Electric Current

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First Edition, 2012

ISBN 978-81-323-4125-3

Published by:
White Word Publications
4735/22 Prakashdeep Bldg,
Ansari Road, Darya Ganj,
Delhi - 110002
Email: info@wtbooks.com

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Chapter 1

Electric Current

Electric current is a flow of electric charge through a medium. This charge is typically carried by moving electrons in a conductor such as wire. It can also be carried by ions in an electrolyte, or by both ions and electrons in a plasma.

The SI unit for measuring the rate of flow of electric charge is the ampere, which is charge flowing through some surface at the rate of one coulomb per second. Electric current is measured using an ammeter.

Symbol

The conventional symbol for current is I, which may seem puzzling. It originates from the French phrase *intensité de courant*, or in English *current intensity*. This phrase is frequently used when discussing the value of an electric current, especially in older texts; modern practice often shortens this to simply *current* but *current intensity* is still used in many recent textbooks. The I symbol was used by André-Marie Ampère himself, after whom the unit of electric current is named, in formulating the eponymous Ampère's force law which he discovered in 1820. The notation travelled from France to England where it became standard, although at least one journal did not change from using C to I until 1896.

Conduction mechanisms in various media

In metallic solids, electricity flows by means of electrons, from lower to higher electrical potential. In other media, any stream of charged objects may constitute an electric current. To provide a definition of current that is independent of the type of charge carriers flowing, *conventional current* is defined to flow in the same direction as positive charges. So in metals where the charge carriers (electrons) are negative, conventional current flows in the opposite direction as the electrons. In conductors where the charge carriers are positive, conventional current flows in the same direction as the charge carriers.

In a vacuum, a beam of ions or electrons may be formed. In other conductive materials, the electric current is due to the flow of both positively and negatively charged particles at the same time. In still others, the current is entirely due to positive charge flow. For example, the electric currents in electrolytes are flows of electrically charged atoms (ions), which exist in both positive and negative varieties. In a common lead-acid electrochemical cell, electric currents are composed of positive hydrogen ions (protons) flowing in one direction, and negative sulfate ions flowing in the other. Electric currents in sparks or plasma are flows of electrons as well as positive and negative ions. In ice and in certain solid electrolytes, the electric current is entirely composed of flowing ions. In a semiconductor it is sometimes useful to think of the current as due to the flow of positive "holes" (the mobile positive charge carriers that are places where the semiconductor crystal is missing a valence electron). This is the case in a p-type semiconductor.

Metals

A solid conductive metal contains mobile, or free electrons, originating in the conduction electrons. These electrons are bound to the metal lattice but no longer to any individual atom. Even with no external electric field applied, these electrons move about randomly due to thermal energy but, on average, there is zero net current within the metal. Given a surface through which a metal wire passes, the number of electrons moving from one side to the other in any period of time is on average equal to the number passing in the opposite direction. As George Gamow put in his science-popularizing book, *One, Two, Three...Infinity* (1947), "The metallic substances differ from all other materials by the fact that the outer shells of their atoms are bound rather loosely, and often let one of their electrons go free. Thus the interior of a metal is filled up with a large number of unattached electrons that travel aimlessly around like a crowd of displaced persons. When a metal wire is subjected to electric force applied on its opposite ends, these free electrons rush in the direction of the force, thus forming what we call an electric current."

When a metal wire is connected across the two terminals of a DC voltage source such as a battery, the source places an electric field across the conductor. The moment contact is made, the free electrons of the conductor are forced to drift toward the positive terminal under the influence of this field. The free electrons are therefore the charge carrier in a typical solid conductor. For an electric current of 1 ampere, 1 coulomb of electric charge (which consists of about 6.242×10^{18} elementary charges) drifts every second through any plane through which the conductor passes.

For a steady flow of charge through a surface, the current I in amperes can be calculated with the following equation:

$$I = \frac{Q}{t},$$

where Q is the electric charge transferred through the surface over some time t. If Q and t are measured in coulombs and seconds respectively, I is in amperes.

More generally, electric current can be represented as the rate at which charge flows through a given surface as:

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}\,.$$

Electrolytes

Electric currents in electrolytes are flows of electrically charged particles (ions). For example, if an electric field is placed across a solution of Na^+ and Cl^- (and conditions are right) the sodium ions move towards the negative electrode (cathode), while the chloride ions move towards the positive electrode (anode). Reactions take place at both electrode surfaces, absorbing each ion.

Water-ice and certain solid electrolytes called proton conductors contain positive hydrogen ions or "protons" which are mobile. In these materials, electric currents are composed of moving protons, as opposed to the moving electrons found in metals.

In certain electrolyte mixtures, brightly-colored ions form the moving electric charges. The slow migration of these ions means that the current is visible.

Gases and plasmas

In air and other ordinary gases below the breakdown field, the dominant source of electrical conduction is via a relatively small number of mobile ions produced by radioactive gases, ultraviolet light, or cosmic rays. Since the electrical conductivity is low, gases are dielectrics or insulators. However, once the applied electric field approaches the breakdown value, free electrons become sufficiently accelerated by the electric field to create additional free electrons by colliding, and ionizing, neutral gas atoms or molecules in a process called avalanche breakdown. The breakdown process forms a plasma that contains a significant number of mobile electrons and positive ions, causing it to behave as an electrical conductor. In the process, it forms a light emitting conductive path, such as a spark, arc or lightning.

Plasma is the state of matter where some of the electrons in a gas are stripped or "ionized" from their molecules or atoms. A plasma can be formed by high temperature, or by application of a high electric or alternating magnetic field as noted above. Due to their lower mass, the electrons in a plasma accelerate more quickly in response to an electric field than the heavier positive ions, and hence carry the bulk of the current.

Vacuum

Since a "perfect vacuum" contains no charged particles, it normally behaves as perfect insulator. However, metal electrode surfaces can cause a region of the vacuum to become conductive by injecting free electrons or ions through either field electron emission or thermionic emission. Thermionic emission occurs when the thermal energy exceeds the metal's work function, while field electron emission occurs when the electric field at the surface of the metal is high enough to cause tunneling, which results in the ejection of free electrons from the metal into the vacuum. Externally heated electrodes are often used to generate an electron cloud as in the filament or indirectly heated cathode of vacuum tubes. Cold electrodes can also spontaneously produce electron clouds via thermionic emission when small incandescent regions (called **cathode spots** or **anode spots**) are formed. These are incandescent regions of the electrode surface that are created by a localized high current flow. These regions may be initiated by field electron emission, but are then sustained by localized thermionic emission once a vacuum arc forms. These small electron-emitting regions can form quite rapidly, even explosively, on a metal surface subjected to a high electrical field. Vacuum tubes and sprytrons are some of the electronic switching and amplifying devices based on vacuum conductivity.

Current density and Ohm's law

Current density is a measure of the density of an electric current. It is defined as a vector whose magnitude is the electric current per cross-sectional area. In SI units, the current density is measured in amperes per square meter.

$$I = \vec{J} \cdot \vec{A}$$

where I is current in the conductor, **J** is the current density, and **A** is the cross-sectional area. The dot product of the two vector quantities (**A** and **J**) is a scalar that represents the electric current.

Current density (current per unit area) J in a material is proportional to the conductivity σ and electric field E in the medium:

$$J = \sigma E$$

Instead of conductivity, reciprocal quantity called resistivity ρ, can be used:

$$J = \frac{E}{\rho}$$

Conduction in semiconductor devices may occur by a combination of electric field (drift) and diffusion, which is proportional to diffusion constant D and charge density α_q. The current density is then:

$$J = \sigma E + Dq\nabla n,$$

with q being the elementary charge and n the electron density. The carriers move in the direction of decreasing concentration, so for electrons a positive current results for a positive density gradient. If the carriers are holes, replace electron density n by the negative of the hole density p.

In linear anisotropic materials, σ, ρ and D are tensors.

In linear materials such as metals, and under low frequencies, the current density across the conductor surface is uniform. In such conditions, Ohm's law states that the current is directly proportional to the potential difference between two ends (across) of that metal (ideal) resistor (or other ohmic device):

$$I = \frac{V}{R},$$

where I is the current, measured in amperes; V is the potential difference, measured in volts; and R is the resistance, measured in ohms. The letter I stands for the German word, "Intensität" meaning "Intensity". For alternating currents, especially at higher frequencies, skin effect causes the current to spread unevenly across the conductor cross-section, with higher density near the surface, thus increasing the apparent resistance.

Drift speed

The mobile charged particles within a conductor move constantly in random directions, like the particles of a gas. In order for there to be a net flow of charge, the particles must also move together with an average drift rate. Electrons are the charge carriers in metals and they follow an erratic path, bouncing from atom to atom, but generally drifting in the opposite direction of the electric field. The speed at which they drift can be calculated from the equation:

$$I = nAvQ,$$

where

I is the electric current
n is number of charged particles per unit volume (or charge carrier density)
A is the cross-sectional area of the conductor
v is the drift velocity, and
Q is the charge on each particle.

Electric currents in solids typically flow very slowly. For example, in a copper wire of cross-section 0.5 mm^2, carrying a current of 5 A, the *drift velocity* of the electrons is on the order of a millimetre per second. To take a different example, in the near-vacuum inside a cathode ray tube, the electrons travel in near-straight lines at about a tenth of the speed of light.

Any accelerating electric charge, and therefore any changing electric current, gives rise to an electromagnetic wave that propagates at very high speed outside the surface of the conductor. This speed is usually a significant fraction of the speed of light, as can be deduced from Maxwell's Equations, and is therefore many times faster than the drift velocity of the electrons. For example, in AC power lines, the waves of electromagnetic energy propagate through the space between the wires, moving from a source to a distant load, even though the electrons in the wires only move back and forth over a tiny distance.

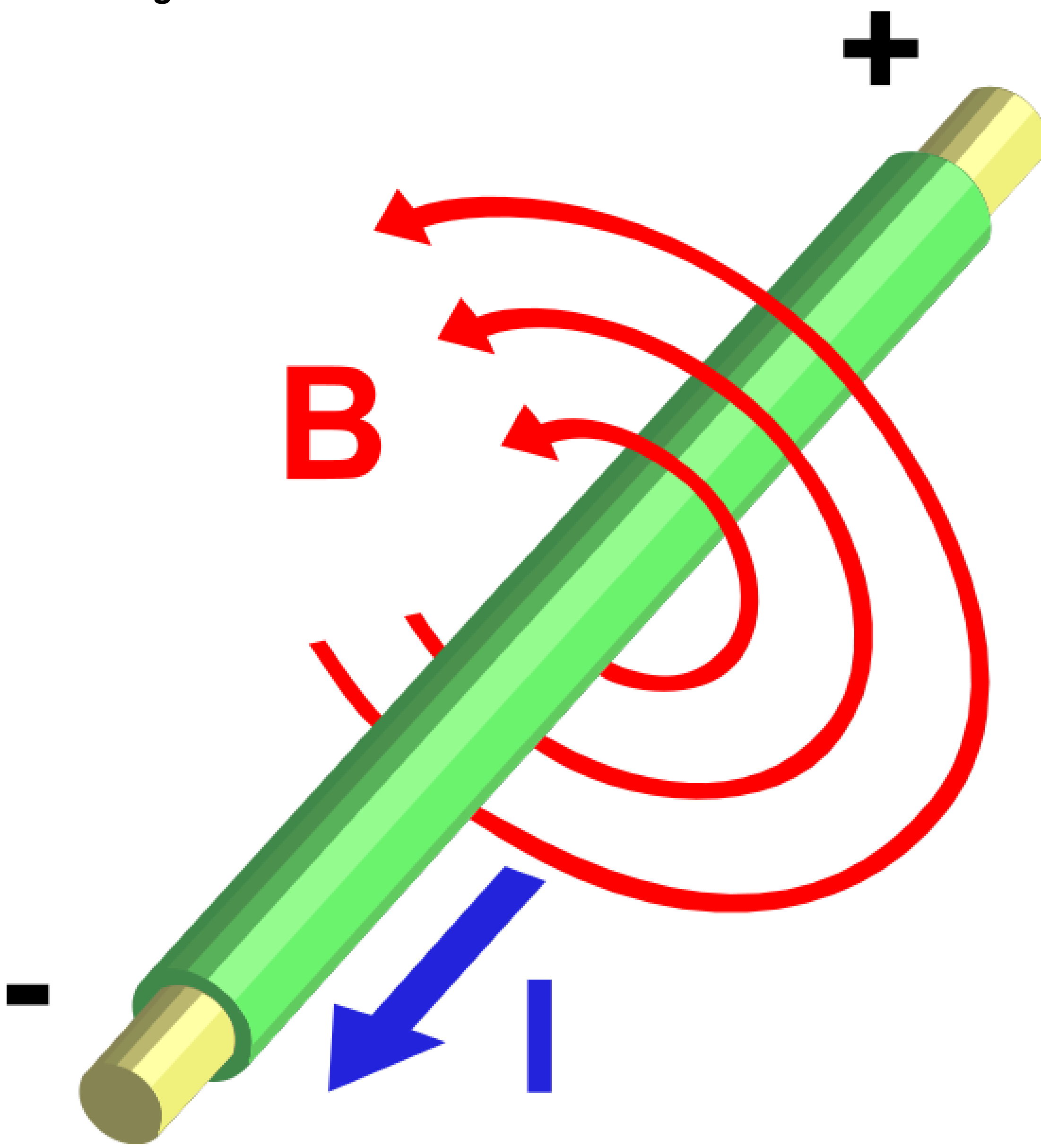
The ratio of the speed of the electromagnetic wave to the speed of light in free space is called the velocity factor, and depends on the electromagnetic properties of the conductor and the insulating materials surrounding it, and on their shape and size.

The magnitudes (but, not the natures) of these three velocities can be illustrated by an analogy with the three similar velocities associated with gases.

- The low drift velocity of charge carriers is analogous to air motion; in other words, winds.
- The high speed of electromagnetic waves is roughly analogous to the speed of sound in a gas (these waves move through the medium much faster than any individual particles do)
- The random motion of charges is analogous to heat - the thermal velocity of randomly vibrating gas particles.

This analogy is extremely simplistic and incomplete: The rapid propagation of a sound wave doesn't impart any change in the air molecules' drift velocity, whereas EM waves do carry the energy to propagate the actual current at a rate which is much, much higher than the electrons' drift velocity. To illustrate the difference: The sound and the change in the air's drift velocity (the force of the wind gust) cross distance at rates equaling the speeds of sound and of mechanical transmission of force (**not higher** than rate of drift velocity); while a change in an EM field and the **change** in current (electrons' drift velocity) both propagate across distance at rates **much higher** than the actual drift velocity. You can hear wind much earlier than the force of the gust reaches you, but you don't observe a change in an EM field earlier than you can observe the change of current.

Electromagnetism



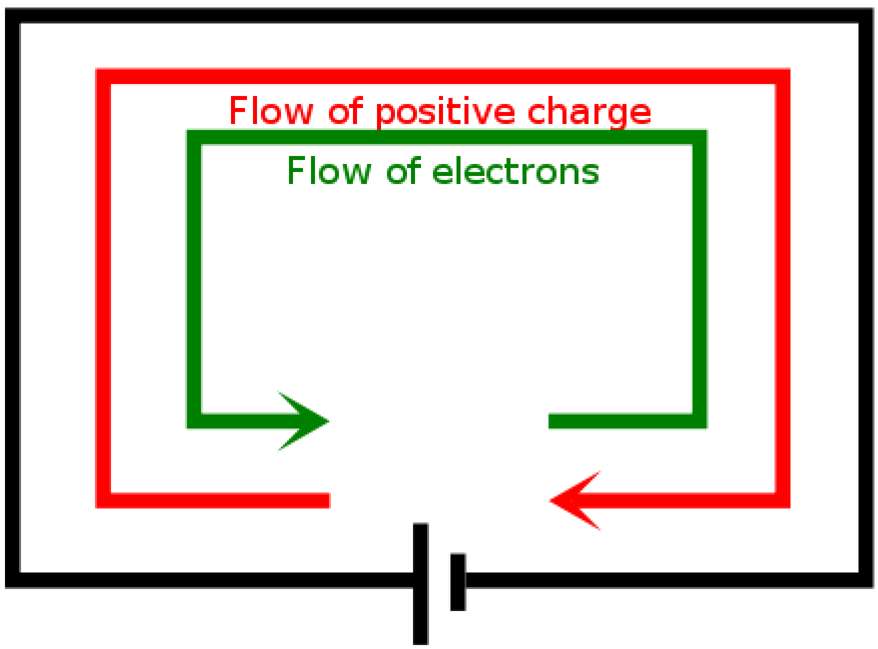
According to Ampère's law, an electric current produces a magnetic field.

Electric current produces a magnetic field. The magnetic field can be visualized as a pattern of circular field lines surrounding the wire.

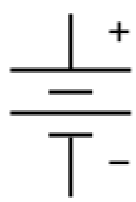
Electric current can be directly measured with a galvanometer, but this method involves breaking the electrical circuit, which is sometimes inconvenient. Current can also be measured without breaking the circuit by detecting the magnetic field associated with the current. Devices used for this include Hall effect sensors, current clamps, current transformers, and Rogowski coils.

The theory of Special Relativity allows one to transform the magnetic field into a static electric field for an observer moving at the same speed as the charge in the diagram. The amount of current is particular to a reference frame.

Conventions



The electrons, the charge carriers in an electrical circuit, flow in the opposite direction of the *conventional* electric current.



The symbol for a battery in a circuit diagram.

A flow of positive charges gives the same *electric* current as a flow of negative charges in the opposite direction. Since current can be the flow of either positive or negative charges, or both, a convention for the direction of current which is independent of the type of charge carriers is needed. Therefore the direction of *conventional current* is defined to be the direction of the flow of positive charges.

In metals, which make up the wires and other conductors in most electrical circuits, the positive charges are immobile, and only the negatively charged electrons flow. Because the electron carries negative charge, the *electron* motion in a metal conductor is in the direction opposite to that of conventional (or *electric*) current.

Reference direction

When analyzing electrical circuits, the actual direction of current through a specific circuit element is usually unknown. Consequently, each circuit element is assigned a current variable with an arbitrarily chosen *reference direction*. When the circuit is solved, the circuit element currents may have positive or negative values. A negative value means that the actual direction of current through that circuit element is opposite that of the chosen reference direction. In electronic circuits the reference current directions are usually chosen so that all currents flow toward ground. This often matches conventional current direction, because in many circuits the power supply voltage is positive with respect to ground.

Occurrences

Natural examples include lightning and the solar wind, the source of the polar auroras (the aurora borealis and aurora australis). The artificial form of electric current is the flow of conduction electrons in metal wires, such as the overhead power lines that deliver electrical energy across long distances and the smaller wires within electrical and electronic equipment. In electronics, other forms of electric current include the flow of electrons through resistors or through the vacuum in a vacuum tube, the flow of ions inside a battery or a neuron, and the flow of holes within a semiconductor.

Current Measurement

Current can be measured using an ammeter.

At the circuit level there are various techniques that can be used to measure current:

- Shunt resistor
- Hall effect current sensor transducers
- Transformer(however dc cannot be measured)
- Magnetoresistive Field Sensors

Chapter 2

AC/DC (Electricity)

The description **AC/DC** refers to equipment designed to be operate on either alternating current (AC) or direct current (DC). This term typically describes certain types of vacuum tube radio or television receivers. AC/DC equipment was necessary because in the early days of vacuum tubes, some regions were supplied with AC power, others with DC. Equipment which is able by its nature to use either AC or DC, e.g., heating devices and incandescent light bulbs, is not usually described as “AC/DC”.

Applicability to early radio and TV

Vacuum tube equipment used a number of tubes, each with a heater requiring a certain amount of electrical power. Tubes require relatively high voltages on some of their electrodes; these voltages can conveniently be derived directly from mains electricity. There are three ways of powering such equipment:

1. AC equipment: a transformer converts mains electricity into both a low-voltage (typically 6.3V) supply connected to the parallel-connected heaters of all the tubes in the equipment, and one or more high-voltage supplies which are rectified and filtered to give high DC voltages required by the equipment. Transformers operate on AC only, so that this type of equipment is AC-only.
2. AC/DC equipment: the heaters of all the tubes are connected in series. All the tubes are rated at the same current (typically 100, 150, 300, or 450mA) but at different voltages. If necessary resistance, which can be a ballast tube (aka barretter), power resistor, or resistive cable, is added so that, when the mains voltage is applied across the chain, the required current flows. With mains voltages of around 220V the voltage drop across and power dissipated by the additional resistance could be quite high, and it was common to use a resistive power cable (mains cord) of defined resistance, running warm, rather than putting a hot resistor inside the case. A rectifier and filter are connected directly to the mains. If the mains power is AC, the rectifier converts it to DC; if DC, the rectifier effectively acts as a conductor. In both cases DC at about the same voltage as the mains is available to drive the circuitry. The tube heaters do just as their name describes and heat the cathodes, whether AC or DC power is applied. There is no transformer to isolate AC/DC equipment from the mains. Much

equipment was built on a metal chassis which had to be connected to one side of the mains. A typical low-cost radio would have 5 tubes, plus a ballast built into an envelope like a tube and easily replaceable.

3. DC-only, from DC mains.

AC/DC equipment was suitable for use on either AC or DC, an important consideration when DC distribution was still used. Manufacturers were able to produce a single range of equipment for all power, and users did not have problems when moving house. Because no power transformer was used, so-called "hot chassis" construction was required; the equipment power supply was conductively connected to the input power source. Any exposed metal on the device connected to the circuit common was also connected to the power supply. For safety, no exposed metal could be connected to the circuit common. Service personnel working on energized equipment had to be mindful that the chassis could be at line potential with respect to earth ground.

If a resistive power cable was used, an inexpert repairer might replace it by a standard cable, or use the wrong length, damaging equipment and risking fire. AC/DC equipment did not require a transformer, and was consequently cheaper, lighter, and smaller than comparable AC equipment. This type of equipment continued to be produced long after AC became the universal standard due to its cost advantage over AC-only, and was only discontinued when vacuum tubes were replaced by low-voltage solid-state electronics.

- Older AC-only equipment uses a bulky, heavy, and expensive 50- or 60-Hz transformer, but the chassis is never live and can be earthed, making for safer operation. Additionally the use of a transformer allowed higher voltages to be generated (e.g., for high-powered audio amplifiers), and allowed multiple independent power supplies from separate transformer windings for different stages.
- DC-only equipment was a little cheaper than AC/DC, but became obsolete as AC power became dominant.

