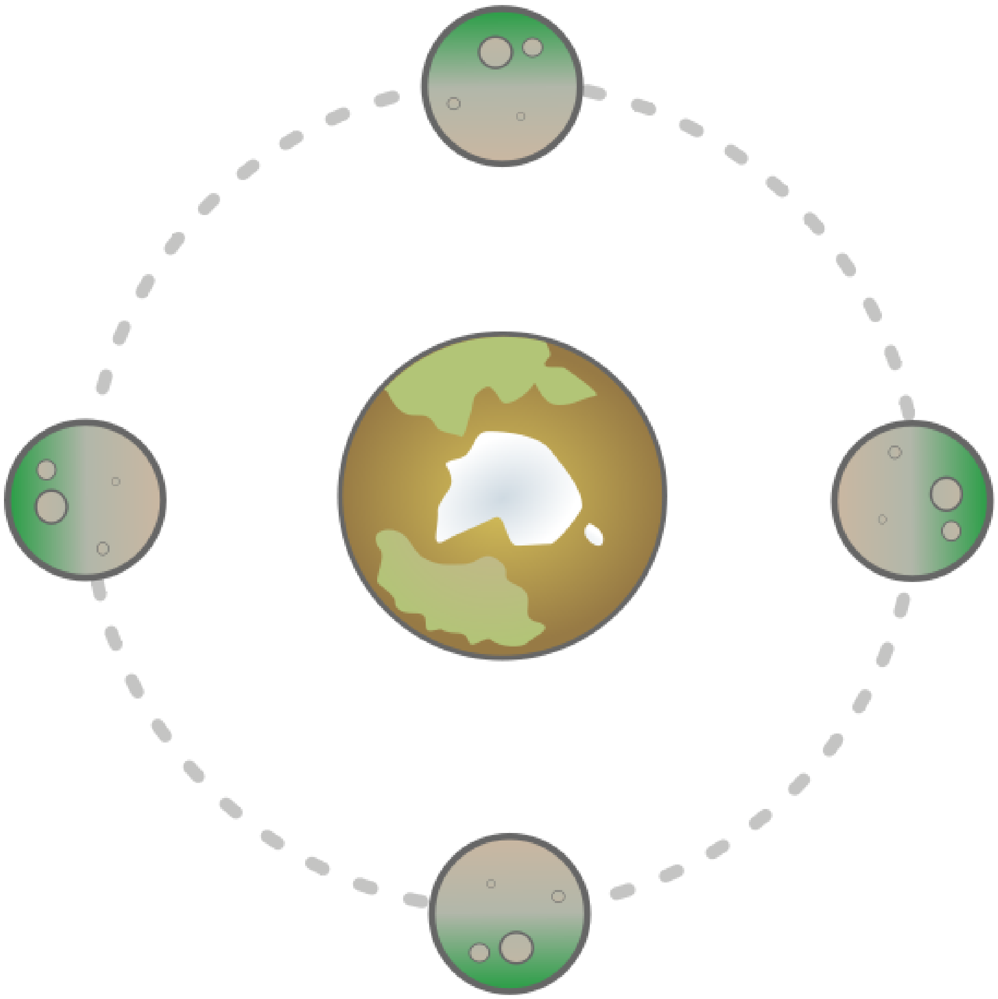
The tidal locking effect is also experienced by the larger body A, but at a slower rate because B's gravitational effect is weaker due to B's smaller size. For example, the Earth's rotation is gradually slowing down because of the Moon, by an amount that becomes noticeable over geological time in some fossils. For similar sized bodies the effect may be of comparable size for both, and both may become tidally locked to each other. The dwarf planet Pluto and its satellite Charon are good examples of this—Charon is only visible from one hemisphere of Pluto and vice versa.

Rotation-orbit resonance

Finally, in some cases where the orbit is eccentric and the tidal effect is relatively weak, the smaller body may end up in an orbital resonance, rather than tidally locked. Here the ratio of rotation period to orbital period is some well-defined fraction different from 1:1. A well known case is the rotation of Mercury—locked to its orbit around the Sun in a 3:2 resonance.

Occurrence

Moons



Due to tidal locking, the inhabitants of the central body will never be able to see its side marked with green.

Most significant moons in the Solar System are tidally locked with their primaries, since they orbit very closely and tidal force increases rapidly (as a cubic) with decreasing distance. Notable exceptions are the irregular outer satellites of the gas giant planets, which orbit much farther away than the large well-known moons.

Pluto and Charon are an extreme example of a tidal lock. Charon is a relatively large moon in comparison to its primary and also has a very close orbit. This has made Pluto also tidally locked to Charon. In effect, these two celestial bodies revolve around each other (their barycenter lies outside of Pluto) as if joined with a rod connecting two opposite points on their surfaces.

The tidal locking situation for asteroid moons is largely unknown, but closely-orbiting binaries are expected to be tidally locked, as well as contact binaries.

Earth's Moon

Since the Moon is 1:1 tidally locked, only one side is visible from Earth.

The Moon's rotation and orbital periods are both just under four weeks, so no matter when the Moon is observed from the Earth the same hemisphere of the Moon is always seen. The far side of the Moon was not seen in its entirety until 1959, when photographs were transmitted from the Soviet spacecraft Luna 3.

Despite the Moon's rotational and orbital periods being exactly locked, about 59% of the moon's total surface may be seen with repeated observations from earth due to the phenomena of librations and parallax. Librations are primarily caused by the Moon's varying orbital speed due to the eccentricity of its orbit: this allows us to see up to about 6° more along its perimeter. Parallax is a geometric effect: at the surface of the Earth we are offset from the line through the centers of Earth and Moon, and because of this we can observe a bit (about 1°) more around the side of the Moon when it is on our local horizon.

Planets

Until radar observations in 1965 proved otherwise, it was thought that Mercury was tidally locked with the Sun. Instead, it turned out that Mercury has a 3:2 spin-orbit resonance, rotating three times for every two revolutions around the Sun; the eccentricity of Mercury's orbit makes this resonance stable. The original reason astronomers thought it was tidally locked was because whenever Mercury was best placed for observation, it was always at the same point in its 3:2 resonance, so showing the same face, which would also be the case if it were tidally locked.

Venus' 583.92-day interval between successive close approaches to the Earth is almost exactly equal to 5 Venusian solar days (precisely, 5.001444 of these), making approximately the same face visible from Earth at each close approach. Whether this relationship arose by chance or is the result of some kind of tidal locking with the Earth is unknown.

Stars

Close binary stars throughout the universe are expected to be tidally locked with each other, and extrasolar planets that have been found to orbit their primaries extremely closely are also thought to be tidally locked to them. An unusual example, confirmed by MOST, is Tau Boötis, a star tidally locked by a planet. The tidal locking is almost certainly mutual.

Timescale

An estimate of the time for a body to become tidally locked can be obtained using the following formula:

$$t_{\text{lock}} \approx \frac{wa^6 IQ}{3Gm_p^2 k_2 R^5}$$

where

- w is the initial spin rate (radians per second)
- a is the semi-major axis of the motion of the satellite around the planet

- $I \approx 0.4 m_s R^2$ is the moment of inertia of the satellite.
- Q is the dissipation function of the satellite.
- G is the gravitational constant
- m_p is the mass of the planet
- m_s is the mass of the satellite
- k_2 is the tidal Love number of the satellite
- R is the radius of the satellite.

Q and k_2 are generally very poorly known except for the Earth's Moon which has $k_2 / Q = 0.0011$. However, for a really rough estimate one can take $Q \approx 100$ (perhaps conservatively, giving overestimated locking times), and

$$k_2 \approx \frac{1.5}{1 + \frac{19\mu}{2\rho g R}},$$

where

- ρ is the density of the satellite
- $g \approx G m_s / R^2$ is the surface gravity of the satellite
- μ is rigidity of the satellite. This can be roughly taken as 3×10^{10} Nm^{-2} for rocky objects and 4×10^{9} Nm^{-2} for icy ones.

As can be seen, even knowing the size and density of the satellite leaves many parameters that must be estimated (especially *w*, *Q*, and μ), so that any calculated locking times obtained are expected to be inaccurate, to even factors of ten. Further, during the tidal locking phase the orbital radius *a* may have been significantly different from that observed nowadays due to subsequent tidal acceleration, and the locking time is extremely sensitive to this value.

Since the uncertainty is so high, the above formulas can be simplified to give a somewhat less cumbersome one. By assuming that the satellite is spherical, $k_2 \ll 1$, *Q* = 100, and it is sensible to guess one revolution every 12 hours in the initial non-locked state (most asteroids have rotational periods between about 2 hours and about 2 days)

$$t_{\text{lock}} \quad \approx \quad 6 \, \frac{a^6 R \mu}{m_s m_p^2} \quad \times 10^{10} \text{ years},$$

with masses in kg, distances in meters, and μ in Nm^{-2}. μ can be roughly taken as 3×10^{10} Nm^{-2} for rocky objects and 4×10^{9} Nm^{-2} for icy ones.

Note the extremely strong dependence on orbital radius *a*.

For the locking of a primary body to its moon as in the case of Pluto, satellite and primary body parameters can be interchanged.

One conclusion is that *other things being equal* (such as Q and μ), a large moon will lock faster than a smaller moon at the same orbital radius from the planet because m_s grows much faster with satellite radius than *R*. A possible example of this is in the Saturn system, where Hyperion is not tidally locked, while the larger Iapetus, which orbits at a greater distance, is. It must be noted, however, that this is not clear cut because Hyperion also experiences strong driving from the nearby Titan, which forces its rotation to be chaotic.

List of known tidally locked bodies

Solar System

Locked to the Sun

- Mercury (in a 3:2 rotation:orbit resonance)

Locked to the Earth

- Moon

Locked to Mars

- Phobos
- Deimos

Locked to Jupiter

- Metis
- Adrastea
- Amalthea
- Thebe
- Io
- Europa
- Ganymede
- Callisto

Locked to Saturn

- Pan
- Atlas
- Prometheus
- Pandora
- Epimetheus
- Janus
- Mimas
- Enceladus

- Telesto
- Tethys
- Calypso
- Dione
- Rhea
- Titan
- Iapetus

Locked to Uranus

- Miranda
- Ariel
- Umbriel
- Titania
- Oberon

Locked to Neptune

- Proteus
- Triton

Locked to Pluto

- Charon (Pluto is itself locked to Charon)

Extra-solar

- Tau Boötis is known to be locked to the close-orbiting giant planet Tau Boötis b.

Bodies likely to be locked

Solar System

Based on comparison between the likely time needed to lock a body to its primary, and the time it has been in its present orbit (comparable with the age of the Solar System for most planetary moons), a number of moons are thought to be locked. However their rotations are not known or not known enough. These are:

Probably locked to Saturn

- Daphnis
- S/2004 S 6
- S/2004 S 4
- S/2004 S 3
- Methone
- Pallene

- Helene
- Polydeuces

Probably locked to Uranus

- Cordelia
- Ophelia
- Bianca
- Cressida
- Desdemona
- Juliet
- Portia
- Rosalind
- Cupid
- Belinda
- Perdita
- Puck
- Mab
- Oberon

Probably locked to Neptune

- Naiad
- Thalassa
- Despina
- Galatea
- Larissa

Extra-solar

- Gliese 581 c may be tidally locked to its parent star Gliese 581.
- Gliese 581 g may be tidally locked to its parent star Gliese 581.
- Gliese 581 b, Gliese 581 d, and Gliese 581 e may be tidally locked to their parent star Gliese 581.

Chapter 5

Tide

High Tide, Alma, New Brunswick in the Bay of Fundy

Low Tide at the same fishing port in Bay of Fundy

Tides are the rise and fall of sea levels caused by the combined effects of the gravitational forces exerted by the Moon and the Sun and the rotation of the Earth.

Most places in the ocean usually experience two high tides and two low tides each day (semidiurnal tide), but some locations experience only one high and one low tide each day (diurnal tide). The times and amplitude of the tides at the coast are influenced by the alignment of the Sun and Moon, by the pattern of tides in the deep ocean (see figure 4) and by the shape of the coastline and near-shore bathymetry.

Because the gravitational field created by the Moon weakens with distance from the moon, it exerts a slightly harder pull on the side of the Earth facing the Moon than on the opposite side. The Moon thus tends to "stretch" the Earth slightly along the line connecting the two bodies. The solid Earth deforms a bit, but ocean water, being fluid, is free to move much more in response to the tidal force, particularly horizontally. As the Earth rotates, the magnitude and direction of the tidal force at any particular point on the Earth's surface change constantly; although the ocean never reaches equilibrium--there is never time for the fluid to "catch up" to the state it would eventually reach if the tidal force were constant--the changing tidal force nonetheless causes rhythmic changes in sea surface height.

The Moon orbits the Earth in the same direction as the Earth rotates on its axis, so it takes slightly more than a day—about 24 hours and 50 minutes—for the Moon to return to the same location in the sky. During this time, it has passed overhead once and underfoot once, so in many places the period of strongest tidal forcing is 12 hours and 25 minutes. The high tides do not necessarily occur when the Moon is overhead or underfoot, but the period of the forcing still determines the time between high tides.

The Sun also exerts on the Earth a gravitational attraction which results in a (less powerful) secondary tidal effect. When the Earth, Moon and Sun are approximately aligned, these two tidal effects reinforce one another, resulting in higher highs and lower lows. This alignment occurs approximately twice a month (at the full moon and new moon). These recurring extreme tides are termed spring tides. Tides with the smallest range are termed neap tides (occurring around the first and last quarter moons).

The tidal forces affect the entire earth, but the movement of the solid Earth is only centimetres. The atmosphere is much more fluid and compressible so its surface moves kilometres, in the sense of the contour level of a particular low pressure in the outer atmosphere.

Tides vary on timescales ranging from hours to years due to numerous influences. To make accurate records, tide gauges at fixed stations measure the water level over time. Gauges ignore variations caused by waves with periods shorter than minutes. These data are compared to the reference (or datum) level usually called mean sea level.

While tides are usually the largest source of short-term sea-level fluctuations, sea levels are also subject to forces such as wind and barometric pressure changes, resulting in storm surges, especially in shallow seas and near coasts.

Tidal phenomena are not limited to the oceans, but can occur in other systems whenever a gravitational field that varies in time and space is present. For example, the solid part of the Earth is affected by tides.

Characteristics

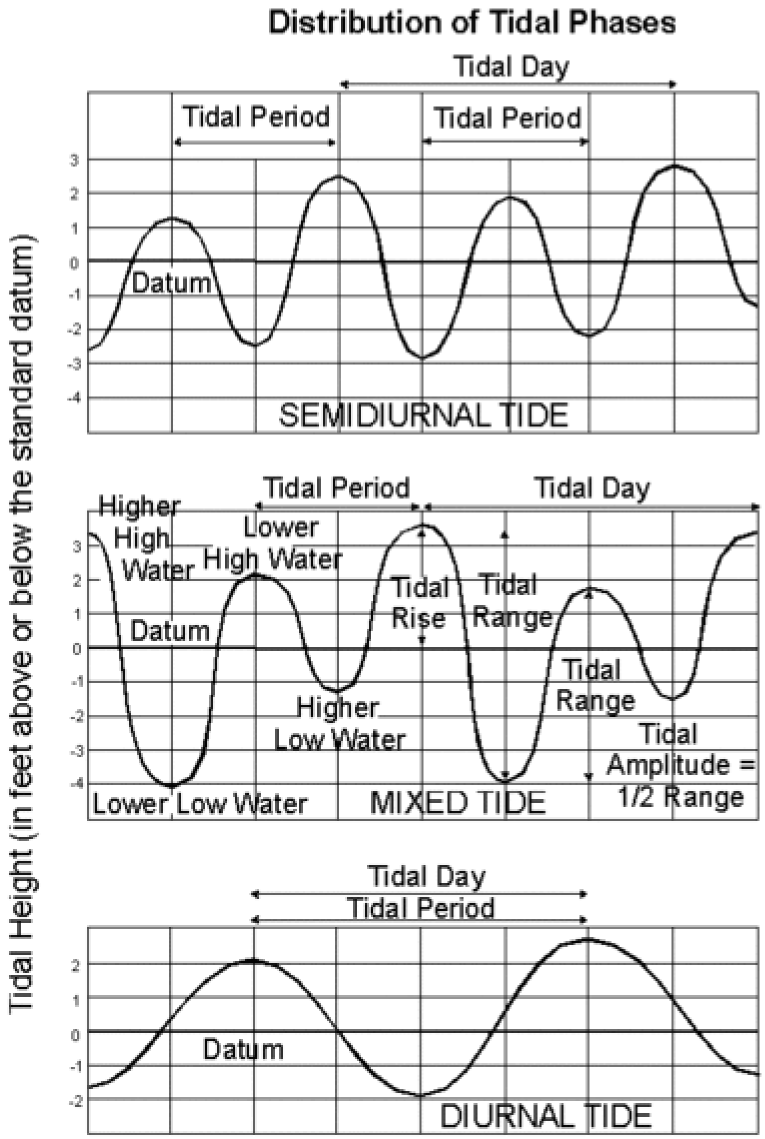


Fig. 1: Types of tides

Tide changes proceed via the following stages:

- Sea level rises over several hours, covering the intertidal zone; flood tide.
- The water rises to its highest level, reaching high tide.
- Sea level falls over several hours, revealing the intertidal zone; ebb tide.
- The water stops falling, reaching low tide.

Tides produce oscillating currents known as tidal streams. The moment that the tidal current ceases is called slack water or slack tide. The tide then reverses direction and is said to be turning. Slack water usually occurs near high water and low water. But there are locations where the moments of slack tide differ significantly from those of high and low water.

Tides are most commonly *semidiurnal* (two high waters and two low waters each day), or *diurnal* (one tidal cycle per day). The two high waters on a given day are typically not the same height (the daily inequality); these are the *higher high water* and the *lower high water* in tide tables. Similarly, the two low waters each day are the *higher low water* and the *lower low water*. The daily inequality is not consistent and is generally small when the Moon is over the equator.

Tidal constituents

Tidal changes are the net result of multiple influences that act over varying periods. These influences are called tidal constituents. The primary constituents are the Earth's rotation, the positions of Moon and the Sun relative to Earth, the Moon's altitude above the Earth, and bathymetry.

Variations with periods of less than half a day are called *harmonic constituents*. Conversely, *long period* constituents cycle over days, months, or years.

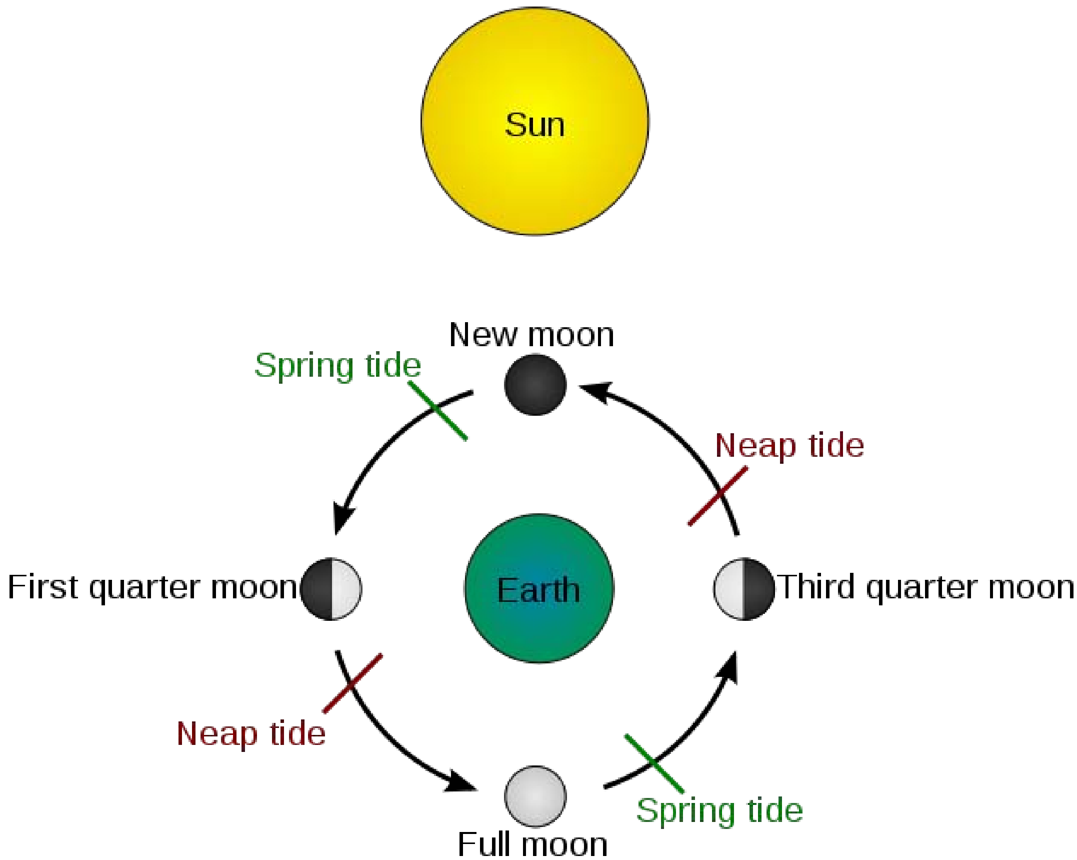
Principal lunar semidiurnal constituent

In most locations, the largest constituent is the "principal lunar semidiurnal", also known as the *M2* (or M_2) tidal constituent. Its period is about 12 hours and 25.2 minutes, exactly half a *tidal lunar day*, which is the average time separating one lunar zenith from the next, and thus is the time required for the Earth to rotate once relative to the Moon. Simple tide clocks track this constituent. The lunar day is longer than the Earth day because the Moon orbits in the same direction the Earth spins. This is analogous to the minute hand on a watch crossing the hour hand at 12:00 and then again at about 1:05 (not at 1:00).

