Shape of the Earth & its Models

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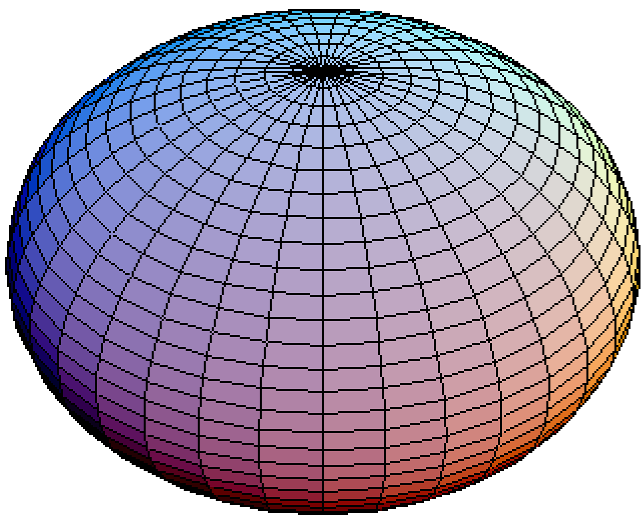
Figure of the Earth



An oblate spheroid

The expression **figure of the Earth** has various meanings in geodesy according to the way it is used and the precision with which the Earth's size and shape is to be defined. The actual topographic surface is most apparent with its variety of land forms and water areas. This is, in fact, the surface on which actual Earth measurements are made. It is not suitable, however, for exact mathematical computations, because the formulas which would be required to take the irregularities into account would necessitate a prohibitive amount of computations. The topographic surface is generally the concern of topographers and hydrographers.

The Pythagorean concept of a spherical Earth offers a simple surface which is mathematically easy to deal with. Many astronomical and navigational computations use it as a surface representing the Earth. While the sphere is a close approximation of the true figure of the Earth and satisfactory for many purposes, to the geodesists interested in the measurement of long distances—spanning continents and oceans—a more exact figure is necessary. Closer approximations range from modelling the shape of the entire Earth as an oblate spheroid or an oblate ellipsoid, to the use of spherical harmonics or local approximations in terms of local reference ellipsoids. The idea of a planar or flat surface for Earth, however, is still acceptable for surveys of small areas, as local topography is more important than the curvature. Plane-table surveys are made for relatively small areas, and no account is taken of the curvature of the Earth. A survey of a city would likely be computed as though the Earth were a plane surface the size of the city. For such small areas, exact positions can be determined relative to each other without considering the size and shape of the total Earth.

The curvature of Earth as seen in Valencia, Spain (Playa de la Malvarrosa)

In the mid- to late- 20th century, research across the geosciences contributed to drastic improvements in the accuracy of the Figure of the Earth. The primary utility (and the motivation for funding, mainly from the military) of this improved accuracy was to provide geographical and gravitational data for the inertial guidance systems of ballistic missiles. This funding also drove the expansion of geoscientific disciplines, fostering the creation and growth of various geoscience departments at many universities.

Ellipsoid of revolution

In geodesy, a **reference ellipsoid** is a mathematically-defined surface that approximates the geoid, the truer figure of the Earth, or other planetary body. Because of their relative simplicity, reference ellipsoids are used as a preferred surface on which geodetic network computations are performed and point coordinates such as latitude, longitude, and elevation are defined.

Ellipsoid parameters

Mathematically, a reference ellipsoid is usually an oblate (flattened) spheroid with two different axes: An equatorial radius (the semi-major axis a), and a polar radius (the semi-minor axis b). More rarely, a scalene ellipsoid with three axes (triaxial—$a_x,\ a_y,\ b$) is used, usually for modeling the smaller, irregularly shaped moons and asteroids. The polar axis here is the same as the axis with the great moment of inertia and is approximately aligned with the rotational axis (not the magnetic nor orbital pole). The geometric center of the ellipsoid is placed at the center of mass of the body being modeled, and not the barycenter in a multi-body system.

In working with elliptic geometry, several parameters are commonly utilized, all of which are trigonometric functions of an ellipse's **angular eccentricity**, $o\!\varepsilon$:

$$o\!\varepsilon = \arccos\left(\frac{b}{a}\right) = 2\arctan\left(\sqrt{\frac{a-b}{a+b}}\right);$$

Due to rotational forces, the equatorial radius is usually larger than the polar radius. This ellipticity or *flattening*, f, determines how close to a true sphere an oblate spheroid is, and is defined as

$$f = \text{ver}(o\!\varepsilon) = 2\sin^2\left(\frac{o\!\varepsilon}{2}\right) = 1 - \cos(o\!\varepsilon) = \frac{a-b}{a}.$$

For Earth, f is around 1/300 (1/298.257223563 as a defined constant in WGS84), and is very gradually decreasing over geologic time scales. For comparison, Earth's Moon is even less elliptical, with a flattening of less than 1/825, while Jupiter is visibly oblate at about 1/15 and one of Saturn's triaxial moons, Telesto, is nearly 1/3 to 1/2.

Such flattening is related to the eccentricity, e, of the cross-sectional ellipse by

$$e^2 = f(2-f) = \sin^2(o\!\varepsilon) = \frac{a^2-b^2}{a^2}.$$

It is traditional when defining a reference ellipsoid to specify the semi-major equatorial radius a(usually in meters) and the inverse of the flattening ratio $1/f$. The semi-minor polar radius is then easily derived.

Coordinates

A primary use of reference ellipsoids is to serve as a basis for a coordinate system of latitude (north/south), longitude (east/west), and elevation (height). For this purpose it is necessary to identify a *zero meridian*, which for Earth is usually the Prime Meridian. For other bodies a fixed surface feature is usually referenced, which for Mars is the meridian passing through the crater Airy-0. It is possible for many different coordinate systems to be defined upon the same reference ellipsoid.

The longitude measures the rotational angle between the zero meridian and the measured point. By convention for the Earth, Moon, and Sun it is expressed as degrees ranging from −180° to +180° For other bodies a range of 0° to 360° is used.

The latitude measures how close to the poles or equator a point is along a meridian, and is represented as angle from −90° to +90°, where 0° is the equator. The common or *geodetic latitude* is the angle between the equatorial plane and a line that is normal to the reference ellipsoid. Depending on the flattening, it may be slightly different from the *geocentric (geographic) latitude*, which is the angle between the equatorial plane and a line from the center of the ellipsoid. For non-Earth bodies the terms *planetographic* and *planetocentric* are used instead.

The coordinates of a geodetic point are customarily stated as geodetic latitude and longitude, i.e., the direction in space of the geodetic normal containing the point, and the height *h* of the point over the reference ellipsoid. If these coordinates, i.e., latitude ϕ, longitude $\lambda$and height *h*, are given, one can compute the *geocentric rectangular coordinates* of the point as follows:

$$X_t = [N + h]\cos(\phi)\cos(\lambda);$$
$$Y_t = [N + h]\cos(\phi)\sin(\lambda);$$
$$Z_t = [\cos^2(\alpha\!\!\!\!\epsilon) N + h]\sin(\phi);$$

where

$$N = N(\phi) = \frac{a}{\sqrt{1 - (\sin(\phi)\sin(\alpha\!\!\!\!\epsilon))^2}}$$

is the ***radius of curvature*** *in the prime vertical*.

In contrast, extracting ϕ, $\lambda$and *h* from the rectangular coordinates usually requires iteration:

Letting $\phi_c = \arctan(\sec^2(\alpha\!\!\!\!\epsilon)\tan(\psi_t))$,

$$\phi_p = \phi_c : \ \phi_c = \arctan\left(\frac{a^2 Z_t \quad + \frac{1}{4}[N(\phi_p)\sin(\phi_p)]^3\sin^2(2\alpha\!\!\!\!\epsilon)}{a^2\sqrt{X_t^2 + Y_t^2} \ - [N(\phi_p)\cos(\phi_p)]^3\sin^2(\alpha\!\!\!\!\epsilon)}\right);$$

Repeat until $\phi_c = \phi_p$; $\phi = \phi_c$.

Or, introducing the **geocentric**, ψ, and **parametric**, or **reduced**, β, latitudes:

$$\psi_t = \arctan\left(\frac{Z_t}{\sqrt{X_t^2 + Y_t^2}}\right)$$ and $\beta_c = \arctan(\sec(\alpha\!\!\!\!\epsilon)\tan(\psi_t))$,

$$\phi_p = \phi_c : \ \phi_c = \arctan\left(\frac{Z_t \quad + b\sin^3(\beta_c)\tan^2(\alpha\!\!\!\!\epsilon)}{\sqrt{X_t^2 + Y_t^2} \ - a\cos^3(\beta_c)\sin^2(\alpha\!\!\!\!\epsilon)}\right);$$

$$\beta_p = \beta_c : \ \beta_c = \arctan(\cos(\alpha\!\!\!\!\epsilon)\tan(\phi_c));$$

Repeat until $\phi_c = \phi_p$ and $\beta_c = \beta_p$:

$$\phi = \phi_c; \quad \beta = \beta_c; \quad \psi = \arctan(\cos(\ae)\tan(\beta)).$$

Once ϕ is determined, then *h* can be isolated:

$$h = \sec(\phi)\sqrt{X_t^2 + Y_t^2} - N \;=\; \csc(\phi)Z_t - \cos^2(\ae)N,$$
$$= \cos(\phi)\sqrt{X_t^2 + Y_t^2} \;+\; \sin(\phi)\left[Z_t + \sin^2(\ae)N\sin(\phi)\right] - N.$$

In contrast, as $[N + h]\cos(\phi)$ cancels out, λ.is a simple extraction from X_t and Y_t:

$$\lambda = \arctan\left(\frac{Y_t}{X_t}\right).$$

Historical Earth ellipsoids

Currently the most common reference ellipsoid used, and that used in the context of the Global Positioning System, is WGS 84.

Traditional reference ellipsoids or *geodetic datums* are defined regionally and therefore non-geocentric, e.g., ED50. Modern geodetic datums are established with the aid of GPS and will therefore be geocentric, e.g., WGS 84.

The following table lists some of the most common ellipsoids:

Name	Equatorial axis (m)	Polar axis (m)	Inverse flattening, $1/f$
Airy 1830	6 377 563.4	6 356 256.9	299.324 975 3
Clarke 1866	6 378 206.4	6 356 583.8	294.978 698 2
Bessel 1841	6 377 397.155	6 356 078.965	299.152 843 4
International 1924	6 378 388	6 356 911.9	296.999 362 1
Krasovsky 1940	6 378 245	6 356 863	298.299 738 1
GRS 1980	6 378 137	6 356 752.3141	298.257 222 101
WGS 1984	6 378 137	6 356 752.3142	298.257 223 563
Sphere (6371 km)	6 371 000	6 371 000	∞

Ellipsoids for other planetary bodies

Reference ellipsoids are also useful for geodetic mapping of other planetary bodies including planets, their satellites, asteroids and comet nuclei. Some well observed bodies such as the Moon and Mars now have quite precise reference ellipsoids.

For rigid-surface nearly-spherical bodies, which includes all the rocky planets and many moons, ellipsoids are defined in terms of the axis of rotation and the mean surface height excluding any atmosphere. Mars is actually egg shaped, where its north and south polar radii differ by approximately 6 km, however this difference is small enough that the average polar radius is used to define its ellipsoid. The Earth's Moon is effectively spherical, having no bulge at its equator. Where possible a fixed observable surface feature is used when defining a reference meridian.

For gaseous planets like Jupiter, an effective surface for an ellipsoid is chosen as the equal-pressure boundary of one bar. Since they have no permanent observable features the choices of prime meridians are made according to mathematical rules.

Small moons, asteroids, and comet nuclei frequently have irregular shapes. For some of these, such as Jupiter's Io, a scalene (triaxial) ellipsoid is a better fit than the oblate spheroid. For highly irregular bodies the concept of a reference ellipsoid may have no useful value, so sometimes a spherical reference is used instead and points identified by planetocentric latitude and longitude. Even that can be problematic for non-convex bodies, such as Eros, in that latitude and longitude don't always uniquely identify a single surface location.

Historical Earth ellipsoids

The reference ellipsoid models listed below have had utility in geodetic work and many are still in use. The older ellipsoids are named for the individual who derived them and the year of development is given. In 1887 the English mathematician Col Alexander Ross Clarke CB FRS RE was awarded the Gold Medal of the Royal Society for his work in determining the figure of the Earth. The international ellipsoid was developed by John Fillmore Hayford in 1910 and adopted by the International Union of Geodesy and Geophysics (IUGG) in 1924, which recommended it for international use.

At the 1967 meeting of the IUGG held in Lucerne, Switzerland, the ellipsoid called GRS-67 (Geodetic Reference System 1967) in the listing was recommended for adoption. The new ellipsoid was not recommended to replace the International Ellipsoid (1924), but was advocated for use where a greater degree of accuracy is required. It became a part of the GRS-67 which was approved and adopted at the 1971 meeting of the IUGG held in Moscow. It is used in Australia for the Australian Geodetic Datum and in South America for the South American Datum 1969.

The GRS-80 (Geodetic Reference System 1980) as approved and adopted by the IUGG at its Canberra, Australia meeting of 1979 is based on the equatorial radius (semi-major axis of Earth ellipsoid) a, total mass GM, dynamic form factor J_2 and angular velocity of rotation ω, making the inverse flattening $1 / f$ a derived quantity. The minute difference in $1 / f$ seen between GRS-80 and WGS-84 results from an unintentional truncation in the latter's defining constants: while the WGS-84 was designed to adhere closely to the GRS-80, incidentally the WGS-84 derived flattening turned out to be slightly different than the GRS-80 flattening because the normalized second degree zonal harmonic gravitational coefficient, that was derived from the GRS-80 value for J2, was truncated to 8 significant digits in the normalization process.

An ellipsoidal model describes only the ellipsoid's geometry and a normal gravity field formula to go with it. Commonly an ellipsoidal model is part of a more encompassing geodetic datum. For example, the older ED-50 (European Datum 1950) is based on the Hayford or International Ellipsoid. WGS-84 is peculiar in that the same name is used for both the complete geodetic reference system and its component ellipsoidal model. Nevertheless the two concepts—ellipsoidal model and geodetic reference system—remain distinct.

Note that the same ellipsoid may be known by different names. It is best to mention the defining constants for unambiguous identification.

Reference ellipsoid name	Equatorial radius (m)	Polar radius (m)	Inverse flattening	Where used
Maupertuis (1738)	6,397,300	6,363,806.283	191	France
Plessis (1817)	6,376,523.0	???	308.64	France
Everest (1830)	6,377,299.365	6,356,098.359	300.80172554	India
Everest 1830 Modified (1967)	6,377,304.063	6,356,103.0390	300.8017	West Malaysia & Singapore
Everest 1830 (1967 Definition)	6,377,298.556	6,356,097.550	300.8017	Brunei & East Malaysia
Airy (1830)	6,377,563.396	6,356,256.909	299.3249646	Britain
Bessel (1841)	6,377,397.155	6,356,078.963	299.1528128	Europe, Japan
Clarke (1866)	6,378,206.4	6,356,583.8	294.9786982	North America
Clarke (1878)	6,378,190	6,356,456	293.4659980	North America
Clarke (1880)	6,378,249.145	6,356,514.870	293.465	France, Africa
Helmert (1906)	6,378,200	6,356,818.17	298.3	
Hayford (1910)	6,378,388	6,356,911.946	297	USA
International (1924)	6,378,388	6,356,911.946	297	Europe
NAD 27 (1927)	6,378,206.4	6,356,583.800	294.978698208	North America
Krassovsky (1940)	6,378,245	6,356,863.019	298.3	Russia
WGS66 (1966)	6,378,145	6,356,759.769	298.25	USA/DoD
Australian National (1966)	6,378,160	6,356,774.719	298.25	Australia
New International (1967)	6,378,157.5	6,356,772.2	298.24961539	
GRS-67 (1967)	6,378,160	6,356,774.516	298.247167427	
South American (1969)	6,378,160	6,356,774.719	298.25	South America

WGS-72 (1972)	6,378,135	6,356,750.52	298.26	USA/DoD
GRS-80 (1979)	6,378,137	6,356,752.3141	298.257222101	Global ITRS
NAD 83	6,378,137	6,356,752.3	298.257024899	North America
WGS-84 (1984)	6,378,137	6,356,752.3142	298.257223563	Global GPS
IERS (1989)	6,378,136	6,356,751.302	298.257	
IERS (2003)	6,378,136.6	6,356,751.9	298.25642	

More complicated figures

The possibility that the Earth's equator is an ellipse rather than a circle and therefore that the ellipsoid is triaxial has been a matter of scientific controversy for many years. Modern technological developments have furnished new and rapid methods for data collection and since the launch of *Sputnik 1*, orbital data have been used to investigate the theory of ellipticity.

A second theory, more complicated than triaxiality, proposed that observed long periodic orbital variations of the first Earth satellites indicate an additional depression at the south pole accompanied by a bulge of the same degree at the north pole. It is also contended that the northern middle latitudes were slightly flattened and the southern middle latitudes bulged in a similar amount. This concept suggested a slightly pear-shaped Earth and was the subject of much public discussion. Modern geodesy tends to retain the ellipsoid of revolution and treat triaxiality and pear shape as a part of the geoid figure: they are represented by the spherical harmonic coefficients C_{22},S_{22} and C_{30}, respectively, corresponding to degree and order numbers 2.2 for the triaxiality and 3.0 for the pear shape.

Geoid

It was stated earlier that measurements are made on the apparent or topographic surface of the Earth and it has just been explained that computations are performed on an ellipsoid. One other surface is involved in geodetic measurement: the geoid. In geodetic surveying, the computation of the geodetic coordinates of points is commonly performed on a reference ellipsoid closely approximating the size and shape of the Earth in the area of the survey. The actual measurements made on the surface of the Earth with certain instruments are however referred to the geoid. The ellipsoid is a mathematically defined regular surface with specific dimensions. The geoid, on the other hand, coincides with that surface to which the oceans would conform over the entire Earth if free to adjust to the combined effect of the Earth's mass attraction (gravitation) and the centrifugal force of the Earth's rotation. As a result of the uneven distribution of the Earth's mass, the geoidal surface is irregular and, since the ellipsoid is a regular surface, the separations between the two, referred to as geoid undulations, geoid heights, or geoid separations, will be irregular as well.

The geoid is a surface along which the gravity potential is everywhere equal and to which the direction of gravity is always perpendicular. The latter is particularly important because optical instruments containing gravity-reference leveling devices are commonly used to make geodetic measurements. When properly adjusted, the vertical axis of the instrument coincides with the direction of gravity and is, therefore, perpendicular to the geoid. The angle between the plumb line which is perpendicular to the geoid (sometimes called "the vertical") and the perpendicular to the ellipsoid (sometimes called "the ellipsoidal normal") is defined as the deflection of the vertical. It has two components: an east-west and a north-south component.

Earth rotation and Earth's interior

Determining the exact figure of the Earth is not only a geodetic operation or a task of geometry, but is also related to geophysics. Without any idea of the Earth's interior, we can state a "constant density" of 5.515 g/cm³ and, according to theoretical arguments, such a body rotating like the Earth would have a flattening of 1:230.

In fact the measured flattening is 1:298.25, which is more similar to a sphere and a strong argument that the Earth's core is *very compact*. Therefore the density must be a function of the depth, reaching from about 2.7 g/cm³ at the surface (rock density of granite, limestone etc.) up to approximately 15 within the inner core. Modern seismology yields a value of 16 g/cm³ at the center of the earth.

Global and regional gravity field

Also with implications for the physical exploration of the Earth's interior is the gravitational field, which can be measured very accurately at the surface and remotely by satellites. True vertical generally does not correspond to theoretical vertical (deflection ranges from 2" to 50") because topography and all *geological masses* disturb the gravitational field. Therefore the gross structure of the earth's crust and mantle can be determined by geodetic-geophysical models of the subsurface.

Volume

The volume of an oblate spheroid is $V = \frac{4}{3}\pi a^2 b;$
(where π is the mathematical constant *pi*, and *a* and *b* are the equatorial and polar radii respectively.)

Using the World Geodetic System's reference ellipsoid, where *a* = 6,378.137 km and *b* = 6,356.7523 km, Earth's volume is calculated as 1,083,210,000,000 km^3, or about 1.08321×10^{12} km^3, in scientific notation.

Earth Radius

Because the Earth is not perfectly spherical, no single value serves as its natural radius. Distances from points on the surface to the center range from 6,353 km to 6,384 km (≈3,947–3,968 mi). Several different ways of modeling the Earth as a sphere each yield a convenient mean radius of 6371 km (≈3,959 mi).

While "radius" normally is a characteristic of perfect spheres, the term as employed here more generally means the distance from some "center" of the Earth to a point on the surface or on an idealized surface that models the Earth. It can also mean some kind of average of such distances. It can also mean the radius of a sphere whose curvature matches the curvature of the ellipsoidal model of the Earth at a given point.

This deals primarily with spherical and ellipsoidal models of the Earth.

The first scientific estimation of the radius of the earth was given by Eratosthenes.

Earth radius is also used as a unit of distance, especially in astronomy and geology. It is usually denoted by $R_\oplus$.

Introduction

Radius and models of the earth

Earth's rotation, internal density variations, and external tidal forces cause it to deviate systematically from a perfect sphere. Local topography increases the variance, resulting in a surface of unlimited complexity. Our descriptions of the Earth's surface must be simpler than reality in order to be tractable. Hence we create models to approximate the Earth's surface, generally relying on the simplest model that suits the need.

Each of the models in common use come with some notion of "radius". Strictly speaking, spheres are the only solids to have radii, but looser uses of the term "radius" are common in many fields, including those dealing with models of the Earth. Viewing models of the Earth from less to more approximate:

- The real surface of the Earth;
- The geoid, defined by mean sea level at each point on the real surface;
- An ellipsoid: geocentric to model the entire earth, or else geodetic for regional work;
- A sphere.

In the case of the geoid and ellipsoids, the fixed distance from any point on the model to the specified center is called *"a radius of the Earth"* or *"the radius of the Earth at that point"*. It is also common to refer to any *mean radius* of a spherical model as *"the radius of the earth"*. On the Earth's real surface, on other hand, it is uncommon to refer to a "radius", since there is no practical need. Rather, elevation above or below sea level is useful.