Regional variations

110-120V was not high enough for high-power audio and television applications, although sufficient to operate low-power audio equipment such as radio receivers. Higher-powered 110-120V tube audio or TV equipment needed higher voltages which had to be stepped up by a transformer power supply, or sometimes a voltage doubler, and so operated off AC only.

Some AC/DC equipment was designed to be switchable to be able to operate off either 110 V AC (possibly with a voltage doubler) or 220-240V AC or DC. Television receivers were produced to run off 240V AC or DC. The voltage was not high enough to power some of the circuits, so it was boosted with energy recovered from the scan coils during flyback. Some details of the way the voltage was boosted are to be found in a technical description of the 1951 Bush TV22. AC/DC TVs were produced well into the color and semiconductor era (sets were tube/semiconductor hybrids).

Motorized tools and appliances

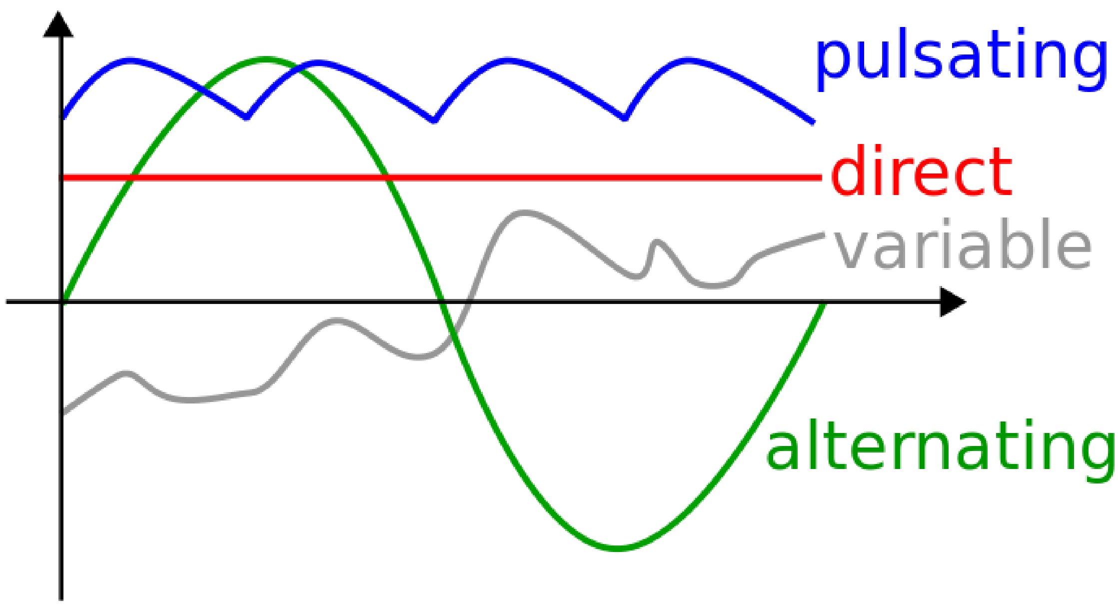
Many hand-held power tools or small kitchen appliances use universal motors. When DC distribution was still used, some motorized appliances were name-plated for operation both on DC and AC systems. Since the universal motor connects its field in series with the armature, the reversal of magnetic fields in both the armature and field occur at the same time, meaning the motor can run equally well on both types of current. Although modern tools and appliances often still use lightweight and powerful universal motors, they are no longer rated for operation on DC systems. Safety standards no longer test for DC operation, switching devices in AC-only appliances are smaller and unsuitable for DC circuits of the same current, and many modern power tools and appliances incorporate AC-only speed control based on thyristors that will not work on direct current supplies.

Modern equipment

Since solid-state electronics displaced vacuum tubes, circuits require high currents at relatively low voltages, and the use of transformers has become almost universal. The decreasing cost of complex electronics, with massive functionality available in a single, cheap, integrated circuit has made it feasible to power equipment safely from either AC or DC mains without a conventional mains transformer: the supply is rectified and filtered if AC, and used to power a high-frequency oscillator whose output is connected to the primary winding of a small, cheap, high-frequency transformer (which is often part of the tuned circuit used by the oscillator) which isolates the circuitry of the equipment from the mains electricity. In principle modern AC/DC equipment would be no more complex or costly than AC-only; in practice mains DC mains electricity is no longer used, and DC operation direct from the mains is irrelevant. Although unrelated to AC/DC as used in the past, modern equipment is often powered from low-voltage DC, typically by a 12V accumulator in a motor vehicle. Much equipment operates from 12V or less, but an inverter can be used to provide AC output; or a DC to DC converter based on a switch-mode power supply (SMPS) for higher DC output. Modern SMPSes can accept a DC input without any problem, though the DC voltage does need to be around 25% higher than the rated RMS AC voltage. Some specialist manufacturers produce electronic DC/DC converters.

Chapter 3

Alternating Current



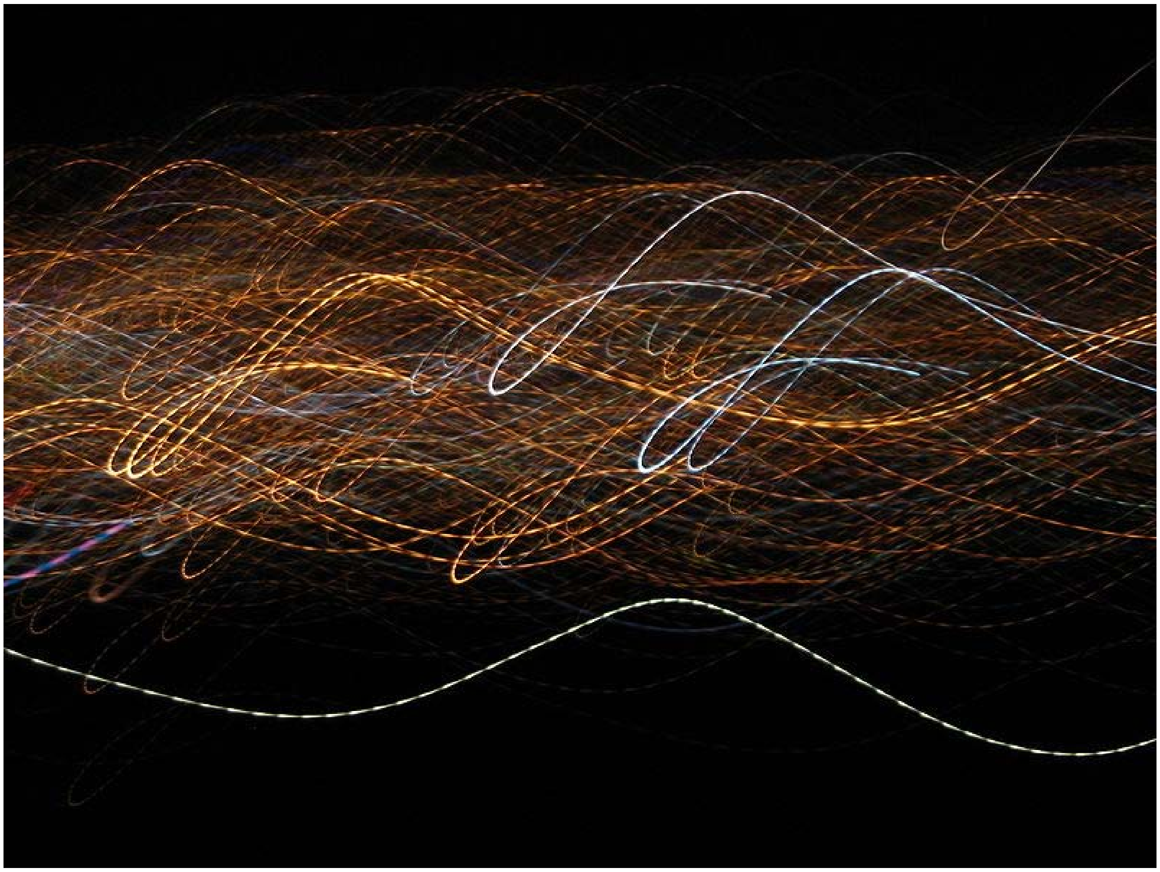
Alternating Current (green curve). The horizontal axis measures time; the vertical, current or voltage.

In **alternating current** (**AC** or **ac**) the movement of electric charge periodically reverses direction. In direct current (DC), the flow of electric charge is only in one direction.

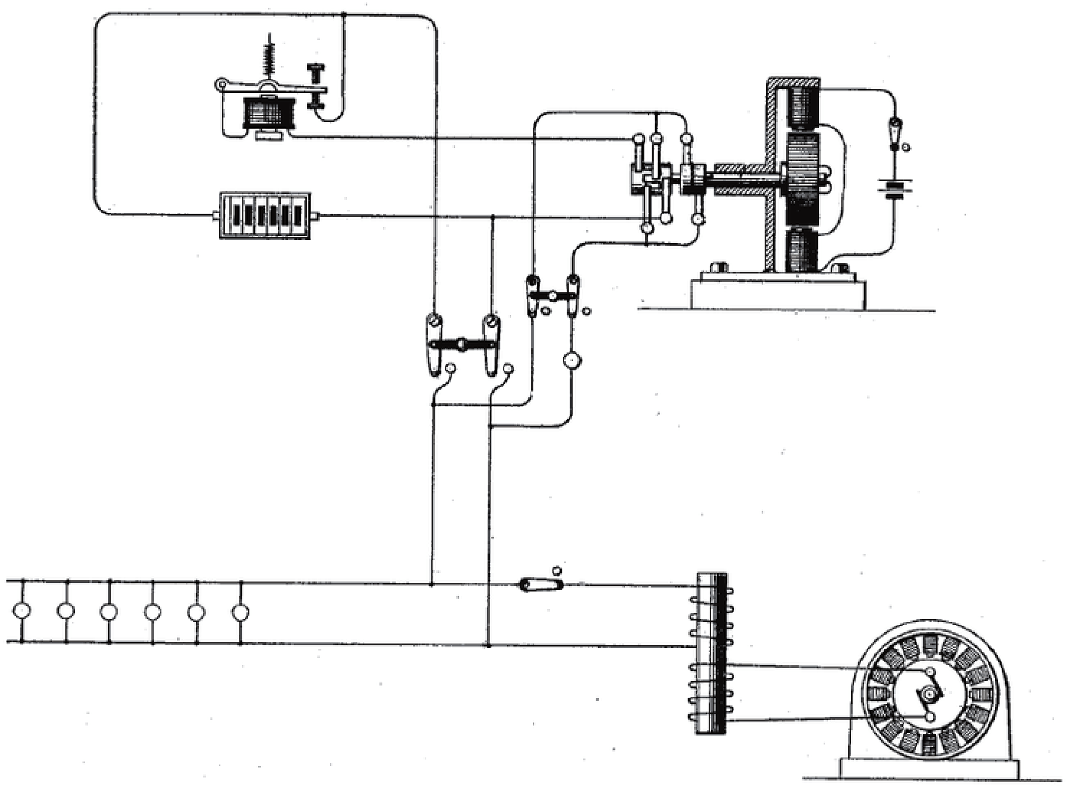
The abbreviations *AC* and *DC* are often used to mean simply *alternating* and *direct*, as when they modify *current* or *voltage*, even though some authors have advised against that usage, pointing out the absurdity of the expanded forms such as *alternating current current*.

AC is the form in which electric power is delivered to businesses and residences. The usual waveform of an AC power circuit is a sine wave. In certain applications, different waveforms are used, such as triangular or square waves. Audio and radio signals carried on electrical wires are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

History



City lights viewed in a motion blurred exposure. The AC blinking causes the lines to be dotted rather than continuous.



Westinghouse Early AC System 1887 (US patent 373035)

The earliest recorded practical application of alternating current is by Guillaume Duchenne, inventor and developer of electrotherapy. In 1855, he announced that AC was superior to direct current for electrotherapeutic triggering of muscle contractions.

A power transformer developed by Lucien Gaulard and John Dixon Gibbs was demonstrated in London in 1881, and attracted the interest of Westinghouse. They also exhibited the invention in Turin in 1884, where it was adopted for an electric lighting system. Many of their designs were adapted to the particular laws governing electrical distribution in the UK.

In 1882, 1884, and 1885 Gaulard and Gibbs applied for patents on their transformer; however, these were overturned due to prior arts of Nikola Tesla and actions initiated by Sebastian Ziani de Ferranti.

Ferranti went into this business in 1882 when he set up a shop in London designing various electrical devices. Ferranti believed in the success of alternating current power distribution early on, and was one of the few experts in this system in the UK. In 1887 the London Electric Supply Corporation (LESCo) hired Ferranti for the design of their power station at Deptford. He designed the building, the generating plant and the distribution system. On its completion in 1891 it was the first truly modern power station, supplying high-voltage AC power that was then "stepped down" for consumer use on each street. This basic system remains in use today around the world. Many homes all over the world still have electric meters with the Ferranti AC patent stamped on them.

William Stanley, Jr. designed one of the first practical devices to transfer AC power efficiently between isolated circuits. Using pairs of coils wound on a common iron core, his design, called an induction coil, was an early transformer. The AC power system used today developed rapidly after 1886, and includes key concepts by Nikola Tesla, who subsequently sold his patent to George Westinghouse. Lucien Gaulard, John Dixon Gibbs, Carl Wilhelm Siemens and others contributed subsequently to this field. AC systems overcame the limitations of the direct current system used by Thomas Edison to distribute electricity efficiently over long distances even though Edison attempted to discredit alternating current as too dangerous during the War of Currents.

The first commercial power plant in the United States using three-phase alternating current was at the Mill Creek No. 1 Hydroelectric Plant near Redlands, California, in 1893 designed by Almirian Decker. Decker's design incorporated 10,000-volt three-phase transmission and established the standards for the complete system of generation, transmission and motors used today.

The Ames Hydroelectric Generating Plant (spring of 1891) and the original Niagara Falls Adams Power Plant (August 25, 1895) were among the first AC-powered hydroelectric plants.

The Jaruga Hydroelectric Power Plant in Croatia was set in operation on 28 August 1895. The two generators (42 Hz, 550 kW each) and the transformers were produced and installed by the Hungarian company Ganz. The transmission line from the power plant to the City of Šibenik was 11.5 kilometers (7.1 mi) long on wooden towers, and the municipal distribution grid 3000 V/110 V included six transforming stations.

Alternating current circuit theory developed rapidly in the latter part of the 19th and early 20th century. Notable contributors to the theoretical basis of alternating current calculations include Charles Steinmetz, James Clerk Maxwell, Oliver Heaviside, and many others. Calculations in unbalanced three-phase systems were simplified by the symmetrical components methods discussed by Charles Legeyt Fortescue in 1918. .

Transmission, distribution, and domestic power supply

AC voltage may be increased or decreased with a transformer. Use of a higher voltage leads to significantly more efficient transmission of power. The power losses in a conductor are a product of the square of the current and the resistance of the conductor, described by the formula

$$P_{\mathrm{L}} = I^2 R\,.$$

This means that when transmitting a fixed power on a given wire, if the current is doubled, the power loss will be four times greater.

The power transmitted is equal to the product of the current and the voltage (assuming no phase difference); that is,

$$P_{\mathrm{T}} = IV \, .$$

Thus, the same amount of power can be transmitted with a lower current by increasing the voltage. It is therefore advantageous when transmitting large amounts of power to distribute the power with high voltages (often hundreds of kilovolts).

High voltage transmission lines deliver power from electric generation plants over long distances using alternating current. These lines are located in eastern Utah.

However, high voltages also have disadvantages, the main one being the increased insulation required, and generally increased difficulty in their safe handling. In a power plant, power is generated at a convenient voltage for the design of a generator, and then stepped up to a high voltage for transmission. Near the loads, the transmission voltage is stepped down to the voltages used by equipment. Consumer voltages vary depending on the country and size of load, but generally motors and lighting are built to use up to a few hundred volts between phases.

The utilization voltage delivered to equipment such as lighting and motor loads is standardized, with an allowable range of voltage over which equipment is expected to operate. Standard power utilization voltages and percentage tolerance vary in the different mains power systems found in the world.

Modern high-voltage, direct-current electric power transmission systems contrast with the more common alternating-current systems as a means for the efficient bulk transmission of electrical power over long distances. HVDC systems, however, tend to be more expensive and less efficient over shorter distances than transformers. Transmission with high voltage direct current was not feasible when Edison, Westinghouse and Tesla were designing their power systems, since there was then no way to economically convert AC power to DC and back again at the necessary voltages.

Three-phase electrical generation is very common. The simplest case is three separate coils in the generator stator that are physically offset by an angle of 120° to each other. Three current waveforms are produced that are equal in magnitude and 120° out of phase to each other. If coils are added opposite to these (60° spacing), they generate the same phases with reverse polarity and so can be simply wired together.

In practice, higher "pole orders" are commonly used. For example, a 12-pole machine would have 36 coils (10° spacing). The advantage is that lower speeds can be used. For example, a 2-pole machine running at 3600 rpm and a 12-pole machine running at 600 rpm produce the same frequency. This is much more practical for larger machines.

If the load on a three-phase system is balanced equally among the phases, no current flows through the neutral point. Even in the worst-case unbalanced (linear) load, the neutral current will not exceed the highest of the phase currents. Non-linear loads (e.g. computers) may require an oversized neutral bus and neutral conductor in the upstream distribution panel to handle harmonics. Harmonics can cause neutral conductor current levels to exceed that of one or all phase conductors.

For three-phase at utilization voltages a four-wire system is often used. When stepping down three-phase, a transformer with a Delta (3-wire) primary and a Star (4-wire, center-earthed) secondary is often used so there is no need for a neutral on the supply side.

For smaller customers (just how small varies by country and age of the installation) only a single phase and the neutral or two phases and the neutral are taken to the property. For larger installations all three phases and the neutral are taken to the main distribution panel. From the three-phase main panel, both single and three-phase circuits may lead off.

Three-wire single-phase systems, with a single center-tapped transformer giving two live conductors, is a common distribution scheme for residential and small commercial buildings in North America. This arrangement is sometimes incorrectly referred to as "two phase". A similar method is used for a different reason on construction sites in the UK. Small power tools and lighting are supposed to be supplied by a local center-tapped transformer with a voltage of 55 V between each power conductor and earth. This significantly reduces the risk of electric shock in the event that one of the live conductors becomes exposed through an equipment fault whilst still allowing a reasonable voltage of 110 V between the two conductors for running the tools.

A third wire, called the bond (or earth) wire, is often connected between non-current-carrying metal enclosures and earth ground. This conductor provides protection from electric shock due to accidental contact of circuit conductors with the metal chassis of portable appliances and tools. Bonding all non-current-carrying metal parts into one complete system ensures there is always a low electrical impedance path to ground sufficient to carry any fault current for as long as it takes for the system to clear the fault. This low impedance path allows the maximum amount of fault current, causing the overcurrent protection device (breakers, fuses) to trip or burn out as quickly as possible, bringing the electrical system to a safe state. All bond wires are bonded to ground at the main service panel, as is the Neutral/Identified conductor if present.

AC power supply frequencies

The frequency of the electrical system varies by country; most electric power is generated at either 50 or 60 Hz. Some countries have a mixture of 50 Hz and 60 Hz supplies, notably Japan.

A low frequency eases the design of low-speed electric motors, particularly for hoisting, crushing and rolling applications, and commutator-type traction motors for applications such as railways, but also causes a noticeable flicker in incandescent lighting and an objectionable flicker in fluorescent lamps. 16.7 Hz power (in former times nominal 16 2/3 cycles per second, practically invariably) is still used in some European rail systems, such as in Austria, Germany, Norway, Sweden and Switzerland. The use of lower frequencies also provided the advantage of lower impedance losses, which are proportional to frequency. The original Niagara Falls generators were built to produce 25 Hz power, as a compromise between low frequency for traction and heavy induction motors, while still allowing incandescent lighting to operate (although with noticeable flicker); most of the 25 Hz residential and commercial customers for Niagara Falls power were converted to 60 Hz by the late 1950s, although some 25 Hz industrial customers still existed as of the start of the 21st century.

Off-shore, military, textile industry, marine, computer mainframe, aircraft, and spacecraft applications sometimes use 400 Hz, for benefits of reduced weight of apparatus or higher motor speeds.

Effects at high frequencies

A direct current flows constantly and uniformly throughout the cross-section of a uniform wire. An alternating current of any frequency is forced away from the wire's center, toward its outer surface. This is because the acceleration of an electric charge in an alternating current produces waves of electromagnetic radiation that cancel the propagation of electricity toward the center of materials with high conductivity. This phenomenon is called skin effect.

At very high frequencies the current no longer flows *in* the wire, but effectively flows *on* the surface of the wire, within a thickness of a few skin depths. The skin depth is the thickness at which the current density is reduced by 63%. Even at relatively low frequencies used for high power transmission (50–60 Hz), non-uniform distribution of current still occurs in sufficiently thick conductors. For example, the skin depth of a copper conductor is approximately 8.57 mm at 60 Hz, so high current conductors are usually hollow to reduce their mass and cost.

Since the current tends to flow in the periphery of conductors, the effective cross-section of the conductor is reduced. This increases the effective AC resistance of the conductor, since resistance is inversely proportional to the cross-sectional area in which the current actually flows. The AC resistance often is many times higher than the DC resistance, causing a much higher energy loss due to ohmic heating (also called I^2R loss).

Techniques for reducing AC resistance

For low to medium frequencies, conductors can be divided into stranded wires, each insulated from one other, and the relative positions of individual strands specially arranged within the conductor bundle. Wire constructed using this technique is called Litz wire. This measure helps to partially mitigate skin effect by forcing more equal current throughout the total cross section of the stranded conductors. Litz wire is used for making high-Q inductors, reducing losses in flexible conductors carrying very high currents at lower frequencies, and in the windings of devices carrying higher radio frequency current (up to hundreds of kilohertz), such as switch-mode power supplies and radio frequency transformers.

Techniques for reducing radiation loss

As written above, an alternating current is made of electric charge under periodic acceleration, which causes radiation of electromagnetic waves. Energy that is radiated is lost. Depending on the frequency, different techniques are used to minimize the loss due to radiation.

