

Electric Power Conversion

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Introduction

In electrical engineering, **power conversion** has a more specific meaning, namely converting electric power from one form to another. This could be as simple as a transformer to change the voltage of AC power, but also includes far more complex systems. The term can also refer to a class of electrical machinery that is used to convert one frequency of electrical power into another frequency.

Power conversion systems often incorporate redundancy and voltage regulation.

One way of classifying power conversion systems is according to whether the input and output are alternating current (AC) or direct current (DC), thus:

- DC to DC
 - DC to DC converter
 - Voltage stabiliser
 - Linear regulator
- AC to DC
 - Rectifier
 - Mains power supply unit (PSU)
 - Switched-mode power supply
- DC to AC
 - Inverter
- AC to AC
 - Transformer/autotransformer
 - Voltage converter
 - Voltage regulator
 - Cycloconverter
 - Variable frequency transformer

There are also devices and methods to convert between power systems designed for single and three-phase operation.

The standard power frequency varies from country to country, and sometimes within a country. In North America and northern South America it is usually 60 hertz (Hz), but in many other parts of the world, is usually 50 Hz. Aircraft often use 400 Hz power, so 50 Hz or 60 Hz to 400 Hz frequency conversion is needed for use in the ground power unit used to power the airplane while it is on the ground.

Certain specialized circuits, such as the flyback transformer for a CRT, can also be considered power converters.

Chapter-1

Inverter

An **inverter** is an electrical device that converts direct current (DC) to alternating current (AC); the converted AC can be at any required voltage and frequency with the use of appropriate transformers, switching, and control circuits.

Solid-state inverters have no moving parts and are used in a wide range of applications, from small switching power supplies in computers, to large electric utility high-voltage direct current applications that transport bulk power. Inverters are commonly used to supply AC power from DC sources such as solar panels or batteries.

There are two main types of inverter. The output of a **modified sine wave** inverter is similar to a square wave output except that the output goes to zero volts for a time before switching positive or negative. It is simple and low cost (~\$0.10USD/Watt) and is compatible with most electronic devices, except for sensitive or specialized equipment, for example certain laser printers. A **pure sine wave** inverter produces a nearly perfect sine wave output (<3% total harmonic distortion) that is essentially the same as utility-supplied grid power. Thus it is compatible with all AC electronic devices. This is the type used in grid-tie inverters. Its design is more complex, and costs 5 or 10 times more per unit power (~\$0.50 to \$1.00USD/Watt). The electrical inverter is a high-power electronic oscillator. It is so named because early mechanical AC to DC converters were made to work in reverse, and thus were "inverted", to convert DC to AC.

The inverter performs the opposite function of a rectifier.

Applications

DC power source utilization



Inverter designed to provide 115 VAC from the 12 VDC source provided in an automobile. The unit shown provides up to 1.2 amperes of alternating current, or enough to power two sixty watt light bulbs.

An inverter converts the DC electricity from sources such as batteries, solar panels, or fuel cells to AC electricity. The electricity can be at any required voltage; in particular it can operate AC equipment designed for mains operation, or rectified to produce DC at any desired voltage.

Grid tie inverters can feed energy back into the distribution network because they produce alternating current with the same wave shape and frequency as supplied by the distribution system. They can also switch off automatically in the event of a blackout.

Micro-inverters convert direct current from individual solar panels into alternating current for the electric grid. They are grid tie designs by default.

Uninterruptible power supplies

An uninterruptible power supply (UPS) uses batteries and an inverter to supply AC power when main power is not available. When main power is restored, a rectifier supplies DC power to recharge the batteries.

Induction heating

Inverters convert low frequency main AC power to a higher frequency for use in induction heating. To do this, AC power is first rectified to provide DC power. The inverter then changes the DC power to high frequency AC power.

HVDC power transmission

With HVDC power transmission, AC power is rectified and high voltage DC power is transmitted to another location. At the receiving location, an inverter in a static inverter plant converts the power back to AC.

Variable-frequency drives

A variable-frequency drive controls the operating speed of an AC motor by controlling the frequency and voltage of the power supplied to the motor. An inverter provides the controlled power. In most cases, the variable-frequency drive includes a rectifier so that DC power for the inverter can be provided from main AC power. Since an inverter is the key component, variable-frequency drives are sometimes called inverter drives or just inverters.

Electric vehicle drives

Adjustable speed motor control inverters are currently used to power the traction motors in some electric and diesel-electric rail vehicles as well as some battery electric vehicles and hybrid electric highway vehicles such as the Toyota Prius and Fisker Karma. Various improvements in inverter technology are being developed specifically for electric vehicle applications. In vehicles with regenerative braking, the inverter also takes power from the motor (now acting as a generator) and stores it in the batteries.

Air conditioning

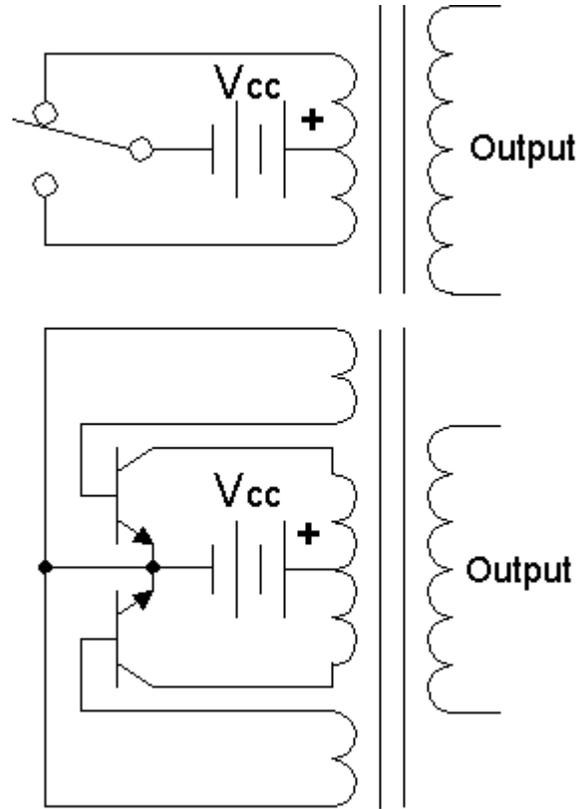
An air conditioner bearing the inverter tag uses a variable-frequency drive to control the speed of the motor and thus the compressor.

The general case

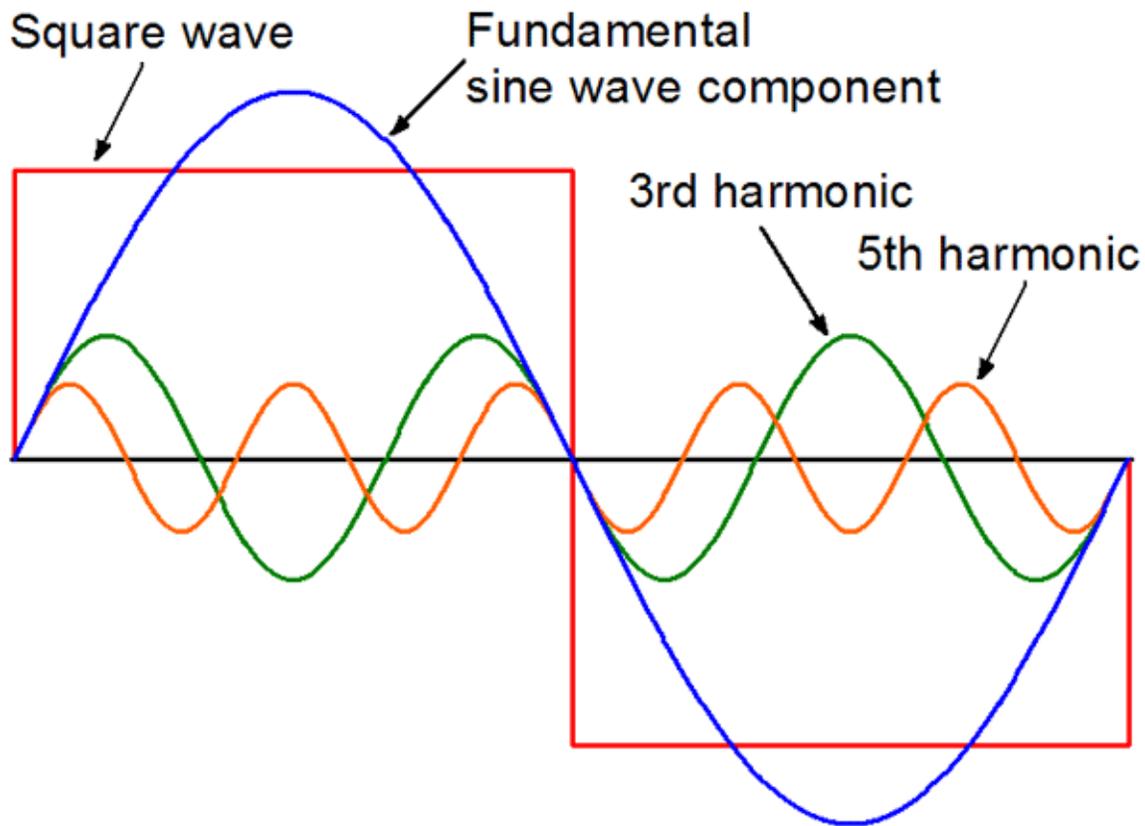
A transformer allows AC power to be converted to any desired voltage, but at the same frequency. Inverters, plus rectifiers for DC, can be designed to convert from any voltage, AC or DC, to any other voltage, also AC or DC, at any desired frequency. The output

power can never exceed the input power, but efficiencies can be high, with a small proportion of the power dissipated as waste heat.

Circuit description



Top: Simple inverter circuit shown with an electromechanical switch and automatic equivalent auto-switching device implemented with two transistors and split winding auto-transformer in place of the mechanical switch.



Square waveform with fundamental sine wave component, 3rd harmonic and 5th harmonic

Basic designs

In one simple inverter circuit, DC power is connected to a transformer through the centre tap of the primary winding. A switch is rapidly switched back and forth to allow current to flow back to the DC source following two alternate paths through one end of the primary winding and then the other. The alternation of the direction of current in the primary winding of the transformer produces alternating current (AC) in the secondary circuit.

The electromechanical version of the switching device includes two stationary contacts and a spring supported moving contact. The spring holds the movable contact against one of the stationary contacts and an electromagnet pulls the movable contact to the opposite stationary contact. The current in the electromagnet is interrupted by the action of the switch so that the switch continually switches rapidly back and forth. This type of electromechanical inverter switch, called a vibrator or buzzer, was once used in vacuum tube automobile radios. A similar mechanism has been used in door bells, buzzers and tattoo guns.

As they became available with adequate power ratings, transistors and various other types of semiconductor switches have been incorporated into inverter circuit designs.

Output waveforms

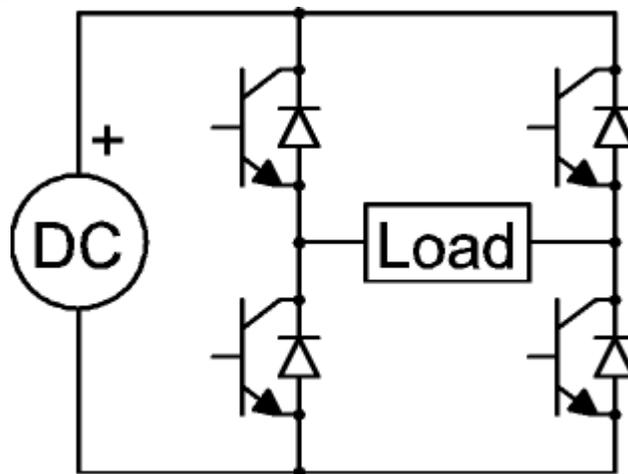
The switch in the simple inverter described above, when not coupled to an output transformer, produces a square voltage waveform due to its simple off and on nature as opposed to the sinusoidal waveform that is the usual waveform of an AC power supply. Using Fourier analysis, periodic waveforms are represented as the sum of an infinite series of sine waves. The sine wave that has the same frequency as the original waveform is called the fundamental component. The other sine waves, called *harmonics*, that are included in the series have frequencies that are integral multiples of the fundamental frequency.

The quality of the inverter output waveform can be expressed by using the Fourier analysis data to calculate the total harmonic distortion (THD). The total harmonic distortion is the square root of the sum of the squares of the harmonic voltages divided by the fundamental voltage:

$$\text{THD} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \dots + V_n^2}}{V_1}$$

The quality of output waveform that is needed from an inverter depends on the characteristics of the connected load. Some loads need a nearly perfect sine wave voltage supply in order to work properly. Other loads may work quite well with a square wave voltage.

Advanced designs

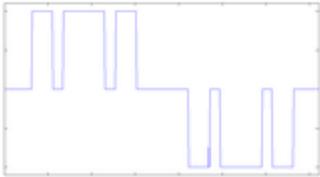


H-bridge inverter circuit with transistor switches and antiparallel diodes

There are many different power circuit topologies and control strategies used in inverter designs. Different design approaches address various issues that may be more or less important depending on the way that the inverter is intended to be used.

The issue of waveform quality can be addressed in many ways. Capacitors and inductors can be used to filter the waveform. If the design includes a transformer, filtering can be applied to the primary or the secondary side of the transformer or to both sides. Low-pass filters are applied to allow the fundamental component of the waveform to pass to the output while limiting the passage of the harmonic components. If the inverter is designed to provide power at a fixed frequency, a resonant filter can be used. For an adjustable frequency inverter, the filter must be tuned to a frequency that is above the maximum fundamental frequency.

Since most loads contain inductance, feedback rectifiers or antiparallel diodes are often connected across each semiconductor switch to provide a path for the peak inductive load current when the switch is turned off. The antiparallel diodes are somewhat similar to the *freewheeling diodes* used in AC/DC converter circuits.

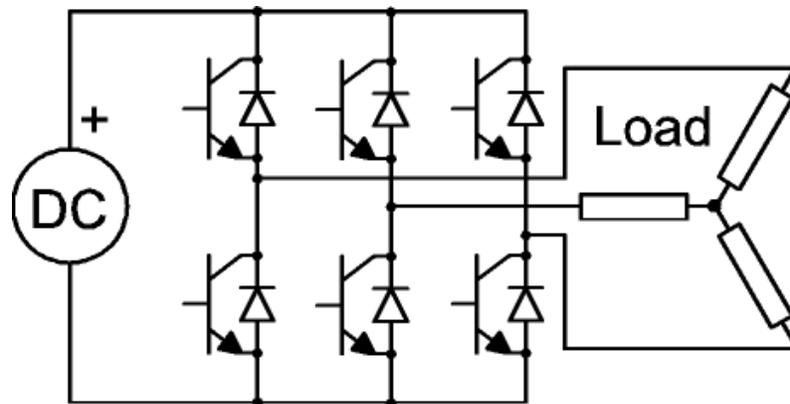
waveform	signal transitions per period	harmonics eliminated	harmonics amplified	System Description	THD
	2	-	-	2-level square wave	~45%
	4	3, 9, 27,...	-	3-level "modified square wave"	> 23.8%
	8			5-level "modified square wave"	> 6.5%
	10	3, 5, 9, 27	7, 11,...	2-level very slow PWM	
	12	3, 5, 9, 27	7, 11,...	3-level very slow PWM	

Fourier analysis reveals that a waveform, like a square wave, that is anti-symmetrical about the 180 degree point contains only odd harmonics, the 3rd, 5th, 7th, etc. Waveforms that have steps of certain widths and heights can attenuate certain lower harmonics at the expense of amplifying higher harmonics. For example, by inserting a zero-voltage step between the positive and negative sections of the square-wave, all of the harmonics that are divisible by three (3rd and 9th, etc.) can be eliminated. That leaves only the 5th, 7th, 11th, 13th etc. The required width of the steps is one third of the period for each of the positive and negative steps and one sixth of the period for each of the zero-voltage steps.

Changing the square wave as described above is an example of pulse-width modulation (PWM). Modulating, or regulating the width of a square-wave pulse is often used as a method of regulating or adjusting an inverter's output voltage. When voltage control is not required, a fixed pulse width can be selected to reduce or eliminate selected harmonics. Harmonic elimination techniques are generally applied to the lowest harmonics because filtering is much more practical at high frequencies, where the filter components can be much smaller and less expensive. *Multiple pulse-width* or *carrier based* PWM control schemes produce waveforms that are composed of many narrow pulses. The frequency represented by the number of narrow pulses per second is called the *switching frequency* or *carrier frequency*. These control schemes are often used in variable-frequency motor control inverters because they allow a wide range of output voltage and frequency adjustment while also improving the quality of the waveform.

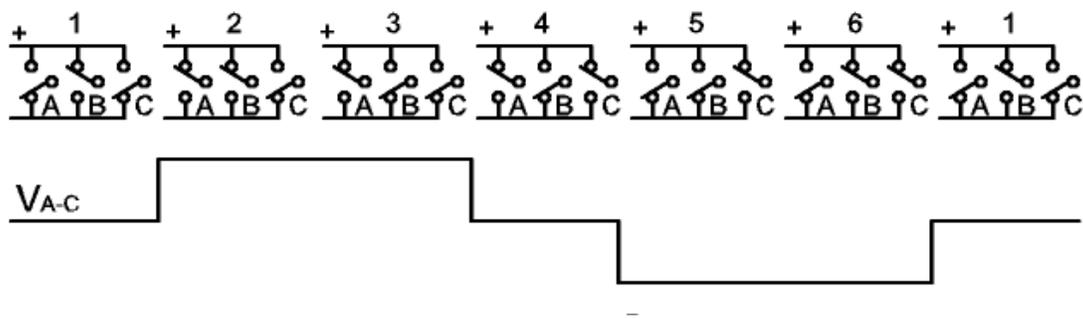
Multilevel inverters provide another approach to harmonic cancellation. Multilevel inverters provide an output waveform that exhibits multiple steps at several voltage levels. For example, it is possible to produce a more sinusoidal wave by having split-rail direct current inputs at two voltages, or positive and negative inputs with a central ground. By connecting the inverter output terminals in sequence between the positive rail and ground, the positive rail and the negative rail, the ground rail and the negative rail, then both to the ground rail, a stepped waveform is generated at the inverter output. This is an example of a three level inverter: the two voltages and ground.

Three phase inverters



3-phase inverter with wye connected load

Three-phase inverters are used for variable-frequency drive applications and for high power applications such as HVDC power transmission. A basic three-phase inverter consists of three single-phase inverter switches each connected to one of the three load terminals. For the most basic control scheme, the operation of the three switches is coordinated so that one switch operates at each 60 degree point of the fundamental output waveform. This creates a line-to-line output waveform that has six steps. The six-step waveform has a zero-voltage step between the positive and negative sections of the square-wave such that the harmonics that are multiples of three are eliminated as described above. When carrier-based PWM techniques are applied to six-step waveforms, the basic overall shape, or *envelope*, of the waveform is retained so that the 3rd harmonic and its multiples are cancelled.



3-phase inverter switching circuit showing 6-step switching sequence and waveform of voltage between terminals A and C

To construct inverters with higher power ratings, two six-step three-phase inverters can be connected in parallel for a higher current rating or in series for a higher voltage rating. In either case, the output waveforms are phase shifted to obtain a 12-step waveform. If additional inverters are combined, an 18-step inverter is obtained with three inverters etc. Although inverters are usually combined for the purpose of achieving increased voltage or current ratings, the quality of the waveform is improved as well.

History

Early inverters

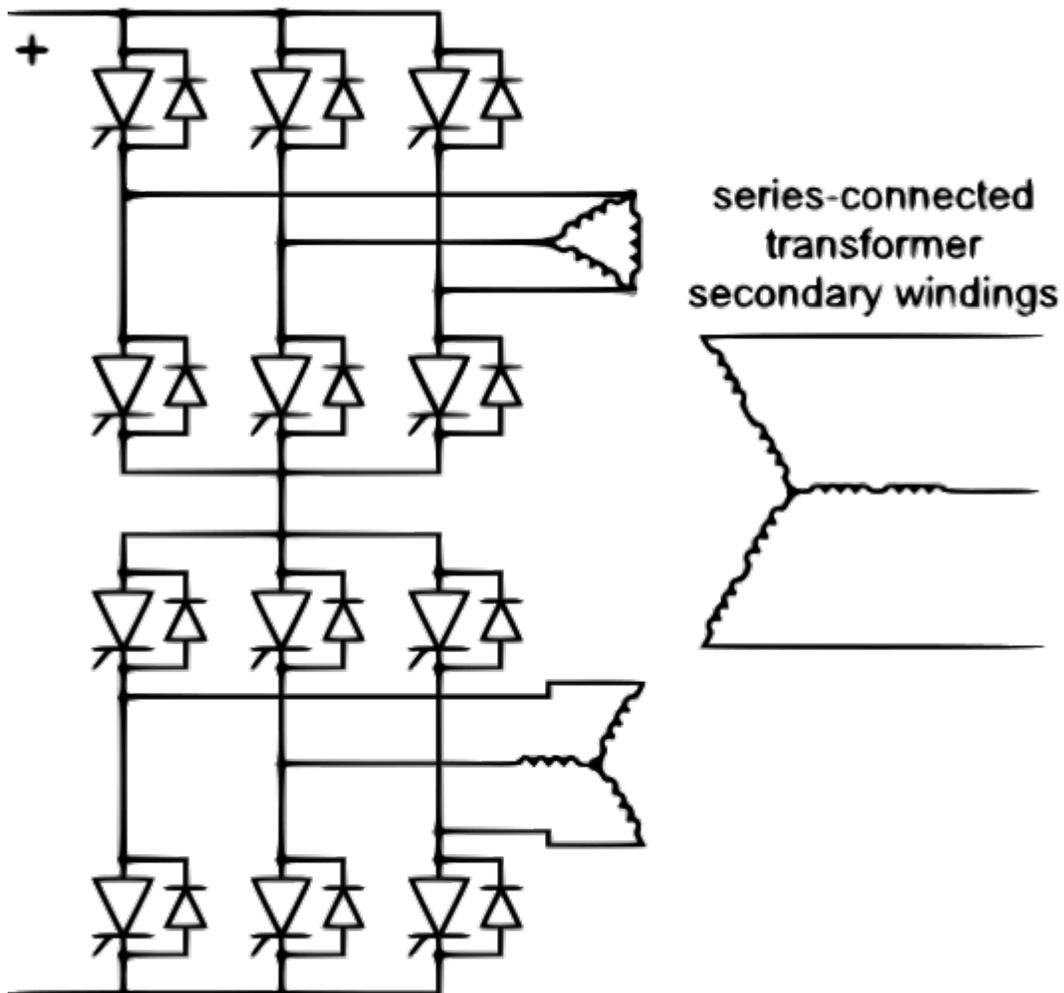
From the late nineteenth century through the middle of the twentieth century, DC-to-AC power conversion was accomplished using rotary converters or motor-generator sets (M-G sets). In the early twentieth century, vacuum tubes and gas filled tubes began to be used as switches in inverter circuits. The most widely used type of tube was the thyatron.

The origins of electromechanical inverters explain the source of the term *inverter*. Early AC-to-DC converters used an induction or synchronous AC motor direct-connected to a generator (dynamo) so that the generator's commutator reversed its connections at exactly the right moments to produce DC. A later development is the synchronous converter, in

which the motor and generator windings are combined into one armature, with slip rings at one end and a commutator at the other and only one field frame. The result with either is AC-in, DC-out. With an M-G set, the DC can be considered to be separately generated from the AC; with a synchronous converter, in a certain sense it can be considered to be "mechanically rectified AC". Given the right auxiliary and control equipment, an M-G set or rotary converter can be "run backwards", converting DC to AC. Hence an inverter is an inverted converter.

Controlled rectifier inverters

Since early transistors were not available with sufficient voltage and current ratings for most inverter applications, it was the 1957 introduction of the thyristor or silicon-controlled rectifier (SCR) that initiated the transition to solid state inverter circuits.



12-pulse line-commutated inverter circuit

The *commutation* requirements of SCRs are a key consideration in SCR circuit designs. SCRs do not turn off or *commutate* automatically when the gate control signal is shut off.

They only turn off when the forward current is reduced to below the minimum holding current, which varies with each kind of SCR, through some external process. For SCRs connected to an AC power source, commutation occurs naturally every time the polarity of the source voltage reverses. SCRs connected to a DC power source usually require a means of forced commutation that forces the current to zero when commutation is required. The least complicated SCR circuits employ natural commutation rather than forced commutation. With the addition of forced commutation circuits, SCRs have been used in the types of inverter circuits described above.

In applications where inverters transfer power from a DC power source to an AC power source, it is possible to use AC-to-DC controlled rectifier circuits operating in the inversion mode. In the inversion mode, a controlled rectifier circuit operates as a line commutated inverter. This type of operation can be used in HVDC power transmission systems and in regenerative braking operation of motor control systems.

Another type of SCR inverter circuit is the current source input (CSI) inverter. A CSI inverter is the dual of a six-step voltage source inverter. With a current source inverter, the DC power supply is configured as a current source rather than a voltage source. The inverter SCRs are switched in a six-step sequence to direct the current to a three-phase AC load as a stepped current waveform. CSI inverter commutation methods include load commutation and parallel capacitor commutation. With both methods, the input current regulation assists the commutation. With load commutation, the load is a synchronous motor operated at a leading power factor.

As they have become available in higher voltage and current ratings, semiconductors such as transistors or IGBTs that can be turned off by means of control signals have become the preferred switching components for use in inverter circuits.

Rectifier and inverter pulse numbers

Rectifier circuits are often classified by the number of current pulses that flow to the DC side of the rectifier per cycle of AC input voltage. A single-phase half-wave rectifier is a one-pulse circuit and a single-phase full-wave rectifier is a two-pulse circuit. A three-phase half-wave rectifier is a three-pulse circuit and a three-phase full-wave rectifier is a six-pulse circuit.

With three-phase rectifiers, two or more rectifiers are sometimes connected in series or parallel to obtain higher voltage or current ratings. The rectifier inputs are supplied from special transformers that provide phase shifted outputs. This has the effect of phase multiplication. Six phases are obtained from two transformers, twelve phases from three transformers and so on. The associated rectifier circuits are 12-pulse rectifiers, 18-pulse rectifiers and so on.

When controlled rectifier circuits are operated in the inversion mode, they would be classified by pulse number also. Rectifier circuits that have a higher pulse number have reduced harmonic content in the AC input current and reduced ripple in the DC output

voltage. In the inversion mode, circuits that have a higher pulse number have lower harmonic content in the AC output voltage waveform.

Chapter-2

Transformer



Pole-mounted power distribution transformer with center-tapped secondary winding (note use of grounded conductor, right, as one leg of the primary feeder). It transforms the high voltage of the overhead distribution wires to the lower voltage used in house wiring.

A **transformer** is a static device that transfers electrical energy from one circuit to another through inductively coupled conductors—the transformer's coils. A varying current in the first or *primary* winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the *secondary* winding. This

varying magnetic field induces a varying electromotive force (EMF) or "voltage" in the secondary winding. This effect is called mutual induction.

If a load is connected to the secondary, an electric current will flow in the secondary winding and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding (V_s) is in proportion to the primary voltage (V_p), and is given by the ratio of the number of turns in the secondary (N_s) to the number of turns in the primary (N_p) as follows:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

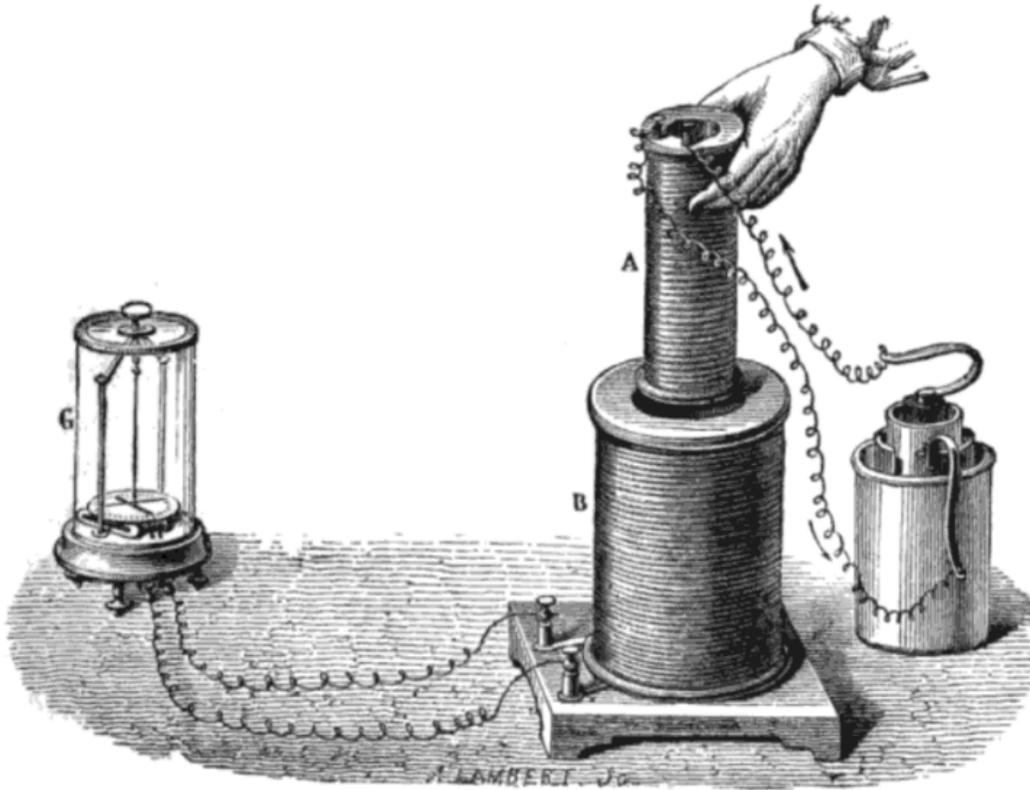
By appropriate selection of the ratio of turns, a transformer thus allows an alternating current (AC) voltage to be "stepped up" by making N_s greater than N_p , or "stepped down" by making N_s less than N_p .

In the vast majority of transformers, the windings are coils wound around a ferromagnetic core, air-core transformers being a notable exception.

Transformers range in size from a thumbnail-sized coupling transformer hidden inside a stage microphone to huge units weighing hundreds of tons used to interconnect portions of power grids. All operate with the same basic principles, although the range of designs is wide. While new technologies have eliminated the need for transformers in some electronic circuits, transformers are still found in nearly all electronic devices designed for household ("mains") voltage. Transformers are essential for high-voltage electric power transmission, which makes long-distance transmission economically practical.

History

Discovery



Faraday's experiment with induction between coils of wire

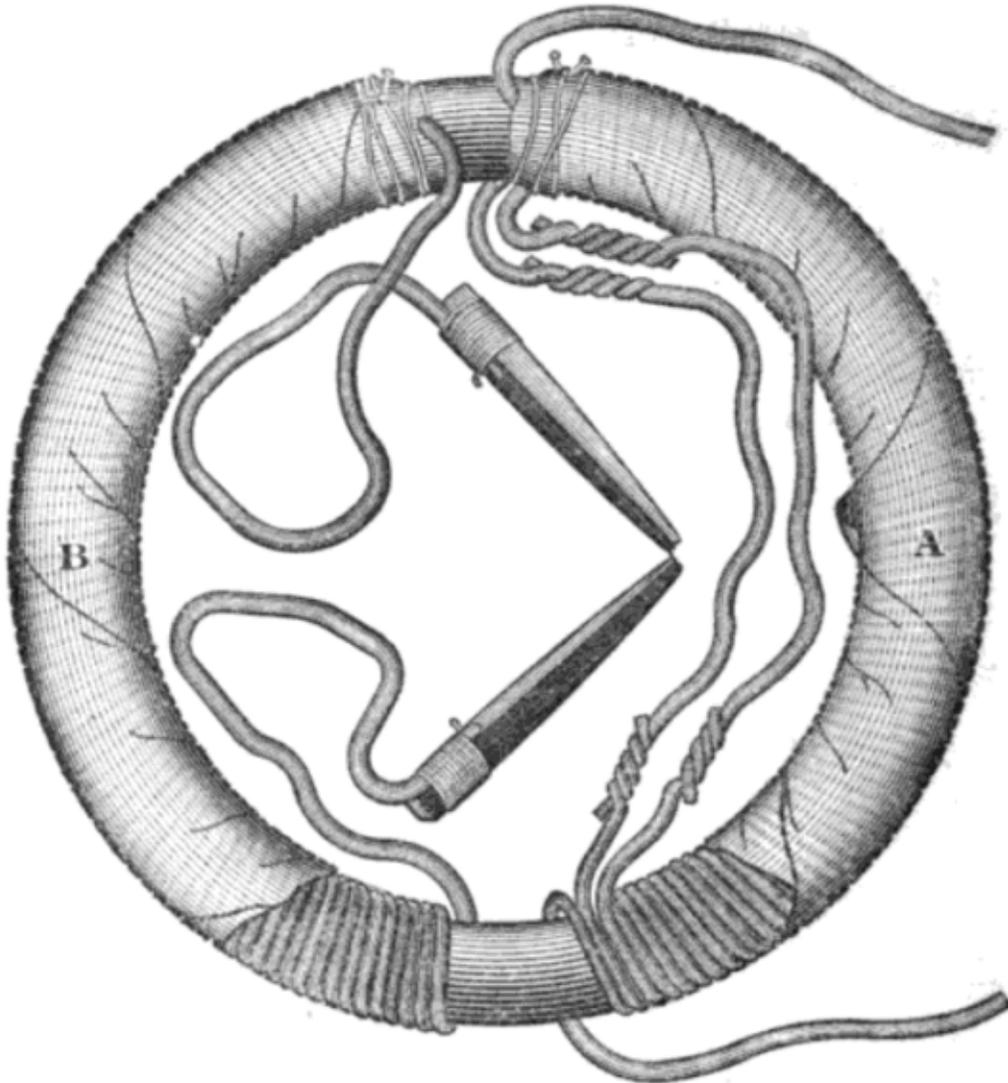
The phenomenon of electromagnetic induction was discovered independently by Michael Faraday and Joseph Henry in 1831. However, Faraday was the first to publish the results of his experiments and thus receive credit for the discovery. The relationship between electromotive force (EMF) or "voltage" and magnetic flux was formalized in an equation now referred to as "Faraday's law of induction":

$$|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right|$$

where $|\mathcal{E}|$ is the magnitude of the EMF in volts and Φ_B is the magnetic flux through the circuit (in webers).

Faraday performed the first experiments on induction between coils of wire, including winding a pair of coils around an iron ring, thus creating the first toroidal closed-core transformer.

Induction coils



Faraday's ring transformer

The first type of transformer to see wide use was the induction coil, invented by Rev. Nicholas Callan of Maynooth College, Ireland in 1836. He was one of the first researchers to realize that the more turns the secondary winding has in relation to the primary winding, the larger is the increase in EMF. Induction coils evolved from scientists' and inventors' efforts to get higher voltages from batteries. Since batteries produce direct current (DC) rather than alternating current (AC), induction coils relied

upon vibrating electrical contacts that regularly interrupted the current in the primary to create the flux changes necessary for induction. Between the 1830s and the 1870s, efforts to build better induction coils, mostly by trial and error, slowly revealed the basic principles of transformers.

In 1876, Russian engineer Pavel Yablochkov invented a lighting system based on a set of induction coils where the primary windings were connected to a source of alternating current and the secondary windings could be connected to several "electric candles" (arc lamps) of his own design. The coils Yablochkov employed functioned essentially as transformers.

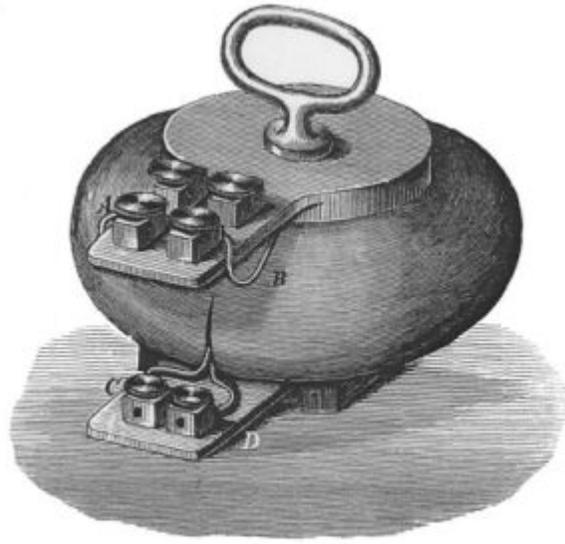
In 1878, the Ganz Company in Hungary began manufacturing equipment for electric lighting and, by 1883, had installed over fifty systems in Austria-Hungary. Their systems used alternating current exclusively and included those comprising both arc and incandescent lamps, along with generators and other equipment.

Lucien Gaulard and John Dixon Gibbs first exhibited a device with an open iron core called a "secondary generator" in London in 1882, then sold the idea to the Westinghouse company in the United States. They also exhibited the invention in Turin, Italy in 1884, where it was adopted for an electric lighting system. However, the efficiency of their open-core bipolar apparatus remained very low.

Induction coils with open magnetic circuits are inefficient for transfer of power to loads. Until about 1880, the paradigm for AC power transmission from a high voltage supply to a low voltage load was a series circuit. Open-core transformers with a ratio near 1:1 were connected with their primaries in series to allow use of a high voltage for transmission while presenting a low voltage to the lamps. The inherent flaw in this method was that turning off a single lamp affected the voltage supplied to all others on the same circuit. Many adjustable transformer designs were introduced to compensate for this problematic characteristic of the series circuit, including those employing methods of adjusting the core or bypassing the magnetic flux around part of a coil.

Efficient, practical transformer designs did not appear until the 1880s, but within a decade the transformer would be instrumental in the "War of Currents", and in seeing AC distribution systems triumph over their DC counterparts, a position in which they have remained dominant ever since.