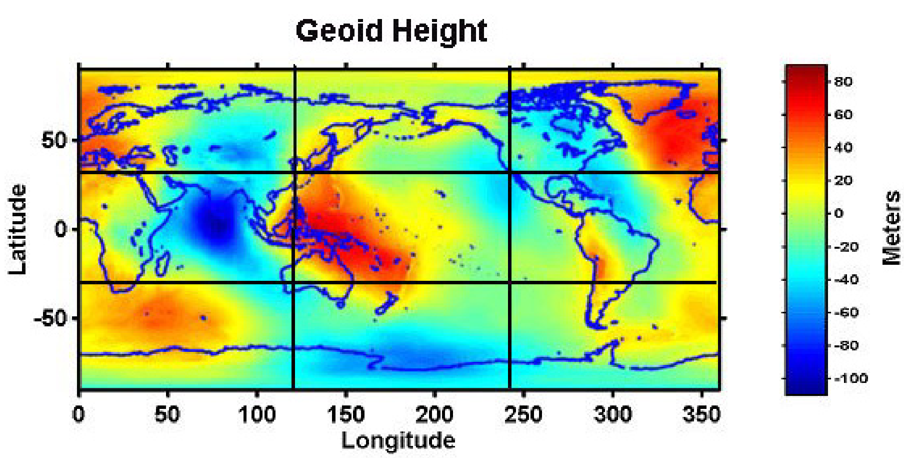
Regardless of model, any radius falls between the polar minimum of about 6,357 km and the equatorial maximum of about 6,378 km (≈3,950 – 3,963 mi). Hence the Earth deviates from a perfect sphere by only a third of a percent, sufficiently close to treat it as a sphere in many contexts and justifying the term "the radius of the Earth". While specific values differ, the concepts here generalize to any major planet.

Physics of Earth's deformation

Rotation of a planet causes it to approximate an *oblate ellipsoid/spheroid* with a bulge at the equator and flattening at the North and South Poles, so that the *equatorial radius a* is larger than the *polar radius b* by approximately *aq* where the *oblateness constant q* is

$$q = \frac{a^3\omega^2}{GM}$$

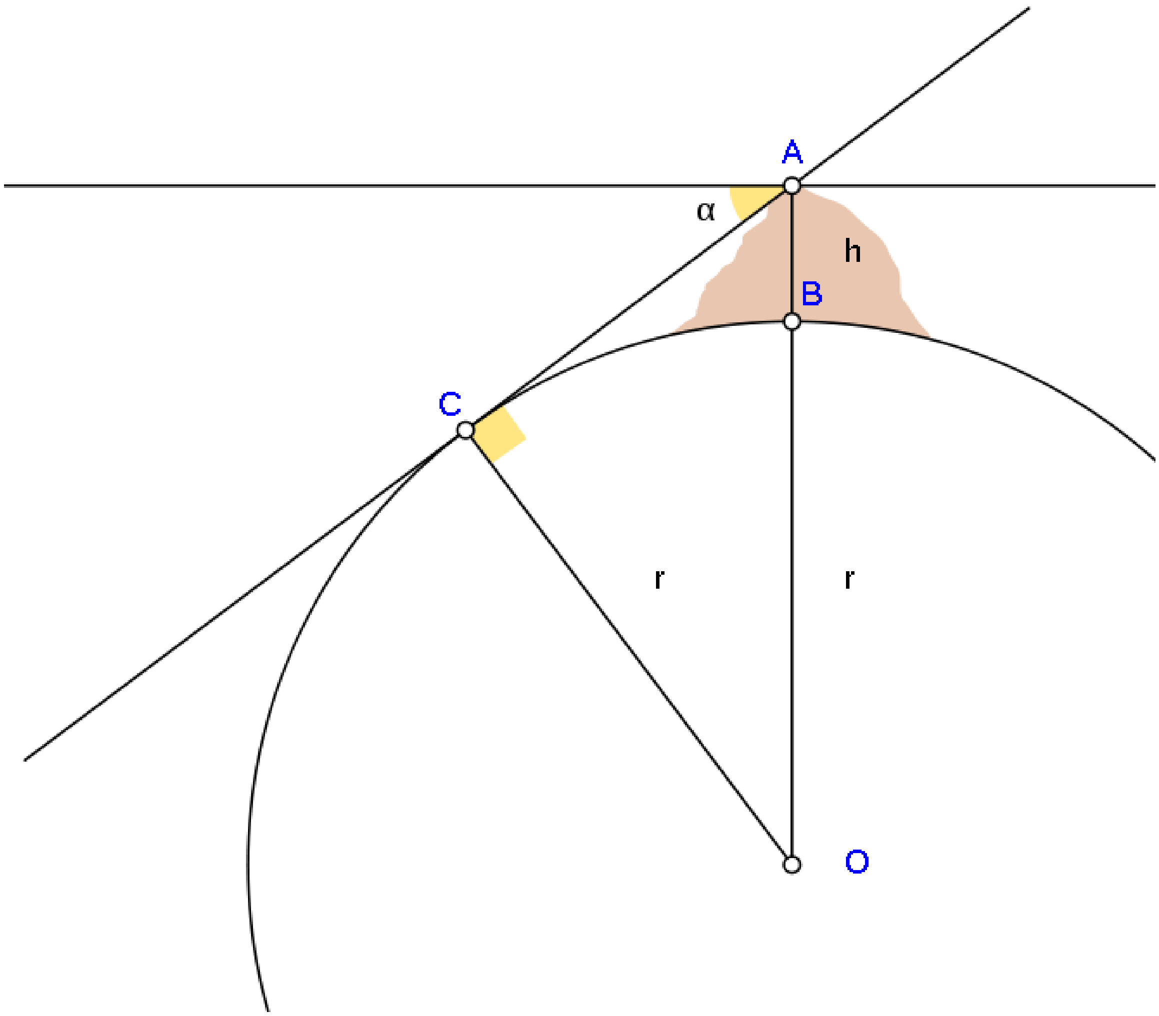
where ω is the angular frequency, *G* is the gravitational constant, and *M* is the mass of the planet. For the Earth $q^{-1} \approx 289$, which is close to the measured inverse flattening $f^{-1} \approx 298.257$. Additionally, the bulge at the equator shows slow variations. The bulge had been declining, but since 1998 the bulge has increased, possibly due to redistribution of ocean mass via currents.



The variation in density and crustal thickness causes gravity to vary on the surface, so that the mean sea level will differ from the ellipsoid. This difference is the *geoid height*, positive above or outside the ellipsoid, negative below or inside. The geoid height variation is under 110 m on Earth. The geoid height can have abrupt changes due to earthquakes (such as the Sumatra-Andaman earthquake) or reduction in ice masses (such as Greenland).

Not all deformations originate within the earth. Gravity of the Moon and Sun cause Earth's surface to undulate by tenths of meters at a point over a nearly 12 hour period.

Radius and local conditions



Biruni's (973-1048) method for calculation of Earth's radius improved accuracy and avoided the need for long walks.

Given local and transient influences on surface height, the values defined below are based on a "general purpose" model, refined as globally precisely as possible within 5 m of reference ellipsoid height, and to within 100 m of mean sea level (neglecting geoid height).

Additionally, the radius can be estimated from the curvature of the Earth at a point. Like a torus the curvature at a point will be largest (tightest) in one direction (North-South on Earth) and smallest (flattest) perpendicularly (East-West). The corresponding radius of curvature depends on location and direction of measurement from that point. A consequence is that a distance to the true horizon at the equator is slightly shorter in the north/south direction than in the east-west direction.

In summary, local variations in terrain prevent the definition of a single absolutely "precise" radius. One can only adopt an idealized model. Since the estimate by Eratosthenes, many models have been created. Historically these models were based on regional topography, giving the best reference ellipsoid for the area under survey. As satellite remote sensing and especially the Global Positioning System rose in importance, true global models were developed which, while not as accurate for regional work, best approximate the earth as a whole.

Fixed radii

The following radii are fixed and do not include a variable location dependence. They are derived from the WGS-84 ellipsoid.

The value for the equatorial radius is defined to the nearest 0.1 meter in WGS-84. The value for the polar radius in this section has been rounded to the nearest 0.1 meter, which is expected to be adequate for most uses. Please refer to the WGS-84 ellipsoid if a more precise value for its polar radius is needed.

The radii in this section are for an idealized surface. Even the idealized radii have an uncertainty of ± 2 meters.. The discrepancy between the ellipsoid radius the radius to a physical location may be significant. When identifying the position of an observable location, the use of more precise values for WGS-84 radii may not yield a corresponding improvement in accuracy.

The symbol given for the named radius is used in the formulae found here.

Equatorial radius

The Earth's equatorial radius a, or semi-major axis, is the distance from its center to the equator and equals 6,378.1370 km (≈3,963.191 mi; ≈3,443.918 nmi). The equatorial radius is often used to compare Earth with other planets.

Polar radius

The Earth's polar radius b, or semi-minor axis, is the distance from its center to the North and South Poles, and equals 6,356.7523 km (≈3,949.903 mi; ≈3,432.372 nmi).

Radii with location dependence

Notable radii

- **Maximum:** The summit of Chimborazo is 6,384.4 km (3,968 mi) from the Earth's center.

- **Minimum:** The floor of the Arctic Ocean is ≈6352.8 km (3,947 mi) from the Earth's center.

Radius at a given geodetic latitude

The distance from the Earth's center to a point on the spheroid surface at geodetic latitude ϕ is:

$$R = R(\phi) = \sqrt{\frac{(a^2\cos(\phi))^2 + (b^2\sin(\phi))^2}{(a\cos(\phi))^2 + (b\sin(\phi))^2}}$$

Radius of curvature

These are based on a oblate ellipsoid.

Eratosthenes used two points, one almost exactly north of the other. The points are separated by distance *D*, and the vertical directions at the two points are known to differ by angle of θ, in radians. A formula used in Eratosthenes' method is

$$R = \frac{D}{\theta}$$

which gives an estimate of radius based on the north-south curvature of the Earth.

Meridional

In particular the Earth's *radius of curvature in the (north-south) meridian* at ϕ is:

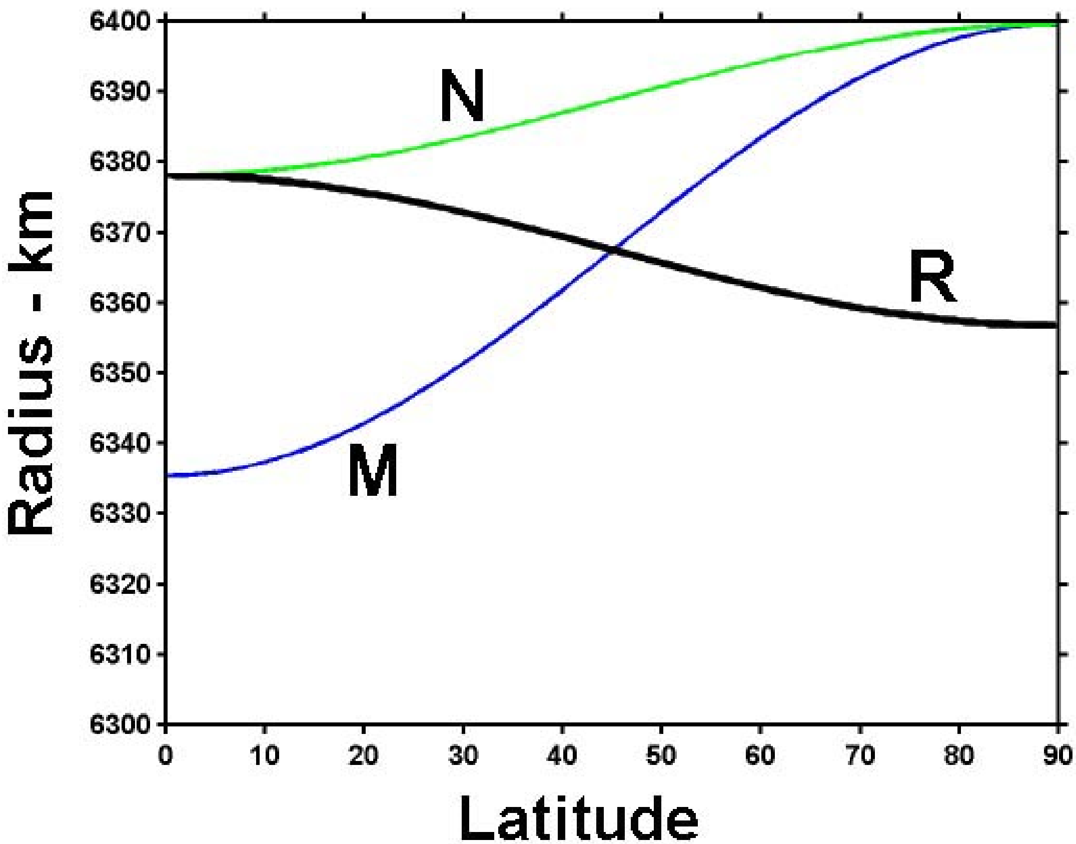
$$M = M(\phi) = \frac{(ab)^2}{((a\cos(\phi))^2 + (b\sin(\phi))^2)^{3/2}}$$

Normal

If one point had appeared due east of the other, one finds the approximate curvature in east-west direction.
This *radius of curvature in the prime vertical*, which is perpendicular, or *normal*, to *M* at geodetic latitude ϕ is:

$$N = N(\phi) = \frac{a^2}{\sqrt{(a\cos(\phi))^2 + (b\sin(\phi))^2}}$$

Note that *N*=*R* at the equator:



At geodetic latitude 48.46791… degrees (e.g., Lèves, Alsace, France), the radius *R* is 20000/π ≈ 6366.1977…, namely the radius of a perfect sphere for which the meridian arc length from the equator to the North Pole is exactly 10000 km, the originally proposed definition of the meter.

The Earth's mean radius of curvature (averaging over all directions) at latitude ϕ is:

$$R_a = \sqrt{MN} = \frac{a^2 b}{(a\cos(\phi))^2 + (b\sin(\phi))^2}$$

The Earth's radius of curvature along a course at geodetic bearing (measured clockwise from north) α, at ϕ is derived from Euler's curvature formula as follows:

$$R_c = \frac{1}{\frac{\cos(\alpha)^2}{M} + \frac{\sin(\alpha)^2}{N}}$$

The Earth's equatorial radius of curvature in the meridian is:

$\frac{b^2}{a}$ = 6335.437 km

The Earth's polar radius of curvature is:

$\frac{a^2}{b}$ = 6399.592 km

Mean radii

The Earth can be modeled as a sphere in many ways. This section describes the common ways. The various radii derived here use the notation and dimensions noted above for the Earth as derived from the WGS-84 ellipsoid; namely,

$a =$ Equatorial radius (6,378.1370 km)
$b =$ Polar radius (6,356.7523 km)

A sphere being a gross approximation of the spheroid, which itself is an approximation of the geoid, units are given here in kilometers rather than the millimeter resolution appropriate for geodesy.

Mean radius

The International Union of Geodesy and Geophysics (IUGG) defines the mean radius (denoted R_1) to be

$$R_1 = \frac{2a + b}{3}$$

For Earth, the mean radius is 6,371.009 km (≈3,958.761 mi; ≈3,440.069 nmi).

Authalic radius

Earth's authalic ("equal area") radius is the radius of a hypothetical perfect sphere which has the same surface area as the reference ellipsoid. The IUGG denotes the authalic radius as R_2.

A closed-form solution exists for a spheroid:

$$R_2 = \sqrt{\frac{a^2 + \frac{ab^2}{\sqrt{a^2-b^2}} \ln\left(\frac{a+\sqrt{a^2-b^2}}{b}\right)}{2}} = \sqrt{\frac{A}{4\pi}}$$

where A is the surface area of the spheroid

For Earth, the authalic radius is 6,371.0072 km (≈3,958.760 mi; ≈3,440.069 nmi).

Volumetric radius

Another spherical model is defined by the volumetric radius, which is the radius of a sphere of volume equal to the ellipsoid. The IUGG denotes the volumetric radius as R_3.

$$R_3 = \sqrt[3]{a^2 b}$$

For Earth, the volumetric radius equals 6,371.0008 km (≈3,958.760 mi; ≈3,440.069 nmi).

Meridional Earth radius

Another mean radius is *rectifying radius*, giving a sphere with circumference equal to the perimeter of the ellipse described by any polar cross section of the ellipsoid. This requires an elliptic integral to find, given the polar and equatorial radii:

$$M_r = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} \sqrt{a^2 \cos^2 \phi + b^2 \sin^2 \phi}\, d\phi .$$

The rectifying radius is equivalent to the meridional mean, which is defined as the average value of *M*:

$$M_r = \frac{2}{\pi} \int_0^{\frac{\pi}{2}} M(\phi)\, d\phi$$

For integration limits of [0…π/2], the integrals for rectifying radius and mean radius evaluate to the same result, which, for Earth, amounts to 6367.4491 km (≈3,956.545 mi; ≈3,438.147 nmi). The meridional mean is well approximated by the semicubic mean of the two axes:

$$M_r \approx \left[\frac{a^{1.5} + b^{1.5}}{2}\right]^{1/1.5}$$

yielding, again, 6367.4491 km; or less accurately by the quadratic mean of the two axes:

$$M_r \approx \sqrt{\frac{a^2 + b^2}{2}} ;$$

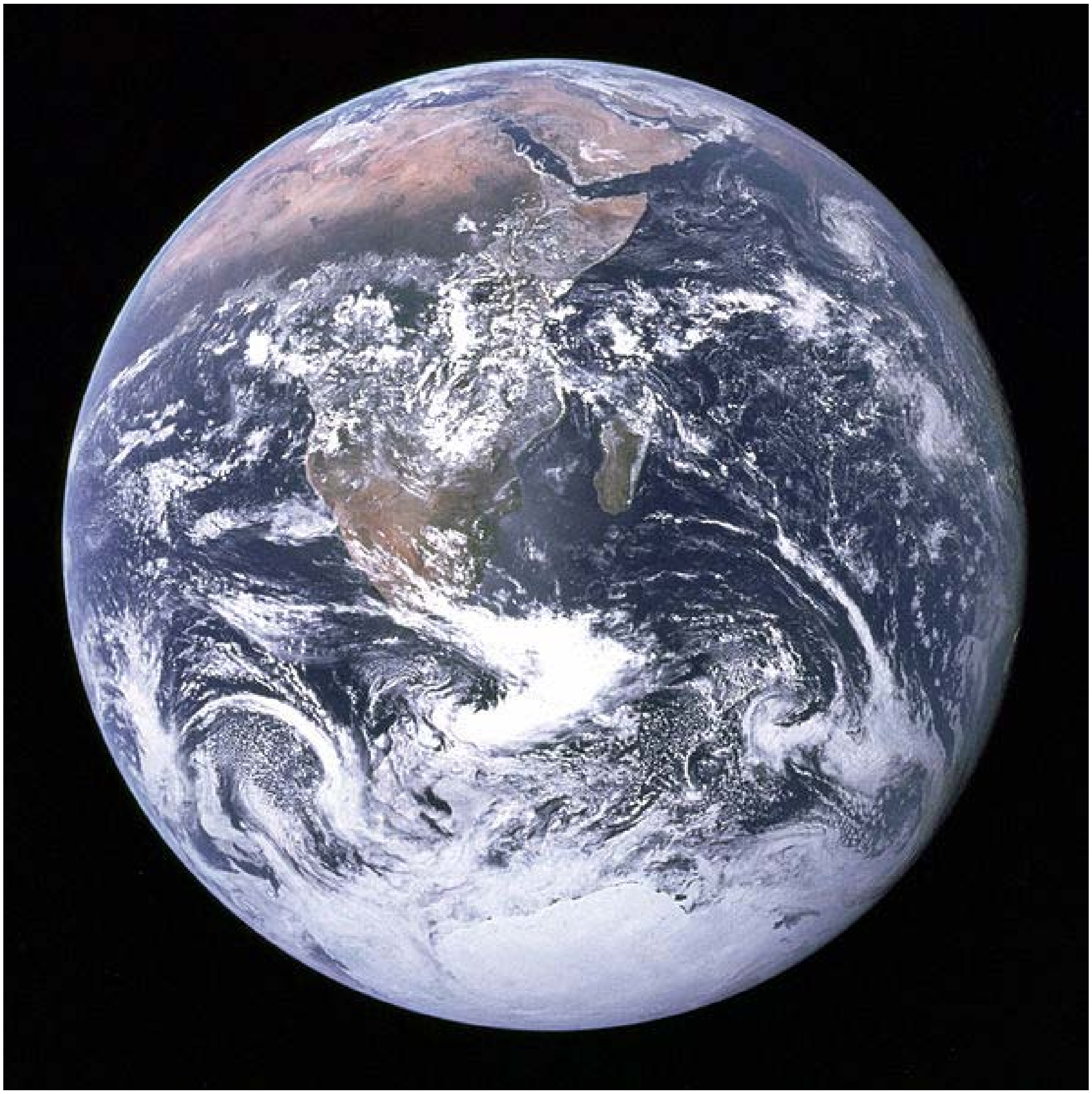
about 6,367.454 km; or even just the mean of the two axes:

$$M_r \approx \frac{a + b}{2} ;$$ about 6,367.445 km.

Chapter-2

Flat Earth

15th century adaptation of a T and O map. This kind of medieval mappa mundi illustrates only the reachable side of a round Earth, since it was thought that no one could cross a torrid clime near the equator to the other half of the globe.



The spherical Earth seen from Apollo 17.

The **Flat Earth** model is a view that the Earth's shape is a flat plane or disk. Most pre-modern cultures have had conceptions of a flat Earth, including ancient Greece until the classical period, the Bronze Age and Iron Age civilizations of the Ancient Near East until the Hellenistic period, Ancient India until the Gupta period (early centuries AD) and China until the 17th century. It was also typically held in the cultures of the New World until the time of European contact, and a flat Earth domed by the firmament in the shape of an inverted bowl is common in pre-scientific societies.

The paradigm of a spherical Earth was developed in ancient Greek astronomy, beginning with Pythagoras (6th century BC), although most Pre-Socratics retained the flat Earth model. Aristotle accepted the spherical shape of the Earth on empirical grounds around

330 BC, and knowledge of the spherical Earth gradually began to spread beyond the Hellenistic world from then on.

The misconception that educated people at the time of Columbus believed in a flat Earth has been referred to as "The Myth of the Flat Earth". In 1945, it was listed by the *Historical Association* (of Britain) as the second of 20 in a pamphlet on common errors in history.

Historical development

Ancient Near East

Imago Mundi Babylonian map, the oldest known world map, 6th century BCE Babylonia.

In early Egyptian and Mesopotamian thought the world was portrayed as a flat disk floating in the ocean. A similar model is found in the Homeric account of the 8th century BCE in which "Okeanos, the personified body of water surrounding the circular surface of the Earth, is the begetter of all life and possibly all the gods,".