Semidiurnal range differences

When there are two high tides each day with different heights (and two low tides also of different heights), the pattern is called a *mixed semidiurnal tide*.

Range variation: springs and neaps



The types of tides

The semidiurnal range (the difference in height between high and low waters over about a half day) varies in a two-week cycle. Around new moon and full moon when the Sun, Moon and Earth form a line (a condition known as syzygy) the tidal force due to the Sun reinforces that due to the Moon. The tide's range is then at its maximum: this is called the *spring tide*, or just *springs*. It is not named after the season but, like that word, derives from an earlier meaning of "jump, burst forth, rise" as in a natural spring. When the Moon is at first quarter or third quarter, the Sun and Moon are separated by 90° when viewed from the Earth, and the solar gravitational force partially cancels the Moon's. At these points in the lunar cycle, the tide's range is at its minimum: this is called the *neap tide*, or *neaps* (a word of uncertain origin). Spring tides result in high waters that are higher than average, low waters that are lower than average, *slack water* time that is shorter than average and stronger tidal currents than average. Neaps result in less extreme tidal conditions. There is about a seven-day interval between springs and neaps.

Lunar altitude

Negative low tide at Ocean Beach in San Francisco

The changing distance separating the Moon and Earth also affects tide heights. When the Moon is at perigee, the range increases, and when it is at apogee, the range shrinks. Every 7½ lunations (the full cycles from full moon to new to full), perigee coincides with either a new or full moon causing perigean spring tides with the largest *tidal range*. If a storm happens to be moving onshore at this time, the consequences (property damage, etc.) can be severe.

Bathymetry

The shape of the shoreline and the ocean floor changes the way that tides propagate, so there is no simple, general rule that predicts the time of high water from the Moon's position in the sky. Coastal characteristics such as underwater bathymetry and coastline shape mean that individual location characteristics affect tide forecasting; actual high water time and height may differ from model predictions due to the coastal morphology's effects on tidal flow. However, for a given location the relationship between lunar altitude and the time of high or low tide (the lunitidal interval) is relatively constant and predictable, as is the time of high or low tide relative to other points on the same coast. For example, the high tide at Norfolk, Virginia, predictably occurs approximately two and a half hours before the Moon passes directly overhead.

Land masses and ocean basins act as barriers against water moving freely around the globe, and their varied shapes and sizes affect the size of tidal frequencies. As a result, tidal patterns vary. For example, in the U.S., the East coast has predominantly semi-diurnal tides, as do Europe's Atlantic coasts, while the West coast predominantly has mixed tides.

Other constituents

These include solar gravitational effects, the obliquity (tilt) of the Earth's equator and rotational axis, the inclination of the plane of the lunar orbit and the elliptical shape of the Earth's orbit of the Sun.

A compound tide (or overtide) results from the shallow-water interaction of its two parent waves.

Phase and amplitude

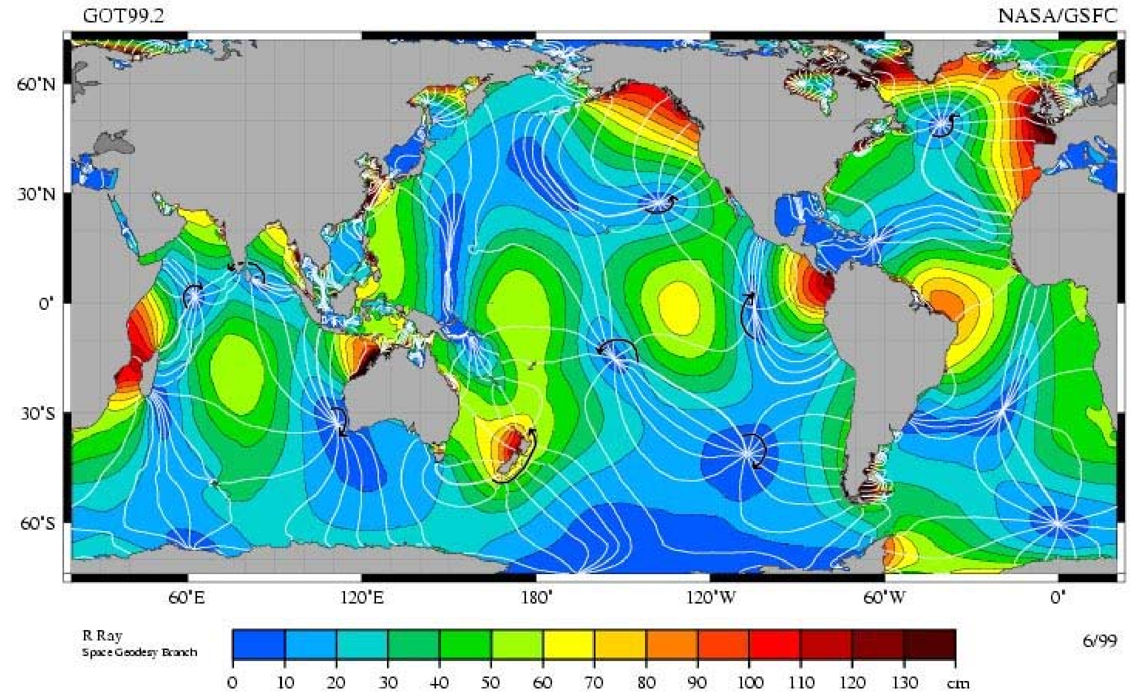


Fig. 4: The M_2 tidal constituent. Amplitude is indicated by color, and the white lines are cotidal differing by 1 hour. The curved arcs around the amphidromic points show the direction of the tides, each indicating a synchronized 6-hour period.

Because the M_2 tidal constituent dominates in most locations, the stage or *phase* of a tide, denoted by the time in hours after high water is a useful concept. Tidal stage is also measured in degrees, with 360° per tidal cycle. Lines of constant tidal phase are called *cotidal lines*, analogous to lines on topographical maps. High water is reached simultaneously along the cotidal lines extending from the coast out into the ocean, and cotidal lines (and hence tidal phases) advance along the coast. Semidiurnal and long phase constituents are measured from high water, diurnal from maximum flood tide. This and the discussion that follows is precisely true only for a single tidal constituent.

For an ocean in the shape of a circular basin enclosed by a coastline, the *cotidal lines* point radially inward and must eventually meet at a common point, the amphidromic point. The amphidromic point is at once cotidal with high and low waters, which is satisfied by *zero* tidal motion. (The rare exception occurs when the tide encircles an island, as it does around New Zealand and Madagascar.) Tidal motion generally lessens moving away from continental coasts, so that crossing the cotidal lines are contours of constant *amplitude* (half the distance between high and low water) which decrease to zero at the amphidromic point. For a semidiurnal tide the amphidromic point can be thought of roughly like the center of a clock face, with the hour hand pointing in the direction of the high water cotidal line, which is directly opposite the low water cotidal line. High water rotates about the amphidromic point once every 12 hours in the direction of rising cotidal lines, and away from ebbing cotidal lines. This rotation is generally clockwise in the southern hemisphere and counterclockwise in the northern hemisphere, and is caused by the Coriolis effect. The difference of cotidal phase from the phase of a reference tide is the *epoch*. The reference tide is the hypothetical constituent equilibrium tide on a landless Earth measured at 0° longitude, the Greenwich meridian.

In the North Atlantic, because the cotidal lines circulate counterclockwise around the amphidromic point, the high tide passes New York harbor approximately an hour ahead of Norfolk harbor. South of Cape Hatteras the tidal forces are more complex, and cannot be predicted reliably based on the North Atlantic cotidal lines.

Physics

History of tidal physics

Tidal physics was important in the early development of heliocentrism and celestial mechanics, with the existence of two daily tides being explained by the Moon's gravity. Later the daily tides were explained more precisely by the interaction of the Moon's gravity and the Sun's gravity to cause the variation of tides.

An early explanation of tides was given by Galileo Galilei in his 1632 *Dialogue Concerning the Two Chief World Systems,* whose working title was *Dialogue on the Tides*. However, the resulting theory was incorrect - he attributed the tides to water sloshing due to the Earth's movement around the Sun, hoping to provide mechanical proof of the Earth's movement - and the value of the theory is disputed, as discussed there. At the same time Johannes Kepler correctly suggested that the Moon caused the tides, based upon ancient observation and correlations, an explanation which was rejected by Galileo. It was originally mentioned in Ptolemy's Tetrabiblos as being derived from ancient observation.

Isaac Newton (1642–1727) was the first person to explain tides scientifically. His explanation of the tides (and many other phenomena) was published in the *Principia* (1687). and used his theory of universal gravitation to account for the tide-generating forces as due to the lunar and solar attractions. Newton and others before Pierre-Simon Laplace worked with an equilibrium theory, largely concerned with an approximation that describes the tides that would occur in a non-inertial ocean evenly covering the whole Earth. The tide-generating force (or its corresponding potential) is still relevant to tidal theory, but as an intermediate quantity rather than as a final result; theory has to consider also the Earth's accumulated dynamic tidal response to the force, a response that is influenced by bathymetry, Earth's rotation, and other factors.

In 1740, the Académie Royale des Sciences in Paris offered a prize for the best theoretical essay on tides. Daniel Bernoulli, Leonhard Euler, Colin Maclaurin and Antoine Cavalleri shared the prize.

Maclaurin used Newton's theory to show that a smooth sphere covered by a sufficiently deep ocean under the tidal force of a single deforming body is a prolate spheroid (essentially a three dimensional oval) with major axis directed toward the deforming body. Maclaurin was the first to write about the Earth's rotational effects on motion. Euler realized that the tidal force's *horizontal* component (more than the vertical) drives the tide. In 1744 Jean le Rond d'Alembert studied tidal equations for the atmosphere which did not include rotation.

Pierre-Simon Laplace formulated a system of partial differential equations relating the ocean's horizontal flow to its surface height, the first major dynamic theory for water tides. The Laplace tidal equations are still in use today. William Thomson, 1st Baron Kelvin, rewrote Laplace's equations in terms of vorticity which allowed for solutions describing tidally-driven coastally-trapped waves, known as Kelvin waves.

Others including Kelvin and Henri Poincaré further developed Laplace's theory. Based on these developments and the lunar theory of E W Brown describing the motions of the moon, Arthur Thomas Doodson developed and published in 1921 the first modern development of the tide-generating potential in harmonic form: Doodson distinguished 388 tidal frequencies. Some of his methods remain in use.

Forces

The tidal force produced by a massive object (Moon, hereafter) on a small particle located on or in an extensive body (Earth, hereafter) is the vector difference between the gravitational force exerted by the Moon on the particle, and the gravitational force that would be exerted on the particle if it were located at the Earth's center of mass. Thus, the tidal force depends not on the strength of the lunar gravitational field, but on its gradient (which falls off approximately as the inverse cube of the distance to the originating gravitational body). The solar *gravitational force* on the Earth is on average 179 times stronger than the lunar, but because the Sun is on average 389 times farther from the Earth, its field gradient is weaker. The solar tidal force is 46% as large as the lunar. More precisely, the lunar tidal acceleration (along the Moon-Earth axis, at the Earth's surface) is about $1.1 \times 10^{-7}\,g$, while the solar tidal acceleration (along the Sun-Earth axis, at the Earth's surface) is about $0.52 \times 10^{-7}\,g$, where g is the gravitational acceleration at the Earth's surface. Venus has the largest effect of the other planets, at 0.000113 times the solar effect.

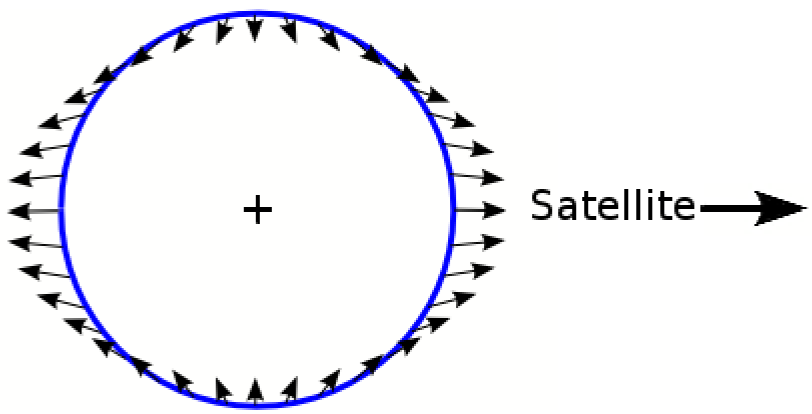


Fig. 6: The lunar gravity differential field at the Earth's surface is known as the tide-generating force. This is the primary mechanism that drives tidal action and explains two equipotential tidal bulges, accounting for two daily high waters.

Tidal forces can also be analyzed this way: each point of the Earth experiences the Moon's radially decreasing gravity differently; they are subject to the *tidal forces* of Figure 6, which dominate. Finally, most importantly, only the tidal forces' *horizontal* components actually tidally accelerate the water particles since there is small resistance. The tidal force on a particle equals about one ten millionth that of Earth's gravitational force.

The ocean's surface is closely approximated by an equipotential surface, (ignoring ocean currents) commonly referred to as the geoid. Since the gravitational force is equal to the potential's gradient, there are no tangential forces on such a surface, and the ocean surface is thus in gravitational equilibrium. Now consider the effect of massive external bodies such as the Moon and Sun. These bodies have strong gravitational fields that diminish with distance in space and which act to alter the shape of an equipotential surface on the Earth. This deformation has a fixed spatial orientation relative to the influencing body. The Earth's rotation relative to this shape causes the daily tidal cycle. Gravitational forces follow an inverse-square law (force is inversely proportional to the square of the distance), but tidal forces are inversely proportional to the cube of the distance. The ocean surface moves to adjust to changing tidal equipotential, tending to rise when the tidal potential is high, which occurs on the part of the Earth nearest to and furthest from the Moon. When the tidal equipotential changes, the ocean surface is no longer aligned with it, so that the apparent direction of the vertical shifts. The surface then experiences a down slope, in the direction that the equipotential has risen.

Laplace's tidal equations

Ocean depths are much smaller than their horizontal extent. Thus, the response to tidal forcing can be modelled using the Laplace tidal equations which incorporate the following features:

1. The vertical (or radial) velocity is negligible, and there is no vertical shear—this is a sheet flow.
2. The forcing is only horizontal (tangential).
3. The Coriolis effect appears as a fictitious lateral forcing proportional to velocity.
4. The surface height's rate of change is proportional to the negative divergence of velocity multiplied by the depth. As the horizontal velocity stretches or compresses the ocean as a sheet, the volume thins or thickens, respectively.

The boundary conditions dictate no flow across the coastline and free slip at the bottom.

The Coriolis effect steers waves to the right in the northern hemisphere and to the left in the southern allowing coastally trapped waves. Finally, a dissipation term can be added which is an analog to viscosity.

Amplitude and cycle time

The theoretical amplitude of oceanic tides caused by the Moon is about 54 centimetres (21 in) at the highest point, which corresponds to the amplitude that would be reached if the ocean possessed a uniform depth, there were no landmasses, and the Earth were rotating in step with the Moon's orbit. The Sun similarly causes tides, of which the theoretical amplitude is about 25 centimetres (9.8 in) (46% of that of the Moon) with a cycle time of 12 hours. At spring tide the two effects add to each other to a theoretical level of 79 centimetres (31 in), while at neap tide the theoretical level is reduced to 29 centimetres (11 in). Since the orbits of the Earth about the Sun, and the Moon about the Earth, are elliptical, tidal amplitudes change somewhat as a result of the varying Earth–Sun and Earth–Moon distances. This causes a variation in the tidal force and theoretical amplitude of about ±18% for the Moon and ±5% for the Sun. If both the Sun and Moon were at their closest positions and aligned at new moon, the theoretical amplitude would reach 93 centimetres (37 in).

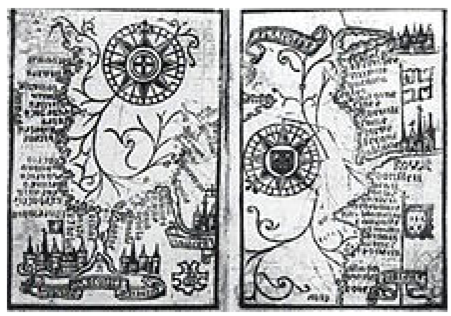
Real amplitudes differ considerably, not only because of depth variations and continental obstacles, but also because wave propagation across the ocean has a natural period of the same order of magnitude as the rotation period: if there were no land masses, it would take about 30 hours for a long wavelength surface wave to propagate along the equator halfway around the Earth (by comparison, the Earth's lithosphere has a natural period of about 57 minutes). Earth tides, which raise and lower the bottom of the ocean, and the tide's own gravitational self attraction are both significant and further complicate the ocean's response to tidal forces.

Dissipation

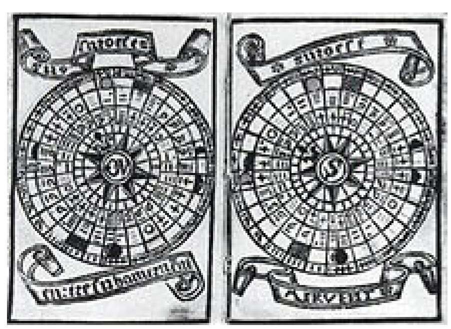
Earth's tidal oscillations introduce dissipation at an average rate of about 3.75 terawatt. About 98% of this dissipation is by marine tidal movement. Dissipation arises as basin-scale tidal flows drive smaller-scale flows which experience turbulent dissipation. This tidal drag creates torque on the Moon that gradually transfers angular momentum to its orbit, and a gradual increase in Earth–Moon separation. The equal and opposite torque on the Earth correspondingly decreases its rotational velocity. Thus, over geologic time, the Moon recedes from the Earth, at about 3.8 centimetres (1.5 in)/year, lengthening the terrestrial day. Day length has increased by about 2 hours in the last 600 million years. Assuming (as a crude approximation) that the deceleration rate has been constant, this would imply that 70 million years ago, day length was on the order of 1% shorter with about 4 more days per year.

Observation and prediction

History



Brouscon's Almanach of 1546: Compass bearings of high waters in the Bay of Biscay (left) and the coast from Brittany to Dover (right).



Brouscon's Almanach of 1546: Tidal diagrams "according to the age of the Moon".

From ancient times, tidal observation and discussion has increased in sophistication, first marking the daily recurrence, then tides' relationship to the Sun and Moon. Pytheas travelled to the British Isles about 325 BC and seems to be the first to have related spring tides to the phase of the Moon.

In the 2nd century BC, the Babylonian astronomer, Seleucus of Seleucia, correctly described the phenomenon of tides in order to support his heliocentric theory. He correctly theorized that tides were caused by the Moon, although he believed that the interaction was mediated by the pneuma. He noted that tides varied in time and strength in different parts of the world. According to Strabo (1.1.9), Seleucus was the first to link tides to the lunar attraction, and that the height of the tides depends on the Moon's position relative to the Sun.