Closed-core transformers and the introduction of parallel connection

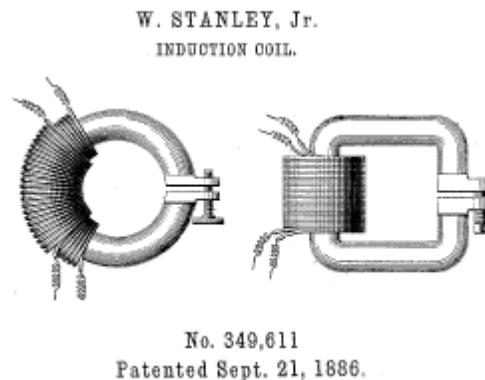


Drawing of Ganz Company's 1885 prototype. Capacity: 1400 VA, frequency: 40 Hz, voltage ratio: 120/72 V



Prototypes of the world's first high-efficiency transformers. They were built by the Z.B.D. team on 16th September 1884.

In the autumn of 1884, Ganz Company engineers Károly Zipernowsky, Ottó Bláthy and Miksa Déri had determined that open-core devices were impracticable, as they were incapable of reliably regulating voltage. In their joint patent application for the "Z.B.D." transformers, they described two designs with closed magnetic circuits: the "closed-core" and "shell-core" transformers. In the closed-core, the primary and secondary windings were wound around a closed iron ring; in the shell-core, the windings were passed *through* the iron core. In both designs, the magnetic flux linking the primary and secondary windings traveled almost entirely within the iron core, with no intentional path through air. The new Z.B.D. transformers reached 98 percent efficiency, which was 3.4 times higher than the open core bipolar devices of Gaulard and Gibbs. When they employed it in parallel connected electric distribution systems, closed-core transformers finally made it technically and economically feasible to provide electric power for lighting in homes, businesses and public spaces. Bláthy had suggested the use of closed-cores, Zipernowsky the use of shunt connections, and Déri had performed the experiments; Bláthy also discovered the transformer formula, $V_s/V_p = N_s/N_p$. The vast majority of transformers in use today rely on the basic principles discovered by the three engineers. They also reportedly popularized the word "transformer" to describe a device for altering the EMF of an electric current, although the term had already been in use by 1882. In 1886, the Ganz Company installed the world's first power station that used AC generators to power a parallel-connected common electrical network, the steam-powered Rome-Cerchi power plant.



Stanley's 1886 design for adjustable gap open-core induction coils

Although George Westinghouse had bought Gaulard and Gibbs' patents in 1885, the Edison Electric Light Company held an option on the U.S. rights for the Z.B.D. transformers, requiring Westinghouse to pursue alternative designs on the same principles. He assigned to William Stanley the task of developing a device for commercial use in United States. Stanley's first patented design was for induction coils with single cores of soft iron and adjustable gaps to regulate the EMF present in the secondary winding. This design was first used commercially in the U.S. in 1886. But Westinghouse soon had his team working on a design whose core comprised a stack of thin "E-shaped" iron plates, separated individually or in pairs by thin sheets of paper or other insulating material. Prewound copper coils could then be slid into place, and

straight iron plates laid in to create a closed magnetic circuit. Westinghouse applied for a patent for the new design in December 1886; it was granted in July 1887.

Other early transformers

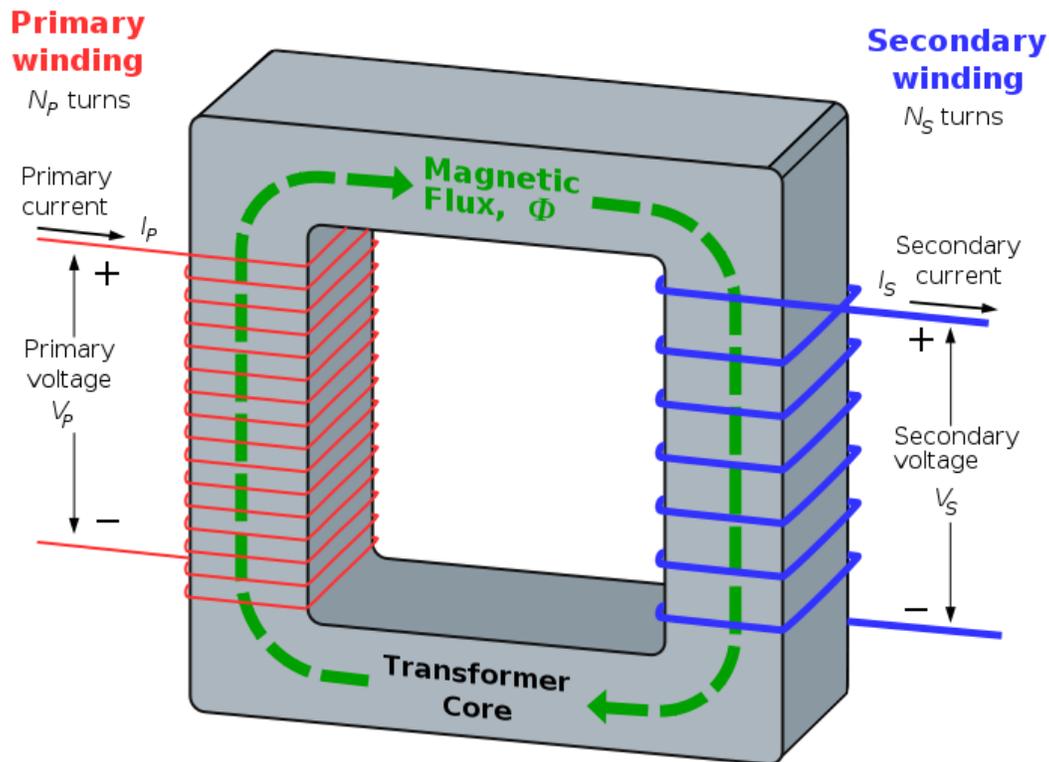
In 1889, Russian-born engineer Mikhail Dolivo-Dobrovolsky developed the first three-phase transformer at the Allgemeine Elektrizitäts-Gesellschaft ("General Electricity Company") in Germany.

In 1891, Nikola Tesla invented the Tesla coil, an air-cored, dual-tuned resonant transformer for generating very high voltages at high frequency.

Audio frequency transformers ("repeating coils") were used by early experimenters in the development of the telephone.

Basic principles

The transformer is based on two principles: first, that an electric current can produce a magnetic field (electromagnetism), and, second that a changing magnetic field within a coil of wire induces a voltage across the ends of the coil (electromagnetic induction). Changing the current in the primary coil changes the magnetic flux that is developed. The changing magnetic flux induces a voltage in the secondary coil.



An ideal transformer

An ideal transformer is shown in the adjacent figure. Current passing through the primary coil creates a magnetic field. The primary and secondary coils are wrapped around a core of very high magnetic permeability, such as iron, so that most of the magnetic flux passes through both the primary and secondary coils.

Induction law

The voltage induced across the secondary coil may be calculated from Faraday's law of induction, which states that:

$$V_s = N_s \frac{d\Phi}{dt},$$

where V_s is the instantaneous voltage, N_s is the number of turns in the secondary coil and Φ is the magnetic flux through one turn of the coil. If the turns of the coil are oriented perpendicular to the magnetic field lines, the flux is the product of the magnetic flux density B and the area A through which it cuts. The area is constant, being equal to the cross-sectional area of the transformer core, whereas the magnetic field varies with time according to the excitation of the primary. Since the same magnetic flux passes through

both the primary and secondary coils in an ideal transformer, the instantaneous voltage across the primary winding equals

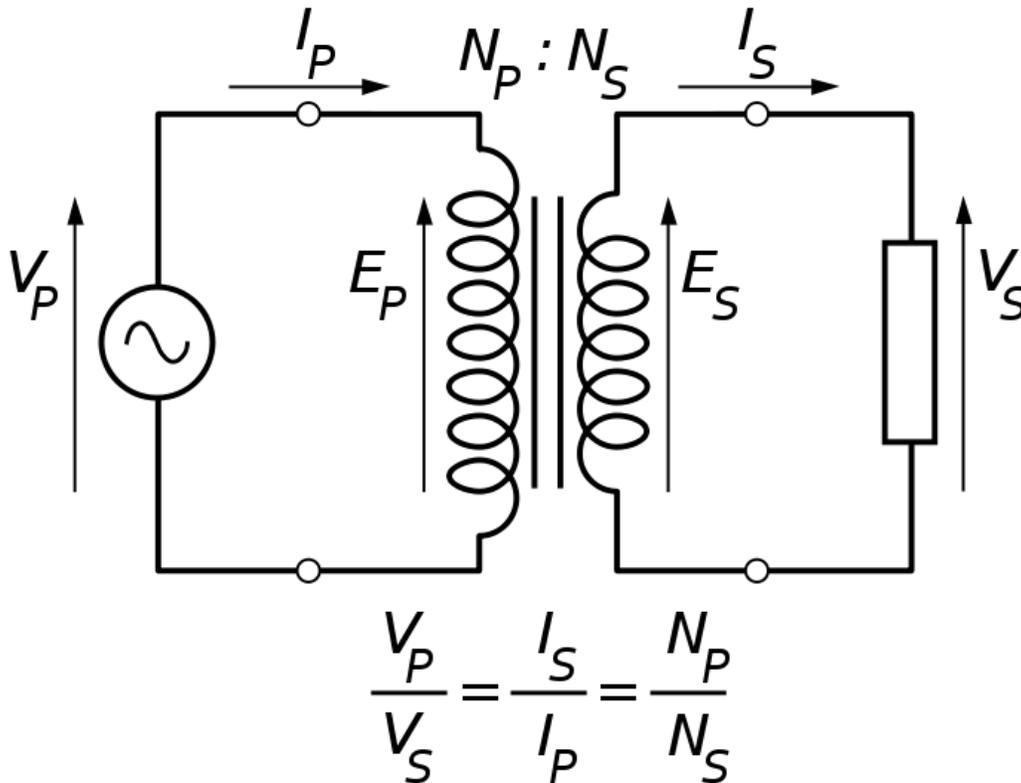
$$V_P = N_P \frac{d\Phi}{dt}.$$

Taking the ratio of the two equations for V_s and V_p gives the basic equation for stepping up or stepping down the voltage

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

N_p/N_s is known as the *turns ratio*, and is the primary functional characteristic of any transformer. In the case of step-up transformers, this may sometimes be stated as the reciprocal, N_s/N_p . *Turns ratio* is commonly expressed as an irreducible fraction or ratio: for example, a transformer with primary and secondary windings of, respectively, 100 and 150 turns is said to have a turns ratio of 2:3 rather than 0.667 or 100:150.

Ideal power equation



The ideal transformer as a circuit element

If the secondary coil is attached to a load that allows current to flow, electrical power is transmitted from the primary circuit to the secondary circuit. Ideally, the transformer is perfectly efficient; all the incoming energy is transformed from the primary circuit to the magnetic field and into the secondary circuit. If this condition is met, the incoming electric power must equal the outgoing power:

$$P_{\text{incoming}} = I_p V_p = P_{\text{outgoing}} = I_s V_s,$$

giving the ideal transformer equation

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}.$$

Transformers normally have high efficiency, so this formula is a reasonable approximation.

If the voltage is increased, then the current is decreased by the same factor. The impedance in one circuit is transformed by the *square* of the turns ratio. For example, if an impedance Z_s is attached across the terminals of the secondary coil, it appears to the primary circuit to have an impedance of $(N_p/N_s)^2 Z_s$. This relationship is reciprocal, so that the impedance Z_p of the primary circuit appears to the secondary to be $(N_s/N_p)^2 Z_p$.

Detailed operation

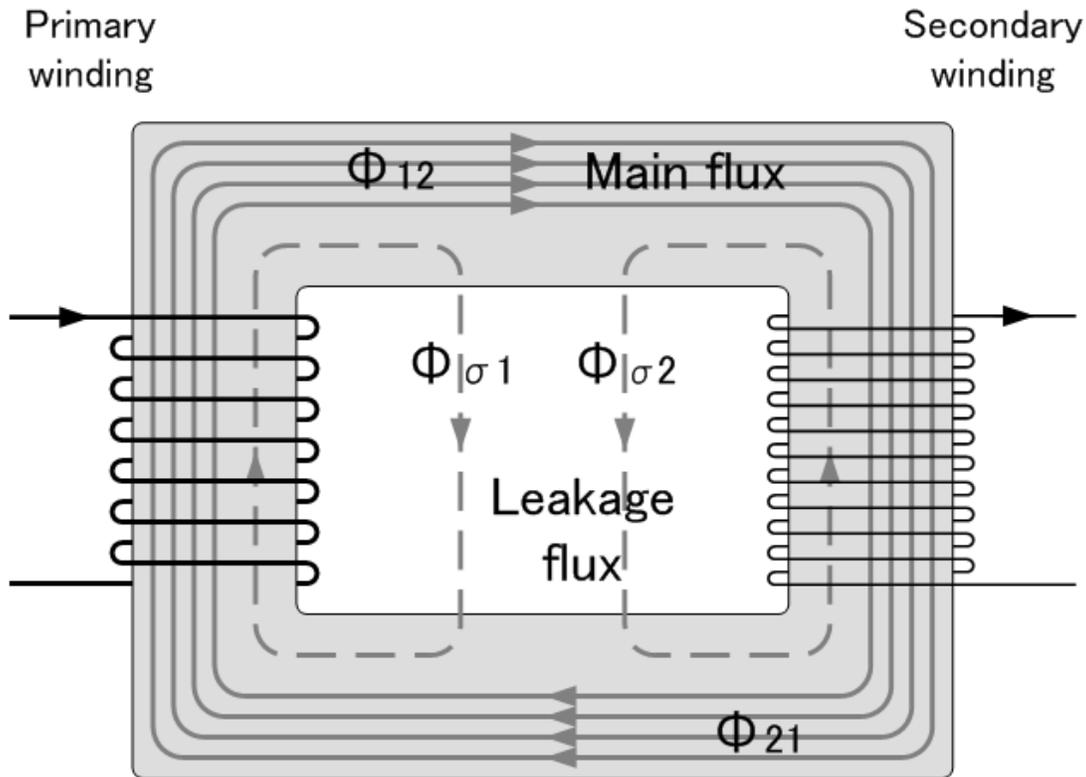
The simplified description above neglects several practical factors, in particular the primary current required to establish a magnetic field in the core, and the contribution to the field due to current in the secondary circuit.

Models of an ideal transformer typically assume a core of negligible reluctance with two windings of zero resistance. When a voltage is applied to the primary winding, a small current flows, driving flux around the magnetic circuit of the core. The current required to create the flux is termed the *magnetizing current*; since the ideal core has been assumed to have near-zero reluctance, the magnetizing current is negligible, although still required to create the magnetic field.

The changing magnetic field induces an electromotive force (EMF) across each winding. Since the ideal windings have no impedance, they have no associated voltage drop, and so the voltages V_p and V_s measured at the terminals of the transformer, are equal to the corresponding EMFs. The primary EMF, acting as it does in opposition to the primary voltage, is sometimes termed the "back EMF". This is due to Lenz's law which states that the induction of EMF would always be such that it will oppose development of any such change in magnetic field.

Practical considerations

Leakage flux



Leakage flux of a transformer

The ideal transformer model assumes that all flux generated by the primary winding links all the turns of every winding, including itself. In practice, some flux traverses paths that take it outside the windings. Such flux is termed *leakage flux*, and results in leakage inductance in series with the mutually coupled transformer windings. Leakage results in energy being alternately stored in and discharged from the magnetic fields with each cycle of the power supply. It is not directly a power loss, but results in inferior voltage regulation, causing the secondary voltage to fail to be directly proportional to the primary, particularly under heavy load. Transformers are therefore normally designed to have very low leakage inductance.

However, in some applications, leakage can be a desirable property, and long magnetic paths, air gaps, or magnetic bypass shunts may be deliberately introduced to a transformer's design to limit the short-circuit current it will supply. Leaky transformers may be used to supply loads that exhibit negative resistance, such as electric arcs, mercury vapor lamps, and neon signs; or for safely handling loads that become periodically short-circuited such as electric arc welders.

Air gaps are also used to keep a transformer from saturating, especially audio-frequency transformers in circuits that have a direct current flowing through the windings.

Leakage inductance is also helpful when transformers are operated in parallel. It can be shown that if the "per-unit" inductance of two transformers is the same (a typical value is 5%), they will automatically split power "correctly" (e.g. 500 kVA unit in parallel with 1,000 kVA unit, the larger one will carry twice the current).

Effect of frequency

Transformer universal EMF equation

If the flux in the core is purely sinusoidal, the relationship for either winding between its **rms voltage** E_{rms} of the winding, and the supply frequency f , number of turns N , core cross-sectional area a and peak magnetic flux density B is given by the universal EMF equation:

$$E_{rms} = \frac{2\pi f N a B_{peak}}{\sqrt{2}} \approx 4.44 f N a B$$

If the flux does not contain even harmonics the following equation can be used for **half-cycle average voltage** E_{avg} of any waveshape:

$$E_{avg} = 4 f N a B_{peak}$$

The time-derivative term in Faraday's Law shows that the flux in the core is the integral with respect to time of the applied voltage. Hypothetically an ideal transformer would work with direct-current excitation, with the core flux increasing linearly with time. In practice, the flux would rise to the point where magnetic saturation of the core occurs, causing a huge increase in the magnetizing current and overheating the transformer. All practical transformers must therefore operate with alternating (or pulsed) current.

The EMF of a transformer at a given flux density increases with frequency. By operating at higher frequencies, transformers can be physically more compact because a given core is able to transfer more power without reaching saturation and fewer turns are needed to achieve the same impedance. However, properties such as core loss and conductor skin effect also increase with frequency. Aircraft and military equipment employ 400 Hz power supplies which reduce core and winding weight. Conversely, frequencies used for some railway electrification systems were much lower (e.g. 16.7 Hz and 25 Hz) than normal utility frequencies (50 – 60 Hz) for historical reasons concerned mainly with the limitations of early electric traction motors. As such, the transformers used to step down the high over-head line voltages (e.g. 15 kV) are much heavier for the same power rating than those designed only for the higher frequencies.

Operation of a transformer at its designed voltage but at a higher frequency than intended will lead to reduced magnetizing current; at lower frequency, the magnetizing current

will increase. Operation of a transformer at other than its design frequency may require assessment of voltages, losses, and cooling to establish if safe operation is practical. For example, transformers may need to be equipped with "volts per hertz" over-excitation relays to protect the transformer from overvoltage at higher than rated frequency.

One example of state-of-the-art design is those transformers used for electric multiple unit high speed trains, particularly those required to operate across the borders of countries using different standards of electrification. The position of such transformers is restricted to being hung below the passenger compartment. They have to function at different frequencies (down to 16.7 Hz) and voltages (up to 25 kV) whilst handling the enhanced power requirements needed for operating the trains at high speed.

Knowledge of natural frequencies of transformer windings is of importance for the determination of the transient response of the windings to impulse and switching surge voltages.

Energy losses

An ideal transformer would have no energy losses, and would be 100% efficient. In practical transformers energy is dissipated in the windings, core, and surrounding structures. Larger transformers are generally more efficient, and those rated for electricity distribution usually perform better than 98%.

Experimental transformers using superconducting windings achieve efficiencies of 99.85%. The increase in efficiency can save considerable energy, and hence money, in a large heavily-loaded transformer; the trade-off is in the additional initial and running cost of the superconducting design.

Losses in transformers (excluding associated circuitry) vary with load current, and may be expressed as "no-load" or "full-load" loss. Winding resistance dominates load losses, whereas hysteresis and eddy currents losses contribute to over 99% of the no-load loss. The no-load loss can be significant, so that even an idle transformer constitutes a drain on the electrical supply and a running cost; designing transformers for lower loss requires a larger core, good-quality silicon steel, or even amorphous steel, for the core, and thicker wire, increasing initial cost, so that there is a trade-off between initial cost and running cost.

Transformer losses are divided into losses in the windings, termed copper loss, and those in the magnetic circuit, termed iron loss. Losses in the transformer arise from:

Winding resistance

Current flowing through the windings causes resistive heating of the conductors. At higher frequencies, skin effect and proximity effect create additional winding resistance and losses.

Hysteresis losses

Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the loss is proportional to the frequency, and is a function of the peak flux density to which it is subjected.

Eddy currents

Ferromagnetic materials are also good conductors, and a core made from such a material also constitutes a single short-circuited turn throughout its entire length. Eddy currents therefore circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and inverse square of the material thickness. Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use laminated or similar cores.

Magnetostriction

Magnetic flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle of the magnetic field, an effect known as magnetostriction. This produces the buzzing sound commonly associated with transformers, and can cause losses due to frictional heating.

Mechanical losses

In addition to magnetostriction, the alternating magnetic field causes fluctuating forces between the primary and secondary windings. These incite vibrations within nearby metalwork, adding to the buzzing noise, and consuming a small amount of power.

Stray losses

Leakage inductance is by itself largely lossless, since energy supplied to its magnetic fields is returned to the supply with the next half-cycle. However, any leakage flux that intercepts nearby conductive materials such as the transformer's support structure will give rise to eddy currents and be converted to heat. There are also radiative losses due to the oscillating magnetic field, but these are usually small.

Dot convention

It is common in transformer schematic symbols for there to be a dot at the end of each coil within a transformer, particularly for transformers with multiple primary and secondary windings. The dots indicate the direction of each winding relative to the others. Voltages at the dot end of each winding are in phase; current flowing into the dot end of a primary coil will result in current flowing out of the dot end of a secondary coil.

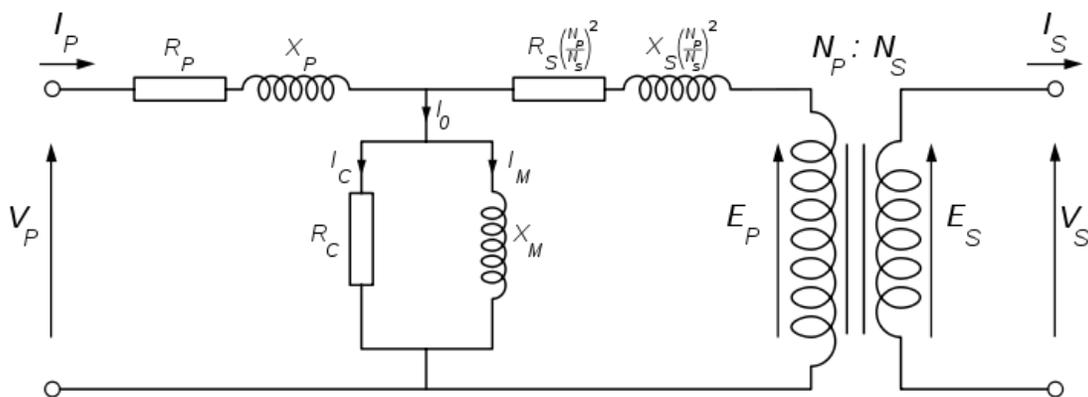
Equivalent circuit

The physical limitations of the practical transformer may be brought together as an equivalent circuit model (shown below) built around an ideal lossless transformer. Power loss in the windings is current-dependent and is represented as in-series resistances R_p and R_s . Flux leakage results in a fraction of the applied voltage dropped without contributing to the mutual coupling, and thus can be modeled as reactances of each leakage inductance X_p and X_s in series with the perfectly coupled region.

Iron losses are caused mostly by hysteresis and eddy current effects in the core, and are proportional to the square of the core flux for operation at a given frequency. Since the core flux is proportional to the applied voltage, the iron loss can be represented by a resistance R_C in parallel with the ideal transformer.

A core with finite permeability requires a magnetizing current I_m to maintain the mutual flux in the core. The magnetizing current is in phase with the flux; saturation effects cause the relationship between the two to be non-linear, but for simplicity this effect tends to be ignored in most circuit equivalents. With a sinusoidal supply, the core flux lags the induced EMF by 90° and this effect can be modeled as a magnetizing reactance (reactance of an effective inductance) X_m in parallel with the core loss component. R_C and X_m are sometimes together termed the *magnetizing branch* of the model. If the secondary winding is made open-circuit, the current I_0 taken by the magnetizing branch represents the transformer's no-load current.

The secondary impedance R_s and X_s is frequently moved (or "referred") to the primary side after multiplying the components by the impedance scaling factor $(N_p/N_s)^2$.



Transformer equivalent circuit, with secondary impedances referred to the primary side

The resulting model is sometimes termed the "exact equivalent circuit", though it retains a number of approximations, such as an assumption of linearity. Analysis may be simplified by moving the magnetizing branch to the left of the primary impedance, an implicit assumption that the magnetizing current is low, and then summing primary and referred secondary impedances, resulting in so-called equivalent impedance.

The parameters of equivalent circuit of a transformer can be calculated from the results of two transformer tests: open-circuit test and short-circuit test.

Types

A wide variety of transformer designs are used for different applications, though they share several common features. Important common transformer types include:

Autotransformer



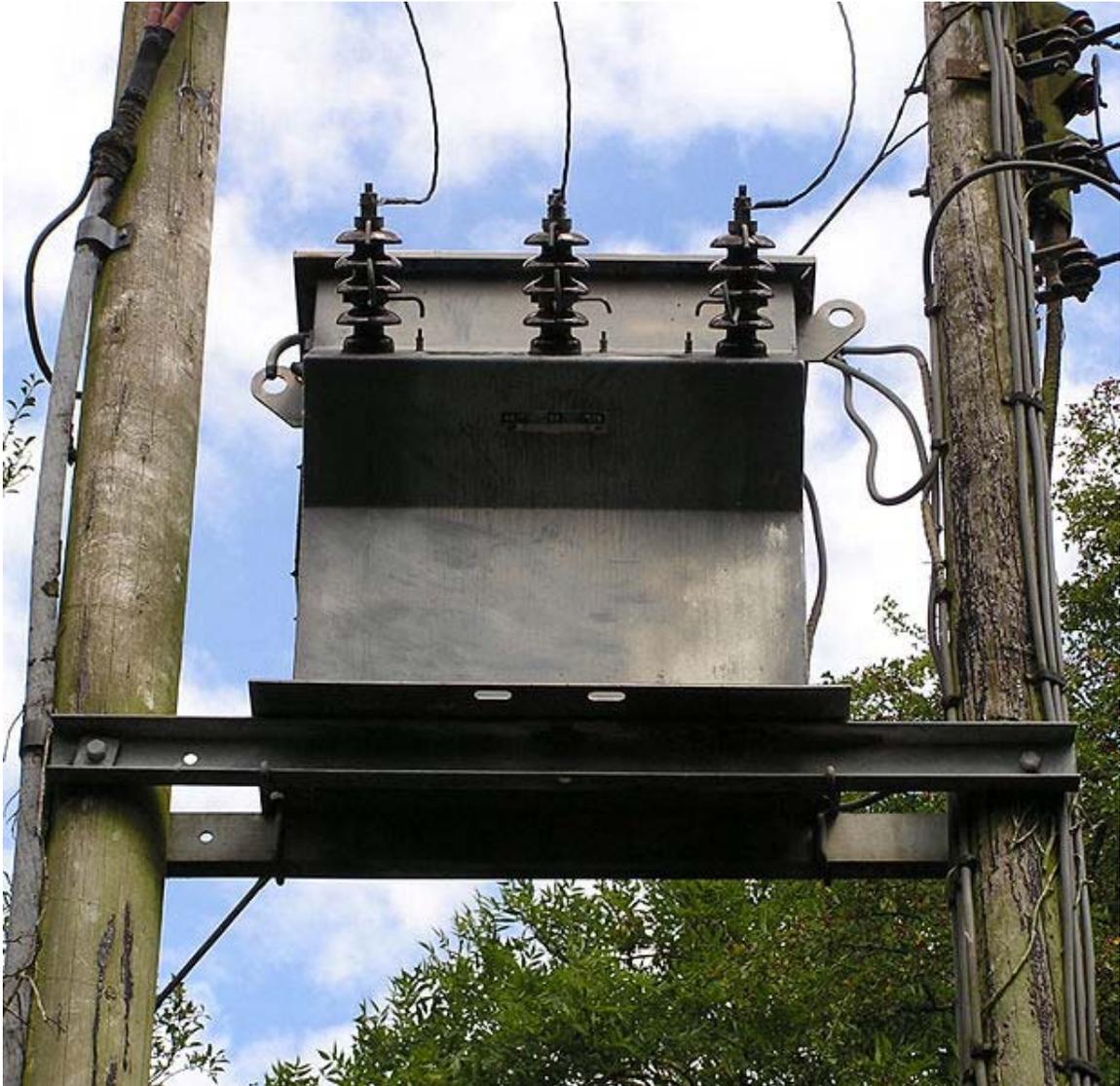
A variable autotransformer

In an autotransformer portions of the same winding act as both the primary and secondary. The winding has at least three taps where electrical connections are made. An autotransformer can be smaller, lighter and cheaper than a standard dual-winding transformer however the autotransformer does not provide electrical isolation.

Autotransformers are often used to step up or down between voltages in the 110-117-120 volt range and voltages in the 220-230-240 volt range, e.g., to output either 110 or 120V (with taps) from 230V input, allowing equipment from a 100 or 120V region to be used in a 230V region.

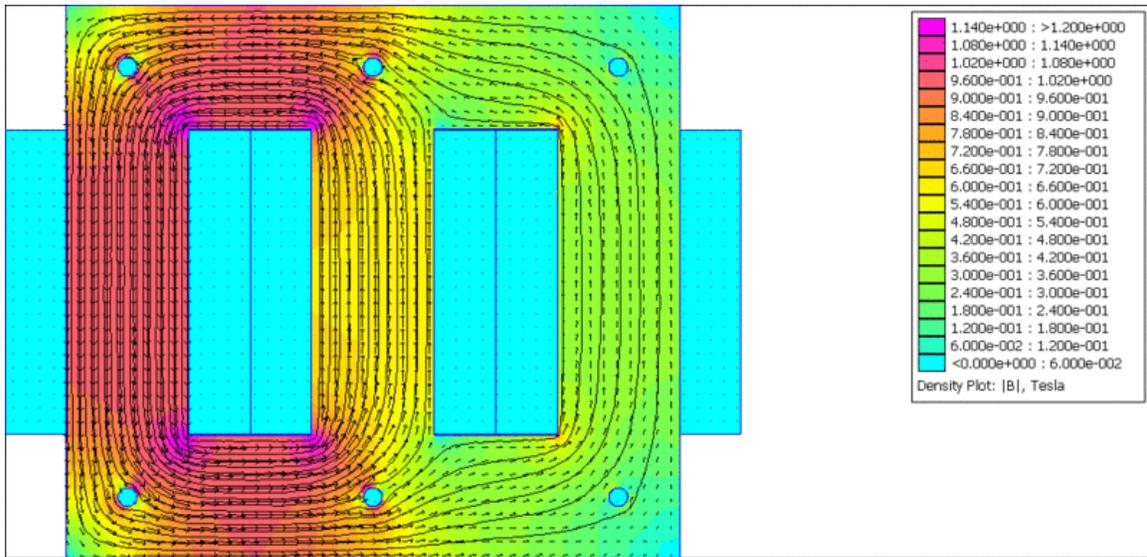
A variable autotransformer is made by exposing part of the winding coils and making the secondary connection through a sliding brush, giving a variable turns ratio. Such a device is often referred to by the trademark name *variac*.

Polyphase transformers



Three-phase step-down transformer mounted between two utility poles

For three-phase supplies, a bank of three individual single-phase transformers can be used, or all three phases can be incorporated as a single three-phase transformer. In this case, the magnetic circuits are connected together, the core thus containing a three-phase flow of flux. A number of winding configurations are possible, giving rise to different attributes and phase shifts. One particular polyphase configuration is the zigzag transformer, used for grounding and in the suppression of harmonic currents.



Screenshot of a FEM simulation of the magnetic flux inside a three-phase power transformer.

Leakage transformers



Leakage transformer

A leakage transformer, also called a stray-field transformer, has a significantly higher leakage inductance than other transformers, sometimes increased by a magnetic bypass or shunt in its core between primary and secondary, which is sometimes adjustable with a set screw. This provides a transformer with an inherent current limitation due to the loose coupling between its primary and the secondary windings. The output and input currents are low enough to prevent thermal overload under all load conditions—even if the secondary is shorted.

Leakage transformers are used for arc welding and high voltage discharge lamps (neon lights and cold cathode fluorescent lamps, which are series-connected up to 7.5 kV AC). It acts then both as a voltage transformer and as a magnetic ballast.

Other applications are short-circuit-proof extra-low voltage transformers for toys or doorbell installations.

Resonant transformers

A resonant transformer is a kind of leakage transformer. It uses the leakage inductance of its secondary windings in combination with external capacitors, to create one or more resonant circuits. Resonant transformers such as the Tesla coil can generate very high voltages, and are able to provide much higher current than electrostatic high-voltage generation machines such as the Van de Graaff generator. One of the applications of the resonant transformer is for the CCFL inverter. Another application of the resonant transformer is to couple between stages of a superheterodyne receiver, where the selectivity of the receiver is provided by tuned transformers in the intermediate-frequency amplifiers.

Audio transformers

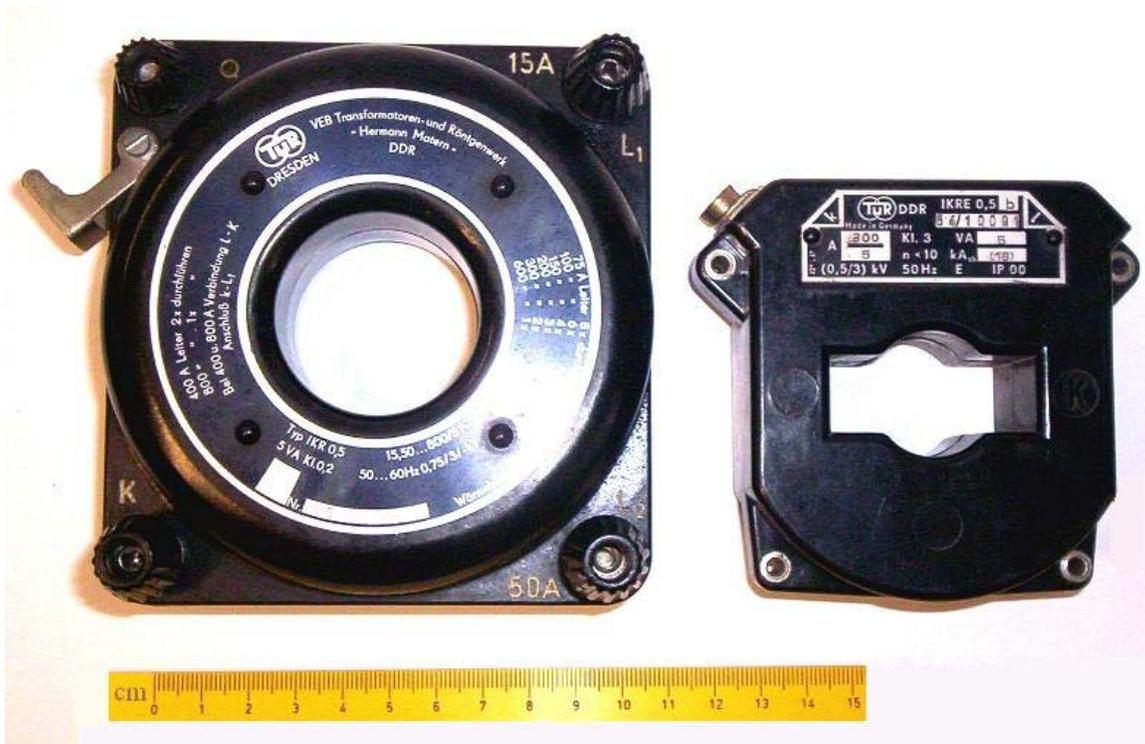
Audio transformers are those specifically designed for use in audio circuits. They can be used to block radio frequency interference or the DC component of an audio signal, to split or combine audio signals, or to provide impedance matching between high and low impedance circuits, such as between a high impedance tube (valve) amplifier output and a low impedance loudspeaker, or between a high impedance instrument output and the low impedance input of a mixing console.

Such transformers were originally designed to connect different telephone systems to one another while keeping their respective power supplies isolated, and are still commonly used to interconnect professional audio systems or system components.

Being magnetic devices, audio transformers are susceptible to external magnetic fields such as those generated by AC current-carrying conductors. "Hum" is a term commonly used to describe unwanted signals originating from the "mains" power supply (typically 50 or 60 Hz). Audio transformers used for low-level signals, such as those from microphones, often include shielding to protect against extraneous magnetically coupled signals.

Instrument transformers

Instrument transformers are used for measuring voltage and current in electrical power systems, and for power system protection and control. Where a voltage or current is too large to be conveniently used by an instrument, it can be scaled down to a standardized, low value. Instrument transformers isolate measurement, protection and control circuitry from the high currents or voltages present on the circuits being measured or controlled.



Current transformers, designed for placing around conductors

A current transformer is a transformer designed to provide a current in its secondary coil proportional to the current flowing in its primary coil.

Voltage transformers (VTs), also referred to as "potential transformers" (PTs), are designed to have an accurately known transformation ratio in both magnitude and phase, over a range of measuring circuit impedances. A voltage transformer is intended to present a negligible load to the supply being measured. The low secondary voltage allows protective relay equipment and measuring instruments to be operated at a lower voltages.

Both current and voltage instrument transformers are designed to have predictable characteristics on overloads. Proper operation of over-current protective relays requires that current transformers provide a predictable transformation ratio even during a short-circuit.