The Hebrew Bible carried forward the ancient Middle Eastern cosmology, such as in the Enuma Elish, which described a flat earth with a solid roof, surrounded by water above and below, as illustrated by references to the "foundations of the earth" and the "circle of the earth" in the following examples:

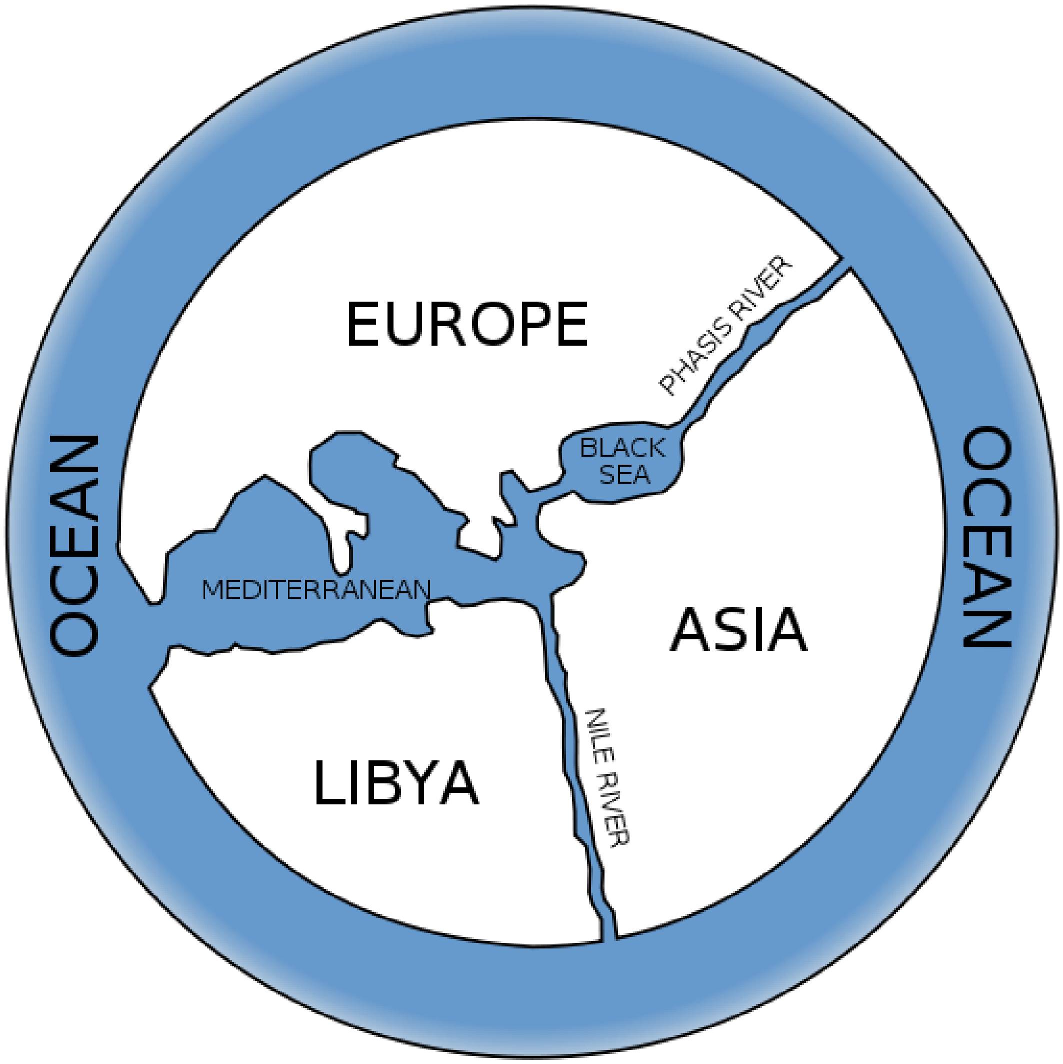
- "He sits enthroned above the circle of the earth, and its people are like grasshoppers. He stretches out the heavens like a canopy, and spreads them out like a tent to live in."
- "For the foundations of the earth are the LORD's; upon them he has set the world."
- "You shall not make for yourself a carved image, or any likeness of anything that is in heaven above, or that is in the earth beneath, or that is in the water under the earth."

There are isolated quotations which can be taken as evidence of something different:

- "He spreads out the northern skies over empty space; he suspends the earth over nothing."

but these are complicated by references to "pillars of the heavens" in subsequent stanzas.

Classical world



Possible rendering of Anaximander's world map

Many pre-Socratic philosophers considered the world to be flat, at least according to Aristotle. According to Aristotle, pre-Socratic philosophers, including Leucippus (c. 440 BC) and Democritus (c. 460–370 BC) believed in a flat Earth. Anaximander (c. 550 BC) believed the Earth to be a short cylinder with a flat, circular top that remained stable because it is the same distance from all things. Anaximenes of Miletus believed that "the earth is flat and rides on air; in the same way the sun and the moon and the other heavenly bodies, which are all fiery, ride the air because of their flatness." Xenophanes of Colophon (c. 500 BC) thought that the Earth was flat, with its upper side touching the air, and the lower side extending without limit. Belief in a flat Earth continued into the 5th-century BC. Anaxagoras (c. 450 BC) agreed that the Earth was flat, and his pupil Archelaus believed that the flat Earth was depressed in the middle like a saucer, to allow for the fact that the Sun does not rise and set at the same time for everyone. Democritus (c. 400 BC) is also said to have believed that the Earth is flat.

Ancient India

In antiquity, a cosmological view prevailed in India that held the Earth is a disc that consists of four continents grouped around the central mountain Meru like the petals of a flower. An outer ocean surrounds these continents. This view was elaborated in traditional Jain cosmology and Buddhist cosmology, which depicts the world (in this case solar system/universe and not Earth) as a vast, flat oceanic disk (of the magnitude of a small planetary system), bounded by mountains, in which the continents are set as small islands. The belief in a disk remained the dominant one in Indian cosmology until the early centuries AD, such as in the Puranas:

In the Puranas the Earth is a flat-bottomed, circular disk, in the center of which is a lofty mountain, Meru.

Ancient China

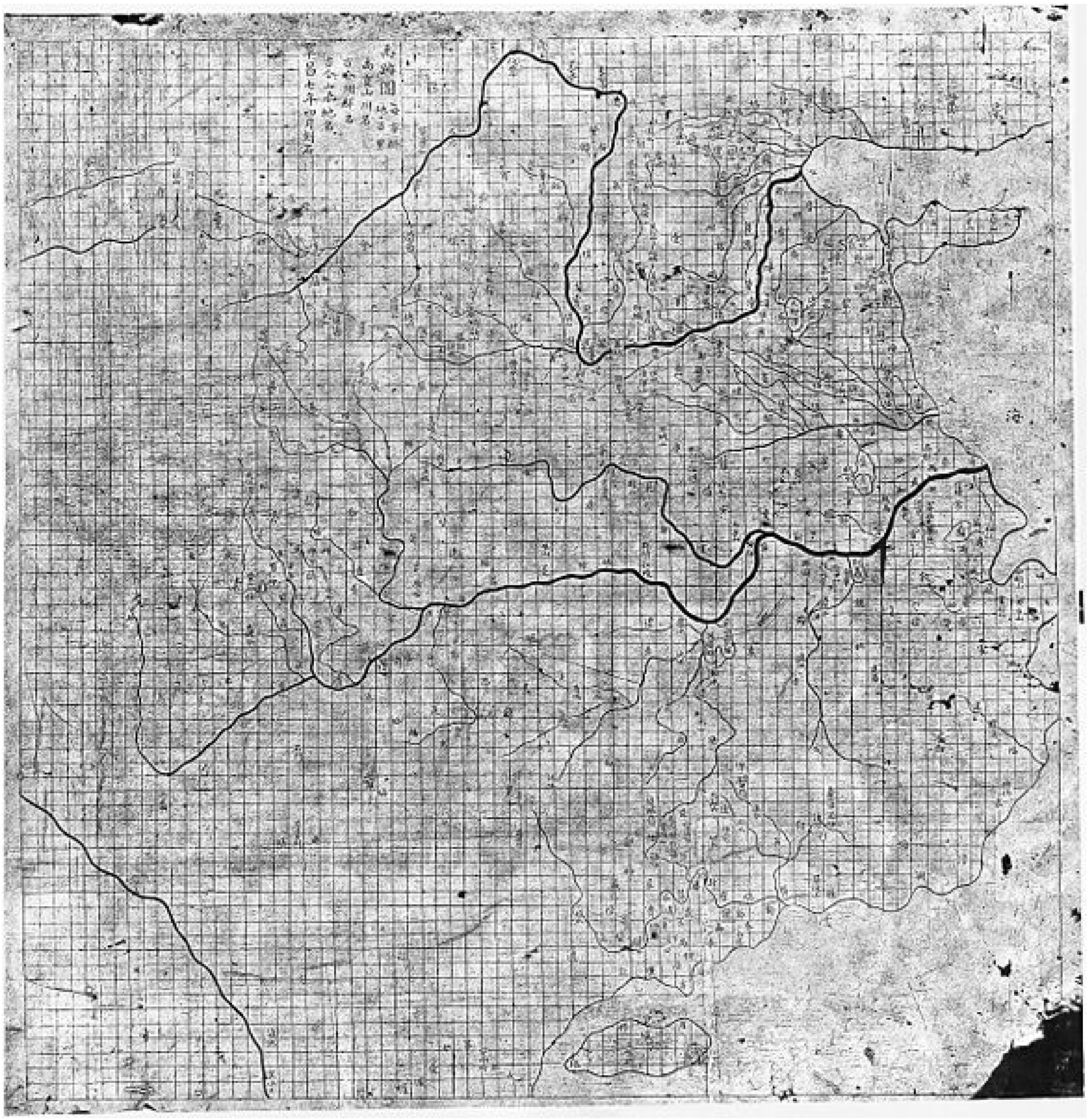
In ancient China, the prevailing belief was that the Earth was flat and square, while the heavens were round, an assumption virtually unquestioned until the introduction of European astronomy in the 17th century. The English sinologist Cullen emphasizes the point that there was no concept of a round Earth in ancient Chinese astronomy:

Chinese thought on the form of the Earth remained almost unchanged from early times until the first contacts with modern science through the medium of Jesuit missionaries in the seventeenth century. While the heavens were variously described as being like an umbrella covering the Earth (the Kai Tian theory), or like a sphere surrounding it (the Hun Tian theory), or as being without substance while the heavenly bodies float freely (the Hsüan yeh theory), the Earth was at all times flat, although perhaps bulging up slightly.

The model of an egg is often used by Chinese astronomers to describe the heavens as spherical and the historian Joseph Needham quotes Zhang Heng (78-139 AD) as saying:

The heavens are like a hen's egg and as round as a crossbow bullet; the earth is like the yoke of the egg, and lies in the centre.

As noted in the book *Huai Nan Zu*, in the 2nd Century BC Chinese astronomers effectively inverted Eratosthenes' calculation of the curvature of the earth in order to calculate the height of the sun above the earth. By assuming the earth to be flat, they arrived at a distance of 100,000 *li*.



The *Yu Ji Tu*, "Map of the Tracks of Yu", carved into stone in 1137, located in the Stele Forest of Xi'an. This 3 ft (0.91 m) squared map features a grid of 100 li squares. China's coastline and river systems are clearly defined and precisely pinpointed on the map.

However this isn't followed up and Zhang Heng is better known for his invention of the rectangular grid or coordinate system which was used for earth maps in the same way as Marinus of Tyre and Ptolemy who were almost exact contemporaries. Cullen comments:

In a passage of Zhang Heng's cosmogony not translated by Needham, Zhang himself says: "Heaven takes its body from the Yang, so it is round and in motion. earth takes its body from the Yin, so it is flat and quiescent". The point of the egg analogy is simply to stress that the earth is completely enclosed by heaven, rather than merely covered from above as the Kai Tian describes. Chinese astronomers, many of them brilliant men by any standards, continued to think in flat-earth terms until the seventeenth century; this surprising fact might be the starting-point for a re-examination of the apparent facility with which the idea of a spherical earth found acceptance in fifth-century B.C. Greece.

Likewise, the 13th century scholar Li Ye, arguing that the movements of the round heaven would be hindered by a square Earth, did not advocate a spherical Earth, but rather that its edge should be rounded off so as to be circular.

Decline of the Flat Earth model

Classical world

When a ship is at the horizon, its lower part is obscured due to the sphericity of the Earth.

It has been suggested that seafarers probably provided the first observational evidence that the Earth was not flat. Greek thalassocracy was receptive to the idea, unlike the Chinese, whose cosmology was firmly land-based.

Semi-circular shadow of Earth on the Moon during the phases of a lunar eclipse

Some ancient authorities in the doxographic tradition credited the Greek philosophers Pythagoras, in the 6th century BC, and Parmenides, in the 5th, with recognizing that the Earth is spherical.

Around 330 BC, Aristotle provided observational evidence for the spherical Earth, The Earth's circumference was first determined around 240 BC by Eratosthenes. Eratosthenes knew that in Syene, in Egypt, the Sun was directly overhead at the summer solstice, while he estimated that the angle formed by a shadow cast by the Sun at Alexandria was 1/50th of a circle. He estimated the distance from Syene to Alexandria as 5,000 stades, and estimated the Earth's circumference was 250,000 stades. Subsequently, ignorance of the size of a stadia caused problems both to the Arabs and to Christopher Columbus.

The Terrestrial Sphere of Crates of Mallus (ca. 150 B.C.).

In the 2nd century BC, Crates of Mallus devised a terrestrial sphere which divided the Earth into four continents, separated by great rivers or oceans, with people presumed to be living in each of the four regions. Opposite the oikumene, the inhabited world, were the *antipodes*, considered unreachable both because of an intervening *torrid zone* (equator) and the ocean. This took a strong hold on the medieval mind.

Lucretius (1st. c. BC) opposed the concept of a spherical Earth, because he considered that in an infinite universe there was no center towards which heavy bodies would tend, thus he considered the idea of animals walking around topsy-turvy under the Earth to be absurd. But by the 1st century AD, Pliny the Elder was in a position to claim that everyone agrees on the spherical shape of Earth, although there continued to be disputes regarding the nature of the antipodes, and how it is possible to keep the ocean in a curved shape. Pliny also considers the possibility of an imperfect sphere, "shaped like a pinecone".

Ptolemy derived his maps from a curved globe and developed the system of latitude, longitude, and climes. His Almagest was written in Greek and only translated into Latin in the 11th century from Arabic translations. But once it was known, it remained the basis of European astronomy throughout the Middle Ages.

In late antiquity such widely read encyclopedists as Macrobius (4th c.) and Martianus Capella (5th c.) discussed the circumference of the sphere of the Earth, its central position in the universe, the difference of the seasons in northern and southern hemispheres, and many other geographical details. In his commentary on Cicero's *Dream of Scipio*, Macrobius described the Earth as a globe of insignificant size in comparison to the remainder of the cosmos.

The Talmud

The Jerusalem Talmud says that Alexander of Macedon (Alexander the Great) was lifted by birds to the point that he saw the curvature of the Earth. This story is mentioned as well by the Tosafos commentary on the Babylonian Talmud. This is used to explain why a statue of a person holding a sphere in his hand is assumed to be an idol. The sphere being held in its hand symbolizing the idol's purported dominion over the world whose shape is a sphere.

Early Christian Church

From Late Antiquity, and from the beginnings of Christian theology, knowledge of the sphericity of the Earth had become widespread. There was considerable misunderstanding illustrated by the debate concerning the possibility of the inhabitants of the antipodes:

Saint Augustine (354–430) argued against assuming people inhabited the antipodes:

But as to the fable that there are Antipodes, that is to say, men on the opposite side of the earth, where the sun rises when it sets to us, men who walk with their feet opposite ours that is on no ground credible. And, indeed, it is not affirmed that this has been learned by historical knowledge, but by scientific conjecture, on the ground that the earth is suspended within the concavity of the sky, and that it has as much room on the one side of it as on the other: hence they say that the part that is beneath must also be inhabited. But they do not remark that, although it be supposed or scientifically demonstrated that the world is of a round and spherical form, yet it does not follow that the other side of the earth is bare of water; nor even, though it be bare, does it immediately follow that it is peopled.

Since these people would have to be descended from Adam, they would have had to travel to the other side of the Earth at some point; Augustine continues:

It is too absurd to say, that some men might have taken ship and traversed the whole wide ocean, and crossed from this side of the world to the other, and that thus even the inhabitants of that distant region are descended from that one first man.

Scholars of Augustine's work have traditionally understood him to have shared the common view of his educated contemporaries that the Earth is spherical, in line with the quotation above, and with Augustine's famous endorsement of science in *De Genesi ad litteram*. That tradition has, however, recently been challenged by Leo Ferrari, who concluded that many of Augustine's passing references to the physical universe imply a belief in an essentially flat Earth "at the bottom of the universe".

The notion that hell was below the earth is stated clearly in early Christian tradition by the belief, still recited by most Christians in the Apostolic and Athanasian creeds, that Christ "descended into hell" between his death and resurrection. The *Harrowing of hell*, combined with the theme of Christ's ascension into heaven, became a favourite motif in medieval art and drama and led to the three tiers of the mystery plays. This was possibly the only source of cosmology available to most unlearned people until literacy became more widespread after the invention of printing.

Some authors and artists less prominent in the Church's history directly opposed the round Earth. After his conversion to Christianity, Lactantius (245–325) became a trenchant critic of all pagan philosophy. In Book III of *The Divine Institutes* he ridicules the notion that there could be inhabitants of the antipodes "whose footsteps are higher than their heads". After presenting some arguments which he attributes to advocates for a spherical heaven and Earth, he writes:

Cosmas Indicopleustes' world picture - flat earth in a Tabernacle.

But if you inquire from those who defend these marvellous fictions, why all things do not fall into that lower part of the heaven, they reply that such is the nature of things, that heavy bodies are borne to the middle, and that they are all joined together towards the middle, as we see spokes in a wheel; but that the bodies that are light, as mist, smoke, and fire, are borne away from the middle, so as to seek the heaven. I am at a loss what to say respecting those who, when they have once erred, consistently persevere in their folly, and defend one vain thing by another;

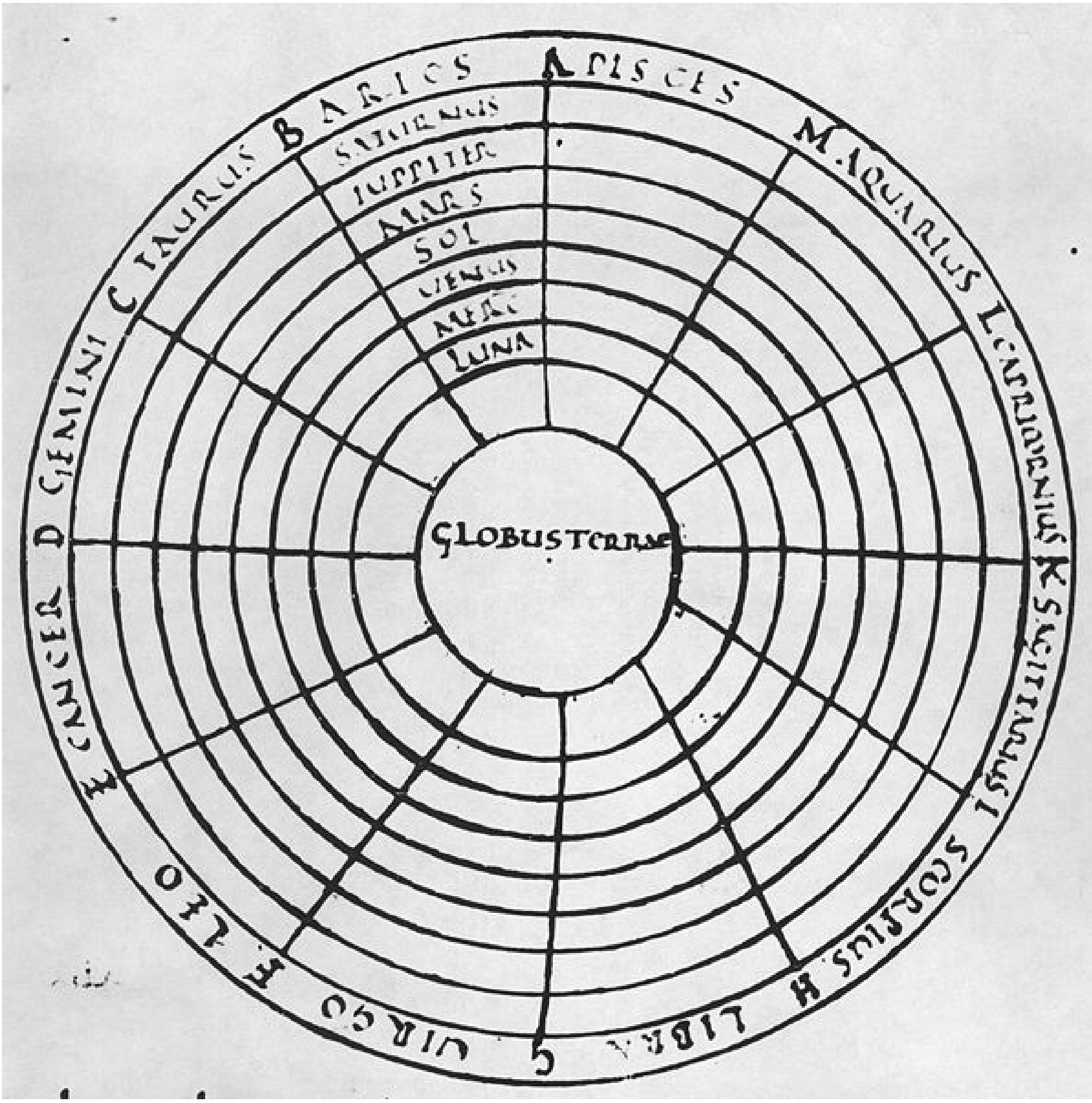
Diodorus of Tarsus (d. 394) may have argued for a flat Earth based on scriptures; however, Diodorus' opinion on the matter is known to us only by a criticism of it by Photius. Severian, Bishop of Gabala (d. 408), wrote that the Earth is flat and the sun does not pass under it in the night, but "travels through the northern parts as if hidden by a wall". The Egyptian monk Cosmas Indicopleustes (547) in his Topographia Christiana, where the Covenant Ark was meant to represent the whole universe, argued on theological grounds that the Earth was flat, a parallelogram enclosed by four oceans.

In his *Homilies Concerning the Statutes* St.John Chrysostom (344–408) explicitly espoused the idea, based on his reading of Scripture, that the Earth floated on the waters gathered below the firmament, and St. Athanasius (c.293–373) expressed similar views in *Against the Heathen*.

A very recent essay by Leone Montagnini, discussing the question of the shape of the Earth from the origins to the late Antiquity, has shown that the Fathers of the Church shared different approaches that paralleled their overall philosophical and theological visions. Those of them who were more close to Platonic visions, like Origen, shared peacefully the geosphericism. A second tradition, including Basil, Ambrose and Augustine, but also Philoponus, accepted the idea of the round Earth and the radial gravity, but in a critical way. In particular they pointed out a number of doubts about the physical reasons of the radial gravity, and hesitated in accepting the physical reasons proposed by Aristotle or Stoicism. However, a "flattist" approach was more or less shared by all the Fathers coming from the Syriac area, who were more inclined to follow the letter of the Old Testament. Diodorus, Severian, and Cosmas Indicopleustes, but also Chrysostom, belonged just to this latter tradition.

At least one early Christian writer, Basil of Caesarea (329–379), believed the matter to be theologically irrelevant.