Twisted pairs

At frequencies up to about 1 GHz, pairs of wires are twisted together in a cable, forming a twisted pair. This reduces losses from electromagnetic radiation and inductive coupling. A twisted pair must be used with a balanced signalling system, so that the two wires carry equal but opposite currents. Each wire in a twisted pair radiates a signal, but it is effectively cancelled by radiation from the other wire, resulting in almost no radiation loss.

Coaxial cables

Coaxial cables are commonly used at audio frequencies and above for convenience. A coaxial cable has a conductive wire inside a conductive tube, separated by a dielectric layer. The current flowing on the inner conductor is equal and opposite to the current flowing on the inner surface of the tube. The electromagnetic field is thus completely contained within the tube, and (ideally) no energy is lost to radiation or coupling outside the tube. Coaxial cables have acceptably small losses for frequencies up to about 5 GHz. For microwave frequencies greater than 5 GHz, the losses (due mainly to the electrical resistance of the central conductor) become too large, making waveguides a more efficient medium for transmitting energy. Coaxial cables with an air rather than solid dielectric are preferred as they transmit power with lower losses.

Waveguides

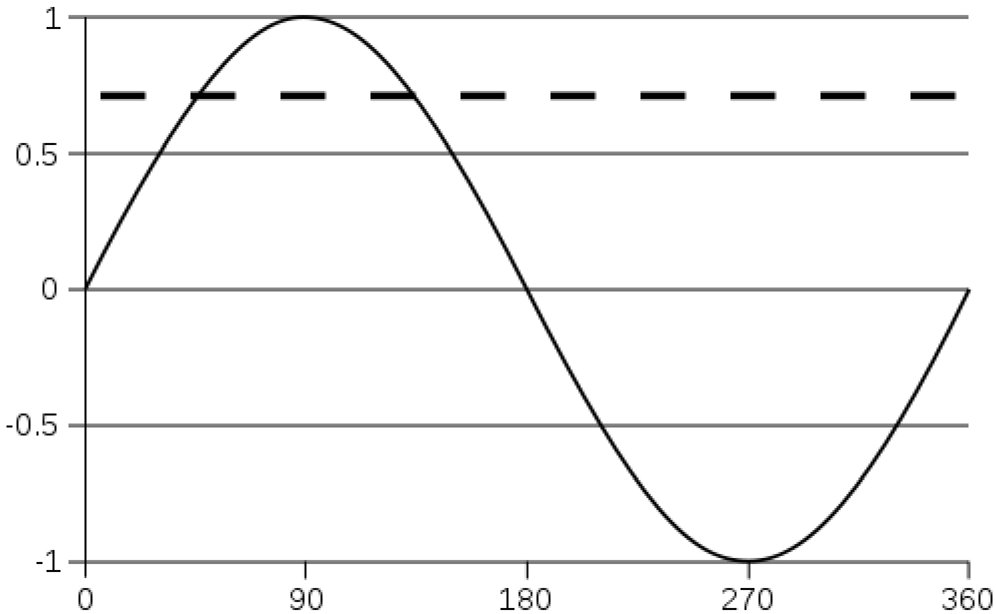
Waveguides are similar to coax cables, as both consist of tubes, with the biggest difference being that the waveguide has no inner conductor. Waveguides can have any arbitrary cross section, but rectangular cross sections are the most common. Because waveguides do not have an inner conductor to carry a return current, waveguides cannot deliver energy by means of an electric current, but rather by means of a *guided* electromagnetic field. Although surface currents do flow on the inner walls of the waveguides, those surface currents do not carry power. Power is carried by the guided electromagnetic fields. The surface currents are set up by the guided electromagnetic fields and have the effect of keeping the fields inside the waveguide and preventing leakage of the fields to the space outside the waveguide.

Waveguides have dimensions comparable to the wavelength of the alternating current to be transmitted, so they are only feasible at microwave frequencies. In addition to this mechanical feasibility, electrical resistance of the non-ideal metals forming the walls of the waveguide cause dissipation of power (surface currents flowing on lossy conductors dissipate power). At higher frequencies, the power lost to this dissipation becomes unacceptably large.

Fiber optics

At frequencies greater than 200 GHz, waveguide dimensions become impractically small, and the ohmic losses in the waveguide walls become large. Instead, fiber optics, which are a form of dielectric waveguides, can be used. For such frequencies, the concepts of voltages and currents are no longer used.

Mathematics of AC voltages



A sine wave, over one cycle (360°). The dashed line represents the root mean square (RMS) value at about 0.707

Alternating currents are accompanied (or caused) by alternating voltages. An AC voltage *v* can be described mathematically as a function of time by the following equation:

$$v(t) = V_{\mathbf{peak}} \cdot \sin(\omega t),$$

where

- $V_{\mathbf{peak}}$ is the peak voltage (unit: volt),
- ω is the angular frequency (unit: radians per second)
  - The angular frequency is related to the physical frequency, f (unit = hertz), which represents the number of cycles per second , by the equation $\omega = 2\pi f$.
- t is the time (unit: second).

The peak-to-peak value of an AC voltage is defined as the difference between its positive peak and its negative peak. Since the maximum value of sin(*x*) is +1 and the minimum value is −1, an AC voltage swings between $+V_{peak}$ and $-V_{peak}$. The peak-to-peak voltage, usually written as V_{pp} or V_{P-P}, is therefore $V_{peak} - (-V_{peak}) = 2V_{peak}$.

Power and root mean square

The relationship between voltage and the power delivered is

$$p(t) = \frac{v^2(t)}{R}$$ where R represents a load resistance.

Rather than using instantaneous power, $p(t)$, it is more practical to use a time averaged power (where the averaging is performed over any integer number of cycles). Therefore, AC voltage is often expressed as a root mean square (RMS) value, written as $V_{\rm rms}$, because

$$P_{\rm time\ averaged} = \frac{{V^2}_{\rm rms}}{R}.$$

For a sinusoidal voltage:

$$V_{\rm rms} = \frac{V_{\rm peak}}{\sqrt{2}}.$$

The factor $\sqrt{2}$ is called the crest factor, which varies for different waveforms.

- For a triangle wave form centered about zero

$$V_{\rm rms} = \frac{V_{\rm peak}}{\sqrt{3}}.$$

- For a square wave form centered about zero

$$V_{\rm rms} = V_{\rm peak}.$$

Example

To illustrate these concepts, consider a 230 V AC mains supply used in many countries around the world. It is so called because its root mean square value is 230 V. This means that the time-averaged power delivered is equivalent to the power delivered by a DC voltage of 230 V. To determine the peak voltage (amplitude), we can rearrange the above equation to:

$$V_{\rm peak} = \sqrt{2}\ V_{\rm rms}.$$

For our 230 V AC, the peak voltage $V_{\rm peak}$ is therefore $230V \times \sqrt{2}$, which is about 325 V. The peak-to-peak value V_{P-P} of the 230 V AC is double that, at about 650 V.

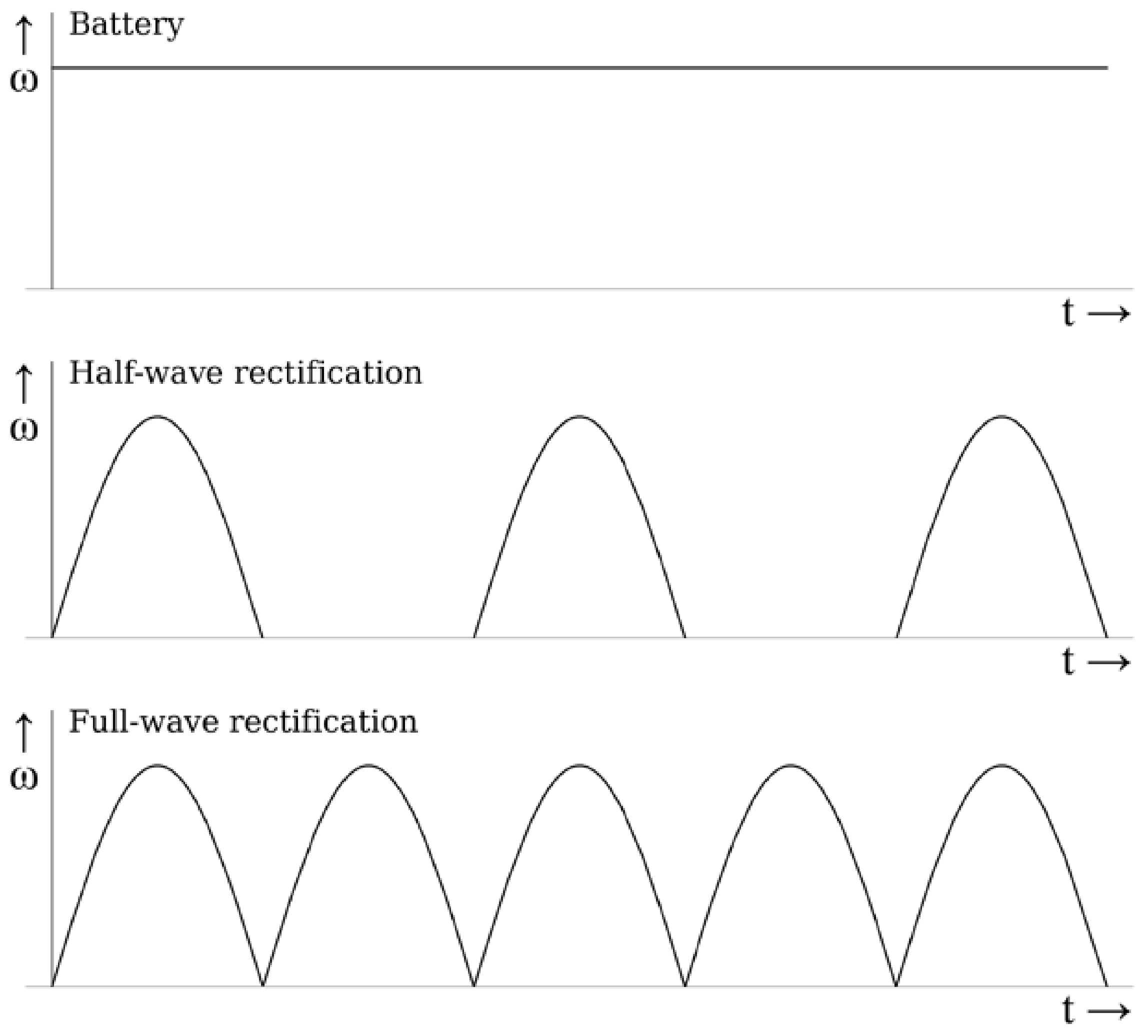
Note that some countries use a frequency of 50 Hz, while others use a frequency of 60 Hz. The calculation to convert from RMS voltage to peak voltage is independent of the frequency.

Chapter 4

Direct Current and Persistent Current

Direct current

Direct current (**DC**) is the unidirectional flow of electric charge. Direct current is produced by such sources as batteries, thermocouples, solar cells, and commutator-type electric machines of the dynamo type. Direct current may flow in a conductor such as a wire, but can also be through semiconductors, insulators, or even through a vacuum as in electron or ion beams. The electric charge flows in a constant direction, distinguishing it from alternating current (AC). A term formerly used for *direct current* was **galvanic current**.



Types of direct current.

Direct current may be obtained from an alternating current supply by use of a current-switching arrangement called a rectifier, which contains electronic elements (usually) or electromechanical elements (historically) that allow current to flow only in one direction. Direct current may be made into alternating current with an inverter or a motor-generator set.

The first commercial electric power transmission (developed by Thomas Edison in the late nineteenth century) used direct current. Because of the significant advantages of alternating current over direct current in transforming and transmission, electric power distribution is nearly all alternating current today. In the mid 1950s, HVDC transmission was developed, which is now replacing the older high voltage alternating current systems. For applications requiring direct current, such as third rail power systems, alternating current is distributed to a substation, which utilizes a rectifier to convert the power to direct current.

Direct current is used to charge batteries, and in nearly all electronic systems, as the power supply. Very large quantities of direct-current power are used in production of aluminum and other electrochemical processes. Direct current is used for some railway propulsion, especially in urban areas. High-voltage direct current is used to transmit large amounts of power from remote generation sites or to interconnect alternating current power grids.

Various definitions

Within electrical engineering, the term *DC* is used to refer to power systems that use only one polarity of voltage or current, and to refer to the constant, zero-frequency, or slowly varying local mean value of a voltage or current. For example, the voltage across a DC voltage source is constant as is the current through a DC current source. The DC solution of an electric circuit is the solution where all voltages and currents are constant. It can be shown that any stationary voltage or current waveform can be decomposed into a sum of a DC component and a zero-mean time-varying component; the DC component is defined to be the expected value, or the average value of the voltage or current over all time.

Although DC stands for "direct current", DC often refers to "constant polarity". Under this definition, DC voltages can vary in time, as seen in the raw output of a rectifier or the fluctuating voice signal on a telephone line.

Some forms of DC (such as that produced by a voltage regulator) have almost no variations in voltage, but may still have variations in output power and current.

Applications

Direct-current installations usually have different types of sockets, switches, and fixtures, mostly due to the low voltages used, from those suitable for alternating current. It is usually important with a direct-current appliance not to reverse polarity unless the device has a diode bridge to correct for this (most battery-powered devices do not).

This symbol is found on many electronic devices that either require or produce direct current.

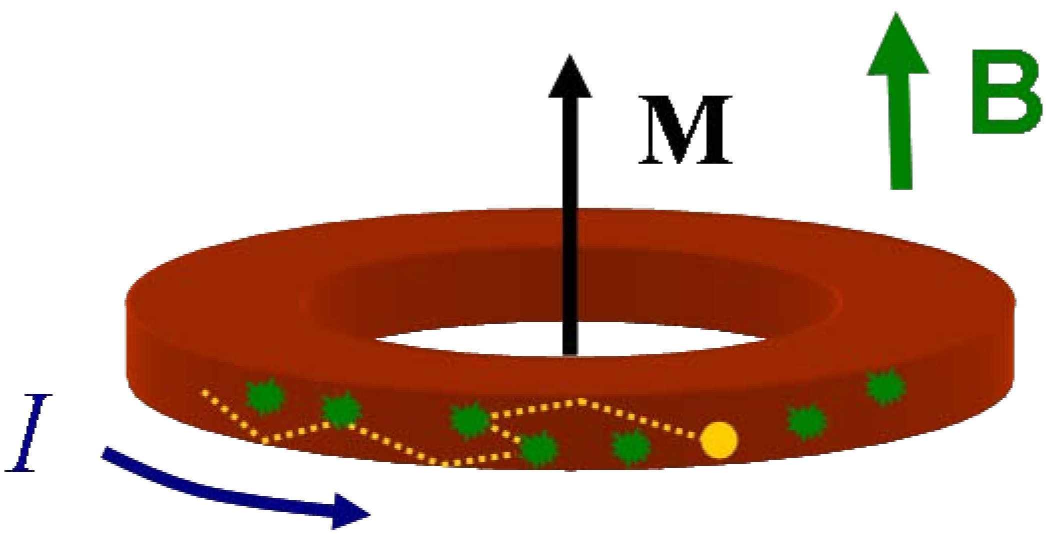
DC is commonly found in many low-voltage applications, especially where these are powered by batteries, which can produce only DC, or solar power systems, since solar cells can produce only DC. Most automotive applications use DC, although the alternator is an AC device which uses a rectifier to produce DC. Most electronic circuits require a

DC power supply. Applications using fuel cells (mixing hydrogen and oxygen together with a catalyst to produce electricity and water as byproducts) also produce only DC.

Many telephones connect to a twisted pair of wires, and internally separate the AC component of the voltage between the two wires (the audio signal) from the DC component of the voltage between the two wires (used to power the phone).

Telephone exchange communication equipment, such as DSLAM, uses standard -48V DC power supply. The negative polarity is achieved by grounding the positive terminal of power supply system and the battery bank. This is done to prevent electrolysis depositions.

Persistent current



Persistent current schematic. The green arrow indicates the direction of static applied magnetic field B which allows a net current I (blue arrow) to flow and create a magnetization M (black arrow) by breaking the symmetry between clockwise and counterclockwise currents. The yellow dot represents an electron traversing the disordered material of the ring (green stars) without dissipation. A typical ring current is 1 nanoampere for a ring diameter of 0.6 micrometer at a temperature below 0.5 kelvin.

Persistent current is a perpetual electrical current, not requiring an external power source, that flows naturally through resistive metal.

The current is the result of a quantum mechanical effect that influences how electrons travel through metals, and arises from the same kind of motion that allows the electrons inside an atom to orbit the nucleus forever. The magnitude of the current becomes appreciable when the size of the metallic system is reduced to the scale of the electron quantum phase coherence length and the thermal length.

Observation

Persistent current was first predicted to be experimentally observable in micrometer-scale rings in 1983 by Markus Büttiker, Yoseph Imry, and Rolf Landauer. Because the effect requires the phase coherence of electrons around the entire ring, the current can not be observed when the ring is interrupted by an ammeter and thus the current must by measured indirectly through its magnetization. Experimental evidence of the observation of persistent currents were first reported in 1990 by a research group at Bell Laboratories using a superconducting resonator to study an array of copper rings. Subsequent measurements using superconducting resonators and extremely sensitive magnetometers known as superconducting quantum interference devices (SQUIDs) produced inconsistent results.

In 2009, physicists at Stanford University using a scanning SQUID and at Yale University using microelectromechanical cantilevers reported measurements of persistent currents in nanoscale gold and aluminum rings respectively that both showed a strong agreement with the simple theory for non-interacting electrons.

> “These are ordinary, non-superconducting metal rings, which we typically think of as resistors, yet these currents will flow forever, even in the absence of an applied voltage.”
>
> — Jack Harris, Associate Professor of Physics and Applied Physics at Yale.

The 2009 measurements both reported greater sensitivity to persistent currents than previous measurements and made several other improvements to persistent current detection. The scanning SQUID's ability to change the position of the SQUID detector relative to the ring sample allowed for a number of rings to be measured on one sample chip and better extraction of the current signal from background noise. The cantilever detector's mechanical detection technique made it possible to measure the rings in a clean electromagnetic environment over a large range of magnetic field and also to measure a number of rings on one sample chip.

Chapter 5

Displacement Current

In electromagnetism, **displacement current** is a quantity that is defined in terms of the rate of change of electric displacement field. Displacement current has the units of electric current density, and it has an associated magnetic field just as actual currents do. However it is not an electric current of moving charges, but a time-varying electric field. In materials, there is also a contribution from the slight motion of charges bound in atoms, dielectric polarization.

The idea was conceived by Maxwell in his 1861 paper On Physical Lines of Force in connection with the displacement of electric particles in a dielectric medium. Maxwell added displacement current to the electric current term in Ampère's Circuital Law. In his 1865 paper A Dynamical Theory of the Electromagnetic Field Maxwell used this amended version of Ampère's Circuital Law to derive the electromagnetic wave equation. This derivation is now generally accepted as an historical landmark in physics by virtue of uniting electricity, magnetism and optics into one single unified theory. The displacement current term is now seen as a crucial addition that completed Maxwell's equations and is necessary to explain many phenomena, most particularly the existence of electromagnetic waves.

Explanation

The electric displacement field is defined as:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P} \ .$$

where:

ε_0 is the permittivity of free space
E is the electric field intensity
P is the polarization of the medium

Differentiating this equation with respect to time defines the *displacement current*, which therefore has two components in a dielectric:

$$J_D = \varepsilon_0 \frac{\partial E}{\partial t} + \frac{\partial P}{\partial t} \ .$$

The first term on the right hand side is present in material media and in free space. It doesn't necessarily involve any actual movement of charge, but it does have an associated magnetic field, just as does a current due to charge motion. Some authors apply the name *displacement current* to only this contribution.

The second term on the right hand side is associated with the polarization of the individual molecules of the dielectric material. Polarization results when the charges in molecules move a little under the influence of an applied electric field. The positive and negative charges in molecules separate, causing an increase in the state of polarization ***P***. A changing state of polarization corresponds to charge movement and so is equivalent to a current.

This polarization is the displacement current as it was originally conceived by Maxwell. Maxwell made no special treatment of the vacuum, treating it as a material medium. For Maxwell, the effect of ***P*** was simply to change the relative permittivity ε_r in the relation $\boldsymbol{D} = \varepsilon_r \varepsilon_0 \boldsymbol{E}$.

The modern justification of displacement current is explained below.

Isotropic dielectric case

In the case of a very simple dielectric material the constitutive relation holds:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \ ,$$

where the permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$,

- ε_r is the relative permittivity of the dielectric and
- ε_0 is the electric constant.

In this equation the use of ε, accounts for the polarization of the dielectric.

The scalar value of displacement current may also be expressed in terms of electric flux:

$$I_{\mathrm{D}} = \varepsilon \frac{\partial \Phi_E}{\partial t}.$$

The forms in terms of ε are correct only for linear isotropic materials. More generally ε may be replaced by a tensor, may depend upon the electric field itself, and may exhibit time dependence (dispersion).

For a linear isotropic dielectric, the polarization ***P*** is given by:

$$\boldsymbol{P} = \varepsilon_0 \chi_e \boldsymbol{E} = \varepsilon_0 (\varepsilon_r - 1) \boldsymbol{E}$$

where χ_e is known as the electric susceptibility of the dielectric. Note that:

$$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e) \varepsilon_0.$$

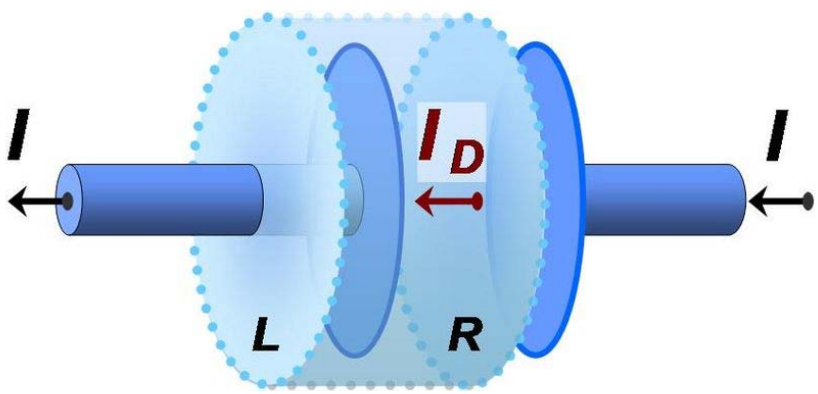
Necessity

Some implications of the displacement current follow, which agree with experimental observation, and with the requirements of logical consistency for the theory of electromagnetism.