Classification

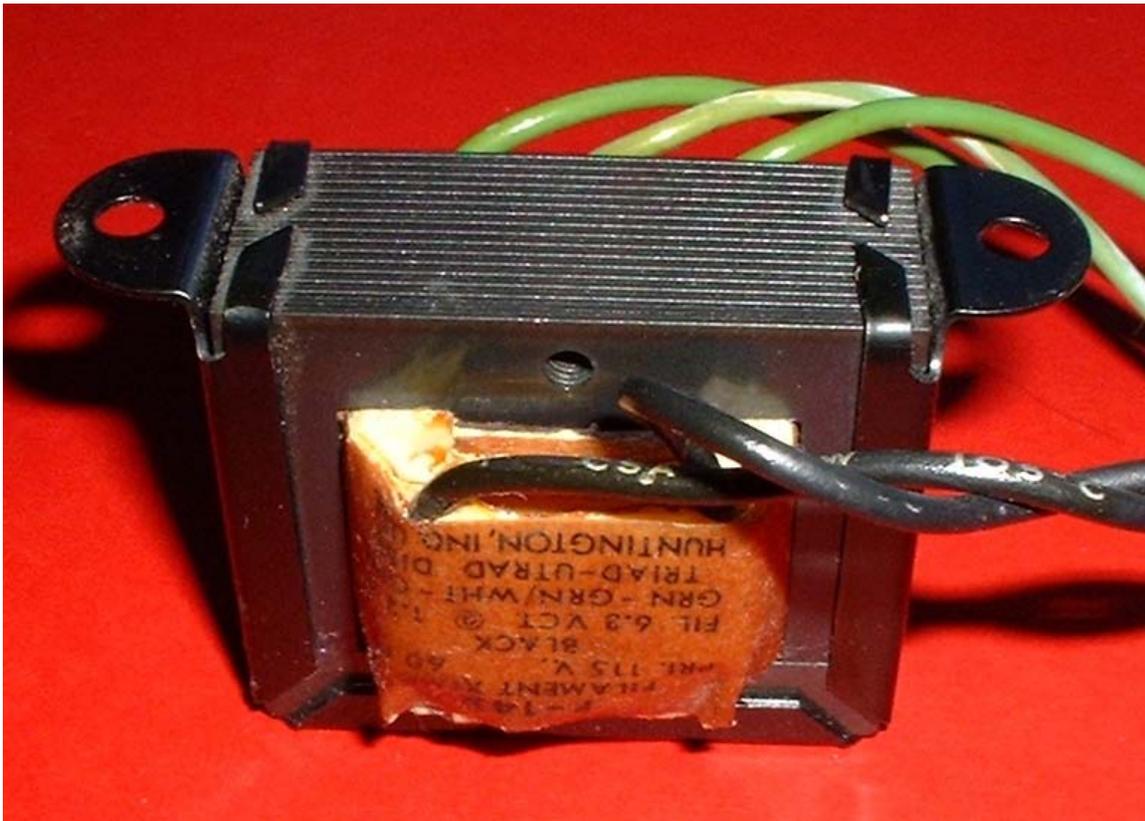
Transformers can be classified in many different ways; an incomplete list is:

- *By power capacity:* from a fraction of a volt-ampere (VA) to over a thousand MVA;
- *By frequency range:* power-, audio-, or radio frequency;
- *By voltage class:* from a few volts to hundreds of kilovolts;

- *By cooling type:* air-cooled, oil-filled, fan-cooled, or water-cooled;
- *By application:* such as power supply, impedance matching, output voltage and current stabilizer, or circuit isolation;
- *By purpose:* distribution, rectifier, arc furnace, amplifier output, etc.;
- *By winding turns ratio:* step-up, step-down, isolating with equal or near-equal ratio, variable, multiple windings.

Construction

Cores



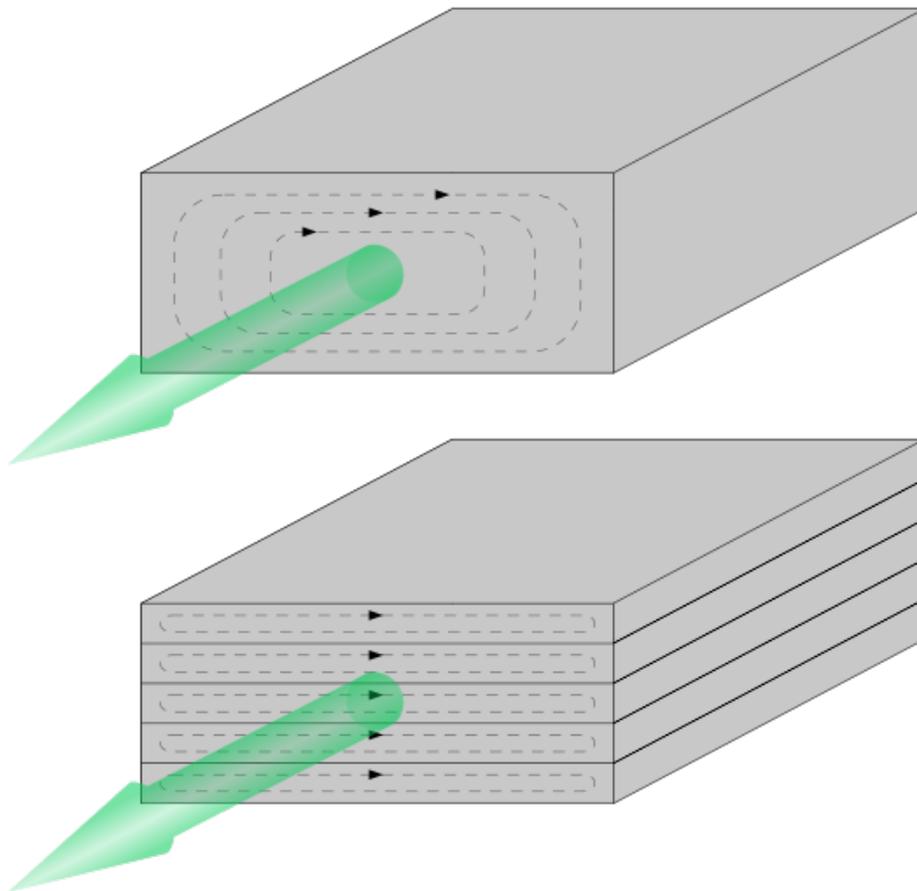
Laminated core transformer showing edge of laminations at top of photo

Laminated steel cores

Transformers for use at power or audio frequencies typically have cores made of high permeability silicon steel. The steel has a permeability many times that of free space, and the core thus serves to greatly reduce the magnetizing current, and confine the flux to a path which closely couples the windings. Early transformer developers soon realized that cores constructed from solid iron resulted in prohibitive eddy-current losses, and their designs mitigated this effect with cores consisting of bundles of insulated iron wires. Later designs constructed the core by stacking layers of thin steel laminations, a principle that has remained in use. Each lamination is insulated from its neighbors by a thin non-

conducting layer of insulation. The universal transformer equation indicates a minimum cross-sectional area for the core to avoid saturation.

The effect of laminations is to confine eddy currents to highly elliptical paths that enclose little flux, and so reduce their magnitude. Thinner laminations reduce losses, but are more laborious and expensive to construct. Thin laminations are generally used on high frequency transformers, with some types of very thin steel laminations able to operate up to 10 kHz.



Laminating the core greatly reduces eddy-current losses

One common design of laminated core is made from interleaved stacks of E-shaped steel sheets capped with I-shaped pieces, leading to its name of "E-I transformer". Such a design tends to exhibit more losses, but is very economical to manufacture. The cut-core or C-core type is made by winding a steel strip around a rectangular form and then bonding the layers together. It is then cut in two, forming two C shapes, and the core assembled by binding the two C halves together with a steel strap. They have the advantage that the flux is always oriented parallel to the metal grains, reducing reluctance.

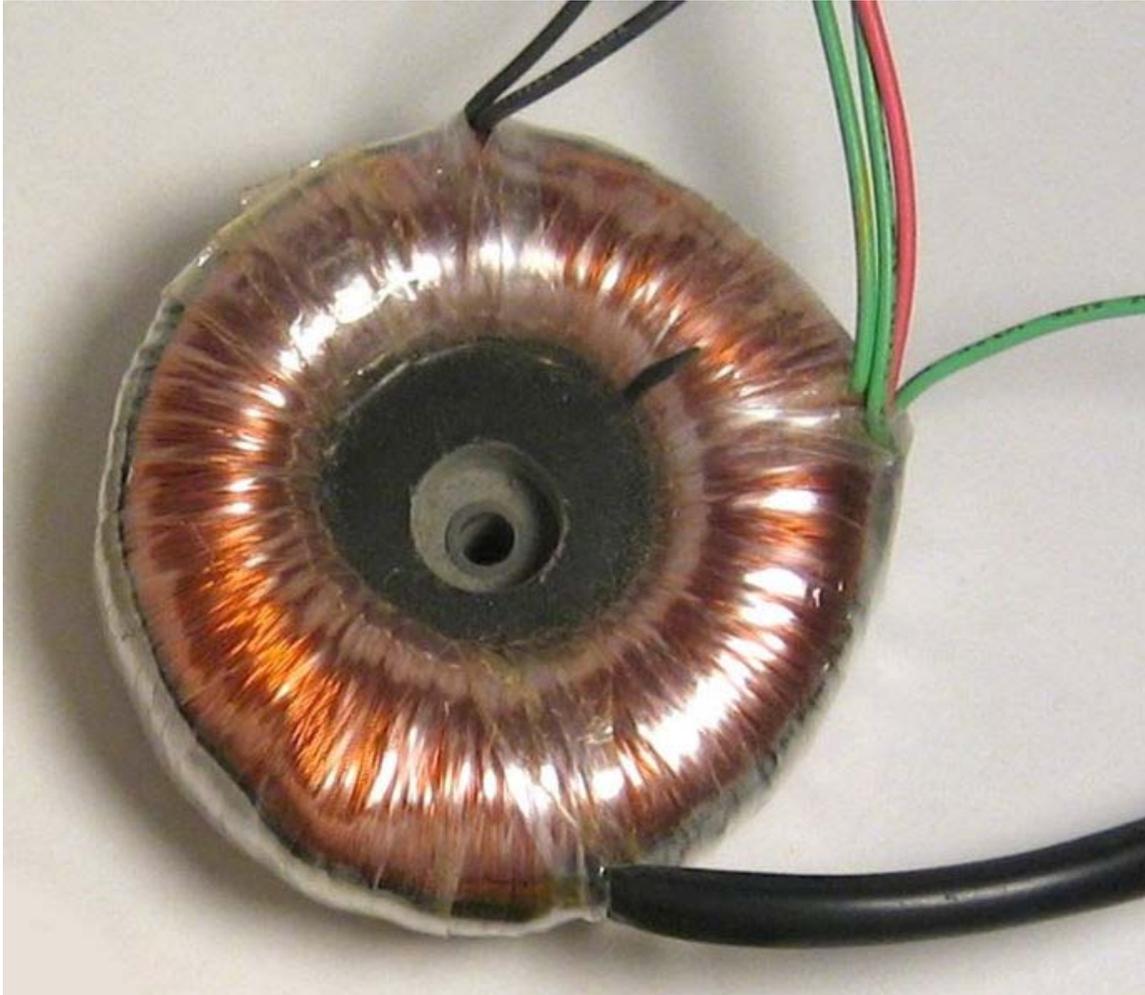
A steel core's remanence means that it retains a static magnetic field when power is removed. When power is then reapplied, the residual field will cause a high inrush current until the effect of the remaining magnetism is reduced, usually after a few cycles of the applied alternating current. Overcurrent protection devices such as fuses must be selected to allow this harmless inrush to pass. On transformers connected to long, overhead power transmission lines, induced currents due to geomagnetic disturbances during solar storms can cause saturation of the core and operation of transformer protection devices.

Distribution transformers can achieve low no-load losses by using cores made with low-loss high-permeability silicon steel or amorphous (non-crystalline) metal alloy. The higher initial cost of the core material is offset over the life of the transformer by its lower losses at light load.

Solid cores

Powdered iron cores are used in circuits (such as switch-mode power supplies) that operate above main frequencies and up to a few tens of kilohertz. These materials combine high magnetic permeability with high bulk electrical resistivity. For frequencies extending beyond the VHF band, cores made from non-conductive magnetic ceramic materials called ferrites are common. Some radio-frequency transformers also have movable cores (sometimes called 'slugs') which allow adjustment of the coupling coefficient (and bandwidth) of tuned radio-frequency circuits.

Toroidal cores



Small toroidal core transformer

Toroidal transformers are built around a ring-shaped core, which, depending on operating frequency, is made from a long strip of silicon steel or permalloy wound into a coil, powdered iron, or ferrite. A strip construction ensures that the grain boundaries are optimally aligned, improving the transformer's efficiency by reducing the core's reluctance. The closed ring shape eliminates air gaps inherent in the construction of an E-I core. The cross-section of the ring is usually square or rectangular, but more expensive cores with circular cross-sections are also available. The primary and secondary coils are often wound concentrically to cover the entire surface of the core. This minimizes the length of wire needed, and also provides screening to minimize the core's magnetic field from generating electromagnetic interference.

Toroidal transformers are more efficient than the cheaper laminated E-I types for a similar power level. Other advantages compared to E-I types, include smaller size (about

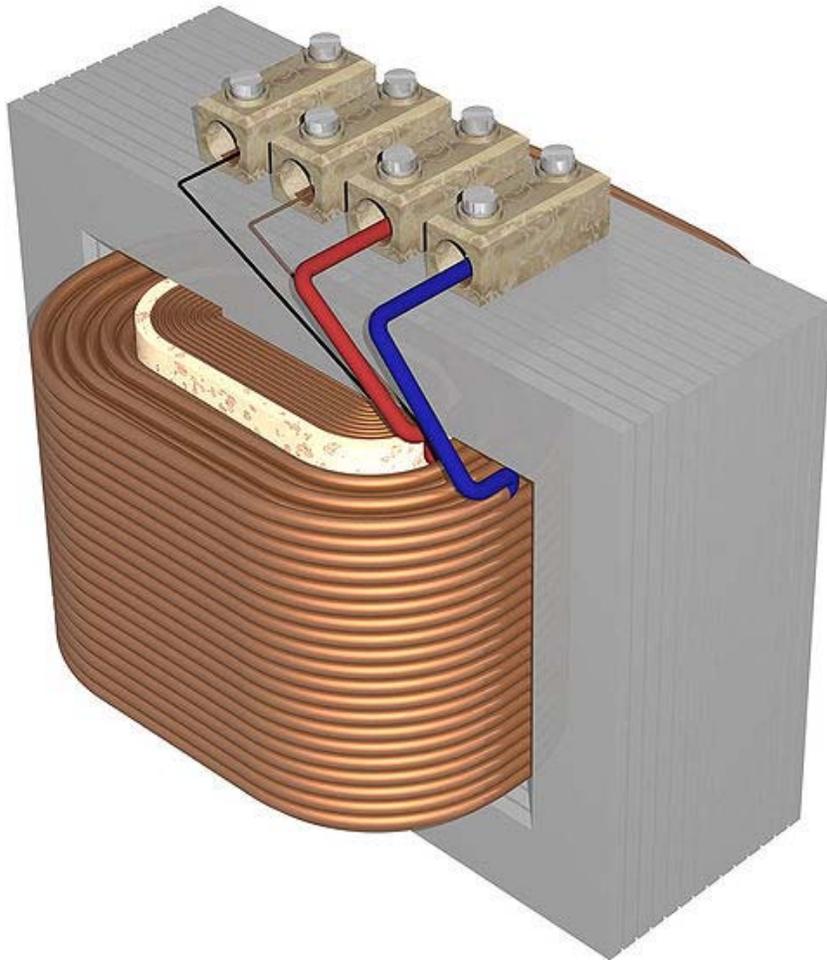
half), lower weight (about half), less mechanical hum (making them superior in audio amplifiers), lower exterior magnetic field (about one tenth), low off-load losses (making them more efficient in standby circuits), single-bolt mounting, and greater choice of shapes. The main disadvantages are higher cost and limited power capacity. Because of the lack of a residual gap in the magnetic path, toroidal transformers also tend to exhibit higher inrush current, compared to laminated E-I types.

Ferrite toroidal cores are used at higher frequencies, typically between a few tens of kilohertz to hundreds of megahertz, to reduce losses, physical size, and weight of switch-mode power supplies. A drawback of toroidal transformer construction is the higher labor cost of winding. This is because it is necessary to pass the entire length of a coil winding through the core aperture each time a single turn is added to the coil. As a consequence, toroidal transformers are uncommon above ratings of a few kVA. Small distribution transformers may achieve some of the benefits of a toroidal core by splitting it and forcing it open, then inserting a bobbin containing primary and secondary windings.

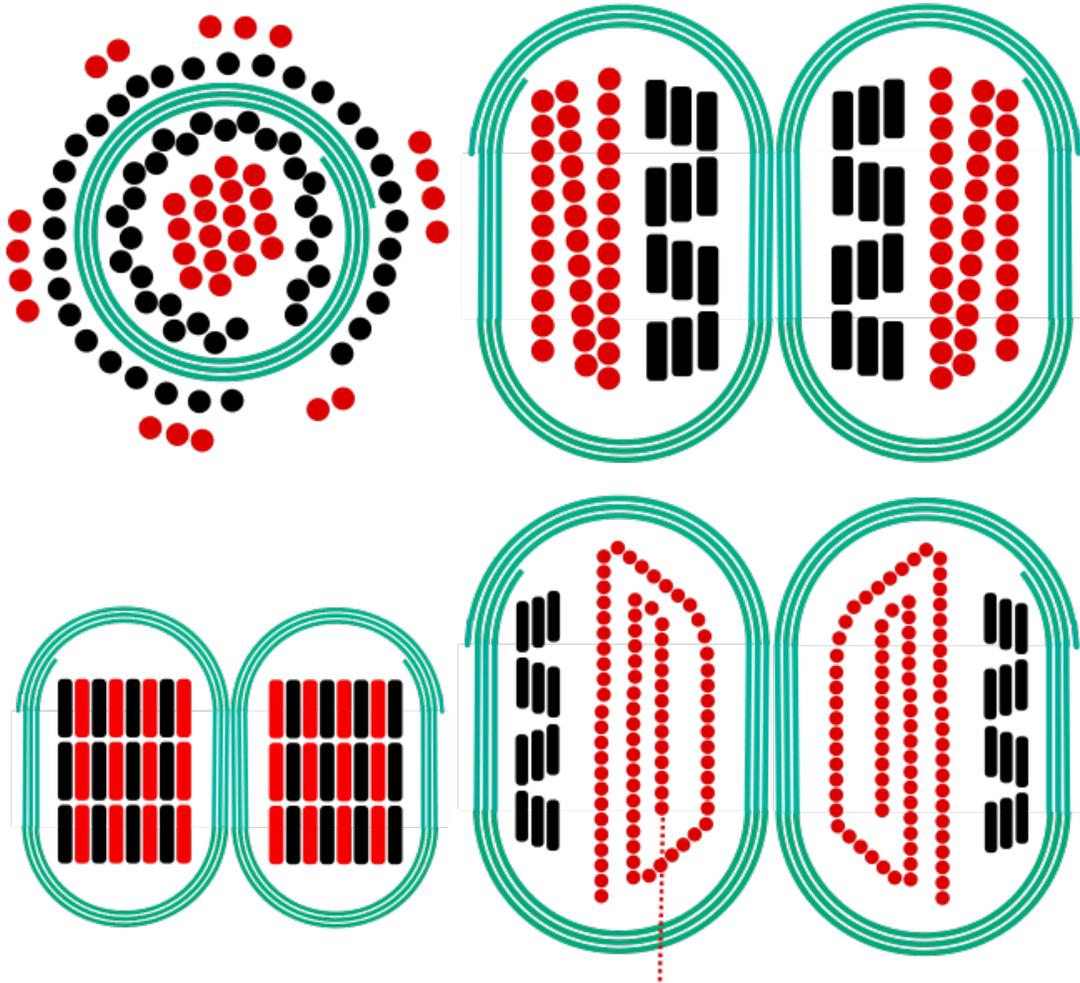
Air cores

A physical core is not an absolute requisite and a functioning transformer can be produced simply by placing the windings near each other, an arrangement termed an "air-core" transformer. The air which comprises the magnetic circuit is essentially lossless, and so an air-core transformer eliminates loss due to hysteresis in the core material. The leakage inductance is inevitably high, resulting in very poor regulation, and so such designs are unsuitable for use in power distribution. They have however very high bandwidth, and are frequently employed in radio-frequency applications, for which a satisfactory coupling coefficient is maintained by carefully overlapping the primary and secondary windings. They're also used for resonant transformers such as Tesla coils where they can achieve reasonably low loss in spite of the high leakage inductance.

Windings



Windings are usually arranged concentrically to minimize flux leakage.



Cut view through transformer windings. White: insulator. Green spiral: Grain oriented silicon steel. Black: Primary winding made of oxygen-free copper. Red: Secondary winding. Top left: Toroidal transformer. Right: C-core, but E-core would be similar. The black windings are made of film. Top: Equally low capacitance between all ends of both windings. Since most cores are at least moderately conductive they also need insulation. Bottom: Lowest capacitance for one end of the secondary winding needed for low-power high-voltage transformers. Bottom left: Reduction of leakage inductance would lead to increase of capacitance.

The conducting material used for the windings depends upon the application, but in all cases the individual turns must be electrically insulated from each other to ensure that the current travels throughout every turn. For small power and signal transformers, in which currents are low and the potential difference between adjacent turns is small, the coils are often wound from enamelled magnet wire, such as Formvar wire. Larger power transformers operating at high voltages may be wound with copper rectangular strip conductors insulated by oil-impregnated paper and blocks of pressboard.

High-frequency transformers operating in the tens to hundreds of kilohertz often have windings made of braided Litz wire to minimize the skin-effect and proximity effect losses. Large power transformers use multiple-stranded conductors as well, since even at low power frequencies non-uniform distribution of current would otherwise exist in high-current windings. Each strand is individually insulated, and the strands are arranged so that at certain points in the winding, or throughout the whole winding, each portion occupies different relative positions in the complete conductor. The transposition equalizes the current flowing in each strand of the conductor, and reduces eddy current losses in the winding itself. The stranded conductor is also more flexible than a solid conductor of similar size, aiding manufacture.

For signal transformers, the windings may be arranged in a way to minimize leakage inductance and stray capacitance to improve high-frequency response. This can be done by splitting up each coil into sections, and those sections placed in layers between the sections of the other winding. This is known as a stacked type or interleaved winding.

Both the primary and secondary windings on power transformers may have external connections, called taps, to intermediate points on the winding to allow selection of the voltage ratio. In distribution transformers the taps may be connected to an automatic on-load tap changer for voltage regulation of distribution circuits. Audio-frequency transformers, used for the distribution of audio to public address loudspeakers, have taps to allow adjustment of impedance to each speaker. A center-tapped transformer is often used in the output stage of an audio power amplifier in a push-pull circuit. Modulation transformers in AM transmitters are very similar.

Certain transformers have the windings protected by epoxy resin. By impregnating the transformer with epoxy under a vacuum, one can replace air spaces within the windings with epoxy, thus sealing the windings and helping to prevent the possible formation of corona and absorption of dirt or water. This produces transformers more suited to damp or dirty environments, but at increased manufacturing cost.

Coolant



Cut-away view of three-phase oil-cooled transformer. The oil reservoir is visible at the top. Radiative fins aid the dissipation of heat.

High temperatures will damage the winding insulation. Small transformers do not generate significant heat and are cooled by air circulation and radiation of heat. Power transformers rated up to several hundred kVA can be adequately cooled by natural convective air-cooling, sometimes assisted by fans. In larger transformers, part of the design problem is removal of heat. Some power transformers are immersed in transformer oil that both cools and insulates the windings. The oil is a highly refined mineral oil that remains stable at transformer operating temperature. Indoor liquid-filled transformers are required by building regulations in many jurisdictions to use a non-flammable liquid, or to be located in fire-resistant rooms. Air-cooled dry transformers are

preferred for indoor applications even at capacity ratings where oil-cooled construction would be more economical, because their cost is offset by the reduced building construction cost.

The oil-filled tank often has radiators through which the oil circulates by natural convection; some large transformers employ forced circulation of the oil by electric pumps, aided by external fans or water-cooled heat exchangers. Oil-filled transformers undergo prolonged drying processes to ensure that the transformer is completely free of water vapor before the cooling oil is introduced. This helps prevent electrical breakdown under load. Oil-filled transformers may be equipped with Buchholz relays, which detect gas evolved during internal arcing and rapidly de-energize the transformer to avert catastrophic failure. Oil-filled transformers may fail, rupture, and burn, causing power outages and losses. Installations of oil-filled transformers usually includes fire protection measures such as walls, oil containment, and fire-suppression sprinkler systems.

Polychlorinated biphenyls have properties that once favored their use as a coolant, though concerns over their environmental persistence led to a widespread ban on their use. Today, non-toxic, stable silicone-based oils, or fluorinated hydrocarbons may be used where the expense of a fire-resistant liquid offsets additional building cost for a transformer vault. Before 1977, even transformers that were nominally filled only with mineral oils may also have been contaminated with polychlorinated biphenyls at 10-20 ppm. Since mineral oil and PCB fluid mix, maintenance equipment used for both PCB and oil-filled transformers could carry over small amounts of PCB, contaminating oil-filled transformers.

Some "dry" transformers (containing no liquid) are enclosed in sealed, pressurized tanks and cooled by nitrogen or sulfur hexafluoride gas.

Experimental power transformers in the 2 MVA range have been built with superconducting windings which eliminates the copper losses, but not the core steel loss. These are cooled by liquid nitrogen or helium.

Insulation drying

Construction of oil-filled transformers requires that the insulation covering the windings be thoroughly dried before the oil is introduced. There are several different methods of drying. Common for all is that they are carried out in vacuum environment. The vacuum makes it difficult to transfer energy (heat) to the insulation. For this there are several different methods. The traditional drying is done by circulating hot air over the active part and cycle this with periods of vacuum (hot-air vacuum drying, HAV). More common for larger transformers is to use evaporated solvent which condenses on the colder active part. The benefit is that the entire process can be carried out at lower pressure and without influence of added oxygen. This process is commonly called vapour-phase drying (VPD).

For distribution transformers, which are smaller and have a smaller insulation weight, resistance heating can be used. This is a method where current is injected in the windings to heat the insulation. The benefit is that the heating can be controlled very well and it is energy efficient. The method is called low-frequency heating (LFH) since the current is injected at a much lower frequency than the nominal of the grid, which is normally 50 or 60 Hz. A lower frequency reduces the effect of the inductance in the transformer, so the voltage can be reduced.

Terminals

Very small transformers will have wire leads connected directly to the ends of the coils, and brought out to the base of the unit for circuit connections. Larger transformers may have heavy bolted terminals, bus bars or high-voltage insulated bushings made of polymers or porcelain. A large bushing can be a complex structure since it must provide careful control of the electric field gradient without letting the transformer leak oil.

Applications



Image of an electrical substation in Melbourne, Australia showing 3 of 5 220kV/66kV transformers, each with a capacity of 185MVA

A major application of transformers is to increase voltage before transmitting electrical energy over long distances through wires. Wires have resistance and so dissipate electrical energy at a rate proportional to the square of the current through the wire. By transforming electrical power to a high-voltage (and therefore low-current) form for transmission and back again afterward, transformers enable economical transmission of power over long distances. Consequently, transformers have shaped the electricity supply industry, permitting generation to be located remotely from points of demand. All but a tiny fraction of the world's electrical power has passed through a series of transformers by the time it reaches the consumer.

Transformers are also used extensively in electronic products to step down the supply voltage to a level suitable for the low voltage circuits they contain. The transformer also electrically isolates the end user from contact with the supply voltage.

Signal and audio transformers are used to couple stages of amplifiers and to match devices such as microphones and record players to the input of amplifiers. Audio transformers allowed telephone circuits to carry on a two-way conversation over a single pair of wires. A balun transformer converts a signal that is referenced to ground to a signal that has balanced voltages to ground, such as between external cables and internal circuits.

The principle of open-circuit (unloaded) transformer is widely used for characterisation of soft magnetic materials, for example in the internationally standardised Epstein frame method.

Chapter-3

Sparse Matrix Converter and Braking Chopper

Sparse matrix converter

The **Sparse Matrix Converter** is an AC/AC converter which offers a reduced number of components, a low-complexity modulation scheme, and low realization effort . Invented in 2001 by Prof Johann W. Kolar , sparse matrix converters avoid the multi step commutation procedure of the conventional matrix converter, improving system reliability in industrial operations. Its principal application is in highly compact integrated AC drives.

Characteristics

- Quasi-Direct AC-AC conversion with no DC link energy storage elements
- Sinusoidal input current in phase with mains voltage
- Zero DC link current commutation scheme resulting in lower modulation complexity and very high reliability
- Low complexity of power circuit / power modules available
- Ultra-Sparse Matrix Converter, does show very low realization effort, in case unidirectional power flow can be accepted (admissible displacement of 30° the input current fundamental and input voltage, as well as for the output voltage fundamental and output current), accordingly, a possible application area would be variable speed PSM drives of low dynamics.

Topologies

Sparse Matrix Converter

Characteristics of the Sparse Matrix Converter topology are 15 Transistors, 18 Diodes, and 7 Isolated Driver Potentials. Compared to the Direct Matrix Converter this topology provides identical functionality, but with a reduced number of power switches and the option of employing an improved zero DC-link current commutation scheme, which provides lower control complexity and higher safety and reliability.

Very Sparse Matrix Converter

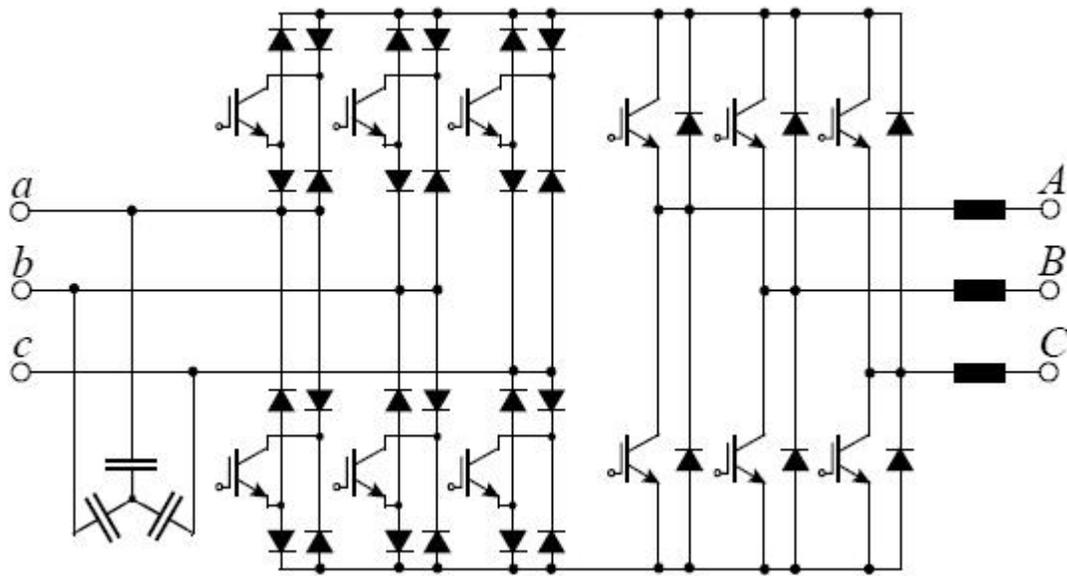


Fig 2: Topology of the very-sparse matrix.

Characteristics of the Very Sparse Matrix Converter topology are 12 Transistors, 30 Diodes, and 10 Isolated Driver Potentials. There are no limitations in functionality compared to the Direct Matrix Converter and Sparse Matrix Converter. Compared to the Sparse Matrix Converter there are fewer transistors but higher conduction losses due to the increased number of diodes in the conduction paths.

Ultra Sparse Matrix Converter

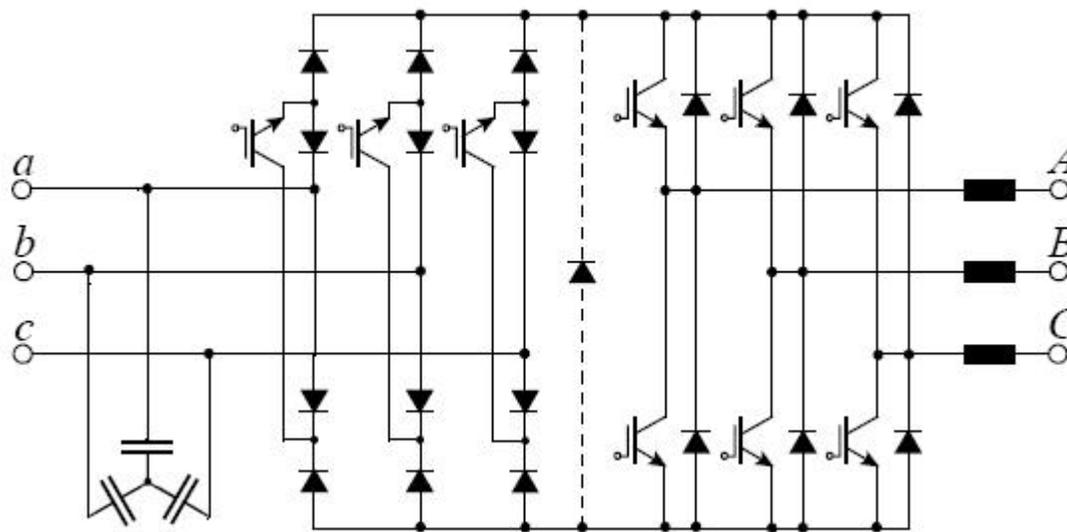


Fig 3: Topology of the ultra-sparse matrix.

Characteristics of the Ultra Sparse Matrix Converter topology are 9 Transistors, 18 Diodes, and 7 Isolated Driver Potentials. The significant limitation of this converter topology compared to the Sparse Matrix Converter is the restriction of its maximal phase displacement between input voltage and input current which is restricted to $\pm 30^\circ$.

Multi-Step Commutation

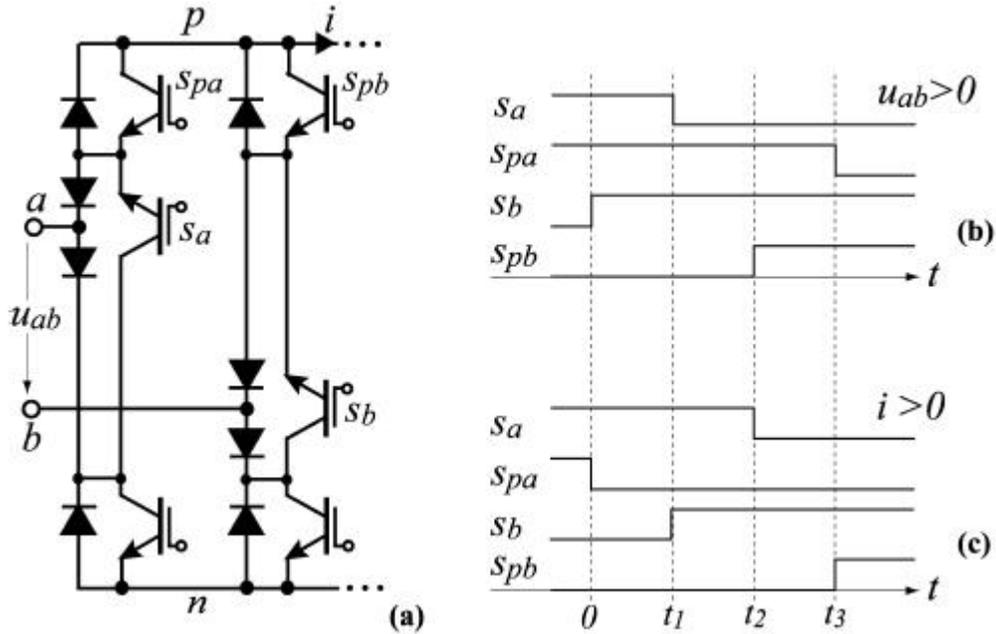


Fig 4: Multistep commutation of the Sparse Matrix Converter rectifier input stage.

This is a commutation scheme, depicted in Fig. 4. For a given switching state of the rectifier input stage, the commutation of the inverter output stage has to be performed in an identical manner to the commutation of a conventional voltage dc-link converter. The basic structure of the commutating bridge legs of the Sparse Matrix Converter is shown in Fig. 4(a). The switch sequence to change the connection of the positive dc-link voltage bus p from input a to input b is shown in Fig. 4(b) and Fig. 4(c). In Fig. 4(b) the assumption is current-independent commutation with $u_{ab} > 0$. In Fig. 4(c) the assumption is voltage-independent commutation with $i > 0$.

A dead time between the turn-off and turn-on of the power transistors of a bridge leg has to be implemented in order to avoid a short circuit of the dc-link voltage. To change the switching state of the Sparse Matrix Converter rectifier input stage for a given inverter switching state, one has to make sure that there is no bidirectional connection between any two input lines. This guarantees that no short-circuiting of an input line-to-line voltage can occur. Additionally a current path must be continuously provided. Therefore multistep commutation schemes, using voltage independent and current independent commutation as known for the Conventional Direct Matrix Converter, can be employed.