In China, Wang Chong (27-100 AD) correlated tide to the moon's movement in the book entitled Lunheng. He noted that "tide's rise and fall follow the moon and vary in magnitude."

The *Naturalis Historia* of Pliny the Elder collates many tidal observations, e.g., the spring tides are a few days after (or before) new and full moon and are highest around the equinoxes, though Pliny noted many relationships now regarded as fanciful. In his *Geography*, Strabo described tides in the Persian Gulf having their greatest range when the Moon was furthest from the plane of the equator. All this despite the relatively small amplitude of Mediterranean basin tides. (The strong currents through the Strait of Messina and between Greece and the island of Euboea through the Euripus puzzled Aristotle). Philostratus discussed tides in Book Five of The Life of Apollonius of Tyana. Philostratus mentions the Moon, but attributes tides to "spirits". In Europe around 730 AD, the Venerable Bede described how the rising tide on one coast of the British Isles coincided with the fall on the other and described the time progression of high water along the Northumbrian coast.

In the 9th century, the Arabian earth-scientist, Al-Kindi (Alkindus), wrote a treatise entitled *Risala fi l-Illa al-Failali l-Madd wa l-Fazr* (*Treatise on the Efficient Cause of the Flow and Ebb*), in which he presents an argument on tides which "depends on the changes which take place in bodies owing to the rise and fall of temperature." He describes a precise laboratory experiment that proved his argument.

The first tide table in China was recorded in 1056 AD primarily for visitors wishing to see the famous tidal bore in the Qiantang River. The first known British tide table is thought to be that of John Wallingford, who died Abbot of St. Albans in 1213, based on high water occurring 48 minutes later each day, and three hours earlier at the Thames mouth than upriver at London.

William Thomson (Lord Kelvin) led the first systematic harmonic analysis of tidal records starting in 1867. The main result was the building of a tide-predicting machine using a system of pulleys to add together six harmonic time functions. It was "programmed" by resetting gears and chains to adjust phasing and amplitudes. Similar machines were used until the 1960s.

The first known sea-level record of an entire spring–neap cycle was made in 1831 on the Navy Dock in the Thames Estuary. Many large ports had automatic tide gage stations by 1850.

William Whewell first mapped co-tidal lines ending with a nearly global chart in 1836. In order to make these maps consistent, he hypothesized the existence of amphidromes where co-tidal lines meet in the mid-ocean. These points of no tide were confirmed by measurement in 1840 by Captain Hewett, RN, from careful soundings in the North Sea.

Timing

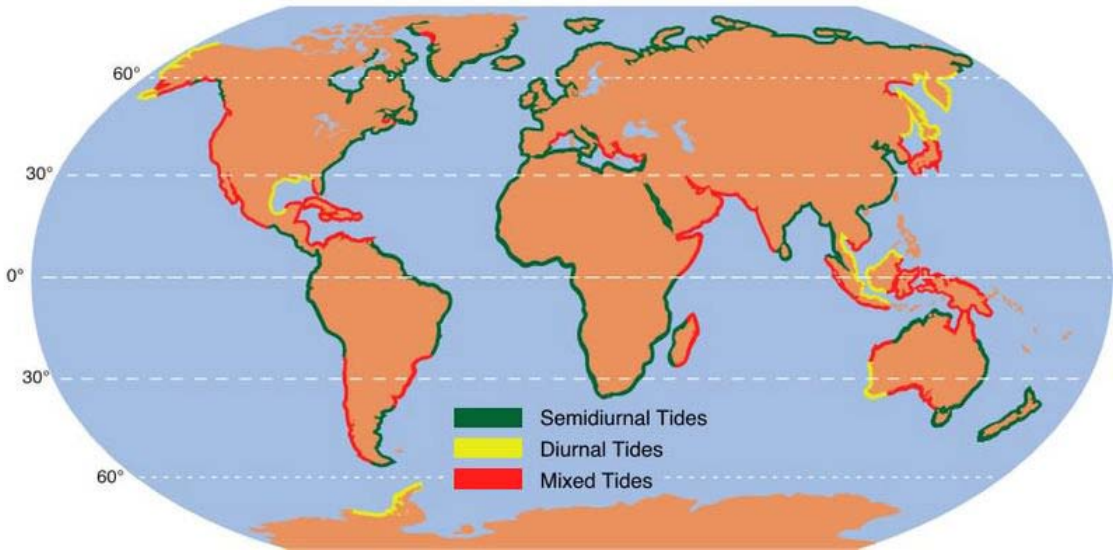


Fig. 7: The same tidal forcing has different results depending on many factors, including coast orientation, continental shelf margin, water body dimensions.

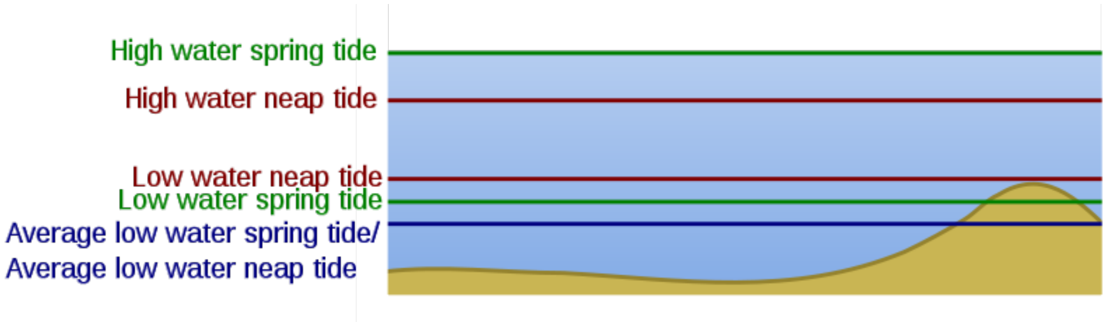
There is a delay between the phases of the Moon and the effect on the tide. Springs and neaps in the North Sea, for example, are two days behind the new/full moon and first/third quarter moon. This is called the tide's *age*.

The local bathymetry greatly influences the tide's exact time and height at a particular coastal point. There are some extreme cases: the Bay of Fundy, on the east coast of Canada, features the world's largest well-documented tidal ranges, 16 metres (52 ft) because of its shape. Some experts believe Ungava Bay in northern Quebec to have even higher tidal ranges, but it is free of pack ice for only about four months every year, while the Bay of Fundy rarely freezes.

Southampton in the United Kingdom has a double high water caused by the interaction between the region's different tidal harmonics. This is contrary to the popular belief that the flow of water around the Isle of Wight creates two high waters. The Isle of Wight is important, however, since it is responsible for the 'Young Flood Stand', which describes the pause of the incoming tide about three hours after low water.

Because the oscillation modes of the Mediterranean Sea and the Baltic Sea do not coincide with any significant astronomical forcing period, the largest tides are close to their narrow connections with the Atlantic Ocean. Extremely small tides also occur for the same reason in the Gulf of Mexico and Sea of Japan. Elsewhere, as along the southern coast of Australia, low tides can be due to the presence of a nearby amphidrome (see figure 4).

Analysis



A regular water level chart

Isaac Newton's theory of gravitation first enabled an explanation of why there were generally two tides a day, not one, and offered hope for detailed understanding. Although it may seem that tides could be predicted via a sufficiently detailed knowledge of the instantaneous astronomical forcings, the actual tide at a given location is determined by astronomical forces accumulated over many days. Precise results require detailed knowledge of the shape of all the ocean basins—their bathymetry and coastline shape.

Current procedure for analysing tides follows the method of harmonic analysis introduced in the 1860s by William Thomson. It is based on the principle that the astronomical theories of the motions of Sun and Moon determine a large number of component frequencies, and at each frequency there is a component of force tending to produce tidal motion, but that at each place of interest on the Earth, the tides respond at each frequency with an amplitude and phase peculiar to that locality. At each place of interest, the tide heights are therefore measured for a period of time sufficiently long (usually more than a year in the case of a new port not previously studied) to enable the response at each significant tide-generating frequency to be distinguished by analysis, and to extract the tidal constants for a sufficient number of the strongest known components of the astronomical tidal forces to enable practical tide prediction. The tide heights are expected to follow the tidal force, with a constant amplitude and phase delay for each component. Because astronomical frequencies and phases can be calculated with certainty, the tide height at other times can then be predicted once the response to the harmonic components of the astronomical tide-generating forces has been found.

The main patterns in the tides are

- the twice-daily variation

- the difference between the first and second tide of a day
- the spring–neap cycle
- the annual variation

The *Highest Astronomical Tide* is the perigean spring tide when both the Sun and the Moon are closest to the Earth.

When confronted by a periodically varying function, the standard approach is to employ Fourier series, a form of analysis that uses sinusoidal functions as a *basis* set, having frequencies that are zero, one, two, three, etc. times the frequency of a particular fundamental cycle. These multiples are called *harmonics* of the fundamental frequency, and the process is termed harmonic analysis. If the basis set of sinusoidal functions suit the behaviour being modelled, relatively few harmonic terms need to be added. Orbital paths are very nearly circular, so sinusoidal variations are suitable for tides.

For the analysis of tide heights, the Fourier series approach has in practice to be made more elaborate than the use of a single frequency and its harmonics. The tidal patterns are decomposed into many sinusoids having many fundamental frequencies, corresponding (as in the lunar theory) to many different combinations of the motions of the Earth, the Moon, and the angles that define the shape and location of their orbits.

For tides, then, *harmonic analysis* is not limited to harmonics of a single frequency. In other words, the harmonies are multiples of many fundamental frequencies, not just of the fundamental frequency of the simpler Fourier series approach. Their representation as a Fourier series having only one fundamental frequency and its (integer) multiples would require many terms, and would be severely limited in the time-range for which it would be valid.

The study of tide height by harmonic analysis was begun by Laplace, William Thomson (Lord Kelvin), and George Darwin. A.T. Doodson extended their work, introducing the *Doodson Number* notation to organise the hundreds of resulting terms. This approach has been the international standard ever since, and the complications arise as follows: the tide-raising force is notionally given by sums of several terms. Each term is of the form

$$A \cdot \cos(w \cdot t + p)$$

where A is the amplitude, w is the angular frequency usually given in degrees per hour corresponding to t measured in hours, and p is the phase offset with regard to the astronomical state at time $t = 0$. There is one term for the Moon and a second term for the Sun. The phase p of the first harmonic for the Moon term is called the lunitidal interval or high water interval. The next step is to accommodate the harmonic terms due to the elliptical shape of the orbits. Accordingly, the value of A is not a constant but also varying with time, slightly, about some average figure. Replace it then by $A(t)$ where A is another sinusoid, similar to the cycles and epicycles of Ptolemaic theory. Accordingly,

$$A(t) = A \cdot (1 + A_a \cdot \cos(w_a \cdot t + p_a)) ,$$

which is to say an average value A with a sinusoidal variation about it of magnitude A_a , with frequency w_a and phase p_a . Thus the simple term is now the product of two cosine factors:

$$A \cdot [1 + A_a \cdot \cos(w_a + p_a)] \cdot \cos(w \cdot t + p)$$

Given that for any x and y

$$\cos(x) \cdot \cos(y) = ½ \cdot \cos(x + y) + ½ \cdot \cos(x – y) ,$$

it is clear that a compound term involving the product of two cosine terms each with their own frequency is the same as *three* simple cosine terms that are to be added at the original frequency and also at frequencies which are the sum and difference of the two frequencies of the product term. (Three, not two terms, since the whole expression is $(1 + \cos(x)) \cdot \cos(y)$.) Consider further that the tidal force on a location depends also on whether the Moon (or the Sun) is above or below the plane of the equator, and that these attributes have their own periods also incommensurable with a day and a month, and it is clear that many combinations result. With a careful choice of the basic astronomical frequencies, the Doodson Number annotates the particular additions and differences to form the frequency of each simple cosine term.

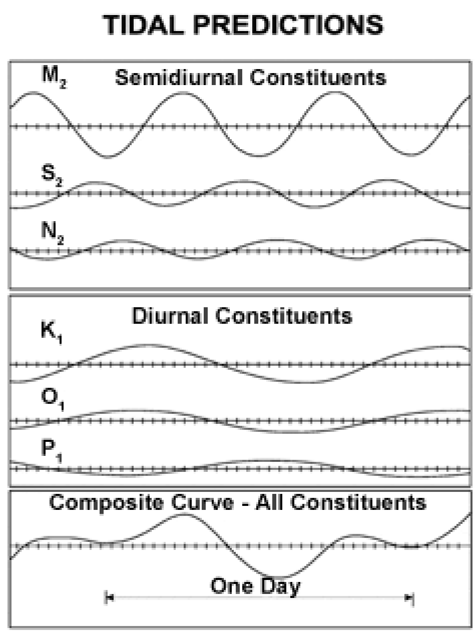


Fig. 8: Tidal prediction summing constituent parts.

Remember that astronomical tides do *not* include weather effects. Also, changes to local conditions (sandbank movement, dredging harbour mouths, etc.) away from those prevailing at the measurement time affect the tide's actual timing and magnitude. Organisations quoting a "highest astronomical tide" for some location may exaggerate the figure as a safety factor against analytical uncertainties, distance from the nearest measurement point, changes since the last observation time, ground subsidence, etc., to avert liability should an engineering work be overtopped. Special care is needed when assessing the size of a "weather surge" by subtracting the astronomical tide from the observed tide.

Careful Fourier data analysis over a nineteen-year period (the *National Tidal Datum Epoch* in the U.S.) uses frequencies called the *tidal harmonic constituents*. Nineteen years is preferred because the Earth, Moon and Sun's relative positions repeat almost exactly in the Metonic cycle of 19 years, which is long enough to include the 18.613 year lunar nodal tidal constituent. This analysis can be done using only the knowledge of the forcing *period*, but without detailed understanding of the mathematical derivation, which means that useful tidal tables have been constructed for centuries. The resulting amplitudes and phases can then be used to predict the expected tides. These are usually dominated by the constituents near 12 hours (the *semidiurnal* constituents), but there are major constituents near 24 hours (*diurnal*) as well. Longer term constituents are 14 day or *fortnightly*, monthly, and semiannual. Semidiurnal tides dominated coastline, but some areas such as the South China Sea and the Gulf of Mexico are primarily diurnal. In the semidiurnal areas, the primary constituents M_2 (lunar) and S_2 (solar) periods differ slightly, so that the relative phases, and thus the amplitude of the combined tide, change fortnightly (14 day period).

In the M_2 plot above, each cotidal line differs by one hour from its neighbors, and the thicker lines show tides in phase with equilibrium at Greenwich. The lines rotate around the amphidromic points counterclockwise in the northern hemisphere so that from Baja California Peninsula to Alaska and from France to Ireland the M_2 tide propagates northward. In the southern hemisphere this direction is clockwise. On the other hand M_2 tide propagates counterclockwise around New Zealand, but this is because the islands act as a dam and permit the tides to have different heights on the islands' opposite sides. (The tides do propagate northward on the east side and southward on the west coast, as predicted by theory.)

The exception is at Cook Strait where the tidal currents periodically link high to low water. This is because cotidal lines 180° around the amphidromes are in opposite phase, for example high water across from low water at each end of Cook Strait. Each tidal constituent has a different pattern of amplitudes, phases, and amphidromic points, so the M_2 patterns cannot be used for other tide components.

Example calculation

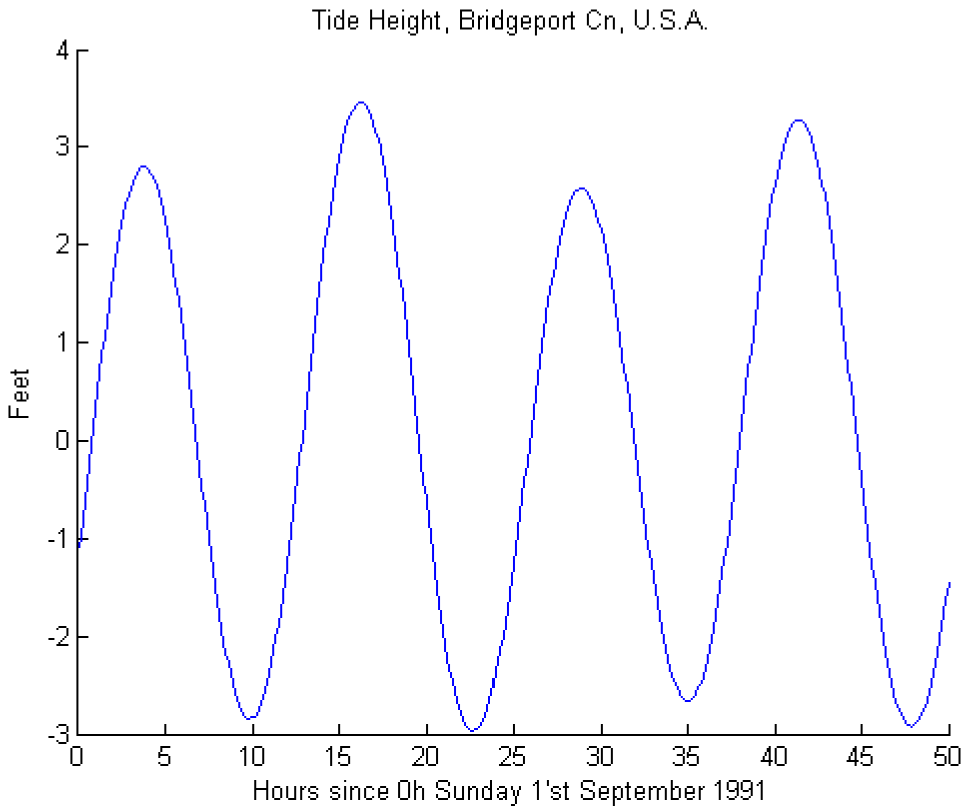


Fig. 9: Tides at Bridgeport, Connecticut, U.S.A. during a 50 hour period.

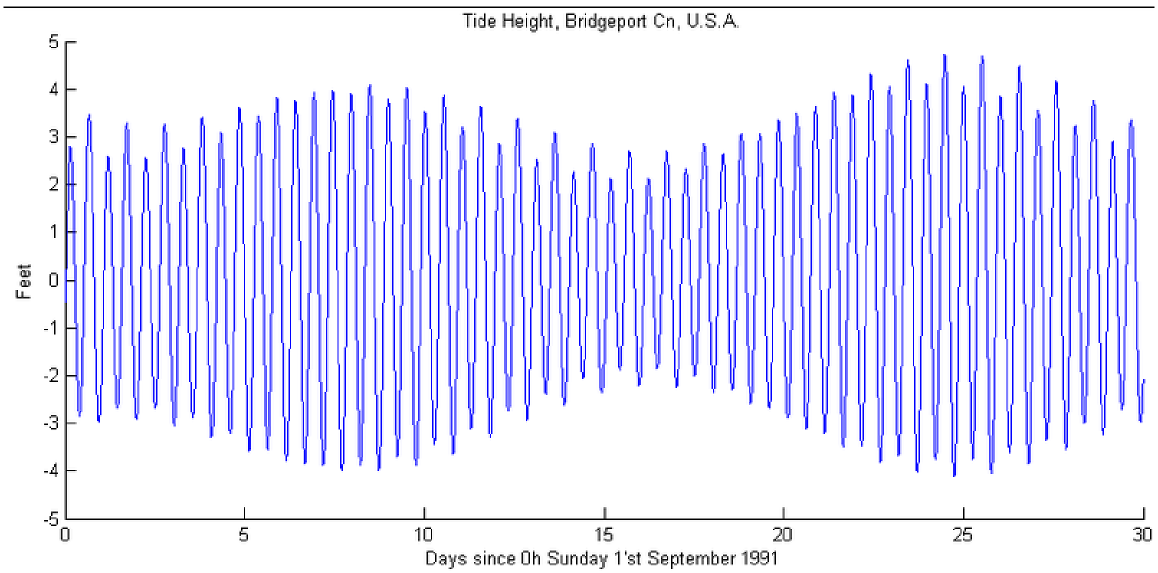


Fig. 10: Tides at Bridgeport, Connecticut, U.S.A. during a 30 day period.