Generalizing Ampère's circuital law

Current in capacitors

An example illustrating the need for the displacement current arises in connection with capacitors with no medium between the plates (in free space). Consider the charging capacitor in the figure. The capacitor is in a circuit that transfers charge (on a wire external to the capacitor) from the left plate to the right plate, charging the capacitor and increasing the electric field between its plates. The same current enters the right plate (say I) as leaves the left plate. Although current is flowing through the capacitor, no actual charge is transported through the vacuum between its plates. Nonetheless, a magnetic field exists between the plates as though a current were present there as well. The explanation is that a *displacement current* I_D flows in the vacuum, and this current produces the magnetic field in the region between the plates according to Ampère's law:



An electrically charging capacitor with an imaginary cylindrical surface surrounding the left-hand plate. Right-hand surface R lies in the space between the plates and left-hand surface L lies to the left of the left plate. No conduction current enters cylinder surface R, while current I leaves through surface L. Consistency of Ampère's law requires a displacement current $I_D = I$ to flow across surface R.

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell} = \mu_0 I_D \ .$$

where

- $\oint_C$ is the closed line integral around some closed curve *C*.
- $\mathbf{B}$ is the magnetic field in tesla.
- $\cdot$ is the vector dot product.
- $\mathrm{d}\boldsymbol{\ell}$ is an infinitesimal element (differential) of the curve *C* (that is, a vector with magnitude equal to the length of the infinitesimal line element, and direction given by the tangent to the curve *C*).
- μ_0 is the magnetic constant also called the permeability of free space.
- I_D is the net displacement current that links the curve *C*.

The magnetic field between the plates is the same as that outside the plates, so the displacement current must be the same as the conduction current in the wires, that is,

$$I_D = I \ ,$$

which extends the notion of current beyond a mere transport of charge.

Next, this displacement current is related to the charging of the capacitor. Consider the current in the imaginary cylindrical surface shown surrounding the left plate. A current, say *I*, passes outward through the left surface *L* of the cylinder, but no conduction current (no transport of real charges) enters the right surface *R*. Notice that the electric field between the plates *E* increases as the capacitor charges. That is, in a manner described by Gauss's law, assuming no dielectric between the plates:

$$Q(t) = \varepsilon_0 \oint_S dS \cdot \boldsymbol{E}(t) \ ,$$

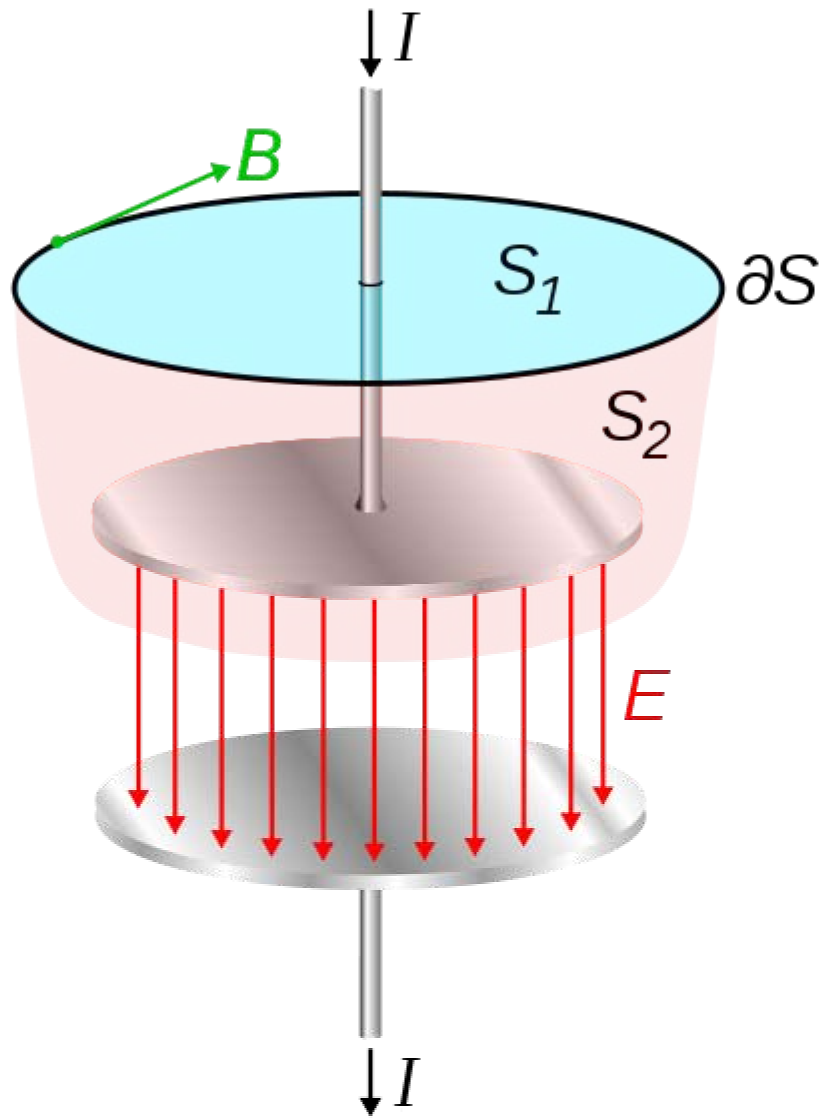
where *S* refers to the imaginary cylindrical surface. Assuming a parallel plate capacitor with uniform electric field, and neglecting fringing effects around the edges of the plates, differentiation provides:

$$\frac{dQ}{dt} = I = \varepsilon_0 \oint_S dS \cdot \frac{\partial \boldsymbol{E}}{\partial t} \approx -S \ \varepsilon_0 \frac{\partial E}{\partial t} \ ,$$

where the sign is negative because charge leaves this plate (the charge is decreasing), and where *S* is the area of the face *R*. The electric field at face *L* is zero because the field due to charge on the right-hand plate is terminated by the equal but opposite charge on the left-hand plate. Under the assumption of a uniform electric field distribution inside the capacitor, the displacement current density J_D is found by dividing by the area of the surface:

$$J_D = \frac{I_D}{S} = -\frac{I}{S} = \varepsilon_0 \frac{\partial E}{\partial t} = \frac{\partial D}{\partial t} \ ,$$

where I is the current leaving the cylindrical surface (which must equal $-I_D$ as the two currents sum to zero) and J_D is the flow of charge per unit area into the cylindrical surface through the face R.



Example showing two surfaces S_1 and S_2 that share the same bounding contour ∂S. However, S_1 is pierced by conduction current, while S_2 is pierced by displacement current.

Combining these results, the magnetic field is found using the integral form of Ampère's law with an arbitrary choice of contour provided the displacement current density term is added to the conduction current density (the Ampère-Maxwell equation):

$$\oint_{\partial S} \boldsymbol{B} \cdot d\boldsymbol{\ell} = \mu_0 \int_S (\boldsymbol{J} + \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}) \cdot d\boldsymbol{S}$$

This equation says that the integral of the magnetic field $\boldsymbol{B}$ around a loop ∂S is equal to the integrated current $\boldsymbol{J}$ through any surface spanning the loop, plus the displacement current term $\varepsilon_0\, \partial \boldsymbol{E} / \partial t$ through the surface. Applying the Ampère-Maxwell equation to surface S_1 we find:

$$B = \frac{\mu_0 I}{2\pi r}$$

However, applying this law to surface S_2, which is bounded by exactly the same curve ∂S, but lies between the plates, provides:

$$B = \frac{\mu_0 I_D}{2\pi r}$$

Any surface that intersects the wire has current I passing through it so Ampère's law gives the correct magnetic field. Also, any surface bounded by the same loop but passing between the capacitor's plates has no charge transport flowing through it, but the $\varepsilon_0\, \partial \boldsymbol{E} / \partial t$ term provides a second source for the magnetic field besides charge conduction current. Because the current is increasing the charge on the capacitor's plates, the electric field between the plates is increasing, and the rate of change of electric field gives the correct value for the field $\boldsymbol{B}$ found above.

Mathematical formulation

In a more mathematical vein, the same results can be obtained from the underlying differential equations. Consider for simplicity a non-magnetic medium where the relative magnetic permeability is unity, and the complication of magnetization current is absent. The current leaving a volume must equal the rate of decrease of charge in a volume. In differential form this continuity equation becomes:

$$\nabla \cdot \boldsymbol{J_f} = -\frac{\partial \rho_f}{\partial t} \ ,$$

where the left side is the divergence of the free current density and the right side is the rate of decrease of the free charge density. However, Ampère's law in its original form states:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J}_f \ ,$$

which implies that the divergence of the current term vanishes, contradicting the continuity equation. (Vanishing of the *divergence* is a result of the mathematical identity that states the divergence of a *curl* is always zero.) This conflict is removed by addition of the displacement current, as then:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right) = \mu_0 \left(\boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t} \right) \ ,$$

and

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{B}) = 0 = \mu_0 \left(\nabla \cdot \boldsymbol{J}_f + \frac{\partial}{\partial t} \boldsymbol{\nabla} \cdot \boldsymbol{D} \right) \ ,$$

which is in agreement with the continuity equation because of Gauss's law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho_f \ .$$

Wave propagation

The added displacement current also leads to wave propagation by taking the curl of the equation for magnetic field. In the particular situation where there is no polarization (***P***=0); which occurs in free space, for example; the displacement current is:

$$\boldsymbol{J_D} = \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

Substituting this form for ***J*** into Ampère's law, and assuming there is no bound or free current density contributing to ***J*** :

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \mu_0 \boldsymbol{J_D} \ ,$$

with the result:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \boldsymbol{\nabla} \times \boldsymbol{E} \ .$$

However,

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial}{\partial t} \boldsymbol{B} \ ,$$

leading to the wave equation:

$$-\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) = \nabla^2 \boldsymbol{B} = \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \boldsymbol{B} = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \boldsymbol{B} \ ,$$

where use is made of the vector identity that holds for any vector field $\boldsymbol{V}(\boldsymbol{r}, t)$:

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{V}) = \boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{V}) - \nabla^2\boldsymbol{V} \ ,$$

and the fact that the divergence of the magnetic field is zero. An identical wave equation can be found for the electric field by taking the *curl*:

$$\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\boldsymbol{E}) = -\frac{\partial}{\partial t}\boldsymbol{\nabla}\times\boldsymbol{B} = -\mu_0\frac{\partial}{\partial t}\left(\boldsymbol{J} + \epsilon_0\frac{\partial}{\partial t}\boldsymbol{E}\right) \ .$$

If ***J, P*** and ρ are zero (as in free space), the result is:

$$\nabla^2\boldsymbol{E} = \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\boldsymbol{E} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\boldsymbol{E} \ .$$

The electric field can be expressed in the general form:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\varphi - \frac{\partial\boldsymbol{A}}{\partial t} \ ,$$

where φ is the electric potential (which can be chosen to satisfy Poisson's equation) and $\boldsymbol{A}$ is a vector potential. The $\nabla\varphi$ component on the right hand side is the Gauss's law component, and this is the component that is relevant to the conservation of charge argument above. The second term on the right-hand side is the one relevant to the electromagnetic wave equation, because it is the term that contributes to the *curl* of $\boldsymbol{E}$. Because of the vector identity that says the *curl* of a *gradient* is zero, $\nabla\varphi$ does not contribute to $\nabla\times\boldsymbol{E}$.

History and interpretation

Maxwell's displacement current was postulated in part III of his 1861 paper 'On Physical Lines of Force'. Few topics in modern physics have caused as much confusion and misunderstanding as that of displacement current. This is in part due to the fact that Maxwell used a sea of molecular vortices in his derivation, while modern textbooks operate on the basis that displacement current can exist in free space. Maxwell's derivation is unrelated to the modern day derivation for displacement current in the vacuum, which is based on consistency between Ampère's law for the magnetic field and the continuity equation for electric charge.

Maxwell's purpose is stated by him at (Part I, p. 161):

I propose now to examine magnetic phenomena from a mechanical point of view, and to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed.

He is careful to point out the treatment is one of analogy:

The author of this method of representation does not attempt to explain the origin of the observed forces by the effects due to these strains in the elastic solid, but makes use of the mathematical analogies of the two problems to assist the imagination in the study of both.

In part III, in relation to displacement current, he says

I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another.

Clearly Maxwell was driving at magnetization even though the same introduction clearly talks about dielectric polarization.

Maxwell concluded, using Newton's equation for the speed of sound (*Lines of Force*, Part III, equation (132)), that "light consists of transverse undulations in the same medium that is the cause of electric and magnetic phenomena."

But although the above quotations point towards a magnetic explanation for displacement current, for example, based upon the divergence of the above *curl* equation, Maxwell's explanation ultimately stressed linear polarization of dielectrics:

This displacement...is the commencement of a current...The amount of displacement depends on the nature of the body, and on the electromotive force so that if h is the displacement, R the electromotive force, and E a coefficient depending on the nature of the dielectric:

$$R = -4\pi \mathrm{E}^2 h \ ;$$

and if r is the value of the electric current due to displacement

$$r = \frac{dh}{dt} \ ,$$

These relations are independent of any theory about the mechanism of dielectrics; but when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement...we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure and recovering its form when the pressure is removed.—Part III – *The theory of molecular vortices applied to statical electricity* , pp. 14–15

With some change of symbols (and units): $r \rightarrow J$, $R \rightarrow -E$ and the material constant $\mathrm{E}^{-2} \rightarrow 4\pi\, \varepsilon_r \varepsilon_0$ these equations take the familiar form:

$$J = \frac{d}{dt}\frac{1}{4\pi \mathrm{E}^2}E = \frac{d}{dt}\varepsilon_r\varepsilon_0 E = \frac{d}{dt}D\ .$$

When it came to deriving the electromagnetic wave equation from displacement current in his 1865 paper A Dynamical Theory of the Electromagnetic Field, he got around the problem of the non-zero divergence associated with Gauss's law and dielectric displacement by eliminating the Gauss term and deriving the wave equation exclusively for the solenoidal magnetic field vector.

Maxwell's emphasis on polarization diverted attention towards the electric capacitor circuit, and led to the common belief that Maxwell conceived of displacement current so as to maintain conservation of charge in an electric capacitor circuit. There are a variety of debatable notions about Maxwell's thinking, ranging from his supposed desire to perfect the symmetry of the field equations to the desire to achieve compatibility with the continuity equation.

So it was that displacement current became associated with capacitors. Once Maxwell's sea of molecular vortices had been abandoned in the 20th century (along with the aether), an interpretation of displacement current evolved that treated free space explicitly, allowing a separation of free space from material media, unlike Maxwell's original concept. The modern displacement current can be derived in connection with ideal capacitors in free space by relating the magnetic field to current using the equation $I = C\ \partial V/\partial t$, where the charging current is $I = \partial Q/\partial t$, Q is electric charge, C is capacitance, and V is voltage.

We can therefore identify three different kinds of displacement current.

1. The displacement current that is associated with polarization of a dielectric.
2. The displacement current that is associated with magnetization and wireless telegraphy (that is, with electromagnetic waves). In this case the electric field term ***E*** will have a zero divergence and will be compatible with the time varying electric field term in Faraday's law of induction.
3. The virtual displacement current that is associated with maintaining a magnetic field in a charging or discharging ideal capacitor in free space despite the solenoidal nature of Ampère's Circuital Law.

(1) and (3), are connected with cable telegraphy and involve a non-zero divergence for ***E***. Interestingly, in 1857, Kirchhoff derived the cable telegraphy equation using the interrelationships between Poisson's equation and the equation of continuity which would connect to (1) and (3) above through capacitor theory. Kirchhoff never used the concept of displacement current. Instead, he manipulated the non-zero divergent ***E*** of Gauss's law with the zero-divergent, time-varying ***E*** of Faraday's law as if they were one and the same thing.

Chapter 6

Streaming Current and Ampere

Streaming current

A **streaming current** and **streaming potential** are two interrelated electrokinetic phenomena studied in the areas of surface chemistry and electrochemistry. They are an electric current or potential which originates when an electrolyte is driven by a pressure gradient through a channel or porous plug with charged walls.

The first observation of the streaming potential is generally attributed to the German physicist Georg Hermann Quincke in 1859.

Streaming currents in well-defined geometries are a sensitive method to characterize the zeta potential of surfaces, which is important in the fields of colloid and interface science. They could in principle be used to generate electrical power, however this process has not been applied so far because of its low efficiency.

Origin of the streaming current

Adjacent to the channel walls, the charge-neutrality of the liquid is violated due to the presence of the electrical double layer: a thin layer of counterions attracted by the charged surface.

The transport of counterions along with the pressure-driven fluid flow gives rise to a net charge transport: the streaming current. The reverse effect, generating a fluid flow by applying a potential difference, is called electroosmotic flow.

Measurement method

A typical setup to measure streaming currents consists of two reversible electrodes placed on either side of a fluidic geometry across which a known pressure difference is applied. When both electrodes are held at the same potential, the streaming current is measured directly as the electric current flowing through the electrodes. Alternatively, the electrodes can be left floating, allowing a streaming potential to build up between the two ends of the channel.

A streaming potential is defined as positive when the electric potential is higher on the high pressure end of the flow system that on the low pressure end.

The value of streaming current observed in a capillary is usually related to the zeta potential through the relation:

$$I_{str} = -\frac{\epsilon_{rs}\epsilon_0 a^2}{\eta}\frac{\Delta P}{L}\zeta.$$

The conduction current, which is equal in magnitude to the streaming current at steady state, is:

$$I_c = K_L a^2 \frac{U_{str}}{L}$$

At steady state, the streaming potential built up across the flow system is given by:

$$U_{str} = \frac{\epsilon_{rs}\epsilon_0\zeta}{\eta K_L}\Delta P$$

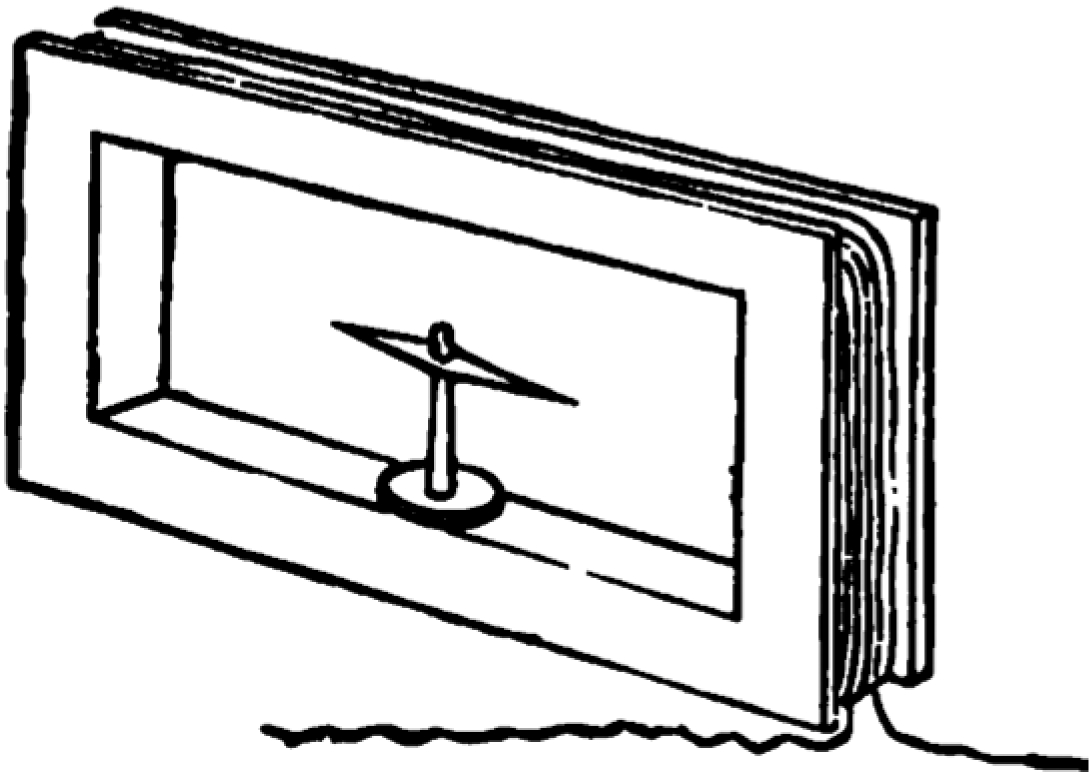
Symbols:

- I_{str} - streaming current under short-circuit conditions, A
- U_{str} - streaming potential at zero net current conditions, V
- I_c - conduction current, A
- ε_{rs} - relative permittivity of the liquid, dimensionless
- ε_0 - electrical permittivity of vacuum, $F \cdot m^{-1}$
- η - dynamic viscosity of the liquid, $kg \cdot m^{-1} \cdot s^{-1}$
- ζ - zeta potential, V
- ΔP - pressure difference, Pa
- L - capillary length, m
- a - capillary radius, m
- K_L - specific conductivity of the bulk liquid, $S \cdot m^{-1}$

The above equations assume that:

- the double layer is not too large compared to the pores or capillaries (i.e., $\kappa a \gg 1$), where κ is the reciprocal of the Debye length
- there is no surface conduction (which typically may become important when the zeta potential is large, e.g., $|\zeta| > 50$ mV)
- there is no electrical double layer polarization.

Ampere



Current can be measured by a galvanometer, via the deflection of a magnetic needle in the magnetic field created by the current.

The **ampere** (symbol: A) is the SI unit of electric current (symbol: I) and is one of the seven SI base units. It is named after André-Marie Ampère (1775–1836), French mathematician and physicist, considered the father of electrodynamics. In practice, its name is often shortened to **amp**.

In practical terms, the ampere is a measure of the amount of electric charge passing a point per unit time. Around 6.241×10^{18} electrons, or one coulomb, passing a given point each second constitutes one ampere.

Definition

Ampère's force law states that there is an attractive force between two parallel wires carrying an electric current. This force is used in the formal definition of the ampere which states that it is "the constant current which will produce an attractive force of 2×10^{-7} newton per metre of length between two straight, parallel conductors of infinite length and negligible circular cross section placed one metre apart in a vacuum".