Zero DC Link Current Commutation

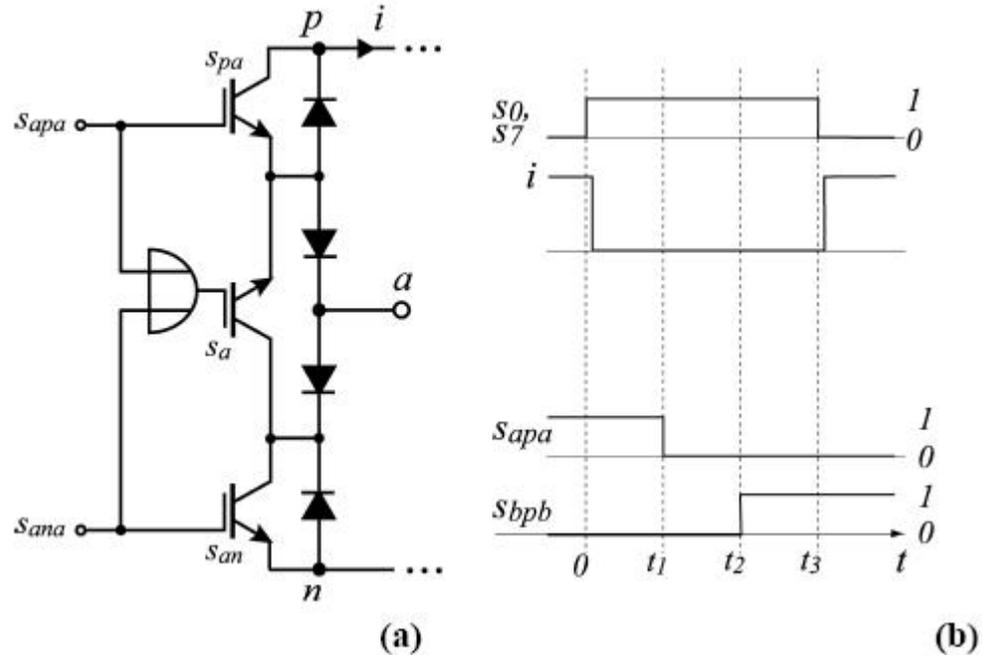


Fig 5: Zero DC link current commutation shown for the Sparse Matrix Converter.

The drawback of the multistep commutation describe before is its complexity. Indirect matrix converters like the Sparse Matrix Converter provide a degree of control freedom that is not available for the Conventional Direct Matrix Converter. This can be used to simplify the complex commutation problem. It has been proposed to switch the inverter stage into a free-wheeling state, and then to commutate the rectifier stage with zero dc-link current. This is shown in Fig. 5.

Fig. 5(a) shows the control of the power transistors in one bridge leg of the Sparse Matrix Converter. Fig. 5(b) shows the switching state sequence where $s_0; s_7 = 1$ indicates free-wheeling operation of the inverter stage. Furthermore, the dc-link current i is shown.

The zero DC link current commutation scheme gives the additional benefit of a reduction in the switching losses of the input stage. One only has to ensure that no overlapping of turn-on intervals of power transistors in a bridge half occurs, because this would result in a short circuit of an input line-to-line voltage.

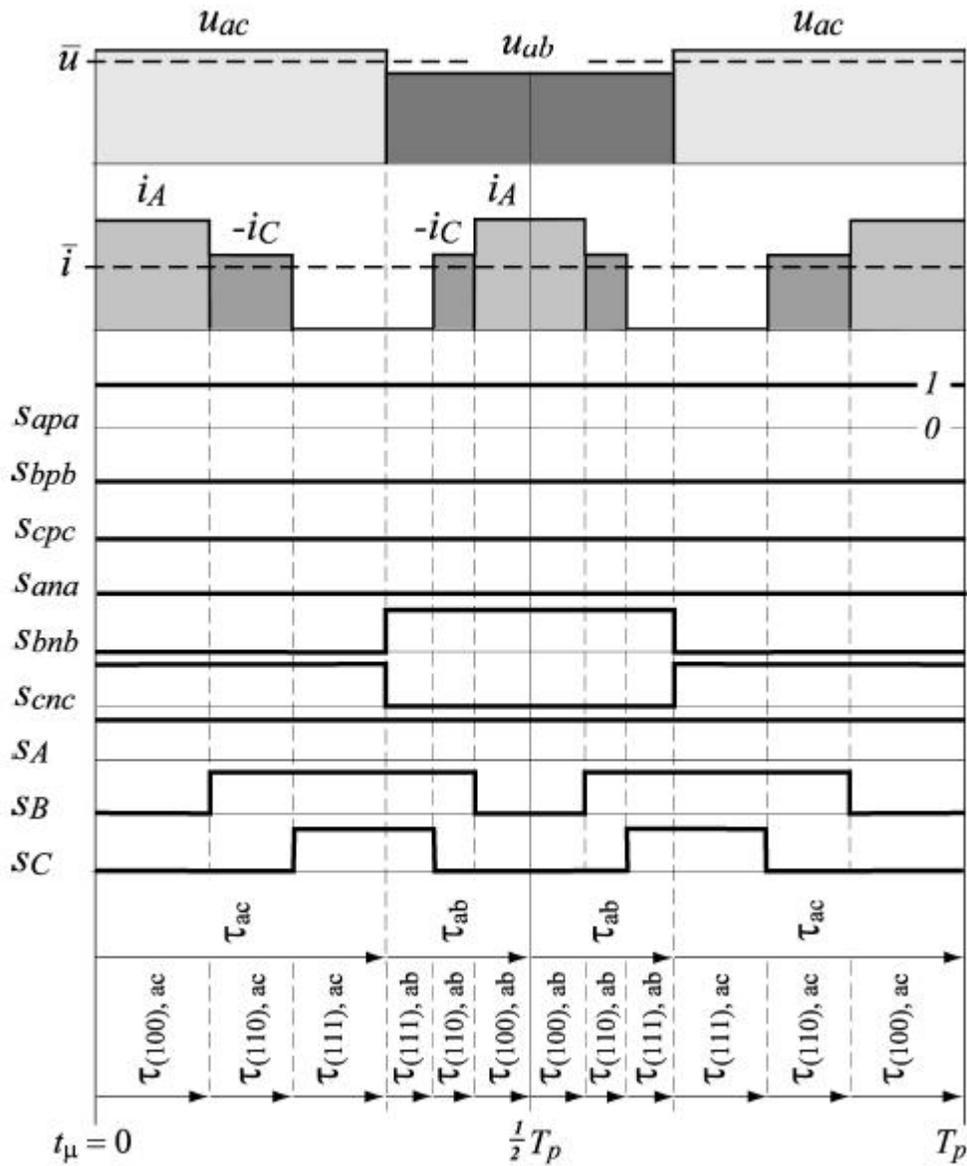


Fig 6: Characteristic voltages and currents and switching states of the Sparse Matrix Converter during on-switching period.

Fig 6 shows the formation of the dc-link voltage \bar{u} and dc-link current \bar{i} within one switching period. Furthermore, it shows as an example the switching functions of the rectifier and inverter stage for φ_1 in interval $(0 \dots \pi / 6)$ and φ_2 in interval $(0 \dots \pi / 6)$. Input stage switching occurs at zero dc-link current. The dc-link current has a constant average value \bar{i} within τ_{ac} and τ_{ab} . The switching state functions are given as s_A, s_B , and s_C . The switching frequency ripple of u_{ac}, u_{ab}, i_A and i_C is neglected.

Braking chopper



A sample of a Braking chopper

Brake choppers are used in the DC voltage intermediate circuits of frequency converters to control voltage when the load feeds energy back to the intermediate circuit. This arises, for example, when a magnetized motor is being roatated by an overhauling load and so functions as a generator feeding power to the DC voltage intermediate circuit.

The concept of flux braking

Flux braking is a method based on motor losses. When braking in the drive system is needed, the motor flux and thus also the magnetizing current component used in the motor are increased. The control of flux can be easily achieved through the direct torque control principle. With DTC the inverter is directly controlled to achieve the desired torque and flux for the motor. During flux braking the motor is under DTC control which guarantees that braking can be made according to the specified speed ramp. This is very different to the DC injection braking typically used in drives. In the DC injection method DC current is injected to the motor so that control of the motor flux is lost during braking. The flux braking method based on DTC enables the motor to shift quickly from braking to motoring power when requested.

In flux braking the increased current means increased losses inside the motor. The braking power is therefore also increased although the braking power delivered to the frequency converter is not increased. The increased current generates increased losses in motor resistances. The higher the resistance value the higher the braking energy dissipation inside the motor. Typically, in low power motors (below 5 kW) the resistance value of the motor is relatively large in respect to the nominal current of the motor. The higher the power or the voltage of the motor the less the resistance value of the motor in respect to motor current. In other words, flux braking is most effective in a low power motor.

Functioning of braking chopper



A large Braking chopper being put to use

The other possibility to limit DC bus voltage is to lead the braking energy to a resistor through a braking chopper. The braking chopper is an electrical switch that connects DC bus voltage to a resistor where the braking energy is converted to heat. The braking choppers are automatically activated when the actual DC bus voltage exceeds a specified level depending on the nominal voltage of the Variable-frequency drive

Benefits of the braking chopper and resistor solutions

1. Simple electrical construction and well-known technology.

2. Low fundamental investment for chopper and resistor.
3. The chopper works even if AC supply is lost. Braking during main power loss may be required. E.g. in elevator or other safety related applications.

Drawbacks of the Braking chopper and resistor

1. The braking energy is wasted if the heated air can not be utilised.
2. The braking chopper and resistors require additional space.
3. May require extra investments in the cooling and heat recovery system.
4. Braking choppers are typically dimensioned for a certain cycle, e.g. 100 % power 1/10 minutes, long braking times require more accurate dimensioning of the braking chopper.
5. Increased risk of fire due to hot resistor and possible dust and chemical components in the ambient air space.
6. The increased DC bus voltage level during braking causes additional voltage stress on motor insulation.

When to apply a braking chopper

1. The braking cycle is needed occasionally.
2. The amount of braking energy with respect to motoring energy is extremely small.
3. Braking operation is needed during main power loss.

When not to use Braking Chopper Resistor

1. The braking is continuous or regularly repeated.
2. The total amount of braking energy is high in respect to the motoring energy needed.
3. The instantaneous braking power is high, e.g. several hundred kW for several minutes.
4. The ambient air includes substantial amounts of dust or other potentially combustible or explosive or metallic components.

Chapter-4

Rectifier

A **rectifier** is an electrical device that converts alternating current (AC), which periodically reverses direction, to direct current (DC), which is in only one direction, a process known as **rectification**. Rectifiers have many uses including as components of power supplies and as detectors of radio signals. Rectifiers may be made of solid state diodes, vacuum tube diodes, mercury arc valves, and other components.

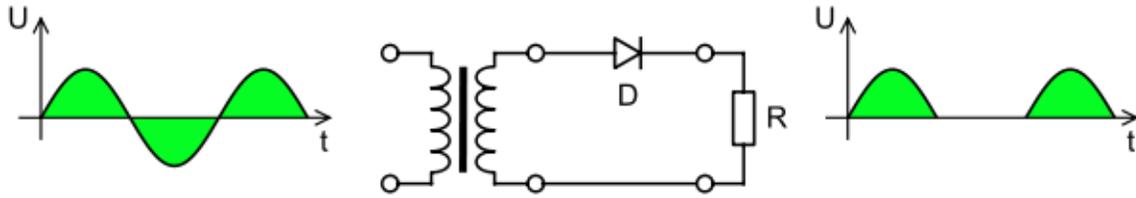
A device which performs the opposite function (converting DC to AC) is known as an inverter.

When only one diode is used to rectify AC (by blocking the negative or positive portion of the waveform), the difference between the term *diode* and the term *rectifier* is merely one of usage, i.e., the term *rectifier* describes a *diode* that is being used to convert AC to DC. Almost all rectifiers comprise a number of diodes in a specific arrangement for more efficiently converting AC to DC than is possible with only one diode. Before the development of silicon semiconductor rectifiers, vacuum tube diodes and copper(I) oxide or selenium rectifier stacks were used.

Early radio receivers, called crystal radios, used a "cat's whisker" of fine wire pressing on a crystal of galena (lead sulfide) to serve as a point-contact rectifier or "crystal detector". Rectification may occasionally serve in roles other than to generate direct current per se. For example, in gas heating systems *flame rectification* is used to detect presence of flame. Two metal electrodes in the outer layer of the flame provide a current path, and rectification of an applied alternating voltage will happen in the plasma, but only while the flame is present to generate it.

Half-wave rectification

In half wave rectification, either the positive or negative half of the AC wave is passed, while the other half is blocked. Because only one half of the input waveform reaches the output, it is very inefficient if used for power transfer. Half-wave rectification can be achieved with a single diode in a one-phase supply, or with three diodes in a three-phase supply.



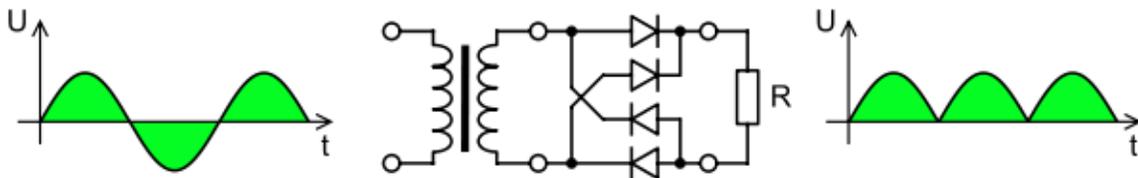
The output DC voltage of a half wave rectifier can be calculated with the following two ideal equations:

$$V_{rms} = \frac{V_{peak}}{2}$$

$$V_{dc} = \frac{V_{peak}}{\pi}$$

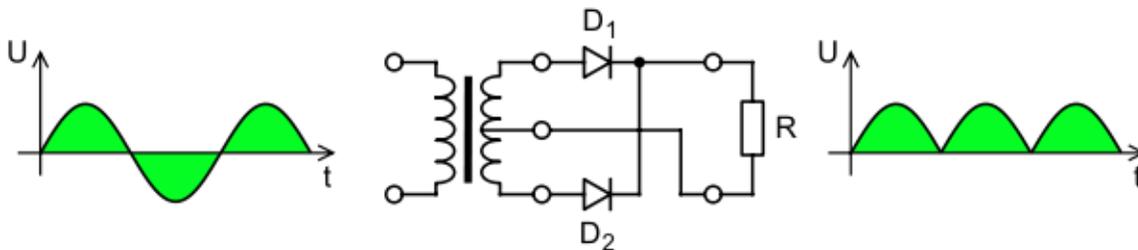
Full-wave rectification

A full-wave rectifier converts the whole of the input waveform to one of constant polarity (positive or negative) at its output. Full-wave rectification converts both polarities of the input waveform to DC (direct current), and is more efficient. However, in a circuit with a non-center tapped transformer, four diodes are required instead of the one needed for half-wave rectification. Four diodes arranged this way are called a diode bridge or bridge rectifier.

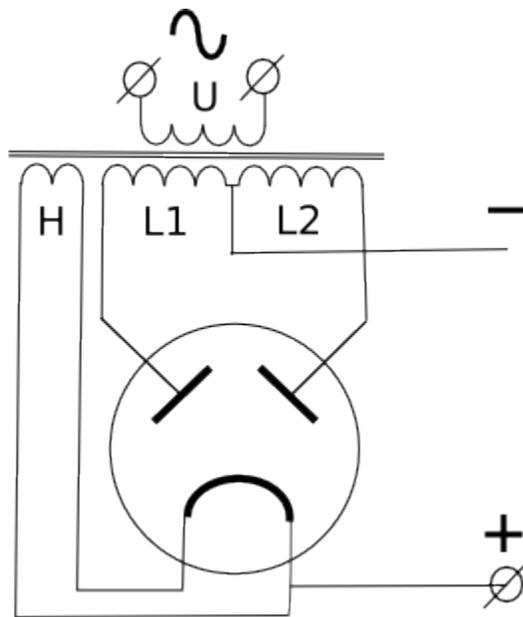


Graetz bridge rectifier: a full-wave rectifier using 4 diodes.

For single-phase AC, if the transformer is center-tapped, then two diodes back-to-back (i.e. anodes-to-anode or cathode-to-cathode) can form a full-wave rectifier. Twice as many windings are required on the transformer secondary to obtain the same output voltage compared to the bridge rectifier above.

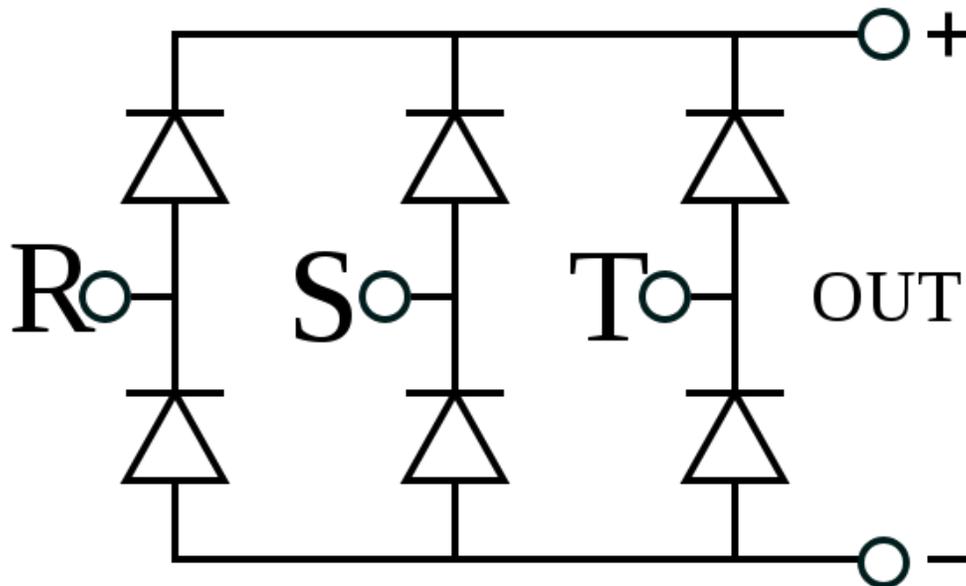


Full-wave rectifier using a center tap transformer and 2 diodes.

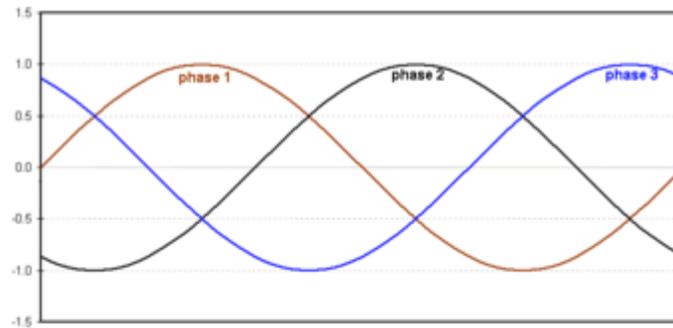


Full-wave rectifier, with vacuum tube having two anodes.

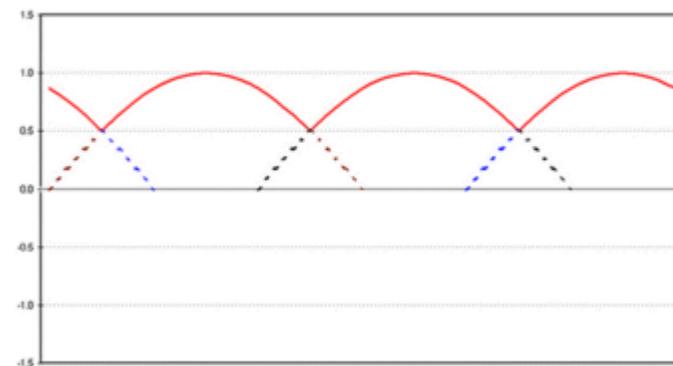
A very common vacuum tube rectifier configuration contained one cathode and twin anodes inside a single envelope; in this way, the two diodes required only one vacuum tube. The 5U4 and 5Y3 were popular examples of this configuration.



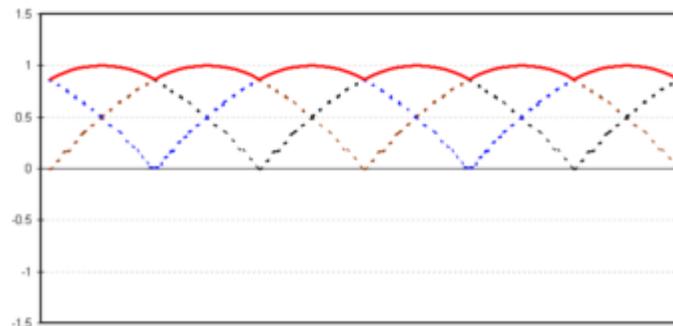
A three-phase bridge rectifier.



3-PHASE AC



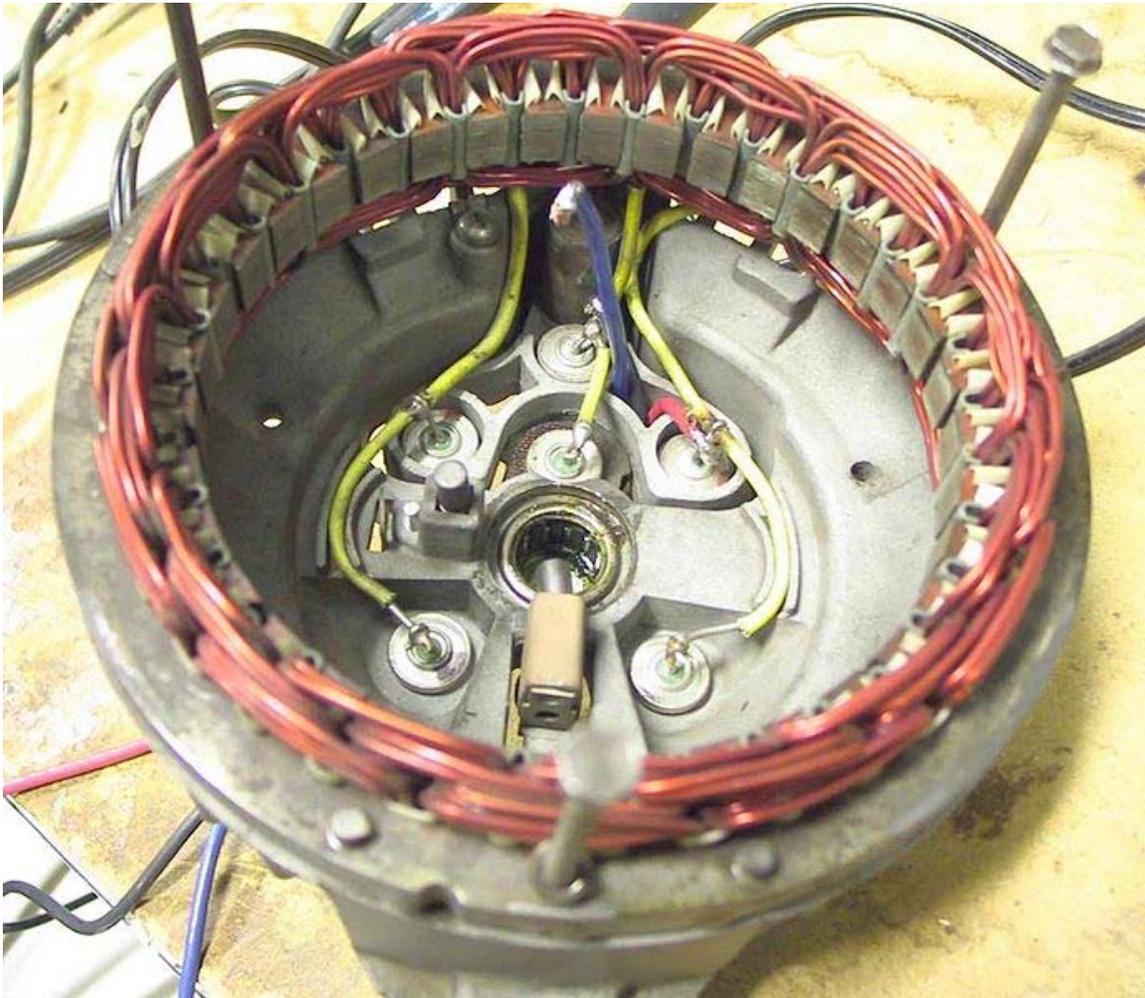
3-PHASE HALF-WAVE RECTIFICATION



3-PHASE FULL WAVE RECTIFICATION

3-phase AC input, half & full wave rectified DC output waveforms

For three-phase AC, six diodes are used. Typically there are three pairs of diodes, each pair, though, is not the same kind of **double diode** that would be used for a full wave single-phase rectifier. Instead the pairs are in series (anode to cathode). Typically, commercially available double diodes have four terminals so the user can configure them as single-phase split supply use, for half a bridge, or for three-phase use.



Disassembled automobile alternator, showing the six diodes that comprise a full-wave three-phase bridge rectifier.

Most devices that generate alternating current (such devices are called alternators) generate three-phase AC. For example, an automobile alternator has six diodes inside it to function as a full-wave rectifier for battery charging applications.

The average and root-mean-square output voltages of an ideal single phase full wave rectifier can be calculated as:

$$V_{dc} = V_{av} = \frac{2V_p}{\pi}$$
$$V_{rms} = \frac{V_p}{\sqrt{2}}$$

Where:

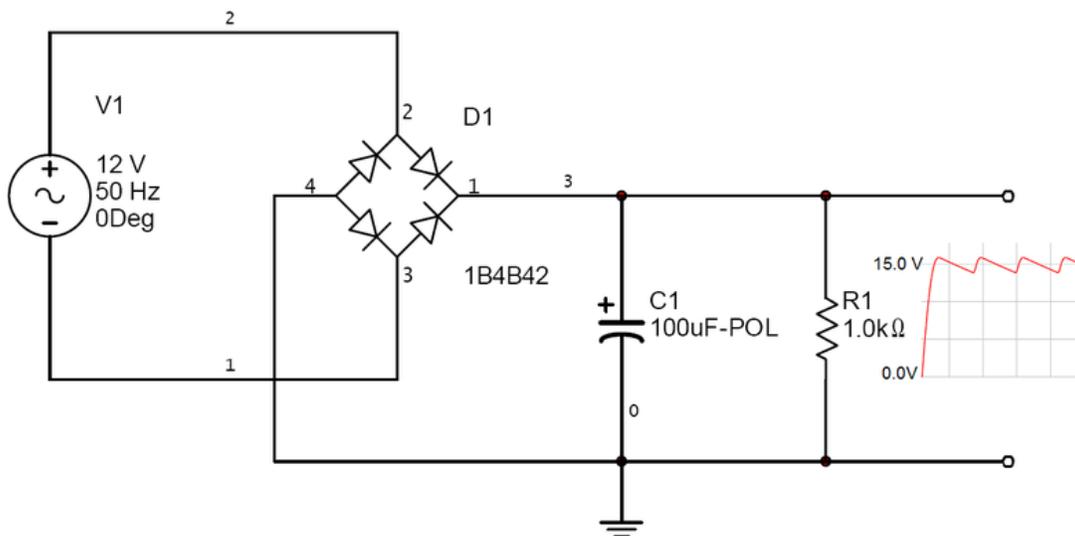
V_{dc}, V_{av} - the average or DC output voltage,
 V_p - the peak value of half wave,
 V_{rms} - the root-mean-square value of output voltage.
 $\pi = \sim 3.14159$

Peak loss

An aspect of most rectification is a loss from the peak input voltage to the peak output voltage, caused by the built-in voltage drop across the diodes (around 0.7 V for ordinary silicon p-n-junction diodes and 0.3 V for Schottky diodes). Half-wave rectification and full-wave rectification using two separate secondaries will have a peak voltage loss of one diode drop. Bridge rectification will have a loss of two diode drops. This may represent significant power loss in very low voltage supplies. In addition, the diodes will not conduct below this voltage, so the circuit is only passing current through for a portion of each half-cycle, causing short segments of zero voltage to appear between each "hump".

Rectifier output smoothing

While half-wave and full-wave rectification suffice to deliver a form of DC output, neither produces constant-voltage DC. In order to produce steady DC from a rectified AC supply, a smoothing circuit or filter is required. In its simplest form this can be just a reservoir capacitor or smoothing capacitor, placed at the DC output of the rectifier. There will still remain an amount of AC ripple voltage where the voltage is not completely smoothed.



RC-Filter Rectifier: This circuit was designed and simulated using Multisim 8 software.

Sizing of the capacitor represents a tradeoff. For a given load, a larger capacitor will reduce ripple but will cost more and will create higher peak currents in the transformer secondary and in the supply feeding it. In extreme cases where many rectifiers are loaded onto a power distribution circuit, it may prove difficult for the power distribution authority to maintain a correctly shaped sinusoidal voltage curve.

For a given tolerable ripple the required capacitor size is proportional to the load current and inversely proportional to the supply frequency and the number of output peaks of the rectifier per input cycle. The load current and the supply frequency are generally outside the control of the designer of the rectifier system but the number of peaks per input cycle can be affected by the choice of rectifier design.

A half-wave rectifier will only give one peak per cycle and for this and other reasons is only used in very small power supplies. A full wave rectifier achieves two peaks per cycle and this is the best that can be done with single-phase input. For three-phase inputs a three-phase bridge will give six peaks per cycle and even higher numbers of peaks can be achieved by using transformer networks placed before the rectifier to convert to a higher phase order.

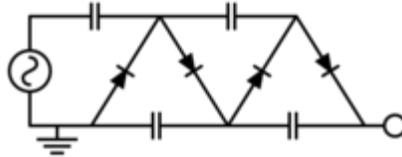
To further reduce this ripple, a capacitor-input filter can be used. This complements the reservoir capacitor with a choke (inductor) and a second filter capacitor, so that a steadier DC output can be obtained across the terminals of the filter capacitor. The choke presents a high impedance to the ripple current.

A more usual alternative to a filter, and essential if the DC load is very demanding of a smooth supply voltage, is to follow the reservoir capacitor with a voltage regulator. The reservoir capacitor needs to be large enough to prevent the troughs of the ripple getting below the voltage the DC is being regulated to. The regulator serves both to remove the last of the ripple and to deal with variations in supply and load characteristics. It would be possible to use a smaller reservoir capacitor (these can be large on high-current power supplies) and then apply some filtering as well as the regulator, but this is not a common strategy. The extreme of this approach is to dispense with the reservoir capacitor altogether and put the rectified waveform straight into a choke-input filter. The advantage of this circuit is that the current waveform is smoother and consequently the rectifier no longer has to deal with the current as a large current pulse, but instead the current delivery is spread over the entire cycle. The downside is that the voltage output is much lower – approximately the average of an AC half-cycle rather than the peak.

Voltage-doubling rectifiers

The simple half wave rectifier can be built in two versions with the diode pointing in opposite directions, one version connects the negative terminal of the output direct to the AC supply and the other connects the positive terminal of the output direct to the AC supply. By combining both of these with separate output smoothing it is possible to get an output voltage of nearly double the peak AC input voltage. This also provides a tap in the middle, which allows use of such a circuit as a split rail supply.

A variant of this is to use two capacitors in series for the output smoothing on a bridge rectifier then place a switch between the midpoint of those capacitors and one of the AC input terminals. With the switch open this circuit will act like a normal bridge rectifier with it closed it will act like a voltage doubling rectifier. In other words this makes it easy to derive a voltage of roughly 320V (+/- around 15%) DC from any mains supply in the world, this can then be fed into a relatively simple switched mode power supply.



Cockcroft Walton Voltage multiplier

Cascaded stages of diodes and capacitors can be added to make a voltage multiplier (Cockcroft-Walton circuit). These circuits can provide a potential several times that of the peak value of the input AC, although limited in current output and regulation. Voltage multipliers are used to provide the high voltage for a CRT in a television receiver, or for powering high-voltage tubes such as image intensifiers or photo multipliers.

Applications

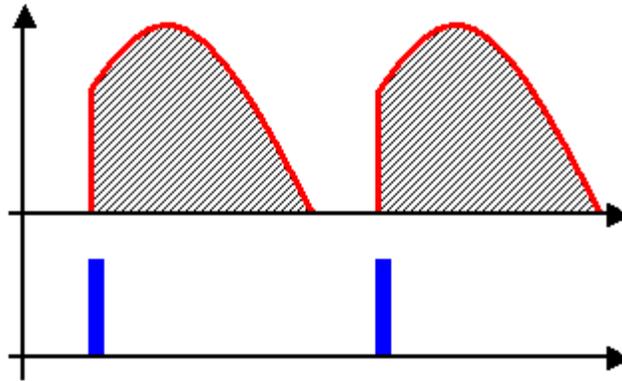


A rectifier diode (silicon controlled rectifier) and associated mounting hardware. The heavy threaded stud helps remove heat.

The primary application of rectifiers is to derive DC power from an AC supply. Virtually all electronic devices require DC, so rectifiers find uses inside the power supplies of virtually all electronic equipment.

Converting DC power from one voltage to another is much more complicated. One method of DC-to-DC conversion first converts power to AC (using a device called an inverter), then use a transformer to change the voltage, and finally rectifies power back to DC.

Rectifiers also find a use in detection of amplitude modulated radio signals. The signal may be amplified before detection, but if un-amplified, a very low voltage drop diode must be used. When using a rectifier for demodulation the capacitor and load resistance must be carefully matched. Too low a capacitance will result in the high frequency carrier passing to the output and too high will result in the capacitor just charging and staying charged.



Output voltage of a full-wave rectifier with controlled thyristors

Rectifiers are also used to supply polarised voltage for welding. In such circuits control of the output current is required and this is sometimes achieved by replacing some of the diodes in bridge rectifier with thyristors, whose voltage output can be regulated by means of phase fired controllers.

Thyristors are used in various classes of railway rolling stock systems so that fine control of the traction motors can be achieved. Gate turn-off thyristors are used to produce alternating current from a DC supply, for example on the Eurostar Trains to power the three-phase traction motors.

Rectification technologies

Electromechanical

Early power conversion systems were purely electro-mechanical in design, since electronic devices were not available to handle significant power. Mechanical rectification systems usually rely on some form of rotation or resonant vibration in order to move quickly enough to match the frequency of the input power source, and cannot operate beyond several thousand cycles per second.

Due to the complexity of mechanical systems, they have traditionally needed a high level of maintenance to keep operating correctly. Moving parts will have friction, which requires lubrication and replacement due to wear. Opening mechanical contacts under load results in electrical arcs and sparks that heat and erode the contacts.

Synchronous rectifier

To convert AC currents into DC current in electric locomotives, a **synchronous rectifier** may be used. It consists of a synchronous motor driving a set of heavy-duty electrical contacts. The motor spins in time with the AC frequency and periodically reverses the connections to the load just when the sinusoidal current goes through a zero-crossing. The contacts do not have to *switch* a large current, but they need to be able to *carry* a large current to supply the locomotive's DC traction motors.

Vibrator

In the past, the vibrators used in battery-to-high-voltage-DC power supplies often contained a second set of contacts that performed synchronous mechanical rectification of the stepped-up voltage.

Motor-generator set

A *motor-generator set*, or the similar *rotary converter*, is not a rectifier in the sense that it doesn't actually *rectify* current, but rather *generates* DC from an AC source. In an "M-G set", the shaft of an AC motor is mechanically coupled to that of a DC generator. The DC generator produces multiphase alternating currents in its armature windings, and a commutator on the armature shaft converts these alternating currents into a direct current output; or a homopolar generator produces a direct current without the need for a commutator. M-G sets are useful for producing DC for railway traction motors, industrial motors and other high-current applications, and were common in many high power D.C. uses (for example, carbon-arc lamp projectors for outdoor theaters) before high-power semiconductors became widely available.

Electrolytic

The electrolytic rectifier was an early device from the 1900s that is no longer used. When two different metals are suspended in an electrolyte solution, it can be found that direct current flowing one way through the metals has less resistance than the other direction. These most commonly used an aluminum anode, and a lead or steel cathode, suspended in a solution of tri-ammonium ortho-phosphate.

The rectification action is due to a thin coating of aluminum hydroxide on the aluminum electrode, formed by first applying a strong current to the cell to build up the coating. The rectification process is temperature sensitive, and for best efficiency should not operate above 86 °F (30 °C). There is also a breakdown voltage where the coating is penetrated and the cell is short-circuited. Electrochemical methods are often more fragile than mechanical methods, and can be sensitive to usage variations which can drastically change or completely disrupt the rectification processes.

Similar electrolytic devices were used as lightning arresters around the same era by suspending many aluminium cones in a tank of tri-ammonium ortho-phosphate solution.

Unlike the rectifier, above, only aluminium electrodes were used, and used on A.C., there was no polarization and thus no rectifier action, but the chemistry was similar.

The modern electrolytic capacitor, an essential component of most rectifier circuit configurations was also developed from the electrolytic rectifier.

Plasma type

Mercury arc

A rectifier used in high-voltage direct current power transmission systems and industrial processing between about 1909 to 1975 is a *mercury arc rectifier* or *mercury arc valve*. The device is enclosed in a bulbous glass vessel or large metal tub. One electrode, the cathode, is submerged in a pool of liquid mercury at the bottom of the vessel and one or more high purity graphite electrodes, called anodes, are suspended above the pool. There may be several auxiliary electrodes to aid in starting and maintaining the arc. When an electric arc is established between the cathode pool and suspended anodes, a stream of electrons flows from the cathode to the anodes through the ionized mercury, but not the other way. [In principle, this is a higher-power counterpart to flame rectification, which uses the same one-way current transmission properties of the plasma naturally present in a flame].

These devices can be used at power levels of hundreds of kilowatts, and may be built to handle one to six phases of AC current. Mercury arc rectifiers have been replaced by silicon semiconductor rectifiers and high power thyristor circuits, from the mid 1970s onward. The most powerful mercury arc rectifiers ever built were installed in the Manitoba Hydro Nelson River Bipole HVDC project, with a combined rating of more than one million kilowatts and 450,000 volts.

Argon gas electron tube

The General Electric Tungar rectifier was an argon gas-filled electron tube device with a tungsten filament cathode and a carbon button anode. It was useful for battery chargers and similar applications from the 1920s until low-cost solid-state rectifiers (the metal rectifiers at first) supplanted it. These were made up to a few hundred volts and a few amperes rating, and in some sizes strongly resembled an incandescent lamp with an additional electrode.

The 0Z4 was a gas-filled rectifier tube commonly used in vacuum tube car radios in the 1940s and 1950s. It was a conventional full wave rectifier tube with two anodes and one cathode, but was unique in that it had no filament (thus the "0" in its type number). The electrodes were shaped such that the reverse breakdown voltage was much higher than the forward breakdown voltage. Once the breakdown voltage was exceeded, the 0Z4 switched to a low-resistance state with a forward voltage drop of about 24 volts.