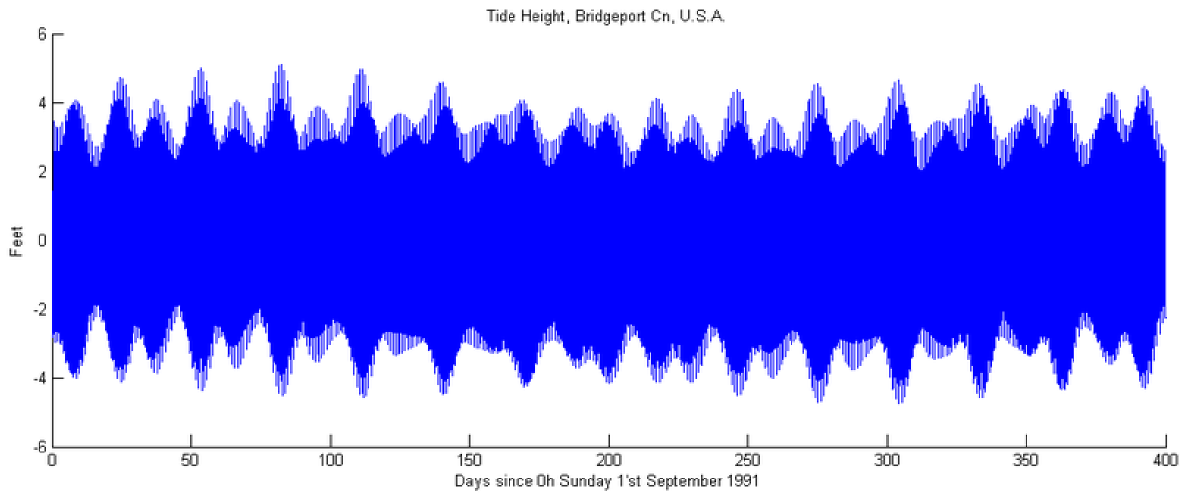


Fig. 11: Tides at Bridgeport, Connecticut, U.S.A. during a 400 day period.

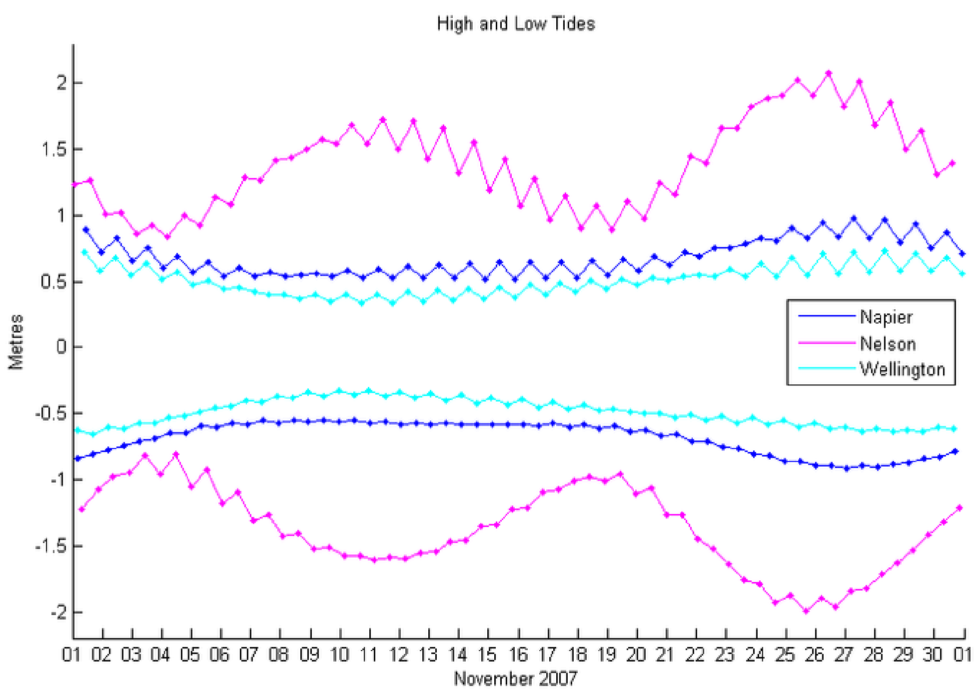


Fig. 12: Two spring tides per month vs. one.

Figure 9 shows the common pattern of two tidal peaks a day as the earth rotates under the moon; because the moon is also moving in its orbit around the earth and in the same sense as the earth's rotation, a point on the earth must rotate slightly further to catch up so that the cycle time is not twelve but 12.4206 hours - a bit over twenty-five minutes extra. The two peaks are not equal: the twin tidal bulges beneath the Moon and on the opposite side of the Earth align with the Moon. Bridgeport is north of the equator, so when the

Moon is north of the equator also and shining upon Bridgeport, Bridgeport is closer to its maximum tide than approximately twelve hours later when Bridgeport is on the opposite side of the Earth from the Moon and the high tide bulge at Bridgeport's longitude has its maximum south of the equator. Thus the two high tides a day alternate in maximum heights: lower high (just under three feet), higher high (just over three feet), and again. Likewise for the low tides.

Figure 10 shows the spring tide/neap tide cycle in tidal amplitudes as the Moon orbits the Earth from being in line (Sun–Earth–Moon, or Sun–Moon–Earth) when the two main influences combine to give the spring tides, to when the two forces are opposing each other as when the angle Moon–Earth–Sun is close to ninety degrees, producing the neap tides. As the Moon moves around its orbit it changes from north of the equator to south of the equator. The alternation in high tide heights becomes smaller, until they are the same (at the lunar equinox, the Moon is above the equator), then redevelops but with the other polarity, waxing to a maximum difference and then waning again.

Figure 11 shows just over a year's worth of tidal height calculations. The Sun also cycles from north to south of the equator, while the Earth–Sun and Earth–Moon distances change on their own cycles. None of the various cycle periods are commensurate.

Current

The tides' influence on current flow is much more difficult to analyse, and data is much more difficult to collect. A tidal height is a simple number which applies to a wide region simultaneously. A flow has both a magnitude and a direction, both of which can vary substantially with depth and over short distances due to local bathymetry. Also, although a water channel's center is the most useful measuring site, mariners object when current-measuring equipment obstructs waterways. A flow proceeding up a curved channel is the same flow, even though its direction varies continuously along the channel. Surprisingly, flood and ebb flows are often not in opposite directions. Flow direction is determined by the upstream channel's shape, not the downstream channel's shape. Likewise, eddies may form in only one flow direction.

Nevertheless, current analysis is similar to tidal analysis: in the simple case, at a given location the flood flow is in mostly one direction, and the ebb flow in another direction. Flood velocities are given positive sign, and ebb velocities negative sign. Analysis proceeds as though these are tide heights.

In more complex situations, the main ebb and flood flows do not dominate. Instead, the flow direction and magnitude trace an ellipse over a tidal cycle (on a polar plot) instead of along the ebb and flood lines. In this case, analysis might proceed along pairs of directions, with the primary and secondary directions at right angles. An alternative is to treat the tidal flows as complex numbers, as each value has both a magnitude and a direction.

Tide flow information is most commonly seen on nautical charts, presented as a table of flow speeds and bearings at hourly intervals, with separate tables for spring and neap tides. The timing is relative to high water at some harbour where the tidal behaviour is similar in pattern, though it may be far away.

As with tide height predictions, tide flow predictions based only on astronomical factors do not incorporate weather conditions, which can *completely* change the outcome.

The tidal flow through Cook Strait between the two main islands of New Zealand is particularly interesting, as the tides on each side of the strait are almost exactly out of phase, so that one side's high water is simultaneous with the other's low water. Strong currents result, with almost zero tidal height change in the strait's center. Yet, although the tidal surge normally flows in one direction for six hours and in the reverse direction for six hours, a particular surge might last eight or ten hours with the reverse surge enfeebled. In especially boisterous weather conditions, the reverse surge might be entirely overcome so that the flow continues in the same direction through three or more surge periods.

A further complication for Cook Strait's flow pattern is that the tide at the north side (e.g. at Nelson) follows the common bi-weekly spring–neap tide cycle (as found along the west side of the country), but the south side's tidal pattern has only *one* cycle per month, as on the east side: Wellington, and Napier.

Figure 12 shows separately the high water and low water height and time, through November 2007; these are *not* measured values but instead are calculated from tidal parameters derived from years-old measurements. Cook Strait's nautical chart offers tidal current information. For instance the January 1979 edition for 41°13·9'S 174°29·6'E (north west of Cape Terawhiti) refers timings to Westport while the January 2004 issue refers to Wellington. Near Cape Terawhiti in the middle of Cook Strait the tidal height variation is almost nil while the tidal current reaches its maximum, especially near the notorious Karori Rip. Aside from weather effects, the actual currents through Cook Strait are influenced by the tidal height differences between the two ends of the strait and as can be seen, only one of the two spring tides at the north end (Nelson) has a counterpart spring tide at the south end (Wellington), so the resulting behaviour follows neither reference harbour.

Power generation

Tidal energy can be extracted by two means: inserting a water turbine into a tidal current, or building ponds that release/admit water through a turbine. In the first case, the energy amount is entirely determined by the timing and tidal current magnitude. However, the best currents may be unavailable because the turbines would obstruct ships. In the second, the impoundment dams are expensive to construct, natural water cycles are completely disrupted, ship navigation is disrupted. However, with multiple ponds, power can be generated at chosen times. So far, there are few installed systems for tidal power generation (most famously, La Rance by Saint Malo, France) which faces many difficulties. Aside from environmental issues, simply withstanding corrosion and biological fouling pose engineering challenges.

Tidal power proponents point out that, unlike wind power systems, generation levels can be reliably predicted, save for weather effects. While some generation is possible for most of the tidal cycle, in practice turbines lose efficiency at lower operating rates. Since the power available from a flow is proportional to the cube of the flow speed, the times during which high power generation is possible are brief.

Navigation

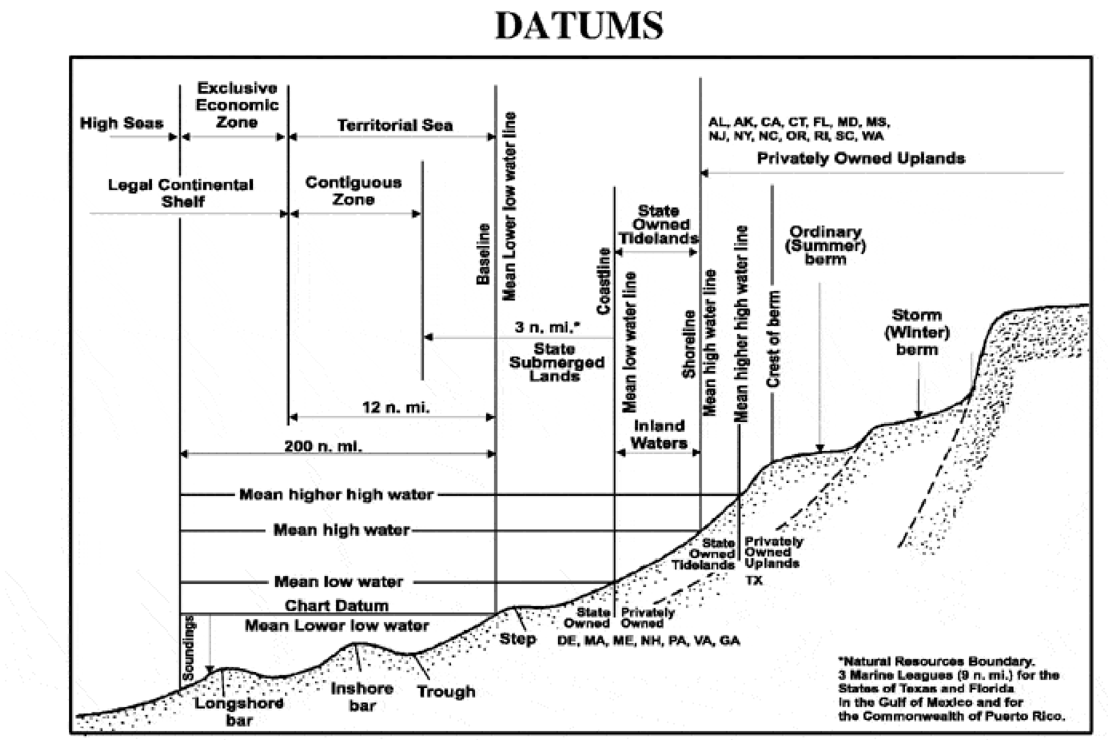


Fig. 13: Civil and maritime uses of tidal data

Tidal flows are important for navigation, and significant errors in position occur if they are not accommodated. Tidal heights are also important; for example many rivers and harbours have a shallow "bar" at the entrance which prevents boats with significant draft from entering at low tide.

Until the advent of automated navigation, competence in calculating tidal effects was important to naval officers. The certificate of examination for lieutenants in the Royal Navy once declared that the prospective officer was able to "shift his tides".

Tidal flow timings and velocities appear in *tide charts* or a tidal stream atlas. Tide charts come in sets. Each chart covers a single hour between one high water and another (they ignore the leftover 24 minutes) and show the average tidal flow for that hour. An arrow on the tidal chart indicates the direction and the average flow speed (usually in knots) for spring and neap tides. If a tide chart is not available, most nautical charts have "tidal diamonds" which relate specific points on the chart to a table giving tidal flow direction and speed.

The standard procedure to counteract tidal effects on navigation is to (1) calculate a "dead reckoning" position (or DR) from travel distance and direction, (2) mark the chart (with a vertical cross like a plus sign) and (3) draw a line from the DR in the tide's direction. The distance the tide moves the boat along this line is computed by the tidal speed, and this gives an "estimated position" or EP (traditionally marked with a dot in a triangle).

Nautical charts display the water's "charted depth" at specific locations with "soundings" and the use of bathymetric contour lines to depict the submerged surface's shape. These depths are relative to a "chart datum", which is typically the water level at the lowest possible astronomical tide (tides may be lower or higher for meteorological reasons) and are therefore the minimum possible water depth during the tidal cycle. "Drying heights" may also be shown on the chart, which are the heights of the exposed seabed at the lowest astronomical tide.

Tide tables list each day's high and low water heights and times. To calculate the actual water depth, add the charted depth to the published tide height. Depth for other times can be derived from tidal curves published for major ports. The rule of twelfths can suffice if an accurate curve is not available. This approximation presumes that the increase in depth in the six hours between low and high water is: first hour — 1/12, second — 2/12, third — 3/12, fourth — 3/12, fifth — 2/12, sixth — 1/12.

Biological aspects

Intertidal ecology

Fig. 14: A rock, seen at low water, exhibiting typical intertidal zonation.

Intertidal ecology is the study of intertidal ecosystems, where organisms live between the low and high water lines. At low water, the intertidal is exposed (or 'emersed') whereas at high water, the intertidal is underwater (or 'immersed'). Intertidal ecologists therefore study the interactions between intertidal organisms and their environment, as well as among the different species. The most important interactions may vary according to the type of intertidal community. The broadest classifications are based on substrates — rocky shore or soft bottom.

Intertidal organisms experience a highly variable and often hostile environment, and have adapted to cope with and even exploit these conditions. One easily visible feature is vertical zonation, in which the community divides into distinct horizontal bands of specific species at each elevation above low water. A species' ability to cope with desiccation determines its upper limit, while competition with other species sets its lower limit.

Humans use intertidal regions for food and recreation. Overexploitation can damage intertidals directly. Other anthropogenic actions such as introducing invasive species and climate change have large negative effects. Marine Protected Areas are one option communities can apply to protect these areas and aid scientific research.

Biological rhythms

The approximately fortnightly tidal cycle has large effects on intertidal organisms. Hence their biological rhythms tend to occur in rough multiples of this period. Many other animals such as the vertebrates, display similar rhythms. Examples include gestation and egg hatching. In humans, the menstrual cycle lasts roughly a month, an even multiple of the tidal period. Such parallels at least hint at the common descent of all animals from a marine ancestor.

Other tides

When oscillating tidal currents in the stratified ocean flow over uneven bottom topography, they generate internal waves with tidal frequencies. Such waves are called *internal tides*.

In addition to oceanic tides, large lakes can experience small tides and even planets can experience *atmospheric tides* and *Earth tides*. These are continuum mechanical phenomena. The first two take place in fluids. The third affects the Earth's thin solid crust surrounding its semi-liquid interior (with various modifications).

Lake tides

Large lakes such as Superior and Erie can experience tides of 1 to 4 cm, but these can be masked by meteorologically induced phenomena such as seiche. The tide in Lake Michigan is described as 0.5 inches to 1.5 inches or 1 and 3/4 inches.

Atmospheric tides

Atmospheric tides are negligible at ground level and aviation altitudes, masked by weather's much more important effects. Atmospheric tides are both gravitational and thermal in origin and are the dominant dynamics from about 80–120 kilometres (50–75 mi) above which the molecular density becomes too low to support fluid behavior.

Earth tides

Earth tides or terrestrial tides affect the entire Earth's mass, which acts similarly to a liquid gyroscope with a very thin crust. The Earth's crust shifts (in/out, east/west, north/south) in response to lunar and solar gravitation, ocean tides, and atmospheric loading. While negligible for most human activities, terrestrial tides' semidiurnal amplitude can reach about 55 centimetres (22 in) at the equator—15 centimetres (5.9 in) is due to the Sun—which is important in GPS calibration and VLBI measurements. Precise astronomical angular measurements require knowledge of the Earth's rotation rate and nutation, both of which are influenced by Earth tides. The semi-diurnal M_2 Earth tides are nearly in phase with the Moon with a lag of about two hours.

Some particle physics experiments must adjust for terrestrial tides. For instance, at CERN and SLAC, the very large particle accelerators account for terrestrial tides. Among the relevant effects are circumference deformation for circular accelerators and particle beam energy. Since tidal forces generate currents in conducting fluids in the Earth's interior, they in turn affect the Earth's magnetic field. Earth tides have also been linked to earthquakes.

Galactic tides

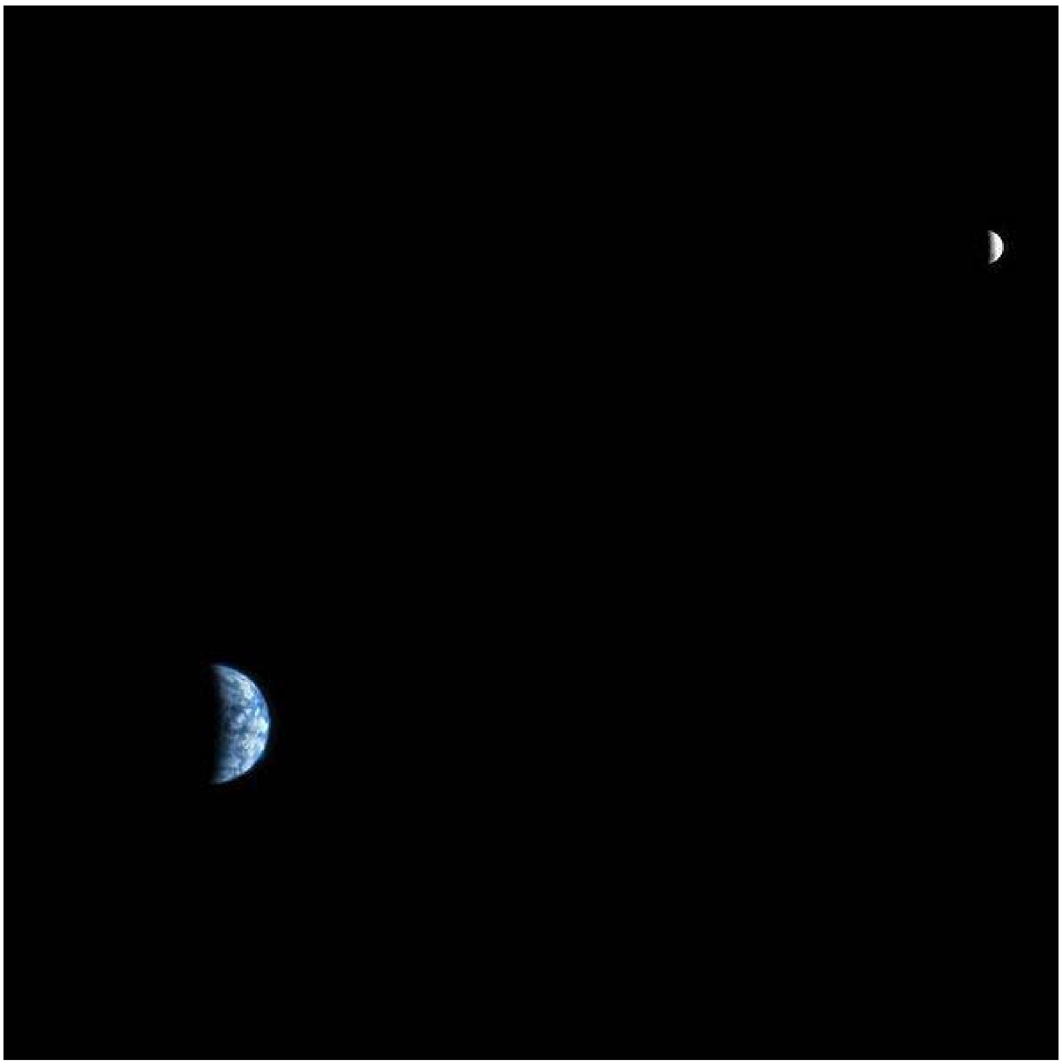
Galactic tides are the tidal forces exerted by galaxies on stars within them and satellite galaxies orbiting them. The galactic tide's effects on the Solar System's Oort cloud are believed to cause 90 percent of long-period comets.