In terms of Ampère's force law,

$$2 \times 10^{-7}\ \frac{\mathrm{N}}{\mathrm{m}} = 2 \times k_A \frac{\mathrm{A} \cdot \mathrm{A}}{\mathrm{m}}$$

so

$$1\ \mathrm{A} = \sqrt{\frac{2 \times 10^{-7}\ \mathrm{N}}{2 \times k_A}}$$

The SI unit of charge, the coulomb, "is the quantity of electricity carried in 1 second by a current of 1 ampere." Conversely, a current of one ampere is one coulomb of charge going past a given point per second**:**

$$1\ \mathrm{A} = 1 \frac{\mathrm{C}}{\mathrm{s}}$$

That is, in general, charge Q is determined by steady current I flowing for a time t as $Q = It$.

History

The ampere was originally defined as one tenth of the CGS system electromagnetic unit of current (now known as the abampere), the amount of current which generates a force of two dynes per centimetre of length between two wires one centimetre apart. The size of the unit was chosen so that the units derived from it in the MKSA system would be conveniently sized.

The "international ampere" was an early realisation of the ampere, defined as the current that would deposit 0.001118000 grams of silver per second from a silver nitrate solution. Later, more accurate measurements revealed that this current is 0.99985 A.

Realisation

The ampere is most accurately realised using a watt balance, but is in practice maintained via Ohm's Law from the units of electromotive force and resistance, the volt and the ohm, since the latter two can be tied to physical phenomena that are relatively easy to reproduce, the Josephson junction and the quantum Hall effect, respectively.

At present, techniques to establish the realisation of an ampere have a relative uncertainty of approximately a few parts in 10^7, and involve realisations of the watt, the ohm and the volt.

Proposed future definition

Rather than a definition in terms of the force between two current-carrying wires, it has been proposed to define the ampere in terms of the rate of flow of elementary charges. Since a coulomb is approximately equal to $6.24150948 \times 10^{18}$ elementary charges, one ampere is approximately equivalent to $6.24150948 \times 10^{18}$ elementary charges per second, such as electrons, moving past a boundary in one second. The proposed change would define 1 A as being the current in the direction of flow of a particular number of elementary charges per second. In 2005, the International Committee for Weights and Measures (CIPM) agreed to study the proposed change. The new definition is expected to be formally proposed at the 25th General Conference on Weights and Measures (CGPM) in 2015.

Everyday examples

The current drawn by typical systems is usually dictated by the power (watts) consumed by the system and voltage supplied. For this reason the examples given below are grouped by voltage level.

Portable gadgets

- Hearing aid (typically 1 mW at 1.4 V): 0.7 mA

Motorcars - 12 V DC

A typical motor car has a 12 V battery. The various accessories that are powered by the battery might include:

- Headlights (typically 60 W): 5 A each.
- Instrument panel light (typically 2 W): 166 mA.
- Starter Motor (typically 1-2 kW): 80-160 A

US domestic supply - 120 V AC

Most United States domestic power suppliers run at 120 V.

Household circuit breakers typically provide a maximum of 15A or 20A of current to a given set of outlets.

European domestic supply - 230 V AC

Most European domestic power supplies run at 230 V, so the current drawn for a particular appliance will be less than for an equivalent United States appliance. This means that lighter (and cheaper) cabling can be used.

The current drawn by a number of typical appliances are:

- Toaster, kettle (2 kW): 9 A
- Immersion heater (16 kW): 20 A
- Tungsten light bulb (60 W - 100 W): 250 mA - 450 mA
- 22 inch/56 cm Portable Television (35 W): 150 mA

Chapter 7

Current Density

Current density is a measure of the density of flow of a conserved charge. Usually the charge is the electric charge, in which case the associated current density is the electric current per unit area of cross section, but the term *current density* can also be applied to other conserved quantities. It is defined as a vector whose magnitude is the current per cross-sectional area.

In SI units, the electric current density is measured in amperes per square metre.

Definition

Electric current is a coarse, average quantity that tells what is happening in an entire wire. The *distribution* of flow of charge is described by the current density:

$$\mathbf{J}(\mathbf{r},t) = qn(\mathbf{r},t)\ \mathbf{v}_d(\mathbf{r},t) = \rho(\mathbf{r},t)\ \mathbf{v}_d(\mathbf{r},t)\,,$$

where

$\mathbf{J}(\mathbf{r},\ t)$ is the current density vector at location $\mathbf{r}$ at time t (SI unit: amperes per square metre),
$n(\mathbf{r},\ t)$ is the particle density in count per volume at location $\mathbf{r}$ at time t (SI unit: m^{-3}),
q is the charge of the individual particles with density n (SI unit: coulombs)
$\rho(\mathbf{r},\ t) = qn(\mathbf{r},\ t)$ is the charge density (SI unit: coulombs per cubic metre), and
$\mathbf{v}_d(\mathbf{r},\ t)$ is the particles' average drift velocity at position $\mathbf{r}$ at time t (SI unit: metres per second)

Importance

Current density is important to the design of electrical and electronic systems.

Circuit performance depends strongly upon the designed current level, and the current density then is determined by the dimensions of the conducting elements. For example, as integrated circuits are reduced in size, despite the lower current demanded by smaller devices, there is trend toward higher current densities to achieve higher device numbers in ever smaller chip areas.

At high frequencies, current density can increase because the conducting region in a wire becomes confined near its surface, the so-called skin effect.

High current densities have undesirable consequences. Most electrical conductors have a finite, positive resistance, making them dissipate power in the form of heat. The current density must be kept sufficiently low to prevent the conductor from melting or burning up, or the insulating material failing. At high current densities the material forming the interconnections actually moves, a phenomenon called *electromigration*. In superconductors excessive current density may generate a strong enough magnetic field to cause spontaneous loss of the superconductive property.

The analysis and observation of current density also is used to probe the physics underlying the nature of solids, including not only metals, but also semiconductors and insulators. An elaborate theoretical formalism has developed to explain many fundamental observations.

The current density is an important parameter in Ampère's circuital law (one of Maxwell's equations), which relates current density to magnetic field.

In special relativity theory, charge and current are combined into a 4-vector.

Approximate calculation of current density

A common approximation to the current density assumes the current simply is proportional to the electric field, as expressed by:

$$\mathbf{J} = \sigma \mathbf{E}$$

where **E** is the electric field and σ is the electrical conductivity.

Conductivity σ is the reciprocal (inverse) of electrical resistivity and has the SI units of siemens per metre (S m^{-1}), and **E** has the SI units of newtons per coulomb (N C^{-1}) or, equivalently, volts per metre (V m^{-1}).

A more fundamental approach to calculation of current density is based upon:

$$\mathbf{J}(\mathbf{r},t) = \int_{-\infty}^{t} \mathrm{d}t' \int \mathrm{d}^3\mathbf{r}' \; \sigma(\mathbf{r}-\mathbf{r}', t-t') \; \mathbf{E}(\mathbf{r}',\; t') \,,$$

indicating the lag in response by the time dependence of σ, and the non-local nature of response to the field by the spatial dependence of σ, both calculated in principle from an underlying microscopic analysis, for example, in the case of small enough fields, the linear response function for the conductive behavior in the material. See, for example, Giuliani or Rammer. The integral extends over the entire past history up to the present time.

As some reflection might indicate, the above conductivity and its associate current density reflect the fundamental mechanisms underlying charge transport in the medium, both in time and over distance.

A Fourier transform in space and time then results in:

$$\mathbf{J}(\mathbf{k},\omega) = \sigma(\mathbf{k},\omega)\ \mathbf{E}(\mathbf{k},\omega)\,,$$

where $\sigma(\mathbf{k},\ \omega)$ is now a complex function.

In many materials, for example, in crystalline materials, the conductivity is a tensor, and the current is not necessarily in the same direction as the applied field. Aside from the material properties themselves, the application of magnetic fields can alter conductive behavior.

Current through a surface

The current through a surface area S perpendicular to the flow can be calculated using a surface integral:

$$I = \int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A}$$

where the current is in fact the integral of the dot product of the current density vector and the differential surface element d**A**, in other words, the net flux of the current density vector field flowing through the surface *S*.

Continuity equation

Because charge is conserved, the net flow out of a chosen volume must equal the net change in charge held inside the volume:

$$\int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A} = -\frac{\mathrm{d}}{\mathrm{d}t}\int_V \rho\, \mathrm{d}V = -\int_V \left(\frac{\partial \rho}{\partial t}\right) \mathrm{d}V\ ,$$

where ρ is the charge density per unit volume, and $d\mathbf{A}$ is a surface element of the surface *S* enclosing the volume *V*. The surface integral on the left expresses the current *outflow* from the volume, and the negatively signed volume integral on the right expresses the *decrease* in the total charge inside the volume. From the divergence theorem,

$$\int_S \mathbf{J} \cdot \mathrm{d}\mathbf{A} = \int_V (\nabla \cdot \mathbf{J}) \mathrm{d}V \ .$$

Hence:

$$\int_V (\nabla \cdot \mathbf{J}) \mathrm{d}V \ = - \int_V \left(\frac{\partial \rho}{\partial t} \right) \mathrm{d}V \ .$$

Because this relation is valid for any volume, no matter how small, no matter where located:

$$\nabla \cdot \mathbf{J} = - \frac{\partial \rho}{\partial t} \ ,$$

which is called the continuity equation.

In practice

- In the domain of electrical wiring (isolated copper), maximum current density can vary from $4 A/mm^2$ for a wire isolated from free air to $6 A/mm^2$ for a wire in free air. If the wire is carrying high frequency currents, depending on its diameter, the skin effect may affect the distribution of the current across the section by concentrating the current on the surface of the conductor. This skin effect plays an important role in Switched-mode power supply transformers where the wires carries high currents and high frequencies (between 10 kHz and 1 MHz). Often in those transformers, the windings are made of multiple isolated wires in parallel with a diameter twice the skin depth and which are twisted together to increase the total skin area and to reduce the impact of the skin effect.

- In the domain of printed circuit boards, for TOP and BOTTOM layers, maximum current density can be as high as $35 A/mm^2$ with a copper thickness of 35 µm. Inner layers cannot dissipate as much power as outer layers; thus it is not a good idea to put high power lines in inner layers.

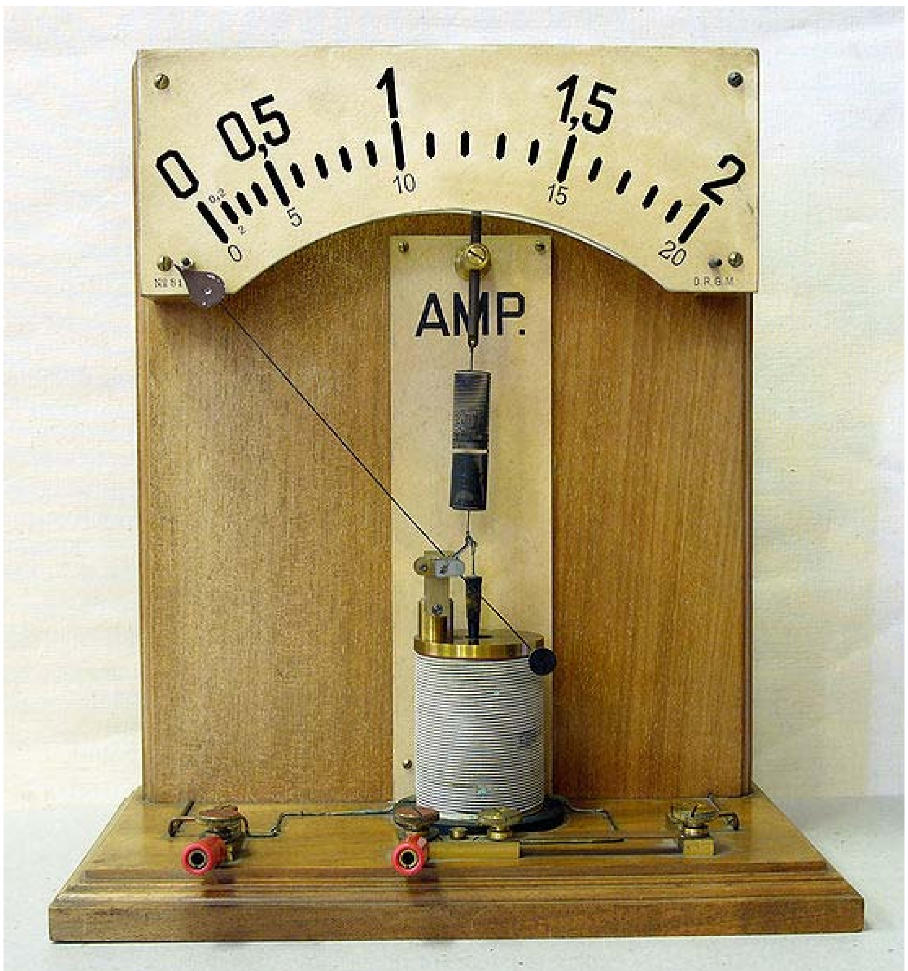
- In the domain of semiconductors, the maximum current density is given by the manufacturer. A common average is $1 mA/\mu m^2$ at 25°C for 180 nm technology. Above the maximum current density, apart from the joule effect, some other effects like electromigration appear in the micrometer scale.

- In biological systems, ion channels regulate the flow of ions (for example, sodium, calcium, potassium) across the membrane in all cells. Current density is measured in pA/pF (picoamperes per picofarad), that is, current divided by capacitance, a de facto measure of membrane area.

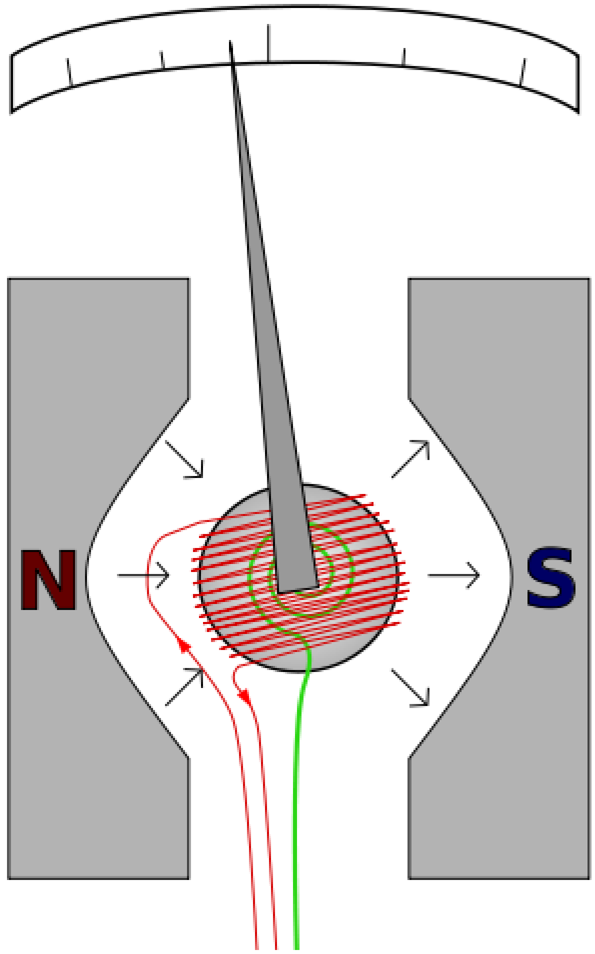
- In gas discharge lamps, such as flashlamps, current density plays an important role in the output spectrum produced. Low current densities produce spectral line emission and tend to favor longer wavelengths. High current densities produce continuum emission and tend to favor shorter wavelengths. Low current densities for flash lamps are generally around $1000 A/cm^2$. High current densities can be more than $4000 A/cm^2$.

Chapter 8

Ammeter

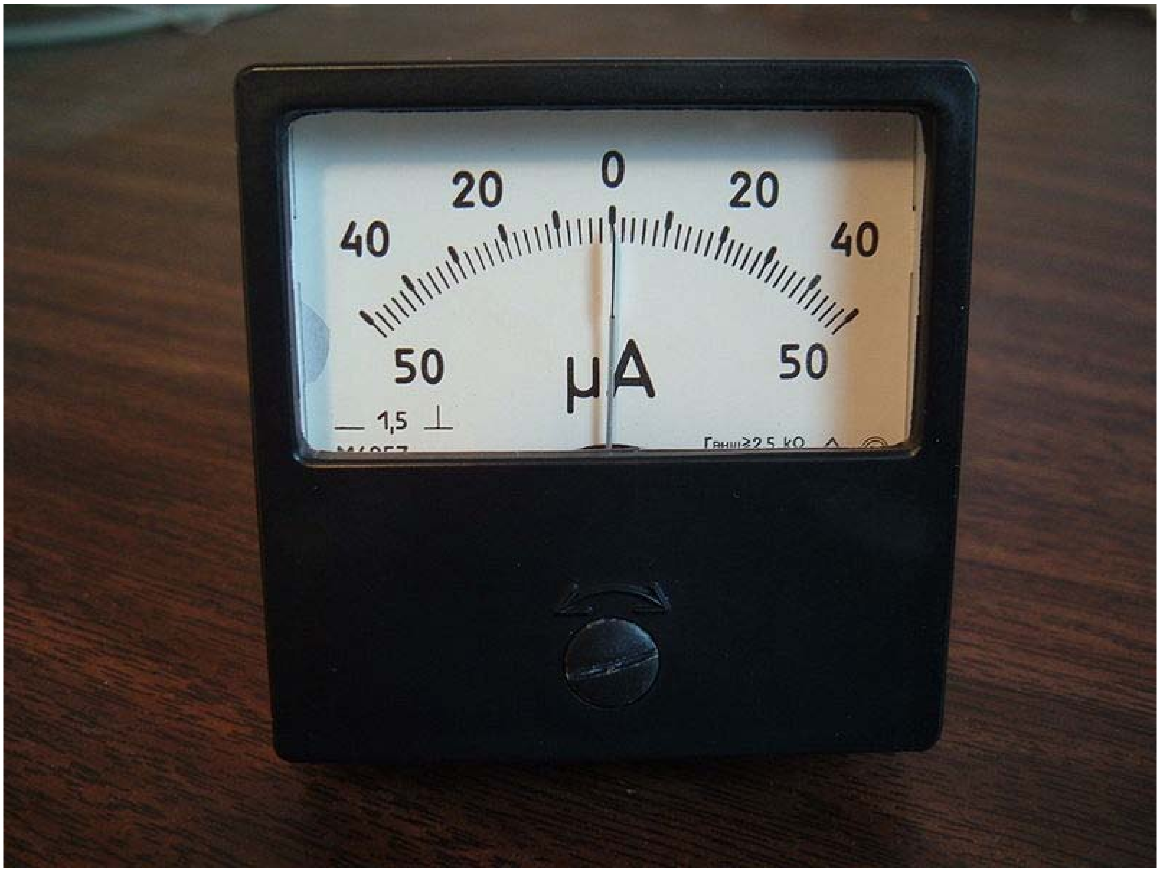


Demonstration model of a moving iron ammeter. As the current through the coil increases, the plunger is drawn further into the coil and the pointer deflects to the right.



Wire carrying current to be measured.
Spring providing restoring force This illustration is conceptual; in a practical meter, the iron core is stationary, and front and rear spiral springs carry current to the coil, which is supported on a rectangular bobbin. Furthermore, the poles of the permanent magnet are arcs of a circle.

Ammeter from the old Penn Station Terminal Service Plant in New York City



Zero-center ammeter

An older moving iron ammeter with its characteristic non-linear scale and with the moving iron ammeter symbol mounted on a small form factor PC.

An **ammeter** is a measuring instrument used to measure the electric current in a circuit. Electric currents are measured in amperes (A), hence the name. Instruments used to measure smaller currents, in the milliampere or microampere range, are designated as *milliammeters* or *microammeters*. Early ammeters were laboratory instruments which relied on the Earth's magnetic field for operation. By the late 19th century, improved instruments were designed which could be mounted in any position and allowed accurate measurements in electric power systems.

History

The relation between electric current, magnetic fields and physical forces was first noted by Hans Christian Ørsted who, in 1820, observed a compass needle was deflected from pointing North when a current flowed in an adjacent wire. The tangent galvanometer was used to measure currents using this effect, where the restoring force returning the pointer to the zero position was provided by the Earth's magnetic field. This made these instruments usable only when aligned with the Earth's field. Sensitivity of the instrument was increased by using additional turns of wire to multiply the effect – the instruments were called "multipliers".

Types

The D'Arsonval galvanometer is a **moving coil** ammeter. It uses magnetic deflection, where current passing through a coil causes the coil to move in a magnetic field. The modern form of this instrument was developed by Edward Weston, and uses two spiral springs to provide the restoring force. By maintaining a uniform air gap between the iron core of the instrument and the poles of its permanent magnet, the instrument has good linearity and accuracy. Basic meter movements can have full-scale deflection for currents from about 25 microamperes to 10 milliamperes and have linear scales.

Moving iron ammeters use a piece of iron which moves when acted upon by the electromagnetic force of a fixed coil of wire. This type of meter responds to both direct and alternating currents (as opposed to the moving coil ammeter, which works on direct current only). The iron element consists of a moving vane attached to a pointer, and a fixed vane, surrounded by a coil. As alternating or direct current flows through the coil and induces a magnetic field in both vanes, the vanes repel each other and the moving vane deflects against the restoring force provided by fine helical springs. The non-linear scale of these meters makes them unpopular.

An **electrodynamic** movement uses an electromagnet instead of the permanent magnet of the d'Arsonval movement. This instrument can respond to both alternating and direct current.

In a **hot-wire ammeter**, a current passes through a wire which expands as it heats. Although these instruments have slow response time and low accuracy, they were sometimes used in measuring radio-frequency current.

Digital ammeter designs use an analog to digital converter (ADC) to measure the voltage across the shunt resistor; the digital display is calibrated to read the current through the shunt.

There is also a whole range of devices referred to as **integrating ammeters**. In these ammeters the amount of current is summed over time giving as a result the product of current and time, which is proportional to the energy transferred with that current. These can be used for energy meters (watt-hour meters) or for estimating the charge of battery or capacitor.

Picoammeter

A **picoammeter**, or pico ammeter, measures very low electrical current, usually from the picoampere range at the lower end to the milliampere range at the upper end. Picoammeters are used for sensitive measurements where the current being measured is below the theoretical limits of sensitivity of other devices, such as Multimeters.

Most picoammeters use a "virtual short" technique and have several different measurement ranges that must be switched between to cover multiple decades of measurement. Other modern picoammeters use log compression and a "current sink" method that eliminates range switching and associated voltage spikes.