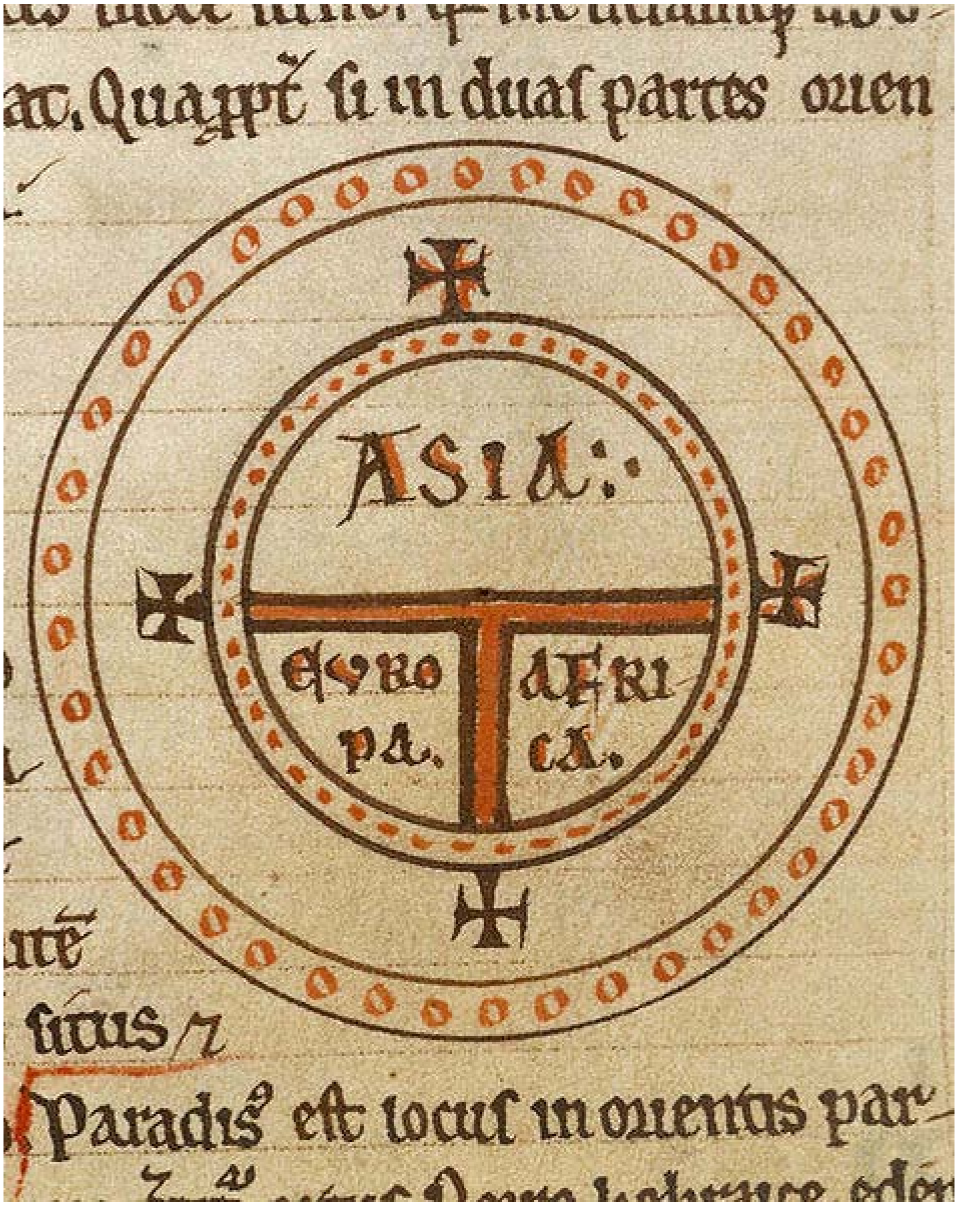
Early Middle Ages



9th century Macrobian cosmic diagram showing the *sphere of the Earth* at the center, (*globus terrae*)

With the end of Roman civilization, Western Europe entered the Middle Ages with great difficulties that affected the continent's intellectual production. Most scientific treatises of classical antiquity (in Greek) were unavailable, leaving only simplified summaries and compilations. Still, many textbooks of the Early Middle Ages supported the sphericity of the Earth. For example: some early medieval manuscripts of Macrobius include maps of the Earth, including the antipodes, zonal maps showing the Ptolemaic climates derived from the concept of a spherical Earth and a diagram showing the Earth (labeled as *globus terrae*, the sphere of the Earth) at the center of the hierarchically ordered planetary spheres. Further examples of such medieval diagrams can be found in medieval manuscripts of the Dream of Scipio. In the Carolingian era, scholars discussed

Macrobius's view of the antipodes. One of them, the Irish monk Dungal, asserted that the tropical gap between our habitable region and the other habitable region to the south was smaller than Macrobius had believed.



12th century T and O map representing the inhabitated world as described by Isidore of Seville in his *Etymologiae*. (chapter 14, *de terra et partibus*).

Europe's view of the shape of the Earth in Late Antiquity and the Early Middle Ages may be best expressed by the writings of early Christian scholars:

- Boethius (c. 480 – 524), who also wrote a theological treatise *On the Trinity*, repeated the Macrobian model of the Earth as an insignificant point in the center of a spherical cosmos in his influential, and widely translated, *Consolation of Philosophy*.

- Bishop Isidore of Seville (560 – 636) taught in his widely read encyclopedia, the *Etymologies,* that the Earth was round and "resembles a wheel". Resembling Anaximander in language and the map that he provided, this was widely interpreted as referring to a flat disc-shaped Earth though some recent writers believe that he considered the Earth to be globular. He did not admit the possibility of antipodes, which he took to mean people dwelling on the opposite side of the Earth, considering them to be legendary and noting that there was no evidence for their existence. "The work remained unsurpassed until the thirteenth century and was regarded as the summit of all knowledge. It became an essential part of European medieval culture. Soon after the invention of typography it appeared many times in print." Isidore's disc-shaped map, essentially unchanged from its predecessors of 1,200 years previously, continued to be used through the Middle Ages by authors, e.g. the 9th century bishop Rabanus Maurus who compared the habitable part of the northern hemisphere (Aristotle's northern temperate clime) with a wheel.
- The monk Bede (c.672 – 735) wrote in his influential treatise on computus, *The Reckoning of Time*, that the Earth was round ('not merely circular like a shield [or] spread out like a wheel, but resembl[ing] more a ball'), explaining the unequal length of daylight from "the roundness of the Earth, for not without reason is it called 'the orb of the world' on the pages of Holy Scripture and of ordinary literature. It is, in fact, set like a sphere in the middle of the whole universe." (*De temporum ratione*, 32). The large number of surviving manuscripts of *The Reckoning of Time*, copied to meet the Carolingian requirement that all priests should study the computus, indicates that many, if not most, priests were exposed to the idea of the sphericity of the Earth. Ælfric of Eynsham paraphrased Bede into Old English, saying "Now the Earth's roundness and the Sun's orbit constitute the obstacle to the day's being equally long in every land."
- Bishop Vergilius of Salzburg (c.700 – 784) was accused by St Boniface for teaching "a perverse and sinful doctrine ... against God and his own soul regarding the sphericity of the earth". Pope Zachary decided that "if it shall be clearly established that he professes belief in another world and other people existing beneath the earth, or in another sun and moon there, thou art to hold a council, and deprive him of his sacerdotal rank, and expel him from the church." The issue as resolved was not the sphericity of the Earth itself, but his apparent belief that there were people living in the antipodes. This raised the theological implication that these were not descended from Adam and hence were not in need of redemption. Vergilius succeeded in freeing himself from that charge; he later became a bishop and was canonised in the 13th century.

12th century depiction of a spherical Earth with the four seasons (book "Liber Divinorum Operum" by Hildegard of Bingen)

A possible non-literary but graphic indication that people in the Middle Ages believed that the Earth (or perhaps the world) was a sphere, is the use of the *orb* (globus cruciger) in the regalia of many kingdoms and of the Holy Roman Empire. It is attested from the time of the Christian late-Roman emperor Theodosius II (423) throughout the Middle Ages; the *Reichsapfel* was used in 1191 at the coronation of emperor Henry VI. However the word 'orbis' means 'circle' and there is no record of a globe as a representation of the Earth since ancient times till that of Martin Behaim in 1492. Additionally it could well be a representation of the entire 'world' or cosmos.

A recent study of medieval concepts of the sphericity of the Earth noted that "since the eighth century, no cosmographer worthy of note has called into question the sphericity of the Earth." However, the work of these intellectuals may not have had significant influence on public opinion, and it is difficult to tell what the wider population may have thought of the shape of the Earth, if they considered the question at all.

High and Late Middle Ages

Picture from a 1550 edition of *On the Sphere of the World*, the most influential astronomy textbook of 13th century Europe.

By the 11th century Europe had learned of Islamic astronomy. The Renaissance of the 12th century from about 1070 started an intellectual revitalization of Europe with strong philosophical and scientific roots, and increased interest in natural philosophy.

Illustration of the spherical Earth in a 14th century copy of *L'Image du monde* (ca. 1246).

Hermannus Contractus (1013–1054) was among the earliest Christian scholars to estimate the circumference of Earth with Eratosthenes' method. Thomas Aquinas (1225–1274), the most important and widely taught theologian of the Middle Ages, believed in a spherical Earth; and he even took for granted his readers also knew the Earth is round. Lectures in the medieval universities commonly advanced evidence in favor of the idea that the Earth was a sphere. Also, "*On the Sphere of the World*", the most influential astronomy textbook of the 13th century and required reading by students in all Western European universities, described the world as a sphere. Thomas Aquinas, in his Summa Theologica, wrote, "The physicist proves the earth to be round by one means, the astronomer by another: for the latter proves this by means of mathematics, e.g. by the shapes of eclipses, or something of the sort; while the former proves it by means of physics, e.g. by the movement of heavy bodies towards the center, and so forth."

The shape of the Earth was not only discussed in scholarly works written in Latin; it was also treated in works written in vernacular languages or dialects and intended for wider audiences. The Norwegian book Konungs Skuggsjá, from around 1250, states clearly that the Earth is round - and that it is night on the other side of the Earth when it is daytime in Norway. The author also discusses the existence of antipodes - and he notes that they (if they exist) will see the Sun in the north of the middle of the day, and that they will have opposite seasons of the people living in the Northern Hemisphere.

However Tattersall shows that in many vernacular works in 12th and 13th century French texts the Earth was considered "round like a table" rather than "round like an apple". "In virtually all the examples quoted...from epics and from non-'historical' romances (that is, works of a less learned character) the actual form of words used suggests strongly a circle rather than a sphere.

Portuguese exploration of Africa and Asia, Columbus voyage to the Americas (1492) and finally Ferdinand Magellan's circumnavigation of the Earth (1519–21) provided the final, practical proofs for the global shape of the Earth.

Ancient India

The idea of a spherical Earth surrounded by the spheres of planets was finally introduced into India during the early centuries AD through the reception of Greek astronomy. Key parts of traditional Indian astronomy, heavily attacked by progressive Indian astronomers, were discarded, while others were attempted to be adapted to the new paradigm. Thus, the circumference of the Indian disk of the Earth was modelled into the equator of the Greeks.

The works of the classical Indian astronomer and mathematician, Aryabhata (476-550 AD), deal with the sphericity of the Earth and the motion of the planets. The final two parts of his Sanskrit magnum opus the Aryabhatiya, which were named the *Kalakriya* ("reckoning of time") and the *Gola* ("sphere"), state that the Earth is spherical and that its circumference is 4,967 yojanas, which in modern units is 39,968 km (24,835 mi), which is close to the current equatorial value of 40,075 km (24,901 mi).

Islamic world

A world map by Muhammad al-Idrisi (1100-1166) for Roger King of Sicily depicting the Earth as round

There has never been in Islam a significant belief in a flat earth. In the 9th century Abbasid Caliphate, most of the Greek works of cosmology from Constantinople, including Aristotle's *De caelo* and Ptolemy's *Almagest* were translated into Arabic in the House of Wisdom in Baghdad. They adopted from Greek astronomy the ideas that the Earth was spherical (Spherical Earth) and at the center of the universe (Geocentric model), that the universe was without void spaces and mainly divided into the wider planetary space composed of the Four Elements on the one hand, and the celestial space beginning from the moon and extending to the edge of the universe on the other. By the time they were re-exported back to Europe they had been enriched by spherical trigonometry, which they used in these calculations and the Arabic numeral system.

Muslim scholars who held to the round Earth theory used it in an impeccably Islamic manner, to calculate the distance and direction from any given point on the Earth to Makkah (Mecca). This determined the Qibla, or Muslim direction of prayer.

Ming China

As late as 1595, the first Jesuit missionary to China, Matteo Ricci, recorded that the Chinese say: "The earth is flat and square, and the sky is a round canopy; they did not succeed in conceiving the possibility of the antipodes". The universal belief in a flat Earth is confirmed by a contemporary Chinese encyclopedia from 1609 illustrating a flat Earth extending over the horizontal diametral plane of a spherical heaven.

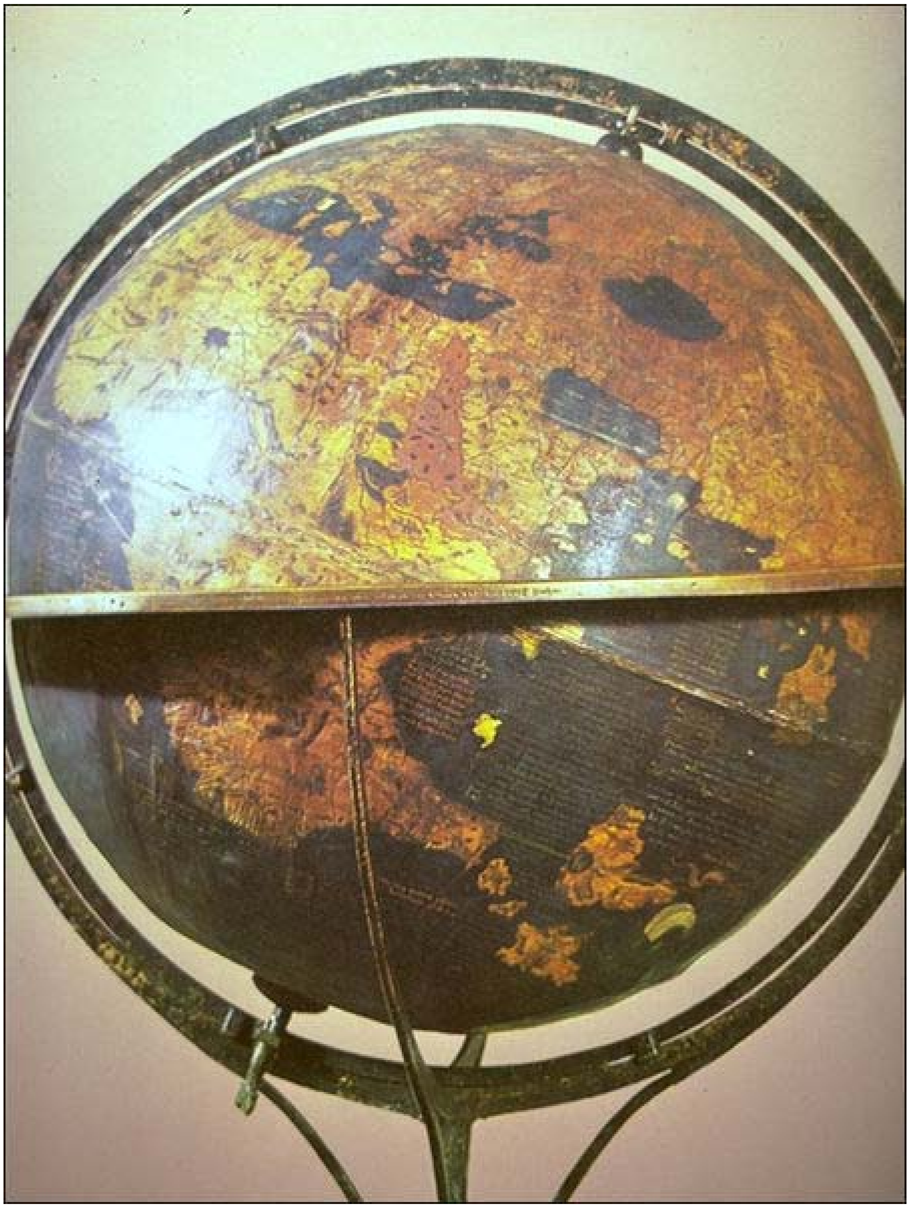
In the 17th century, the idea of a spherical Earth spread in China due to the influence of the Jesuits, who held high positions as astronomers at the imperial court. The *Ge Chi Cao* treatise of Xiong Ming-yu (1648) showed a printed picture of the Earth as a spherical globe, with the text stating that "the round earth certainly has no square corners". The text also pointed out that sailing ships could return to their port of origin after circumnavigating the waters of the Earth.

The influence of the map is distinctly Western, as traditional maps of Chinese cartography held the graduation of the sphere at 365.25 degrees, while the Western graduation was of 360 degrees. Also of interest to note is on one side of the world, there is seen towering Chinese pagodas, while on the opposite side (upside-down) there were European cathedrals. Western influence of geographical knowledge was used by Xiong to enforce what he believed had already been argued by earlier Chinese astronomers. However, the French sinologist Jean-Claude Martzloff regards this as a retrospective interpretation:

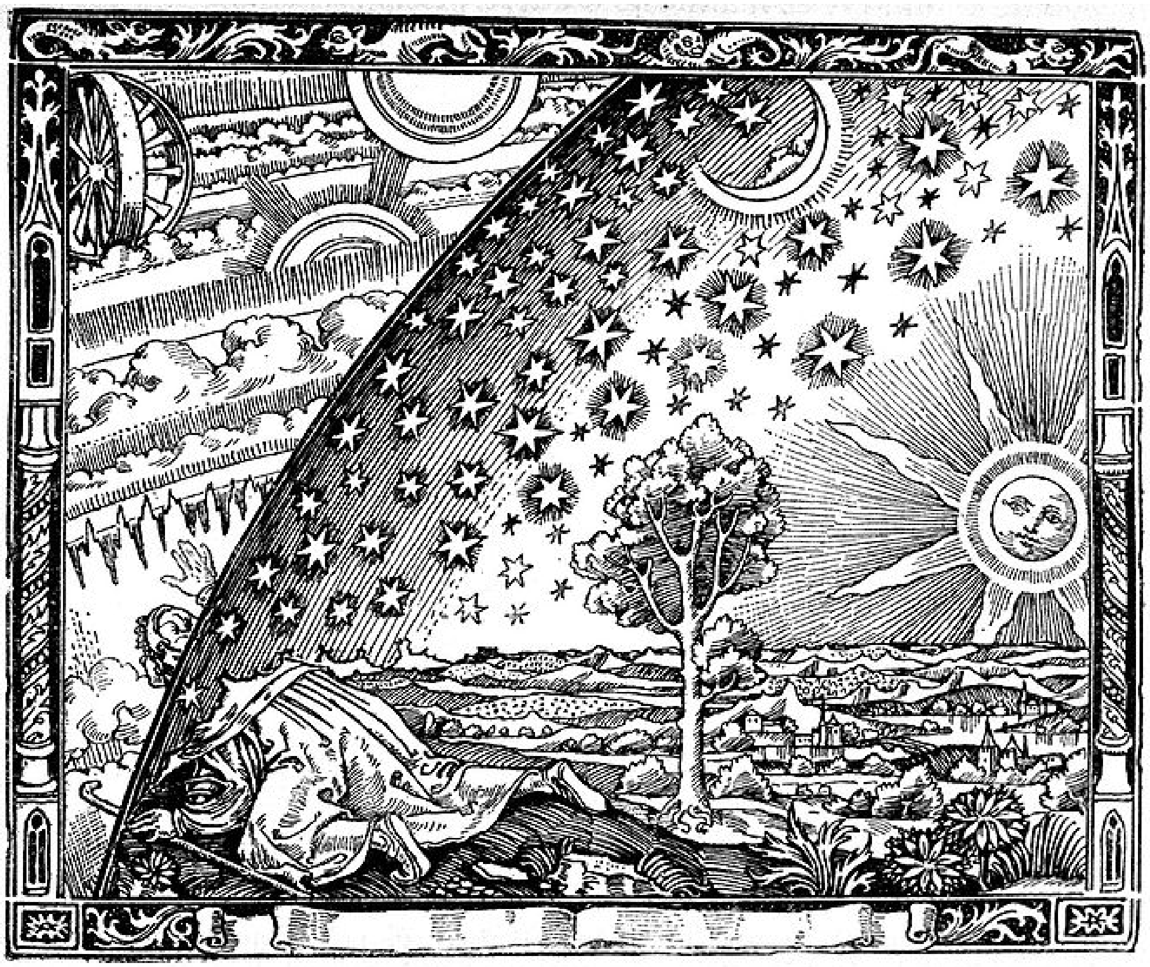
European astronomy was so much judged worth consideration that numerous Chinese authors developed the idea that the Chinese of antiquity had anticipated most of the novelties presented by the missionaries as European discoveries, for example, the rotundity of the earth and the “heavenly spherical star carrier model.” Making skillful use of philology, these authors cleverly reinterpreted the greatest technical and literary works of Chinese antiquity. From this sprang a new science wholly dedicated to the demonstration of the Chinese origin of astronomy and more generally of all European science and technology.

Modern period

"Myth of the Flat Earth" in modern historiography



Martin Behaim's *Erdapfel*, the oldest surviving terrestial globe and finished before the news of the discovery of the Americas had reached Europe (1492), demonstrates that knowledge of the round Earth was common on the continent before.



Un missionnaire du moyen âge raconte qu'il avait trouvé le point
où le ciel et la Terre se touchent...

The famous "Flat Earth" Flammarion woodcut originates with Flammarion's 1888 *L'atmosphère: météorologie populaire* (p. 163)

Illustration of the spherical Earth in a 14th century copy of *L'Image du monde* (ca. 1246).

The **myth of the Flat Earth** is the modern misconception that the prevailing cosmological view during the Middle Ages saw the Earth as flat, instead of spherical.

"The idea that educated men at the time of Columbus believed that the earth was flat, and that this belief was one of the obstacles to be overcome by Columbus before he could get his project sanctioned, remains one of the hardiest errors in teaching."

During the early Middle Ages, virtually all scholars maintained the spherical viewpoint first expressed by the Ancient Greeks. By the 14th century, belief in a flat earth among the educated was essentially dead. Flat-Earth models were in fact held at earlier (pre-medieval) times, before the spherical model became commonly accepted in Hellenistic astronomy.

However, among Medieval artists, depictions of a flat earth remained common. The exterior of the famous triptych The Garden of Earthly Delights by Hieronymus Bosch is a Renaissance example in which a disc-shaped earth is shown floating inside a transparent sphere.

According to Stephen Jay Gould, "there never was a period of 'flat earth darkness' among scholars (regardless of how the public at large may have conceptualized our planet both then and now). Greek knowledge of sphericity never faded, and all major medieval scholars accepted the earth's roundness as an established fact of cosmology."

Historians of science David Lindberg and Ronald Numbers point out that "there was scarcely a Christian scholar of the Middle Ages who did not acknowledge [Earth's] sphericity and even know its approximate circumference".