Misapplications

Tsunamis, the large waves that occur after earthquakes, are sometimes called *tidal waves*, but this name is given by their *resemblance* to the tide, rather than any actual link to the tide. Other phenomena unrelated to tides but using the word *tide* are rip tide, storm tide, hurricane tide, and black or red tides.

Chapter 6

Tidal Acceleration



A picture of the Earth and the Moon from Mars. The presence of the moon (which has about 1/81 the mass of the Earth), is slowing Earth's rotation and lengthening the day by about 2 ms every one hundred years.

Tidal acceleration is an effect of the tidal forces between an orbiting natural satellite (*e.g.* the Moon), and the primary planet that it orbits (*e.g.* the Earth). The "acceleration" is usually negative, as it causes a gradual slowing and recession of a satellite in a prograde orbit away from the primary, and a corresponding slowdown of the primary's rotation. The process eventually leads to tidal locking of first the smaller, and later the larger body. The Earth-Moon system is the best studied case.

The similar process of **tidal deceleration** occurs for satellites that have an orbital period that is shorter than the primary's rotation period, or that orbit in a retrograde direction.

Earth-Moon system

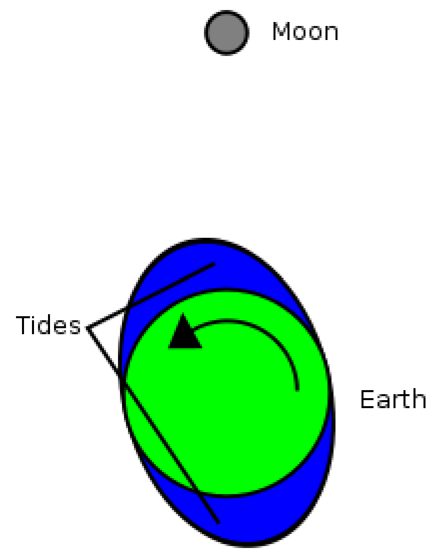
Discovery history of the secular acceleration

Edmond Halley was the first to suggest, in 1695, that the mean motion of the Moon was apparently getting faster, by comparison with ancient eclipse observations, but he gave no data. (It was not yet known in Halley's time that what is actually occurring includes a slowing-down of the Earth's rate of rotation: Ephemeris time - History. When measured as a function of mean solar time rather than uniform time, the effect appears as a positive acceleration.) In 1749 Richard Dunthorne confirmed Halley's suspicion after re-examining ancient records, and produced the first quantitative estimate for the size of this apparent effect: a centurial rate of +10" (arcseconds) in lunar longitude (a surprisingly good result for its time, not far different from values assessed later, e.g. in 1786 by de Lalande, and to compare with values from about 10" to nearly 13" being derived about century later.)

Pierre-Simon Laplace produced in 1786 a theoretical analysis giving a basis on which the Moon's mean motion should accelerate in response to perturbational changes in the eccentricity of the orbit of the Earth around the Sun. Laplace's initial computation accounted for the whole effect, thus seeming to tie up the theory neatly with both modern and ancient observations.

However, in 1854, J C Adams caused the question to be re-opened by finding an error in Laplace's computations: it turned out that only about half of the Moon's apparent acceleration could be accounted for on Laplace's basis by the change in the Earth's orbital eccentricity. Adams' finding provoked a sharp astronomical controversy that lasted some years, but the correctness of his result, agreed by other mathematical astronomers including C E Delaunay, was eventually accepted. The question depended on correct analysis of the lunar motions, and received a further complication with another discovery, around the same time, that another significant long-term perturbation that had been calculated for the Moon (supposedly due to the action of Venus) was also in error, was found on re-examination to be almost negligible, and practically had to disappear from the theory. A part of the answer was suggested independently in the 1860s by Delaunay and by William Ferrel: tidal retardation of the Earth's rotation rate was lengthening the unit of time and causing a lunar acceleration that was only apparent.

It took some time for the astronomical community to accept the reality and the scale of tidal effects. But eventually it became clear that three effects are involved, when measured in terms of mean solar time. Beside the effects of perturbational changes in the Earth's orbital eccentricity, as found by Laplace and corrected by Adams, there are two tidal effects (a combination first suggested by Emmanuel Liais). First there is a real retardation of the Moon's angular rate of orbital motion, due to tidal exchange of angular momentum between the Earth and Moon. This increases the Moon's angular momentum around the Earth (and moves the Moon to a higher orbit with a slower period). Secondly there is an apparent increase in the Moon's angular rate of orbital motion (when measured in terms of mean solar time). This arises from the Earth's loss of angular momentum and the consequent increase in length of day.



A diagram of the Earth-Moon system showing how the tidal bulge is pushed ahead by the Earth's rotation. This offset bulge exerts a net torque on the Moon, boosting it while slowing the Earth's rotation.

Effects of Moon's gravity

Because the Moon's mass is a considerable fraction of that of the Earth (about 1:81), the two bodies can be regarded as a double planet system, rather than as a planet with a satellite. The plane of the Moon's orbit around the Earth lies close to the plane of the Earth's orbit around the Sun (the ecliptic), rather than in the plane perpendicular to the axis of rotation of the Earth (the equator) as is usually the case with planetary satellites. The mass of the Moon is sufficiently large, and it is sufficiently close, to raise tides in the matter of the Earth. In particular, the water of the oceans bulges out along both ends of an axis passing through the centers of the Earth and Moon. The average tidal bulge closely follows the Moon in its orbit, and the Earth rotates under this tidal bulge in just over a day. However, the rotation drags the position of the tidal bulge ahead of the position directly under the Moon. As a consequence, there exists a substantial amount of mass in the bulge that is offset from the line through the centers of the Earth and Moon. Because of this offset, a portion of the gravitational pull between Earth's tidal bulges and the Moon is perpendicular to the Earth-Moon line, *i.e.* there exists a torque between the Earth and the Moon. This boosts the Moon in its orbit, and decelerates the rotation of the Earth.

So the result is that the mean solar day, which is nominally 86400 seconds long, is actually getting longer when measured in SI seconds with stable atomic clocks. (The SI second, when adopted, was already a little shorter than the current value of the second of mean solar time.) The small difference accumulates every day, which leads to an increasing difference between our clock time (Universal Time) on the one hand, and Atomic Time and Ephemeris Time on the other hand. This makes it necessary to insert a leap second at irregular intervals.

If other effects were ignored, tidal acceleration would continue until the rotational period of the Earth matched the orbital period of the Moon. At that time, the Moon would always be overhead of a single fixed place on Earth. Such a situation already exists in the Pluto-Charon system. However, the slowdown of the Earth's rotation is not occurring fast enough for the rotation to lengthen to a month before other effects make this irrelevant: About 2.1 billion years from now, the continual increase of the Sun's radiation will cause the Earth's oceans to vaporize, removing the bulk of the tidal friction and acceleration. Even without this, the slowdown to a month-long day would still not have been completed by 4.5 billion years from now when the Sun will evolve into a red giant and possibly destroy both the Earth and Moon. (Tidal acceleration and solar mass loss is also moving the Earth outward from the Sun, but it is unknown whether it will be enough to save it from destruction.)

Tidal acceleration is one of the few examples in the dynamics of the solar system of a so-called **secular perturbation** of an orbit, *i.e.* a perturbation that continuously increases with time and is not periodic. Up to a high order of approximation, mutual gravitational perturbations between major or minor planets only cause periodic variations in their orbits, that is, parameters oscillate between maximum and minimum values. The tidal effect gives rise to a quadratic term in the equations, which leads to unbounded growth. In the mathematical theories of the planetary orbits that form the basis of ephemerides, quadratic and higher order secular terms do occur, but these are mostly Taylor expansions of very long time periodic terms. The reason that tidal effects are different is that unlike distant gravitational perturbations, friction is an essential part of tidal acceleration, and leads to permanent loss of energy from the dynamical system in the form of heat. In other words, we do not have a Hamiltonian system here.

Angular momentum and energy

The gravitational torque between the Moon and the tidal bulge of the Earth causes the Moon to be promoted in its orbit, and the Earth to be decelerated in its rotation. As in any physical process within an isolated system, total energy and angular momentum are conserved. Effectively, energy and angular momentum are transferred from the rotation of the Earth to the orbital motion of the Moon (however, most of the energy lost by the Earth is converted to heat, and only about one 30th is transferred to the Moon). The

Moon moves farther away from the Earth, so its potential energy (in the Earth's gravity well) increases. It stays in orbit, and from Kepler's 3rd law it follows that its velocity actually decreases, so the tidal action on the Moon actually causes a deceleration of its motion across the celestial sphere. Although its kinetic energy decreases, its potential energy increases by a larger amount. The tidal force has a component in the direction of the Moon's motion, and therefore increases its energy, but the non-tidal part of the Earth's gravity pulls (on average) slightly backwards on the Moon (which on average has a slight outward velocity), so the net result is that the Moon slows down. The Moon's orbital angular momentum increases.

The rotational angular momentum of the Earth decreases and consequently the length of the day increases. The *net* tide raised on Earth by the Moon is dragged ahead of the Moon by Earth's much faster rotation. **Tidal friction** is required to drag and maintain the bulge ahead of the Moon, and it dissipates the excess energy of the exchange of rotational and orbital energy between the Earth and Moon as heat. If the friction and heat dissipation were not present, the Moon's gravitational force on the tidal bulge would rapidly (within two days) bring the tide back into synchronization with the Moon, and the Moon would no longer recede. Most of the dissipation occurs in a turbulent bottom boundary layer in shallow seas such as the European shelf around the British Isles, the Patagonian shelf off Argentina, and the Bering Sea.

The dissipation of energy by tidal friction averages about 3.75 terawatts, of which 2.5 terawatts are from the principal M_2 lunar component and the remainder from other components, both lunar and solar.

An *equilibrium tidal bulge* does not really exist on Earth because the continents do not allow this mathematical solution to take place. Oceanic tides actually rotate around the oceans basin as vast *gyres* around several *amphidromic points* where no tide exists. The Moon pulls on each individual undulation as Earth rotates—some undulations are ahead of the Moon, others are behind it, while still others are on either side. The "bulges" that actually do exist for the Moon to pull on (and which pull on the Moon) are the net result of integrating the actual undulations over all the world's oceans. Earth's *net* (or *equivalent*) equilibrium tide has an amplitude of only 3.23 cm, which is totally swamped by oceanic tides that can exceed one metre.

Historical evidence

This mechanism has been working for 4.5 billion years, since oceans first formed on the Earth. There is geological and paleontological evidence that the Earth rotated faster and that the Moon was closer to the Earth in the remote past. *Tidal rhythmites* are alternating layers of sand and silt laid down offshore from estuaries having great tidal flows. Daily, monthly and seasonal cycles can be found in the deposits. This geological record is consistent with these conditions 620 million years ago: the day was 21.9±0.4 hours, and there were 13.1±0.1 synodic months/year and 400±7 solar days/year. The length of the year has remained virtually unchanged during this period because no evidence exists that the constant of gravitation has changed. The average recession rate of the Moon between then and now has been 2.17±0.31 cm/year, which is about half the present rate.

Quantitative description of the Earth-Moon case

The motion of the Moon can be followed with an accuracy of a few centimeters by lunar laser ranging (LLR). Laser pulses are bounced off mirrors on the surface of the moon, emplaced during the Apollo missions of 1969 to 1972 and by Lunokhod 2 in 1973. Measuring the return time of the pulse yields a very accurate measure of the distance. These measurements are fitted to the equations of motion. This yields numerical values for the Moon's secular acceleration in longitude and the rate of change of the semimajor axis of the Earth-Moon ellipse. From the period 1970–2007, the results are:

> $-25.85''/cy^2$ in ecliptic longitude
> (*cy* is centuries, here taken to the square)
> +38.14 mm/yr in the mean Earth-Moon distance

This is consistent with results from satellite laser ranging (SLR), a similar technique applied to artificial satellites orbiting the Earth, which yields a model for the gravitational field of the Earth, including that of the tides. The model accurately predicts the changes in the motion of the Moon.

Finally, ancient observations of solar eclipses give fairly accurate positions for the Moon at those moments. Studies of these observations give results consistent with the value quoted above.

The other consequence of tidal acceleration is the deceleration of the rotation of the Earth. The rotation of the Earth is somewhat erratic on all time scales (from hours to centuries) due to various causes. The small tidal effect cannot be observed in a short period, but the cumulative effect on the Earth's rotation as measured with a stable clock (ephemeris time, atomic time) of a shortfall of even a few milliseconds every day becomes readily noticeable in a few centuries. Since some event in the remote past, more days and hours have passed (as measured in full rotations of the Earth) (Universal Time) than as measured with stable clocks calibrated to the present, longer length of the day (ephemeris time). This is known as ΔT. Recent values can be obtained from the International Earth Rotation and Reference Systems Service (IERS). A table of the actual length of the day in the past few centuries is also available.

From the observed change in the Moon's orbit, the corresponding change in the length of the day can be computed:

> +2.3 ms/cy
> (*cy* is centuries).

However, from historical records over the past 2700 years the following average value is found:

$+1.70 \pm 0.05$ ms/cy

The corresponding cumulative value is a parabola having a coefficient of T^2 (time in centuries squared) of:

$\Delta T = +31$ s/cy²

Opposing the tidal deceleration of the Earth is a mechanism that is in fact accelerating the rotation. The Earth is not a sphere, but rather an ellipsoid that is flattened at the poles. SLR has shown that this flattening is decreasing. The explanation is, that during the ice age large masses of ice collected at the poles, and depressed the underlying rocks. The ice mass started disappearing over 10000 years ago, but the Earth's crust is still not in hydrostatic equilibrium and is still rebounding (the relaxation time is estimated to be about 4000 years). As a consequence, the polar diameter of the Earth increases, and since the mass and density remain the same, the volume remains the same; therefore the equatorial diameter is decreasing. As a consequence, mass moves closer to the rotation axis of the Earth. This means that its moment of inertia is decreasing. Because its total angular momentum remains the same during this process, the rotation rate increases. This is the well-known phenomenon of a spinning figure skater who spins ever faster as she retracts her arms. From the observed change in the moment of inertia the acceleration of rotation can be computed: the average value over the historical period must have been about −0.6 ms/cy. This largely explains the historical observations.

Other cases of tidal acceleration

Most natural satellites of the planets undergo tidal acceleration to some degree (usually small), except for the two classes of tidally decelerated bodies. In most cases, however, the effect is small enough that even after billions of years most satellites will not actually be lost. The effect is probably most pronounced for Mars' second moon Deimos, which may become an Earth-crossing asteroid after it leaks out of Mars' grip. The effect also arises between different components in a binary star.

Tidal deceleration

This comes in two varieties:

1. *Fast satellites*: Some inner moons of the gas giant planets and Phobos orbit within the synchronous orbit radius so that their orbital period is shorter than their planet's rotation. In this case the tidal bulges raised by the moon on their planet lag behind the moon, and act to *decelerate* it in its orbit. The net effect is a decay of that moon's orbit as it gradually spirals towards the planet. The planet's rotation also speeds up slightly in the process. In the distant future these moons will impact the planet or cross within their Roche limit and be tidally disrupted into fragments. However, all such moons in the solar system are very small bodies and the tidal bulges raised by them on the planet are also small, so the effect is usually weak and the orbit decays slowly. The moons affected are:

    - *Around Mars*: Phobos
    - *Around Jupiter*: Metis and Adrastea
    - *Around Saturn*: none, except for the ring particles (like Jupiter, Saturn is a very rapid rotator but has no satellites close enough)
    - *Around Uranus*: Cordelia, Ophelia, Bianca, Cressida, Desdemona, Juliet, Portia, Rosalind, Cupid, Belinda, and Perdita
    - *Around Neptune*: Naiad, Thalassa, Despina, Galatea and Larissa
2. *Retrograde satellites*: All retrograde satellites experience tidal deceleration to some degree because the moon's orbital motion and the planet's rotation are in opposite directions, causing restoring forces from their tidal bulges. A difference to the previous "fast satellite" case here is that the planet's rotation is also slowed down rather than sped up (angular momentum is still conserved because in such a case the values for the planet's rotation and the moon's revolution have opposite signs). The only satellite in the Solar System for which this effect is non-negligible is Neptune's moon Triton. All the other retrograde satellites are on distant orbits and tidal forces between them and the planet are negligible.

The planet Venus is believed to have no satellites chiefly because any hypothetical satellites would have suffered deceleration long ago, from either cause; Venus has a very slow *and* retrograde rotation.

Chapter 7

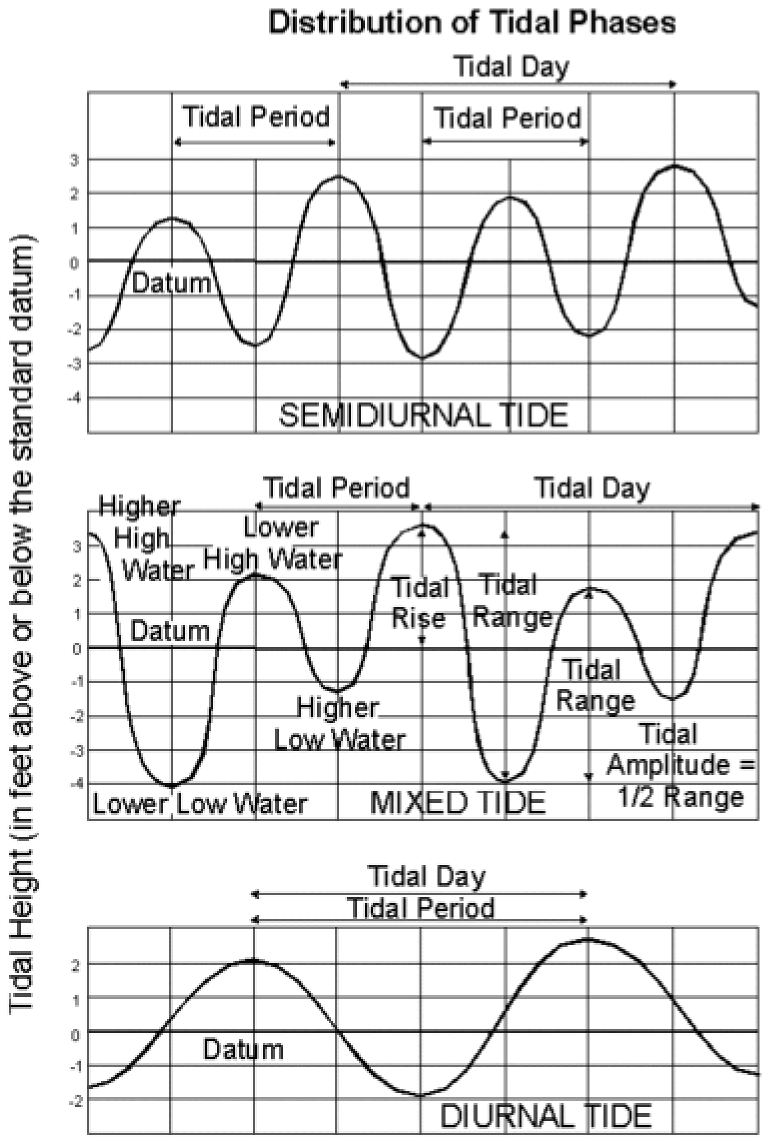
Tidal Power

Tidal power, also called **tidal energy**, is a form of hydropower that converts the energy of tides into electricity or other useful forms of power. The first large-scale tidal power plant (the Rance Tidal Power Station) started operation in 1966.