Application

The majority of ammeters are either connected in series with the circuit carrying the current to be measured (for small fractional amperes), or have their shunt resistors connected similarly in series. In either case, the current passes through the meter or (mostly) through its shunt. They must not be connected to a source of voltage; they are designed for minimal burden, which refers to the voltage drop across the ammeter, which is typically a small fraction of a volt. They are almost a short circuit.

Ordinary Weston-type meter movements can measure only milliamperes at most, because the springs and practical coils can carry only limited currents. To measure larger currents, a resistor called a *shunt* is placed in parallel with the meter. The resistances of shunts is in the integer to fractional milliohm range. Nearly all of the current flows through the shunt, and only a small fraction flows through the meter. This allows the meter to measure large currents. Traditionally, the meter used with a shunt has a full-scale deflection (FSD) of 50 mV, so shunts are typically designed to produce a voltage drop of 50 mV when carrying their full rated current.

Zero-center ammeters are used for applications requiring current to be measured with both polarities, common in scientific and industrial equipment. Zero-center ammeters are also commonly placed in series with a battery. In this application, the charging of the battery deflects the needle to one side of the scale (commonly, the right side) and the discharging of the battery deflects the needle to the other side. A special type of zero-center ammeter for testing high currents in cars and trucks has a pivoted bar magnet that moves the pointer, and a fixed bar magnet to keep the pointer centered with no current. The magnetic field around the wire carrying current to be measured deflects the moving magnet.

Since the ammeter shunt has a very low resistance, mistakenly wiring the ammeter in parallel with a voltage source will cause a short circuit, at best blowing a fuse, possibly damaging the instrument and wiring, and exposing an observer to injury.

In AC circuits, a current transformer converts the magnetic field around a conductor into a small AC current, typically either 1 A or 5 A at full rated current, that can be easily read by a meter. In a similar way, accurate AC/DC non-contact ammeters have been constructed using Hall effect magnetic field sensors. A portable hand-held clamp-on ammeter is a common tool for maintenance of industrial and commercial electrical equipment, which is temporarily clipped over a wire to measure current. Some recent types have a parallel pair of magnetically-soft probes that are placed on either side of the conductor.

Chapter 9

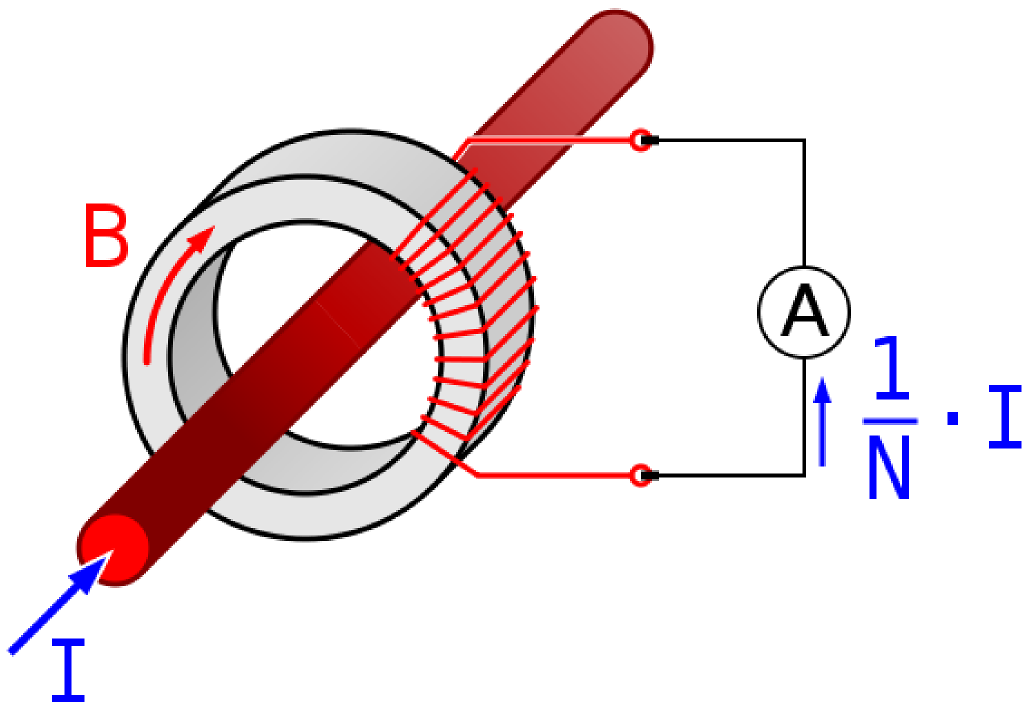
Current Transformer and Current Clamp

Current transformer

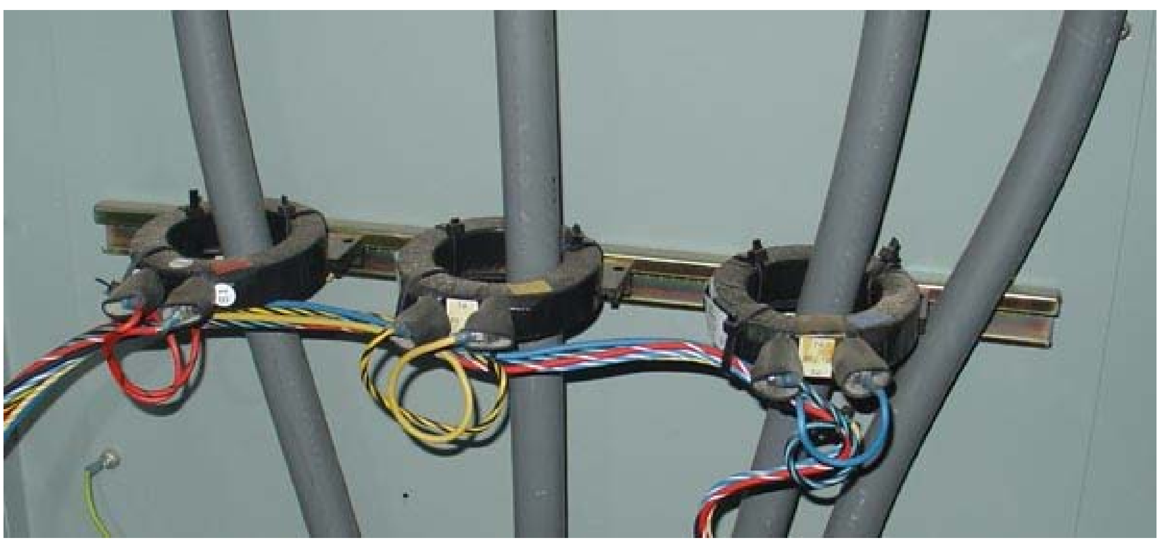
A CT for operation on a 110 kV grid

In electrical engineering, a **current transformer** (**CT**) is used for measurement of electric currents. Current transformers, together with **voltage transformers** (**VT**) (**potential transformers** (**PT**)), are known as **instrument transformers**. When current in a circuit is too high to directly apply to measuring instruments, a current transformer produces a reduced current accurately proportional to the current in the circuit, which can be conveniently connected to measuring and recording instruments. A current transformer also isolates the measuring instruments from what may be very high voltage in the monitored circuit. Current transformers are commonly used in metering and protective relays in the electrical power industry.

Design



SF_6 110 kV current transformer TGFM series, Russia



Current transformers used in metering equipment for three-phase 400 ampere electricity supply

Like any other transformer, a current transformer has a primary winding, a magnetic core, and a secondary winding. The alternating current flowing in the primary produces a magnetic field in the core, which then induces a current in the secondary winding circuit. A primary objective of current transformer design is to ensure that the primary and secondary circuits are efficiently coupled, so that the secondary current bears an accurate relationship to the primary current.

The most common design of CT consists of a length of wire wrapped many times around a silicon steel ring passed over the circuit being measured. The CT's primary circuit therefore consists of a single 'turn' of conductor, with a secondary of many hundreds of turns. The primary winding may be a permanent part of the current transformer, with a heavy copper bar to carry current through the magnetic core. Window-type current transformers are also common, which can have circuit cables run through the middle of an opening in the core to provide a single-turn primary winding. When conductors passing through a CT are not centered in the circular (or oval) opening, slight inaccuracies may occur.

Shapes and sizes can vary depending on the end user or switchgear manufacturer. Typical examples of low voltage single ratio metering current transformers are either ring type or plastic moulded case. High-voltage current transformers are mounted on porcelain bushings to insulate them from ground. Some CT configurations slip around the bushing of a high-voltage transformer or circuit breaker, which automatically centers the conductor inside the CT window.

The primary circuit is largely unaffected by the insertion of the CT. The rated secondary current is commonly standardized at 1 or 5 amperes. For example, a 4000:5 CT would provide an output current of 5 amperes when the primary was passing 4000 amperes. The secondary winding can be single ratio or multi ratio, with five taps being common for multi ratio CTs. The load, or burden, of the CT should be of low resistance. If the voltage time integral area is higher than the core's design rating, the core goes into saturation towards the end of each cycle, distorting the waveform and affecting accuracy.

Usage

Current transformers are used extensively for measuring current and monitoring the operation of the power grid. Along with voltage leads, revenue-grade CTs drive the electrical utility's watt-hour meter on virtually every building with three-phase service and single-phase services greater than 200 amp.

The CT is typically described by its current ratio from primary to secondary. Often, multiple CTs are installed as a "stack" for various uses. For example, protection devices and revenue metering may use separate CTs to provide isolation between metering and protection circuits, and allows current transformers with different characteristics (accuracy, overload performance) to be used for the different purposes.

Safety precautions

Care must be taken that the secondary of a current transformer is not disconnected from its load while current is flowing in the primary, as the transformer secondary will attempt to continue driving current across the effectively infinite impedance. This will produce a high voltage across the open secondary (into the range of several kilovolts in some cases), which may cause arcing. The high voltage produced will compromise operator and equipment safety and permanently affect the accuracy of the transformer.

Accuracy

The accuracy of a CT is directly related to a number of factors including:

- Burden
- Burden class/saturation class
- Rating factor
- Load
- External electromagnetic fields
- Temperature and
- Physical configuration.
- The selected tap, for multi-ratio CTs

For the IEC standard, accuracy classes for various types of measurement are set out in IEC 60044-1, Classes 0.1, 0.2s, 0.2, 0.5, 0.5s, 1, and 3. The class designation is an approximate measure of the CT's accuracy. The ratio (primary to secondary current) error of a Class 1 CT is 1% at rated current; the ratio error of a Class 0.5 CT is 0.5% or less. Errors in phase are also important especially in power measuring circuits, and each class has an allowable maximum phase error for a specified load impedance. Current transformers used for protective relaying also have accuracy requirements at overload currents in excess of the normal rating to ensure accurate performance of relays during system faults.

Burden

The load, or burden, in a CT metering circuit is the (largely resistive) impedance presented to its secondary winding. Typical burden ratings for IEC CTs are 1.5 VA, 3 VA, 5 VA, 10 VA, 15 VA, 20 VA, 30 VA, 45 VA & 60 VA. As for ANSI/IEEE burden ratings are B-0.1, B-0.2, B-0.5, B-1.0, B-2.0 and B-4.0. This means a CT with a burden rating of B-0.2 can tolerate up to 0.2 Ω of impedance in the metering circuit before its output current is no longer a fixed ratio to the primary current. Items that contribute to the burden of a current measurement circuit are switch-blocks, meters and intermediate conductors. The most common source of excess burden in a current measurement circuit is the conductor between the meter and the CT. Often, substation meters are located significant distances from the meter cabinets and the excessive length of small gauge conductor creates a large resistance. This problem can be solved by using CT with 1 ampere secondaries which will produce less voltage drop between a CT and its metering devices.

Knee-point voltage

The **knee-point voltage** of a current transformer is the magnitude of the secondary voltage after which the output current ceases to follow the input current. This means that the one-to-one or proportional relationship between the input and output is no longer within rated accuracy. The output current increases abruptly even with small increment in the input, if the voltage across the secondary terminals exceeds the knee-point voltage. The knee-point voltage is not applicable for metering current transformers, the concept of knee point voltage is pertinent to protect current transformers only since they are necessarily exposed to high currents during faults.

Rating factor

Rating factor is a factor by which the nominal full load current of a CT can be multiplied to determine its absolute maximum measurable primary current. Conversely, the minimum primary current a CT can accurately measure is "light load," or 10% of the nominal current (there are, however, special CTs designed to measure accurately currents as small as 2% of the nominal current). The rating factor of a CT is largely dependent upon ambient temperature. Most CTs have rating factors for 35 degrees Celsius and 55 degrees Celsius. It is important to be mindful of ambient temperatures and resultant rating factors when CTs are installed inside pad-mounted transformers or poorly ventilated mechanical rooms. Recently, manufacturers have been moving towards lower nominal primary currents with greater rating factors. This is made possible by the development of more efficient ferrites and their corresponding hysteresis curves. This is a distinct advantage over previous CTs because it increases their range of accuracy, since the CTs are most accurate between their rated current and rating factor.

Special designs

Specially constructed *wideband current transformers* are also used (usually with an oscilloscope) to measure waveforms of high frequency or pulsed currents within pulsed power systems. One type of specially constructed wideband transformer provides a voltage output that is proportional to the measured current. Another type (called a Rogowski coil) requires an external integrator in order to provide a voltage output that is proportional to the measured current. Unlike CTs used for power circuitry, wideband CTs are rated in output volts per ampere of primary current.

Standards

Depending on the ultimate clients requirement, there are two main standards to which current transformers are designed. IEC 60044-1 (BSEN 60044-1) & IEEE C57.13 (ANSI), although the Canadian & Australian standards are also recognised.

Current clamp

In electrical and electronic engineering, a **current clamp** or **current probe** is an electrical device having two jaws which open to allow clamping around an electrical conductor. This allows properties of the electric current in the conductor to be measured, without having to make physical contact with it, or to disconnect it for insertion through the probe. Current clamps are usually used to read the magnitude of a sinusoidal current (as invariably used in alternating current (AC) power distribution systems), but in conjunction with more advanced instrumentation the phase and waveform are available. Very high alternating currents (1000 A and more) are easily read with an appropriate meter; direct currents, and very low AC currents (milliamperes) are more difficult to measure.

Types of current clamp

Current transformer

The most common form of current clamp comprises a split ferrite ring. A wire coil is wound round one or both halves, forming the secondary winding of a current transformer. The conductor to be measured forms the primary. Like any transformer this type works only with AC or pulse waveforms, with some examples extending into the megahertz range.

This type may also be used to inject current into the conductor, for example in EMC susceptibility testing to induce an interference current. Usually, the injection probe is specifically designed for this purpose.

Iron vane

In the iron vane type, the magnetic flux in the core directly affects a moving iron vane, allowing both AC and DC to be measured, and gives a true RMS value for non-sinusoidal AC waveforms. Due to its physical size it is generally limited to power transmission frequencies up to around 100 Hz.

The vane is usually fixed directly to the display mechanism of an analogue (moving pointer) clamp meter.

Hall effect

The Hall effect type is more sensitive and is able to measure both DC and AC, in some examples up to the kilohertz (thousands of hertz) range. This type is often used with oscilloscopes, and with high-end computerized digital multimeters.

Clamp meter

A multimeter with built in clamp.
Pushing the large button at the bottom opens the lower jaw of the clamp, allowing the clamp to be placed around a conductor.

An electrical meter with integral AC current clamp is known as a clamp meter, clamp-on ammeter or tong tester.

In order to use a clamp meter, only one conductor is normally passed through the probe; if more than one conductor is passed through then the measurement would be the vector sum of the currents flowing in the conductors and would depend on the phase relationship of the currents. In particular if the clamp is closed around a two-conductor cable carrying power to equipment the same current flows down one conductor and up the other, with a net current of zero. Clamp meters are often sold with a device that is plugged in between the power outlet and the device to be tested. The device is essentially a short extension cord with the two conductors separated, so that the clamp can be placed around only one conductor.

The reading produced by a conductor carrying a very low current can be increased by winding the conductor around the clamp several times; the meter reading divided by the number of turns is the current, with some loss of accuracy due to inductive effects.

Clamp meters are used by electricians, sometimes with the clamp incorporated into a general purpose multimeter.

It is simple to measure very high currents (hundreds of amperes) with the appropriate current transformer. Accurate measurement of low currents (a few milliamperes) with a current transformer clamp is more difficult.

An iron vane type clamp-on ammeter

Less-expensive clamp meters use a rectifier circuit which actually reads mean current, but is calibrated to display the RMS current corresponding to the measured mean, giving a correct RMS reading only if the current is a sine wave. For other waveforms readings will be incorrect; when these simpler meters are used with non-sinusoidal loads such as the ballasts used with fluorescent lamps or high-intensity discharge lamps or most modern computer and electronic equipment, readings can be quite inaccurate. Meters which respond to true RMS rather than mean current are described as "true RMS".

Typical hand-held Hall effect units can read currents as low as 200 mA, and units that can read down to 1 mA are available.

The Columbia tong test ammeter, manufactured by Weschler Instruments, is an example of the iron vane type, used for measuring large AC currents up to 1000 amperes. The iron jaws of the meter direct the magnetic field surrounding the conductor to an iron vane that is attached to the needle of the meter. The iron vane moves in proportion to the strength to the magnetic field and thus produces a meter indication proportional to the current. This type of ammeter can measure both AC and DC currents and provides a true RMS current measurement of non-sinusoidal or distorted AC waveforms. Interchangeable meter movements can be installed in the clamping assembly to provide various full-scale current values up to 1000 amperes. The iron vane is in a small cylinder that is inserted in a space at the hinged end of the clamp-on jaws. Several jaw sizes are available for clamping around large conductors and bus bars up to 4½ inches (110 mm) wide.

Power meter, energy analyzer

Clamp meters are used in some meters to measure electrical power and energy. The clamp measures the current and other circuitry the voltage; the true power is the product of the instantaneous voltage and current integrated over a cycle. Comprehensive meters designed to measure many parameters of electrical energy (power factor, distortion, instantaneous power as a function of time, phase relationships, etc.), energy analyzers, use this principle. With an appropriate instrument measurements may be made on three-phase, as well as single-phase, power systems.

Chapter 10

Current Mirror

A **current mirror** is a circuit designed to copy a current through one active device by controlling the current in another active device of a circuit, keeping the output current constant regardless of loading. The current being 'copied' can be, and sometimes is, a varying signal current. Conceptually, an ideal current mirror is simply an ideal current amplifier. The current mirror is used to provide bias currents and active loads to circuits.

Mirror characteristics

There are three main specifications that characterize a current mirror. The first is the current level it produces. The second is its AC output resistance, which determines how much the output current varies with the voltage applied to the mirror. The third specification is the minimum voltage drop across the mirror necessary to make it work properly. This minimum voltage is dictated by the need to keep the output transistor of the mirror in active mode. The range of voltages where the mirror works is called the **compliance range** and the voltage marking the boundary between good and bad behavior is called the **compliance voltage**. There are also a number of secondary performance issues with mirrors, for example, temperature stability.

Practical approximations

For small-signal analysis the current mirror can be approximated by its equivalent Norton impedance .

In large-signal hand analysis, a current mirror usually is approximated simply by an ideal current source. However, an ideal current source is unrealistic in several respects:

- it has infinite AC impedance, while a practical mirror has finite impedance
- it provides the same current regardless of voltage, that is, there are no compliance range requirements
- it has no frequency limitations, while a real mirror has limitations due to the parasitic capacitances of the transistors
- the ideal source has no sensitivity to real-world effects like noise, power-supply voltage variations and component tolerances.

Circuit realizations of current mirrors

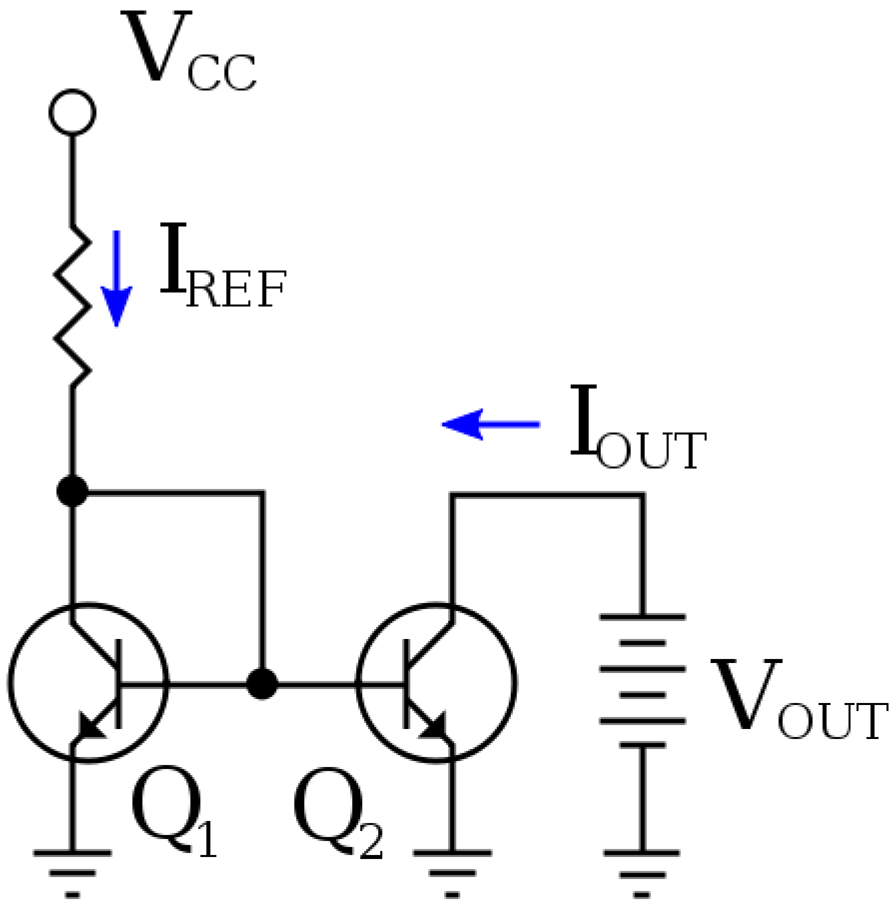


Figure 1: A current mirror implemented with npn bipolar transistors using a resistor to set the reference current I_{REF}; V_{CC} = supply voltage

Basic bipolar transistor mirror

The simplest bipolar current mirror consists of two transistors connected as shown in Figure 1. Transistor Q_1 is connected to ground. Its collector-base voltage is zero as shown. Consequently, the voltage drop across Q_1 is V_{BE}, that is, this voltage is set by the diode law and Q_1 is said to be **diode connected**. It is important to have Q_1 in the circuit instead of a simple diode, because Q_1 sets V_{BE} for the transistor Q_2. If Q_1 and Q_2 are matched, that is, have substantially the same device properties, and if the mirror output voltage is chosen so the collector-base voltage of Q_2 also is zero, then the V_{BE}-value set by Q_1 results in an emitter current in the matched Q_2 that is the same as the emitter current in Q_1. Because Q_1 and Q_2 are matched, their β_0-values also agree, making the mirror output current the same as the collector current of Q_1. The current delivered by the mirror for arbitrary collector-base reverse bias V_{CB} of the output transistor is given by

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CB}}{V_A}\right),$$

where V_T = thermal voltage, I_S = reverse saturation current, or scale current; V_A = Early voltage. This current is related to the reference current I_{REF} when the output transistor V_{CB} = 0 V by:

$$I_{REF} = I_C\left(1 + \frac{2}{\beta_0}\right) ,$$

as found using Kirchhoff's current law at the collector node of Q_1:

$$I_{REF} = I_C + I_{B1} + I_{B2}.$$

The reference current supplies the collector current to Q_1 and the base currents to both transistors — when both transistors have zero base-collector bias, the two base currents are equal, $I_{B1}=I_{B2}=I_B$.

$$I_{REF} = I_C + I_B + I_B = I_C + 2I_B = I_C\left(1 + \frac{2}{\beta_0}\right) ,$$

Parameter β_0 is the transistor β-value for V_{CB} = 0 V.