Historian Jeffrey Burton Russell says the flat earth error flourished most between 1870 and 1920, and had to do with the ideological setting created by struggles over evolution. Russell claims "with extraordinary [sic] few exceptions no educated person in the history of Western Civilization from the third century B.C. onward believed that the earth was flat," and credits histories by John William Draper, Andrew Dickson White, and Washington Irving for popularizing the flat-earth myth.

Origin

In *Inventing the Flat Earth: Columbus and Modern Historians*, Jeffrey Russell claims that the Flat Earth theory is a fable used to impugn pre-modern civilization, especially that of the Middle Ages in Europe.

James Hannam wrote:

The myth that people in the Middle Ages thought the earth is flat appears to date from the 17th century as part of the campaign by Protestants against Catholic teaching. But it gained currency in the 19th century, thanks to inaccurate histories such as John William Draper's *History of the Conflict Between Religion and Science* (1874) and Andrew Dickson White's *History of the Warfare of Science with Theology in Christendom* (1896). Atheists and agnostics championed the conflict thesis for their own purpose ...

Conflict between religion and science

The 19th century was a period in which the perception of an antagonism between religion and science was especially strong. The disputes surrounding the Darwinian revolution contributed to the birth of the conflict thesis, a view of history according to which any interaction between religion and science almost inevitably would lead to open hostility, with religion usually taking the part of the aggressor against new scientific ideas.

Irving's biography of Columbus

In 1828, Washington Irving's highly romanticised and inaccurate biography, *The Life and Voyages of Christopher Columbus*, was published and mistaken by many for a scholarly work. In Book III, Chapter II of this biography, Irving gave a largely fictional account of the meetings of a commission established by the Spanish sovereigns to examine Columbus's proposals. One of his more fanciful embellishments was a highly unlikely tale that the more ignorant and bigoted members on the commission had raised scriptural objections to Columbus's assertions that the Earth was spherical.

But in reality, the issue in the 1490s was not the shape of the Earth, but its size, and the position of the east coast of Asia, as Irving in fact points out. Historical estimates from

Ptolemy onwards placed the coast of Asia about 180° east of the Canary Islands. Columbus adopted an earlier (and rejected) distance of 225°, added 28° (based on Marco Polo's travels), and then placed Japan another 30° further east. Starting from Cape St. Vincent in Portugal, Columbus made Eurasia stretch 283° to the east, leaving the Atlantic as only 77° wide. Since he planned to leave from the Canaries (9° further west), his trip to Japan would only have to cover 68° of longitude.

Furthermore, Columbus mistakenly used a much shorter length for a degree (he substituted the shorter 1480 m Italian "mile" for the longer 2177 m Arabic "mile"), making his degree (and the circumference of the Earth) about 75% of what it really was. The combined effect of these mistakes was that Columbus estimated the distance to Japan to be only about 5,000 km (or only to the eastern edge of the Caribbean) while the true figure is about 20,000 km. The Spanish scholars may not have known the exact distance to the east coast of Asia, but they certainly knew that it was significantly further than Columbus' projection; and this was the basis of the criticism in Spain and Portugal, whether academic or amongst mariners, of the proposed voyage.

The disputed point, therefore, was not the shape of the Earth, nor the idea that going west would eventually lead to Japan and China, but the ability of European ships to sail that far across open seas. The small ships of the day (Columbus' three ships varied between 20.5 and 23.5 m – or 67 to 77 feet – in length and carried about 90 men) simply could not carry enough food and water to reach Japan. In fact, the ships barely reached the eastern Caribbean islands. Already the crews were mutinous, not because of some fear of "sailing off the edge", but because they were running out of food and water with no chance of any new supplies within sailing distance. They were on the edge of starvation. What saved Columbus, of course, was the unknown existence of the Americas precisely at the point he thought he would reach Japan. His ability to resupply with food and water from the Caribbean islands allowed him to return safely to Europe. Otherwise his crews would have died, and the ships foundered. The academics were right: it was not possible for a 1492 ship to sail west across open oceans directly to Japan; mariners would die long before their proposed arrival.

Letronne, Whewell and Flammarion

In 1834, a few years after the publication of Irving's book, Jean Antoine Letronne, a French academic of strong antireligious ideas, misrepresented the church fathers and their medieval successors as believing in a flat earth, in his *On the Cosmographical Ideas of the Church Fathers*. Then, In 1837, the English philosopher of science William Whewell first identified, in his *History of the Inductive Sciences*, two minimally significant characters named Lactantius (245-325, also mocked by Copernicus' in De revolutionibus of 1543, as someone who *speaks quite childishly about the Earth's shape, when he mocks those who declared that the Earth has the form of a globe*) and Cosmas Indicopleustes, who wrote his "Christian Topography" in 547-549. Whewell pointed to them as evidence of a medieval belief in a Flat Earth, and other historians quickly followed him, even though it was hard to find other examples.

The widely circulated woodcut of a man poking his head through the firmament of a flat Earth to view the mechanics of the spheres, executed in the style of the 16th century was published in Camille Flammarion's *L'Atmosphère: Météorologie Populaire* (Paris, 1888, p. 163). The woodcut illustrates the statement in the text that a medieval missionary claimed that "he reached the horizon where the Earth and the heavens met", an anecdote that may be traced back to Voltaire. In its original form, the woodcut included a decorative border that places it in the 19th century; in later publications, some claiming that the woodcut did, in fact, date to the 16th century, the border was removed. Flammarion, according to anecdotal evidence, had commissioned the *Flammarion woodcut* himself.

Promulgation

Accounts in modern textbooks

The popularized version of the misconception that people before the Age of Discovery believed that Earth was flat persists in the popular imagination, and was repeated in some widely read textbooks.

An American schoolbook by Emma Miller Bolenius published in 1919 has this introduction to the suggested reading for Columbus Day (12 October):

- When Columbus lived, people thought that the earth was flat. They believed the Atlantic Ocean to be filled with monsters large enough to devour their ships, and with fearful waterfalls over which their frail vessels would plunge to destruction. Columbus had to fight these foolish beliefs in order to get men to sail with him. He felt sure the earth was round.

Previous editions of Thomas Bailey's *The American Pageant* stated that *"The superstitious sailors [of Columbus' crew] ... grew increasingly mutinous...because they were fearful of sailing over the edge of the world"*; however, no such historical account is known. Actually, sailors were probably among the first to know of the curvature of Earth from daily observations — seeing how shore landscape features (or masts of other ships) gradually descend/ascend near the horizon, and the biggest fear among Columbus' crew would have been that the ocean was too big to cross leading to starvation.

Accounts in popular media

The 1937 popular song, *They All Laughed* contains the couplet "They all laughed at Christopher Columbus/When he said the world was round".

In Walt Disney's 1963 animation *The Sword in the Stone*, wizard Merlin (who has travelled into the future) explains to his apprentice that "One day they will discover that the earth is round".

In a 2008 speech at Google, Salman Rushdie describes popular Western European perceptions fifty years before Columbus: that a ship sailing west would find first ship-eating monsters, then mud replacing the waters, and finally, the edge of the world.

Pre-19th century writings

It should be noted, however, that French dramatist Cyrano de Bergerac in chapter 5 of his *The Other World The Societies and Governments of the Moon* quotes St. Augustine as saying "that in his day and age the earth was as flat as a stove lid and that it floated on water like half of a sliced orange." Robert Burton, in his *The Anatomy of Melancholy* wrote: "Virgil, sometimes bishop of Saltburg (as Aventinus *anno* 745 relates) by Bonifacius bishop of Mentz was therefore called in question, because he held antipodes (which they made a doubt whether Christ died for) and so by that means took away the seat of hell, or so contracted it, that it could bear no proportion to heaven, and contradicted that opinion of Austin [St. Augustine], Basil, Lactantius that held the earth round as a trencher (whom Acosta and common experience more largely confute) but not as a ball." Thus, there is evidence that accusations of flatearthism, though somewhat whimsical (Burton ends his digression with a legitimate quotation of St. Augustine: "Better doubt of things concealed, than to contend about uncertainties, where Abraham's bosom is, and hell fire") were used to discredit opposing authorities several centuries before the 19th. Another early mention in literature is Ludvig Holberg's comedy Erasmus Montanus (1723). Erasmus Montanus meets considerable opposition when he claims the Earth is round, since all the peasants hold it to be flat. He is not allowed to marry his fiancée until he cries "The earth is flat as a pancake".

Recognition as a misconception

Since the early 20th century, a number of books and articles have been devoted to documenting the flat earth error as one of a number of widespread misconceptions in popular views of the Middle Ages.

Jeffrey Burton Russell published monographs on the topic in 1991 and 1997. Louise Bishop (2008) asserts that virtually every thinker and writer of the thousand-year medieval period affirmed the spherical shape of the earth.

Modern flat-Earthers

In the modern era, belief in a flat Earth has been expressed by isolated individuals and groups, but no scientists of note.

English writer Samuel Rowbotham (1816–1885), writing under the pseudonym "Parallax," produced a pamphlet called *Zetetic Astronomy* in 1849 arguing for a flat Earth and published results of many experiments that tested the curvatures of water over a long drainage ditch, followed by another called *The inconsistency of Modern Astronomy and its Opposition to the Scripture*. One of his supporters, John Hampden, lost a bet to Alfred Russell Wallace in the famous Bedford Level Experiment, which attempted to prove it.

Rowbotham also produced studies that purported to show the effects of ships disappearing below the horizon could be explained by the laws of perspective in relation to the human eye.

In 1883 he founded Zetetic Societies in England and New York, to which he shipped a thousand copies of *Zetetic Astronomy*. Challenges were issued in the New York Daily Graphic offering $10,000 to charity to anyone proving the Earth revolved on axes.

William Carpenter, a printer originally from Greenwich, England, was a supporter of Rowbotham and published *Theoretical Astronomy Examined and Exposed - Proving the Earth not a Globe* in eight parts from 1864 under the name *Common Sense*. He later emigrated to Baltimore where he published *A hundred proofs the Earth is not a Globe* in 1885. He argues that:

- "There are rivers that flow for hundreds of miles towards the level of the sea without falling more than a few feet — notably, the Nile, which, in a thousand miles, falls but a foot. A level expanse of this extent is quite incompatible with the idea of the Earth's convexity. It is, therefore, a reasonable proof that Earth is not a globe."
- "If the Earth were a globe, a small model globe would be the very best - because the truest - thing for the navigator to take to sea with him. But such a thing as that is not known: with such a toy as a guide, the mariner would wreck his ship, of a certainty!, This is a proof that Earth is not a globe."

John Jasper, the black ex-slave preacher said to have preached to more people than any Southern clergyman of his generation, echoed his friend Carpenter's sentiments in his most famous sermon "Der Sun do move and the Earth Am Square", preached over 250 times always by invitation.

In Brockport, N.Y, in 1887, M.C. Flanders argued the case of a flat Earth for three nights against two scientific gentlemen defending sphericity. Five townsmen chosen as judges voted unanimously for a flat Earth at the end. The case was reported in the Brockport Democrat.

Prof' Joe Holden of Maine, a former justice of the peace, gave numerous lectures in New England and lectured on flat Earth theory at the Columbian Exposition in Chicago. His fame stretched to North Carolina where the Statesville *Semi-weekly Landmark* recorded at his death in 1900: 'We hold to the doctrine that the earth is flat ourselves and we regret exceedingly to learn that one of our members is dead'.

After Rowbotham's death, Lady Elizabeth Blount created the Universal Zetetic Society in 1893 in England and created a journal called *Earth not a Globe Review,* which sold for twopence, as well as one called *Earth* which only lasted from 1901 to 1904. She held that the Bible was the unquestionable authority on the natural world and argued that one could not be a Christian and believe the Earth to be a globe. Well-known members included E. W. Bullinger of the Trinitarian Bible Society, *Edward Haughton*, senior moderator in natural science in Trinity College, Dublin and an archbishop. She repeated Rowbotham's experiments, generating some interesting counter-experiments, but interest declined after the First World War.

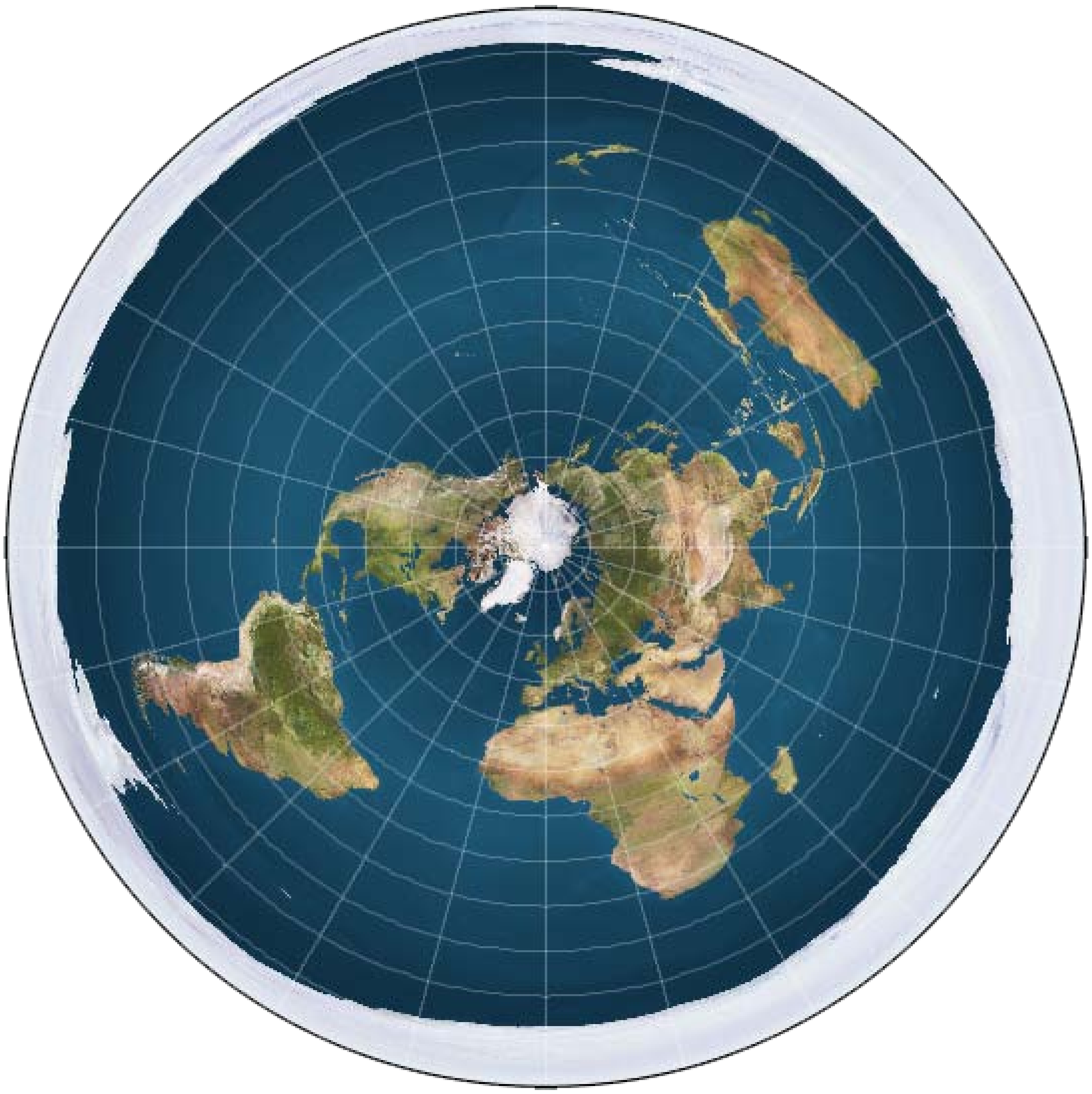
In 1898, during his solo circumnavigation of the world, Joshua Slocum encountered a group of flat-Earthers in Durban. Three Boers, one of them a clergyman, presented Slocum with a pamphlet in which they set out to prove that the world was flat. Paul Kruger, President of the Transvaal Republic, advanced the same view: "You don't mean *round* the world, it is impossible! You mean *in* the world. Impossible!"

Wilbur Glenn Voliva, who in 1906 took over the Christian Catholic Church, a pentecostal sect that established a utopian community at Zion, Illinois, preached flat Earth doctrine from 1915 onwards and used a photograph of a twelve mile stretch of the shoreline at Lake Winnebago, Wisconsin taken three feet above the waterline to prove his point. When the airship Italia disappeared on an expedition to the North Pole in 1928 he warned the world's press that it had sailed over the edge of the world. He offered a $5,000 award for proving the Earth is not flat, under his own conditions. Teaching a globular Earth was banned in the Zion schools and the message was transmitted on his WCBD radio station.

Modern biblical literalists have swung to the other extreme over flat Earth theories. For example, the *Biblical Astronomer* website, while supporting a geocentric model of the Earth, is clear on this, although its description of the foundations shows ingenuity: "In summary, the Bible teaches that the earth is basically a sphere in shape; that there are pillars which undergird the world and which we conclude to be the crystalline rock corresponding to what we commonly call the mantle."

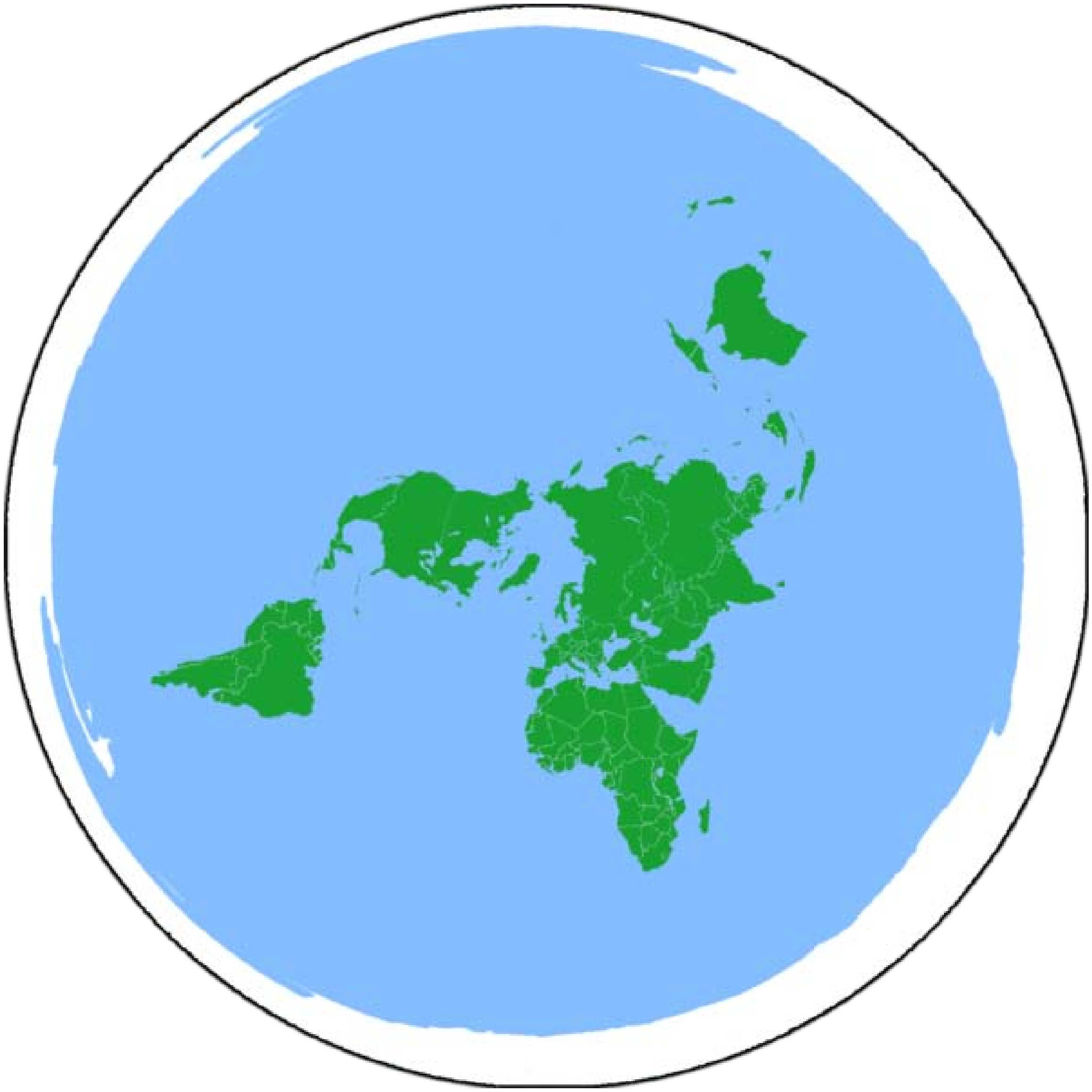
Mohammed Yusuf, creator of the Boko Haram Islamic sect in Nigeria stated his belief in a flat earth

Flat Earth Society



This flat Earth model depicts Antarctica as an ice wall surrounding a disk shaped Earth

Final Frontier Voyager or Flat Earth Society, by George Grie.



The Flat Earth model depicts Antarctica as an ice wall surrounding a disk shaped Earth

The **Flat Earth Society** (also known as the **International Flat Earth Society** or the **International Flat Earth Research Society**) is an organization that seeks to further the belief that the Earth is flat, contrary to the scientifically proven fact that it is a spheroid. The modern organization was founded by Englishman Samuel Shenton in 1956, and later led by Charles K. Johnson, who based the organization in his home in Lancaster, California. The formal society was inactive after Johnson's death in 2001, but was recently resurrected by its new president Daniel Shenton.

Origins - the Zetetic societies

The belief that the Earth was flat was typical of ancient cosmologies until about the 4th century BC, when the Ancient Greek philosophers proposed the idea that the Earth was a sphere, or at least rounded in shape. Aristotle was one of the first thinkers to propose a spherical Earth in 330 BC. By the early Middle Ages, it was widespread knowledge throughout Europe that the Earth was a sphere.

Modern hypotheses supporting a flat Earth originated with English inventor Samuel Rowbotham (1816–1884). Based on his interpretation of certain biblical passages, Rowbotham published a 16-page pamphlet, which he later expanded into a 430-page book, *Earth Not a Globe*, expounding his views. According to Rowbotham's system, which he called "Zetetic Astronomy", the earth is a flat disc centered at the North Pole and bounded along its southern edge by a wall of ice (Antarctica), with the sun and moon 3000 miles (4800 km) and the "cosmos" 3100 miles (5000 km) above earth.

Rowbotham and his followers gained attention by engaging in public debates with leading scientists of the day. One such debate, involving the prominent naturalist Alfred Russel Wallace, concerned the Bedford Level experiment (and later led to several lawsuits for fraud and libel). He created a Zetetic Society in England and New York, shipping over a thousand copies of *Zetetic Astronomy*.

After Rowbotham's death, Lady Elizabeth Blount, wife of the explorer Sir Walter de Sodington Blount, established a Universal Zetetic Society, published a magazine entitled *The Earth Not a Globe Review*, and remained active well into the early part of the 20th century. A flat Earth journal, *Earth: a Monthly Magazine of Sense and Science*, was published between 1901–1904, edited by Lady Blount. In 1901, she repeated Rowbotham's Bedford Level Experiment and photographed the effect, sparking a correspondence in the magazine English Mechanic with several counter-claims. Later it achieved some notoriety by being involved in a scam involving dental practices. After World War I, the movement underwent a slow decline.