Vacuum tube (valve)

Since the discovery of the Edison effect or thermionic emission, various vacuum tube devices have been developed to rectify alternating currents. Low-power devices are used as signal detectors, first used in radio by Fleming in 1904. Many vacuum-tube devices also used vacuum rectifiers in their power supplies, for example the All American Five radio receiver. Vacuum rectifiers were made for very high voltages, such as the high voltage power supply for the cathode ray tube of television receivers, and the kenotron used for power supply in X-ray equipment. However, vacuum rectifiers generally had low current capacity owing to the maximum current density that could be obtained by electrodes heated to temperatures compatible with long life. Another limitation of the vacuum tube rectifier was that the heater power supply often required special arrangements to insulate it from the high voltages of the rectifier circuit.

Solid state

Crystal detector

The cat's-whisker detector, using a crystal such as galena, was the earliest type of solid state diode.

Selenium and copper oxide rectifiers

Once common until replaced by more compact and less costly silicon solid-state rectifiers, these units used stacks of metal plates and took advantage of the semiconductor properties of selenium or copper oxide. While selenium rectifiers were lighter in weight and used less power than comparable vacuum tube rectifiers, they had the disadvantage of finite life expectancy, increasing resistance with age, and were only suitable to use at low frequencies. Both selenium and copper oxide rectifiers have somewhat better tolerance of momentary voltage transients than silicon rectifiers.

Typically these rectifiers were made up of stacks of metal plates or washers, held together by a central bolt, with the number of stacks determined by voltage; each cell was rated for about 20 volts. An automotive battery charger rectifier might have only one cell: the high-voltage power supply for a vacuum tube might have dozens of stacked plates. Current density in an air-cooled selenium stack was about 600 mA per square inch of active area (about 90 mA per square centimeter).

Silicon and germanium diodes

In the modern world, silicon diodes are the most widely used rectifiers and have largely replaced earlier germanium diodes.

Recent developments

High-speed rectifiers

Researchers at Idaho National Laboratory (INL) have proposed high-speed rectifiers that would sit at the center of spiral nanoantennas and convert infrared frequency electricity from AC to DC. Infrared frequencies range from 0.3 to 400 terahertz.

Unimolecular rectifiers

A Unimolecular rectifier is a single organic molecule which functions as a rectifier. The technology is still in the experimental stage.

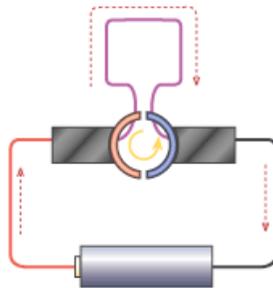
Chapter-5

Commutator

A **commutator** is a rotary electrical switch in certain types of electric motors or electrical generators that periodically reverses the current direction between the rotor and the external circuit. In a motor, it applies power to the best location on the rotor, and in a generator, picks off power similarly. As a switch, it has exceptionally long life, considering the number of circuit makes and breaks that occur in normal operation.

A commutator is a common feature of direct current rotating machines. By reversing the current direction in the moving coil of a motor's armature, a steady rotating force (torque) is produced. Similarly, in a generator, reversing of the coil's connection to the external circuit provides unidirectional—direct—current to the external circuit. The first commutator-type direct current machine was built by Hippolyte Pixii in 1832, based on a suggestion by André-Marie Ampère.

Principle of Operation



As the rotor turns, the current in the winding reverses every time the commutator makes half a turn. This reversal of the winding current compensates for the fact that the winding has also rotated half a turn relative to the fixed magnetic field (not shown). The current in the winding causes the fixed magnetic field to exert a rotational force (a torque) on the winding, making it turn. As the rotor's field comes close to aligning itself with that of the stator, the commutator switches the rotor's polarity, so the motor is perpetually trying to settle, so to speak.

Note that all practical commutators have at least three segments, and in some instances (such as the N.Y. City transit system's old rotary AC-to-DC converters), up to several hundred. In these elementary diagrams, there is a dead position where the motor will not start.

For the image to the right, when the brushes make contact across both commutator segments, the commutator is short-circuited and current passes directly from one brush to the other across the commutator, doing no work in the rotor windings, and drawing a destructive fault current from the power source. As well, practical rotors have more turns in their windings. For the image to the left, there is a dead spot when the brushes cross the insulation between the two segments and no current flows. In either case, in a motor, the rotor cannot begin to spin if it is stopped in this position.

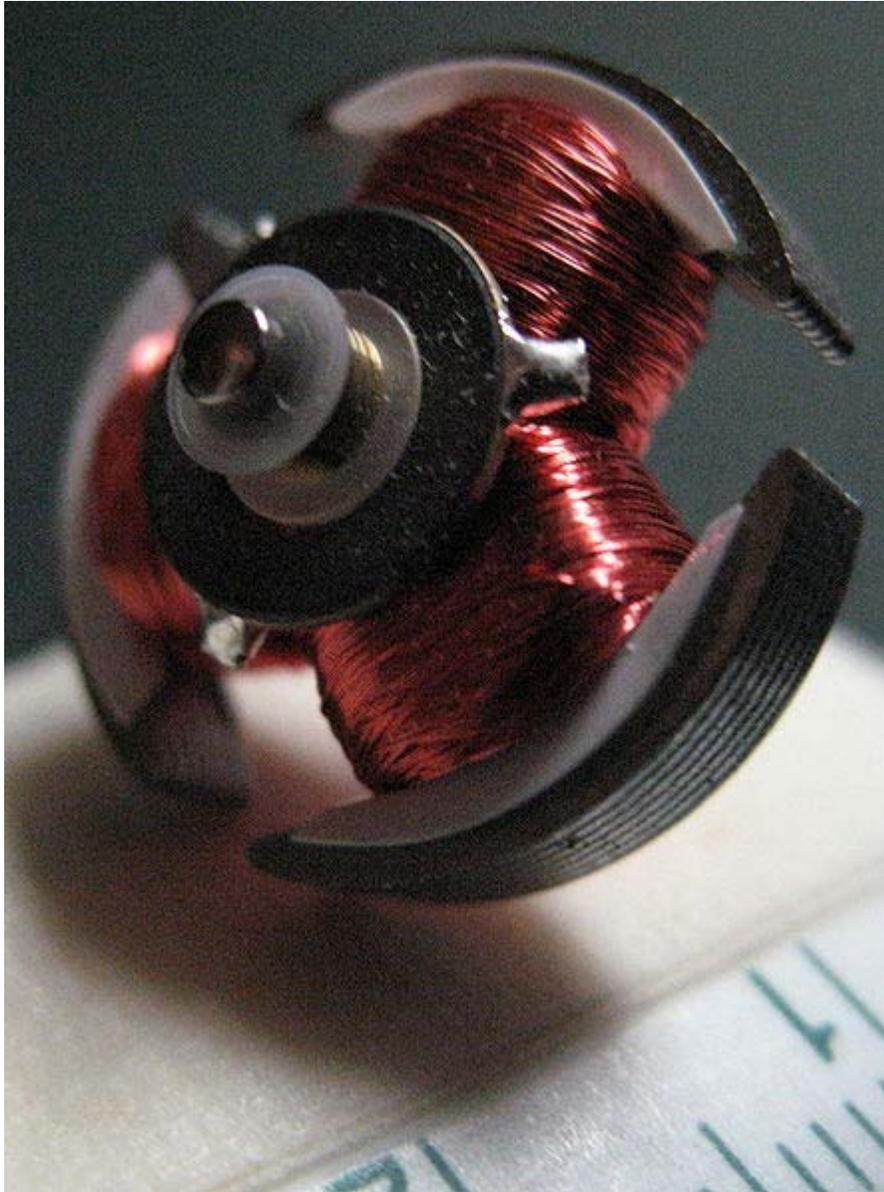
Simplest practical commutator

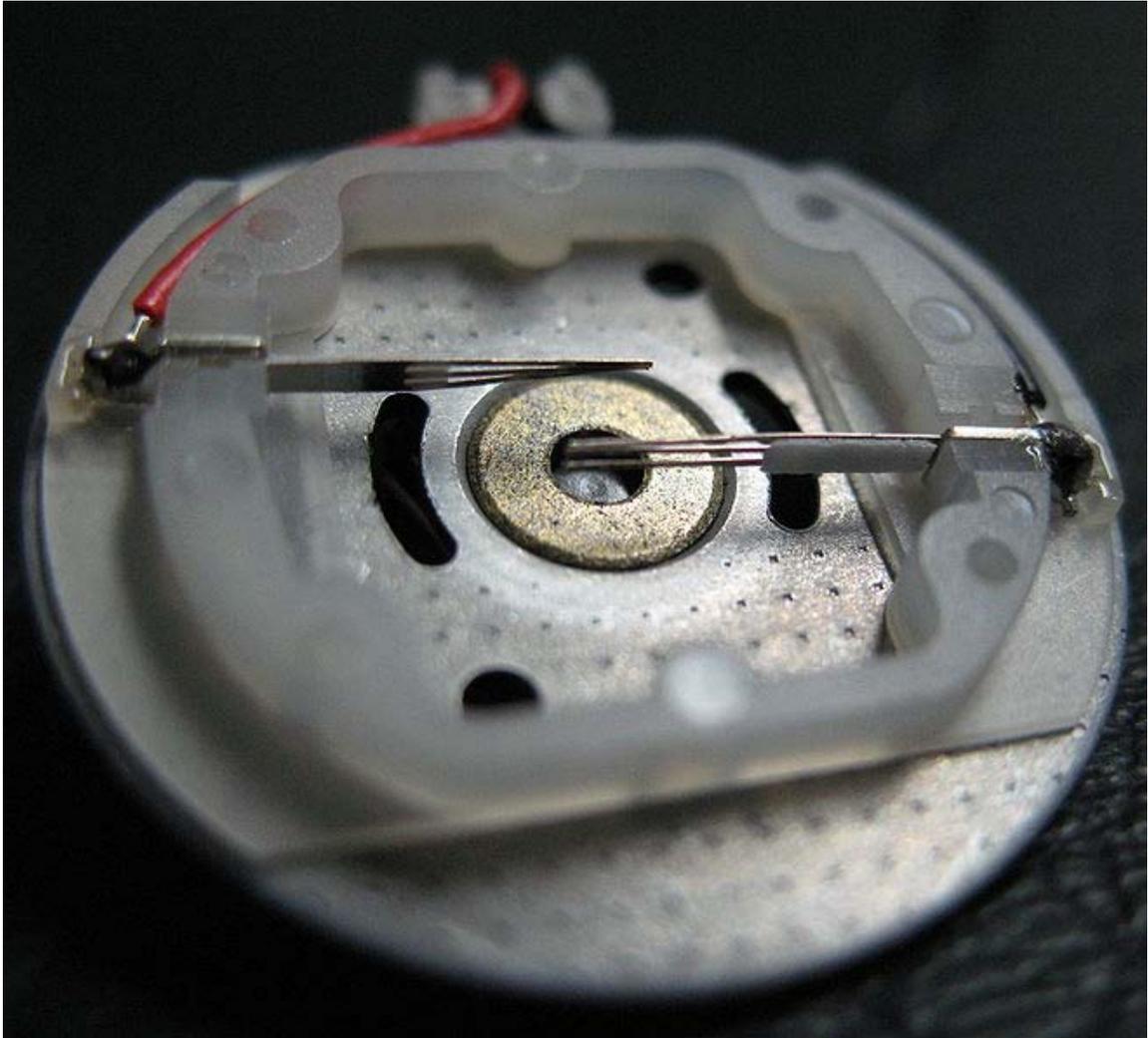
This has three segments, and the rotor has three poles. The left image shows the three rotor poles with their windings. The commutator is near the end of the shaft, as it points up and to the left. It is a metal cylinder (note the yellowish reflection) with three equally spaced cuts parallel to the shaft, and has white plastic discs on both ends. Each segment connects to the nearest junction between two of the three rotor coils.

In the middle illustration, the brushes (in this instance, flat metal springs; carbon brushes are not needed at the low voltages used by such motors as these) are the two straight horizontal pieces; when assembled, the brushes are under tension, slightly away from each other, to stay in contact with the commutator. Power connects to two solder terminals on the outside of the end disc shown in this image. Those terminals are likely to be the same pieces of metal as the brushes themselves.

Inside the exterior metal cylinder is a hollow cylindrical permanent magnet with its south pole opposite its north pole. Interaction between the rotor and that magnet's field is what makes the motor spin. This motor's diameter is greater than its length, something uncommon in motors of this sort. In other sorts of motors, it is typical. Considering that it was used to spin the disc in a CD drive, short length was quite important.

This type of motor is widely used in small toys, models, and electromechanical/electronic devices.



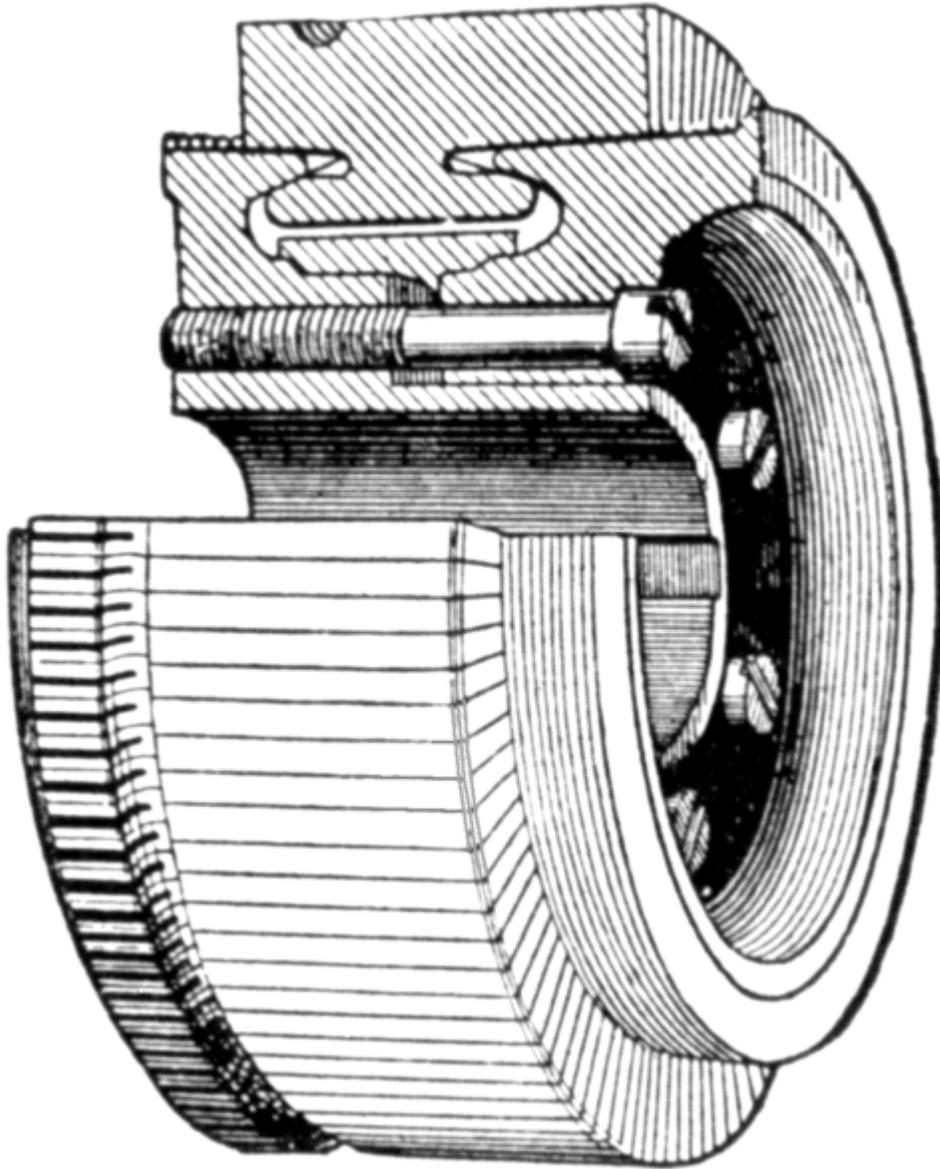




Although the rotor can potentially stop in a position where two commutator segments touch one brush, this only de-energizes one of the three rotor arms while the other two are correctly powered. The motor produces sufficient torque with the two powered rotor arms to begin spinning the rotor, and no direct shorting can occur between the commutator brushes.

Although, so far, this explanation has assumed a permanent-magnet field (or a wound field with the electromagnet fed by DC), so-called universal motors in appliances such as vacuum cleaners have wound fields, and operate well on AC. Power goes to both the field and the brushes, so the magnetic fields of both rotor and stator reverse together. These motors also operate on DC, hence the term "universal".

Ring/Segment Construction



Cross-section of a commutator that can be disassembled for repair.

A commutator typically consists of a set of copper segments, fixed around part of the circumference of the rotating part of the machine (the *rotor*), and a set of spring-loaded brushes fixed to the stationary frame of the machine. The external source of current (for a motor) or electrical load (for a generator) is connected to the brushes. For small equipment the commutator segments can be stamped from sheet metal. For very large equipment the segments are made from a copper casting that is then machined into the final shape.

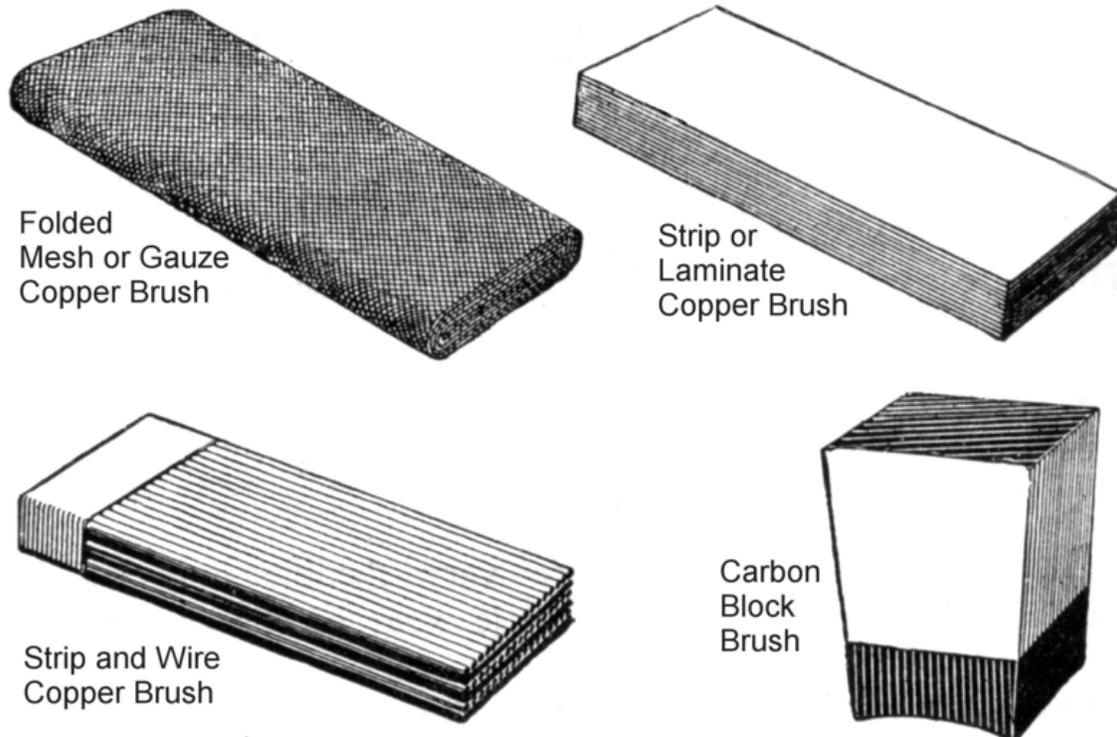
Each conducting segment on the armature of the commutator is insulated from adjacent segments. Initially when the technology was first developed, mica was used as an insulator between commutation segments. Later materials research into polymers brought the development of plastic spacers which are more durable and less prone to cracking, and have a higher and more uniform breakdown voltage than mica.

The segments are held onto the shaft using a dovetail shape on the edges or underside of each segment, using insulating wedges around the perimeter of each commutation segment. Due to the high cost of repairs, for small appliance and tool motors the segments are typically crimped permanently in place and cannot be removed; when the motor fails it is simply discarded and replaced. On very large industrial motors it is economical to be able to replace individual damaged segments, and so the end-wedge can be unscrewed and individual segments removed and replaced.

Commutator segments are connected to the coils of the armature, with the number of coils (and commutator segments) depending on the speed and voltage of the machine. Large motors may have hundreds of segments.

Friction between the segments and the brushes eventually causes wear to both surfaces. Carbon brushes, being made of a softer material, wear faster and may be designed to be replaced easily without dismantling the machine. Older copper brushes caused more wear to the commutator, causing deep grooving and notching of the surface over time. The commutator on small motors (say, less than a kilowatt rating) is not designed to be repaired through the life of the device. On large industrial equipment, the commutator may be re-surfaced with abrasives, or the rotor may be removed from the frame, mounted in a large metal lathe, and the commutator resurfaced by cutting it down to a smaller diameter. The largest of equipment can include a lathe turning attachment directly over the commutator.

Brush Construction



Various types of copper and carbon brushes.

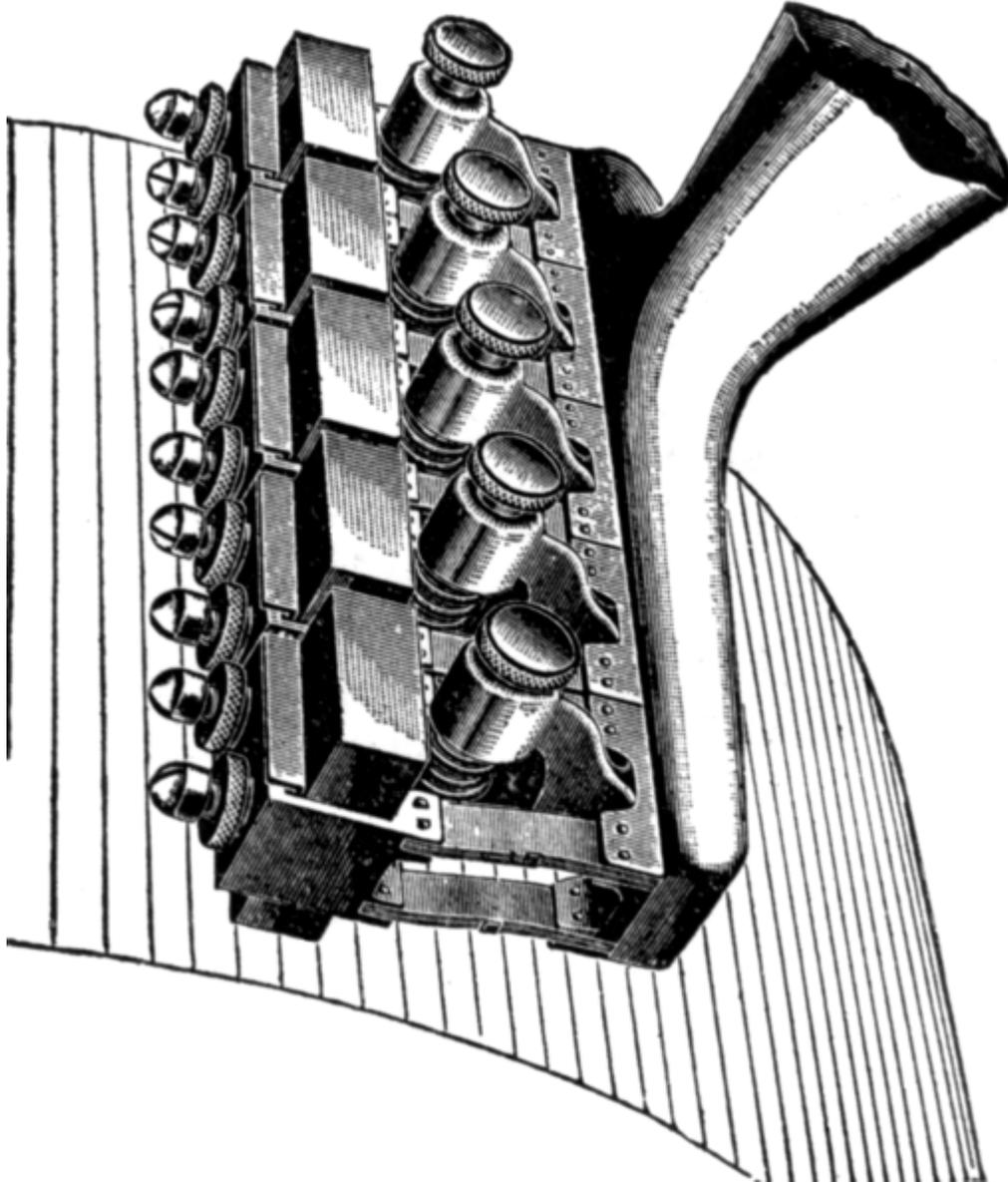
Early in the development of dynamos and motors, copper brushes were used to contact the surface of the commutator. However, these hard metal brushes tended to scratch and groove the smooth commutator segments, eventually requiring resurfacing of the commutator. As the copper brushes wear away, the dust and pieces of the brush could wedge between commutator segments, shorting them and reducing the efficiency of the device. Fine copper wire mesh or gauze provided better surface contact with less segment wear, but gauze brushes were more expensive than strip or wire copper brushes. The copper brush was eventually replaced by the carbon brush.

Carbon brushes tend to wear more evenly than copper brushes, and the soft carbon causes far less damage to the commutator segments. There is less sparking with carbon as compared to copper, and as the carbon wears away, the higher resistance of carbon results in fewer problems from the dust collecting on the commutator segments.

Copper and carbon are each better suited for a particular purpose. Copper brushes perform better with very low voltages and high current, while carbon brushes are better for high voltage and low current. Copper brushes typically carry 150 to 200 amperes per square inch of contact surface, while carbon only carries 40 to 70 amperes per square inch. The higher resistance of carbon also results in a greater voltage drop of 0.8 to 1.0 volts per contact, or 1.6 to 2.0 volts across the commutator.

Modern rotating machines with commutators now use carbon brushes, which may have copper powder mixed in to improve conductivity. Metallic copper brushes would only be found in toy or very small motors, such as the one illustrated above.

Brush Holders



Compound carbon brush holder, with individual clamps and tension adjustments for each block of carbon.

A spring is typically used with the brush, to maintain constant contact with the commutator. As the brush and commutator wear down, the spring steadily pushes the brush downwards towards the commutator. Eventually the brush wears small and thin enough that steady contact is no longer possible or it is no longer securely held in the brush holder, and so the brush must be replaced.

It is common for a flexible power cable to be directly attached to the brush, because current flowing through the support spring causes heating, which may lead to a loss of metal temper and a loss of the spring tension.

When a commutated motor or generator uses more power than a single brush is capable of conducting, an assembly of several brush holders is mounted in parallel across the surface of the very large commutator.

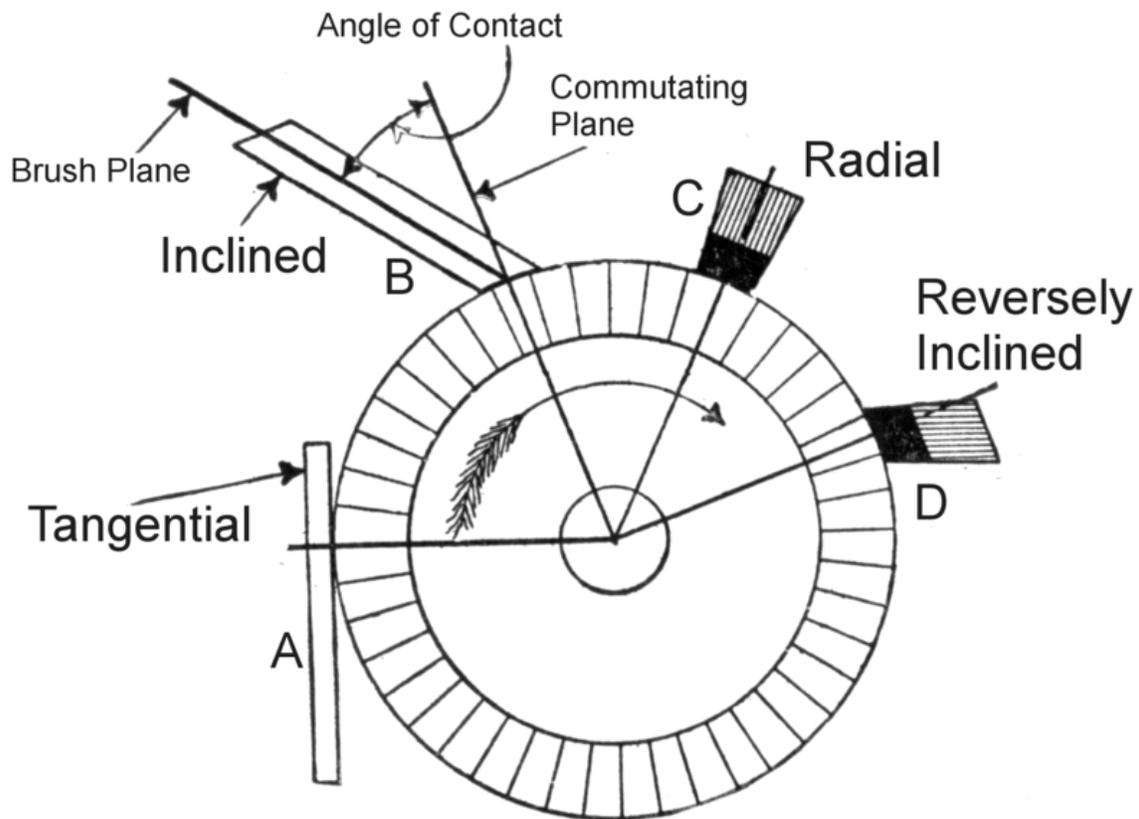
This parallel holder distributes current evenly across all the brushes, and permits a careful operator to remove a bad brush and replace it with a new one, even as the machine continues to spin fully powered and under load.

High power, high current commutated equipment is now uncommon, due to the less complex design of alternating current generators that permits a low current, high voltage spinning field coil to energize high current fixed-position stator coils. This permits the use of very small singular brushes in the alternator design. In this instance, the rotating contacts are continuous rings, called slip rings, and, of course, no switching happens.

Modern devices using carbon brushes usually have a maintenance-free design that requires no adjustment throughout the life of the device, using a fixed-position brush holder slot and a combined brush-spring-cable assembly that fits into the slot. Replacement simply involves pulling out the old brush and inserting a new one.

Older commutator motors sometimes had all brushes mounted on movable frames so that the position of the brushes in relation to the magnetic fields of the stator poles could be adjusted manually.

Brush Contact Angle

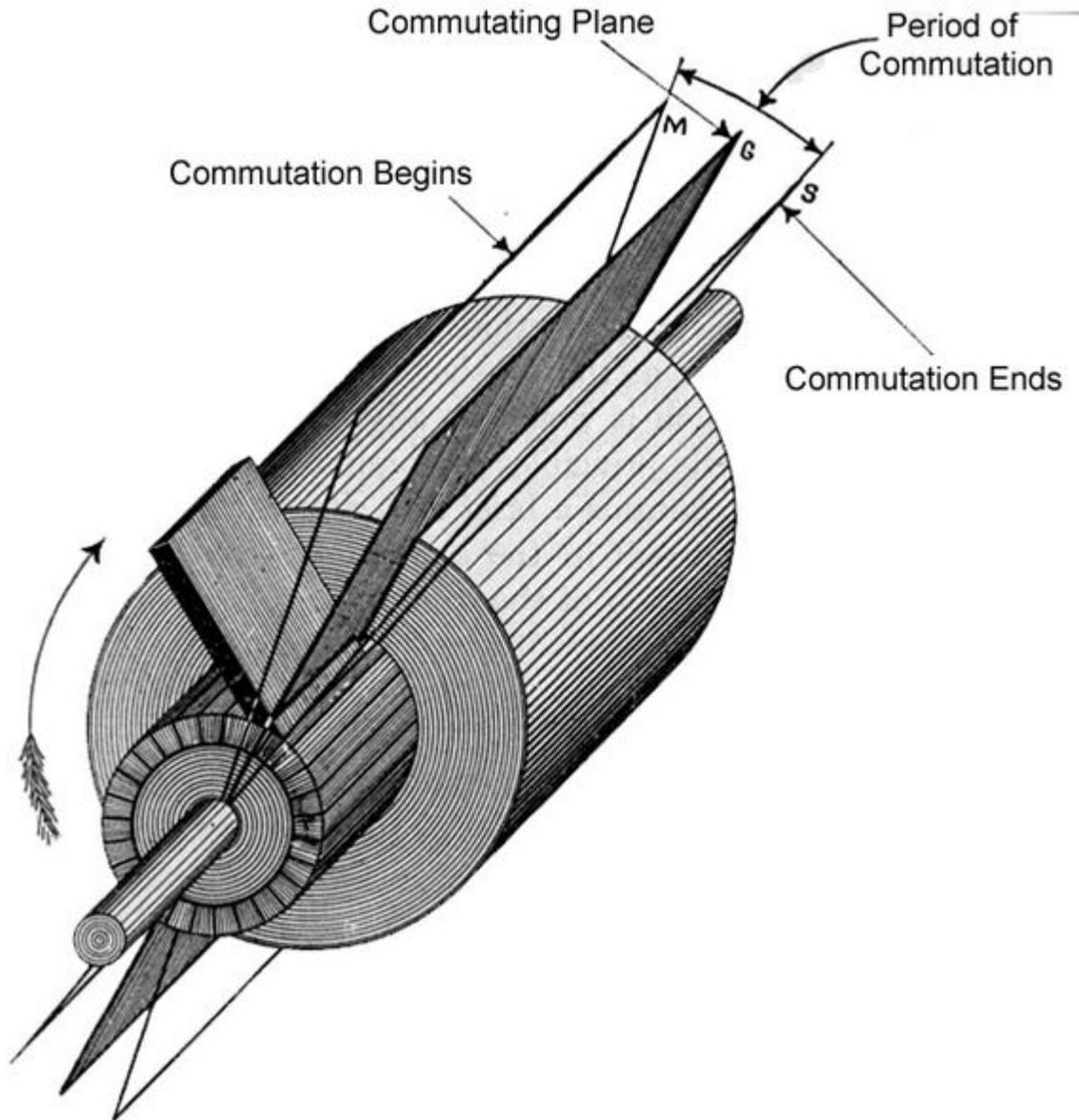


Brush angle definitions

The different brush types make contact with the commutator in different ways. Because copper brushes have the same hardness as the commutator segments, the rotor cannot be spun backwards against the ends of copper brushes without the copper digging into the segments and causing severe damage. Consequently strip/laminate copper brushes only make tangential contact with the commutator, while copper mesh and wire brushes use an inclined contact angle touching their edge across the segments of a commutator that can spin in only one direction.

The softness of carbon brushes permits direct radial end-contact with the commutator without damage to the segments, permitting easy reversal of rotor direction, without the need to reorient the brush holders for operation in the opposite direction. Although never reversed, common appliance motors that use wound rotors, commutators and brushes have radial-contact brushes. In the case of a reaction-type carbon brush holder, carbon brushes may be reversely inclined with the commutator so that the commutator tends to push against the carbon for firm contact.

The Commutating Plane

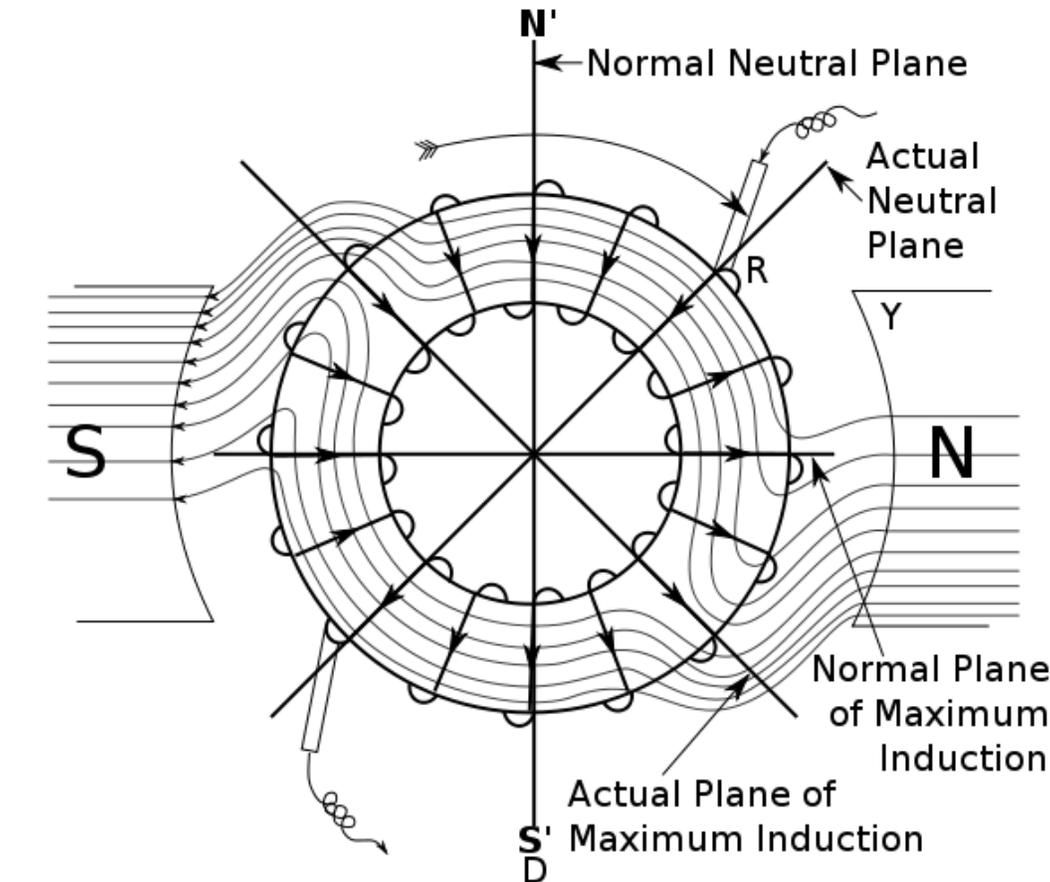


Commutating plane definitions.