Output resistance

If V_{CB} is greater than zero in output transistor Q_2, the collector current in Q_2 will be somewhat larger than for Q_1 due to the Early effect. In other words, the mirror has a finite output (or Norton) resistance given by the r_O of the output transistor, namely:

$$R_N = r_O = \frac{V_A + V_{CB}}{I_C},$$

where V_A = Early voltage and V_{CB} = collector-to-base bias.

Compliance voltage

To keep the output transistor active, $V_{CB} \geq 0$ V. That means the lowest output voltage that results in correct mirror behavior, the compliance voltage, is $V_{OUT} = V_{CV} = V_{BE}$ under bias conditions with the output transistor at the output current level I_C and with V_{CB} = 0 V or, inverting the *I-V* relation above:

$$V_{CV} = V_T \ln\left(\frac{I_C}{I_S} + 1\right) ,$$

where V_T = thermal voltage and I_S = reverse saturation current (scale current).

Extensions and complications

When Q_2 has $V_{CB} > 0$ V, the transistors no longer are matched. In particular, their β-values differ due to the Early effect, with

$$\beta_1 = \beta_0 \text{ and } \beta_2 = \beta_0 \left(1 + \frac{V_{CB}}{V_A}\right)$$

where V_A is the Early voltage and β_0 = transistor β for V_{CB} = 0 V. Besides the difference due to the Early effect, the transistor β-values will differ because the β_0-values depend on current, and the two transistors now carry different currents**.**

Further, Q_2 may get substantially hotter than Q_1 due to the associated higher power dissipation. To maintain matching, the temperature of the transistors must be nearly the same. In integrated circuits and transistor arrays where both transistors are on the same die, this is easy to achieve. But if the two transistors are widely separated, the precision of the current mirror is compromised.

Additional matched transistors can be connected to the same base and will supply the same collector current. In other words, the right half of the circuit can be duplicated several times with various resistor values replacing R_2 on each. Note, however, that each additional right-half transistor "steals" a bit of collector current from Q_1 due to the non-zero base currents of the right-half transistors. This will result in a small reduction in the programmed current.

An example of a mirror with emitter degeneration to increase mirror resistance is found in two-port networks.

For the simple mirror shown in the diagram, typical values of β will yield a current match of 1% or better.

Basic MOSFET current mirror

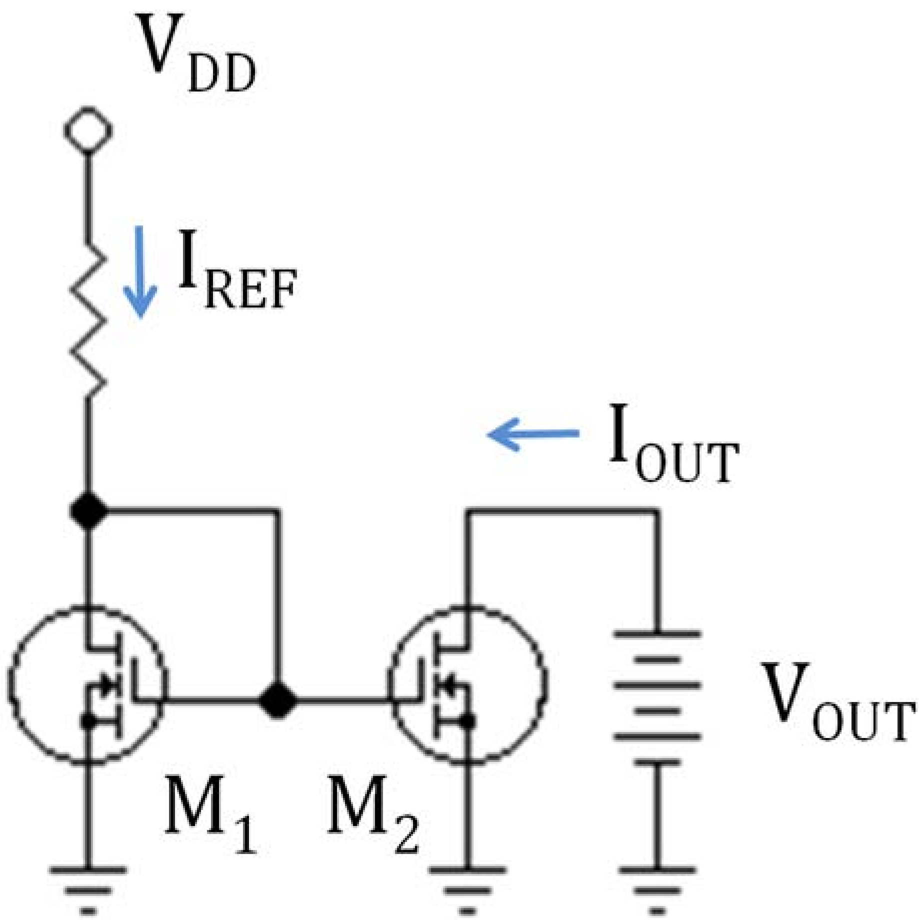


Figure 2: An n-channel MOSFET current mirror with a resistor to set the reference current I_{REF}; V_{DD} is the supply voltage

The basic current mirror can also be implemented using MOSFET transistors, as shown in Figure 2. Transistor M_1 is operating in the saturation or active mode, and so is M_2. In this setup, the output current I_{OUT} is directly related to I_{REF}, as discussed next.

The drain current of a MOSFET I_D is a function of both the gate-source voltage and the drain-to-gate voltage of the MOSFET given by $I_D = f(V_{GS}, V_{DG})$, a relationship derived from the functionality of the MOSFET device. In the case of transistor M_1 of the mirror, $I_D = I_{REF}$. Reference current I_{REF} is a known current, and can be provided by a resistor as shown, or by a "threshold-referenced" or "self-biased" current source to ensure that it is constant, independent of voltage supply variations.

Using V_{DG}=0 for transistor M_1, the drain current in M_1 is $I_D = f(V_{GS}, V_{DG}=0)$, so we find: $f(V_{GS}, 0) = I_{REF}$, implicitly determining the value of V_{GS}. Thus I_{REF} sets the value of V_{GS}.

The circuit in the diagram forces the same V_{GS} to apply to transistor M_2. If M_2 also is biased with zero V_{DG} and provided transistors M_1 and M_2 have good matching of their properties, such as channel length, width, threshold voltage *etc.*, the relationship $I_{OUT} = f(V_{GS}, V_{DG}=0)$ applies, thus setting $I_{OUT} = I_{REF}$; that is, the output current is the same as the reference current when $V_{DG}=0$ for the output transistor, and both transistors are matched.

The drain-to-source voltage can be expressed as $V_{DS}=V_{DG}+V_{GS}$. With this substitution, the Shichman-Hodges model provides an approximate form for function $f(V_{GS},V_{DG})$:

$$I_d = f\ (V_{GS}, V_{DG}) = \tfrac{1}{2}K_p\left(\tfrac{W}{L}\right)(V_{GS} - V_{th})^2(1 + \lambda V_{DS})$$
$$= \tfrac{1}{2}K_p\left(\tfrac{W}{L}\right)(V_{GS} - V_{th})^2\ (1 + \lambda(V_{DG} + V_{GS}))$$

where, K_p is a technology related constant associated with the transistor, W/L is the width to length ratio of the transistor, V_{GS} is the gate-source voltage, V_{th} is the threshold voltage, λ is the channel length modulation constant, and V_{DS} is the drain source voltage.

Output resistance

Because of channel-length modulation, the mirror has a finite output (or Norton) resistance given by the r_o of the output transistor, namely:

$$R_N = r_o = \frac{1/\lambda + V_{DS}}{I_D},$$

where λ = channel-length modulation parameter and V_{DS} = drain-to-source bias.

Compliance voltage

To keep the output transistor resistance high, $V_{DG} \geq 0$ V. That means the lowest output voltage that results in correct mirror behavior, the compliance voltage, is $V_{OUT} = V_{CV} = V_{GS}$ for the output transistor at the output current level with $V_{DG} = 0$ V, or using the inverse of the f-function, f^{-1}:

$$V_{CV} = V_{GS}(\text{for } I_D \text{ at } V_{DG} = 0V) = f^{-1}(I_D) \text{ with } V_{DG} = 0.$$

For Shichman-Hodges model, f^{-1} is approximately a square-root function.

Extensions and reservations

A useful feature of this mirror is the linear dependence of f upon device width W, a proportionality approximately satisfied even for models more accurate than the Shichman-Hodges model. Thus, by adjusting the ratio of widths of the two transistors, multiples of the reference current can be generated.

It must be recognized that the Shichman-Hodges model is accurate only for rather dated technology, although it often is used simply for convenience even today. Any quantitative design based upon new technology uses computer models for the devices that account for the changed current-voltage characteristics. Among the differences that must be accounted for in an accurate design is the failure of the square law in V_{gs} for voltage dependence and the very poor modeling of V_{ds} drain voltage dependence provided by λV_{ds}. Another failure of the equations that proves very significant is the inaccurate dependence upon the channel length L. A significant source of L-dependence stems from λ, as noted by Gray and Meyer, who also note that λ usually must be taken from experimental data.

Feedback assisted current mirror

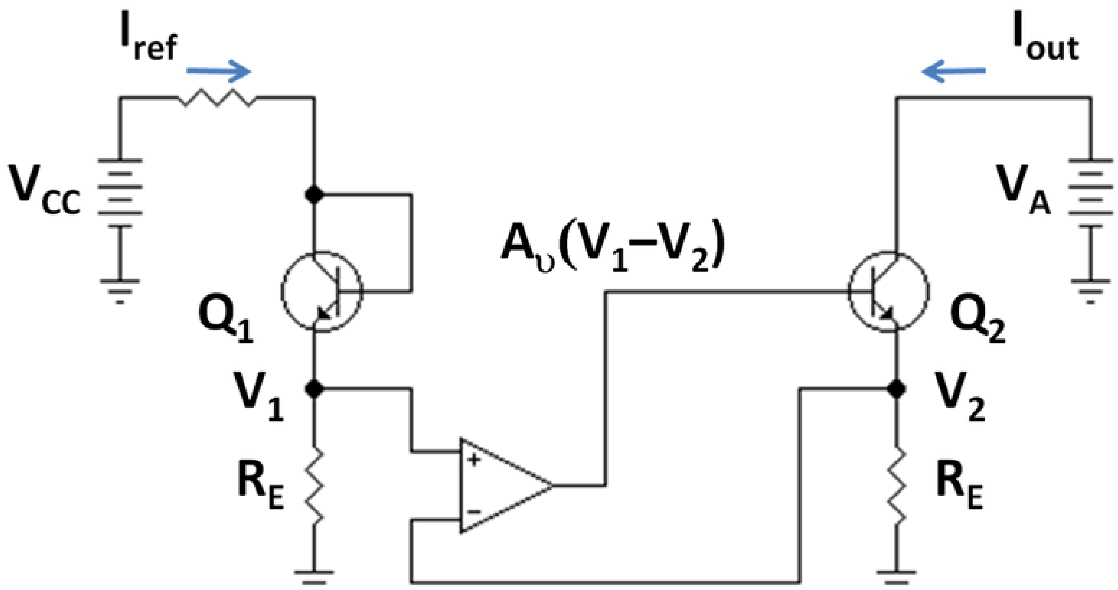


Figure 3: Gain-boosted current mirror with op amp feedback to increase output resistance

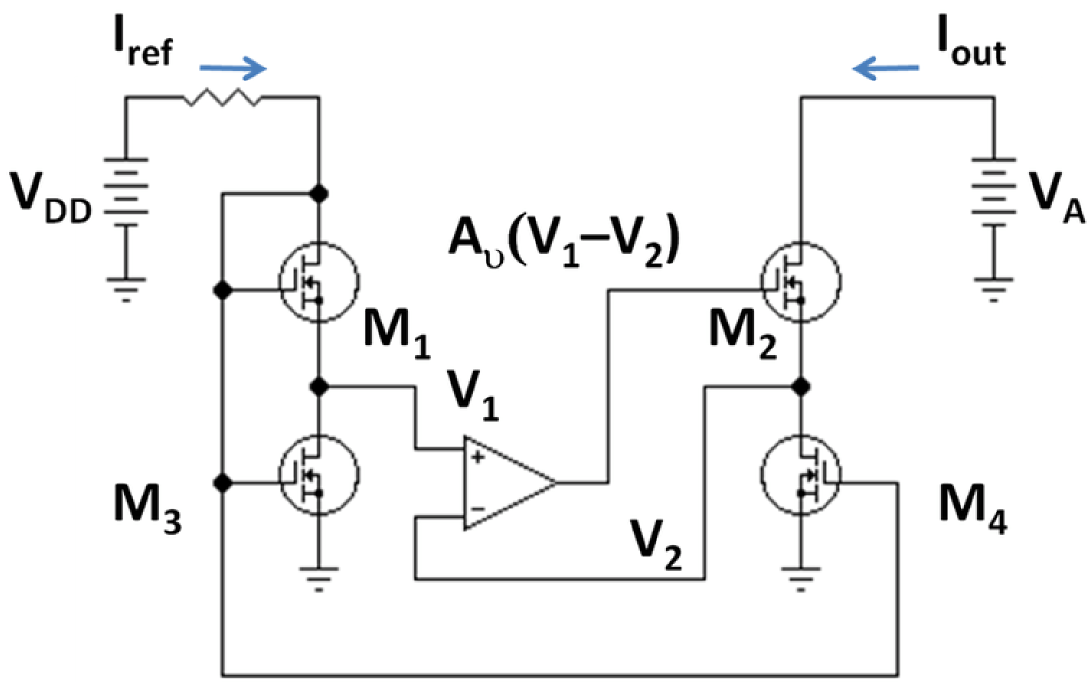


Figure 4: MOSFET version of wide-swing current mirror; M_1 and M_2 are in active mode, while M_3 and M_4 are in Ohmic mode and act like resistors

Figure 3 shows a mirror using negative feedback to increase output resistance. Because of the op amp, these circuits are sometimes called **gain-boosted current mirrors**. Because they have relatively low compliance voltages, they also are called **wide-swing current mirrors**. A variety of circuits based upon this idea are in use, particularly for MOSFET mirrors because MOSFETs have rather low intrinsic output resistance values. A MOSFET version of Figure 3 is shown in Figure 4 where MOSFETs M_3 and M_4 operate in Ohmic mode to play the same role as emitter resistors R_E in Figure 3, and MOSFETs M_1 and M_2 operate in active mode in the same roles as mirror transistors Q_1 and Q_2 in Figure 3. An explanation follows of how the circuit in Figure 3 works.

The operational amplifier is fed the difference in voltages V_1 - V_2 at the top of the two emitter-leg resistors of value R_E. This difference is amplified by the op amp and fed to the base of output transistor Q_2. If the collector base reverse bias on Q_2 is increased by increasing the applied voltage V_A, the current in Q_2 increases, increasing V_2 and decreasing the difference V_1 - V_2 entering the op amp. Consequently, the base voltage of Q_2 is decreased, and V_{BE} of Q_2 decreases, counteracting the increase in output current.

If the op amp gain A_v is large, only a very small difference V_1 - V_2 is sufficient to generate the needed base voltage V_B for Q_2, namely

$$V_1 - V_2 = \frac{V_B}{A_v} \ .$$

Consequently, the currents in the two leg resistors are held nearly the same, and the output current of the mirror is very nearly the same as the collector current I_{C1} in Q_1, which in turn is set by the reference current as

$$I_{ref} = I_{C1}(1 + 1/\beta_1) \ ,$$

where β_1 for transistor Q_1 and β_2 for Q_2 differ due to the Early effect if the reverse bias across the collector-base of Q_2 is non-zero.

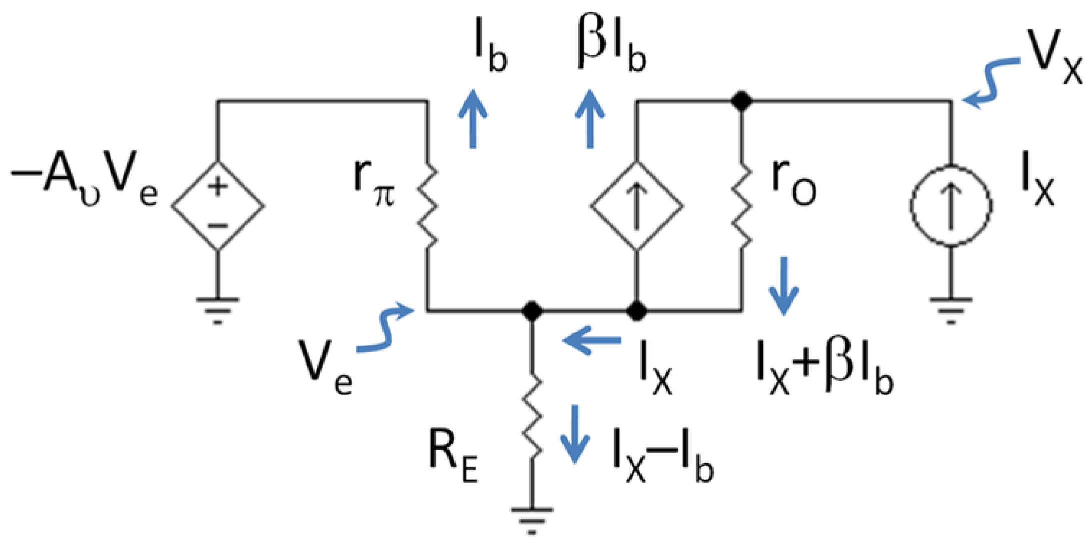


Figure 5: Small-signal circuit to determine output resistance of mirror; transistor Q_2 is replaced with its hybrid-pi model; a test current I_X at the output generates a voltage V_X, and the output resistance is $R_{out} = V_X / I_X$.

Output resistance

An idealized treatment of output resistance is given in the footnote. A small-signal analysis for an op amp with finite gain A_v but otherwise ideal is based upon Figure 5 (β, r_O and r_π refer to Q_2). To arrive at Figure 5, notice that the positive input of the op amp in Figure 3 is at AC ground, so the voltage input to the op amp is simply the AC emitter voltage V_e applied to its negative input, resulting in a voltage output of $-A_v V_e$. Using Ohm's law across the input resistance r_π determines the small-signal base current I_b as:

$$I_b = \frac{V_e}{r_\pi/(A_v + 1)} \ .$$

Combining this result with Ohm's law for R_E, V_e can be eliminated, to find:

$$I_b = I_X \frac{R_E}{R_E + \frac{r_\pi}{A_v+1}} \ .$$

Kirchhoff's voltage law from the test source I_X to the ground of R_E provides:

$$V_X = (I_X + \beta I_b) r_O + (I_X - I_b) R_E \ .$$

Substituting for I_b and collecting terms the output resistance R_{out} is found to be:

$$R_{out} = \frac{V_X}{I_X} = r_O \left(1 + \beta \frac{R_E}{R_E + r_\pi/(A_v+1)} \right) + R_E \| \frac{r_\pi}{A_v+1} \ .$$

For a large gain $A_v >> r_\pi / R_E$ the maximum output resistance obtained with this circuit is

$$R_{out} = (\beta + 1) r_O \ ,$$

a substantial improvement over the basic mirror where $R_{out} = r_O$.

The small-signal analysis of the MOSFET circuit of Figure 4 is obtained from the bipolar analysis by setting $\beta = g_m r_\pi$ in the formula for R_{out} and then letting $r_\pi \rightarrow \infty$. The result is

$$R_{out} = r_O \left(1 + g_m R_E (A_v + 1) \right) + R_E \ .$$

This time, R_E is the resistance of the source-leg MOSFETs M_3, M_4. Unlike Figure 3, however, as A_v is increased (holding R_E fixed in value), R_{out} continues to increase, and does not approach a limiting value at large A_v.

Compliance voltage

For Figure 3, a large op amp gain achieves the maximum R_{out} with only a small R_E. A low value for R_E means V_2 also is small, allowing a low compliance voltage for this mirror, only a voltage V_2 larger than the compliance voltage of the simple bipolar mirror. For this reason this type of mirror also is called a *wide-swing current mirror*, because it allows the output voltage to swing low compared to other types of mirror that achieve a large R_{out} only at the expense of large compliance voltages.

With the MOSFET circuit of Figure 4, like the circuit in Figure 3, the larger the op amp gain A_v, the smaller R_E can be made at a given R_{out}, and the lower the compliance voltage of the mirror.