Flat Earth Society

In 1956, Samuel Shenton, a signwriter by trade and a Fellow of the Royal Geographic Society created the International Flat Earth Society as a successor to the Universal Zetetic Society and ran it as "organizing secretary" from his home in Dover, in Britain. Due to Shenton's interest in alternative science and technology the emphasis on religious arguments was less than in the predecessor society.

This was just before the launch of the first artificial satellite and when satellite images taken from outer space showed the Earth as a sphere rather than flat, the society was undaunted; Shenton remarked: "It's easy to see how a photograph like that could fool the untrained eye."

However it was not till the advent of manned spaceflight that Shenton managed to attract wide publicity, being featured in the *New York Times* in January and June 1964, when the epithet "flat-earther" was also slung across the floor of the House of Commons of the United Kingdom in both directions.

The society also took the position that the Apollo Moon landings were a hoax, staged by Hollywood and based on a script by Arthur C. Clarke, a position also held by others not connected to the Flat Earth Society. On hearing this, Clarke sent a facetious letter to NASA's chief administrator:

"Dear Sir, on checking my records, I see that I have never received payment for this work. Could you please look into this matter with some urgency? Otherwise you will be hearing from my solicitors, Messrs Geldsnatch, Geldsnatch and Blubberclutch".

In 1969, Shenton persuaded Ellis Hillman, a Polytechnic lecturer, to become president of the Flat Earth Society, but there is little evidence of any activity on his part until after Shenton's death, when he added most of Shenton's library to the archives of the Science Fiction Foundation which he helped to establish.

Shenton died in 1971 and Charles K. Johnson, inheriting part of Shenton's library from Shenton's wife, established and became the president of the International Flat Earth Research Society of America and Covenant People's Church in California. Under his leadership, over the next three decades, the Flat Earth Society grew in size from a few members to about 3,000. Johnson distributed newsletters, flyers, maps, and other promotional materials to anyone who asked for them, and managed all membership applications together with his wife, Marjory, who was also a flat-earther. The most famous of these newsletters was *Flat Earth News,* which was a quarterly four page tabloid. Johnson paid for this through annual dues of members, which ranged from $6 to $10 over the course of his leadership.

Some headlines from *Flat Earth News* during the '70s and early '80s:

- "Whole World Deceived... Except the Very Elect" (Dec. 1977)
- "Australia Not Down Under" (May 1978)
- "Sun Is a Light 32 Miles Across" (Dec. 1978)
- "The Earth Has No Motion" (Jun. 1979)
- "Nikita Krushchev Father of NASA" (Mar. 1980)
- "Galileo Was a Liar" (Dec. 1980)
- "Science Insults Your Intelligence" (Sep. 1980)
- "World IS Flat, and That's That" (Sep. 1980)
- "The Earth Is Not a Ball; Gravity Does Not Exist" (Mar. 1981)

United Nations flag

The most recent world model propagated by the Flat Earth Society holds that humans live on a disc, with the North Pole at its center and a 150-foot (45 m) high wall of ice at the outer edge. The resulting map resembles the symbol of the United Nations, which Johnson used as evidence for his position. In this model, the sun and moon are each a mere 32 miles (52 km) in diameter.

A newsletter from the society gives some insight into Johnson's thinking:

Aim: To carefully observe, think freely, rediscover forgotten fact and oppose theoretical dogmatic assumptions. To help establish the United States...of the world on this flat earth. Replace the science religion...with SANITY.
The International Flat Earth Society is the oldest continuous Society existing on the world today. It began with the Creation of the Creation. First the water...the face of the deep...without form or limits...just Water. Then the Land sitting in and on the Water, the Water then as now being flat and level, as is the very Nature of Water. There are, of course, mountains and valleys on the Land but since most of the World is Water, we say, "The World is Flat". Historical accounts and spoken history tell us the Land part may have been square, all in one mass at one time, then as now, the magnetic north being the Center. Vast cataclysmic events and shaking no doubt broke the land apart, divided the Land to be our present continents or islands as they exist today. One thing we know for sure about this world...the known inhabited world is Flat, Level, a Plain World.
We maintain that what is called 'Science' today and 'scientists' consist of the same old gang of witch doctors, sorcerers, tellers of tales, the 'Priest-Entertainers' for the common

people. 'Science' consists of a weird, way-out occult concoction of gibberish theory-theology...unrelated to the real world of facts, technology and inventions, tall buildings and fast cars, airplanes and other Real and Good things in life; technology is not in any way related to the web of idiotic scientific theory. ALL inventors have been anti-science. The Wright brothers said: "Science theory held us up for years. When we threw out all science, started from experiment and experience, then we invented the airplane." By the way, airplanes all fly level on this Plane earth.

The Flat Earth Society recruited members by attacking the United States government and all of its agencies, particularly NASA. Much of the society's literature in its early days focused on interpreting the Bible literally to mean that the Earth is flat, although they did attempt to offer scientific explanations and evidence.

The group rose to about 2,000 members during its peak under Charles K. Johnson. The organization faced overwhelming scientific evidence and public opinion that maintained that the Earth is a sphere. The term "flat-earther" became commonly used to refer to an individual who stubbornly adheres to discredited or outmoded ideas.

The society began to decline in the 1990s, and was further affected by a fire at the house of Charles K. Johnson which destroyed all of the records and contacts of members of the Flat Earth Society. Johnson's wife, who helped manage the database, died shortly thereafter. Charles K. Johnson himself died on March 19, 2001.

Flat Earth Society of Canada

The Flat Earth Society of Canada was set up by Leo Ferrari (1927-2010), a philosophy professor at the St Thomas University in 1970, together with Raymond Fraser and Alden Nowlan and was active till about 1984. Their aims were quite different from other flat earth societies. They believed 'a prevailing problem of the new technological age was the willingness of people to accept theories "on blind faith and to reject the evidence of their own senses."'

They published a newsletter, *The Official Chronicle* and promoted their ideas more widely through television and press. Its primary aims were "to combat the fallacious deification of the circle," "to restore man's confidence in the validity of his own perceptions", and "to spearhead man's escape from his metaphysical and geometrical prison."

Flat Earth Society today

As of September 2009, two web-based discussion forums exist. The Flat Earth Society is also represented on Twitter and Facebook.

Some of the postings appear to exhibit a genuine belief in the flat earth idea, and a FAQ is available.

One web site that claims to represent the Flat Earth Society states that it has existed as an organization since 1547.

Current proponents of the Flat Earth Society do not have a central alternative theory; different members have unique ideas on how the Earth is constructed. Some advocate the idea that the Earth is utterly flat, while others advocate a disk construction.

Daniel Shenton has resurrected the Flat Earth society. In an article in The Guardian, Shenton says he has recruited 60 members. The report notes that he has a "website [which] features scanned issues of the society's newsletter, the Flat Earth News, from the 1970s and 80s."

Chapter-3

Spherical Earth

Medieval artistic representation of a spherical Earth - with compartments representing earth, air, and water (c. 1400).

The concept of a **spherical Earth** dates back to ancient Greek philosophy from around the 6th century BC, but remained a matter of philosophical speculation until the 3rd century BC when Hellenistic astronomy established the spherical shape of the earth as a physical given. The Greek paradigm was gradually adopted, either directly or via later European science, by all major civilizations and world regions, becoming the dominant concept of modern times. The final, practical demonstration of Earth's sphericity was achieved by Ferdinand Magellan / Juan Sebastian Elcano's expedition's circumnavigation (1519−1521).

The concept of a spherical Earth displaced earlier beliefs in a flat Earth: In early Mesopotamian mythology, the world was portrayed as a flat disk floating in the ocean and surrounded by a spherical sky, and this forms the premise for early world maps like those of Anaximander and Hecataeus of Miletus. Other speculations on the shape of Earth include a seven-layered ziggurat or cosmic mountain, alluded to in the Avesta and ancient Persian writings, or a wheel, bowl, or four-cornered plane alluded to in the Rigveda.

As determined by modern instruments, a sphere approximates the Earth's shape to within one part in 300. An oblate ellipsoid shape with a flattening of 1/300 matches even more precisely. Recent measurements from satellites suggest that the Earth is, in fact, slightly pear-shaped.

History

Early Greek and Hellenistic development

Though the earliest evidence of a spherical Earth comes from ancient Greek sources, there is no account of how the sphericity of the Earth was discovered. A plausible explanation is that it was "the experience of travellers that suggested such an explanation for the variation in the observable altitude and the change in the area of circumpolar stars, a change that was quite drastic between Greek settlements" around the eastern Mediterranean Sea, particularly those between the Nile Delta and the Crimea.

Pythagoras

Early Greek philosophers alluded to a spherical Earth, though with some ambiguity. Pythagoras (6th century BC) was among those said to have originated the idea, but this may reflect the ancient Greek practice of ascribing every discovery to one or another of their ancient wise men. Some idea of the sphericity of the Earth seems to have been known to both Parmenides and Empedocles in the 5th century BC, and although the idea cannot reliably be ascribed to Pythagoras, it may, nevertheless have been formulated in the Pythagorean school in the 5th century BC. After the 5th century BC, no Greek writer of repute thought the world was anything but round.

Herodotus

In *The Histories*, written 431–425 BC, Herodotus dismisses a report of the sun observed shining from the north. This arises when discussing the circumnavigation of Africa undertaken by Phoenicians under Necho II c. 610–595 BC. (*The Histories*, 4.43) His dismissive comment attests to a widespread ignorance of the ecliptic's inverted declination in a southern hemisphere.

Plato

Plato (427–347 BC) travelled to southern Italy to study Pythagorean mathematics. When he returned to Athens and established his school, Plato also taught his students that Earth was a sphere. If man could soar high above the clouds, Earth would resemble *"one of* *those balls which have leather coverings in twelve pieces, and is decked with various colours, of which the colours used by painters on earth are in a manner samples."* In Timaeus, his one work that was available throughout the Middle Ages in Latin, we read that the Creator "made the world in the form of a globe, round as from a lathe, having its extremes in every direction equidistant from the centre, the most perfect and the most like itself of all figures", though the word "world" normally refers to the universe.

Aristotle

Round Earth umbra during a lunar eclipse

Aristotle (384–322 BC) was Plato's prize student and *"the mind of the school."* Aristotle observed *"there are stars seen in Egypt and [...] Cyprus which are not seen in the northerly regions."* Since this could only happen on a curved surface, he too believed Earth was a sphere *"of no great size, for otherwise the effect of so slight a change of place would not be quickly apparent."* (*De caelo*, 298a2–10)

Aristotle provided physical and observational arguments supporting the idea of a spherical Earth:

- Every portion of the Earth tends toward the center until by compression and convergence they form a sphere. (*De caelo*, 297a9–21)
- Travelers going south see southern constellations rise higher above the horizon; and
- The shadow of Earth on the Moon during a lunar eclipse is round. (*De caelo*, 297b31–298a10)

The concepts of symmetry, equilibrium and cyclic repetition permeated Aristotle's work. In his *Meteorology* he divided the world into five climatic zones: two temperate areas separated by a torrid zone near the equator, and two cold inhospitable regions, *"one near our upper or northern pole and the other near the ... southern pole,"* both impenetrable and girdled with ice (*Meteorologica*, 362a31–35). Although no humans could survive in the frigid zones, inhabitants in the southern temperate regions could exist.

Eratosthenes

Eratosthenes (276–194 BC) estimated Earth's circumference around 240 BC. He had heard that in Syene the Sun was directly overhead at the summer solstice whereas in Alexandria it still cast a shadow. Using the differing angles the shadows made as the basis of his trigonometric calculations he estimated a circumference of around 250,000 *stades*. The length of a 'stade' is not precisely known, but Eratosthenes' figure only has an error of around five to ten percent. Eratosthenes used rough estimates and round numbers, but depending on the length of the stadion, his result is within a margin of between 2% and 20% of the actual meridional circumference, 40,008 kilometres (24,860 mi). Note that Eratosthenes could only measure the circumference of the Earth by assuming that the distance to the Sun is so great that the rays of sunlight are essentially parallel.

Seleucus of Seleucia

Seleucus of Seleucia (c. 190 BC), who lived in the Seleucia region of Mesopotamia, stated that the Earth is spherical (and actually orbits the Sun, influenced by the heliocentric theory of Aristarchus of Samos).

Posidonius

Posidonius (c. 135 – 51 BC) put faith in Eratosthenes's method, though by observing the star Canopus, rather than the sun in establishing the Earth's circumference. In Ptolemy's *Geographia*, his result was favoured over that of Erastosthenes. Posidonius furthermore expressed the distance of the sun in earth radii.

Strabo

When a ship is at the horizon its lower part is invisible due to Earth's curvature. This was one of the first arguments favoring a round-Earth model.

It has been suggested that seafarers probably provided the first observational evidence that the Earth was not flat, based on observations of the horizon. This argument was put forward by the geographer Strabo (c. 64 BC – 24 AD), who suggested that the spherical shape of the Earth was probably known to seafarers around the Mediterranean Sea since at least the time of Homer, citing a line from the *Odyssey* as indicating that the poet Homer was already aware of this as early as the 7th or 8th century BC. Strabo cited various phenomena observed at sea as suggesting that the Earth was spherical. He observed that elevated lights or areas of land were visible to sailors at greater distances than those less elevated, and stated that the curvature of the sea was obviously responsible for this.

Claudius Ptolemy

Claudius Ptolemy (90–168 AD) lived in Alexandria, the centre of scholarship in the second century. In the Almagest, which remained the standard work of astronomy for 1,400 years, he advanced many arguments for the sphericity of the Earth. Among them was the observation that when sailing towards mountains, they seem to rise from the sea, indicating that they were hidden by the curved surface of the sea. He also gives separate arguments that the Earth is curved north-south and that it is curved east-west.

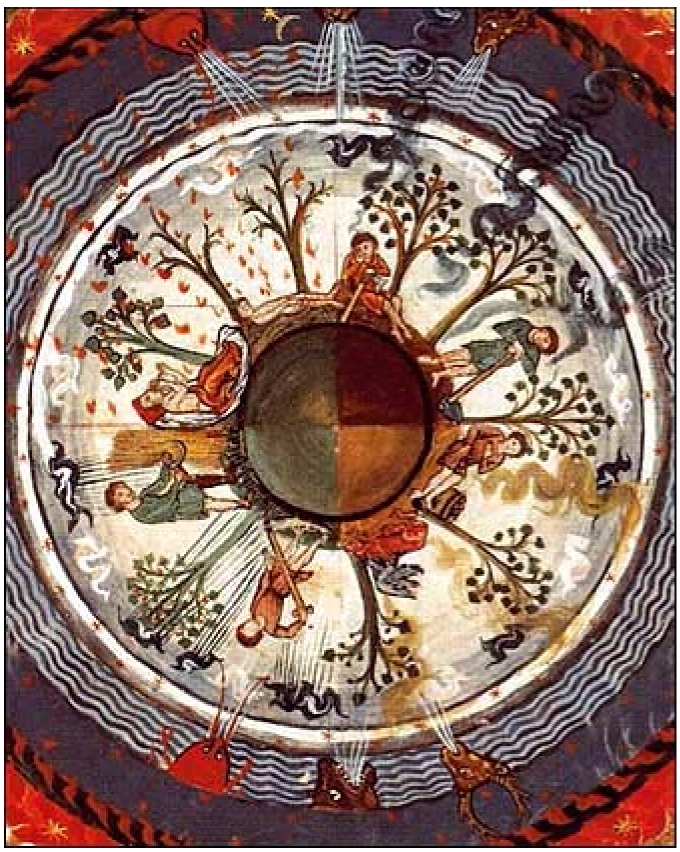
He also produced an eight-volume *Geographia* dealing with the earth. The first part of the *Geographia* is a discussion of the data and of the methods he used. As with the model of the solar system in the *Almagest*, Ptolemy put all this information into a grand scheme. He assigned coordinates to all the places and geographic features he knew, in a grid that spanned the globe (although most of this has been lost). Latitude was measured from the equator, as it is today, but Ptolemy preferred to express it as the length of the longest day rather than degrees of arc (the length of the midsummer day increases from 12h to 24h as you go from the equator to the polar circle). He put the meridian of 0 longitude at the most western land he knew, the Canary Islands.

Geographia indicated the countries of "Serica" and "Sinae" (China) at the extreme right, beyond the island of "Taprobane" (Sri Lanka, oversized) and the "Aurea Chersonesus" (Southeast Asian peninsula).

Ptolemy also devised and provided instructions on how to create maps both of the whole inhabited world (*oikoumenè*) and of the Roman provinces. In the second part of the *Geographia* he provided the necessary topographic lists, and captions for the maps. His *oikoumenè* spanned 180 degrees of longitude from the Canary Islands in the Atlantic Ocean to China, and about 81 degrees of latitude from the Arctic to the East Indies and deep into Africa. Ptolemy was well aware that he knew about only a quarter of the globe.

Global diffusion

Spread to the west



Spherical earth with the four seasons. Illustration in 12th century book *Liber Divinorum Operum* by Hildegard of Bingen

From its Greek origins, the idea of a spherical earth, along with much of Greek astronomical thought, slowly spread across the globe and ultimately became the adopted view in all major astronomical traditions: In the west, the idea came naturally to the

Romans through the lengthy process of cross-fertilization with Greek civilization. Many Roman authors such as Cicero and Pliny refer in their works to the rotundity of the earth as a matter of course.

Treated as self-evident by early Christian authorities like Aurelius Augustinus, the idea survived into the medieval corpus of knowledge and became increasingly traceable with the rise of scholasticism and medieval learning. A non-exhaustive list of more than a hundred Latin and vernacular writers from late antiquity to the 15th century, who were aware that the earth was spherical, has been compiled by Reinhard Krüger, professor for Romance literature at the University of Stuttgart:

Portuguese exploration of Africa and Asia, Columbus voyage to the Americas (1492) and finally Ferdinand Magellan's circumnavigation of the earth (1519–21) provided the final, practical proofs for the global shape of the earth, while European colonists planted the idea in the American colonies.

Spread to the east

With the rise of Greek culture in the east, Hellenistic astronomy filtered eastwards to ancient India where its profound influence became apparent in the early centuries AD. The Greek concept of a spherical earth surrounded by the spheres of planets, vehemently supported by astronomers like Varahamihira and Brahmagupta, supplanted the long-standing Indian cosmological belief into a flat and circular earth disk.

Islamic astronomy inherited the idea of a spherical earth from the Greek astronomical tradition. The Islamic theoretical framework largely relied on the fundamental contributions of Aristotle (*De caelo*) and Ptolemy (*Almagest*), both of which worked with the premise that the earth was spherical and at the center of the universe (geocentric model).

In the 17th century, the idea of a spherical earth, now considerably advanced by Western Astronomy, ultimately spread to Ming China, when Jesuit missionaries, who held high positions as astronomers at the imperial court, successfully challenged the Chinese belief that the earth was flat and square.

Other ancient views

Rigveda

While Indian cosmology long maintained the belief in a disk-shaped earth, K. V. Sarma mentions in passing that the *Rigveda* contains speculations on the idea of a spherical self-supporting earth, but does not give more details nor references. It is impossible to date when any particular piece of the Rigveda was composed. While the core of the text probably dates to around 1400 BC, many revisions and redactions are thought to have been incorporated around 600 BC. Nothing was written down until the 3rd century BC at earliest, and possibly much later.

Aryabhata

The works of the classical Indian astronomer and mathematician, Aryabhata (476–550 AD), deal with the sphericity of the Earth and the motion of the planets. The final two parts of his Sanskrit magnum opus, the *Aryabhatiya*, which were named the *Kalakriya* ("reckoning of time") and the *Gola* ("sphere"), state that the Earth is spherical and that its circumference is 4,967 yojanas, which in modern units is 39,968 km, which is only 62 km less than the current value of 40,030 km. He also stated that the apparent rotation of the celestial objects was due to the actual rotation of the Earth, calculating the length of the sidereal day to be 23 hours, 56 minutes and 4.1 seconds, which is also surprisingly accurate. It is likely that Aryabhata's results influenced European astronomy, because the 8th century Arabic version of the Aryabhatiya was translated into Latin in the 13th century.

Anania Shirakatsi

Anania Shirakatsi (Armenian: Անանիա Շիրակացի), also known as Ananias of Sirak, (610–685) was an Armenian scholar, mathematician, and geographer. His most famous works are *Geography Guide* (‘Ashharatsuyts’ in Armenian), and *Cosmography* (Tiezeragitutiun). He described the world as "being like an egg with a spherical yolk (the globe) surrounded by a layer of white (the atmosphere) and covered with a hard shell (the sky)."

Medieval Europe

Isidore of Seville

Bishop Isidore of Seville (560–636) taught in his widely read encyclopedia, the Etymologies, that the Earth was round. While some writers have thought he referred to a disc-shaped Earth; his other writings make it clear that he considered the Earth to be spherical. He didn't admit the possibility of people dwelling at the antipodes, considering them as legendary and noting that there was no evidence for their existence. Isidore's representation continued to be used through the Middle Ages by authors clearly favouring a spherical Earth, e.g. the 9th century bishop Rabanus Maurus who compared the habitable part of the northern hemisphere (Aristotle's northern temperate clime) with a wheel, imagined as a slice of the whole sphere.

Bede the Venerable

The monk Bede (c. 672–735) wrote in his influential treatise on computus, *The Reckoning of Time*, that the Earth was round, explaining the unequal length of daylight from "the roundness of the Earth, for not without reason is it called 'the orb of the world' on the pages of Holy Scripture and of ordinary literature. It is, in fact, set like a sphere in the middle of the whole universe." (De temporum ratione, 32). The large number of surviving manuscripts of The Reckoning of Time, copied to meet the Carolingian requirement that all priests should study the computus, indicates that many, if not most, priests were exposed to the idea of the sphericity of the Earth. Ælfric of Eynsham paraphrased Bede into Old English, saying "Now the Earth's roundness and the Sun's orbit constitute the obstacle to the day's being equally long in every land."

Bede was lucid about earth's sphericity, writing "We call the earth a globe, not as if the shape of a sphere were expressed in the diversity of plains and mountains, but because, if all things are included in the outline, the earth's circumference will represent the figure of a perfect globe... For truly it is an orb placed in the center of the universe; in its width it is like a circle, and not circular like a shield but rather like a ball, and it extends from its center with perfect roundness on all sides."

Hildegard Von Bingen

Saint Hildegard (Hildegard von Bingen, 1098–1179), depicts the spherical earth several times in her work *Liber Divinorum Operum*.

Johannes of Sacrobosco

Johannes de Sacrobosco (c. 1195 – c. 1256 AD) wrote a famous work on Astronomy called Tractatus de Sphaera, based on Ptolemy, in which he considers the Earth to be spherical.