Although not yet widely used, tidal power has potential for future electricity generation. Tides are more predictable than wind energy and solar power. Among sources of renewable energy, tidal power has traditionally suffered from relatively high cost and limited availability of sites with sufficiently high tidal ranges or flow velocities, thus constricting its total availability. However, many recent technological developments and improvements, both in design (e.g. dynamic tidal power, tidal lagoons) and turbine technology (e.g. new axial turbines, crossflow turbines), indicate that the total availability of tidal power may be much higher than previously assumed, and that economic and environmental costs may be brought down to competitive levels.

Historically, tide mills have been used, both in Europe and on the Atlantic coast of North America. The earliest occurrences date from the Middle Ages, or even from Roman times.

Generation of tidal energy



Variation of tides over a day

Tidal power is the only form of energy which derives directly from the relative motions of the Earth–Moon system, and to a lesser extent from the Earth–Sun system. Tidal forces produced by the Moon and Sun, in combination with Earth's rotation, are responsible for the generation of the tides. Other sources of energy originate directly or indirectly from the Sun, including fossil fuels, conventional hydroelectric, wind, biofuels, wave power and solar. Nuclear energy makes use of Earth's mineral deposits of fissile elements, while geothermal power uses the Earth's internal heat which comes from a combination of residual heat from planetary accretion (about 20%) and heat produced through radioactive decay (80%).

Tidal energy is extracted from the relative motion of large bodies of water. Periodic changes of water levels, and associated tidal currents, are due to the gravitational attraction of the Sun and Moon. Magnitude of the tide at a location is the result of the changing positions of the Moon and Sun relative to the Earth, the effects of Earth rotation, and the local geography of the sea floor and coastlines.

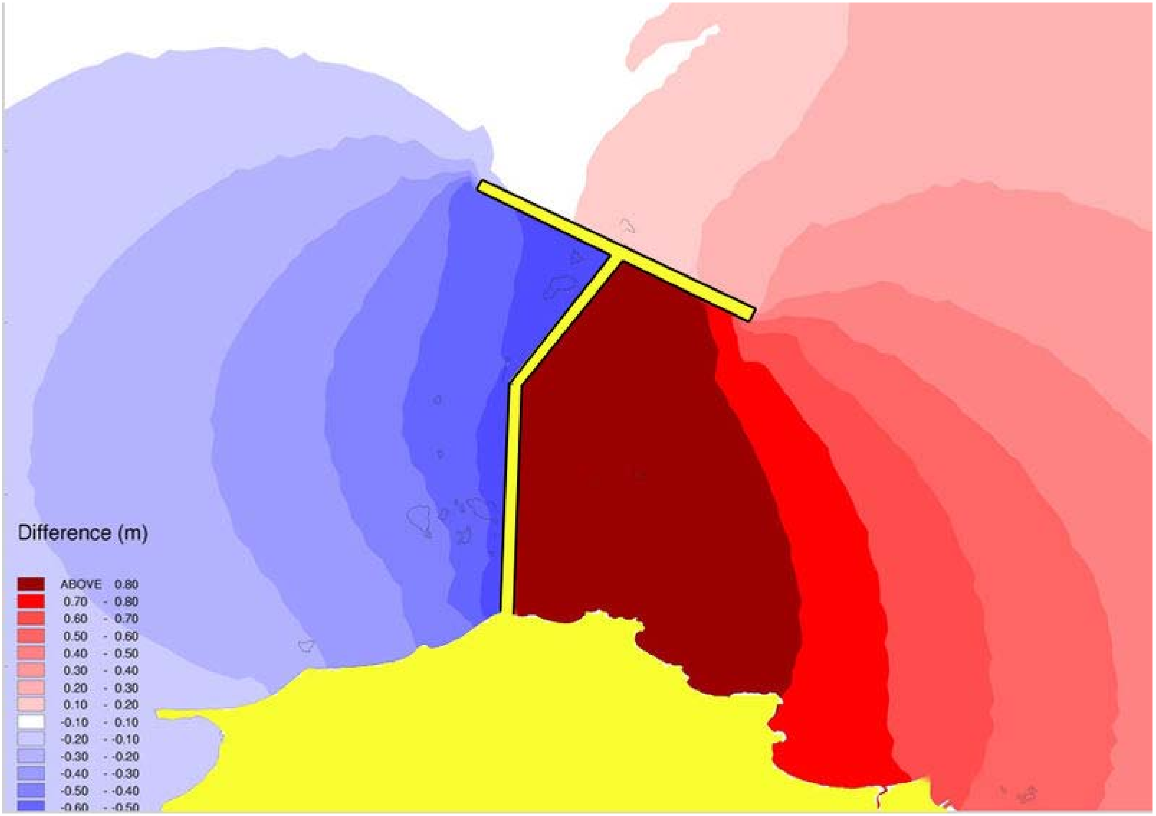
Because the Earth's tides are ultimately due to gravitational interaction with the Moon and Sun and the Earth's rotation, tidal power is practically inexhaustible and classified as a renewable energy resource.

A tidal generator uses this phenomenon to generate electricity. Greater tidal variation or tidal current velocities can dramatically increase the potential for tidal electricity generation.

The movement of the tides causes a continual loss of mechanical energy in the Earth–Moon system due to pumping of water through the natural restrictions around coastlines, and consequent viscous dissipation at the seabed and in turbulence. This loss of energy has caused the rotation of the Earth to slow in the 4.5 billion years since formation. During the last 620 million years the period of rotation has increased from 21.9 hours to the 24 hours we see now; in this period the Earth has lost 17% of its rotational energy. While tidal power may take additional energy from the system, increasing the rate of slowdown, the effect would be noticeable over millions of years only, thus being negligible.

Generating methods

The world's first commercial-scale and grid-connected tidal stream generator – SeaGen – in Strangford Lough. The strong wake shows the power in the tidal current.



Top-down view of a DTP dam. Blue and dark red colors indicate low and high tides, respectively.

Tidal power can be classified into three generating methods:

Tidal stream generator

Tidal stream generators (or TSGs) make use of the kinetic energy of moving water to power turbines, in a similar way to wind turbines that use moving air. This method is gaining in popularity because of the lower cost and lower ecological impact compared to tidal barrages.

Tidal barrage

Tidal barrages make use of the potential energy in the difference in height (or *head*) between high and low tides. Barrages are essentially dams across the full width of a tidal estuary, and suffer from very high civil infrastructure costs, a worldwide shortage of viable sites and environmental issues.

Dynamic tidal power

Dynamic tidal power (or DTP) is a theoretical generation technology that would exploit an interaction between potential and kinetic energies in tidal flows. It proposes that very long dams (for example: 30–50 km length) be built from coasts straight out into the sea or ocean, without enclosing an area. Tidal phase differences are introduced by the dam, leading to a significant water level differential (at least 2–3 meters) in shallow coastal seas featuring strong coast-parallel oscillating tidal currents such as found in the UK, China and Korea. Each dam would generate power at a scale of 6 - 15 GW.

Current and future tidal power schemes

- The first tidal power station was the Rance tidal power plant built over a period of 6 years from 1960 to 1966 at La Rance, France. It has 240 MW installed capacity.
- The first tidal power site in North America is the Annapolis Royal Generating Station, Annapolis Royal, Nova Scotia, which opened in 1984 on an inlet of the Bay of Fundy. It has 20 MW installed capacity.
- The Jiangxia Tidal Power Station, south of Hangzhou in China has been operational since 1985, with current installed capacity of 3.2 MW. More tidal power is planned near the mouth of the Yalu River.
- The first in-stream tidal current generator in North America (Race Rocks Tidal Power Demonstration Project) was installed at Race Rocks on southern Vancouver Island in September 2006. The next phase in the development of this tidal current generator will be in Nova Scotia.
- A small project was built by the Soviet Union at Kislaya Guba on the Barents Sea. It has 0.4 MW installed capacity. In 2006 it was upgraded with a 1.2MW experimental advanced orthogonal turbine.
- Jindo Uldolmok Tidal Power Plant in South Korea is a tidal stream generation scheme planned to be expanded progressively to 90 MW of capacity by 2013. The first 1 MW was installed in May 2009.
- A 1.2 MW SeaGen system became operational in late 2008 on Strangford Lough in Northern Ireland.
- 254 MW Sihwa Lake Tidal Power Plant in South Korea is under construction and planned to be completed by the end of 2010.
- The contract for an 812 MW tidal barrage near Ganghwa Island north-west of Incheon has been signed by Daewoo. Completion is planned for 2015.

- A 1,320 MW barrage built around islands west of Incheon is proposed by the Korean government, with projected construction start in 2017.

- Other South Korean projects include barrages planned for Garorim Bay, Ansanman, and Swaseongho, and tidal generation associated with the Saemangeum reclamation project. The barrages are all in the multiple-hundred megawatts range.

- The Indian state of Gujarat is planning to host South Asia's first commercial-scale tidal power station. The company Atlantis Resources is to install a 50MW tidal farm in the Gulf of Kutch on India's west coast, with construction starting early in 2012.

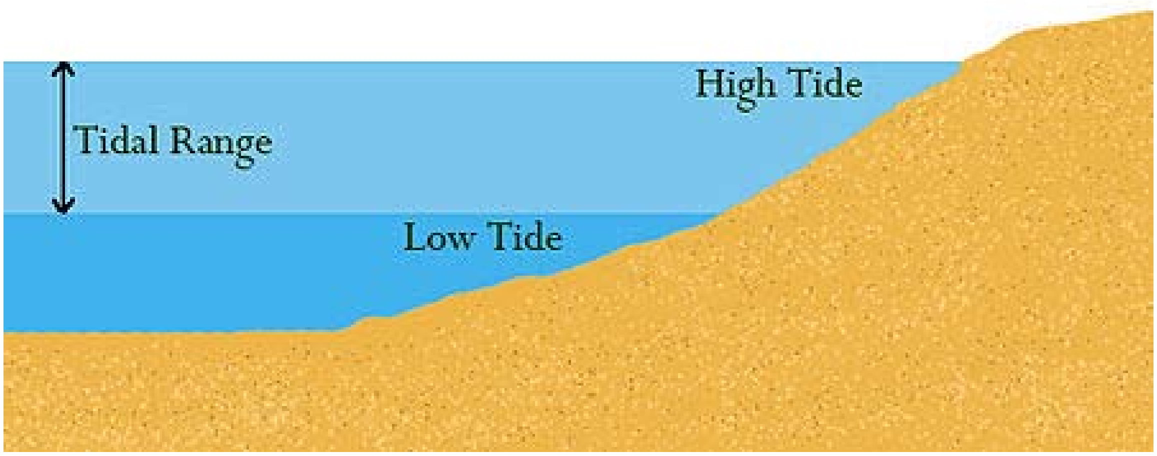
- Estimates for new tidal barrages in England give the potential generation at 5.6GW mean power.

Country	Place	Mean tidal range (m)	Area of basin (km²)	Maximum capacity (MW)
United Kingdom	River Severn	7.8	450	8,640
Russia	Penzhinskaya Bay	6.0	20,500	87,000

Chapter 8

Tidal Range & Amphidromic Point

Tidal Range



The tidal range is difference between the high tide and the low tide.

The **tidal range** is the vertical difference between the high tide and the succeeding low tide. Tides are the rise and fall of sea levels caused by the combined effects of the gravitational forces exerted by the Moon and the Sun and the rotation of the Earth. The tidal range is not constant, but changes depending on where the sun and the moon are.

The most extreme tidal range occurs around the time of the full or new moons, when the gravitational forces of both the Sun and Moon are in phase reinforcing each other in the same direction (new moon), or are exactly the opposite phase (full). This type of tide is known as a spring tide. During neap tides, when the Moon and Sun's gravitational force vectors act in quadrature (making a right angle to the Earth's orbit), the difference between high and low tides is smaller. Neap tides occur during the first and last quarters of the moon's phases. The largest annual tidal range can be expected around the time of the Equinox, if coincidental with a spring tide.

Tidal data for coastal areas is published by the National Hydrographic service of the country concerned Tidal data is based on astronomical phenomena and is predictable. Storm force winds blowing from a steady direction for a prolonged time interval combined with low barometric pressure can increase the tidal range particularly in narrow bays. Such weather related effects on the tide, which can cause ranges in excess of predicted values and can cause localized flooding are not calculable in advance.

Geography

The typical tidal range in the open ocean is about 0.6 meters (2 feet). Closer to the coast, this range is much greater. Coastal tidal ranges vary globally and can differ anywhere from near zero to over 11 meters (38 feet). The exact range depends on the volume of water adjacent to the coast, and the geography of the basin the water sits in. Larger bodies of water have higher ranges, and the geography can act as a funnel amplifying or dispersing the tide. The world's largest tidal range of 11.7 meters (38.4 feet) occurs at Burntcoat Head in the Bay of Fundy,Eastern Canada. The Bristol Channel, between England and Wales, regularly experiences tidal ranges of up to 14 meters. The top 50 locations with the largest tidal ranges world-wide are listed by the National Oceanic and Atmospheric Administration of the US.

Some of the smallest tidal ranges occur in the Mediterranean, Baltic, and Caribbean Seas. A point within a tidal system where the tidal range is almost zero is called an amphidromic point.

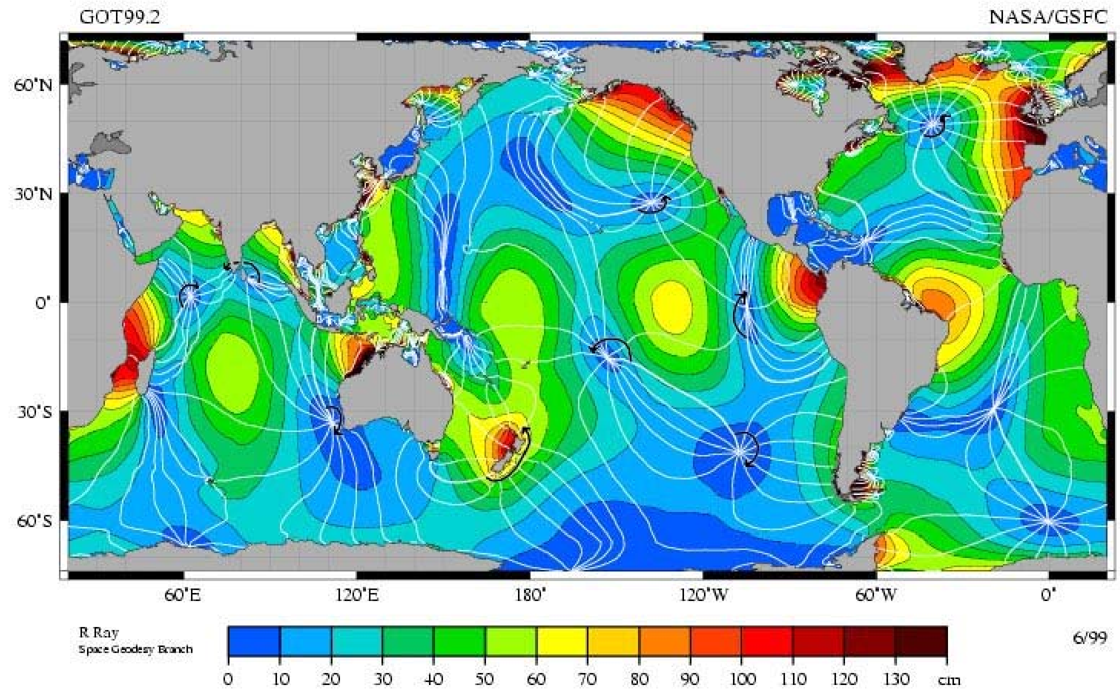
Classification

The tidal range has been classified as:

- **Micromareal**, when the tidal range is lower than 2 meters.
- **Mesomareal**, when the tidal range is between 2 meters and 4 meters.
- **Macromareal**, when the tidal range is higher than 4 meters.

Amphidromic Point

An **amphidromic point** is a point within a tidal system where the tidal range is almost zero. The tidal range (the height difference between high tide and low tide) is zero at the amphidromic point and increases with distance from this point. These points are called nodes.



The M_2 tidal constituent, the amplitude indicated by color. The white lines are cotidal lines spaced at phase intervals of 30° (a bit over 1 hr). The amphidromic points are the dark blue areas where the lines come together.

Amphidromic points occur because of the Coriolis effect and interference within oceanic basins, seas and bays creating a wave pattern — called an **amphidromic system** — which rotates around the amphidromic point. At the amphidromic point, there is no vertical movement from tidal action. There can be tidal currents as the water levels on either side of the amphidromic point are not the same.

In most locations M_2 is the largest (semidiurnal) tidal constituent, with an amplitude of roughly half of the full tidal range. Cotidal points means they reach high tide at the same time and low tide at the same time. In the accompanying figure, the low tide lags or leads by 1 hr 2 min from its neighboring lines. Where the lines meet are amphidromes and the tide rotates around them; for example: along the Chilean coast, and from southern Mexico to Peru the tide propagates southward, while from Baja California to Alaska the tide propagates northward.

Amphidromic points in the M_2 *tidal constituent*

Based on the accompanying figure, the set of clockwise amphidromic points includes:

- north of the Seychelles
- near Enderby Land
- off Perth
- east of New Guinea

- south of Easter Island
- west of the Galapagos Islands
- north of Queen Maud Land

Anti-clockwise amphidromic points include:

- near Sri Lanka
- north of New Guinea
- at Tahiti
- between Mexico and Hawaii
- near the Leeward Islands
- east of Newfoundland
- midway between Rio de Janeiro and Angola
- east of Iceland

The islands of Madagascar and New Zealand are amphidromic points in the sense that the tide goes around them (counterclockwise in both cases) in about 12 and a half hours, but the amplitude of the tides on their coasts is in some places large.

Chapter 9

Tidal Bore

The tidal bore in Upper Cook Inlet, Alaska

A **tidal bore** (or simply **bore** in context, or also **aegir**, **eagre**, or **eygre**) is a tidal phenomenon in which the leading edge of the incoming tide forms a wave (or waves) of water that travel up a river or narrow bay against the direction of the river or bay's current. As such, it is a true *tidal wave* and not to be confused with a tsunami, which is a large ocean wave traveling primarily on the open ocean.

The phenomenon

Bores occur in relatively few locations worldwide, usually in areas with a large tidal range (typically more than 6 metres (20 ft) between high and low water), and where incoming tides are funneled into a shallow, narrowing river or lake via a broad bay. The funnel-like shape not only increases the tidal range, but it can also decrease the duration of the flood tide, down to a point where the flood appears as a sudden increase in the water level. Note the tidal bore takes place during the flood tide and never during the ebb tide.

A tidal bore may take on various forms, ranging from a single breaking wavefront with a roller — somewhat like a hydraulic jump — to ‘undular bores’, comprising a smooth wavefront followed by a train of secondary waves (*whelps*). Large bores can be particularly very unsafe for shipping,but also present opportunities for river surfing.

Two key features of a tidal bore are the intense turbulence and turbulent mixing generated during the bore propagation, as well as its rumbling noise. The visual observations of tidal bores highlight the turbulent nature of the surging waters. The tidal bore induces a strong turbulent mixing in the estuarine zone, and the effects may be felt along considerable distances. The velocity observations indicate a rapid deceleration of the flow associated with the passage of the bore as well as large velocity fluctuations. A tidal bore creates a powerful roar that combines the sounds caused by the turbulence in the bore front and whelps, entrained air bubbles in the bore roller, sediment erosion beneath the bore front and of the banks, scouring of shoals and bars, and impacts on obstacles. The bore rumble is heard far away because its low frequencies can travel over long distances. The low-frequency sound is a characteristic feature of the advancing roller in which the air bubbles entrapped in the large-scale eddies are acoustically active and play the dominant role in the rumble sound generation.

The word *bore* derives through Old English from the Old Norse word *bára*, meaning a wave or swell.

Rivers with tidal bores

Rivers that have been known to exhibit bores include those listed below.