The contact point where a brush touches the commutator is referred to as the *commutating plane*. In order to conduct sufficient current to or from the commutator, the brush contact area is not a thin line but instead a rectangular patch across the segments. Typically the brush is wide enough to span 2.5 commutator segments. This means that two adjacent segments are electrically connected by the brush when it contacts both.

On the left is an exaggerated example of how the field is distorted by the rotor. On the right, iron filings show the distorted field across the rotor.

In a real motor or generator, the field around the rotor is never perfectly uniform. Instead, the rotation of the rotor induces field effects which drag and distort the magnetic lines of the outer non-rotating stator.



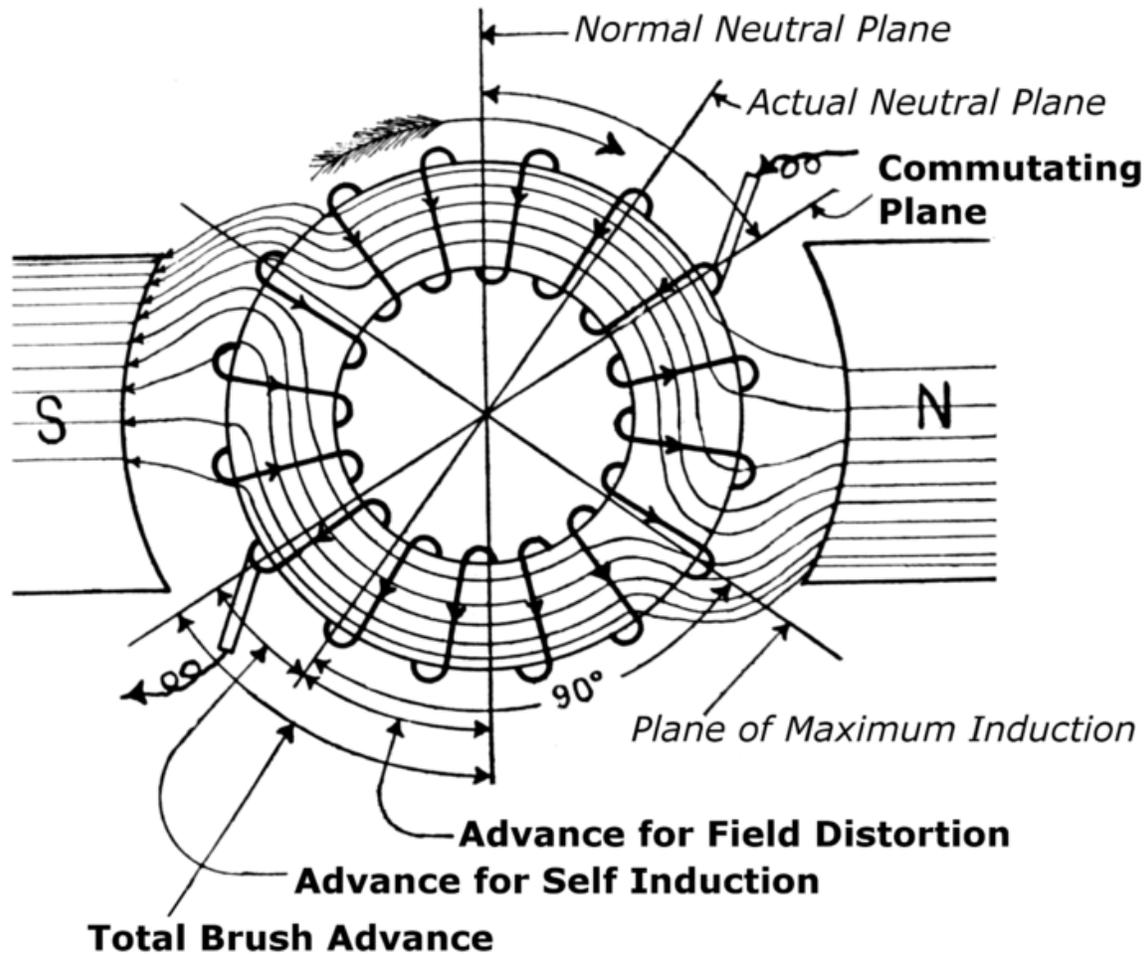
Actual position of the commutating plane to compensate for field distortion.

The faster the rotor spins, the further this degree of field distortion. Because a motor or generator operates most efficiently with the rotor field at right angles to the stator field, it is necessary to either retard or advance the brush position to put the rotor's field into the correct position to be at a right angle to the distorted field.

These field effects are reversed when the direction of spin is reversed. It is therefore difficult to build an efficient reversible commutated dynamo, since for highest field strength it is necessary to move the brushes to the opposite side of the normal neutral plane.

The effect can be considered to be analogous to timing advance in an internal combustion engine. Generally a dynamo that has been designed to run at a certain fixed speed will have its brushes permanently fixed to align the field for highest efficiency at that speed.

Further Compensation for Self-Induction



Brush advance for Self-Induction.

In a coil of wire, the magnetic field of each wire compounds together to form a magnetic field that tends to resist changes in current, as if the current had inertia. This is known as *self-induction*.

In the coils of the rotor, there is a tendency for current to continue to flow for a brief moment after the brush has been reached. This energy is wasted as heat due to the brush spanning across several commutator segments and the current short-circuiting across the segments.

Spurious resistance is an apparent increase in the resistance in the armature winding, which is proportional to the speed of the armature, and is due to the lagging of the current.

In order to minimize sparking at the brushes due to this short-circuiting, the brushes are advanced a few degrees further yet, beyond the advance for field distortions. This moves the rotor winding undergoing commutation slightly forward into the stator field which has magnetic lines in the opposite direction and which oppose the field in the stator. This opposing field helps to reverse the lagging self-inducting current in the stator.

So even for a rotor which is at rest and initially requires no compensation for spinning field distortions, the brushes should still be advanced beyond the perfect 90-degree angle as taught in so many beginners textbooks, in order to compensate for self-induction.

Limitations and alternatives

While commutators are widely applied in direct current machines, up to several thousand kilowatts in rating, they have limitations.

Brushes and copper segments wear. On small machines the brushes may last as long as the product (small power tools, appliances, etc.) but larger machines will require regular replacement of brushes and occasional resurfacing of the commutator. Brush-type motors may not be suitable for long service on aerospace equipment where maintenance is not possible.

The efficiency of direct current machines is limited by the "brush drop" due to the resistance of the sliding contact. This may be several volts, making low-voltage direct-current machines very inefficient. The friction of the brush on the commutator also absorbs some of the energy of the machine.

Lastly, the current density in the brush is limited and the maximum voltage on each segment of the commutator is also limited. Very large direct current machines, say, more than several megawatts rating, cannot be built with commutators. The largest motors and generators, of hundreds of megawatt ratings, are all alternating-current machines.

With the widespread availability of power semiconductors, it is now economical to provide electronic switching of the current in the motor windings. These "brushless direct current" motors eliminate the commutator; these can be likened to AC machines with a built-in DC to AC inverter. In these motors, rotor position determines when the stator windings switch polarity. Operating life is limited only by bearing wear, if other factors are not adverse.

Repulsion induction motors

These are single-phase AC-only motors with higher starting torque than can be obtained with split-phase starting windings, and before high-capacitance (non-polar, relatively

high-current electrolytic) starting capacitors became practical. They have a conventional wound stator as with any induction motor, but the wire-wound rotor is much like that with a conventional commutator. Brushes opposite each other are connected to each other (not to an external circuit), and transformer action induces currents into the rotor that develop torque by repulsion.

One variety, notable for having an adjustable speed, runs continuously with brushes in contact, while another uses repulsion only for high starting torque and in some cases lifts the brushes once the motor is running fast enough. In the latter case, all commutator segments are connected together as well, before the motor attains running speed.

Once at speed, the rotor windings become functionally equivalent to the squirrel-cage structure of a conventional induction motor, and the motor runs as such.

Web ref. gives a nice, concise description

Laboratory commutators

Commutators were used as simple forward-off-reverse switches for electrical experiments in physics laboratories. There are two well-known historical types :

Ruhmkorff commutator

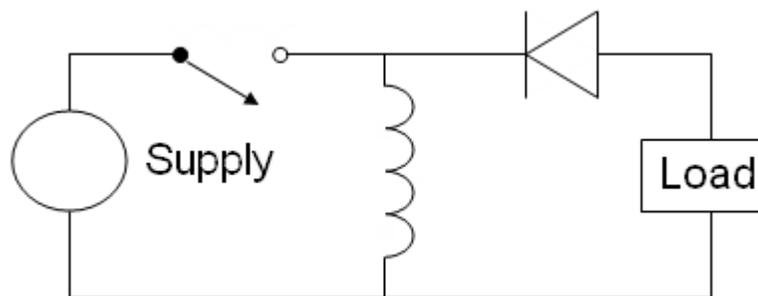
This is similar in design to the commutators used in motors and dynamos. It was usually constructed of brass and ivory (later ebonite).

Pohl commutator

This consisted of a block of wood or ebonite with four wells, containing mercury, which were cross-connected by copper wires. The output was taken from a pair of curved copper wires which were moved to dip into one or other pair of mercury wells.

Chapter-6

Buck–boost Converter



The basic schematic of a buck–boost converter.

Two different topologies are called **buck–boost converter**. Both of them can produce an output voltage much larger (in absolute magnitude) than the input voltage. Both of them can produce a wide range of output voltage from that maximum output voltage to almost zero.

- The inverting topology – The output voltage is of the opposite polarity as the input
- A buck (step-down) converter followed by a boost (step-up) converter – The output voltage is of the same polarity as the input, and can be lower or higher than the input. Such a non-inverting buck-boost converter may use a single inductor that is used as both the buck inductor and the boost inductor.

This page describes the inverting topology.

The **buck–boost converter** is a type of DC-to-DC converter that has an output voltage magnitude that is either greater than or less than the input voltage magnitude. It is a switched-mode power supply with a similar circuit topology to the boost converter and the buck converter. The output voltage is adjustable based on the duty cycle of the switching transistor. One possible drawback of this converter is that the switch does not have a terminal at ground; this complicates the driving circuitry. Also, the polarity of the output voltage is opposite the input voltage. Neither drawback is of any consequence if the power supply is isolated from the load circuit (if, for example, the supply is a battery)

as the supply and diode polarity can simply be reversed. The switch can be on either the ground side or the supply side.

Principle of operation

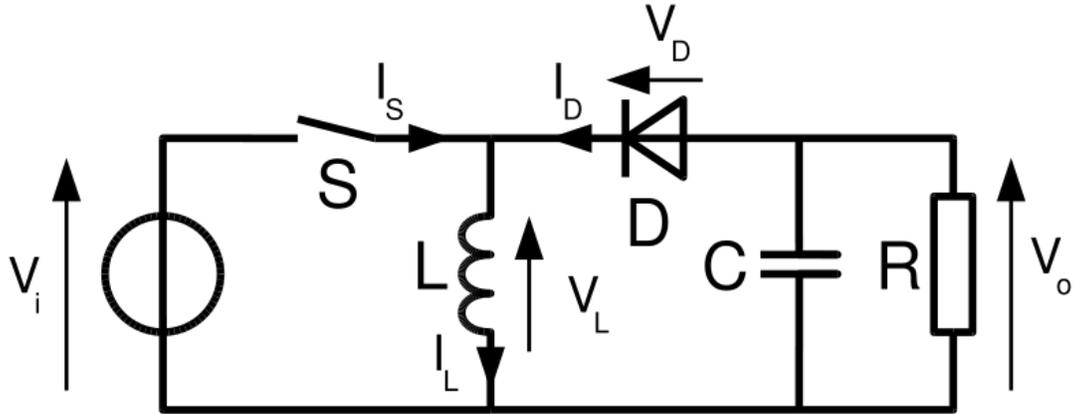
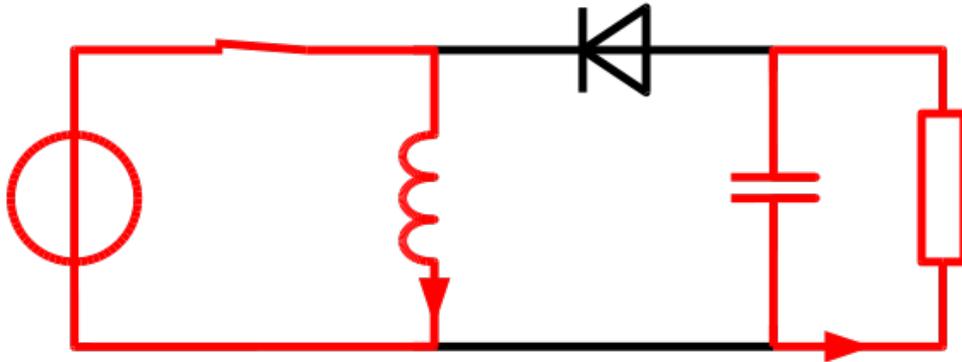


Fig. 1: Schematic of a buck–boost converter.

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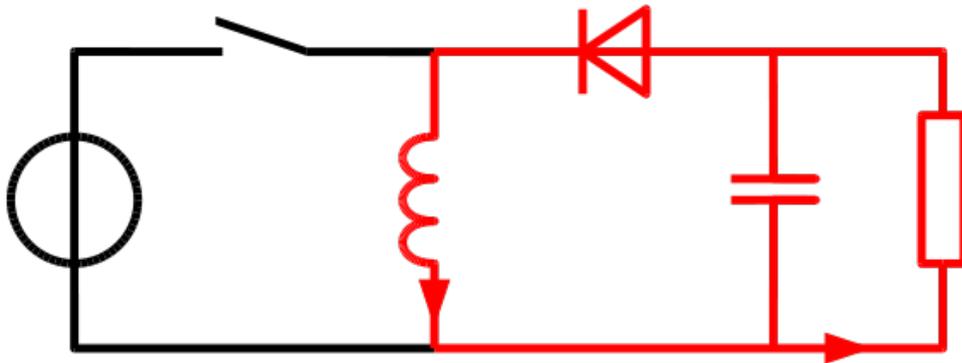


Fig. 2: The two operating states of a buck–boost converter: When the switch is turned-on, the input voltage source supplies current to the inductor and the capacitor supplies current to the resistor (output load). When the switch is opened (providing energy is stored into the inductor), the inductor supplies current to the load via the diode D.

The basic principle of the buck–boost converter is fairly simple (see figure 2):

- while in the On-state, the input voltage source is directly connected to the inductor (L). This results in accumulating energy in L. In this stage, the capacitor supplies energy to the output load.
- while in the Off-state, the inductor is connected to the output load and capacitor, so energy is transferred from L to C and R.

Compared to the buck and boost converters, the characteristics of the buck–boost converter are mainly:

- polarity of the output voltage is opposite to that of the input;

- the output voltage can vary continuously from 0 to $-\infty$ (for an ideal converter). The output voltage ranges for a buck and a boost converter are respectively 0 to V_i and V_i to ∞ .

Continuous Mode

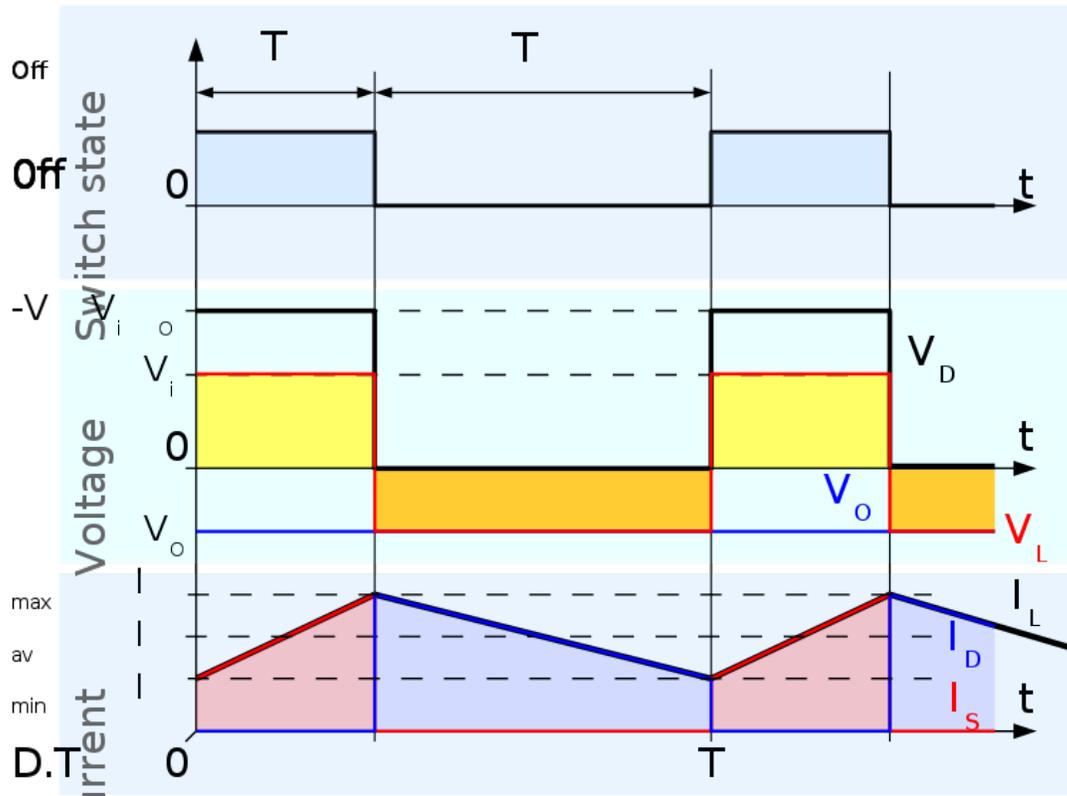


Fig 3: Waveforms of current and voltage in a buck–boost converter operating in continuous mode.

If the current through the inductor L never falls to zero during a commutation cycle, the converter is said to operate in continuous mode. The current and voltage waveforms in an ideal converter can be seen in Figure 3.

From $t=0$ to $t=DT$, the converter is in On-State, so the switch S is closed. The rate of change in the inductor current (I_L) is therefore given by

$$\frac{dI_L}{dt} = \frac{V_i}{L}$$

At the end of the On-state, the increase of I_L is therefore:

$$\Delta I_{L\text{On}} = \int_0^{DT} dI_L = \int_0^{DT} \frac{V_i}{L} dt = \frac{V_i DT}{L}$$

D is the duty cycle. It represents the fraction of the commutation period T during which the switch is On. Therefore D ranges between 0 (S is never on) and 1 (S is always on).

During the Off-state, the switch S is open, so the inductor current flows through the load. If we assume zero voltage drop in the diode, and a capacitor large enough for its voltage to remain constant, the evolution of I_L is:

$$\frac{dI_L}{dt} = \frac{V_o}{L}$$

Therefore, the variation of I_L during the Off-period is:

$$\Delta I_{L\text{Off}} = \int_0^{(1-D)T} dI_L = \int_0^{(1-D)T} \frac{V_o}{L} dt = \frac{V_o (1-D) T}{L}$$

As we consider that the converter operates in steady-state conditions, the amount of energy stored in each of its components has to be the same at the beginning and at the end of a commutation cycle. As the energy in an inductor is given by:

$$E = \frac{1}{2} L I_L^2$$

it is obvious that the value of I_L at the end of the Off state must be the same as the value of I_L at the beginning of the On-state, i.e. the sum of the variations of I_L during the on and the off states must be zero:

$$\Delta I_{L\text{On}} + \Delta I_{L\text{Off}} = 0$$

Substituting $\Delta I_{L\text{On}}$ and $\Delta I_{L\text{Off}}$ by their expressions yields:

$$\Delta I_{L\text{On}} + \Delta I_{L\text{Off}} = \frac{V_i DT}{L} + \frac{V_o (1-D) T}{L} = 0$$

This can be written as:

$$\frac{V_o}{V_i} = \left(\frac{-D}{1-D} \right)$$

This in return yields that:

$$D = \frac{V_o}{V_o - V_i}$$

From the above expression it can be seen that the polarity of the output voltage is always negative (as the duty cycle goes from 0 to 1), and that its absolute value increases with D , theoretically up to minus infinity as D approaches 1. Apart from the polarity, this converter is either step-up (as a boost converter) or step-down (as a buck converter). This is why it is referred to as a buck–boost converter.

Discontinuous Mode

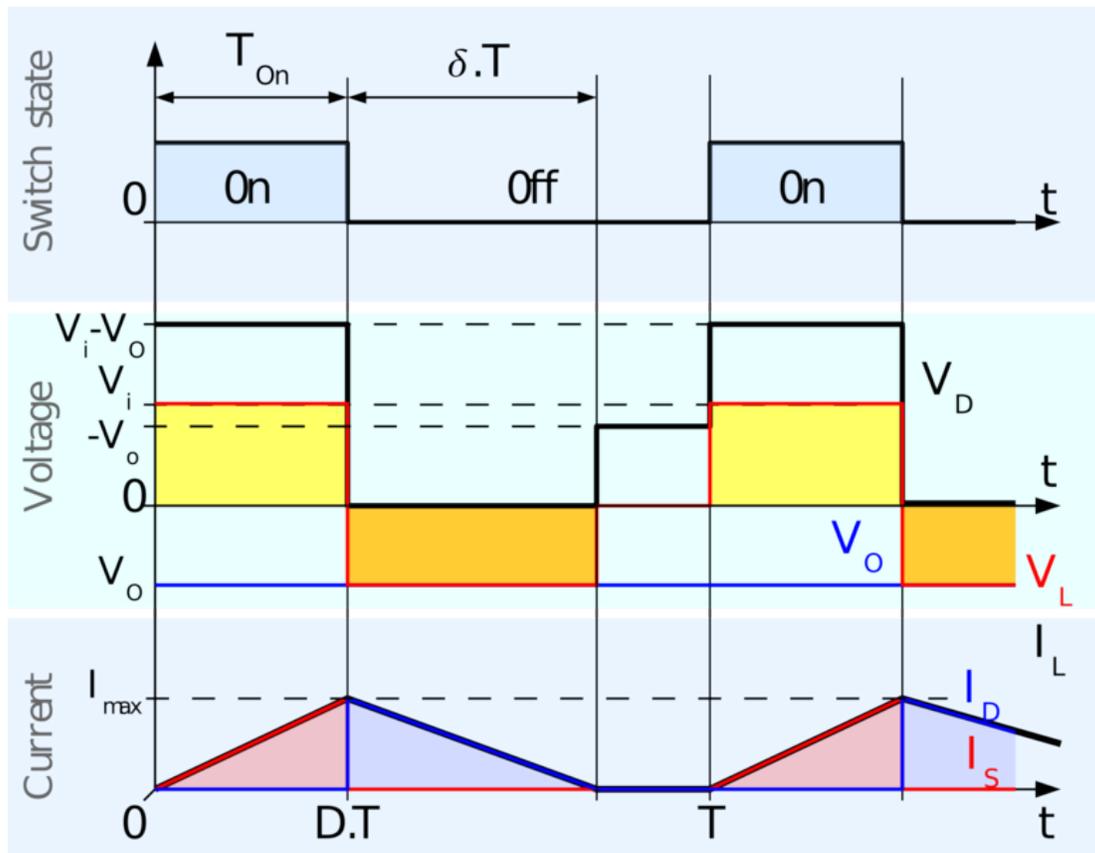


Fig 4: Waveforms of current and voltage in a buck–boost converter operating in discontinuous mode.

In some cases, the amount of energy required by the load is small enough to be transferred in a time smaller than the whole commutation period. In this case, the current through the inductor falls to zero during part of the period. The only difference in the principle described above is that the inductor is completely discharged at the end of the commutation cycle (see waveforms in figure 4). Although slight, the difference has a strong effect on the output voltage equation. It can be calculated as follows:

As the inductor current at the beginning of the cycle is zero, its maximum value $I_{L_{\max}}$ (at $t = DT$) is

$$I_{L_{\max}} = \frac{V_i DT}{L}$$

During the off-period, I_L falls to zero after $\delta.T$:

$$I_{L_{\max}} + \frac{V_o \delta T}{L} = 0$$

Using the two previous equations, δ is:

$$\delta = -\frac{V_i D}{V_o}$$

The load current I_o is equal to the average diode current (I_D). As can be seen on figure 4, the diode current is equal to the inductor current during the off-state. Therefore, the output current can be written as:

$$I_o = \bar{I}_D = \frac{I_{L_{\max}} \delta}{2}$$

Replacing $I_{L_{\max}}$ and δ by their respective expressions yields:

$$I_o = -\frac{V_i DT}{2L} \frac{V_i D}{V_o} = -\frac{V_i^2 D^2 T}{2LV_o}$$

Therefore, the output voltage gain can be written as:

$$\frac{V_o}{V_i} = -\frac{V_i D^2 T}{2LI_o}$$

Compared to the expression of the output voltage gain for the continuous mode, this expression is much more complicated. Furthermore, in discontinuous operation, the output voltage not only depends on the duty cycle, but also on the inductor value, the input voltage and the output current.

Limit between continuous and discontinuous modes

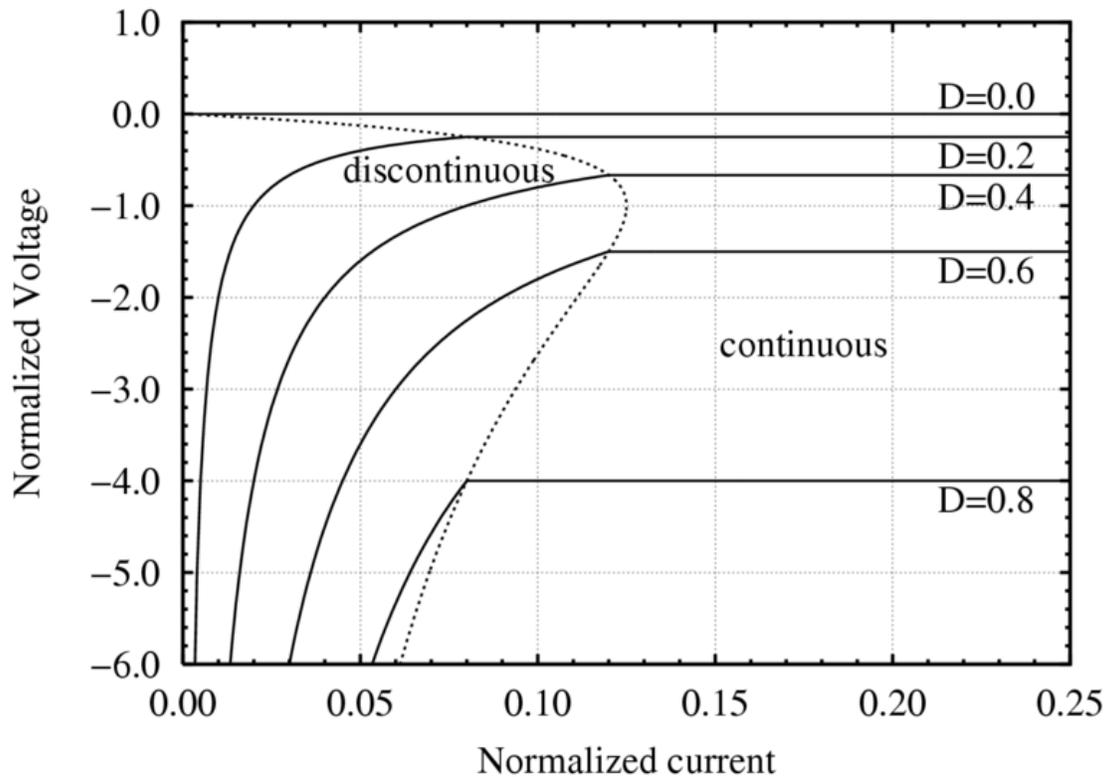


Fig 5: Evolution of the normalized output voltage with the normalized output current in a buck–boost converter.

As told at the beginning, the converter operates in discontinuous mode when low current is drawn by the load, and in continuous mode at higher load current levels. The limit between discontinuous and continuous modes is reached when the inductor current falls to zero exactly at the end of the commutation cycle. with the notations of figure 4, this corresponds to :

$$\begin{aligned} DT + \delta T &= T \\ D + \delta &= 1 \end{aligned}$$

In this case, the output current $I_{o\lim}$ (output current at the limit between continuous and discontinuous modes) is given by:

$$I_{o\lim} = \bar{I}_D = \frac{I_{L\max}}{2} (1 - D)$$

Replacing $I_{L\max}$ by the expression given in the *discontinuous mode* section yields:

$$I_{o\lim} = \frac{V_i D T}{2L} (1 - D)$$

As $I_{o\lim}$ is the current at the limit between continuous and discontinuous modes of operations, it satisfies the expressions of both modes. Therefore, using the expression of the output voltage in continuous mode, the previous expression can be written as:

$$I_{o\lim} = \frac{V_i D T}{2L} \frac{V_i}{V_o} (-D)$$

Let's now introduce two more notations:

- the normalized voltage, defined by $|V_o| = \frac{V_o}{V_i}$. It corresponds to the gain in voltage of the converter;
- the normalized current, defined by $|I_o| = \frac{L}{T V_i} I_o$. The term $\frac{T V_i}{L}$ is equal to the maximum increase of the inductor current during a cycle; i.e., the increase of the inductor current with a duty cycle $D=1$. So, in steady state operation of the converter, this means that $|I_o|$ equals 0 for no output current, and 1 for the maximum current the converter can deliver.

Using these notations, we have:

- in continuous mode, $|V_o| = \frac{D}{1-D}$;
- in discontinuous mode, $|V_o| = -\frac{D^2}{2|I_o|}$;
- the current at the limit between continuous and discontinuous mode is $I_{o\lim} = \frac{V_i T}{2L} D(1-D) = \frac{I_{o\lim}}{2|I_o|} D(1-D)$. Therefore the locus of the limit between continuous and discontinuous modes is given by $\frac{1}{2|I_o|} D(1-D) = 1$.

These expressions have been plotted in figure 5. The difference in behaviour between the continuous and discontinuous modes can be seen clearly.

Non-ideal circuit

Effect of parasitic resistances

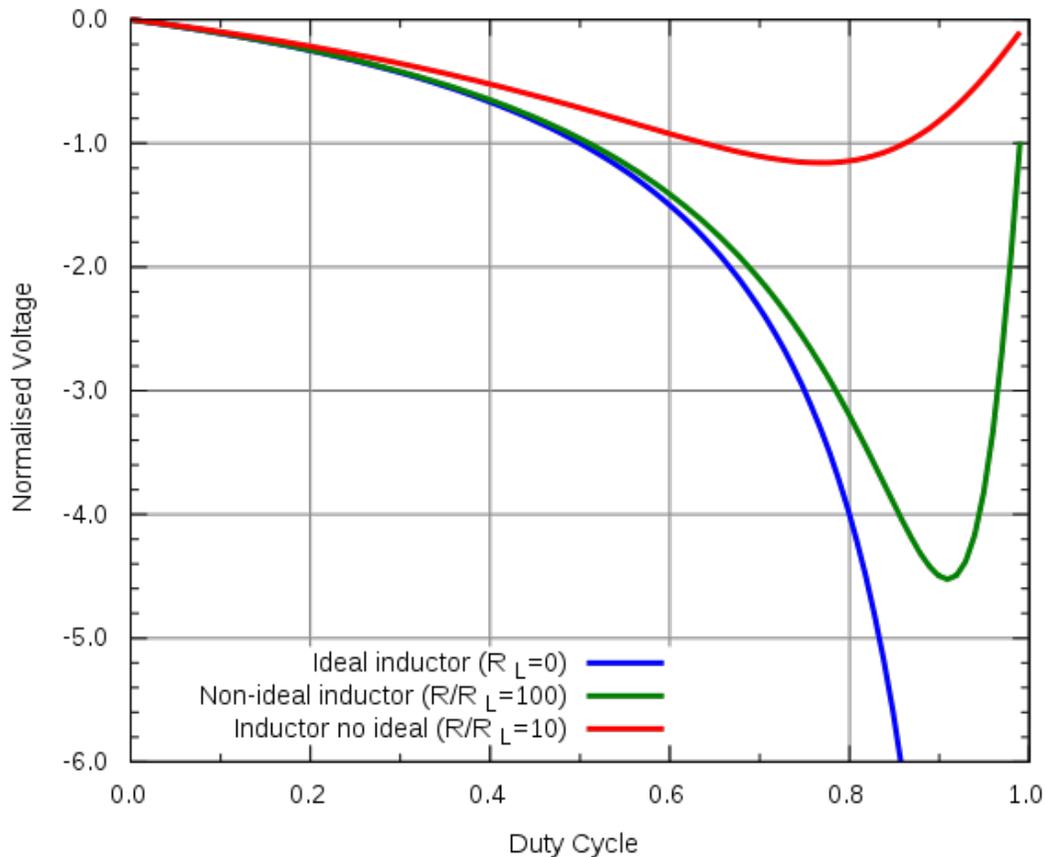


Fig 6: Evolution of the output voltage of a buck–boost converter with the duty cycle when the parasitic resistance of the inductor increases.

In the analysis above, no dissipative elements (resistors) have been considered. That means that the power is transmitted without losses from the input voltage source to the load. However, parasitic resistances exist in all circuits, due to the resistivity of the materials they are made from. Therefore, a fraction of the power managed by the converter is dissipated by these parasitic resistances.

For the sake of simplicity, we consider here that the inductor is the only non-ideal component, and that it is equivalent to an inductor and a resistor in series. This assumption is acceptable because an inductor is made of one long wound piece of wire, so it is likely to exhibit a non-negligible parasitic resistance (R_L). Furthermore, current flows through the inductor both in the on and the off states.

Using the state-space averaging method, we can write:

$$V_i = \bar{V}_L + \bar{V}_S$$

where \bar{V}_L and \bar{V}_S are respectively the average voltage across the inductor and the switch over the commutation cycle. If we consider that the converter operates in steady-state, the average current through the inductor is constant. The average voltage across the inductor is:

$$\bar{V}_L = L \frac{d\bar{I}_L}{dt} + R_L \bar{I}_L = R_L \bar{I}_L$$

When the switch is in the on-state, $V_S=0$. When it is off, the diode is forward biased (we consider the continuous mode operation), therefore $V_S=V_i-V_o$. Therefore, the average voltage across the switch is:

$$\bar{V}_S = D \cdot 0 + (1 - D)(V_i - V_o) = (1 - D)(V_i - V_o)$$

The output current is the opposite of the inductor current during the off-state. the average inductor current is therefore:

$$\bar{I}_L = \frac{-I_o}{1 - D}$$

Assuming the output current and voltage have negligible ripple, the load of the converter can be considered as purely resistive. If R is the resistance of the load, the above expression becomes:

$$\bar{I}_L = \frac{-V_o}{(1 - D)R}$$

Using the previous equations, the input voltage becomes:

$$V_i = R_L \frac{-V_o}{(1 - D)R} + (1 - D)(V_i - V_o)$$

This can be written as:

$$\frac{V_o}{V_i} = \frac{-D}{\frac{R_L}{R(1-D)} + 1 - D}$$

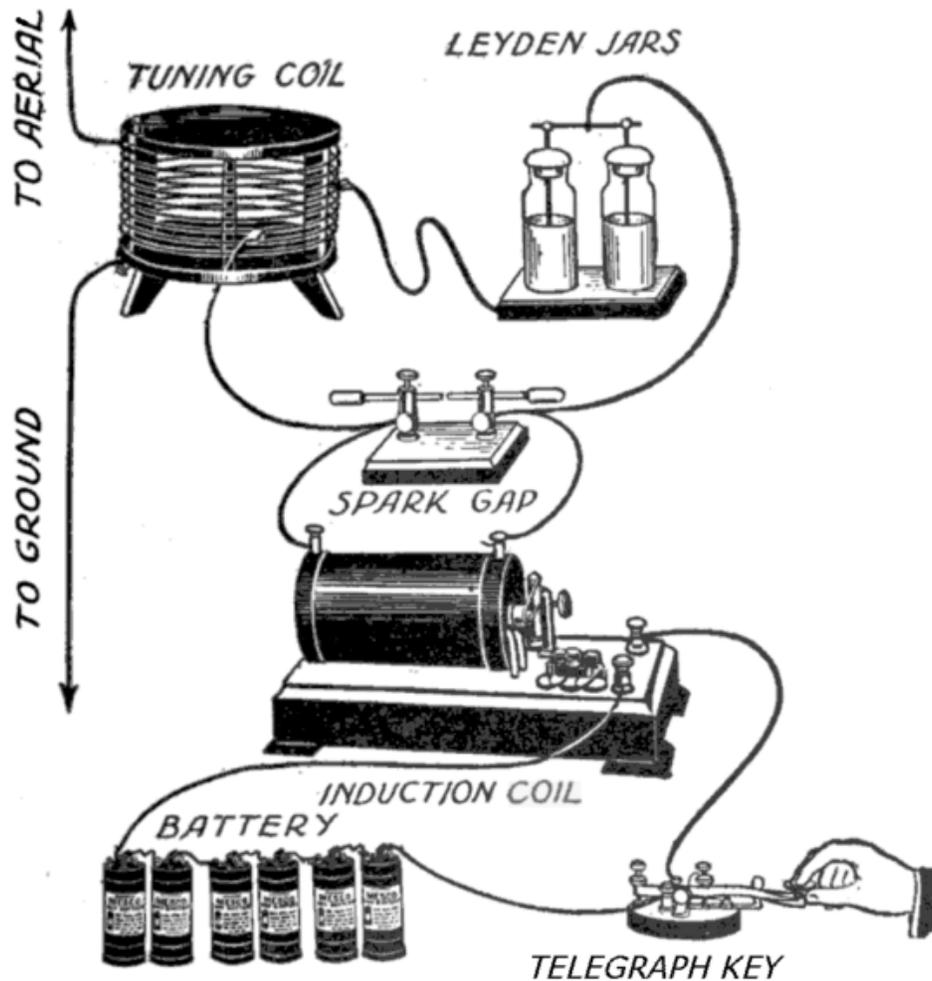
If the inductor resistance is zero, the equation above becomes equal to the one of the *ideal* case. But as R_L increases, the voltage gain of the converter decreases compared to the ideal case. Furthermore, the influence of R_L increases with the duty cycle. This is summarized in figure 6.