Other sources

John Gower prepares to shoot the world, a sphere with compartments representing earth, air, and water (*Vox Clamantis*, around 1400)

Dante's Divine Comedy, written in Italian in the early 14th century, portrays Earth as a sphere, discussing implications such as the different stars visible in the southern hemisphere, the altered position of the sun, and the various timezones of the Earth. Also, the Elucidarium of Honorius Augustodunensis (c. 1120), an important manual for the instruction of lesser clergy, which was translated into Middle English, Old French, Middle High German, Old Russian, Middle Dutch, Old Norse, Icelandic, Spanish, and several Italian dialects, explicitly refers to a spherical Earth. Likewise, the fact that Bertold von Regensburg (mid-13th century) used the spherical Earth as a sermonic illustration shows that he could assume this knowledge among his congregation. The sermon was held in the vernacular German, and thus was not intended for a learned audience.

Reinhard Krüger, a professor for Romance literature at the University of Stuttgart (Germany), has compiled a list of more than 100 medieval Latin and vernacular writers from the late antiquity to the 15th century—79 known by name—whom he identified as knowing that the Earth was spherical.

Medieval Islamic scholars

Early Islamic scholars recognized earth's sphericity, leading Muslim mathematicians to develop spherical trigonometry in order to further mensuration and to calculate the distance and direction from any given point on the Earth to Mecca. This determined the Qibla, or Muslim direction of prayer.

Al-Ma'mun

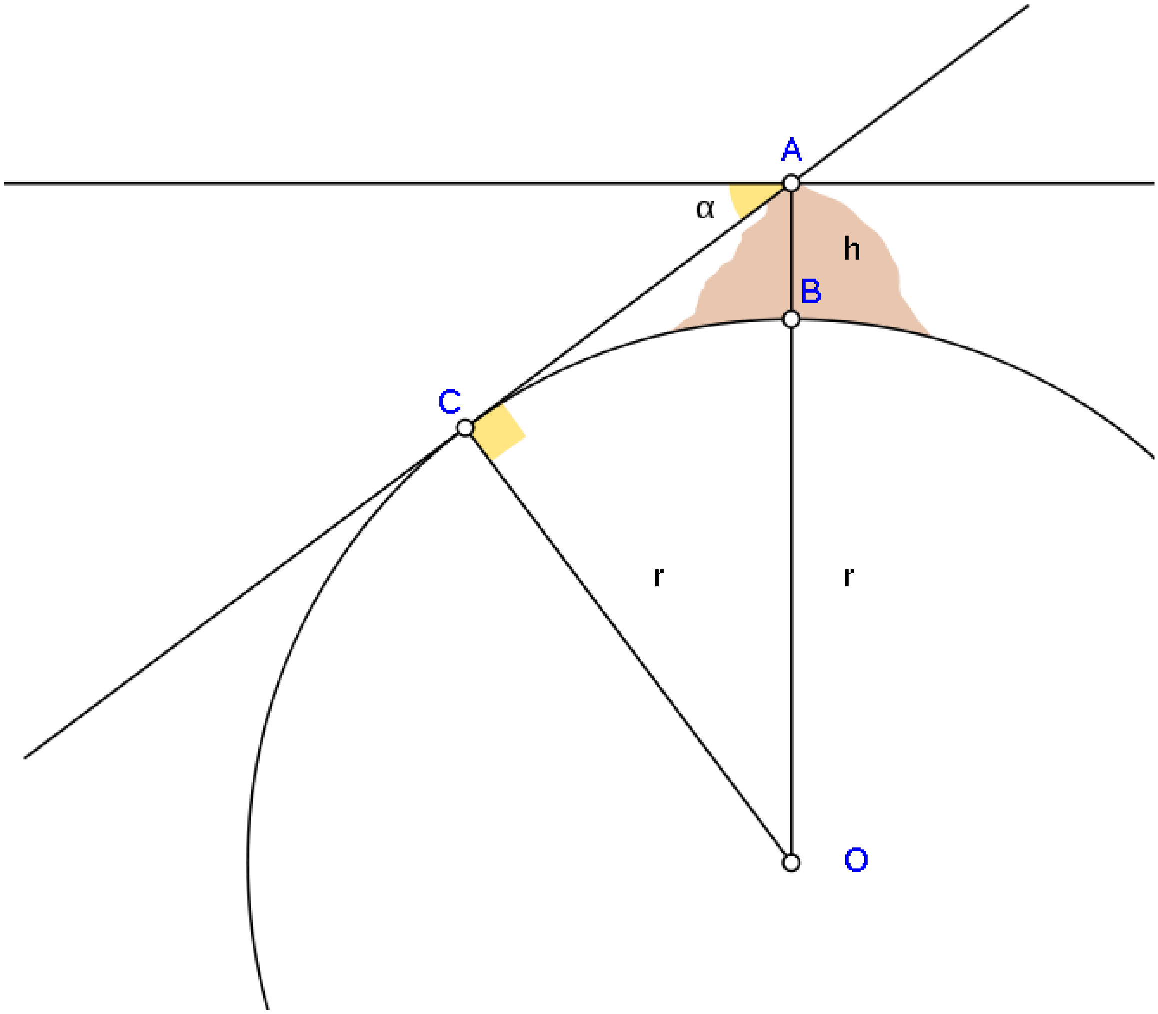
Around 830 AD, Caliph Al-Ma'mun commissioned a group of Muslim astronomers and Muslim geographers to measure the distance from Tadmur (Palmyra) to al-Raqqah, in modern Syria. They found the cities to be separated by one degree of latitude and the distance between them to be 66 2/3 miles and thus calculated the Earth's circumference to be 24,000 miles.

Another estimate given by his astronomers was 56 2/3 Arabic miles per degree, which corresponds to 111.8 km per degree and a circumference of 40,248 km, very close to the currently modern values of 111.3 km per degree and 40,068 km circumference, respectively.

Al-Farghānī

Al-Farghānī (Latinized as Alfraganus) was a Persian astronomer of the 9th century involved in measuring the diameter of the Earth, and commissioned by Al-Ma'mun. His estimate given above for a degree (56 2/3 Arabic miles) was much more accurate than the 60 2/3 Roman miles (89.7 km) given by Ptolemy. Christopher Columbus uncritically used Alfraganus's figure as if it were in Roman miles instead of in Arabic miles, in order to prove a smaller size of the Earth than that propounded by Ptolemy.

Biruni



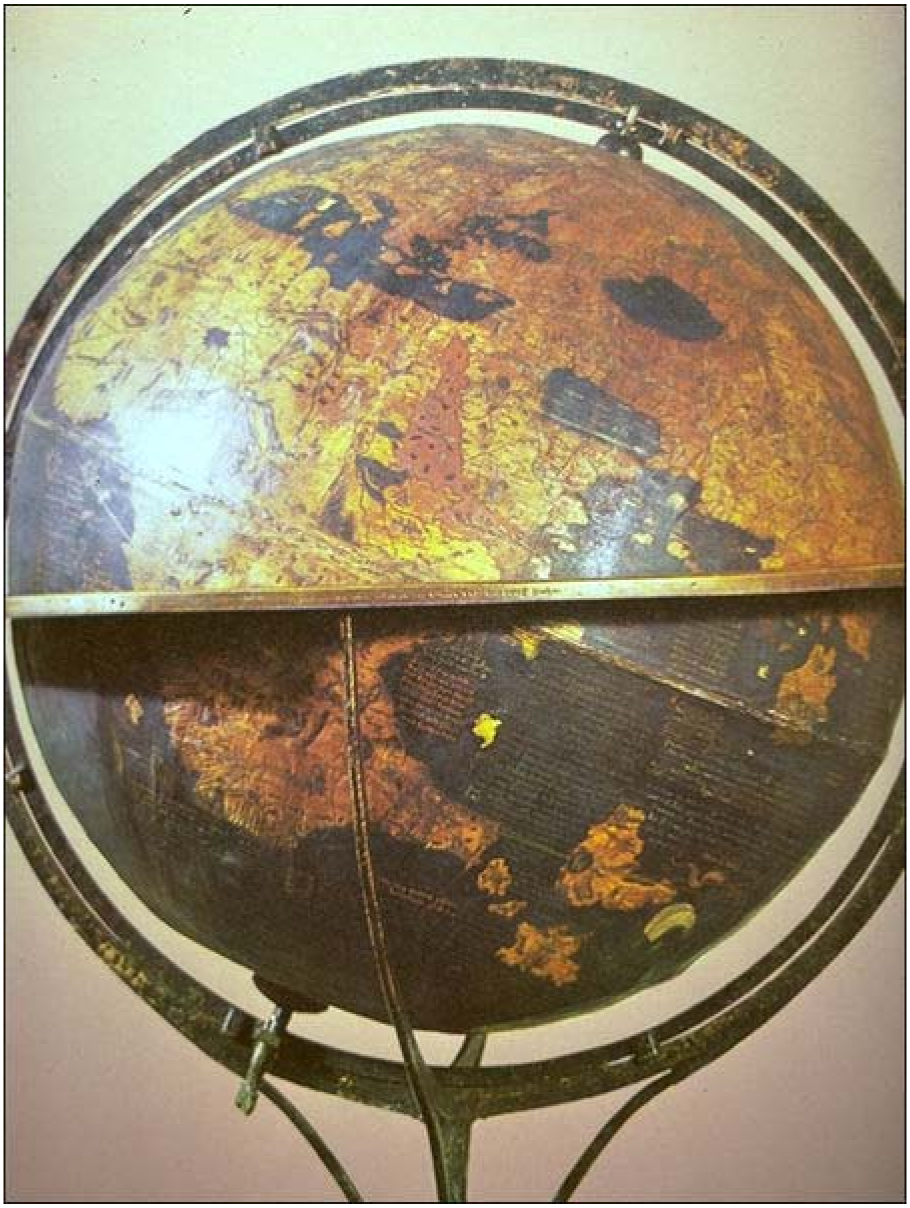
Biruni's Method for calculation of Earth's radius

Abu Rayhan Biruni (973-1048) used a new method to accurately compute the Earth's circumference, by which he arrived at a value that was close to modern values for the Earth's circumference. His estimate of 6,339.9 km for the Earth radius was only 16.8 km less than the modern value of 6,356.7 km. In contrast to his predecessors who measured the Earth's circumference by sighting the Sun simultaneously from two different locations, Biruni developed a new method of using trigonometric calculations based on the angle between a plain and mountain top which yielded more accurate measurements of the Earth's circumference and made it possible for it to be measured by a single person from a single location. Biruni's method was intended to avoid "walking across hot, dusty deserts" and the idea came to him when he was on top of a tall mountain in India. From the top of the mountain, he sighted the angle to the horizon which, along with the mountain's height (which he calculated beforehand), allowed him to calculate the curvature of the Earth. He also made use of algebra to formulate trigonometric equations and used the astrolabe to measure angles.

John J. O'Connor and Edmund F. Robertson write in the *MacTutor History of Mathematics archive*:

"Important contributions to geodesy and geography were also made by Biruni. He introduced techniques to measure the earth and distances on it using triangulation. He found the radius of the earth to be 6339.6 km, a value not obtained in the West until the 16th century. His *Masudic canon* contains a table giving the coordinates of six hundred places, almost all of which he had direct knowledge."

Circumnavigation of the globe



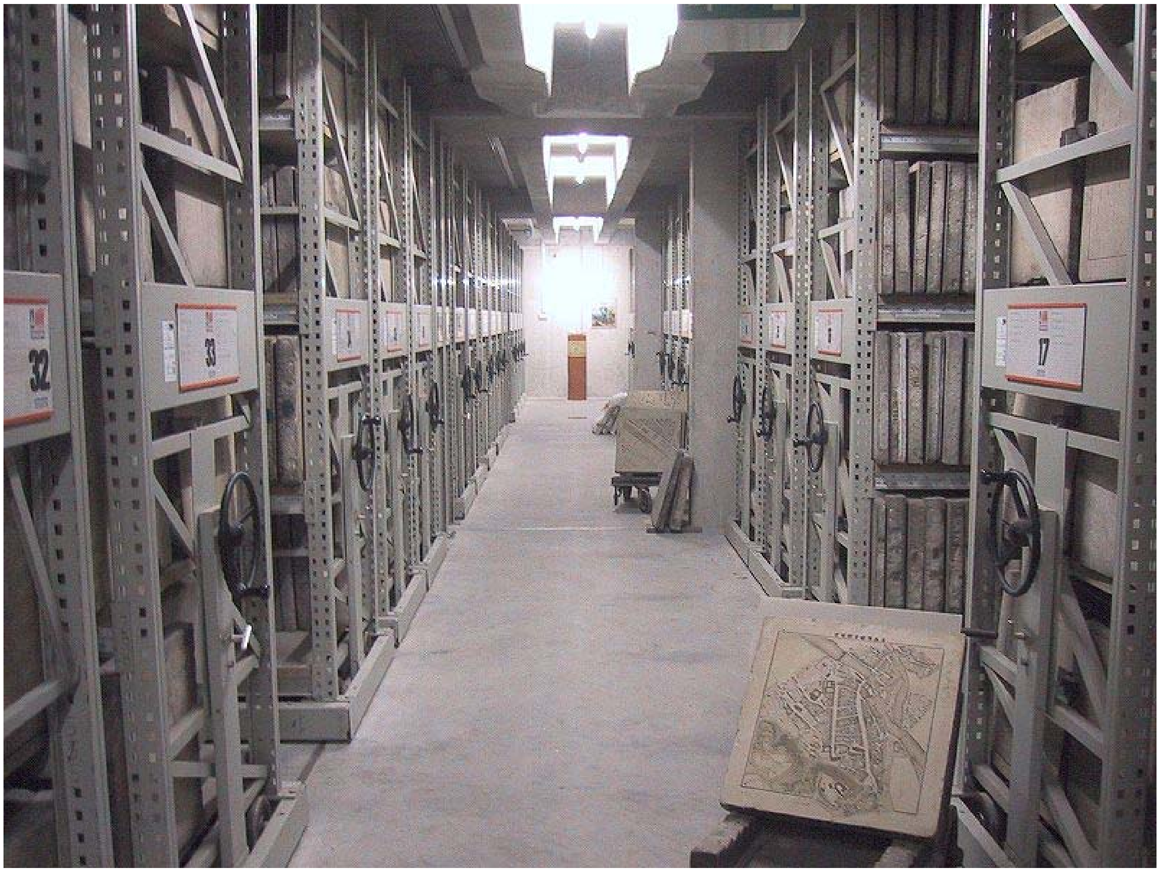
The *Erdapfel*, the oldest surviving terrestial globe (1492/93)

The first direct demonstration of Earth's sphericity came in the form of the first circumnavigation in history, an expedition captained by Portuguese explorer Ferdinand Magellan. The expedition was financed by the Spanish Crown. On August 10, 1519, the five ships under Magellan's command departed from Seville. They crossed the Atlantic Ocean, passed through the Strait of Magellan, crossed the Pacific, and arrived in Cebu, where Magellan was killed by Philippine natives in a battle. His second in command, the Spaniard Juan Sebastián Elcano, continued the expedition and, on September 6, 1522, arrived at Seville, completing the circumnavigation. Charles I of Spain, in recognition of his feat, gave Elcano a coat of arms with the motto *Primus circumdedisti me* (in Latin, "You went around me first").

A circumnavigation alone does not prove that the earth is spherical. It could be cylindric or irregularly globular or one of many other shapes. Still, combined with trigonometric evidence of the form used by Eratosthenes 1,700 years prior, the Magellan expedition removed any reasonable doubt in educated circles in Europe.

Geodesy

An old geodetic pillar (1855) at Ostend, Belgium



A Munich archive with lithography plates of maps of Bavaria

Geodesy also named **geodetics**, a branch of earth sciences, is the scientific discipline that deals with the measurement and representation of the Earth, including its gravitational field, in a three-dimensional time-varying space. Geodesists also study geodynamical phenomena such as crustal motion, tides, and polar motion. For this they design global and national control networks, using space and terrestrial techniques while relying on datums and coordinate systems.

Definition

Geodesy (from Greek *γεωδαισία – geodaisia*, lit. "division of the Earth") is primarily concerned with positioning within the temporally varying gravity field. Somewhat obsolete nowadays, geodesy in the German speaking world is divided into "Higher Geodesy" ("Erdmessung" or "höhere Geodäsie"), which is concerned with measuring the Earth on the global scale, and "Practical Geodesy" or "Engineering Geodesy" ("Ingenieurgeodäsie"), which is concerned with measuring specific parts or regions of the Earth, and which includes surveying.

The shape of the Earth is to a large extent the result of its rotation, which causes its equatorial bulge, and the competition of geological processes such as the collision of plates and of vulcanism, resisted by the Earth's gravity field. This applies to the solid surface, the liquid surface (dynamic sea surface topography) and the Earth's atmosphere. For this reason, the study of the Earth's gravity field is called physical geodesy by some.

History

Geoid and reference ellipsoid

The geoid is essentially the figure of the Earth abstracted from its topographical features. It is an idealized equilibrium surface of sea water, the mean sea level surface in the absence of currents, air pressure variations etc. and continued under the continental masses. The geoid, unlike ellipsoid, is irregular and too complicated to serve as the computational surface on which to solve geometrical problems like point positioning. The geometrical separation between the geoid and the reference ellipsoid is called the geoidal undulation. It varies globally between ±110 m.

A reference ellipsoid, customarily chosen to be the same size (volume) as the geoid, is described by its semi-major axis (equatorial radius) a and flattening f. The quantity $f = (a-b)/a$, where b is the semi-minor axis (polar radius), is a purely geometrical one. The mechanical ellipticity of the Earth (dynamical flattening, symbol J_2) can be determined to high precision by observation of satellite orbit perturbations. Its relationship with the geometrical flattening is indirect. The relationship depends on the internal density distribution, or, in simplest terms, the degree of central concentration of mass.

The 1980 Geodetic Reference System (GRS80) posited a 6,378,137 m semi-major axis and a 1:298.257 flattening. This system was adopted at the XVII General Assembly of the International Union of Geodesy and Geophysics (IUGG). It is essentially the basis for geodetic positioning by the Global Positioning System and is thus also in extremely widespread use outside the geodetic community.

The numerous other systems which have been used by diverse countries for their maps and charts are gradually dropping out of use as more and more countries move to global, geocentric reference systems using the GRS80 reference ellipsoid.

Coordinate systems in space

The locations of points in three-dimensional space are most conveniently described by three cartesian or rectangular coordinates, X,Y and Z. Since the advent of satellite positioning, such coordinate systems are typically geocentric: the Z axis is aligned with the Earth's (conventional or instantaneous) rotation axis.

Prior to satellite geodesy era, the coordinate systems associated with a geodetic datum attempted to be geocentric, but their origins differed from the geocentre by hundreds of metres, due to regional deviations in the direction of the plumbline (vertical). These regional geodetic datums, such as ED50 (European Datum 1950) or NAD83 (North American Datum 1983) have ellipsoids associated with them that are regional 'best fits' to the geoids within their areas of validity, minimising the deflections of the vertical over these areas.

It is only because GPS satellites orbit about the geocentre, that this point becomes naturally the origin of a coordinate system defined by satellite geodetic means, as the satellite positions in space are themselves computed in such a system.

Geocentric coordinate systems used in geodesy can be divided naturally into two classes:

1. Inertial reference systems, where the coordinate axes retain their orientation relative to the fixed stars, or equivalently, to the rotation axes of ideal gyroscopes; the X axis points to the vernal equinox
2. Co-rotating, also ECEF ("Earth Centred, Earth Fixed"), where the axes are attached to the solid body of the Earth. The X axis lies within the Greenwich observatory's meridian plane.

The coordinate transformation between these two systems is described to good approximation by (apparent) sidereal time, which takes into account variations in the Earth's axial rotation (length-of-day variations). A more accurate description also takes polar motion into account, a phenomenon closely monitored by geodesists.

Coordinate systems in the plane

In surveying and mapping, important fields of application of geodesy, two general types of coordinate systems are used in the plane:

1. Plano-polar, in which points in a plane are defined by a distance s from a specified point along a ray having a specified direction α with respect to a base line or axis;
2. Rectangular, points are defined by distances from two perpendicular axes called x and y. It is geodetic practice—contrary to the mathematical convention—to let the x axis point to the North and the y axis to the East.

Rectangular coordinates in the plane can be used intuitively with respect to one's current location, in which case the x axis will point to the local North. More formally, such coordinates can be obtained from three-dimensional coordinates using the artifice of a map projection. It is *not* possible to map the curved surface of the Earth onto a flat map surface without deformation. The compromise most often chosen—called a conformal projection—preserves angles and length ratios, so that small circles are mapped as small circles and small squares as squares.

An example of such a projection is UTM (Universal Transverse Mercator). Within the map plane, we have rectangular coordinates x and y. In this case the North direction used for reference is the *map* North, not the *local* North. The difference between the two is called *meridian convergence*.

It is easy enough to "translate" between polar and rectangular coordinates in the plane: let, as above, direction and distance be α and *s* respectively, then we have

$$\begin{aligned} x &= s\cos\alpha \\ y &= s\sin\alpha \end{aligned}$$

The reverse transformation is given by:

$$\begin{aligned} s &= \sqrt{x^2+y^2} \\ \alpha &= \arctan(y/x). \end{aligned}$$

Heights

In geodesy, point or terrain *heights* are "above sea level", an irregular, physically defined surface. Therefore a height should ideally *not* be referred to as a coordinate. It is more like a physical quantity, and though it can be tempting to treat height as the vertical coordinate *z*, in addition to the horizontal coordinates *x* and *y*, and though this actually is a good approximation of physical reality in small areas, it quickly becomes invalid for regional considerations.

Heights come in the following variants:

1. Orthometric heights
2. Normal heights
3. Geopotential heights

Each has its advantages and disadvantages. Both orthometric and normal heights are heights in metres above sea level, whereas geopotential numbers are measures of potential energy (unit: $m^2 s^{-2}$) and not metric. Orthometric and normal heights differ in the precise way in which mean sea level is conceptually continued under the continental masses. The reference surface for orthometric heights is the geoid, an equipotential surface approximating mean sea level.

None of these heights is in any way related to **geodetic** or **ellipsoidial** heights, which express the height of a point above the reference ellipsoid. Satellite positioning receivers typically provide ellipsoidal heights, unless they are fitted with special conversion software based on a model of the geoid.

Geodetic data

Because geodetic point coordinates (and heights) are always obtained in a system that has been constructed itself using real observations, geodesists introduce the concept of a *geodetic datum*: a physical realization of a coordinate system used for describing point locations. The realization is the result of *choosing* conventional coordinate values for one or more *datum points*.

In the case of height datums, it suffices to choose *one* datum point: the reference bench mark, typically a tide gauge at the shore. Thus we have vertical datums like the NAP (Normaal Amsterdams Peil), the North American Vertical Datum 1988 (NAVD88), the Kronstadt datum, the Trieste datum, and so on.

In case of plane or spatial coordinates, we typically need several datum points. A regional, ellipsoidal datum like ED50 can be fixed by prescribing the undulation of the geoid and the deflection of the vertical in *one* datum point, in this case the Helmert Tower in Potsdam. However, an overdetermined ensemble of datum points can also be used.