Asia

- Ganges–Brahmaputra, India, Bangladesh
- Indus River, India, Pakistan
- Qiantang River, China, which has the world's largest bore, up to 9 metres (30 ft) high, traveling at up to 40 kilometres (25 mi) per hour.
- Batang Lupar or Lupar River, near Sri Aman, Malaysia. The tidal bore is locally known as *benak* .
- Bono, Kampar River, Indonesia. The phenomenon is feared by the locals to sink ships. It is reported to break up to 130 kilometres (81 mi) inland.

Australia

- Styx River, Queensland, Australia

- Daly River, Northern Territory, Australia

Europe

United Kingdom

The Trent Aegir seen from West Stockwith, Nottinghamshire, 20 September 2005

The Trent Aegir at Gainsborough, Lincolnshire, 20 September 2005

- River Dee, Wales / England
- River Mersey
- The Severn bore on the River Severn, Wales / England up to 2 metres (6.6 ft) high
- The Trent Aegir on the River Trent, up to 1.5 metres (4.9 ft) high, England. Also other tributaries of the Humber Estuary
- River Parrett
- River Welland
- River Kent
- River Great Ouse
- River Ouse, Yorkshire, like the Trent bore, this is also known as "the Aegir".
- River Eden
- River Esk
- River Nith
- River Lune, Lancashire
- River Ribble, Lancashire

Tidal bore on the River Ribble.

France

The phenomenon is generally named *un mascaret* in French but some other local names are preferred.

- Seine, locally named *la barre*, had a significant bore until the 1960s. Since then it has been practically eliminated by dredging and river training .
- Baie du Mont Saint Michel including Couesnon, Sélune, Sée.
- Arguenon
- Baie de la Frênaye
- Vire
- Sienne
- Vilaine, locally named *le mascarin*
- Dordogne
- Garonne

North America

United States

Tidal bore on the Petitcodiac River

- The Turnagain arm of Cook Inlet, Alaska. Up to 2 metres (6.6 ft) and 20 km/h.

Canada

Most rivers draining into the upper Bay of Fundy between Nova Scotia and New Brunswick have tidal bores. Notable ones include:

- The Petitcodiac River. Formerly the highest bore in North America at over 2 metres (6.6 ft); however, causeway construction and extensive silting reduced it to little more than a ripple, until the causeway gates were opened on April 14, 2010 as part of the Petitcodiac River Restoration project and the tidal bore began to grow again.

- The Shubenacadie River, also off the Bay of Fundy in Nova Scotia. When the tidal bore approaches, completely drained riverbeds are filled. It has claimed the lives of several tourists who were in the riverbeds when the bore came in. Tour boat operators offer rafting excursions in summer.
- The bore is fastest and highest on some of the smaller rivers that connect to the Bay including the River Hebert and Maccan River on Cumberland Basin, the St. Croix, Herbert and Kennetcook Rivers in the Minas Basin, and the Salmon River in Truro.

Mexico

There is a tidal bore on the Sea of Cortez in Mexico at the entrance of the Colorado River.

South America

- Amazon River in Brazil and Orinoco River in Venezuela, up to 4 metres (13 ft) high, running at up to 13 miles per hour (21 km/h). It is known locally as the *pororoca*.
- Mearim River in Brazil.
- Araguari River in Brazil.

Lakes with tidal bores

Lakes with an ocean inlet can also exhibit tidal bores.

North America

- Nitinat Lake on Vancouver Island has a sometimes dangerous tidal bore at Nitinat Narrows where the lake meets the Pacific Ocean. The lake is popular with windsurfers due to its consistent winds.

Chapter 10

Theory of Tides

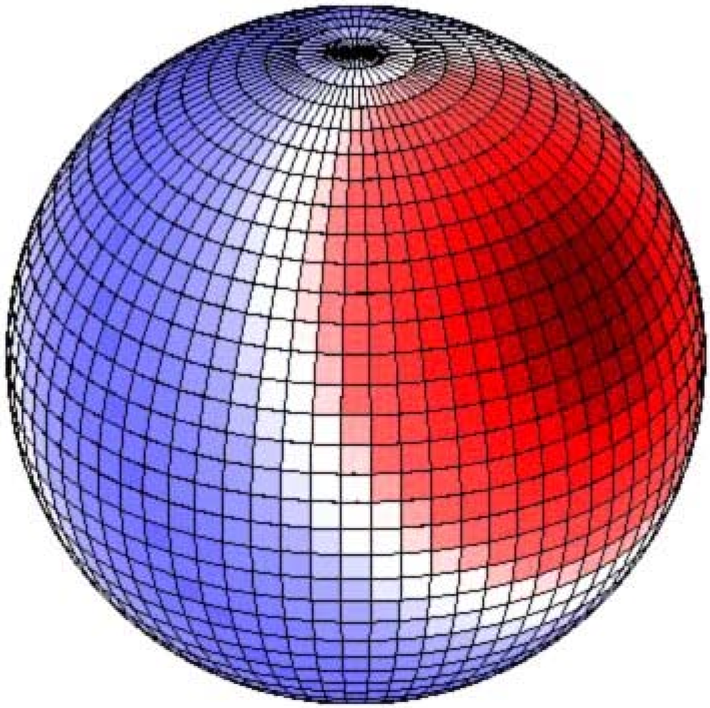
The **theory of tides** is the application of continuum mechanics to interpret and predict the tidal deformations of planetary and satellite bodies and their atmospheres and oceans, under the gravitational loading of another astronomical body or bodies. It commonly refers to the fluid dynamic motions for the Earth's oceans.

Origin of theory

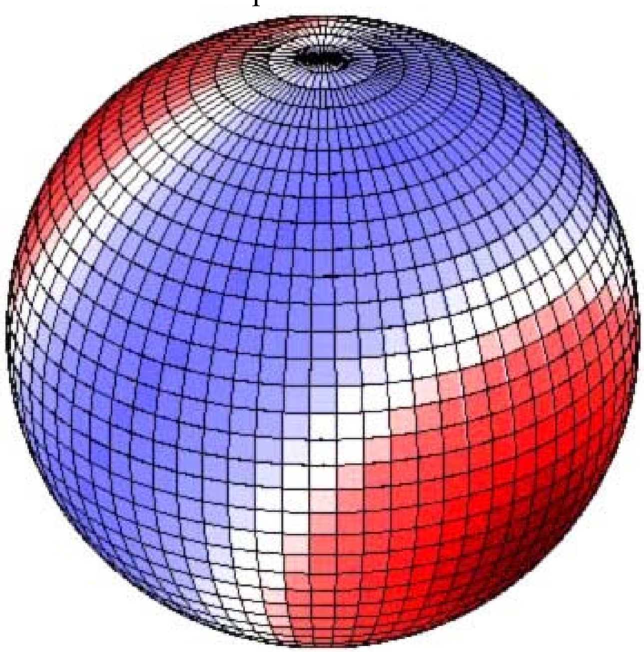
In 1616, Galileo Galilei wrote *Discourse on the Tides* (in Italian: *Discorso del flusso e reflusso del mare*), a paper in which he tried to explain the occurrence of the tides as the result of the Earth's rotation around the Sun. However, Galileo's theory was, in the later Newtonian terms, an error. Later analysis over the centuries had led to the current tidal physics.

Tidal physics

Tidal forcing



A. Lunar gravitational potential: this depicts the Moon directly over 30° N (or 30° S) viewed from above the Northern Hemisphere.



B. This view shows same potential from 180° from view *A*. Viewed from above the Northern Hemisphere. Red up, blue down.

The forces discussed here apply to body (Earth tides), oceanic and atmospheric tides. Atmospheric tides on Earth, however, tend to be dominated by forcing due to solar heating.

On the planet (or satellite) experiencing tidal motion consider a point at latitude φ and longitude λ at distance *a* from the center of mass, then this point can be written in cartesian coordinates as $\mathbf{p} = a\mathbf{x}$ where

$$\mathbf{x} = (\cos\lambda\cos\varphi, \sin\lambda\cos\varphi, \sin\varphi).$$

Let δ be the declination and α be the right ascension of the deforming body, the Moon for example, then the vector direction is

$$\mathbf{x}_m = (\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta),$$

and r_m be the orbital distance between the center of masses and M_m the mass of the body. Then the force on the point is

$$\mathbf{F}_a = \frac{GM_m(r_m\mathbf{x}_m - a\mathbf{x})}{R^3}.$$

where $R = \|r_m\mathbf{x}_m - a\mathbf{x}\|$ For a circular orbit the angular momentum ω centripetal acceleration balances gravity at the planetary center of mass

$$Mr_{cm}\omega^2 = \frac{GMM_m}{r_m^2}.$$

where r_{cm} is the distance between the center of mass for the orbit and planet and *M* is the planetary mass. Consider the point in the reference fixed without rotation, but translating at a fixed translation with respect to the center of mass of the planet. The body's centripetal force acts on the point so that the total force is

$$\mathbf{F}_p = \frac{GM_m(r_m\mathbf{x}_m - a\mathbf{x})}{R^3} - r_{cm}\omega^2\mathbf{x}_m.$$

Substituting for center of mass acceleration, and reordering

$$\mathbf{F}_p = GM_m r_m\left(\frac{1}{R^3} - \frac{1}{r_m^3}\right)\mathbf{x}_m - \frac{(GM_m a\mathbf{x})}{R^3}.$$

In ocean tidal forcing, the radial force is not significant, the next step is to rewrite the $\mathbf{x}_m$ coefficient. Let $\varepsilon = \frac{a}{r_m}$ then

$$R = r_m\sqrt{1+\varepsilon^2-2\varepsilon(\mathbf{x}_m,\mathbf{x})}$$

where $(\mathbf{x}_m,\mathbf{x}) = \cos z$ is the inner product determining the angle z of the deforming body or Moon from the zenith. This means that

$$\left(\frac{1}{R^3}-\frac{1}{r_m^3}\right) \approx \frac{3\varepsilon\cos z}{r_m^3},$$

if ε is small. If particle is on the surface of the planet then the local gravity is $g = \frac{GM}{a^2}$ and set $\mu = \frac{M_m}{M}$.

$$\mathbf{F}_p = 3g\mu\varepsilon^3\cos z\mathbf{x}_m - \frac{(g\mu a^3\mathbf{x})}{R^3} + O(\varepsilon^4),$$

which is a small fraction of g. Note also that force is attractive toward the Moon when the $z < \pi / 2$ and repulsive when $z > \pi / 2$.

This can also be used to derive a tidal potential.

Laplace's tidal equations

in 1776, Pierre-Simon Laplace formulated a single set of linear partial differential equations, for tidal flow described as a barotropic two-dimensional sheet flow. Coriolis effects are introduced as well as lateral forcing by gravity. Laplace obtained these equations by simplifying the fluid dynamic equations. But they can also be derived from energy integrals via Lagrange's equation.

For a fluid sheet of average thickness D, the vertical tidal elevation ς, as well as the horizontal velocity components u and v (in the latitude φ and longitude λ directions, respectively) satisfy **Laplace's tidal equations**:

$$\frac{\partial\zeta}{\partial t} + \frac{1}{a\cos(\varphi)}\left[\frac{\partial}{\partial\lambda}(uD) + \frac{\partial}{\partial\varphi}\left(vD\cos(\varphi)\right)\right] = 0,$$

$$\frac{\partial u}{\partial t} - v\left(2\Omega\sin(\varphi)\right) + \frac{1}{a\cos(\varphi)}\frac{\partial}{\partial\lambda}\left(g\zeta+U\right) = 0 \qquad \text{and}$$

$$\frac{\partial v}{\partial t} + u\left(2\Omega\sin(\varphi)\right) + \frac{1}{a}\frac{\partial}{\partial\varphi}\left(g\zeta+U\right) = 0,$$

where Ω is the angular frequency of the planet's rotation, g is the planet's gravitational acceleration at the mean ocean surface, and U is the external gravitational tidal-forcing potential.

William Thomson (Lord Kelvin) rewrote Laplace's momentum terms using the curl to find an equation for vorticity. Under certain conditions this can be further rewritten as a conservation of vorticity.

Tidal analysis and prediction

Harmonic analysis

Laplace's improvements in theory were substantial, but they still left prediction in an approximate state. This position changed in the 1860s when the local circumstances of tidal phenomena were more fully brought into account by William Thomson's application of Fourier analysis to the tidal motions. Thomson's work in this field was then further developed and extended by George Darwin: Darwin's work was based on the lunar theory current in his time. His symbols for the tidal harmonic constituents are still used. Darwin's harmonic developments of the tide-generating forces were later brought up to date with modern developments by A T Doodson whose development of the tide generating potential (TGP) in harmonic form was carried out and published in 1921: Doodson distinguished 388 tidal frequencies. Doodson's analysis of 1921 was based on the then-latest lunar theory of E W Brown.

Doodson devised a practical system for specifying the different harmonic components of the tide-generating potential, the Doodson Numbers, a system still in use.

Since the mid-twentieth century further analysis has generated many more terms than Doodson's 388. About 62 constituents are of sufficient size to be considered for possible use in marine tide prediction, but sometimes many less even than that can predict tides to useful accuracy. The calculations of tide predictions using the harmonic constituents are laborious, and from the 1870s to about the 1960s they were carried out using a mechanical tide-predicting machine, a special-purpose form of analog computer now superseded in this work by digital electronic computers that can be programmed to carry out the same computations.

Tidal constituents

Tidal constituents combine to give an endlessly-varying aggregate because of their different and incommensurable frequencies: the effect is visualized in an animation of the American Mathematical Society illustrating the way in which the components used to be mechanically combined in the tide-predicting machine. Amplitudes of tidal constituents are given below for the following example locations:

ME Eastport,
MS Biloxi,

PR San Juan,
AK Kodiak,
CA San Francisco, and
HI Hilo.

Higher harmonics	**Darwin**	**Period**	**Phase**	**Doodson coefs**				**Doodson**	**Amplitude at example location (cm)**						**NOAA**
Species	**Symbol**	**(hr)**	**rate(°/hr)**	***n_1*** (L)	n_2 (m)	n_3 (y)	n_4 (mp)	**number**	**ME**	**MS**	**PR**	**AK**	**CA**	**HI**	**order**
Shallow water overtides of principal lunar	M_4	6.210300601	57.9682084	4				455.555	6.0	0.6		0.9	2.3		**5**
Shallow water overtides of principal lunar	M_6	4.140200401	86.9523127	6				655.555	5.1	0.1		1.0			**7**
Shallow water terdiurnal	MK_3	8.177140247	44.0251729	3	1			365.555				0.5	1.9		**8**
Shallow water overtides of principal solar	S_4	6	60	4	4	-4		491.555		0.1					**9**
Shallow water quarter diurnal	MN_4	6.269173724	57.4238337	4	-1		1	445.655	2.3			0.3	0.9		**10**
Shallow water overtides of principal solar	S_6	4	90	6	6	-6		*		0.1					12
Lunar terdiurnal	M_3	8.280400802	43.4761563	3				355.555					0.5		32
Shallow water terdiurnal	2"MK$_3$	8.38630265	42.9271398	3	-1			345.555	0.5			0.5	1.4		34
Shallow water eighth diurnal	M_8	3.105150301	115.9364166	8				855.555	0.5	0.1					36
Shallow water quarter diurnal	MS_4	6.103339275	58.9841042	4	2	-2		473.555	1.8			0.6	1.0		37
Semi-diurnal	**Darwin**	**Period**	**Phase**	**Doodson coefs**				**Doodson**	**Amplitude at example location (cm)**						**NOAA**
Species	**Symbol**	**(hr)**	**(°/hr)**	n_1 (L)	n_2 (m)	n_3 (y)	n_4 (mp)	**number**	**ME**	**MS**	**PR**	**AK**	**CA**	**HI**	**order**
Principal lunar semidiurnal	M_2	12.4206012	28.9841042	2				255.555	268.7	3.9	15.9	97.3	58.0	23.0	**1**
Principal solar semidiurnal	S_2	12	30	2	2	-2		273.555	42.0	3.3	2.1	32.5	13.7	9.2	**2**
Larger lunar elliptic semidiurnal	N_2	12.65834751	28.4397295	2	-1		1	245.655	54.3	1.1	3.7	20.1	12.3	4.4	**3**
Larger lunar evectional	ν_2	12.62600509	28.5125831	2	-1	2	-1	247.455	12.6	0.2	0.8	3.9	2.6	0.9	11
Variational	MU_2	12.8717576	27.9682084	2	-2	2		237.555	2.0	0.1	0.5	2.2	0.7	0.8	13
Lunar elliptical semidiurnal second-order	2"N$_2$	12.90537297	27.8953548	2	-2		2	235.755	6.5	0.1	0.5	2.4	1.4	0.6	14
Smaller lunar evectional	λ_2	12.22177348	29.4556253	2	1	-2	1	263.655	5.3		0.1	0.7	0.6	0.2	16
Larger solar elliptic	T_2	12.01644934	29.9589333	2	2	-3		272.555	3.7	0.2	0.1	1.9	0.9	0.6	27
Smaller solar elliptic	R_2	11.98359564	30.0410667	2	2	-1		274.555	0.9			0.2	0.1	0.1	28
Shallow water semidiurnal	$2SM_2$	11.60695157	31.0158958	2	4	-4		291.555	0.5						31
Smaller lunar elliptic semidiurnal	L_2	12.19162085	29.5284789	2	1		-1	265.455	13.5	0.1	0.5	2.4	1.6	0.5	33
Lunisolar semidiurnal	K_2	11.96723606	30.0821373	2	2			275.555	11.6	0.9	0.6	9.0	4.0	2.8	35

Diurnal	**Darwin**	**Period**	**Phase**	**Doodson coefs**				**Doodson**	**Amplitude at example location (cm)**						**NOAA**
Species	**Symbol**	**(hr)**	**(°/hr)**	$\boldsymbol{n_1}$ (L)	n_2 (m)	n_3 (y)	n_4 (mp)	**number**	**ME**	**MS**	**PR**	**AK**	**CA**	**HI**	**order**
Lunar diurnal	K_1	23.93447213	15.0410686	1	1			165.555	15.6	16.2	9.0	39.8	36.8	16.7	**'4**
Lunar diurnal	O_1	25.81933871	13.9430356	1	-1			145.555	11.9	16.9	7.7	25.9	23.0	9.2	**6**
Lunar diurnal	OO_1	22.30608083	16.1391017	1	3			185.555	0.5	0.7	0.4	1.2	1.1	0.7	15
Solar diurnal	S_1	24	15	1	1	-1		164.555	1.0		0.5	1.2	0.7	0.3	17
Smaller lunar elliptic diurnal	M_1	24.84120241	14.4920521	1				155.555	0.6	1.2	0.5	1.4	1.1	0.5	18
Smaller lunar elliptic diurnal	J_1	23.09848146	15.5854433	1	2		-1	175.455	0.9	1.3	0.6	2.3	1.9	1.1	19
Larger lunar evectional diurnal	ρ	26.72305326	13.4715145	1	-2	2	-1	137.455	0.3	0.6	0.3	0.9	0.9	0.3	25
Larger lunar elliptic diurnal	Q_1	26.868350	13.3986609	1	-2		1	135.655	2.0	3.3	1.4	4.7	4.0	1.6	26
Larger elliptic diurnal	$2Q_1$	28.00621204	12.8542862	1	-3		2	125.755	0.3	0.4	0.2	0.7	0.4	0.2	29
Solar diurnal	P_1	24.06588766	14.9589314	1	1	-2		163.555	5.2	5.4	2.9	12.6	11.6	5.1	30

Long period	**Darwin**	**Period**	**Phase**	**Doodson coefs**				**Doodson**	**Amplitude at example location (cm)**						**NOAA**
Species	**Symbol**	**(hr)**	**(°/hr)**	n_1 (L)	n_2 (m)	n_3 (y)	n_4 (mp)	**number**	**ME**	**MS**	**PR**	**AK**	**CA**	**HI**	**order**
Lunar monthly	M_m	661.3111655	0.5443747	0	1		-1	65.455			0.7	1.9			20
Solar semiannual	S_{sa}	4383.076325	0.0821373	0		2		57.555	1.6		2.1	1.5	3.9		21
Solar annual	S_a	8766.15265	0.0410686	0		1		56.555			5.5	7.8	3.8	4.3	22
Lunisolar synodic fortnightly	M_{sf}	354.3670666	1.0158958	0	2	-2		73.555				1.5			23
Lunisolar fortnightly	M_f	327.8599387	1.0980331	0	2			75.555			1.4	2.0		0.7	24