Chapter-7

Spark-gap Transmitter

A **spark-gap transmitter** is a device for generating radio frequency electromagnetic waves using a spark gap.

These devices served as the transmitters for most wireless telegraphy systems for the first three decades of radio (1887–1916) and the first demonstrations of practical radio were carried out using them. In later years (prior to the development of practical vacuum tube systems) somewhat more efficient transmitters were developed based on high-speed Alexanderson alternators and Poulsen Arc generators, but spark transmitters were still preferred by most operators. This was both because of their uncomplicated design and the fact that the transmitter stopped generating a carrier as soon as the Morse Code key was released, allowing the operator to "listen through" for a reply. With the other types of transmitter, the carrier wave could not be controlled so easily, and elaborate measures were required both to modulate the carrier and to separate the receiving antenna from the transmitting antenna, to keep transmitter leakage from de-sensitizing the receiver. After WWI, greatly improved transmitters based on vacuum tubes became available, which overcame these problems, and by the late 1920s the only spark transmitters still in regular operation were "legacy" installations on Naval vessels. Even when vacuum tube based transmitters had been installed, many vessels retained their crude but reliable spark transmitters as an emergency backup. However, by 1940, the technology was no longer used for communication. Use of the spark-gap transmitter led to many radio operators being nicknamed "Sparks" long after spark transmitters ceased to be used. Even today, the German verb "funken", literally, "to spark", also means "to send a radio message/signal".



Pictorial diagram of a simple spark-gap transmitter showing examples of the early electronic components used. From a 1917 boy's book, it is typical of the low power transmitters homebuilt by thousands of amateurs to explore the exciting new technology of radio.

History

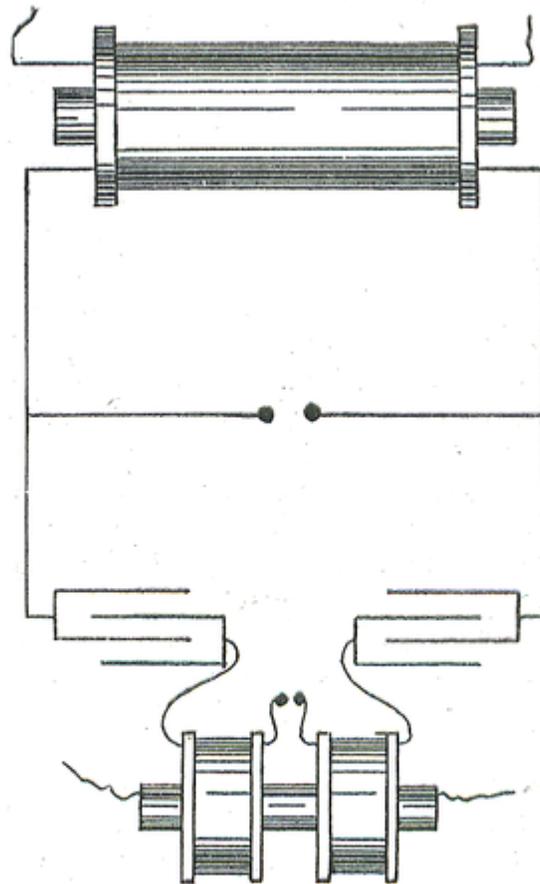
The history of radio shows that the spark gap transmitter was the product of many people, often working in competition. In 1862 James Clerk Maxwell predicted the propagation of electromagnetic waves through a vacuum.

In 1887, David E. Hughes used a spark gap to generate radio signals, achieving a detectable range of approximately 500 metres.

In 1888 physicist Heinrich Hertz set out to scientifically verify Maxwell's predictions. Hertz used a tuned spark gap transmitter and a tuned spark gap detector (consisting of a loop of wire connected to a small spark gap) located a few meters away. In a series of

UHF experiments, Hertz verified that electromagnetic waves were being produced by the transmitter. When the transmitter sparked, small sparks also appeared across the receiver's spark gap, which could be seen under a microscope.

Nikola Tesla introduced his radio system in 1893 and later developed the so-called "loose coupler" system which was a major technological breakthrough. It produced a far more coherent carrier wave, generated far less interference, worked with much greater efficiency, required much lower operating voltages and could be operated in any weather conditions.



One form of Nikola Tesla's Spark-gap transmitter

Tesla pursued the application of his high voltage high frequency technology to radio. By tuning a receiving coil to the specific frequency used in the transmitting coil, he showed that the radio receiver's output could be greatly magnified through resonant action. Tesla was one of the first to patent a means to reliably produce radio frequencies (e.g., U.S. Patent 447,920, "Method of Operating Arc-Lamps" (March 10, 1891)). Tesla also invented a variety of rotary, cooled, and quenched spark gaps capable of handling high power.

Marconi began experimenting with wireless telegraphy in the early 1890s. In 1895 he succeeded in transmitting over a distance of 1 1/4 miles. His first transmitter consisted of an induction coil connected between a wire antenna and ground, with a spark gap across it. Every time the induction coil pulsed, the antenna would be momentarily charged up to tens (sometimes hundreds) of thousands of volts until the spark gap started to arc over. This acted as a switch, essentially connecting the charged antenna to ground, producing a very brief burst of electromagnetic radiation.

While the various early systems of spark transmitters worked well enough to prove the concept of wireless telegraphy, the primitive spark gap assemblies used had some severe shortcomings. The biggest problem was that the maximum power that could be transmitted was directly determined by how much electrical charge the antenna could hold. Because the capacitance of practical antennas is quite small, the only way to get a reasonable power output was to charge it up to very high voltages. However, this made transmission impossible in rainy or even damp conditions. Also, it necessitated a quite wide spark gap, with a very high electrical resistance, with the result that most of the electrical energy was used simply to heat up the air in the spark gap.

The other problem was that, due to the very brief duration of each burst of electromagnetic radiation, the system radiated an extremely "dirty" signal sideband-wise, which was almost impossible to tune out if the listener wanted to monitor a different station. This signal-bleed played a role in the inability of the *RMS Titanic* and the *SS Californian* to communicate with each other in 1912.

Despite these flaws, Marconi was able to generate sufficient interest from the British Admiralty in these originally crude systems to eventually finance the development of a commercial wireless telegraph service between United States and Europe using vastly improved equipment.

Reginald Fessenden's first attempts to transmit voice employed a spark transmitter operating at approximately 10,000 sparks/second. To modulate this transmitter he inserted a carbon microphone in series with the supply lead. He experienced great difficulty in achieving intelligible sound. At least one high-powered audio transmitter used water cooling for the microphone.

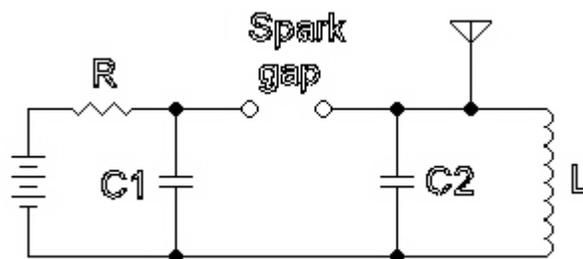
In 1905 a "state of the art" spark gap transmitter generated a signal having a wavelength between 250 meters (1.2 MHz) and 550 meters (545 kHz). 600 meters (500 kHz) became the International distress frequency. The receivers were simple unamplified Magnetic

Detectors or electrolytic detectors. This later gave way to the famous and more sensitive galena crystal sets. Tuners were primitive or nonexistent. Early amateur radio operators built low power spark gap transmitters using the spark coil from Ford Model T automobiles. But a typical commercial station in 1916 might include a 1/2 kW transformer that supplied 14,000 volts, an eight section capacitor, and a rotary gap capable of handling a peak current of several hundred amperes.

Shipboard installations usually used a DC motor (usually run off the ship's DC lighting supply) to drive an alternator whose AC output was then stepped up to 10,000–14,000 volts by a transformer. This was a very convenient arrangement, since the signal could be easily modulated by simply connecting a relay between the relatively low voltage alternator output and the transformer's primary winding, and activating it with the morse key. (Lower-powered units sometimes used the morse key to directly switch the AC, but this required a heavier key making it more difficult to operate).

Spark gap transmitters generate fairly broad-band signals. As the more efficient transmission mode of continuous waves (CW) became easier to produce and band crowding and interference worsened, spark-gap transmitters and damped waves were legislated off the new shorter wavelengths by international treaty, and replaced by Poulsen arc converters and high frequency alternators which developed a sharply defined transmitter frequency. These approaches later yielded to vacuum tube technology and the 'electric age' of radio would end. Long after they stopped being used for communications, spark gap transmitters were employed for radio jamming. As late as 1955, a Japanese radio-controlled toy bus used a spark transmitter and coherer receiver; the spark was visible behind a sheet of blue transparent plastic. Spark gap oscillators are still used to generate high frequency high voltage to initiate welding arcs in gas tungsten arc welding. Powerful spark gap pulse generators are still used to simulate EMPs. Most high power gas-discharge street lamps (mercury and sodium vapor) still use modified spark transmitters as switch-on ignitors.

Operation



A typical spark transmitter circuit.

Legend:

capacitor - C_1 and C_2 ;

resistor - R;

inductor - L.

The spark transmitter is very simple in operation, but it presented significant technical problems mostly due to very large induced EMF when the spark struck, which caused breakdown of the insulation in the primary transformer. To overcome this the construction of even low-power sets was very solid. The damped wave output was very wasteful of bandwidth, and this limited the number of stations that could communicate effectively without interfering with each other.

In its simplest form, a spark-gap transmitter consists of a spark gap connected across an oscillatory circuit consisting of a capacitor and an inductor in series or parallel. In a typical transmitter circuit, a high voltage source (shown in the schematic as a battery, but usually a high voltage transformer) charges a capacitor (C1 in figure) through a resistor until the spark gap discharges, then a pulse of current passes through the capacitor (C2 in figure). The inductor and capacitor after the gap form a resonant circuit. After being excited by the current pulse, the oscillation rapidly decays because energy is radiated from the antenna. Because of the rapid onset and decay of the oscillation, the RF pulse occupies a large band of frequencies.

The function of the spark gap is to present initially a high resistance to the circuit to allow the capacitor to charge. When the breakdown voltage of the gap is reached, it then presents a low resistance to the circuit causing the capacitor to discharge. The discharge through the conducting spark takes the form of a damped oscillation, at a frequency determined by the resonant frequency of the LC circuit.

Spark gaps

A simple **spark gap** consists of two conducting electrodes separated by a gap immersed within a gas (typically air). When a sufficiently high voltage is applied, a spark will bridge the gap, ionizing the gas and drastically reducing its electrical resistance. An electric current then flows until the path of ionized gas is broken or the current is reduced below a minimum value called the 'holding current'. This usually occurs when the voltage across the gap drops sufficiently, but the process may also be assisted by cooling the spark channel or by physically separating the electrodes. This breaks the conductive filament of ionized gas, allowing the capacitor to recharge, and permitting the recharging/discharging cycle to repeat. The action of ionizing the gas is quite sudden and violent (*disruptive*), and it creates a sharp sound (ranging from a *snap* for a spark plug, to a loud *bang* for a wider gap). The spark gap also liberates light and heat.

Quenching the arc

Quenching refers to the act of extinguishing the previously established arc within the spark gap. This is considerably more difficult than initiating spark breakdown in the gap. As transmitter power was increased, the problem of quenching arose.

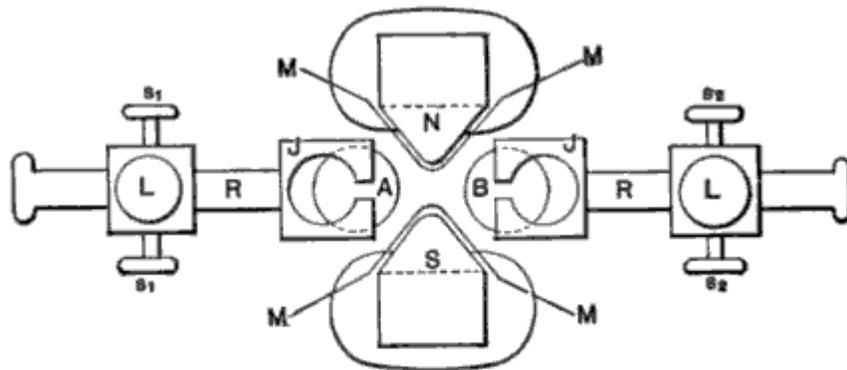
A cold, non-firing spark gap contains no ionized gases. Once the voltage across the gap reaches its breakdown voltage, gas molecules in the gap are very quickly ionized along a path, creating a hot electric arc, or plasma, that consists of large numbers of ions and free

electrons between the electrodes. The arc also heats part of the electrodes to incandescence. The incandescent regions contribute free electrons via thermionic emission, and (easily ionized) metal vapor. The mixture of ions and free electrons in the plasma is highly conductive, resulting in a sharp drop in the gap's electrical resistance. This highly conductive arc supports efficient tank circuit oscillations. However, the oscillating current also sustains the arc and, until it can be extinguished, the tank capacitor cannot be recharged for the next pulse.

Several methods were applied to quench the arc.

- Jets of air that cool, stretch, and literally 'blow out' the plasma,
- multi-plate discharger of Max Wien to cool the arcs in medium power spark sets, known as the "whistling spark" for its distinctive signal,
- using a different gas, such as hydrogen, that quenches more efficiently by providing more effective electrode cooling,
- a magnetic field (from a pair of permanent magnets or poles of an electromagnets) oriented at right angles to the gap to stretch and cool the arc.

Magnetic



ARRANGEMENT OF IMPROVED DISCHARGER AND MAGNET.

A magnetic blowout

Spark gaps used in early radio transmitters varied in construction, depending on the power to be handled. Some were fairly simple, consisting of one or more fixed (*static*) gaps connected in series, while others were significantly more complex. Because sparks were quite hot and erosive, electrode wear and cooling were constant problems.

Rotary gaps

The need to extinguish arcs in increasingly higher power transmitters led to the development of the rotating spark gap. These devices were used with an alternating current power supply, produced a more regular spark, and could handle more power than conventional static spark gaps. The inner rotating metal disc typically had a number of

studs on its outer edge. A discharge would take place when two of the studs lined up with the two outer contacts which carried the high voltage. The resulting arcs were rapidly stretched, cooled, and broken as the disk rotated.

Rotary gaps were operated in two modes, synchronous and asynchronous. A synchronous gap was driven by a synchronous AC motor so that it ran at a fixed speed, and the gap fired in direct relation to the waveform of the A.C. supply that recharged the tank capacitor. The point in the waveform where the gaps were closest was changed by adjusting the rotor position on the motor shaft relative to the stator's studs. By properly adjusting the synchronous gap, it was possible to have the gap fire only at the voltage peaks of the input current. This technique allowed the tank circuit to fire only at successive voltage peaks, thereby delivering maximum energy from the fully charged tank capacitor each time the gap fired. The *break rate* was thus fixed at twice the incoming power frequency (typically, 100 to 120 breaks/second). When properly engineered and adjusted, synchronous spark gap systems delivered the largest amount of power to the antenna. However, electrode wear would progressively change the gap's *firing point*, so synchronous gaps were somewhat temperamental and difficult to maintain.

Asynchronous gaps were considerably more common. In an asynchronous gap, the rotation of the motor had no fixed relationship relative to the incoming AC waveform. Asynchronous gaps worked quite well and were much easier to maintain. By using a larger number of rotating studs or a higher rotational speed, many asynchronous gaps operated at break rates in excess of 400 breaks/second. Since the gap could be fired more often than the input waveform switched polarity, the tank capacitor was charged and discharged more rapidly than a synchronous gap. However, each discharge would occur at a varying voltage that was almost always lower than the consistent peak voltage obtained from a synchronous gap.

Rotary gaps also served to alter the tone of the transmitter, since changing either the number of studs or the rotational speed changed the spark discharge frequency which was audible in receivers with detectors that could detect the modulation on the spark signal. This enabled listeners to distinguish between different transmitters that were nominally tuned to the same frequency. A typical high-power multiple spark system (as it was also called) used a 9-to-24-inch-diameter (230 to 610 mm) rotating commutator with six to twelve studs per wheel, typically switching several thousand volts.

The output of a rotary spark gap transmitter was turned on and off by the operator using a special kind of telegraph key that switched power going to the high voltage power supply. The key was designed with large contacts to carry the heavy current that flowed into the low voltage (primary) side of the high voltage transformer (often in excess of 20 amps). Alternatively a relay was used to do the actual switching.

Chapter-8

Buck Converter

A **buck converter** is a step-down DC to DC converter. Its design is similar to the step-up boost converter, and like the boost converter it is a switched-mode power supply that uses two switches (a transistor and a diode), an inductor and a capacitor.

The simplest way to reduce the voltage of a DC supply is to use a linear regulator (such as a 7805), but linear regulators waste energy as they operate by bleeding off excess power as heat. Buck converters, on the other hand, can be remarkably efficient (95% or higher for integrated circuits), making them useful for tasks such as converting the 12–24 V typical battery voltage in a laptop down to the few volts needed by the processor.

Theory of operation

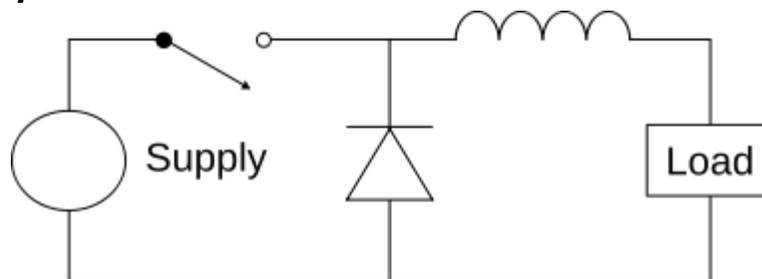
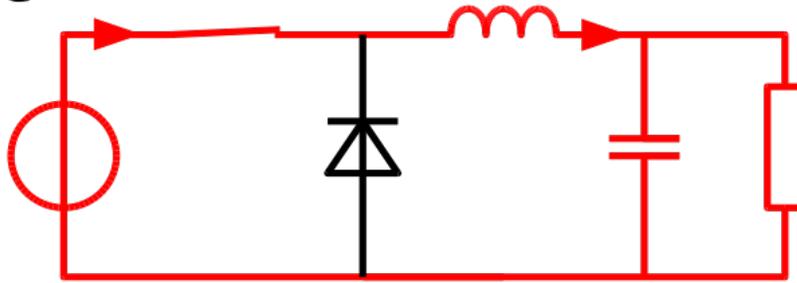


Fig. 1: Buck converter circuit diagram.

On-State



Off-State

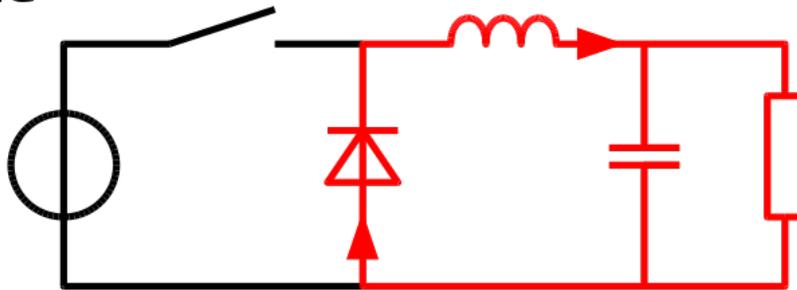


Fig. 2: The two circuit configurations of a buck converter: On-state, when the switch is closed, and Off-state, when the switch is open.

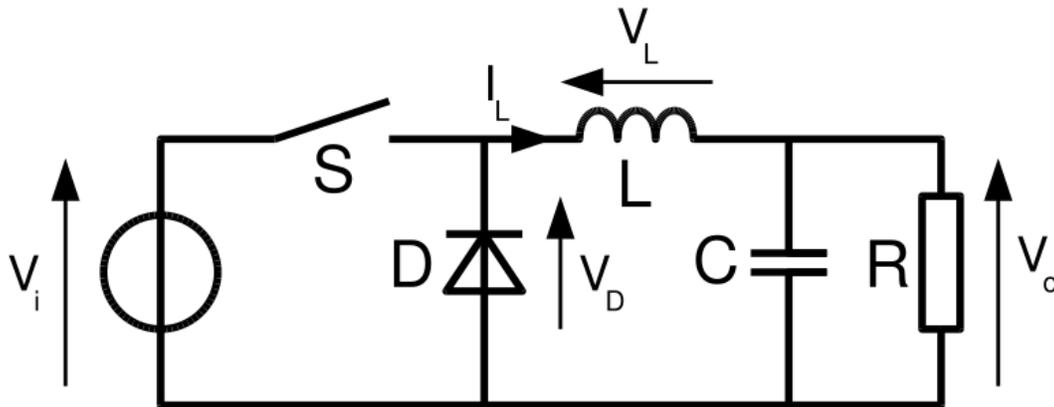


Fig. 3: Naming conventions of the components, voltages and current of the buck converter.

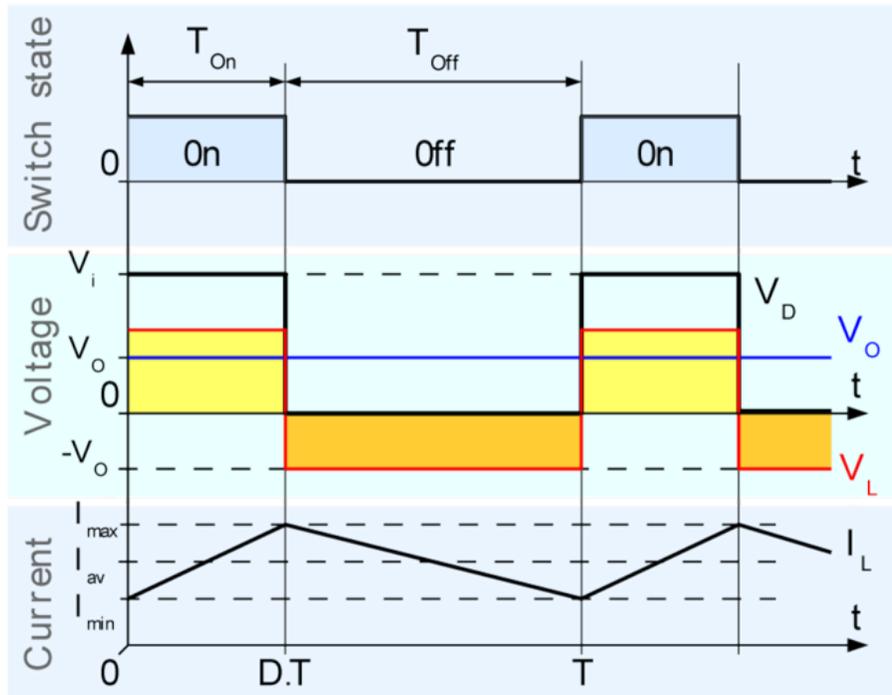


Fig. 4: Evolution of the voltages and currents with time in an ideal buck converter operating in continuous mode.

The operation of the buck converter is fairly simple, with an inductor and two switches (usually a transistor and a diode) that control the inductor. It alternates between connecting the inductor to source voltage to store energy in the inductor and discharging the inductor into the load.

Continuous mode

A buck converter operates in continuous mode if the current through the inductor (I_L) never falls to zero during the commutation cycle. In this mode, the operating principle is described by the chronogram in figure 4:

- When the switch pictured above is closed (On-state, top of figure 2), the voltage across the inductor is $V_L = V_i - V_o$. The current through the inductor rises linearly. As the diode is reverse-biased by the voltage source V , no current flows through it;
- When the switch is opened (off state, bottom of figure 2), the diode is forward biased. The voltage across the inductor is $V_L = -V_o$ (neglecting diode drop). Current I_L decreases.

The energy stored in inductor L is

$$E = \frac{1}{2}L \times I_L^2$$

Therefore, it can be seen that the energy stored in L increases during On-time (as I_L increases) and then decreases during the Off-state. L is used to transfer energy from the input to the output of the converter.

The rate of change of I_L can be calculated from:

$$V_L = L \frac{dI_L}{dt}$$

With V_L equal to $V_i - V_o$ during the On-state and to $-V_o$ during the Off-state. Therefore, the increase in current during the On-state is given by:

$$\Delta I_{L_{on}} = \int_0^{t_{on}} \frac{V_L}{L} dt = \frac{(V_i - V_o)}{L} t_{on}$$

Identically, the decrease in current during the Off-state is given by:

$$\Delta I_{L_{off}} = \int_0^{t_{off}} \frac{V_L}{L} dt = -\frac{V_o}{L} t_{off}$$

If we assume that the converter operates in steady state, the energy stored in each component at the end of a commutation cycle T is equal to that at the beginning of the cycle. That means that the current I_L is the same at $t=0$ and at $t=T$ (see figure 4). Therefore,

So we can write from the above equations:

$$\frac{(V_i - V_o)}{L} t_{on} - \frac{V_o}{L} t_{off} = 0$$

It is worth noting that the above integrations can be done graphically: In figure 4, $\Delta I_{L_{on}}$ is proportional to the area of the yellow surface, and $\Delta I_{L_{off}}$ to the area of the orange surface, as these surfaces are defined by the inductor voltage (red) curve. As these surfaces are simple rectangles, their areas can be found easily: $(V_i - V_o) t_{on}$ for the yellow rectangle and $-V_o t_{off}$ for the orange one. For steady state operation, these areas must be equal.

As can be seen on figure 4, $t_{on} = DT$ and $t_{off} = (1-D)T$. D is a scalar called the *duty cycle* with a value between 0 and 1. This yields:

$$(V_i - V_o)DT - V_o(1 - D)T = 0$$

$$\Rightarrow V_o - DV_i = 0$$

$$\Rightarrow D = \frac{V_o}{V_i}$$

From this equation, it can be seen that the output voltage of the converter varies linearly with the duty cycle for a given input voltage. As the duty cycle D is equal to the ratio between t_{On} and the period T , it cannot be more than 1. Therefore, $V_o \leq V_i$. This is why this converter is referred to as *step-down converter*.

So, for example, stepping 12 V down to 3 V (output voltage equal to a fourth of the input voltage) would require a duty cycle of 25%, in our theoretically ideal circuit.

Discontinuous mode

In some cases, the amount of energy required by the load is small enough to be transferred in a time lower than the whole commutation period. In this case, the current through the inductor falls to zero during part of the period. The only difference in the principle described above is that the inductor is completely discharged at the end of the commutation cycle (see figure 5). This has, however, some effect on the previous equations.

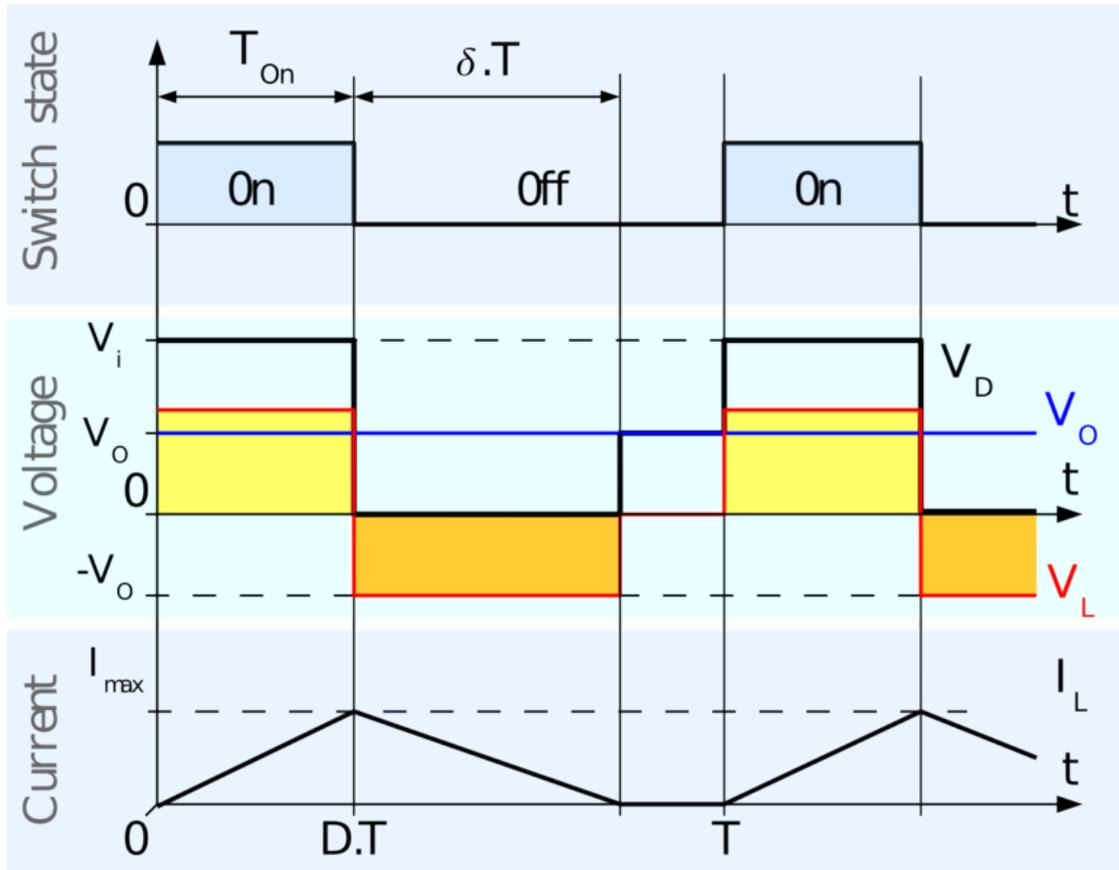


Fig. 5: Evolution of the voltages and currents with time in an ideal buck converter operating in discontinuous mode.

We still consider that the converter operates in steady state. Therefore, the energy in the inductor is the same at the beginning and at the end of the cycle (in the case of discontinuous mode, it is zero). This means that the average value of the inductor voltage (V_L) is zero; i.e., that the area of the yellow and orange rectangles in figure 5 are the same. This yields:

$$(V_i - V_o) DT - V_o \delta T = 0$$

So the value of δ is:

$$\delta = \frac{V_i - V_o}{V_o} D$$

The output current delivered to the load (I_o) is constant, as we consider that the output capacitor is large enough to maintain a constant voltage across its terminals during a commutation cycle. This implies that the current flowing through the capacitor has a zero average value. Therefore, we have :

$$\bar{I}_L = I_o$$

Where \bar{I}_L is the average value of the inductor current. As can be seen in figure 5, the inductor current waveform has a triangular shape. Therefore, the average value of I_L can be sorted out geometrically as follow:

$$\begin{aligned}\bar{I}_L &= \left(\frac{1}{2} I_{L_{max}} DT + \frac{1}{2} I_{L_{max}} \delta T \right) \frac{1}{T} \\ &= \frac{I_{L_{max}} (D + \delta)}{2} \\ &= I_o\end{aligned}$$

The inductor current is zero at the beginning and rises during t_{on} up to $I_{L_{max}}$. That means that $I_{L_{max}}$ is equal to:

$$I_{L_{Max}} = \frac{V_i - V_o}{L} DT$$

Substituting the value of $I_{L_{max}}$ in the previous equation leads to:

$$I_o = \frac{(V_i - V_o) DT (D + \delta)}{2L}$$

And substituting δ by the expression given above yields:

$$I_o = \frac{(V_i - V_o) DT \left(D + \frac{V_i - V_o}{V_o} D \right)}{2L}$$

This expression can be rewritten as:

$$V_o = V_i \frac{1}{\frac{2LI_o}{D^2 V_i T} + 1}$$

It can be seen that the output voltage of a buck converter operating in discontinuous mode is much more complicated than its counterpart of the continuous mode.

Furthermore, the output voltage is now a function not only of the input voltage (V_i) and the duty cycle D , but also of the inductor value (L), the commutation period (T) and the output current (I_o).

From discontinuous to continuous mode (and vice versa)

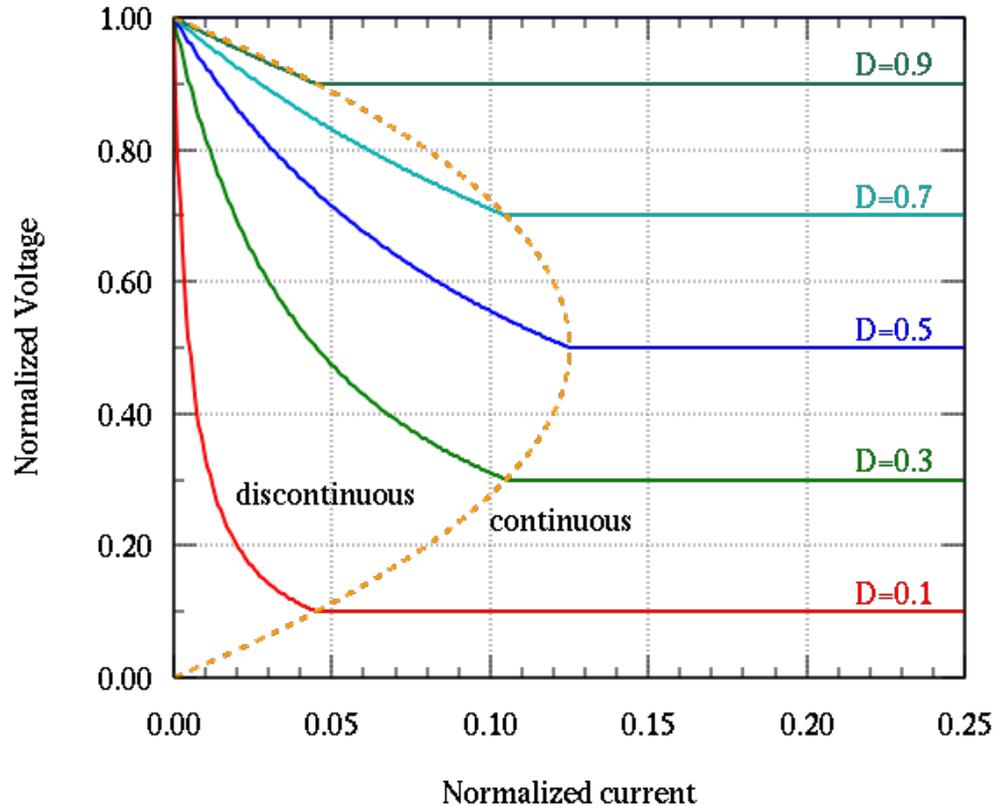


Fig. 6: Evolution of the normalized output voltages with the normalized output current.

As mentioned at the beginning, the converter operates in discontinuous mode when low current is drawn by the load, and in continuous mode at higher load current levels. The limit between discontinuous and continuous modes is reached when the inductor current falls to zero exactly at the end of the commutation cycle. with the notations of figure 5, this corresponds to :

$$DT + \delta T = T$$

$$\Rightarrow D + \delta = 1$$

Therefore, the output current (equal to the average inductor current) at the limit between discontinuous and continuous modes is (see above):

$$I_{olim} = \frac{I_{Lmax}}{2} (D + \delta) = \frac{I_{Lmax}}{2}$$

Substituting I_{Lmax} by its value:

$$I_{o\lim} = \frac{V_i - V_o}{2L} DT$$

On the limit between the two modes, the output voltage obeys both the expressions given respectively in the continuous and the discontinuous sections. In particular, the former is

$$V_o = DV_i$$

So $I_{o\lim}$ can be written as:

$$I_{o\lim} = \frac{V_i (1 - D)}{2L} DT$$

Let's now introduce two more notations:

- the normalized voltage, defined by $|V_o| = \frac{V_o}{V_i}$. It is zero when $V_o = 0$, and 1 when $V_o = V_i$;
- the normalized current, defined by $|I_o| = \frac{L}{TV_i} I_o$. The term $\frac{TV_i}{L}$ is equal to the maximum increase of the inductor current during a cycle; i.e., the increase of the inductor current with a duty cycle $D=1$. So, in steady state operation of the converter, this means that $|I_o|$ equals 0 for no output current, and 1 for the maximum current the converter can deliver.

Using these notations, we have:

- in continuous mode:

$$|V_o| = D$$

- in discontinuous mode:

$$\begin{aligned} |V_o| &= \frac{1}{\frac{2LI_o}{D^2V_iT} + 1} \\ &= \frac{1}{\frac{2|I_o|}{D^2} + 1} \\ &= \frac{D^2}{2|I_o| + D^2} \end{aligned}$$

the current at the limit between continuous and discontinuous mode is:

$$I_{o_{lim}} = \frac{V_i}{2L} D (1 - D) T$$

$$= \frac{I_o}{2|I_o|} D (1 - D)$$

Therefore, the locus of the limit between continuous and discontinuous modes is given by:

$$\frac{(1 - D) D}{2|I_o|} = 1$$

These expressions have been plotted in figure 6. From this, it is obvious that in continuous mode, the output voltage does only depend on the duty cycle, whereas it is far more complex in the discontinuous mode. This is important from a control point of view.