Other current mirrors

There are many sophisticated current mirrors that have higher output resistances than the basic mirror (more closely approach an ideal mirror with current output independent of output voltage) and produce currents less sensitive to temperature and device parameter variations and to circuit voltage fluctuations. These multi-transistor mirror circuits are used both with bipolar and MOS transistors. These circuits include:

- the Widlar current source
- the Wilson current source
- the cascoded current sources

Chapter 11

Hydraulic Analogy

The electronic–**hydraulic analogy** (derisively referred to as the **drain-pipe theory** by Oliver Heaviside) is the most widely used analogy for "electron fluid" in a metal conductor. Since electric current is invisible and the processes at play in electronics are often difficult to demonstrate, the various electronic components are represented by hydraulic equivalents. Electricity (as well as heat) was originally understood to be a kind of fluid, and the names of certain electric quantities (such as current) are derived from hydraulic equivalents. Like all analogies, it demands an intuitive and competent understanding of the baseline paradigms (electronics and hydraulics).

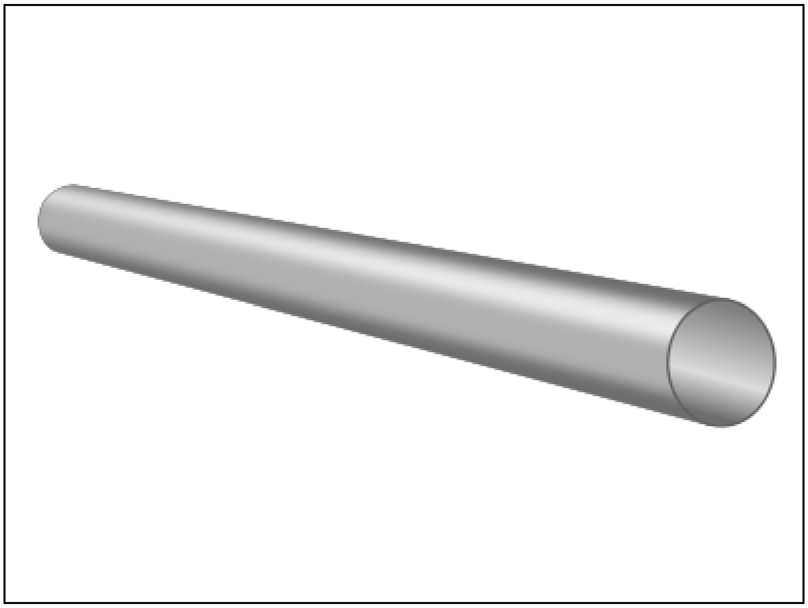
Basic ideas

There are two basic paradigms:

- Version with pressure induced by gravity. Large tanks of water are held up high, or are filled to differing water levels, and the potential energy of the water head is the pressure source. This is reminiscent of electrical diagrams with an up arrow pointing to +V, grounded pins that otherwise are not shown connecting to anything, and so on.
- Completely enclosed version with pumps providing pressure only; no gravity. This is reminiscent of a circuit diagram with a voltage source shown and the wires actually completing a circuit.

Applications: Flow and pressure variables can be calculated in fluid flow network with the use of the hydraulic ohm analogy. The method can be applied to both steady and transient flow situations.

Component equivalents



A simple pipe.

Wires

A relatively wide pipe completely filled with water is equivalent to a piece of wire. When comparing to a piece of wire, the pipe should be thought of as having semi-permanent caps on the ends. Connecting one end of a wire to a circuit is equivalent to forcibly un-capping one end of the pipe and attaching it to another pipe. With few exceptions (such as a high-voltage power source), a wire with only one end attached to a circuit will do nothing; the pipe remains capped on the free end, and thus adds nothing to the circuit.

Electric potential

Equivalent to pressure.

Voltage

Also called *potential difference*. A difference in pressure between two points. Or in open hydraulic troughs and tanks, a difference in water height or "head". Usually measured in volts.

Electric charge

Equivalent to a quantity of water.

Current

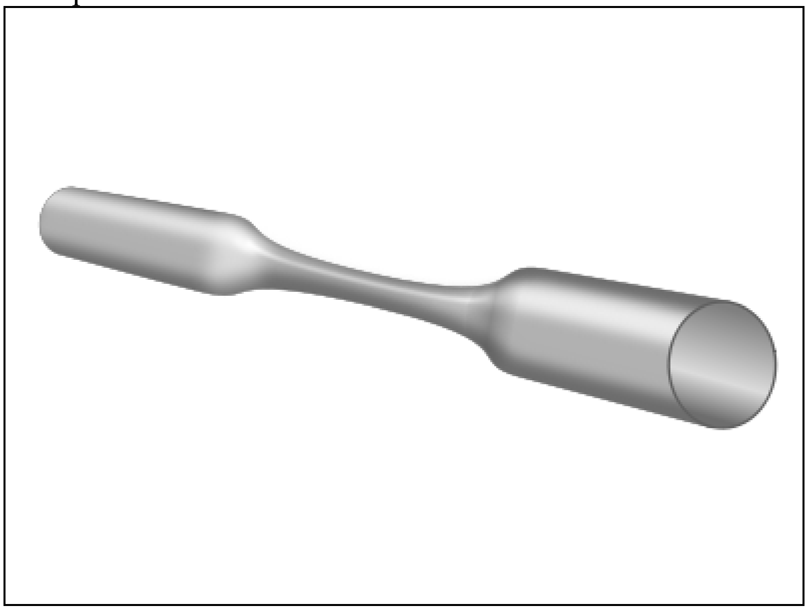
Equivalent to a hydraulic volume flow rate; that is, the volumetric quantity of flowing water over time. Usually measured in amperes.

Ideal voltage source

A dynamic pump with feedback control. A pressure meter on both sides shows that regardless of the current being produced, this kind of pump produces constant pressure difference.

Ideal current source

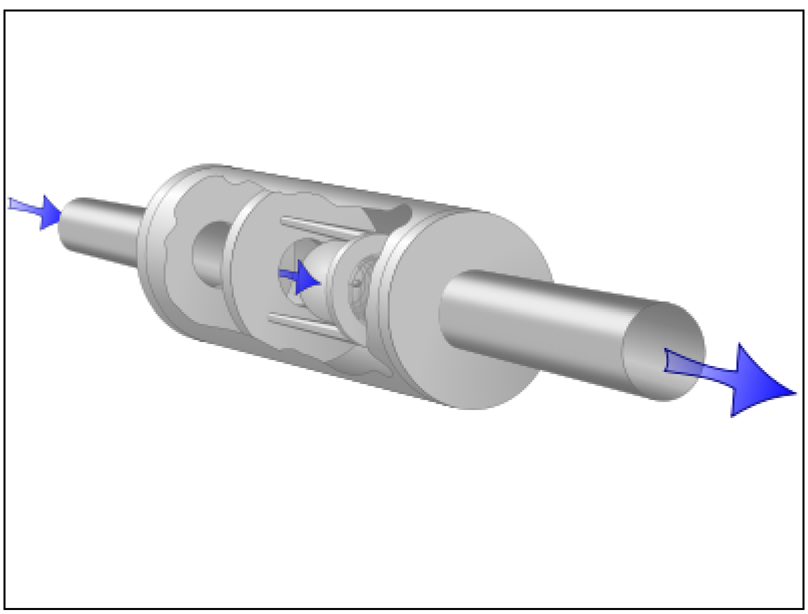
A positive displacement pump. A current meter (little paddle wheel) shows that when this kind of pump is driven at a constant speed, it maintains a constant speed of the little paddle wheel.



A simple pipe with a constricted region.

Resistor

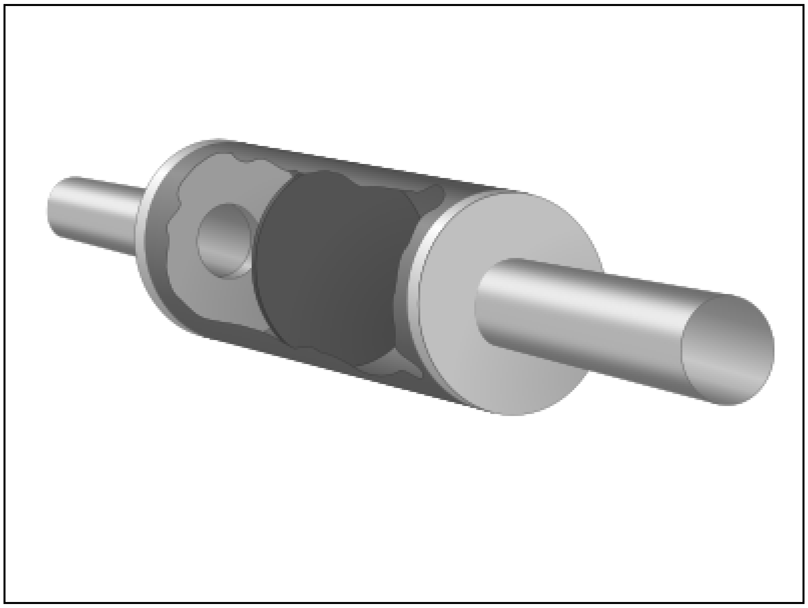
A constriction in the bore of the pipe which requires more pressure to pass the same amount of water. All pipes have some resistance to flow, just like all wires have some resistance to current.



A simple one-way ball-type check valve, in its "open" state.

Diode

Equivalent to a one-way check valve with a slightly leaky valve seat. Like a diode, a small pressure difference is needed before the valve opens. And like a diode, too much reverse bias can damage or destroy the valve assembly.



A tank divided by a rubber diaphragm.

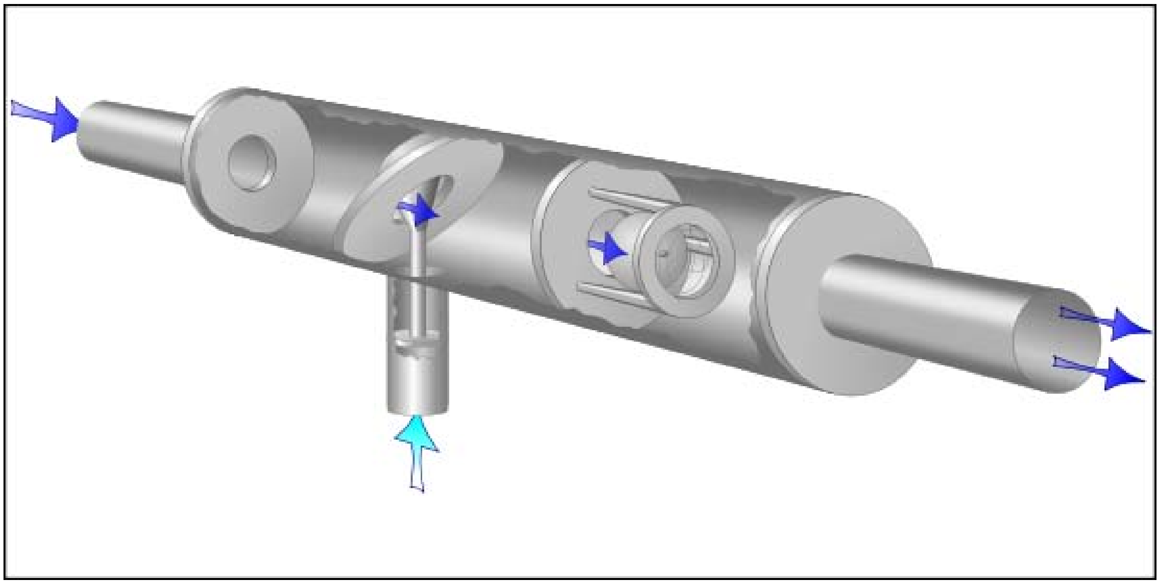
Capacitor

A tank with one connection at each end and a rubber sheet dividing the tank in two lengthwise. When water is forced into one pipe, equal water is simultaneously forced out the other pipe, yet no water can penetrate the rubber diaphragm. Energy is stored by the stretching of the rubber. As more current flows "through" the capacitor, the back-pressure (voltage) becomes greater, thus current "leads" voltage in a capacitor. As the back-pressure from the stretched rubber approaches the applied pressure, the current becomes less and less. Thus capacitors "filter out" constant pressure differences and slowly-varying, low-frequency pressure differences, while allowing rapid changes in pressure to pass through.

Inductor

A heavy paddle wheel placed in the current. The mass of the wheel and the size of the blades restrict the water's ability to rapidly change its rate of flow (current) through the wheel due to the effects of inertia, but, given time, a constant flowing stream will pass mostly unimpeded through the wheel, as it turns at the same speed as the water flow. The mass and surface area of the wheel and its blades is analogous to inductance, and friction between its axle and the axle bearings correspond to the resistance that accompanies any non-superconducting inductor. An alternative Inductor model is simply a long pipe, perhaps coiled into a spiral for convenience. This fluid-inertia device is used in real life as an essential component of a hydraulic ram. The inertia of the water flowing through the pipe produces the inductance effect; inductors "filter out" rapid changes in flow, while allowing slow variations in current to be passed through. The drag imposed by the

walls of the pipe is somewhat analogous to parasitic resistance.
In either model, the pressure difference (voltage) across the device must be present before the current will start moving, thus in inductors voltage "leads" current. As the current increases, approaching the limits imposed by its own internal friction and of the current that the rest of the circuit can provide, the pressure drop across the device becomes lower and lower.



A pressure-actuated valve combined with a one-way check valve.

Transistor

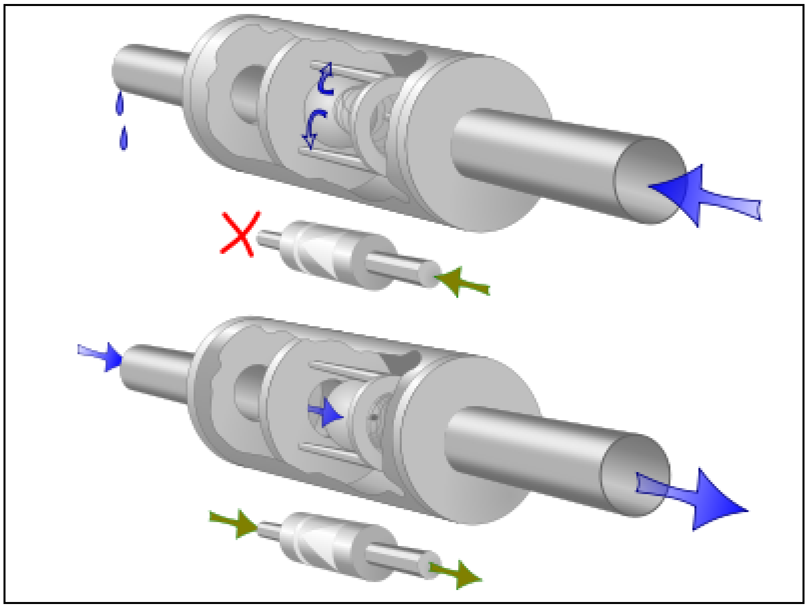
A valve in which a diaphragm controlled by a low-current signal (either constant current for a BJT or constant pressure for a FET) moves a plunger which affects the current through another section of pipe.

CMOS

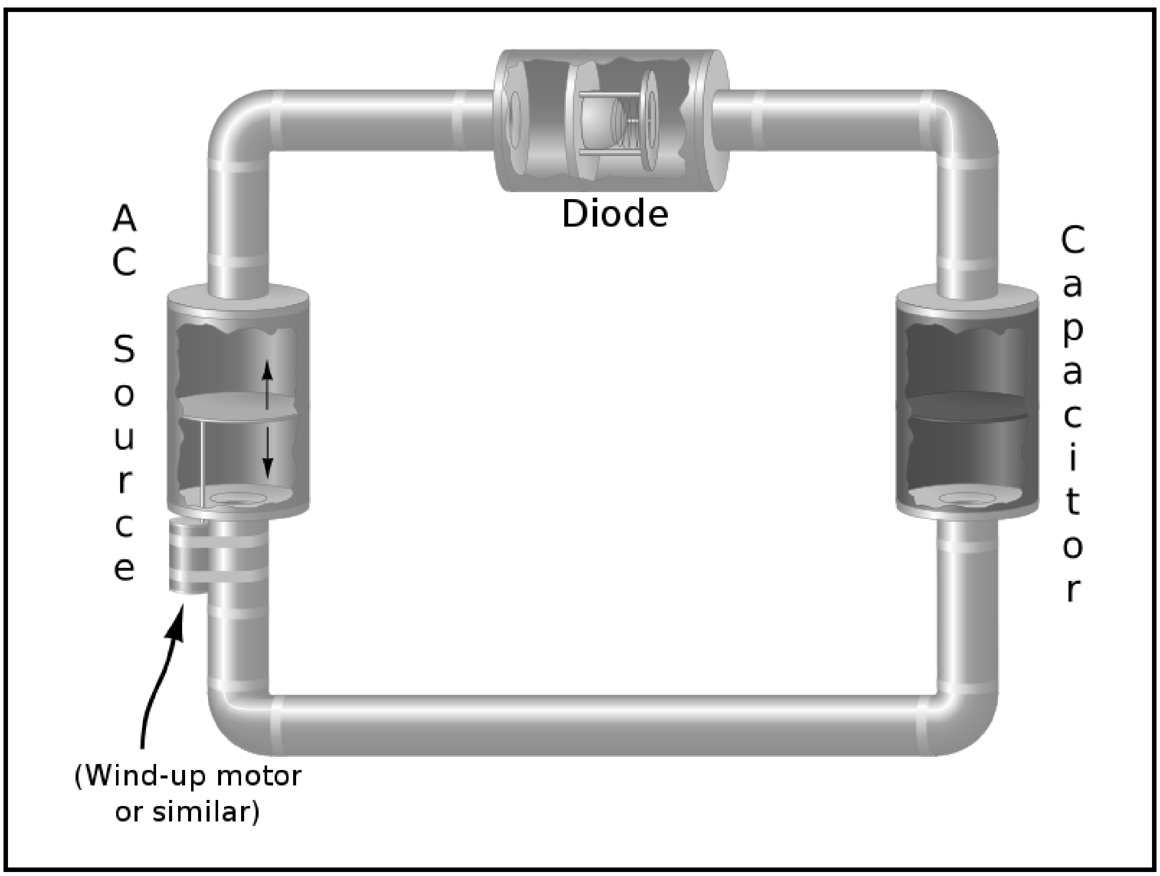
A combination of two MOSFET transistors. As the input pressure changes, the pistons allow the output to connect to either zero or positive pressure.

Memristor

A needle valve operated by a flow meter. As water flows through in the forward direction, the needle valve restricts flow more; as water flows the other direction, the needle valve opens further providing less resistance.



Like a one-way check valve, a diode blocks current that flows the wrong way. Current that flows the right way goes through almost unchanged.



A simple A/C circuit consisting of an oscillating pump, a "diode" valve, and a "capacitor" tank. Any kind of motor could be used here to drive the pump, as long as it oscillates.

Principle equivalents

EM wave speed (velocity of propagation)

Speed of sound in water. When a light switch is flipped, the electric wave travels very quickly through the wires.

Charge flow speed (drift velocity)

Particle speed of water. The moving charges themselves move rather slowly.

DC

Constant flow of water in a circuit of pipe

Low frequency AC

Water oscillating back and forth in a pipe

Higher-frequency AC and transmission lines

Sound being transmitted through the water pipes

Inductive spark

Used in induction coils, similar to water hammer, caused by the inertia of water

Equation examples

Some examples of equivalent electrical and hydraulic equations:

type	**hydraulic**	**electric**	**thermal**
quantity	volume V [m^3]	charge q [C]	heat Q [J]
potential	pressure p [Pa=J/m^3]	potential φ [V=J/C]	temperature T [K=J/k_B]
flux	Volumetric flow rate Φ_V [m^3/s]	current I [A=C/s]	heat transfer rate $\dot{Q}$[J/s]
flux density	velocity v [m/s]	current density j [C/(m^2·s) = A/m²]	heat flux $\dot{Q}''$[W/m^2]
linear model	Poiseuille's law $\Phi_V = \frac{\pi r^4}{8\eta}\frac{\Delta p^\star}{\ell}$	Ohm's law $j = -\sigma\nabla\phi$	Fourier's law $\dot{Q}'' = \kappa\nabla T$

If the differential equations have the same form , the response will be similar .

Limits to the analogy

If taken too far, the water analogy can create misconceptions. For it to be useful, we must remain aware of the regions where electricity and water behave very differently.

Fields

Electrons can push or pull other distant electrons via their fields, while water molecules experience forces only from direct contact with other molecules. For this reason, waves in water travel at the speed of sound, but waves in a sea of charge will travel much faster as the forces from one electron are applied to many distant electrons and not to only the neighbors in direct contact. In a hydraulic transmission line, the energy flows as mechanical waves through the water, but in an electric transmission line the energy flows as fields in the space surrounding the wires, and does not flow inside the metal. Also, an accelerating electron will drag its neighbors along while attracting them, both because of magnetic forces.

Charge

Unlike water, movable charge carriers can be positive or negative, and conductors can exhibit an overall positive or negative net charge. The mobile carriers in electric currents are usually electrons, but sometimes they are positive ions or single protons (H+ ions).

Leaking pipes

If a hole is made in an open hydraulic system, the water can pour out. But the movable charges present within electrical conductors are always attracted to unmoving opposite charges in the material. The "electric fluid" can be forcibly removed from metals, but enormous voltages arise if even a tiny amount is removed. For this reason, the surfaces of conductors act as if they always have a high energy-barrier preventing leaks. Also for this reason, continuing electric currents require closed loops rather than hydraulics' open source/sink resembling spigots and buckets.

Fluid velocity and resistance of metals

As with water hoses, the carrier drift velocity in conductors is directly proportional to current. However, water only experiences drag via the pipes' inner surface, while charges are slowed at all points within a metal. Also, typical velocity of charge carriers within a conductor is less than centimeters per minute, and the "electrical friction" is extremely high. If charges ever flowed as fast as water can flow in pipes, the electric current would be immense, and the conductors would become incandescently hot and perhaps vaporize. To model the resistance and the charge-velocity of metals, perhaps a pipe packed with sponge, or a narrow straw filled with syrup, would be a better analogy than a large-diameter water pipe.

Quantum Mechanics

Conductors and insulators contain charges at more than one quantized level of atomic orbit energy, while the water in one region of a pipe can only have a single value of pressure. For this reason there is no hydraulic explanation for such things as a battery's charge pumping ability, a diode's voltage drop, solar cell functions, Peltier effect, etc., however equivalent devices can be designed which exhibit similar responses, although some of the mechanisms would only serve to regulate the flow curves rather than to contribute to the component's primary function.