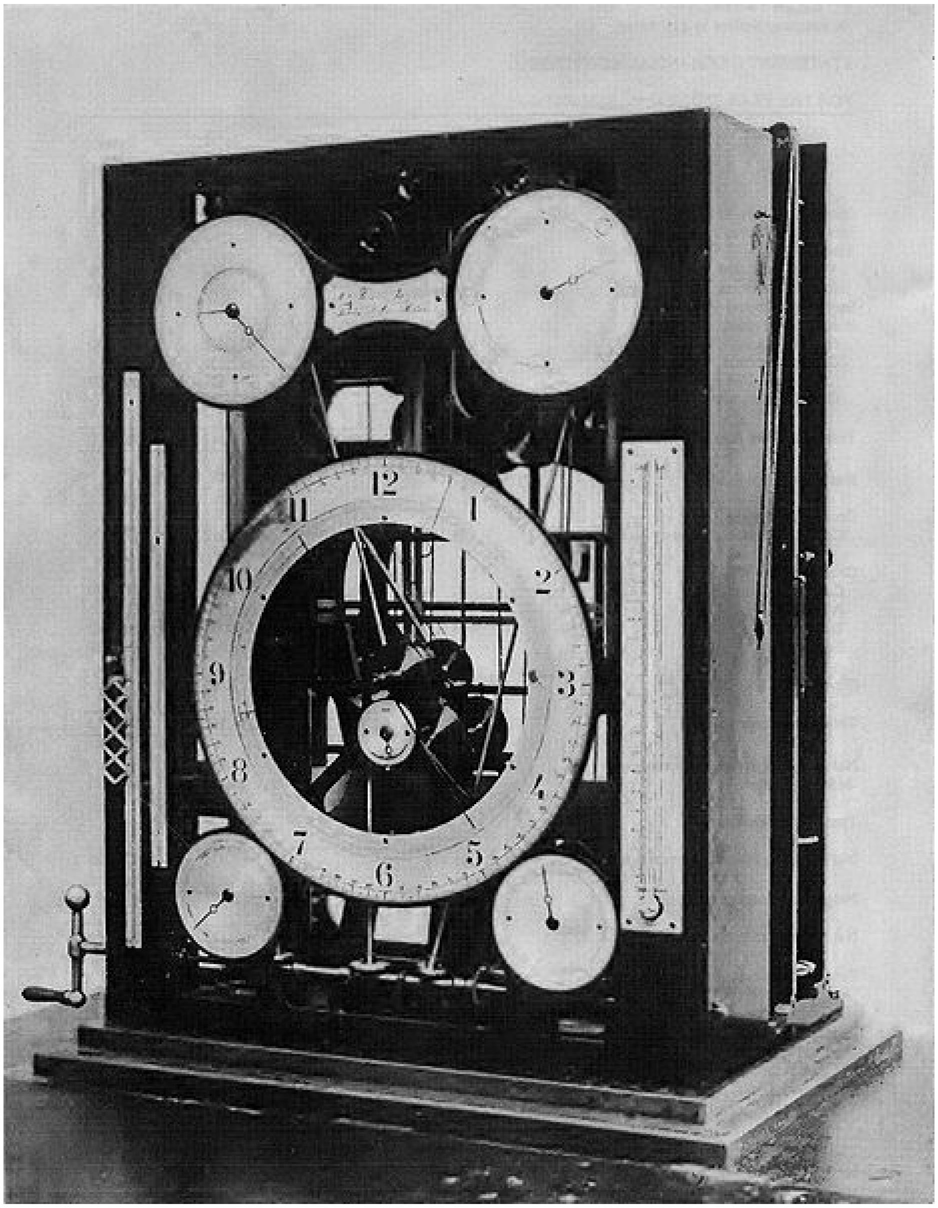
Chapter 11

Tide-Predicting Machine

10-component tide-predicting machine of 1872-3, conceived by Sir William Thomson (Lord Kelvin), and designed by Thomson and collaborators, at the Science Museum, South Kensington, London

A **tide-predicting machine** was a special-purpose mechanical analog computer of the late 19th and early 20th centuries, constructed and set up to predict the ebb and flow of sea tides and the irregular variations in their heights – which change in mixtures of rhythms, that never (in the aggregate) repeat themselves exactly. Its purpose was to shorten the laborious and error-prone computations of tide-prediction. Such machines usually provided predictions valid from hour to hour and day to day for a year or more ahead.

The first tide-predicting machine, designed and built in 1872-3, and followed by two larger machines on similar principles in 1876 and 1879, was conceived by Sir William Thomson (who later became Lord Kelvin). Thomson had previously, during the 1860s, introduced the method of harmonic analysis of tidal patterns. The first machine was designed by Thomson with the collaboration of Edward Roberts (assistant at the UK Nautical Almanac Office), and of Alexander Légé, who constructed it.



William Ferrel's tide-predicting machine of 1881-2, now at the Smithsonian National Museum of American History

In the US, another tide-predicting machine on a different pattern (shown right) was designed by William Ferrel and built in 1881-2. Developments and improvements continued in the UK, US and Germany through the first half of the 20th century. The machines became widely used for constructing official tidal predictions for general marine navigation. They came to be regarded as of military strategic importance during World War I, and again during the second World War, when the US No.2 Tide Predicting Machine, described below, was classified, along with the data that it produced, and used to predict tides for the D-day Normandy landings and all the island landings in the Pacific war. Military interest in such machines continued even for some time afterwards. They were made obsolete by digital electronic computers that can be programmed to carry out similar computations, but the tide-predicting machines continued in use until the 1960s and 1970s.

Several examples of tide-predicting machines remain on display as museum pieces, occasionally put into operation for demonstration purposes, monuments to the mathematical and mechanical ingenuity of their creators.

Background to the problem solved by the machines

Modern scientific study of tides dates back to Isaac Newton's 'Principia' of 1687, in which he applied the theory of gravitation to make a first approximation of the effects of the Moon and Sun on the Earth's tidal waters. The approximation developed by Newton and his successors of the next 90 years is known as the 'equilibrium theory' of tides.

Beginning in the 1770s, Pierre-Simon Laplace made a fundamental advance on the equilibrium approximation by bringing into consideration non-equilibrium dynamical aspects of the motion of tidal waters that occurs in response to the tide-generating forces due to the Moon and Sun.

Laplace's improvements in theory were substantial, but they still left prediction in an approximate state. This position changed in the 1860s when the local circumstances of tidal phenomena were more fully brought into account by William Thomson's application of Fourier analysis to the tidal motions. Thomson's work in this field was then further developed and extended by George Darwin: Darwin's work was based on the lunar theory current in his time. His symbols for the tidal harmonic constituents are still used. Darwin's harmonic developments of the tide-generating forces were later brought by A T Doodson up to date and extended in light of the new and more accurate lunar theory of E W Brown that remained current through most of the twentieth century.

The state to which the science of tide-prediction had arrived by the 1870s can be summarized: Astronomical theories of the Moon and Sun had identified the frequencies and strengths of different components of the tide-generating force. But effective prediction at any given place called for measurement of an adequate sample of local tidal observations, to show the local tidal response at those different frequencies, in amplitude and phase. Those observations had then to be analyzed, to derive the coefficients and phase angles. Then, for purposes of prediction, those local tidal constants had to be recombined, each with a different component of the tide-generating forces to which it applied, and at each of a sequence of future dates and times, and then the different elements finally collected together to obtain their aggregate effects. In the age when calculations were done by hand and brain, with pencil and paper and tables, this was recognized as an immensely laborious and error-prone undertaking.

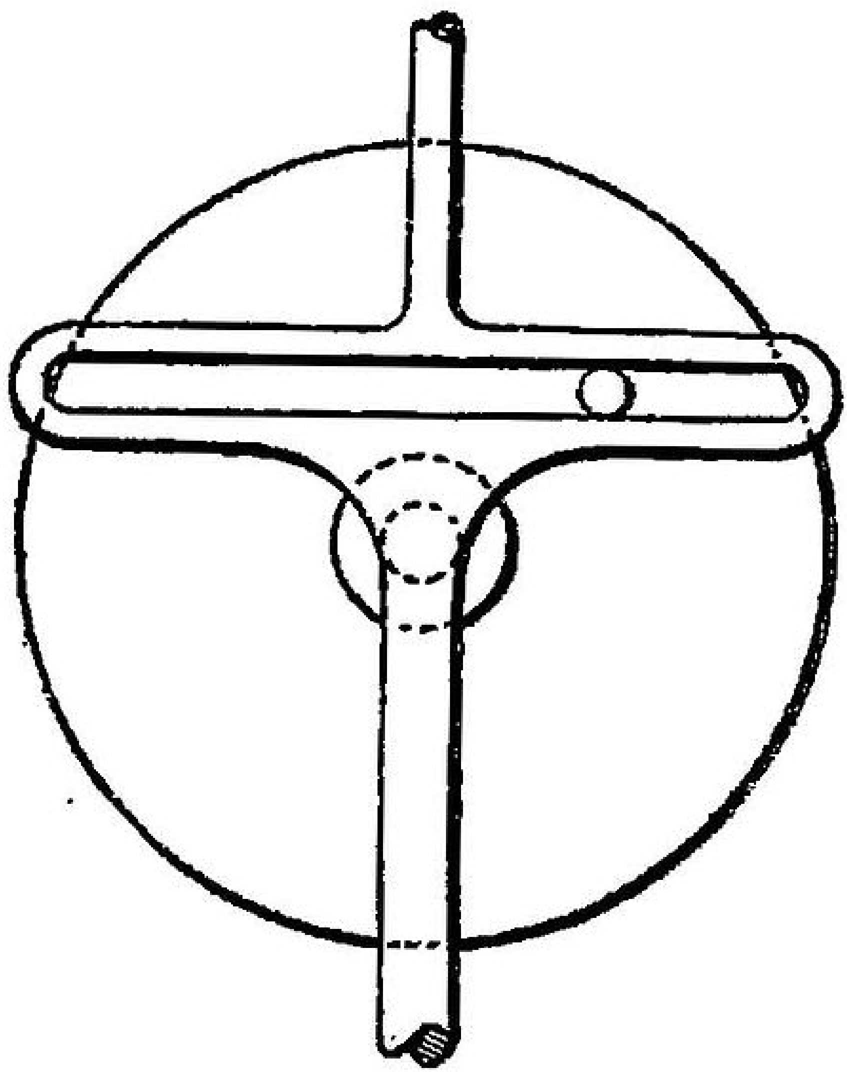
Thomson recognized that what was needed was a convenient and preferably automated way to evaluate repeatedly the sum of tidal terms such as:

$$A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2) + A_3 \cos(\omega_3 t + \phi_3) + \dots$$

containing 10, 20 or even more trigonometrical terms, so that the computation could conveniently be repeated in full for each of a very large number of different chosen values of the date/time *t*. This was the core of the problem solved by the tide-predicting machines.

How they worked to predict the tides

Thomson conceived his aim as to construct a mechanism that would evaluate this trigonometrical sum physically, e.g. as the vertical position of a pen that could then plot a curve on a moving band of paper.

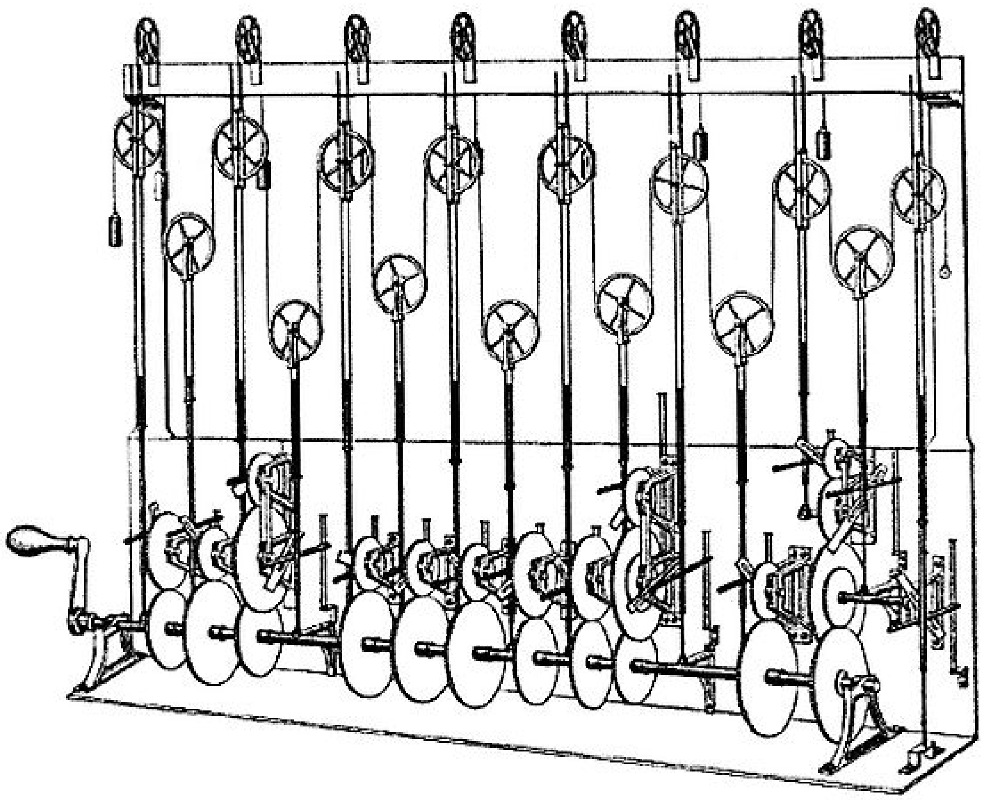


mechanism for generating sinusoidal motion component

There were several mechanisms available to him for converting rotary motion into sinusoidal motion. One of them is shown in the schematic (right). A rotating drive-wheel is fitted with an off-center peg. A shaft with a horizontally-slotted section is free to move vertically up and down. The wheel's off-center peg is located in the slot. As a result, when the peg moves around with the wheel, it can make the shaft move up and down within limits. This arrangement shows that when the drive-wheel rotates uniformly, say clockwise, the shaft moves sinusoidally up and down. The vertical position of the center of the slot, at any time t, can then be expressed as $A_1\cos(\omega_1 t + \varphi_1)$, where A_1 is the radial distance from the wheel's center to the peg, ω_1 is the rate at which the wheel turns (in radians per unit of time), and φ_1 is the starting phase angle of the peg, measured in radians from the 12 o'clock position to the angular position where the peg was at time zero.

This arrangement makes a physical analog of just one trigonometrical term. Thomson needed to construct a physical sum of many such terms.

At first he inclined to use gears. Then he discussed the problem with engineer Beauchamp Tower before the British Association meeting in 1872, and Tower suggested the use of a device that (as he remembered) was once used by Wheatstone. It was a chain running alternately over and under a sequence of pulleys on movable shafts. The chain was fixed at one end, and the other (free) end was weighted to keep it taut. As each shaft moved up or down it would take up or release a corresponding length of the chain. The movements in position of the free (movable) end of the chain represented the sum of the movements of the different shafts. The movable end was kept taut, and fitted with a pen and a moving band of paper on which the pen plotted a tidal curve. In some designs, the movable end of the line was connected instead to a dial and scale from which tidal heights could be read off.



Thomson's design for the third tide-predicting machine, 1879-81

One of Thomson's designs for the calculating part of a tide-predicting machine is shown in the figure (right), closely similar to the third machine of 1879-81. A long cord, with one end held fixed, passed vertically upwards and over a first upper pulley, then vertically downwards and under the next, and so on. These pulleys were all moved up and down by cranks, and each pulley took in or let out cord according to the direction in which it moved. These cranks were all moved by trains of wheels gearing into the wheels fixed on a drive shaft. The greatest number of teeth on any wheel was 802 engaging with another of 423. All the other wheels had comparatively small numbers of teeth. A flywheel of great inertia enabled the operator to turn the machine fast, without jerking the pulleys, and so to run off a year's curve in about twenty-five minutes. The machine shown in the figure was arranged for fifteen constituents in all.

Thomson acknowledged that the use of an over-and-under arrangement of the flexible line that summed the motion components was suggested to him in August 1872 by engineer Beauchamp Tower.

Online demonstration of the mechanism

An online demonstration is available to show the principle of operation of a 7-component version of a tide-predicting machine otherwise like Thomson's (Kelvin's) original design. The animation shows part of the operation of the machine: the motions of several pulleys can be seen, each moving up and down to simulate one of the tidal frequencies; and the animation also shows how these sinusoidal motions were generated by wheel rotations and how they were combined to form the resulting tidal curve. Not shown in the animation is the way in which the individual motions were generated in the machine at the correct relative frequencies, by gearing in the correct ratios, or how the amplitudes and starting phase angles for each motion were set in an adjustable way. These amplitudes and starting phase angles represented the local tidal constants, separately reset, and different for each place for which predictions were to be made. Also, in the real Thomson machines, to save on motion and wear of the other parts, the shaft and pulley with the largest expected motion (for the M2 tide component at twice per lunar day) was mounted nearest to the pen, and the shaft and pulley representing the smallest component was at the other end, nearest to the point of fixing of the flexible cord or chain, to minimize unnecessary motion in the most part of the flexible cord.

History of their building and use

The first tide-predicting machines 1872-1883

The first tide predicting machine, designed in 1872 and of which a model was exhibited at the British Association meeting in 1873 (for computing 8 tidal components), followed in 1875-6 by a machine on a slightly larger scale (for computing 10 tidal components), was designed by Sir William Thomson (who later became Lord Kelvin). The 10-component machine and results obtained from it were shown at the Paris Exhibition in 1878. An enlarged and improved version of the machine, for computing 20 tidal components, was built for the Government of India in 1879, and then modified in 1881 to extend it to compute 24 harmonic components.

In these machines, the prediction was delivered in the form of a continuous graphical pen-plot of tidal height against time. The plot was marked with hour- and noon-marks, and was made by the machine on a moving band of paper as the mechanism was turned. A year's tidal predictions for a given place, usually a chosen sea-port, could be plotted by the 1876 and 1879 machines in about four hours (but the drives had to be rewound during that time).

In 1881-2, another tide predicting machine, operating quite differently, was designed by William Ferrel and built in Washington under Ferrel's direction by E G Fischer (who later designed the successor machine described below, which was in operation at the US Coast and Geodetic Survey from 1912 until the 1960s). Ferrel's machine delivered predictions by telling the times and heights of successive high and low waters, shown by pointer-readings on dials and scales. These were read by an operator who copied the readings on to forms, to be sent to the printer of the US tide-tables.

These machines had to be set with local tidal constants special to the place for which predictions were to be made. Such numbers express the local tidal response to individual components of the global tide-generating potential, at different frequencies. This local response, shown in the timing and the height of tidal contributions at different frequencies, is a result of local and regional features of the coasts and sea-bed. The tidal constants are usually evaluated from local histories of tide-gauge observations, by harmonic analysis based on the principal tide-generating frequencies as shown by the global theory of tides and the underlying lunar theory.

Thomson was also responsible for originating the method of harmonic tidal analysis, and for devising a harmonic analyzer machine, which partly mechanized the evaluation of the constants from the gauge readings.

Development and improvement based on the experience of these early machines continued through the first half of the 20th century.

British Tide Predictor No.2, after initial use to generate data for Indian ports, was used for tide prediction for the British empire beyond India, and transferred to the National Physical Laboratory in 1903. British Tide Predictor No.3 was sold to the French Government in 1900 and used to generate French tide tables.

US Tide Predicting Machine No.2 ("Old Brass Brains") was designed in the 1890s, completed and brought into service in 1912, used for several decades including during the second World War, and retired in the 1960s.

Tide-predicting machines were built in Germany during World War I, and again in the period 1935-8.

Two of the last to be built were:

- a TPM built in 1947 for the Norwegian Hydrographic Service by Chadburn of Liverpool, and designed to compute 30 tidal harmonic constituents; used until 1975 to compute official Norwegian Tide Tables, before being superseded by digital computing.
- the Doodson-Légé TPM built in 1949,
- an East German TPM built 1953-5.

Tide predicting machines on display

They can be seen in London, Washington, Liverpool, and elsewhere, including the Deutches Museum in Munich.