Changing the coordinates of a point set referring to one datum, so to make them refer to another datum, is called a *datum transformation*. In the case of vertical datums, this consists of simply adding a constant shift to all height values. In the case of plane or spatial coordinates, datum transformation takes the form of a similarity or *Helmert transformation*, consisting of a rotation and scaling operation in addition to a simple translation. In the plane, a Helmert transformation has four parameters; in space, seven.

A note on terminology

In the abstract, a coordinate system as used in mathematics and geodesy is, e.g., in ISO terminology, referred to as a *coordinate system*. International geodetic organizations like the IERS (International Earth Rotation and Reference Systems Service) speak of a *reference system*.

When these coordinates are realized by choosing datum points and fixing a geodetic datum, ISO uses the terminology *coordinate reference system*, while IERS speaks of a *reference frame*. A datum transformation again is referred to by ISO as a *coordinate transformation*. (ISO 19111: Spatial referencing by coordinates).

Point positioning

Geodetic Control Mark (example of a deep benchmark)

Point positioning is the determination of the coordinates of a point on land, at sea, or in space with respect to a coordinate system. Point position is solved by computation from measurements linking the known positions of terrestrial or extraterrestrial points with the unknown terrestrial position. This may involve transformations between or among astronomical and terrestrial coordinate systems.

The known points used for point positioning can be triangulation points of a higher order network, or GPS satellites.

Traditionally, a hierarchy of networks has been built to allow point positioning within a country. Highest in the hierarchy were triangulation networks. These were densified into networks of traverses (polygons), into which local mapping surveying measurements, usually with measuring tape, corner prism and the familiar red and white poles, are tied.

Nowadays all but special measurements (e.g., underground or high precision engineering measurements) are performed with GPS. The higher order networks are measured with static GPS, using differential measurement to determine vectors between terrestrial points. These vectors are then adjusted in traditional network fashion. A global polyhedron of permanently operating GPS stations under the auspices of the IERS is used to define a single global, geocentric reference frame which serves as the "zero order" global reference to which national measurements are attached.

For surveying mappings, frequently Real Time Kinematic GPS is employed, tying in the unknown points with known terrestrial points close by in real time.

One purpose of point positioning is the provision of known points for mapping measurements, also known as (horizontal and vertical) control. In every country, thousands of such known points exist and are normally documented by the national mapping agencies. Surveyors involved in real estate and insurance will use these to tie their local measurements to.

Geodetic problems

In geometric geodesy, two standard problems exist:

First geodetic problem

Given a point (in terms of its coordinates) and the direction (azimuth) and distance from that point to a second point, determine (the coordinates of) that second point.

Second (inverse) geodetic problem

Given two points, determine the azimuth and length of the line (straight line, arc or geodesic) that connects them.

In the case of plane geometry (valid for small areas on the Earth's surface) the solutions to both problems reduce to simple trigonometry. On the sphere, the solution is significantly more complex, e.g., in the inverse problem the azimuths will differ between the two end points of the connecting great circle, arc, i.e. the geodesic.

On the ellipsoid of revolution, geodesics may be written in terms of elliptic integrals, which are usually evaluated in terms of a series expansion.

In the general case, the solution is called the geodesic for the surface considered. The differential equations for the geodesic can be solved numerically.

Geodetic observational concepts

Here we define some basic observational concepts, like angles and coordinates, defined in geodesy (and astronomy as well), mostly from the viewpoint of the local observer.

- The *plumbline* or *vertical* is the direction of local gravity, or the line that results by following it. It is slightly curved.

- The *zenith* is the point on the celestial sphere where the direction of the gravity vector in a point, extended upwards, intersects it. More correct is to call it a <direction> rather than a point.
- The *nadir* is the opposite point (or rather, direction), where the direction of gravity extended downward intersects the (invisible) celestial sphere.
- The celestial *horizon* is a plane perpendicular to a point's gravity vector.
- *Azimuth* is the direction angle within the plane of the horizon, typically counted clockwise from the North (in geodesy and astronomy) or South (in France).
- *Elevation* is the angular height of an object above the horizon, Alternatively zenith distance, being equal to 90 degrees minus elevation.
- *Local topocentric coordinates* are azimuth (direction angle within the plane of the horizon) and elevation angle (or zenith angle) and distance.
- The North *celestial pole* is the extension of the Earth's (precessing and nutating) instantaneous spin axis extended Northward to intersect the celestial sphere. (Similarly for the South celestial pole.)
- The *celestial equator* is the intersection of the (instantaneous) Earth equatorial plane with the celestial sphere.
- A *meridian plane* is any plane perpendicular to the celestial equator and containing the celestial poles.
- The *local meridian* is the plane containing the direction to the zenith and the direction to the celestial pole.

Geodetic measurements

The level is used for determining height differences and height reference systems, commonly referred to mean sea level. The traditional spirit level produces these practically most useful heights above sea level directly; the more economical use of GPS instruments for height determination requires precise knowledge of the figure of the geoid, as GPS only gives heights above the GRS80 reference ellipsoid. As geoid knowledge accumulates, one may expect use of GPS heighting to spread.

The theodolite is used to measure horizontal and vertical angles to target points. These angles are referred to the local vertical. The tacheometer additionally determines, electronically or electro-optically, the distance to target, and is highly automated to even robotic in its operations. The method of free station position is widely used.

For local detail surveys, tacheometers are commonly employed although the old-fashioned rectangular technique using angle prism and steel tape is still an inexpensive alternative. Real-time kinematic (RTK) GPS techniques are used as well. Data collected are tagged and recorded digitally for entry into a Geographic Information System (GIS) database.

Geodetic GPS receivers produce directly three-dimensional coordinates in a geocentric coordinate frame. Such a frame is, e.g., WGS84, or the frames that are regularly produced and published by the International Earth Rotation and Reference Systems Service (IERS).

GPS receivers have almost completely replaced terrestrial instruments for large-scale base network surveys. For Planet-wide geodetic surveys, previously impossible, we can still mention Satellite Laser Ranging (SLR) and Lunar Laser Ranging (LLR) and Very Long Baseline Interferometry (VLBI) techniques. All these techniques also serve to monitor Earth rotation irregularities as well as plate tectonic motions.

Gravity is measured using gravimeters. Basically, there are two kinds of gravimeters. *Absolute* gravimeters, which nowadays can also be used in the field, are based directly on measuring the acceleration of free fall (for example, of a reflecting prism in a vacuum tube). They are used for establishing the vertical geospatial control. Most common *relative* gravimeters are spring based. They are used in gravity surveys over large areas for establishing the figure of the geoid over these areas. Most accurate relative gravimeters are *superconducting* gravimeters, and these are sensitive to one thousandth of one billionth of the Earth surface gravity. Twenty-some superconducting gravimeters are used worldwide for studying Earth tides, rotation, interior, and ocean and atmospheric loading, as well as for verifying the Newtonian constant of gravitation.

Units and measures on the ellipsoid

Geographical latitude and longitude are stated in the units degree, minute of arc, and second of arc. They are *angles*, not metric measures, and describe the *direction* of the local normal to the reference ellipsoid of revolution. This is *approximately* the same as the direction of the plumbline, i.e., local gravity, which is also the normal to the geoid surface. For this reason, astronomical position determination – measuring the direction of the plumbline by astronomical means – works fairly well provided an ellipsoidal model of the figure of the Earth is used.

One geographical mile, defined as one minute of arc on the equator, equals 1,855.32571922 m. One nautical mile is one minute of astronomical latitude. The radius of curvature of the ellipsoid varies with latitude, being the longest at the pole and the shortest at the equator as is the nautical mile.

A metre was originally defined as the 40-millionth part of the length of a meridian (the target wasn't quite reached in actual implementation, so that is off by 0.02% in the current definitions). This means that one kilometre is roughly equal to (1/40,000) * 360 * 60 meridional minutes of arc, which equals 0.54 nautical mile, though this is not exact because the two units are defined on different bases (the international nautical mile is defined as exactly 1,852 m, corresponding to a rounding of 1000/0.54 m to four digits).

Temporal change

In geodesy, temporal change can be studied by a variety of techniques. Points on the Earth's surface change their location due to a variety of mechanisms:

- Continental plate motion, plate tectonics
- Episodic motion of tectonic origin, esp. close to fault lines

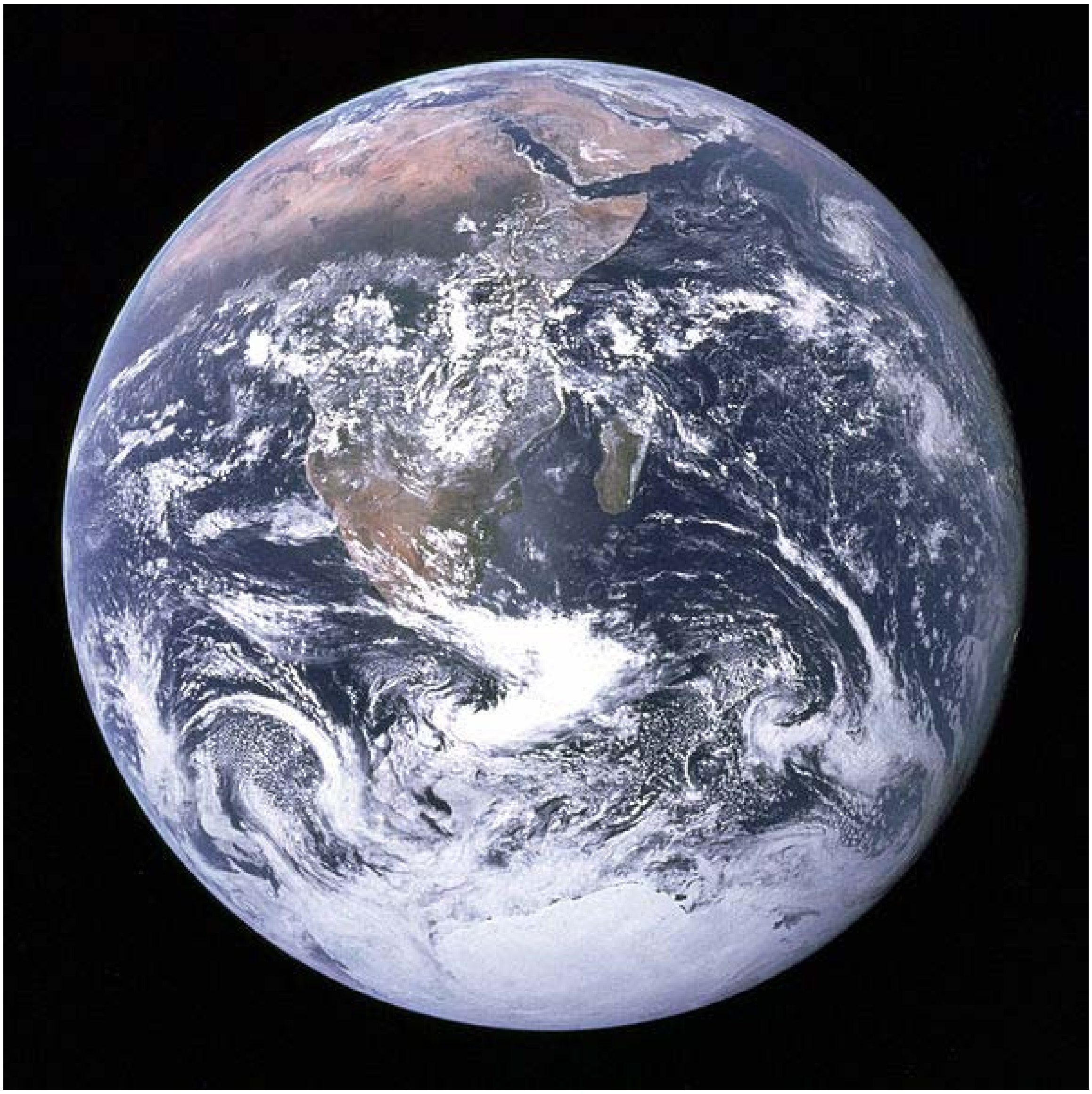
- Periodic effects due to Earth tides
- Postglacial land uplift due to isostatic adjustment
- Various anthropogenic movements due to, for instance, petroleum or water extraction or reservoir construction.

The science of studying deformations and motions of the Earth's crust and the solid Earth as a whole is called geodynamics. Often, study of the Earth's irregular rotation is also included in its definition.

Techniques for studying geodynamic phenomena on the global scale include:

- satellite positioning by GPS and other such systems,
- Very Long Baseline Interferometry (VLBI)
- satellite and lunar laser ranging
- Regionally and locally, precise levelling,
- precise tacheometers,
- monitoring of gravity change,
- Interferometric synthetic aperture radar (InSAR) using satellite images, etc.

Spherical models



The Earth as seen from the Apollo 17 mission.

There are several reasonable ways to approximate Earth's shape as a sphere. Most preserve a different feature of an ellipsoid that closely models the real Earth in order to compute the radius of the spherical model.

Chapter-4

Geographical Distance

Geographical distance is the distance measured along the surface of the earth. The formulae here calculate distances between points which are defined by geographical coordinates in terms of latitude and longitude.

An abstraction

Calculating the distance between geographical coordinates is based on some level of abstraction; it does not provide an *exact* distance, which is unattainable if one attempted to account for every irregularity in the surface of the earth. Common abstractions for the surface between two geographic points are:

- Flat surface;
- Spherical surface;
- Ellipsoidal surface.

All abstractions above ignore changes in elevation.

Nomenclature

Distance, D, is calculated between two points, P_1 and P_2. The geographical coordinates of the two points, as (latitude, longitude) pairs, are (ϕ_1, λ_1) and (ϕ_2, λ_2), respectively. Which of the two points is designated as P_1 is not important for the calculation of distance.

Latitude and longitude coordinates on maps are usually expressed in degrees. In the given forms of the formulae below, one or more values *must* be expressed in the specified units to obtain the correct result. Where geographic coordinates are used as the argument of a trigonometric function, the values may be expressed in any angular units compatible with the method used to determine the value of the trigonometric function. Many electronic calculators allow calculations of trigonometric functions in either degrees or radians. The calculator mode must be compatible with the units used for geometric coordinates.

Differences in latitude and longitude are labeled and calculated as follows:

$$\Delta\phi = \phi_2 - \phi_1;$$
$$\Delta\lambda = \lambda_2 - \lambda_1.$$

It is not important whether the result is positive or negative when used in the formulae below.

"Mean latitude" is labeled and calculated as follows:

$$\phi_m = \frac{\phi_1 + \phi_2}{2}.$$

Colatitude is labeled and calculated as follows:

For latitudes expressed in radians:
$$\theta = \frac{\pi}{2} - \phi;$$
For latitudes expressed in degrees:
$$\theta = 90^\circ - \phi.$$

Unless specified otherwise, the radius of the earth for the calculations below is:

R= 6,371.009 kilometers = 3,958.761 statute miles = 3,440.069 nautical miles.

D= Distance between the two points, as measured along the surface of the earth and in the same units as the value used for radius unless specified otherwise.

Effects of the singularities and discontinuity of longitude/latitude

Longitude has singularities at the Poles (longitude is undefined) and discontinuities at the ±180° meridian. Also, planar projections of the circles of constant latitude are highly curved near the Poles. Hence, the above equations for delta latitude/longitude and mean latitude may not give the expected answer for positions near or at the Poles or positions on either side of the 180° meridian. This could affect the accuracy or correctness of the following formulae at these positions (since the formulae are based on these three equations). If calculations that are valid for all Earth positions are needed, consider replacing latitude/longitude with n-vector.

Flat-surface formulae

A planar approximation for the surface of the earth may be useful over small distances. The accuracy of distance calculations using this approximation become increasingly inaccurate as:

- The separation between the points becomes greater;

- A point becomes closer to a geographic pole.

The shortest distance between two points in plane is a straight line. The Pythagorean theorem is used to calculate the distance between points in a plane.

Even over short distances, the accuracy of geographic distance calculations which assume a flat Earth depend on the method by which the latitude and longitude coordinates have been projected onto the plane. The projection of global latitude and longitude coordinates onto a plane is the realm of cartography.

The formulae presented in this section provide varying degrees of accuracy.

Pythagorean formula with parallel meridians

This formula is not accurate enough for general use. This formula might be used with acceptable accuracy for points near the equator.

$$D = R\sqrt{(\Delta\phi)^2 + (\Delta\lambda)^2};$$

where $\Delta\phi$ and $\Delta\lambda$ are in radians.
To change latitude or longitude to radians use following formulae:

- latitude in radians = (latitude in degrees)*PI/180
- longitude in radians = (longitude in degrees)*PI/180

where PI is Ludolph's number (3.14159...)

Spherical Earth projected to a plane

This formula takes into account the variation in distance between meridians with latitude:

$$D = R\sqrt{(\Delta\phi)^2 + (\cos(\phi_m)\Delta\lambda)^2};$$

where:
$\Delta\phi$ and $\Delta\lambda$ are in radians;
ϕ_m must be in units compatible with the method used for determining $\cos(\phi_m)$.
To change latitude or longitude to radians use following formulae:

- latitude in radians = (latitude in degrees)*PI/180
- longitude in radians = (longitude in degrees)*PI/180

where PI is Ludolph's number (3.14159...)
Note: This approximation is very fast and produces fairly accurate result for small distances. Also, when ordering locations by distance, such as in a database query,

it is much faster to order by squared distance, eliminating the need for computing the square root.

Ellipsoidal Earth projected to a plane

The FCC prescribes essentially the following formulae in 47 CFR 73.208 for distances not exceeding 475 km /295 miles:

$$D = \sqrt{(K_1 \Delta\phi)^2 + (K_2 \Delta\lambda)^2};$$

where
D= Distance in kilometers;
$\Delta\phi$ and $\Delta\lambda$ are in degrees;
ϕ_m must be in units compatible with the method used for determining $\cos(\phi_m)$;
$K_1 = 111.13209 - 0.56605\cos(2\phi_m) + 0.00120\cos(4\phi_m)$;
$K_2 = 111.41513\cos(\phi_m) - 0.09455\cos(3\phi_m) + 0.00012\cos(5\phi_m)$.
It may be interesting to note that:
$K_1 = M\frac{\pi}{180}$= kilometers per degree of latitude difference;
$K_2 = \cos(\phi_m)N\frac{\pi}{180}$= kilometers per degree of longitude difference;
where M and N are the ***meridional*** and its perpendicular, or "***n****ormal*", radii of curvature (the expressions in the FCC formula are derived from the binomial series expansion form of M and N, set to the *Clarke 1866* reference ellipsoid).

Polar coordinate flat-Earth formula

$$D = R\sqrt{\theta_1^2 + \theta_2^2 - 2\theta_1\theta_2\cos(\Delta\lambda)};$$

where the colatitude values are in radians. For a latitude measured in degrees, the colatitude in radians may be calculated as follows: $\theta = \frac{\pi}{180}(90^\circ - \phi).$

Spherical-surface formulae

If we are willing to accept a possible error of 0.5%, we can use formulas of spherical trigonometry on the sphere that best approximates the surface of the earth.

The shortest distance along the surface of a sphere between two points on the surface is along the great-circle which contains the two points.

Tunnel distance

A tunnel between points on Earth is defined by a line through three-dimensional space between the points of interest. For a spherical Earth, this line is also the chord of the great circle between the points. For points near each other, the tunnel distance is only slightly less than the great-circle distance.

The great circle chord length may be calculated as follows for the corresponding unit sphere:

$$\Delta X = \cos(\theta_2)\cos(\lambda_2) - \cos(\theta_1)\cos(\lambda_1);$$
$$\Delta Y = \cos(\theta_2)\sin(\lambda_2) - \cos(\theta_1)sin(\lambda_1);$$
$$\Delta Z = \sin(\theta_2) - \sin(\theta_1);$$
$$C_h = \sqrt{(\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2}.$$

The tunnel distance between points on the surface of a spherical Earth is: $D = RC_h$

Great-circle distance

The great-circle distance article presents formulae for calculating the exact distance along a great-circle. The great-circle distance article includes a worked example for calculating distances by this method.

Ellipsoidal-surface formulae

An ellipsoidal approximation for the surface of the earth may be useful over great distances. The shortest distance along the surface of an ellipsoid between two points on the surface is along the geodesic.

Vincenty's formulae

The Vincenty's formulae article presents an algorithm for calculating the geodesic distance between two points on an ellipsoid. The results are accurate to about 0.5 mm; however, the algorithm fails to converge for points that are nearly anti-podal. However, the article also gives Vincenty's formulae for the Direct problem (given initial latitude and longitude and the distance and initial direction of a geodesic line, find the lat-lon of the endpoint); those formulae always converge, enabling us to find a near-antipodal geodesic distance by successive approximation.

Bowring's formulae

Vincenty's formulae are intended to maintain accuracy to a millimeter over any distance; if we limit distance to 100-150 km we can get the same accuracy with Bowring's much simpler formulae.

Bowring's formulae appeared in the magazine *Surveying and Mapping* in 1981. In the linked pdf, "e'^2" is the second eccentricity squared, a constant for the chosen spheroid;

$$e'^2 = \frac{2r-1}{(r-1)^2}$$

where r is the reciprocal of the flattening (r = 298.257223563 for the WGS84 spheroid). $\phi_1$and $\phi_2$are the latitudes of the two points, $\lambda_1$and $\lambda_2$are the longitudes; $\Delta\phi$is the difference in latitude (which at one point must be in radians). Calculate A, B, C and w on the first page of the pdf, then skip to "Inverse Problem" on the second page.

Lambert's formulae

Lambert's formulae are also much simpler than Vincenty's and give accuracy on the order of 10 meters over thousands of kilometers.

Lambert's formulae appeared in the Journal of the Washington Academy of Sciences in 1942. First convert the latitudes ϕ_1, $\phi_2$of the two points to reduced latitudes ψ_1, ψ_2

$$\tan\psi = \frac{r-1}{r}\tan\phi$$

> where r is the reciprocal of the flattening (r = 298.257223563 for the WGS84 spheroid).

Then calculate the central angle σ in radians between two points $(\psi_1,\ \lambda_1)$and $(\psi_2,\ \lambda_2)$on a sphere in the usual way (law of cosines or haversine formula), with longitudes $\lambda_1$and $\lambda_2$being the same on the sphere as on the spheroid.

$$P = \frac{\psi_1 + \psi_2}{2}$$

$$Q = \frac{\psi_2 - \psi_1}{2}$$

$$X = (\sigma - \sin\sigma)\frac{\sin^2 P\cos^2 Q}{\cos^2\frac{\sigma}{2}}$$

$$Y = (\sigma + \sin\sigma)\frac{\cos^2 P\sin^2 Q}{\sin^2\frac{\sigma}{2}}$$

$$distance = a(\sigma - \frac{X+Y}{2r})$$

where *a* is the equatorial radius of the chosen spheroid.

On the GRS 80 spheroid Lambert's formula is off by

0 North 0 West to 40 North 120 West, 12.6 meters;
0N 0W to 40N 60W, 6.6 meters;
40N 0W to 40N 60W, 0.85 meter.