Non ideal circuit

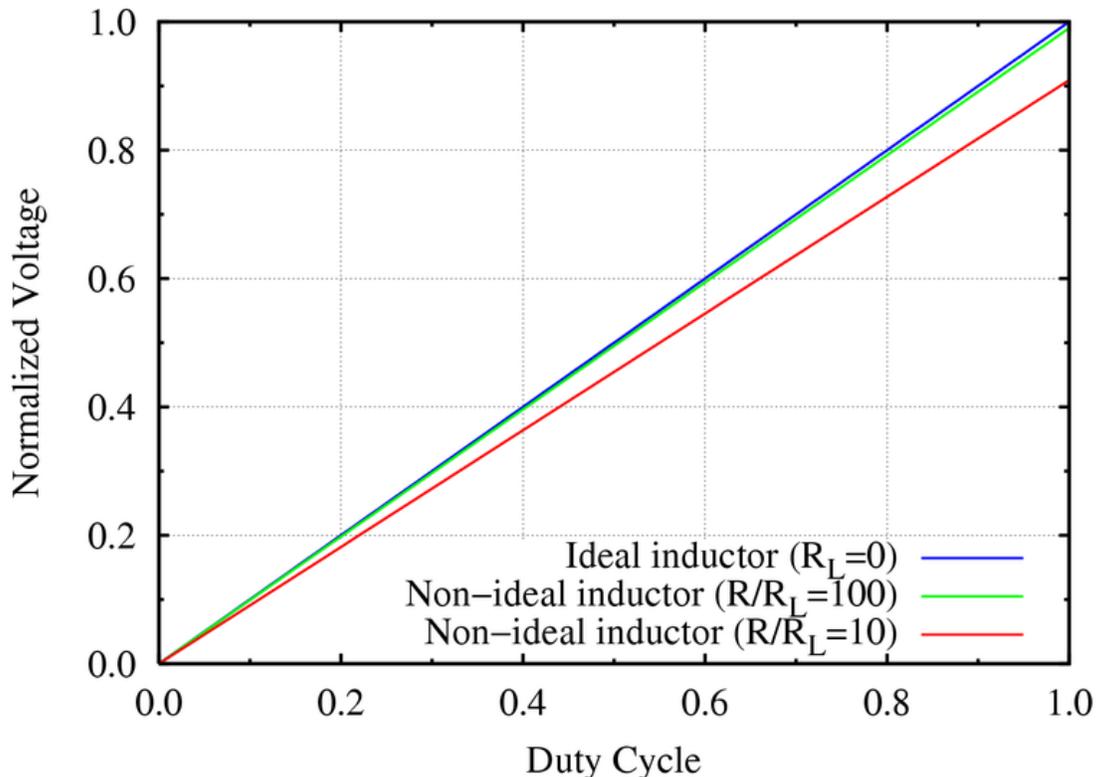


Fig. 7: Evolution of the output voltage of a buck converter with the duty cycle when the parasitic resistance of the inductor increases.

The previous study was conducted with the following assumptions:

- The output capacitor has enough capacitance to supply power to the load (a simple resistance) without any noticeable variation in its voltage.
- The voltage drop across the diode when forward biased is zero
- No commutation losses in the switch nor in the diode

These assumptions can be fairly far from reality, and the imperfections of the real components can have a detrimental effect on the operation of the converter.

Output voltage ripple

Output voltage ripple is the name given to the phenomenon where the output voltage rises during the On-state and falls during the Off-state. Several factors contribute to this including, but not limited to, switching frequency, output capacitance, inductor, load and any current limiting features of the control circuitry. At the most basic level the output voltage will rise and fall as a result of the output capacitor charging and discharging:

$$dV_o = \frac{idT}{C}$$

During the Off-state, the current in this equation is the load current. In the On-state the current is the difference between the switch current (or source current) and the load current. The duration of time (dT) is defined by the duty cycle and by the switching frequency.

For the On-state:

$$dT_{on} = DT = \frac{D}{f}$$

For the Off-state:

$$dT_{off} = (1 - D)T = \frac{1 - D}{f}$$

Qualitatively, as the output capacitor or switching frequency increase, the magnitude of the ripple decreases. Output voltage ripple is typically a design specification for the power supply and is selected based on several factors. Capacitor selection is normally determined based on cost, physical size and non-idealities of various capacitor types. Switching frequency selection is typically determined based on efficiency requirements, which tends to decrease at higher operating frequencies, as described below in Effects of non-ideality on the efficiency. Higher switching frequency can also reduce efficiency and possibly raise EMI concerns.

Output voltage ripple is one of the disadvantages of a switching power supply, and can also be a measure of its quality.

Effects of non-ideality on the efficiency

A simplified analysis of the buck converter, as described above, does not account for non-idealities of the circuit components nor does it account for the required control circuitry. Power losses due to the control circuitry is usually insignificant when compared with the losses in the power devices (switches, diodes, inductors, etc.) The non-idealities of the power devices account for the bulk of the power losses in the converter.

Both static and dynamic power losses occur in any switching regulator. Static power losses include I^2R (conduction) losses in the wires or PCB traces, as well as in the switches and inductor, as in any electrical circuit. Dynamic power losses occur as a result of switching, such as the charging and discharging of the switch gate, and are proportional to the switching frequency.

It is useful to begin by calculating the duty cycle for a non-ideal buck converter, which is:

$$D = \frac{V_o + (V_{SYNCSW} + V_L)}{V_i - V_{SWITCH} + V_{SYNCSW}}$$

where:

- V_{SWITCH} is the voltage drop on the power switch,
- V_{SYNCSW} is the voltage drop on the synchronous switch or diode, and
- V_L is the voltage drop on the inductor.

The voltage drops described above are all static power losses which are dependent primarily on DC current, and can therefore be easily calculated. For a transistor in saturation or a diode drop, V_{SWITCH} and V_{SYNCSW} may already be known, based on the properties of the selected device.

$$V_{SWITCH} = I_{SWITCH}R_{ON} = DI_oR_{ON}$$

$$V_{SYNCSW} = I_{SYNCSW}R_{ON} = (1 - D)I_oR_{ON}$$

$$V_L = I_LR_{DCR}$$

where:

- R_{ON} is the ON-resistance of each switch (R_{DSON} for a MOSFET), and
- R_{DCR} is the DC resistance of the inductor.

The careful reader will note that the duty cycle equation is somewhat recursive. A rough analysis can be made by first calculating the values V_{SWITCH} and V_{SYNCSW} using the ideal duty cycle equation.

Switch resistance, for components such as the power MOSFET, and forward voltage, for components such as the insulated-gate bipolar transistor (IGBT) can be determined by referring to datasheet specifications.

In addition, power loss occurs as a result of leakage currents. This power loss is simply

$$P_{LEAKAGE} = I_{LEAKAGE}V$$

where:

- $I_{LEAKAGE}$ is the leakage current of the switch, and
- V is the voltage across the switch.

Dynamic power losses are due to the switching behavior of the selected pass devices (MOSFETs, power transistors, IGBTs, etc.). These losses include turn-on and turn-off switching losses and switch transition losses.

Switch turn-on and turn-off losses are easily lumped together as

$$P_{SW} = \frac{VI_o(t_{RISE} + t_{FALL})}{6T}$$

where:

- V is the voltage across the switch while the switch is off,
- t_{RISE} and t_{FALL} are the switch rise and fall times, and
- T is the switching period.

But this doesn't take into account the parasitic capacitance of the MOSFET which makes the *Miller plate*. Then, the switch losses will be more like:

$$P_{SW} = \frac{VI_o(t_{RISE} + t_{FALL})}{2T}$$

When a MOSFET is used for the lower switch, additional losses may occur during the time between the turn-off of the high-side switch and the turn-on of the low-side switch, when the body diode of the low-side MOSFET conducts the output current. This time, known as the non-overlap time, prevents "shootthrough", a condition in which both switches are simultaneously turned on. The onset of shootthrough generates severe power loss and heat. Proper selection of non-overlap time must balance the risk of shootthrough with the increased power loss caused by conduction of the body diode. When a diode is used for the lower switch, diode forward turn-on time can reduce efficiency and lead to voltage overshoot.

Power loss on the body diode is also proportional to switching frequency and is

$$P_{BODYDIODE} = V_F I_o t_{NO} f_{SW}$$

where:

- V_F is the forward voltage of the body diode, and
- t_{NO} is the selected non-overlap time.

Finally, power losses occur as a result of the power required to turn the switches on and off. For MOSFET switches, these losses are dominated by the gate charge, essentially the energy required to charge and discharge the capacitance of the MOSFET gate between the threshold voltage and the selected gate voltage. These switch transition losses occur primarily in the gate driver, and can be minimized by selecting MOSFETs with low gate charge, by driving the MOSFET gate to a lower voltage (at the cost of increased MOSFET conduction losses), or by operating at a lower frequency.

$$P_{GATEDRIVE} = Q_G V_{GS} f_{SW}$$

where:

- Q_G is the gate charge of the selected MOSFET, and
- V_{GS} is the peak gate-source voltage.

It is essential to remember that, for N-MOSFETs, the high-side switch must be driven to a higher voltage than V_i . Therefore V_G will nearly always be different for the high-side and low-side switches.

A complete design for a buck converter includes a tradeoff analysis of the various power losses. Designers balance these losses according to the expected uses of the finished design. A converter expected to have a low switching frequency does not require switches with low gate transition losses; a converter operating at a high duty cycle requires a low-side switch with low conduction losses.

Specific structures

Synchronous rectification

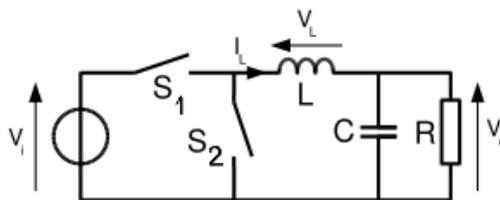


Fig. 8: Simplified schematic of a synchronous converter, in which D is replaced by a second switch, S_2

A synchronous buck converter is a modified version of the basic buck converter circuit topology in which the diode, D, is replaced by a second switch, S₂. This modification is a tradeoff between increased cost and improved efficiency.

In a standard buck converter, the freewheeling diode turns on, on its own, shortly after the switch turns off, as a result of the rising voltage across the diode. This voltage drop across the diode results in a power loss which is equal to

$$P_D = V_D(1 - D)I_o$$

where:

- V_D is the voltage drop across the diode at the load current I_o ,
- D is the duty cycle, and
- I_o is the load current.

By replacing diode D with switch S₂, which is advantageously selected for low losses, the converter efficiency can be improved. For example, a MOSFET with very low $R_{DS(on)}$ might be selected for S₂, providing power loss on switch 2 which is

$$P_{S2} = I_o^2 R_{DS(on)}(1 - D)$$

By comparing these equations the reader will note that in both cases, power loss is strongly dependent on the duty cycle, D. It stands to reason that the power loss on the freewheeling diode or lower switch will be proportional to its on-time. Therefore, systems designed for low duty cycle operation will suffer from higher losses in the freewheeling diode or lower switch, and for such systems it is advantageous to consider a synchronous buck converter design.

Without actual numbers the reader will find the usefulness of this substitution to be unclear. Consider a computer power supply, where the input is 5 V, the output is 3.3 V, and the load current is 10A. In this case, the duty cycle will be 66% and the diode would be on for 34% of the time. A typical diode with forward voltage of 0.7 V would suffer a power loss of 2.38 W. A well-selected MOSFET with $R_{DS(on)}$ of 0.015 Ω , however, would waste only 0.51 W in conduction loss. This translates to improved efficiency and reduced heat loss.

Another advantage of the synchronous converter is that it is bi-directional, which lends itself to applications requiring regenerative braking. When power is transferred in the "reverse" direction, it acts much like a boost converter.

The advantages of the synchronous buck converter do not come without cost. First, the lower switch typically costs more than the freewheeling diode. Second, the complexity of the converter is vastly increased due to the need for a complementary-output switch driver.

Such a driver must prevent both switches from being turned on at the same time, a fault known as "shootthrough." The simplest technique for avoiding shootthrough is a time delay between the turn-off of S_1 to the turn-on of S_2 , and vice versa. However, setting this time delay long enough to ensure that S_1 and S_2 are never both on will itself result in excess power loss. An improved technique for preventing this condition is known as adaptive "non-overlap" protection, in which the voltage at the switch node (the point where S_1 , S_2 and L are joined) is sensed to determine its state. When the switch node voltage passes a preset threshold, the time delay is started. The driver can thus adjust to many types of switches without the excessive power loss this flexibility would cause with a fixed non-overlap time.

Multiphase buck

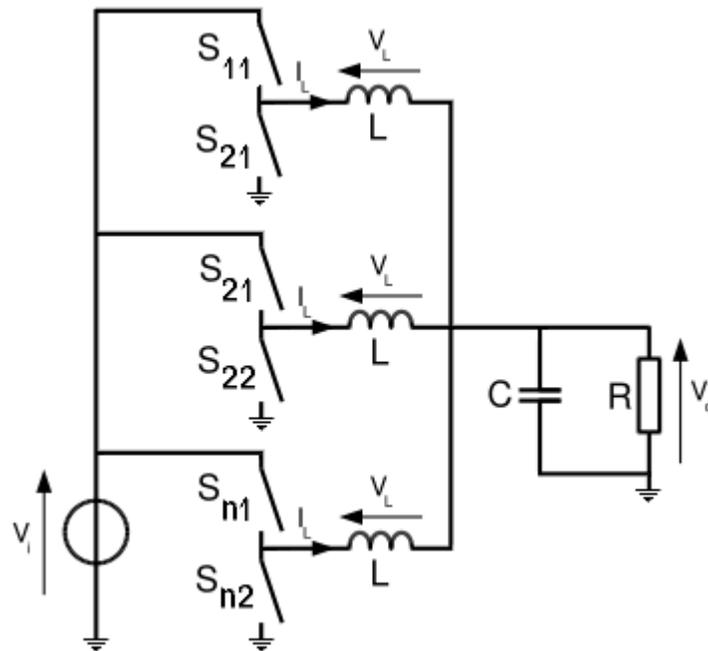


Fig. 9: Schematic of a generic synchronous n -phase buck converter.

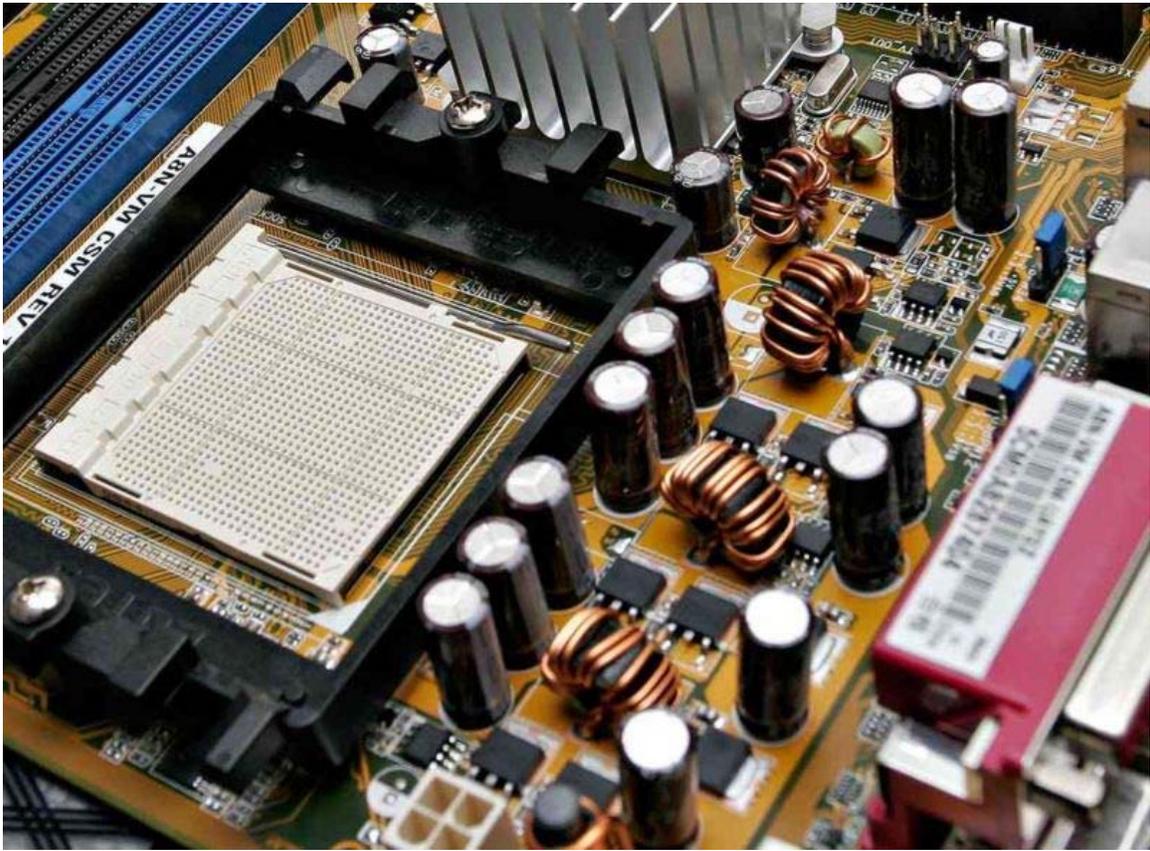


Fig. 10: Closeup picture of a multiphase CPU power supply for an AMD Socket 939 processor. The three phases of this supply can be recognized by the three black toroidal inductors in the foreground. The smaller inductor below the heat sink is part of an input filter.

The multiphase buck converter is circuit topology where the basic buck converter circuit are placed in parallel between the input and load. Each of the n "phases" is turned on at equally spaced intervals over the switching period. This circuit is typically used with the synchronous buck topology, described above.

The primary advantage of this type of converter is that it can respond to load changes as quickly as if it switched at n times as fast, without the increase in switching losses that that would cause. Thus, it can respond to rapidly changing loads, such as modern microprocessors.

There is also a significant decrease in switching ripple. Not only is there the decrease due to the increased effective frequency, but any time that n times the duty cycle is an integer, the switching ripple goes to 0; the rate at which the inductor current is increasing in the phases which are switched on exactly matches the rate at which it is decreasing in the phases which are switched off.

Another advantage is that the load current is split among the n phases of the multiphase converter. This load splitting allows the heat losses on each of the switches to be spread across a larger area.

This circuit topology is used in computer power supplies to convert the 12 V_{DC} power supply to a lower voltage (around 1 V), suitable for the CPU. Modern CPU power requirements can exceed 200W, can change very rapidly, and have very tight ripple requirements, less than 10mV. Typical motherboard power supplies use 3 or 4 phases, although control IC manufacturers allow as many as 6 phases

One major challenge inherent in the multiphase converter is ensuring the load current is balanced evenly across the n phases. This current balancing can be performed in a number of ways. Current can be measured "losslessly" by sensing the voltage across the inductor or the lower switch (when it is turned on). This technique is considered lossless because it relies on resistive losses inherent in the buck converter topology. Another technique is to insert a small resistor in the circuit and measure the voltage across it. This approach is more accurate and adjustable, but incurs several costs—space, efficiency and money.

Finally, the current can be measured at the input. Voltage can be measured losslessly, across the upper switch, or using a power resistor, to approximate the current being drawn. This approach is technically more challenging, since switching noise cannot be easily filtered out. However, it is less expensive than emplacing a sense resistor for each phase.

Efficiency factors

Conduction losses that depend on load:

- Resistance when the transistor or MOSFET switch is conducting.
- Diode forward voltage drop (usually 0.7 V or 0.4 V for schottky diode)
- Inductor winding resistance
- Capacitor equivalent series resistance

Switching losses:

- Voltage-Ampere overlap loss
- Frequency_{switch}*CV² loss
- Reverse latency loss

Losses due driving MOSFET gate and controller consumption. Transistor leakage current losses, and controller standby consumption.

Impedance Matching

A buck converter can be used to maximize the power transfer through the use of impedance matching. An application of this is in a "maximum power point tracker" commonly used in photovoltaic systems.

By the equation for electric power:

$$V_o I_o = \eta V_i I_i$$

where:

- V_o is the output voltage
- I_o is the output current
- η is the power efficiency (ranging from 0 to 1)
- V_i is the input voltage
- I_i is the input current

By Ohm's Law:

$$I_o = V_o / Z_o$$

$$I_i = V_i / Z_i$$

where:

- Z_o is the output impedance
- Z_i is the input impedance

Substituting these expressions for I_o and I_i into the power equation yields:

$$V_o^2 / Z_o = \eta V_i^2 / Z_i$$

As was previously shown for the continuous mode, (where $I_L > 0$):

$$V_o = D V_i$$

where: D is the duty cycle

Substituting this equation for V_o into the previous equation, yields:

$$(D V_i)^2 / Z_o = \eta V_i^2 / Z_i$$

which reduces to:

$$D^2 / Z_o = \eta / Z_i$$

and finally:

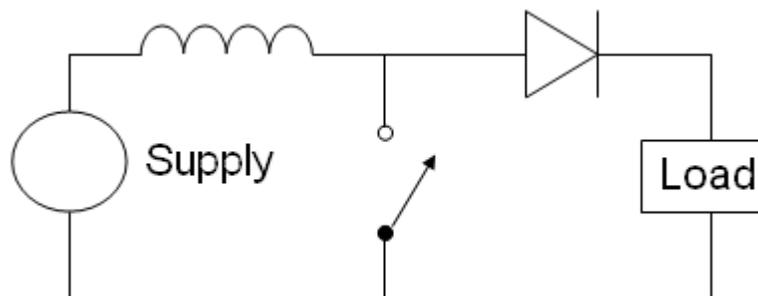
$$D = \sqrt{\eta Z_o / Z_i}$$

This shows that it is possible to adjust the impedance ratio by adjusting the duty cycle. This is particularly useful in applications where the impedance(s) are dynamically changing.

Chapter-9

Boost Converter

A **boost converter (step-up converter)** is a power converter with an output DC voltage greater than its input DC voltage. It is a class of switching-mode power supply (SMPS) containing at least two semiconductor switches (a diode and a transistor) and at least one energy storage element. Filters made of capacitors (sometimes in combination with inductors) are normally added to the output of the converter to reduce output voltage ripple.



The basic schematic of a boost converter. The switch is typically a MOSFET, IGBT, or BJT.

Overview

Power can also come from DC sources such as batteries, solar panels, rectifiers and DC generators. A process that changes one DC voltage to a different DC voltage is called DC to DC conversion. A boost converter is a DC to DC converter with an output voltage greater than the source voltage. A boost converter is sometimes called a step-up converter since it “steps up” the source voltage. Since power ($P = VI$) must be conserved, the output current is lower than the source current.

A boost converter is used as the voltage increase mechanism in the circuit known as the 'Joule thief'. This circuit topology is used with low power battery applications, and is aimed at the ability of a boost converter to 'steal' the remaining energy in a battery. This energy would otherwise be wasted since the low voltage of a nearly depleted battery makes it unusable for a normal load.*

- This energy would otherwise remain untapped because many applications do not allow enough current to flow through a load when voltage decreases. This voltage decrease occurs as batteries become depleted, and is a characteristic of the ubiquitous alkaline battery. Since ($P = V^2 / R$) as well, and R tends to be stable, power available to the load goes down significantly as voltage decreases.

History

For high efficiency, the SMPS switch must turn on and off quickly and have low losses. The advent of a commercial semiconductor switch in the 1950s represented a major milestone that made SMPSs such as the boost converter possible. Semiconductor switches turned on and off more quickly and lasted longer than other switches such as vacuum tubes and electromechanical relays. The major DC to DC converters were developed in the early 1960s when semiconductor switches had become available. The aerospace industry's need for small, lightweight, and efficient power converters led to the converter's rapid development.

Switched systems such as SMPS are a challenge to design since its model depends on whether a switch is opened or closed. R.D. Middlebrook from Caltech in 1977 published the models for DC to DC converters used today. Middlebrook averaged the circuit configurations for each switch state in a technique called state-space averaging. This simplification reduced two systems into one. The new model led to insightful design equations which helped SMPS growth.

Applications

Battery powered systems often stack cells in series to achieve higher voltage. However, sufficient stacking of cells is not possible in many high voltage applications due to lack of space. Boost converters can increase the voltage and reduce the number of cells. Two battery-powered applications that use boost converters are hybrid electric vehicles (HEV) and lighting systems.

The NHW20 model Toyota Prius HEV uses a 500 V motor. Without a boost converter, the Prius would need nearly 417 cells to power the motor. However, a Prius actually uses only 168 cells and boosts the battery voltage from 202 V to 500 V. Boost converters also power devices at smaller scale applications, such as portable lighting systems. A white LED typically requires 3.3 V to emit light, and a boost converter can step up the voltage from a single 1.5 V alkaline cell to power the lamp. Boost converters can also produce higher voltages to operate cold cathode fluorescent tubes (CCFL) in devices such as LCD backlights and some flashlights.

Circuit analysis

Operating principle

The key principle that drives the boost converter is the tendency of an inductor to resist changes in current. When being charged it acts as a load and absorbs energy (somewhat like a resistor), when being discharged, it acts as an energy source (somewhat like a battery). The voltage it produces during the discharge phase is related to the rate of change of current, and not to the original charging voltage, thus allowing different input and output voltages.

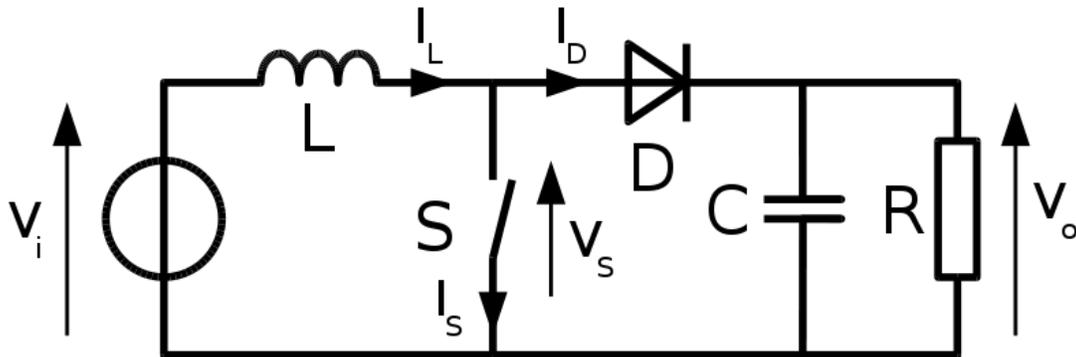
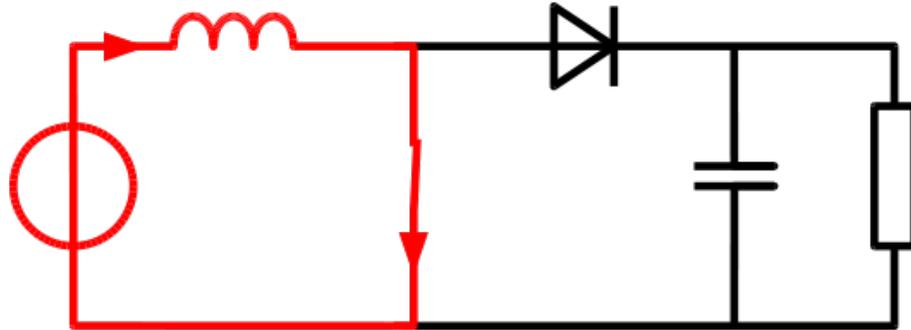


Fig. 1: Boost converter schematic

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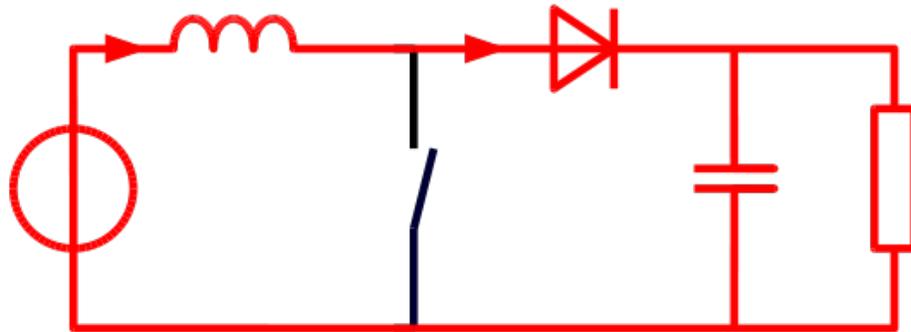


Fig. 2: The two configurations of a boost converter, depending on the state of the switch S.

The basic principle of a Boost converter consists of 2 distinct states (see figure 2):

- in the On-state, the switch S (see figure 1) is closed, resulting in an increase in the inductor current;
- in the Off-state, the switch is open and the only path offered to inductor current is through the flyback diode D, the capacitor C and the load R. This results in transferring the energy accumulated during the On-state into the capacitor.
- The input current is the same as the inductor current as can be seen in figure 2. So it is not discontinuous as in the buck converter and the requirements on the input filter are relaxed compared to a buck converter.

Continuous mode

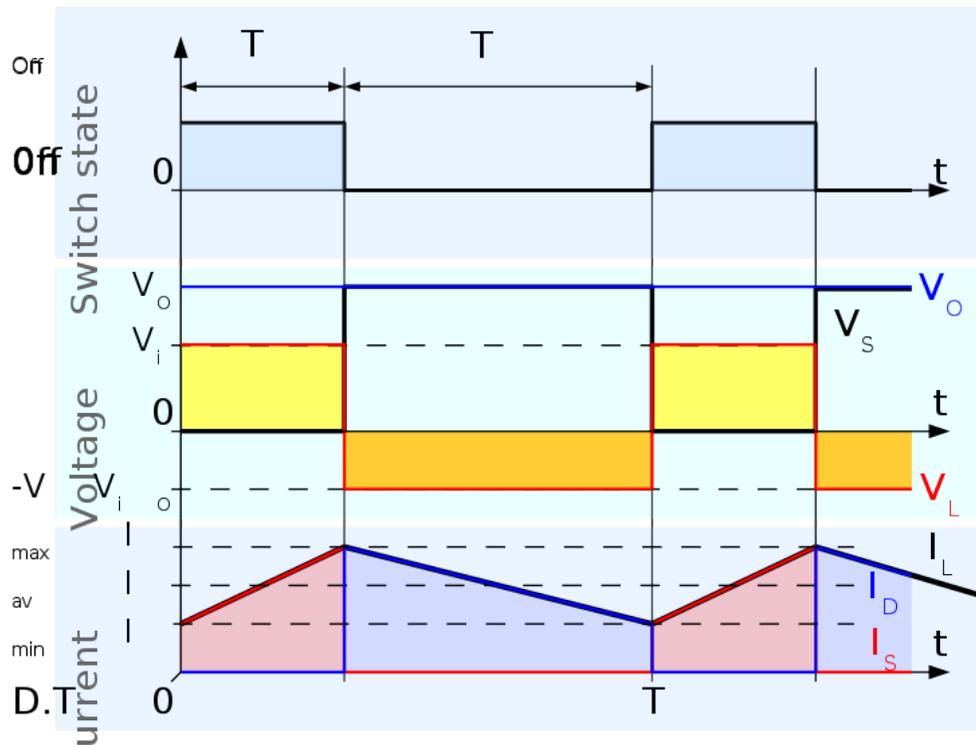


Fig. 3: Waveforms of current and voltage in a boost converter operating in continuous mode.

When a boost converter operates in continuous mode, the current through the inductor (I_L) never falls to zero. Figure 3 shows the typical waveforms of currents and voltages in a converter operating in this mode. The output voltage can be calculated as follows, in the case of an ideal converter (i.e. using components with an ideal behaviour) operating in steady conditions:

During the On-state, the switch S is closed, which makes the input voltage (V_i) appear across the inductor, which causes a change in current (I_L) flowing through the inductor during a time period (t) by the formula:

$$\frac{\Delta I_L}{\Delta t} = \frac{V_i}{L}$$

At the end of the On-state, the increase of I_L is therefore:

$$\Delta I_{L_{On}} = \frac{1}{L} \int_0^{DT} V_i dt = \frac{DT}{L} V_i$$

D is the duty cycle. It represents the fraction of the commutation period T during which the switch is On. Therefore D ranges between 0 (S is never on) and 1 (S is always on).

During the Off-state, the switch S is open, so the inductor current flows through the load. If we consider zero voltage drop in the diode, and a capacitor large enough for its voltage to remain constant, the evolution of I_L is:

$$V_i - V_o = L \frac{dI_L}{dt}$$

Therefore, the variation of I_L during the Off-period is:

$$\Delta I_{L\text{Off}} = \int_0^{(1-D)T} \frac{(V_i - V_o) dt}{L} = \frac{(V_i - V_o)(1 - D)T}{L}$$

As we consider that the converter operates in steady-state conditions, the amount of energy stored in each of its components has to be the same at the beginning and at the end of a commutation cycle. In particular, the energy stored in the inductor is given by:

$$E = \frac{1}{2} L I_L^2$$

So, the inductor current has to be the same at the start and end of the commutation cycle. This means the overall change in the current (the sum of the changes) is zero:

$$\Delta I_{L\text{On}} + \Delta I_{L\text{Off}} = 0$$

Substituting $\Delta I_{L\text{On}}$ and $\Delta I_{L\text{Off}}$ by their expressions yields:

$$\Delta I_{L\text{On}} + \Delta I_{L\text{Off}} = \frac{V_i D T}{L} + \frac{(V_i - V_o)(1 - D)T}{L} = 0$$

This can be written as:

$$\frac{V_o}{V_i} = \frac{1}{1 - D}$$

Which in turns reveals the duty cycle to be:

$$D = 1 - \frac{V_i}{V_o}$$

From the above expression it can be seen that the output voltage is always higher than the input voltage (as the duty cycle goes from 0 to 1), and that it increases with D, theoretically to infinity as D approaches 1. This is why this converter is sometimes referred to as a *step-up* converter.

Discontinuous mode

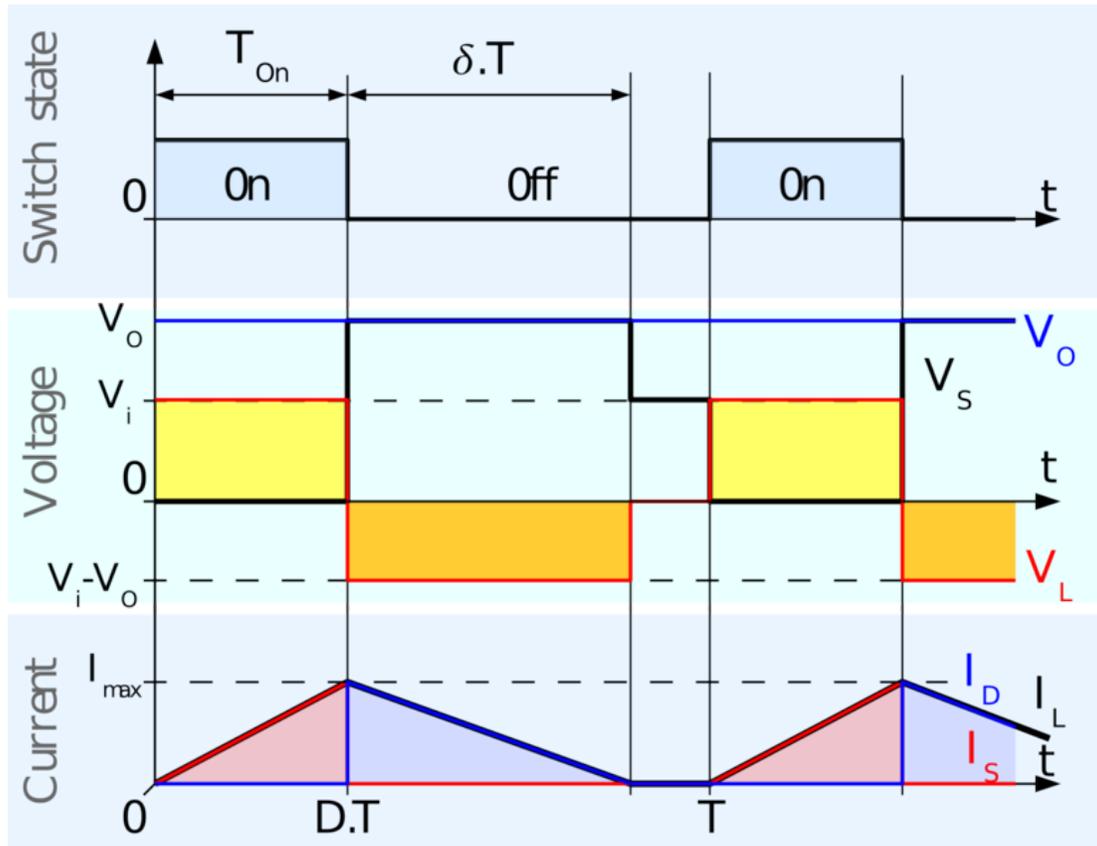


Fig. 4: Waveforms of current and voltage in a boost converter operating in discontinuous mode.

In some cases, the amount of energy required by the load is small enough to be transferred in a time smaller than the whole commutation period. In this case, the current through the inductor falls to zero during part of the period. The only difference in the principle described above is that the inductor is completely discharged at the end of the commutation cycle (see waveforms in figure 4). Although slight, the difference has a strong effect on the output voltage equation. It can be calculated as follows:

As the inductor current at the beginning of the cycle is zero, its maximum value $I_{L_{Max}}$ (at $t = DT$) is

$$I_{L_{Max}} = \frac{V_i DT}{L}$$

During the off-period, I_L falls to zero after δT :

$$I_{L_{Max}} + \frac{(V_i - V_o) \delta T}{L} = 0$$

Using the two previous equations, δ is:

$$\delta = \frac{V_i D}{V_o - V_i}$$

The load current I_o is equal to the average diode current (I_D). As can be seen on figure 4, the diode current is equal to the inductor current during the off-state. Therefore the output current can be written as:

$$I_o = \bar{I}_D = \frac{I_{L_{max}} \delta}{2}$$

Replacing $I_{L_{max}}$ and δ by their respective expressions yields:

$$I_o = \frac{V_i D T}{2L} \cdot \frac{V_i D}{V_o - V_i} = \frac{V_i^2 D^2 T}{2L (V_o - V_i)}$$

Therefore, the output voltage gain can be written as follows:

$$\frac{V_o}{V_i} = 1 + \frac{V_i D^2 T}{2L I_o}$$

Compared to the expression of the output voltage for the continuous mode, this expression is much more complicated. Furthermore, in discontinuous operation, the output voltage gain not only depends on the duty cycle, but also on the inductor value, the input voltage, the switching frequency, and the output current.